Magical Properties of 2540 km Baseline

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- Sushant Raut, Ravi Shanker Singh and S. Uma Sankar, arXiv:0908.3741 (accepted in Physics Letters B)
- Aniket Joglekar, Suprabh Prakash, Sushant Raut and S. Uma Sankar, arXiv:1011.1146

Also

- Amol Dighe, Srubabati Goswami and Shamayita Ray, arXiv:1009.1093 (accepted in Physical Review Letters)

Outstanding Problems of Neutrino Physics

- Nature of neutrinos (Dirac vs Majorana),
- Absolute mass of neutrinos,
- Value of the mixing angle θ_{13},
- Mass pattern or mass hierarchy of neutrinos,
  (whether $m_3 \gg m_2 > m_1$ or $m_2 > m_1 \gg m_3$)
- CP Violation in neutrino sector and determination of the phase δ_{CP}.

The last three quantities can be determined from neutrino oscillation experiments.

Measurement of θ_{13}

θ_{13} can be measured in reactor neutrino experiments via $\bar{\nu}_e$ disappearance or in accelerator neutrino experiments via $\nu_\mu \to \nu_e$ appearance.

Chooz experiment has set an upper limit $\sin^2 2\theta_{13} \leq 0.1$. Double-Chooz, Daya Bay and RENO are expected to improve upon this. Daya Bay will measure θ_{13} if $\sin^2 2\theta_{13} \geq 0.01$.

The accelerator experiments T2K and NOνA also can measure θ_{13}. However, this measurement is complicated by the parameter degeneracies. The measurement of θ_{13} by the reactor experiments is free of these degeneracies.

In the following, we will assume that the reactor neutrinos will determine non-zero θ_{13}.

$\nu_\mu \to \nu_e$ oscillation probability

The three flavour expression for the $\nu_\mu \to \nu_e$ oscillation probability, including the matter efffects, is

$$
\begin{aligned}
P_{\mu e} &= C_0 \frac{\sin^2((1-\hat{A})\Delta)}{(1-\hat{A})^2} \\
&+ \alpha\, C_1 \frac{\sin((1-\hat{A})\Delta)}{(1-\hat{A})}\ \frac{\sin(\hat{A}\Delta)}{\hat{A}} + \alpha^2\, C_2 \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2},
\end{aligned}
$$

where $\alpha = \Delta_{21}/|\Delta_{31}| \approx 0.03$, $\Delta = (1.27\Delta_{31}L/E)$ and $\hat{A} = A/\Delta_{31}$ and

$$
\begin{aligned}
C_0 &= \sin^2\theta_{23}\sin^2 2\theta_{13} \\
C_1 &= \cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}\cos(\Delta + \delta_{CP}) \\
C_2 &= \sin^2 2\theta_{12}\cos^2\theta_{23}
\end{aligned}
$$

