

# Modeling of Hadronic Interactions

## *Lecture 2*

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# Outline

## **Lecture 1 – Low- and intermediate-energy interactions**

- Particle production threshold: resonances
- Intermediate energies: two-string models
- Extension to nuclei and photons

## **Lecture 2 – Interactions at very high energy**

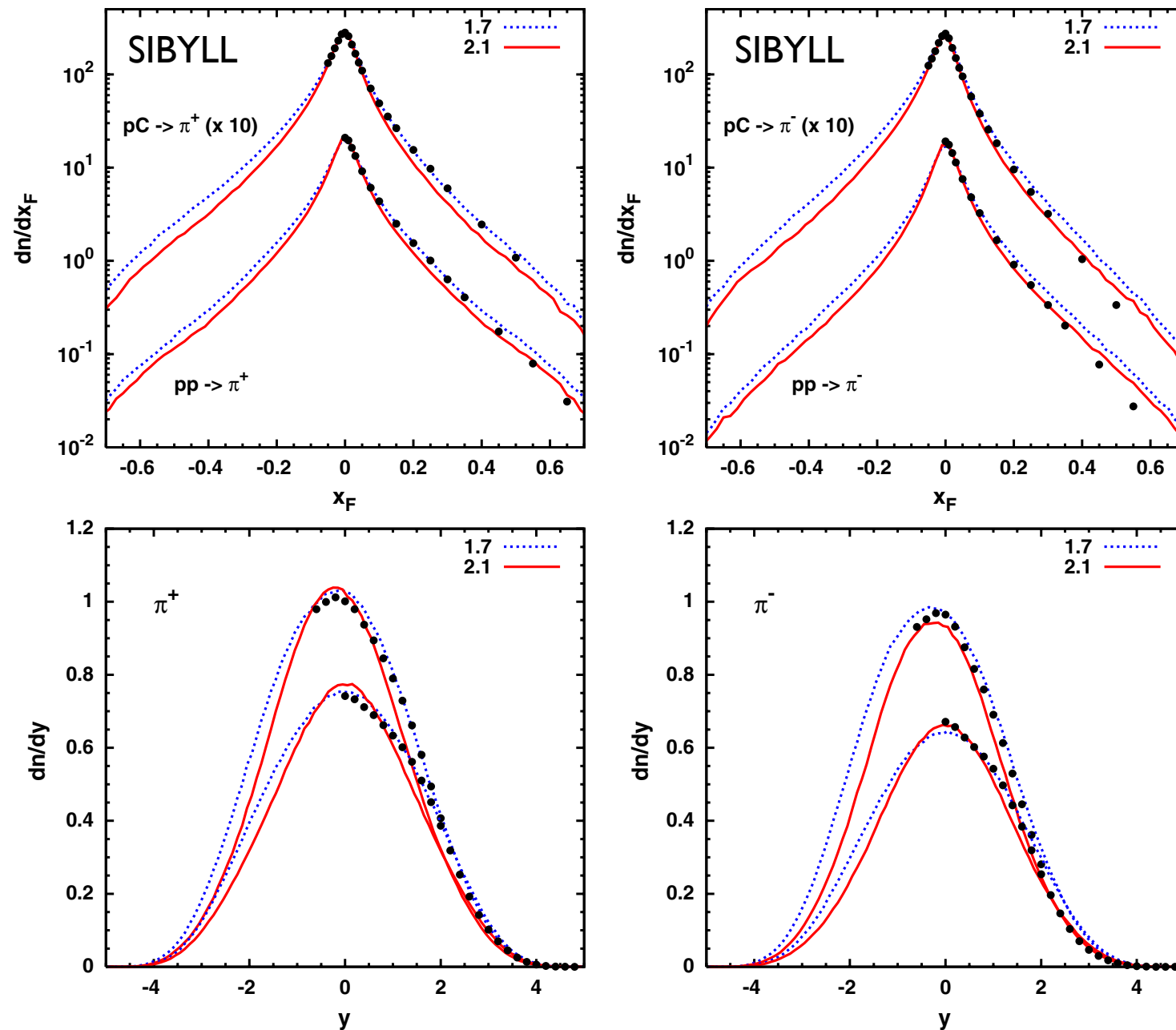
- Jets and minijets, multiple interactions
- Unitarization and saturation scenarios
- Comparison of models and uncertainties of extrapolations

## **Lecture 3 – Air shower phenomenology and accelerator data**

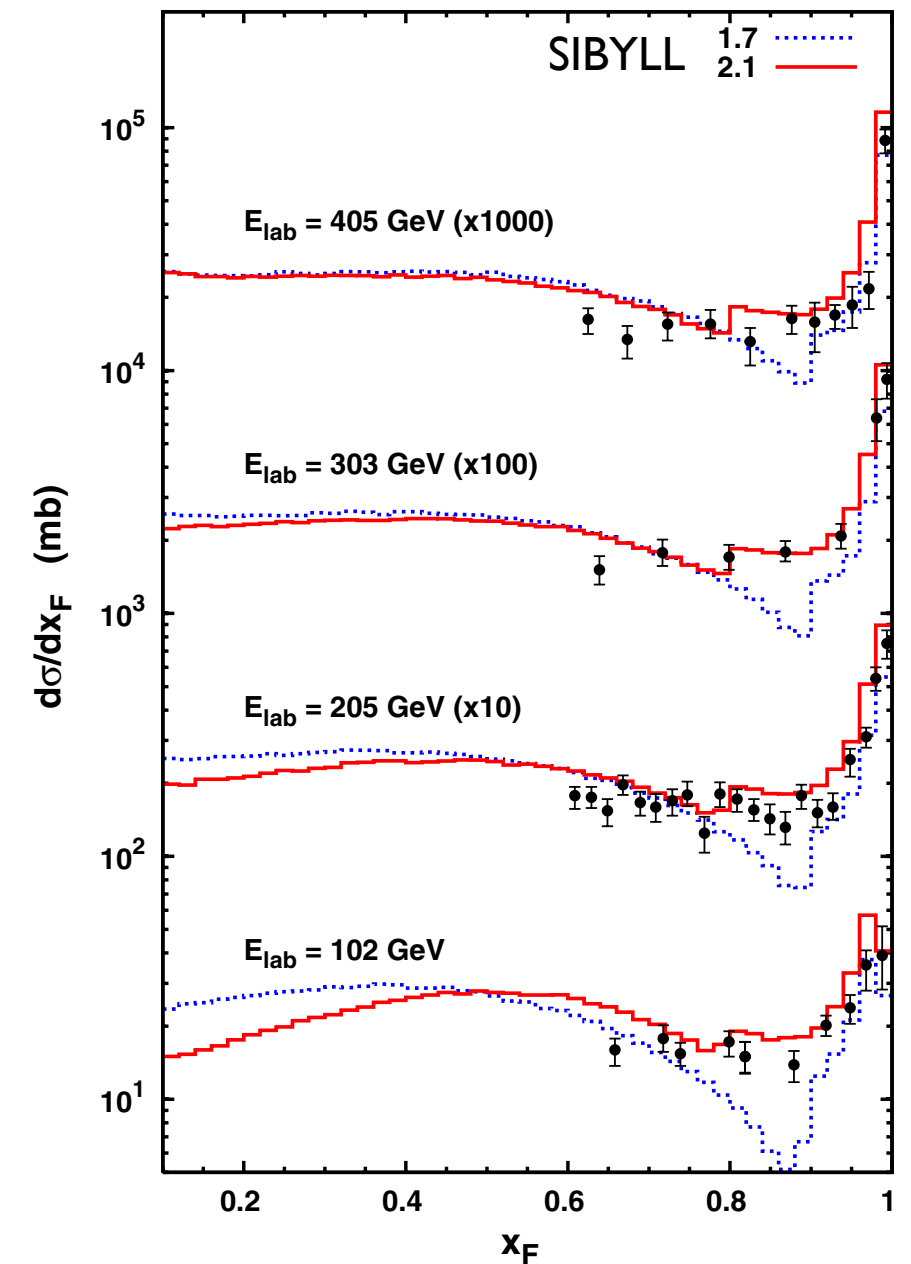
- Relation between hadronic interactions and air showers
- Accelerator experiments & discrimination potential of LHC
- Comparison of model predictions with accelerator data

# SIBYLL: central & leading particle production

NA49 p-p and p-C at 158 GeV

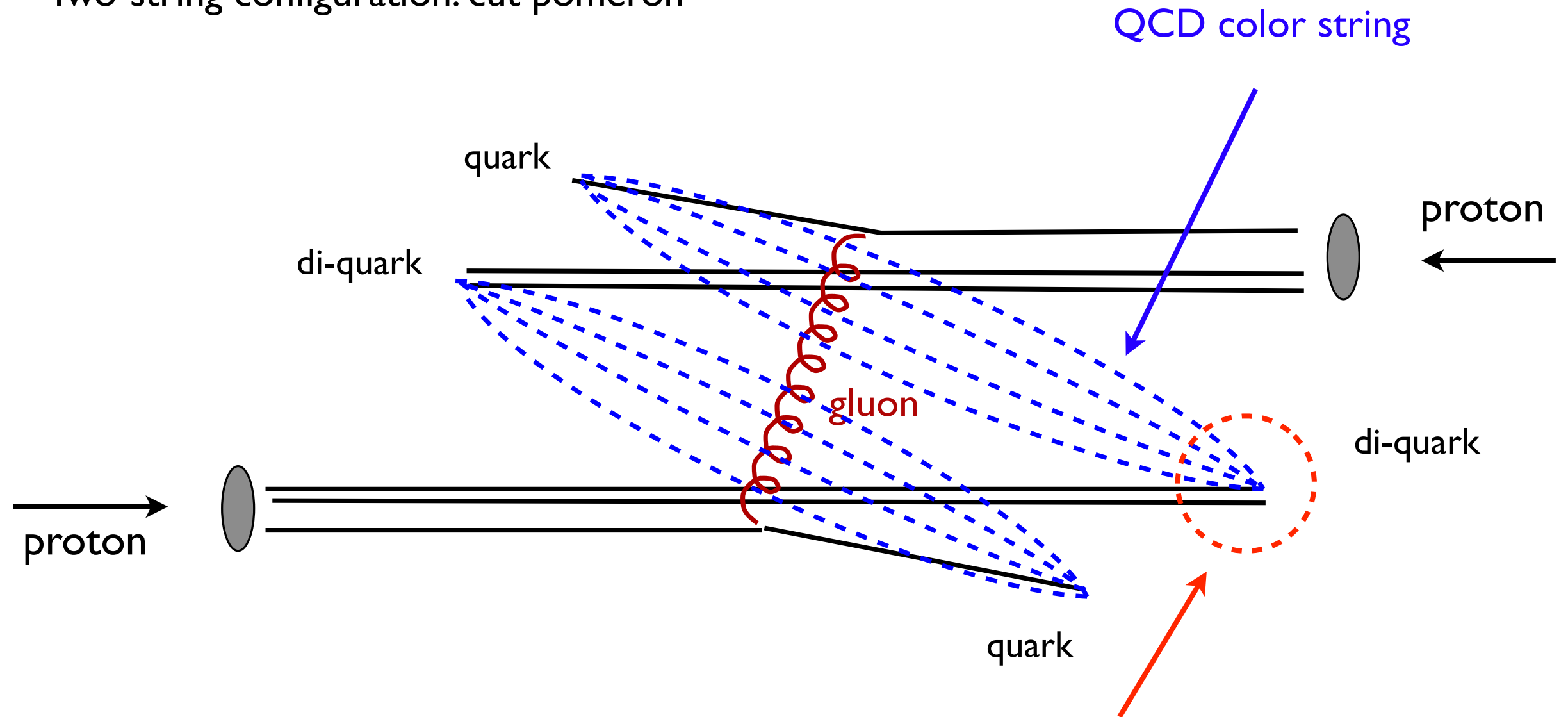


leading proton distributions



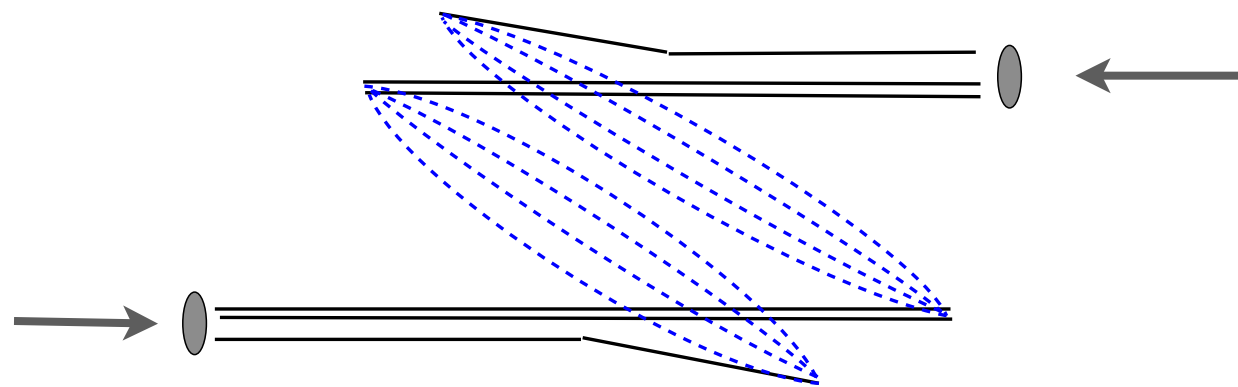
# Simplest interaction scenario: single gluon exchange