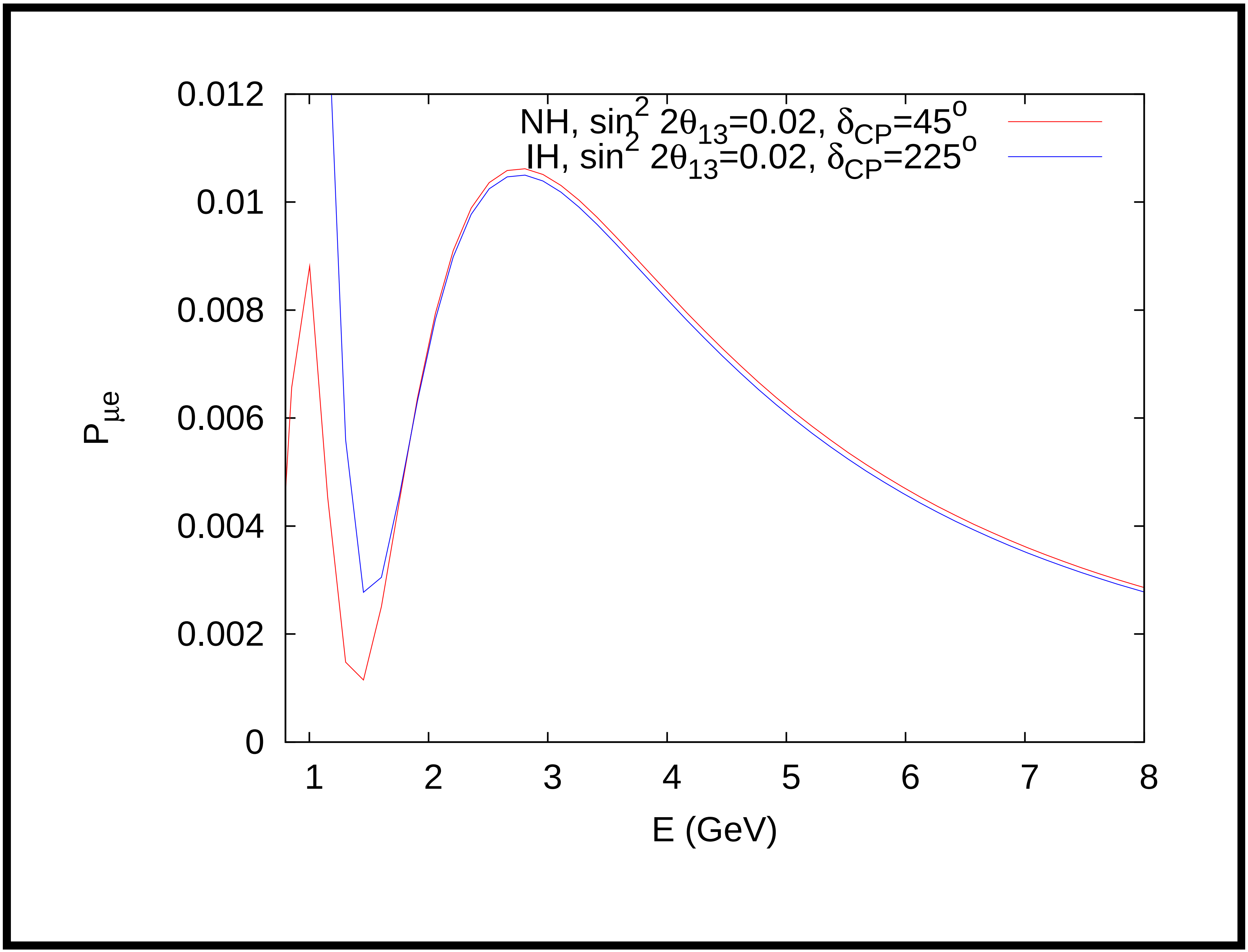
Hierarchy-δ_{CP} Degeneracy

The matter term A is positive for neutrinos and negative for anti-neutrinos. Hence its presence induces a **CP Violation like** change in $P_{\mu e}$.

Δ_{31} is positive for normal hierarchy (NH) and is negative for inverted hierarchy (IH). Hence, if we use a neutrino beam, the term $\hat{A}$ is positive for NH and negative for IH. (For anti-neutrinos, the situation is reversed).

Because of the change in the sign of $\hat{A}$, the values of $P_{\mu e}$ are, in general, different for different hierarchies. Thus a measurement of $P_{\mu e}$ in a long baseline experiment can be used to determine neutrino mass hierarchy.

However, this ability is compromised by our lack of knowledge of δ_{CP}. It is possible to have degenerate solutions such that
$P_{\mu e}(NH, \delta^1_{CP}) = P_{\mu e}(IH, \delta^2_{CP})$.



NH, $\sin^2 2\theta_{13}$=0.02, δ_{CP}=45^o
IH, $\sin^2 2\theta_{13}$=0.02, δ_{CP}=225^o
$P_{\mu e}$
E (GeV)
0.012
0.01
0.008
0.006
0.004
0.002
0
1
2
3
4
5
6
7
8

Magic Baseline

Lack of any knowledge of δ_{CP} limits our ability to determine the mass hierarchy. We can overcome this limitation by choosing neutrino energy or the baseline of the experiment in such a way that δ_{CP} dependence of $P_{\mu e}$ vanishes.

If we choose $\sin(\hat{A}\Delta) = 0$ or $\hat{A}\Delta = \pi$, both Δ_{31} and E drop out of the condition and we get a condition on the baseline L as a function of G_F and ρ. This leads to the famous **magic** baseline condition $L \approx 7500$ km.

At this distance, $P_{\mu e}$ is independent of δ_{CP} for both NH and IH. Also, $P_{\mu e}$ is quite large for NH and is essentially 0 for IH over a wide range of energy.

Hence measurement of $P_{\mu e}$ (or equivalently $P_{e\mu}$) at magic baseline, will lead to a clean measurement of hierarachy. But, such an experiment, has no handle on δ_{CP}.

Moreover, the distance of magic baseline is quite large. The fluxes at such a distance will be quite small. Sufficient fluxes may be achieved only with a futuristic accelerator such a muon storage ring.

Shorter Magic Baseline

Here we consider the magical properties of a much shorter baseline.

δ_{CP} dependence of $P_{\mu e}$ can be made to vanish by choosing $\sin((1-\hat{A})\Delta) = 0$. Unlike the magic baseline condition, this condition can hold for only one hierarchy at a time. (arXiv:0908.3741)

We demand that the above condition hold for IH. This translates into

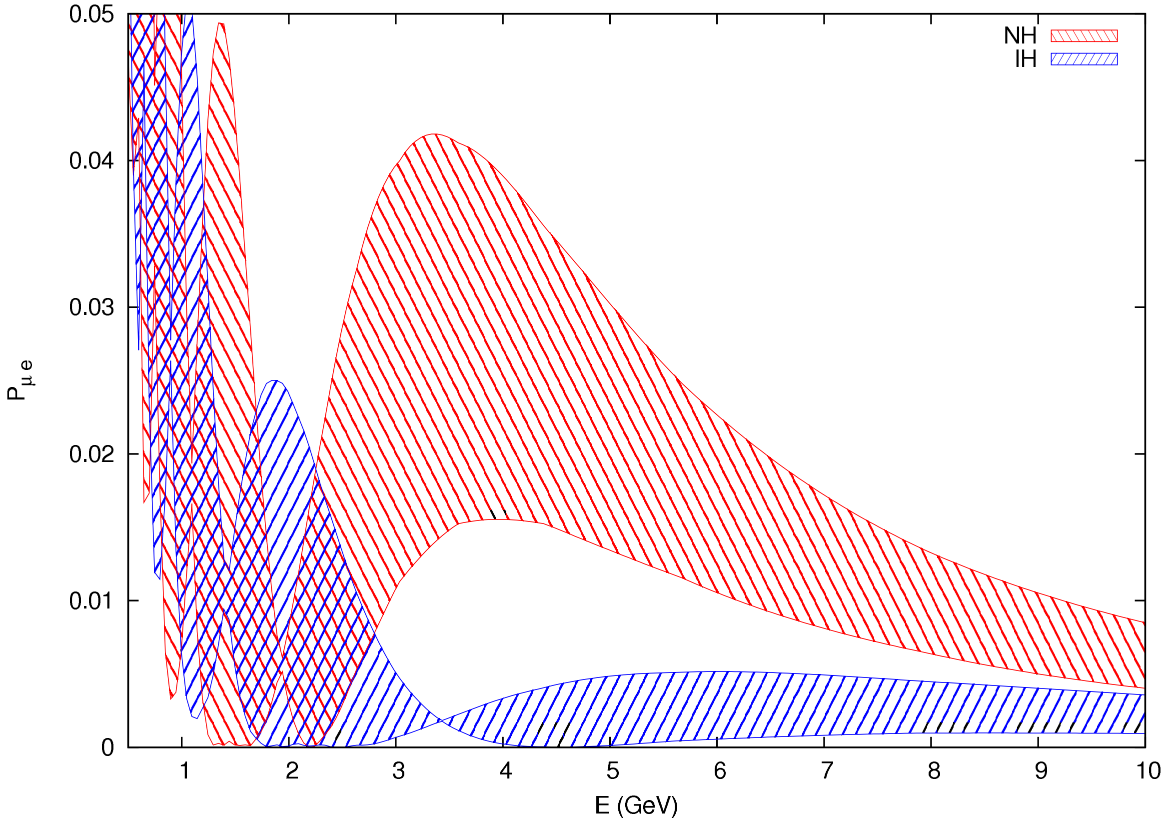
$$1.27(|\Delta_{31}| + A)L/E = \pi.$$

It also makes $P_{\mu e}$ is very small (≈ 0.002), for IH.