Two-string configuration: cut pomeron



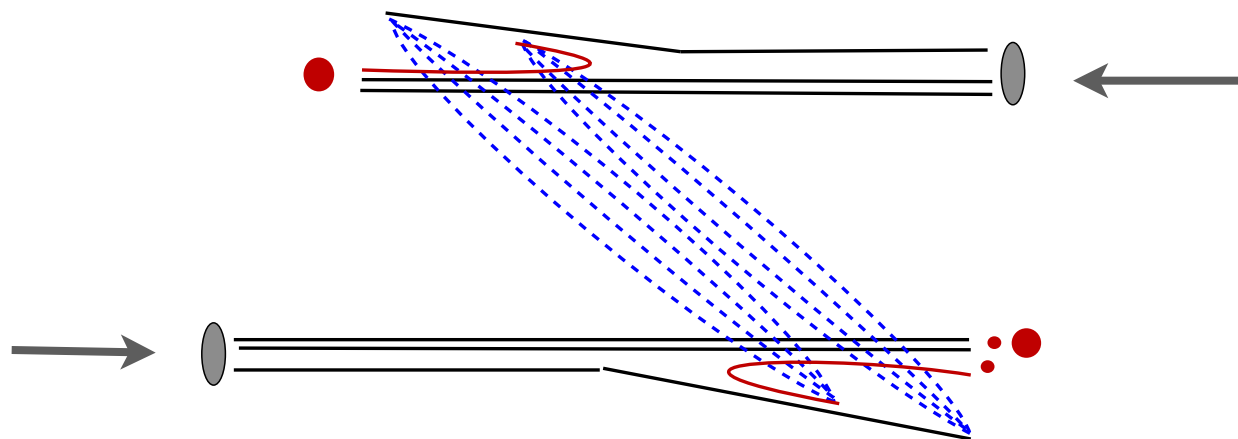
**Leading particle production:** first particle in string fragmentation contains quarks/diquarks from beam particle

# Different implementations



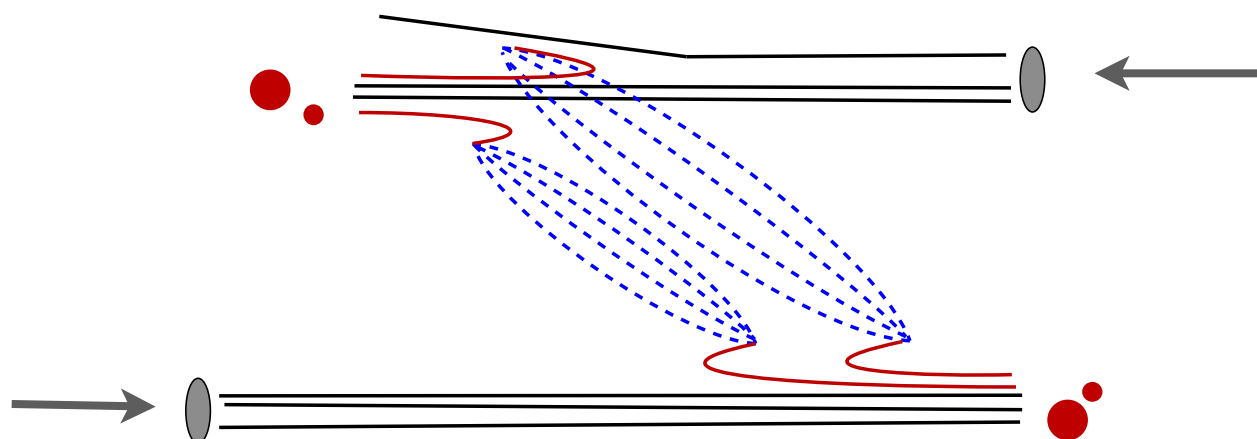
## SIBYLL:

strings connected to valence quarks;  
first fragmentation step with harder  
fragmentation function



## QGSJET:

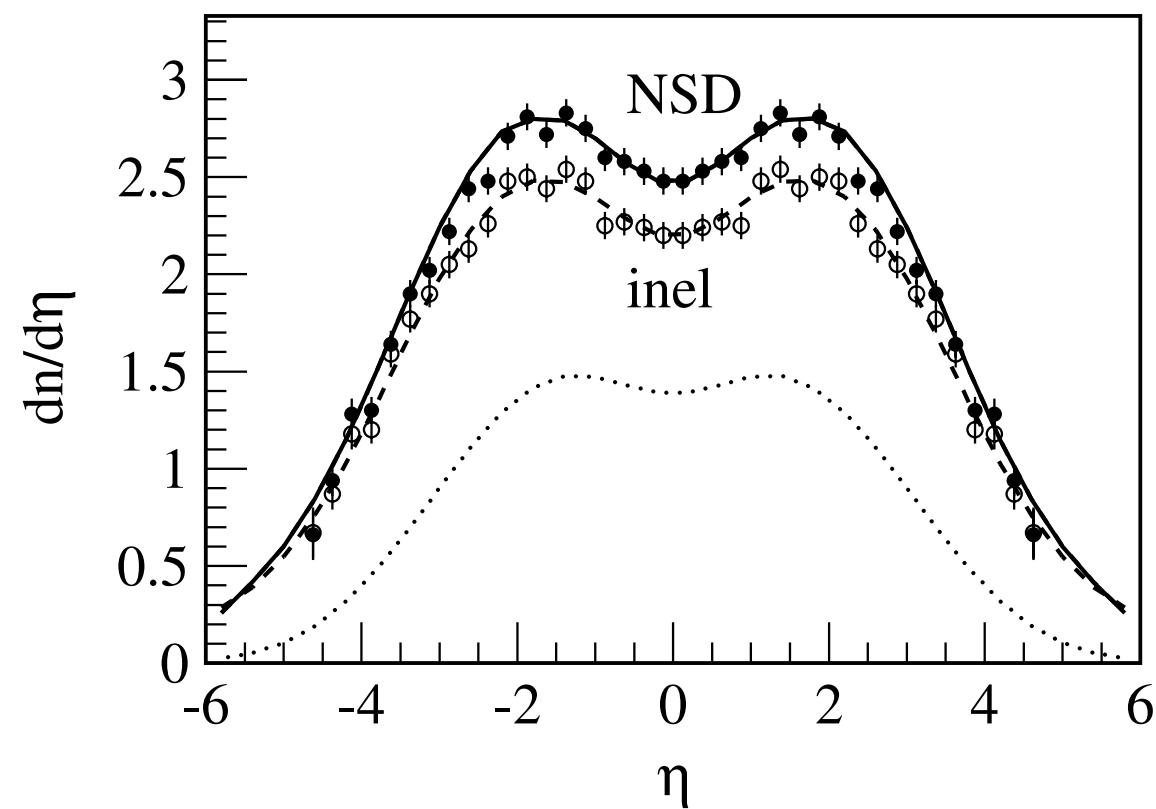
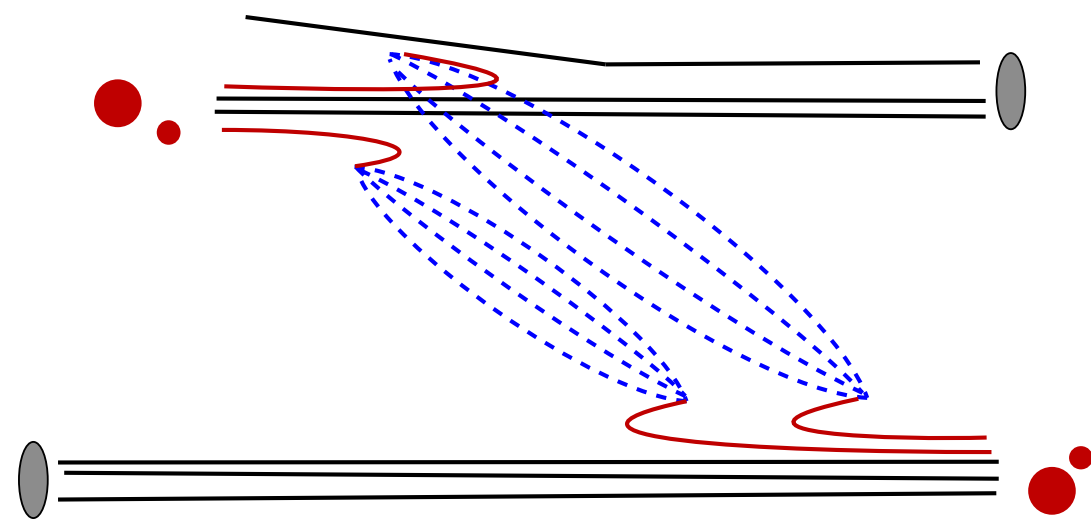
fixed probability of strings connected to  
valence quarks or sea quarks;  
explicit construction of remnant hadron



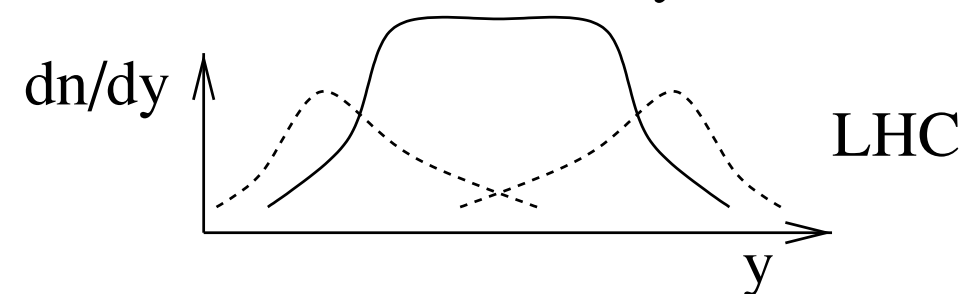
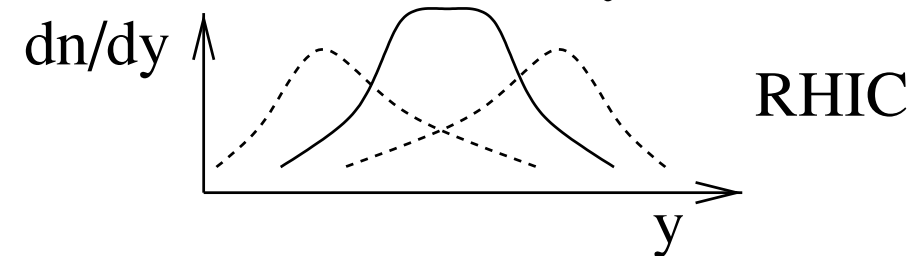
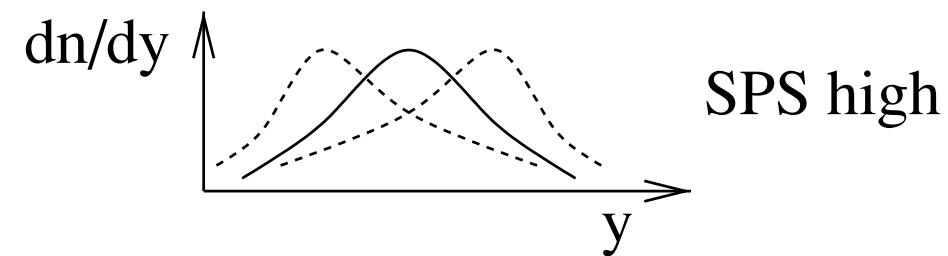
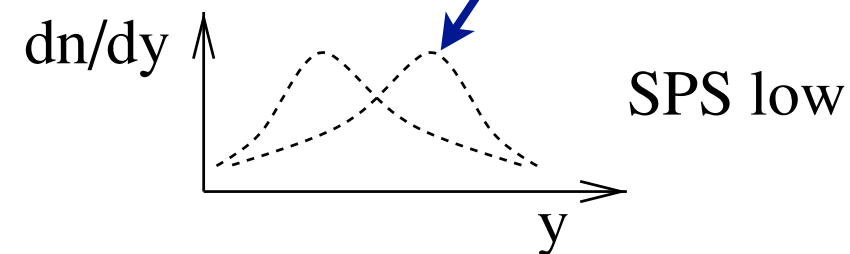
## EPOS:

strings always connected to sea quarks;  
bags of sea and valence quarks fragmented  
statistically