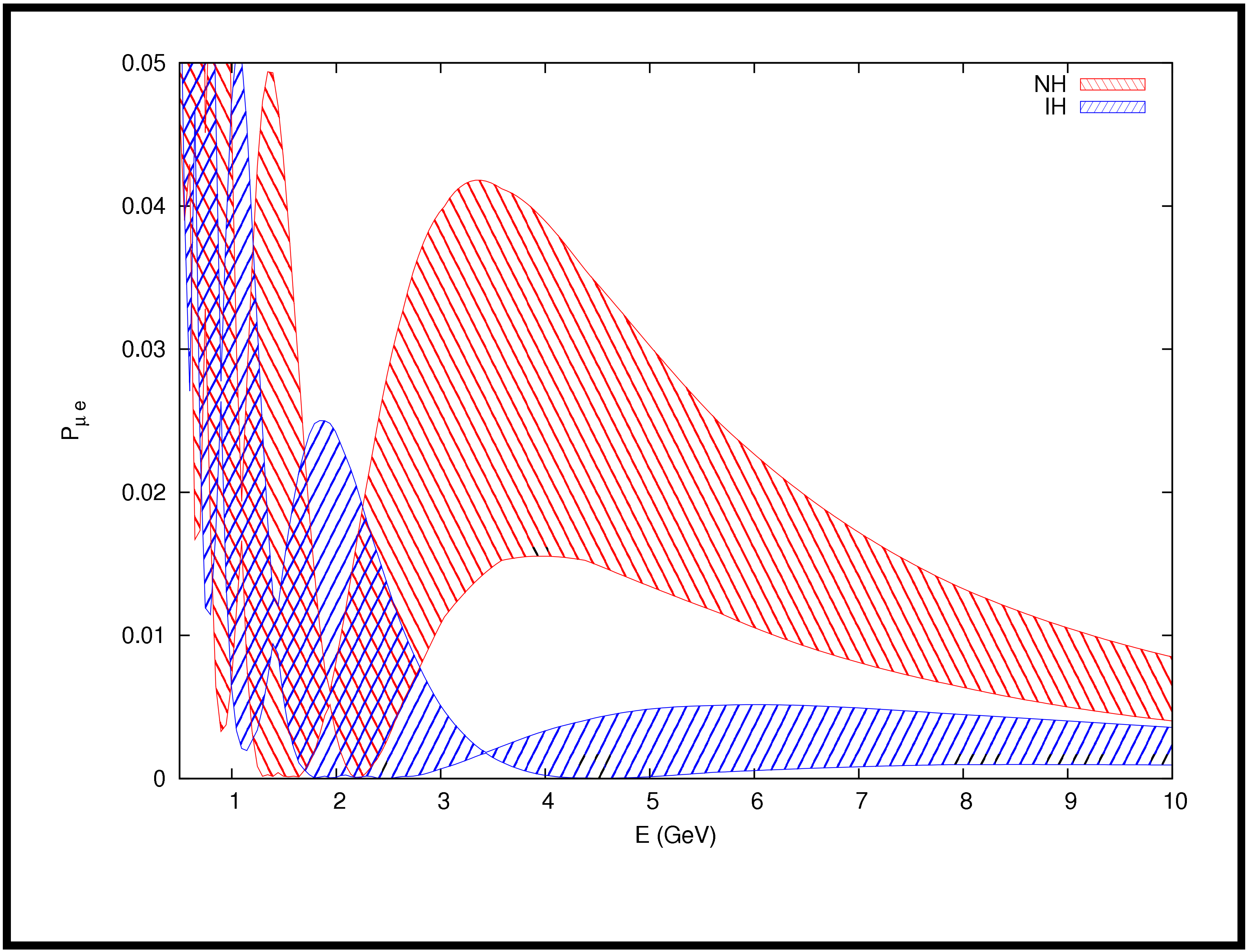
In addition, we demand that, for the same energy E and baseline L, $P_{\mu e}$ should be close to maximum for NH. This gives us the condition

$$1.27(|\Delta_{31}| - A)L/E = \pi/2.$$

Solving the above two equations give $L = 2540$ km and $E = 3.3$ GeV.



0.05
0.04
0.03
0.02
0.01
0
$P_{\mu e}$
1
2
3
4
5
6
7
8
9
10
E (GeV)
NH
IH



NH
IH
$P_{\mu e}$
E (GeV)
0
0.01
0.02
0.03
0.04
0.05
1
2
3
4
5
6
7
8
9
10

Bimagic Properties

In arXiv:1009.1093, it was pointed out that this baseline has additional magic properties.

One can make the demand that $P_{\mu e}$ for NH be vanishingly small and that for IH should be close to maximal. This changes the conditions to

$$1.27(|\Delta_{31}| + A)L/E = 3\pi/2.$$

$$1.27(|\Delta_{31}| - A)L/E = \pi.$$

Solving these equations, again gives $L = 2540$ km but a different energy $E = 1.9$ GeV.

Thus, with a broad band beam, it is possible to have significant event rates for both NH and IH.

Superbeam Realization

In arXiv:1011.1146, we considered the following configuration.

- Source: NuMI like source in medium energy option with a power of approximately 1 MW (10×10^{20} POT/yr).
- Detector: NOνA like detector (totally active scintillator detector) with 100 kton mass, located at a distance 2540 km away from the source at a location 7 mr off-axis.