# EPOS: remnant vs. string contributions

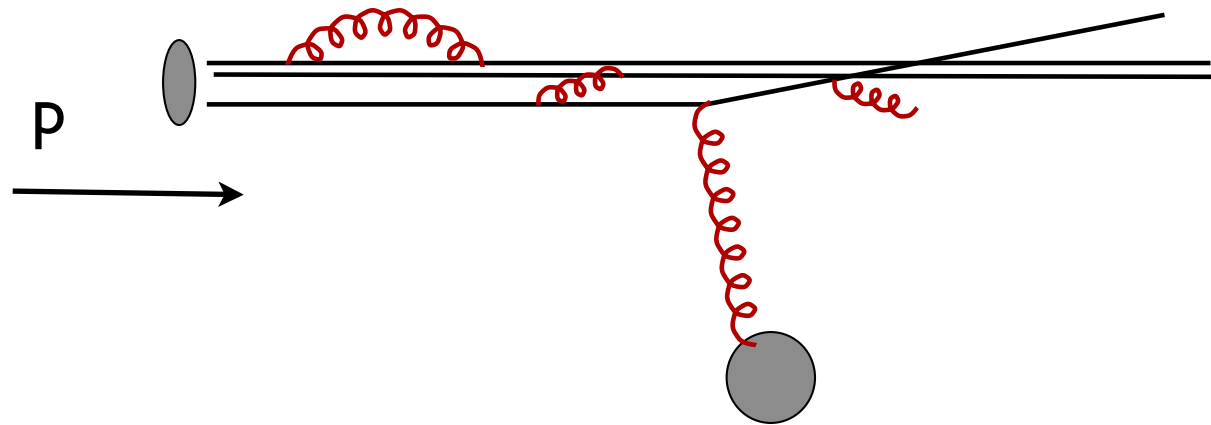


remnant contributions



EPOS: change from remnant-dominated to string-dominated particle production

# Transition from intermediate to high energy

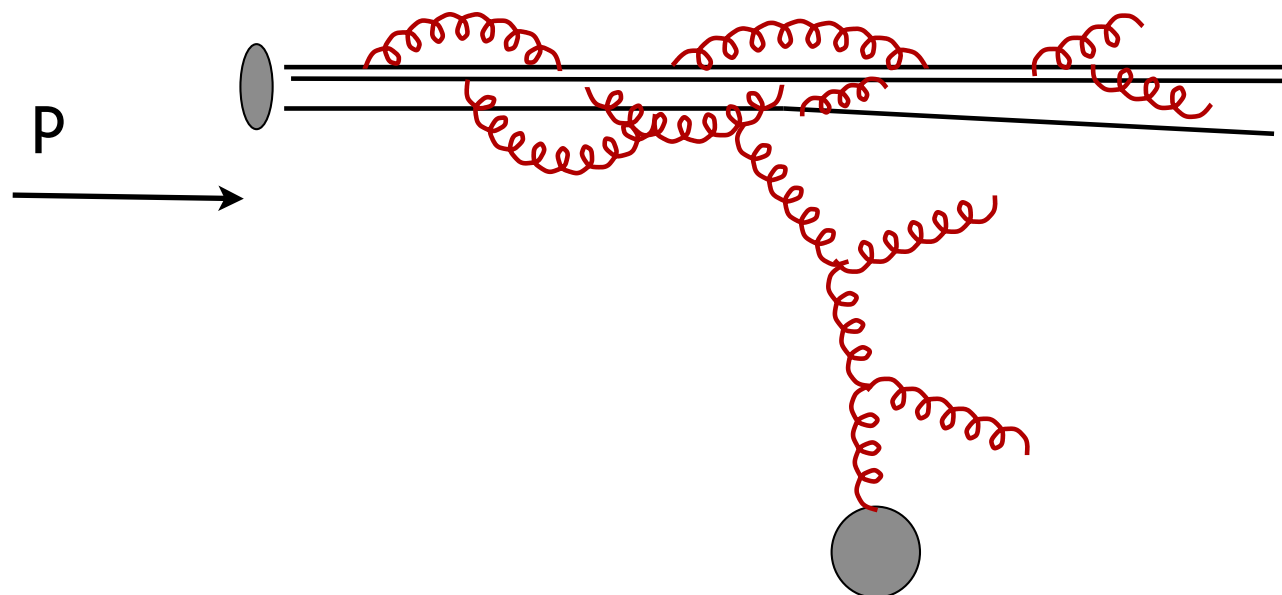


## Intermediate energy:

- $E_{\text{lab}} < 1,500 \text{ GeV}$
- $E_{\text{cm}} < 50 \text{ GeV}$
- dominated by valence quarks

Lifetime of fluctuations

$$\Delta t \approx \frac{1}{\Delta E} = \frac{1}{\sqrt{p^2 + m^2} - p} = \frac{1}{p(\sqrt{1 + m^2/p^2} - 1)} \approx \frac{2p}{m^2}$$

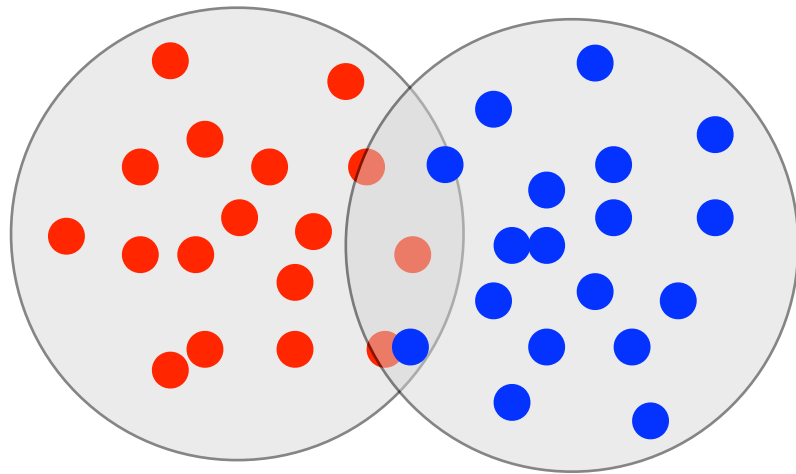


## High energy regime:

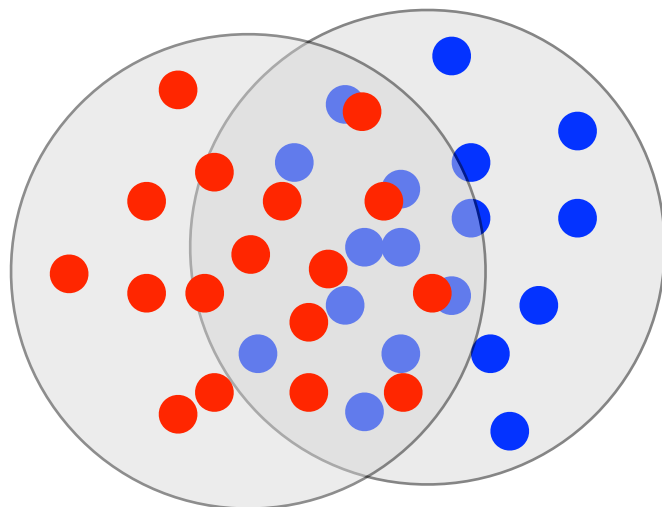
- $E_{\text{lab}} > 21,000 \text{ GeV}$
- $E_{\text{cm}} > 200 \text{ GeV}$
- dominated by gluons and sea quarks

# Geometric interpretation of collision at high energy

Hadrons filled with partons from fluctuations  
frozen in time (relative to time scale of interaction)



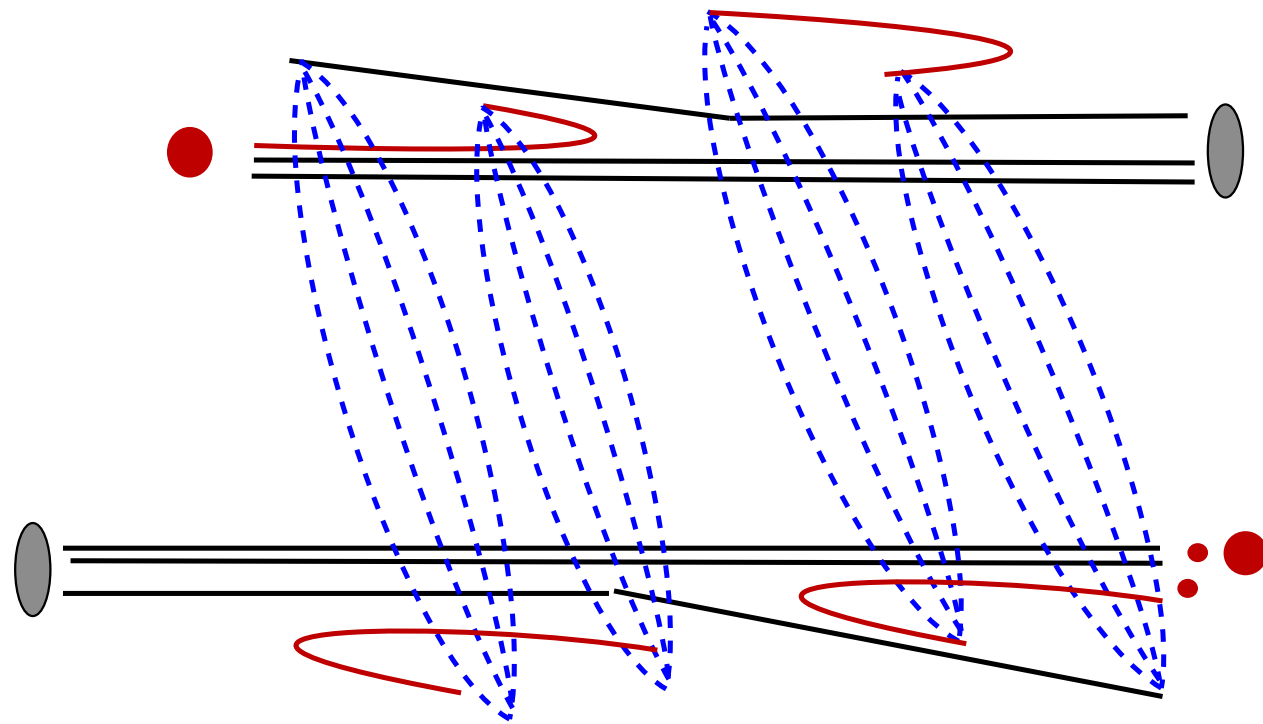
**Peripheral collision:**  
only one parton-pair interacting



**Central collision:**  
several parton-pairs interacting



# Interaction of two parton pairs

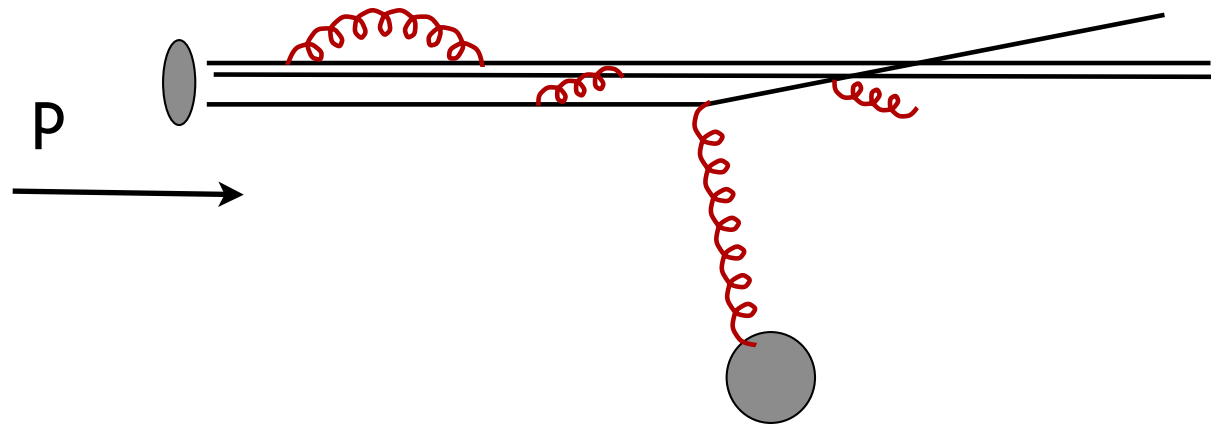


**Two soft interactions**

## **Generic diagram of interaction of two parton pairs**

- gluon exchange between each pair produces two strings
- sea quarks needed for string ends (different combinations possible)
- other sea quark pairs possible but not explicitly simulated
- each string fragments into hadrons with small transverse momenta

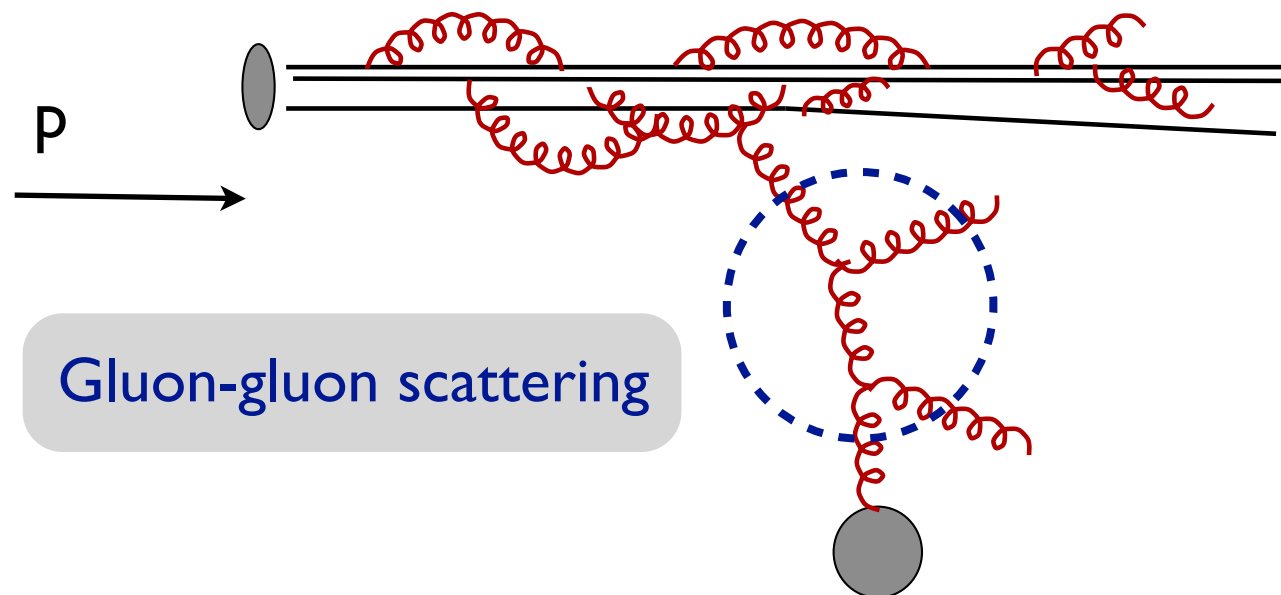
# Transition from intermediate to high energy



## Intermediate energy:

- $E_{\text{lab}} < 1,500 \text{ GeV}$
- $E_{\text{cm}} < 50 \text{ GeV}$
- dominated by valence quarks

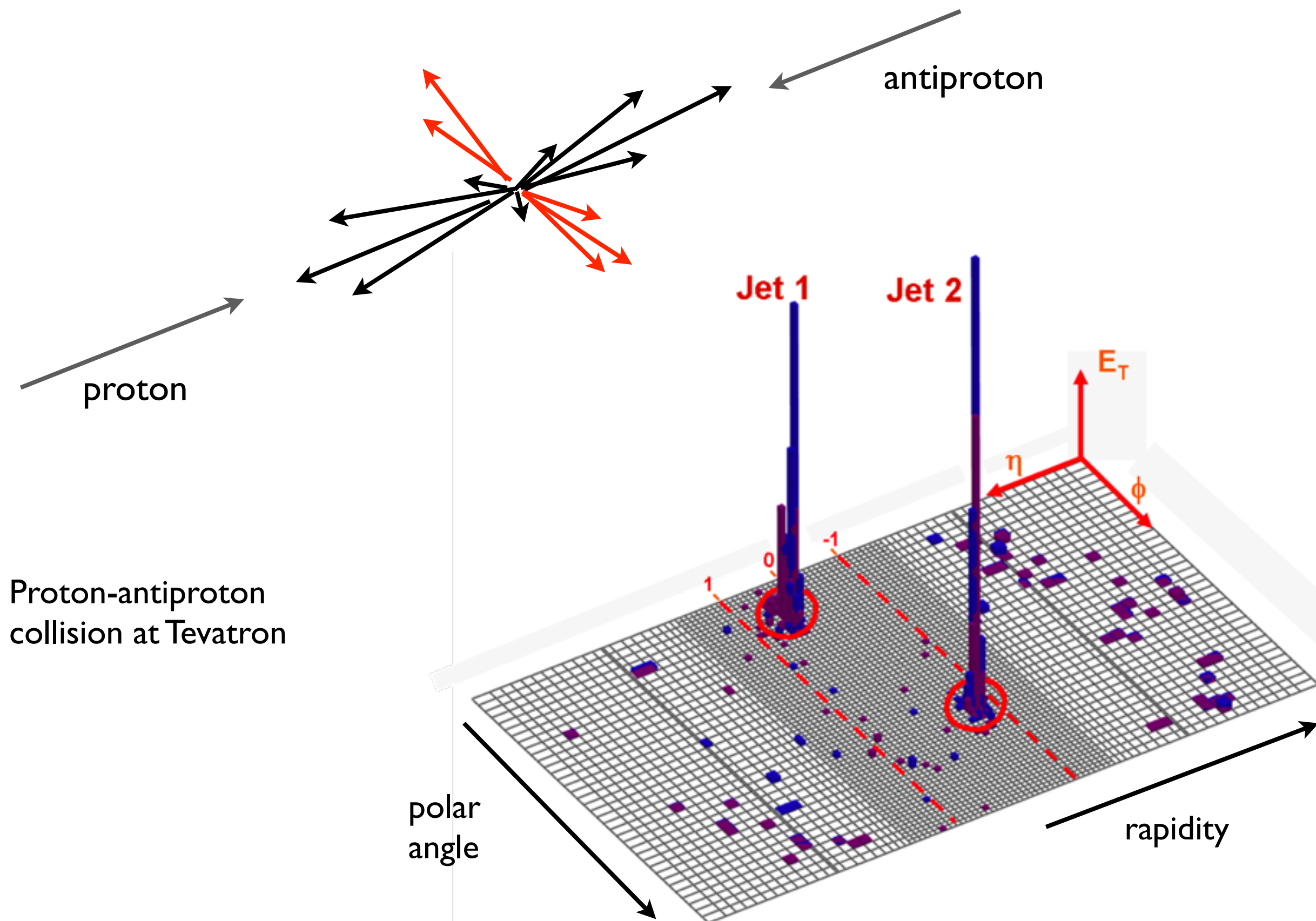
Lifetime of fluctuations  $\Delta t \approx \frac{1}{\Delta E} = \frac{1}{\sqrt{p^2 + m^2} - p} = \frac{1}{p(\sqrt{1 + m^2/p^2} - 1)} \approx \frac{2p}{m^2}$



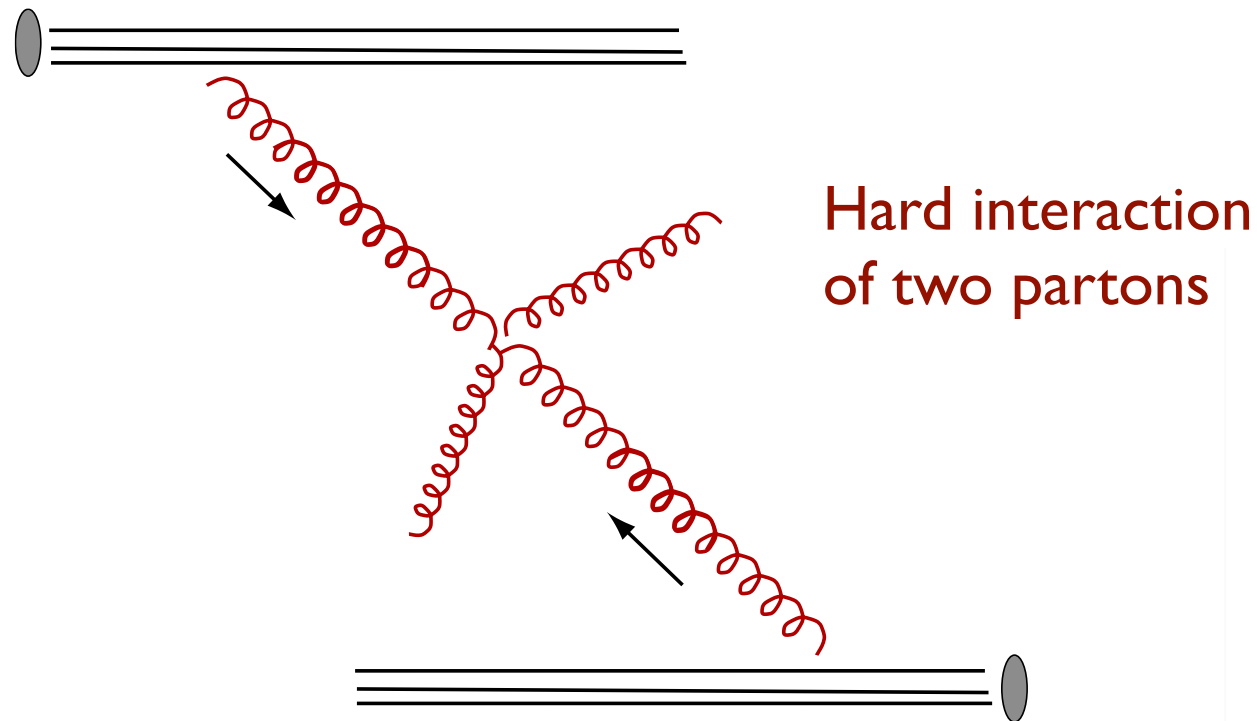
## High energy regime:

- $E_{\text{lab}} > 21,000 \text{ GeV}$
- $E_{\text{cm}} > 200 \text{ GeV}$
- dominated by gluons and sea quarks

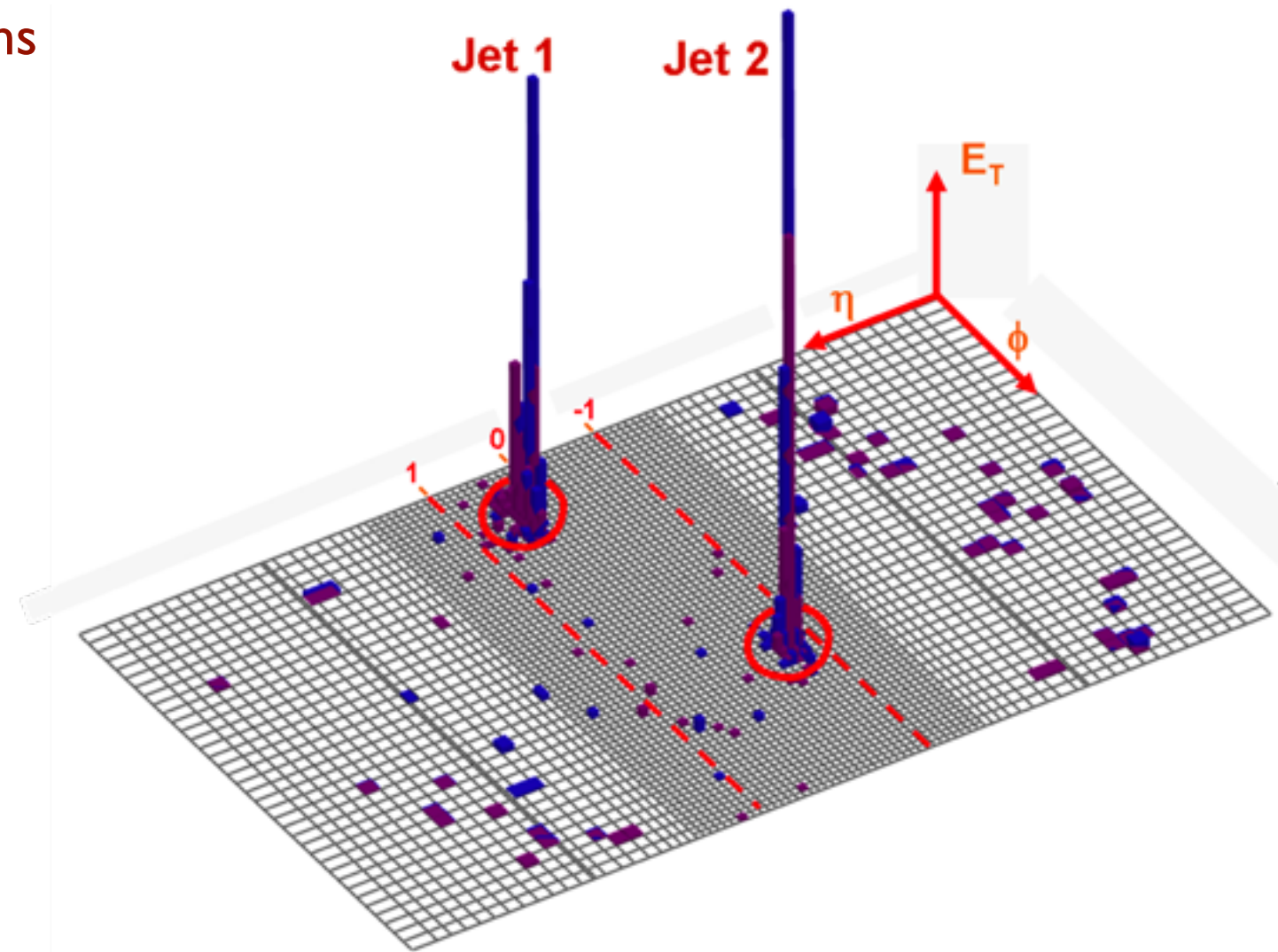
# Scattering of quarks and gluons: jet production