These numbers were chosen so that this configuration has exposure 1000 (kton-POT/yr) as opposed to 100 (kton-POT/yr) for NOνA. Since the source-distance for this detector is 3 times that of NOνA, the number of events in this detector will be similar to that of NOνA.

In addition, we assume that θ_{13} is measured by the reactor neutrino experiments.

Calculation

We did the calculation using GLoBES, using the crosssection and effciencies of NOνA, inputted in there.

We calculated the fluxes using the program given at Messier's website http://enrico1.physics.indiana.edu/messier/off-axis/spectra/

We assumed that this experiment will run in neutrino mode only. We computed the ability of this experiment, in conjunction with reactor neutrino data, to determine the neutrino mass hierarchy.

The time needed to obtain a $3\ \sigma$ distinction between the hierarchies is listed in the table in the next slide.

Results

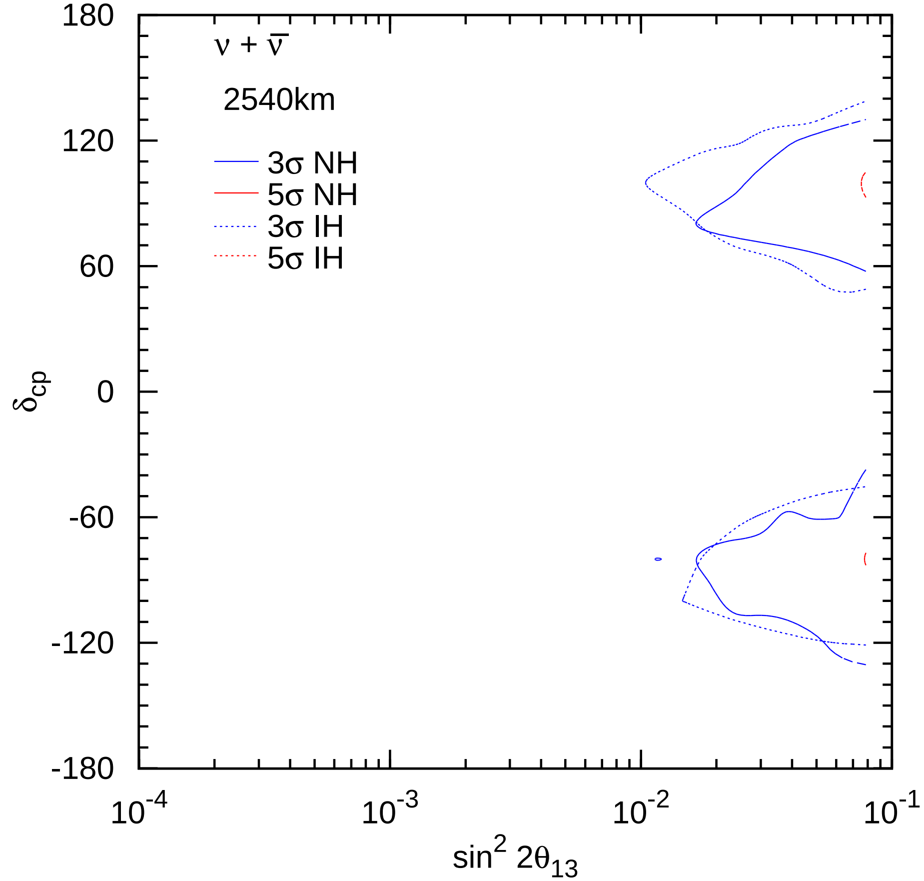
$\sin^2 2\theta_{13}$ (true)	Exposure time(NH)	Exposure time(IH)
0.10	0.022	0.048
0.09	0.026	0.057
0.08	0.031	0.068
0.07	0.040	0.082
0.06	0.051	0.105
0.05	0.070	0.137
0.04	0.104	0.195
0.03	0.180	0.420
0.02	0.425	2.600
0.01	2.95	4.8

We note from the above table that this set up has outstanding capability to distinguish between the two hierarchies.

In particular, if $\sin^2 2\theta_{13} \geq 0.03$, the hierarchy can be determined in 0.4 years (or in 4 years if the exposure is one tenth what we assumed).

That is: If NuMI like beam is aimed at NOνA detector from a distance of 2540 km, then NOνA can determine neutrino mass hierarchy in 4 years **independent** of the value of δ_{CP}.

In addition, this experiment is also sensitive to δ_{CP}. Its ability to determine non-zero δ_{CP} is shown in the next slide.



ν + ν̄
2540km
3σ NH
5σ NH
3σ IH
5σ IH
180
120
60
0
-60
-120
-180
δcp
10-4
10-3
10-2
10-1
sin2 2θ13