# Interpretation within perturbative QCD



QCD predictions known for parton-parton cross sections

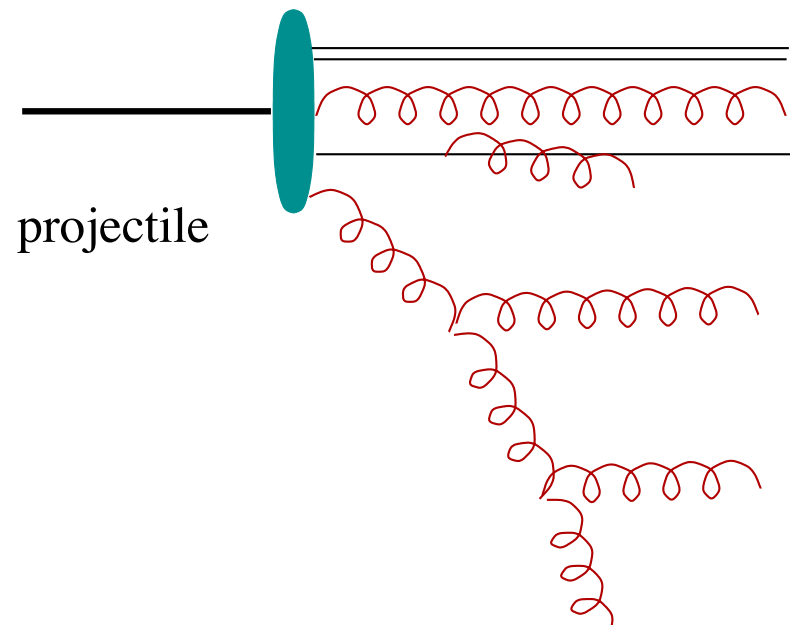


## Terminology

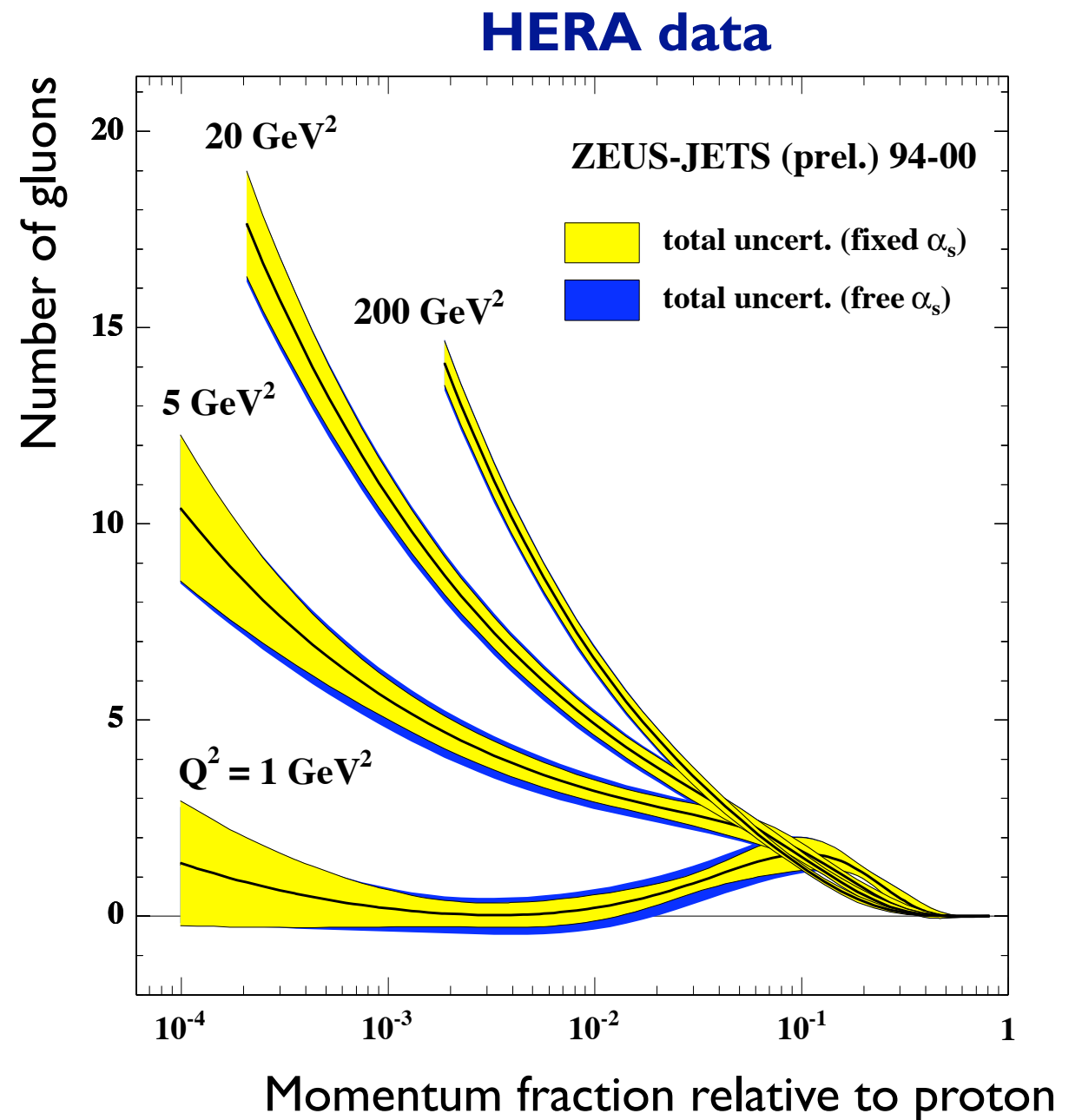
Soft interaction: no large momentum transfer

Hard interaction: large momentum transfer ( $|t| > 2 \text{ GeV}^2$ )

# Perturbative QCD predictions for parton densities



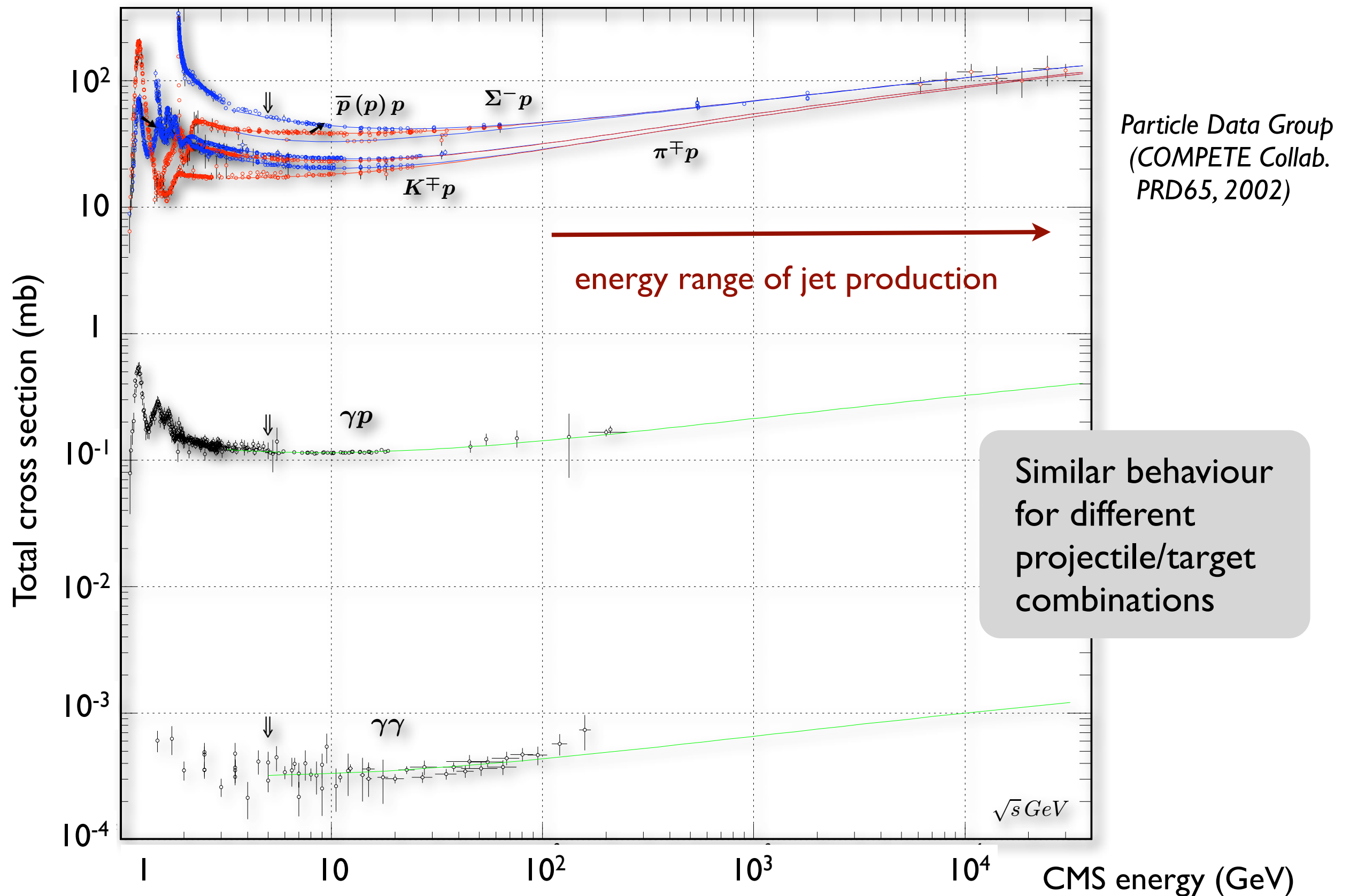
Evolution of parton number given by DGLAP equation (and non-linear versions of it)



$$\frac{df_i(x, Q^2)}{d \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \sum_j f_j(y, Q^2) P_{j \rightarrow i} \left( \frac{x}{y} \right)$$

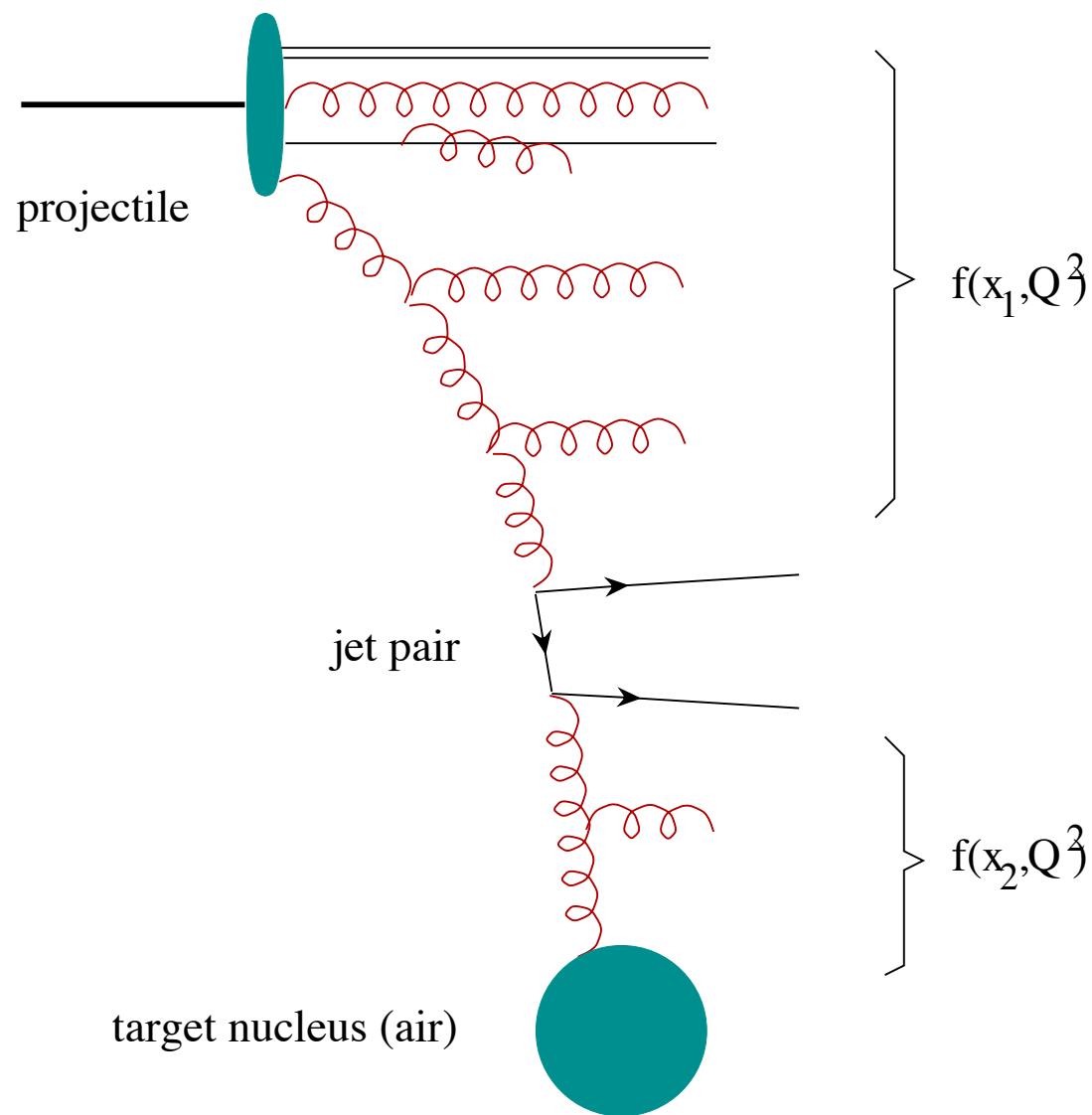
Prediction of perturbative QCD

# Minijet model: rise of cross section due to jet production

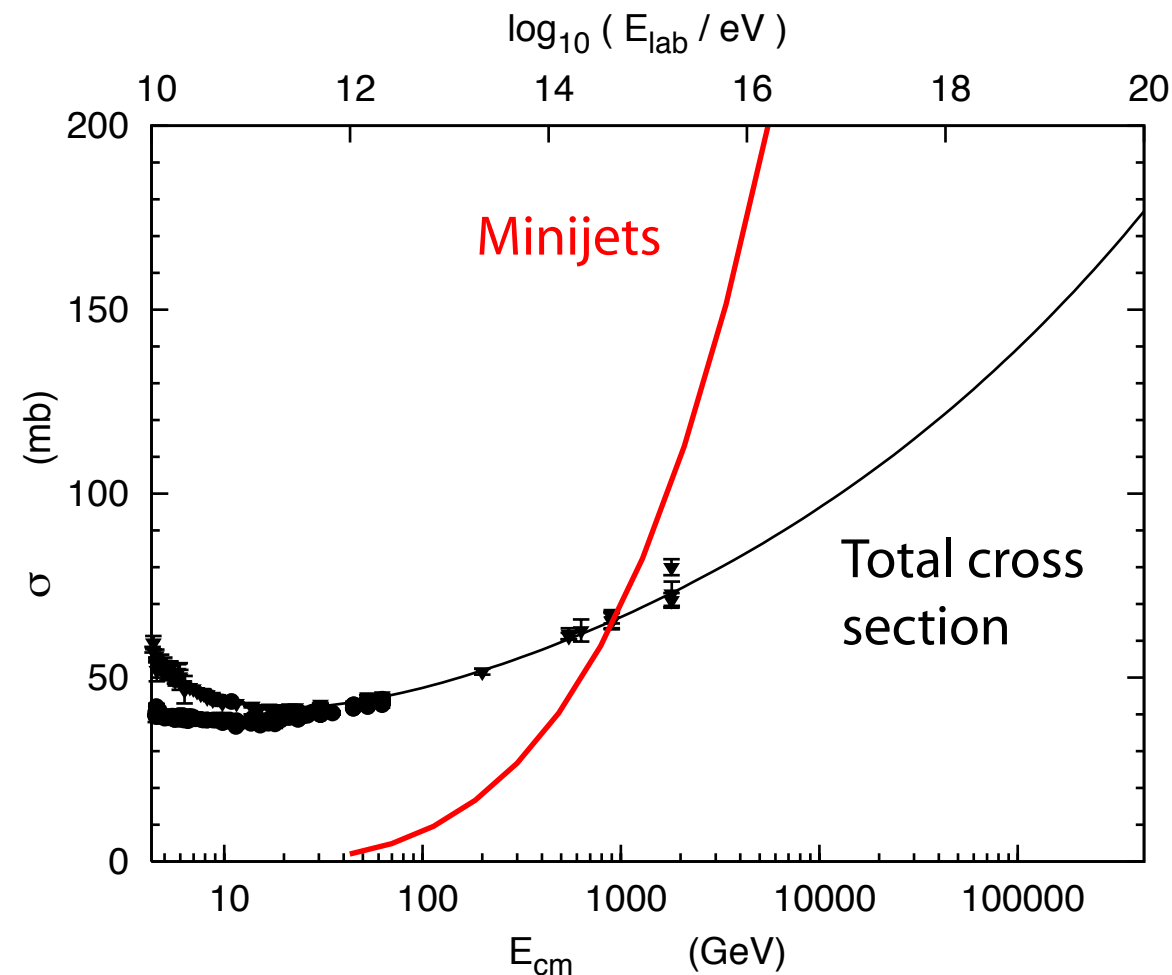




# QCD parton model: minijets



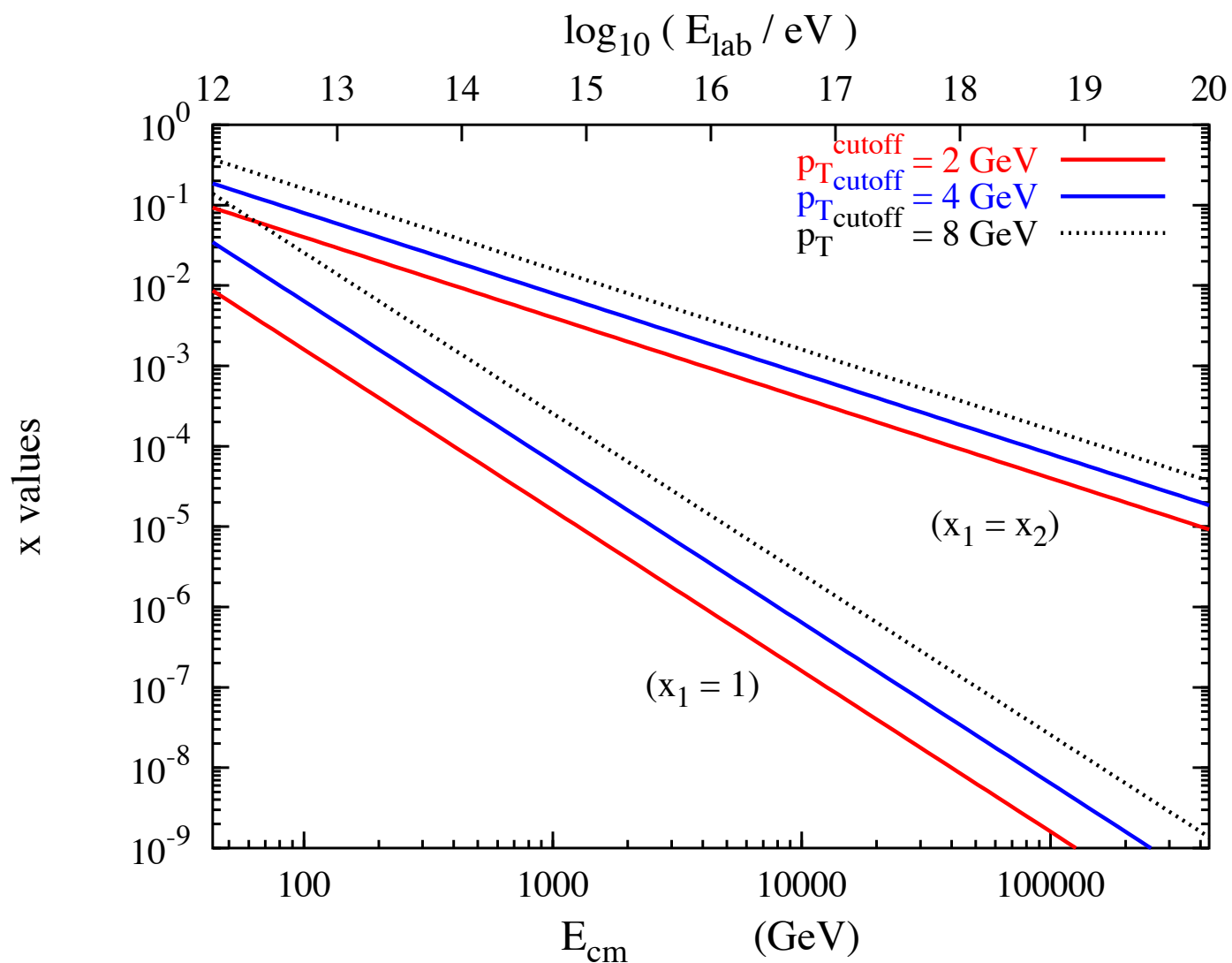
## Proton-proton cross section



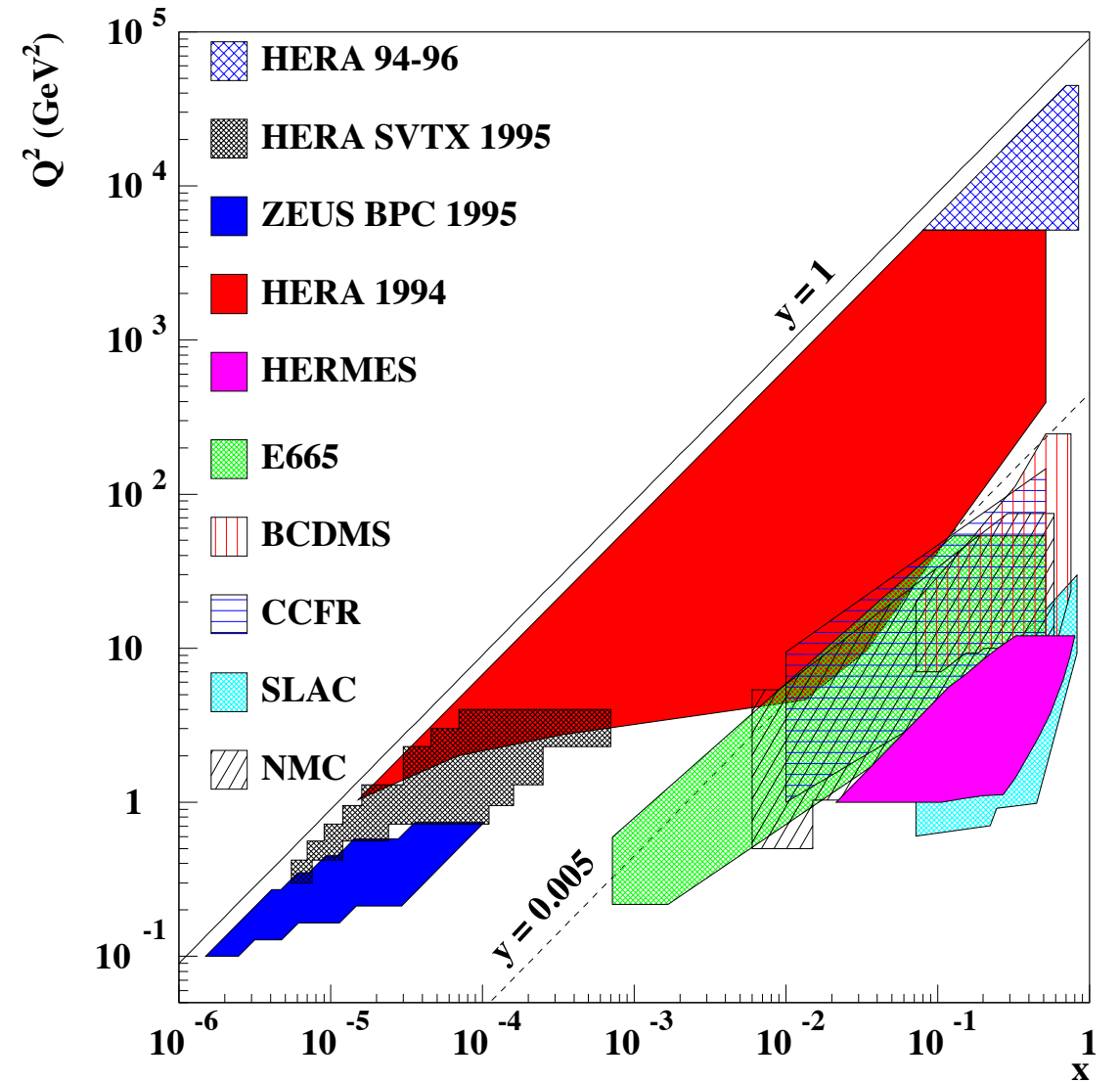
$$\sigma_{QCD} = \sum_{i,j,k,l} \frac{1}{1 + \delta_{kl}} \int dx_1 dx_2 \int_{p_{\perp}^{\text{cutoff}}} dp_{\perp}^2 f_i(x_1, Q^2) f_j(x_2, Q^2) \frac{d\sigma_{i,j \rightarrow k,l}}{dp_{\perp}}$$

# Parton densities not known at very low x

Range of x values (momentum fractions) needed in calculation



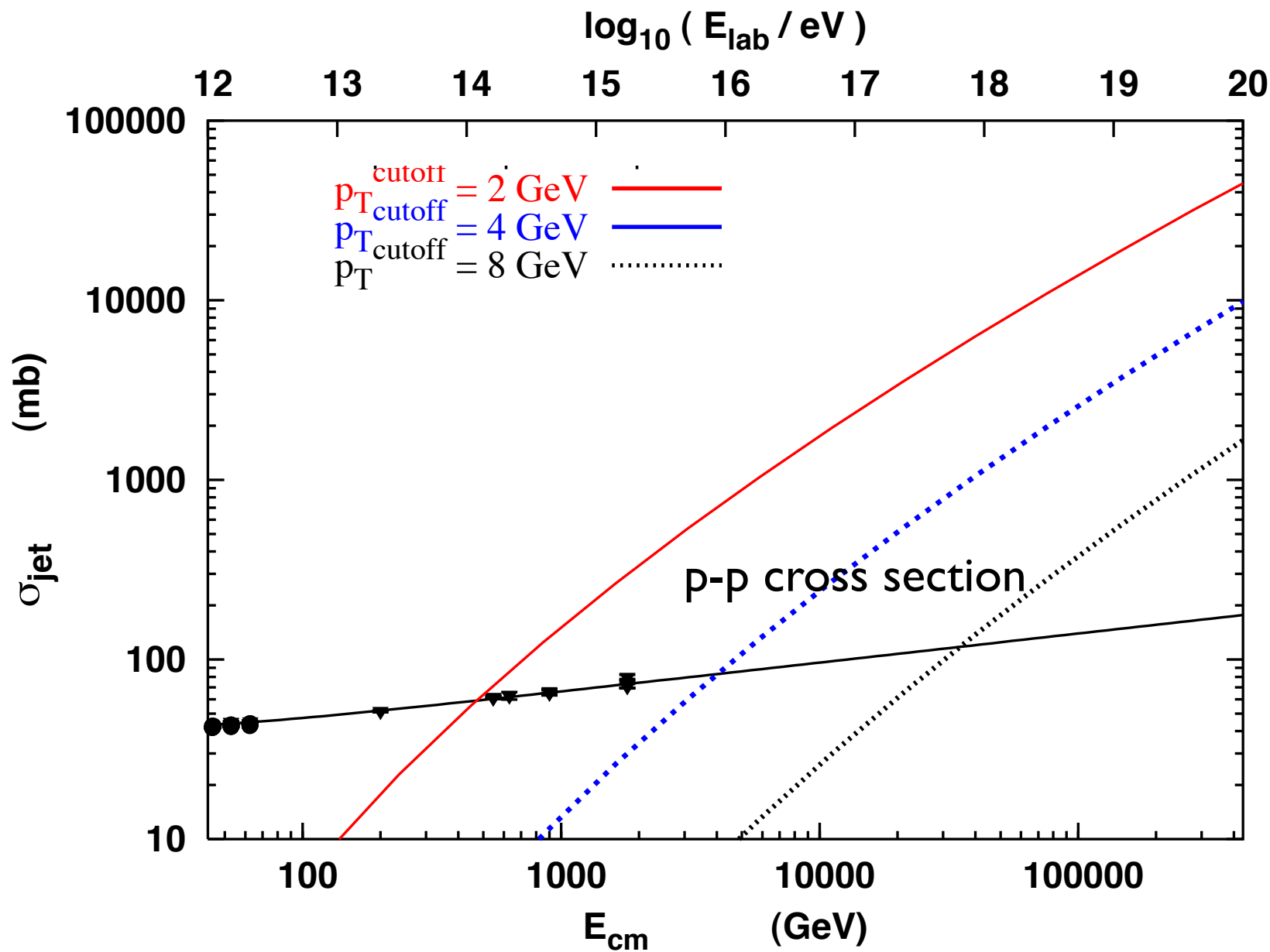
## HERA measurement range



$$\hat{s} = x_1 x_2 s \geq 4p_{\perp}^2$$



# Dependence on transverse momentum cutoff



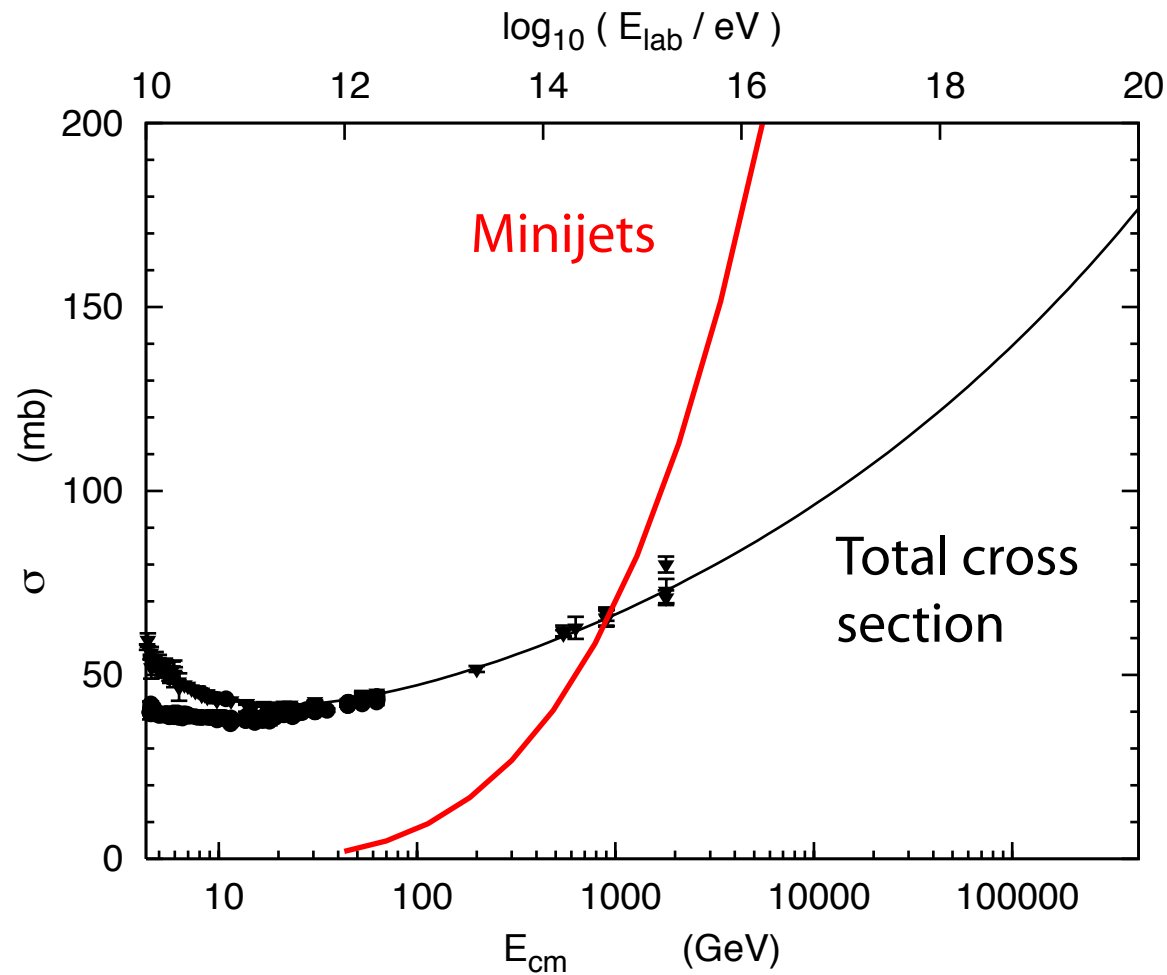
Factor  $\sim 150$

Selecting different values for cutoff does not help

Cross section of new interaction process of minijet production cannot simply be added to cross section from soft interactions

# Solution: Multiple parton-parton interactions

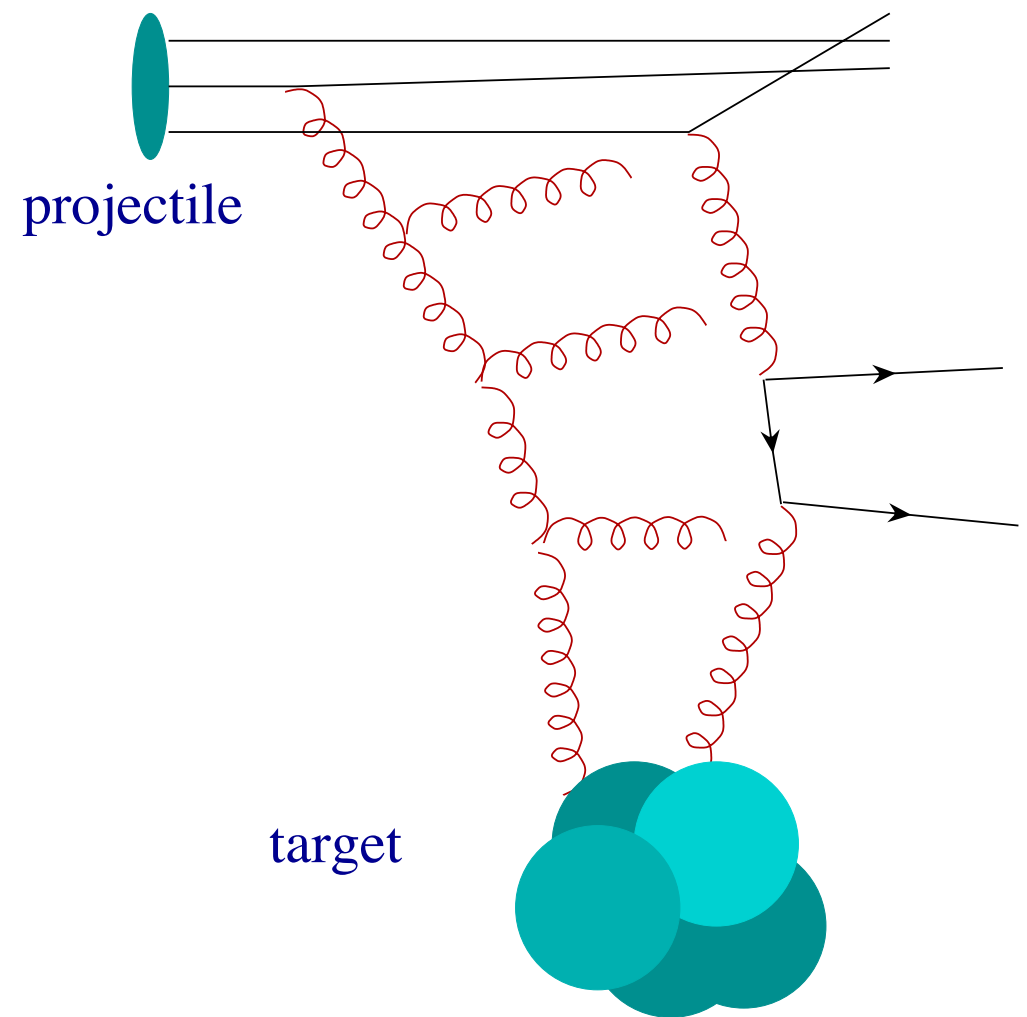
## Proton-proton cross section



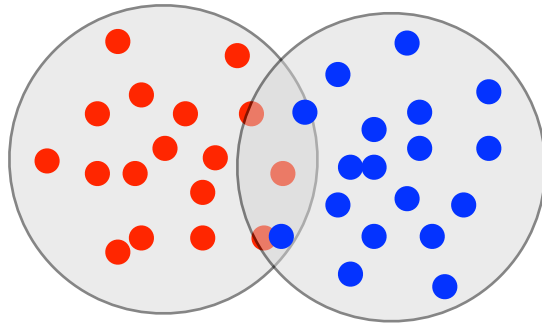
Average number  
of minijet pairs

$$\langle n_{\text{jet}} \rangle = \frac{\sigma_{\text{QCD}}}{\sigma_{\text{ine}}}$$

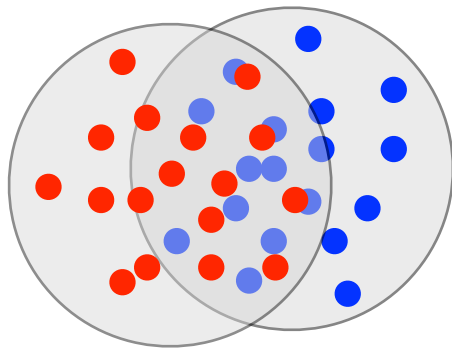
QCD prediction:  
**inclusive** cross section



# Poissonian probability distribution



**Peripheral collision:**  
only very few parton-pairs interacting



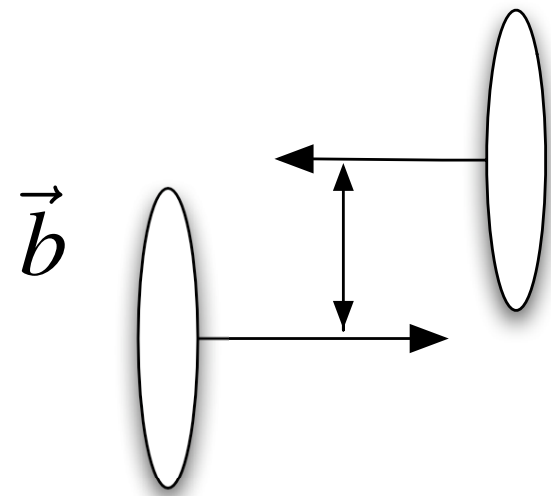
**Central collision:**  
many parton-pairs interacting

$$P_n = \frac{\langle n_{\text{hard}}(\vec{b}) \rangle^n}{n!} \exp \left( -\langle n_{\text{hard}}(\vec{b}) \rangle \right)$$

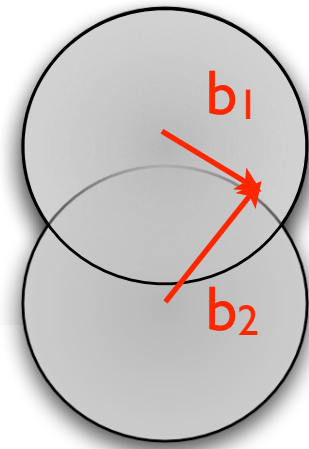
Need to know mean number of interactions  
as function of impact parameter

mean number of  
interactions for given  
impactparameter of  
collision

# Minijet model: underlying ideas (unitarization)



(side view)



(view long beam axis)

Overlap function

$$A(s, \vec{b}) = \int d^2\vec{b}_1 d^2\vec{b}_2 A_p(s, \vec{b}_1) A_p(s, \vec{b}_2) \delta(\vec{b} - \vec{b}_1 - \vec{b}_2)$$

Independent interactions:  
Poisson distribution

$$P_n = \frac{\langle n_{\text{hard}}(\vec{b}) \rangle^n}{n!} \exp\left(-\langle n_{\text{hard}}(\vec{b}) \rangle\right)$$

$$\langle n_{\text{hard}}(\vec{b}) \rangle = \sigma_{\text{QCD}} A(s, \vec{b})$$

$$\sigma_{\text{ine}} = \int d^2\vec{b} \sum_{n=1}^{\infty} P_n = \int d^2\vec{b} \left(1 - \exp\{-\sigma_{\text{QCD}} A(s, \vec{b})\}\right)$$

# Classic minijet model: free parameters

$$\sigma_{\text{ine}} = \int d^2\vec{b} \left( 1 - \exp \left\{ -\sigma_{\text{soft}} A_{\text{soft}}(s, \vec{b}) - \sigma_{\text{QCD}} A_{\text{hard}}(s, \vec{b}) \right\} \right)$$

- Soft cross section
- Transverse momentum cutoff for QCD cross section
- Soft and hard profile functions

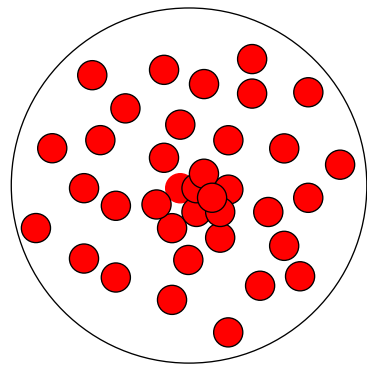
Classic Minijet Model (SIBYLL 1.7):

- Soft cross section const.
- Transverse momentum cutoff energy-independent
- Profile functions taken from form factor

$$A_{pp}(\mathbf{v}, \vec{B}) = \frac{\mathbf{v}^2}{96\pi} (\mathbf{v}|\vec{B}|)^3 K_3(\mathbf{v}|\vec{B}|)$$

$$A_{\pi p}(\mathbf{v}, \mu_\pi, \vec{B}) = \frac{1}{4\pi} \frac{\mathbf{v}^2 \mu_\pi^2}{\mu_\pi^2 - \mathbf{v}^2} \left( (\mathbf{v}|\vec{B}|) K_1(\mathbf{v}|\vec{B}|) - \frac{2\mathbf{v}^2}{\mu_\pi^2 - \mathbf{v}^2} \left[ K_0(\mathbf{v}|\vec{B}|) - K_0(\mu_\pi|\vec{B}|) \right] \right)$$

# Problem: Very high parton densities (saturation)

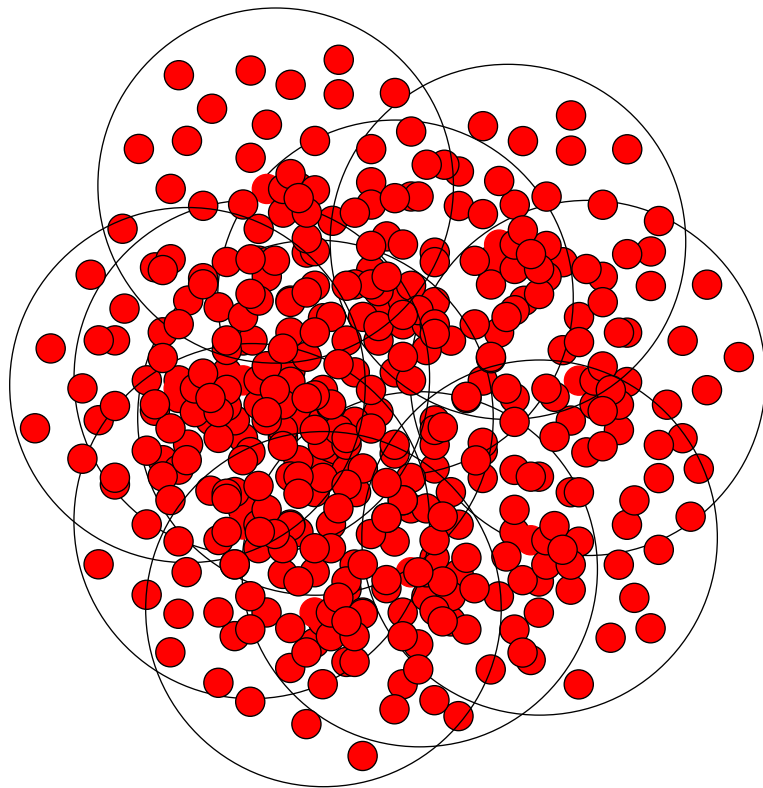


nucleon

## Saturation:

- parton wave functions overlap
- number of partons does not increase anymore at low x
- extrapolation to very high energy unclear

## Simple geometric criterion



nucleus

*RHIC data very important*

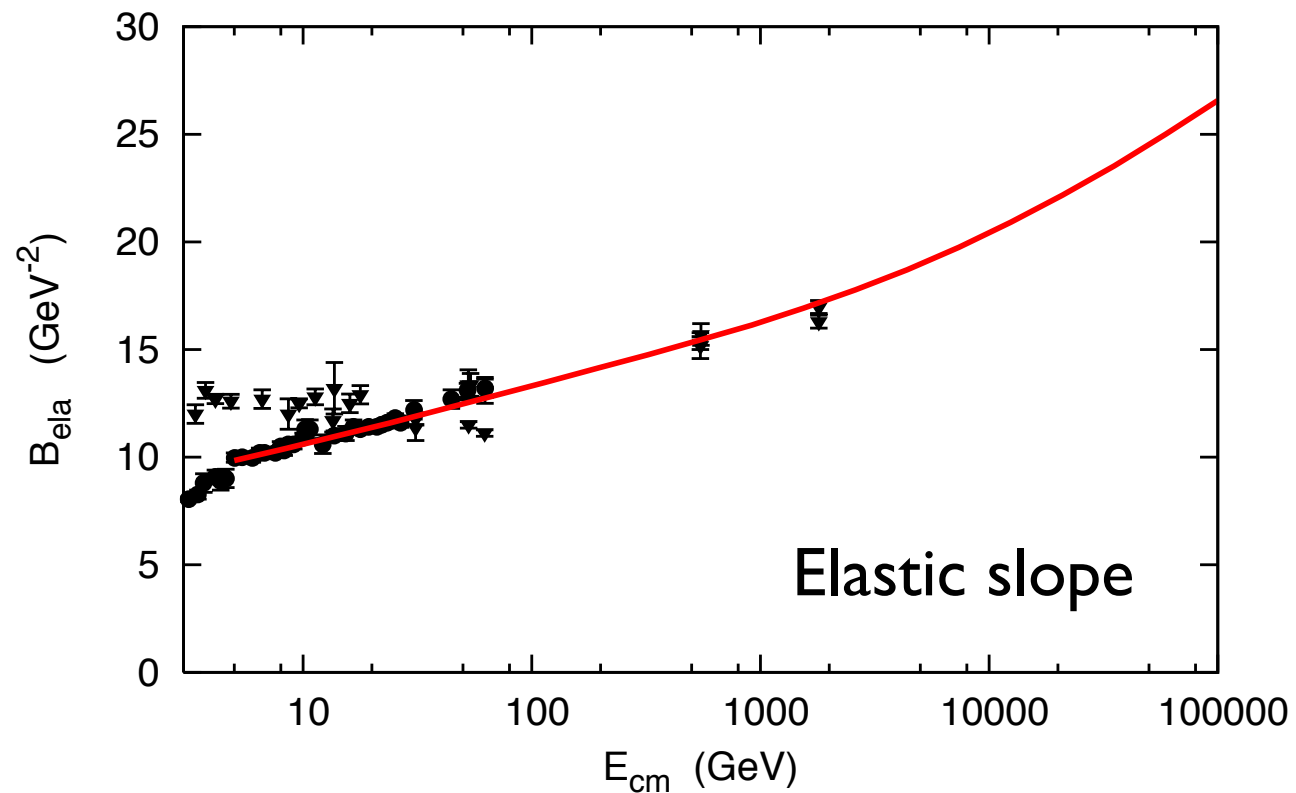
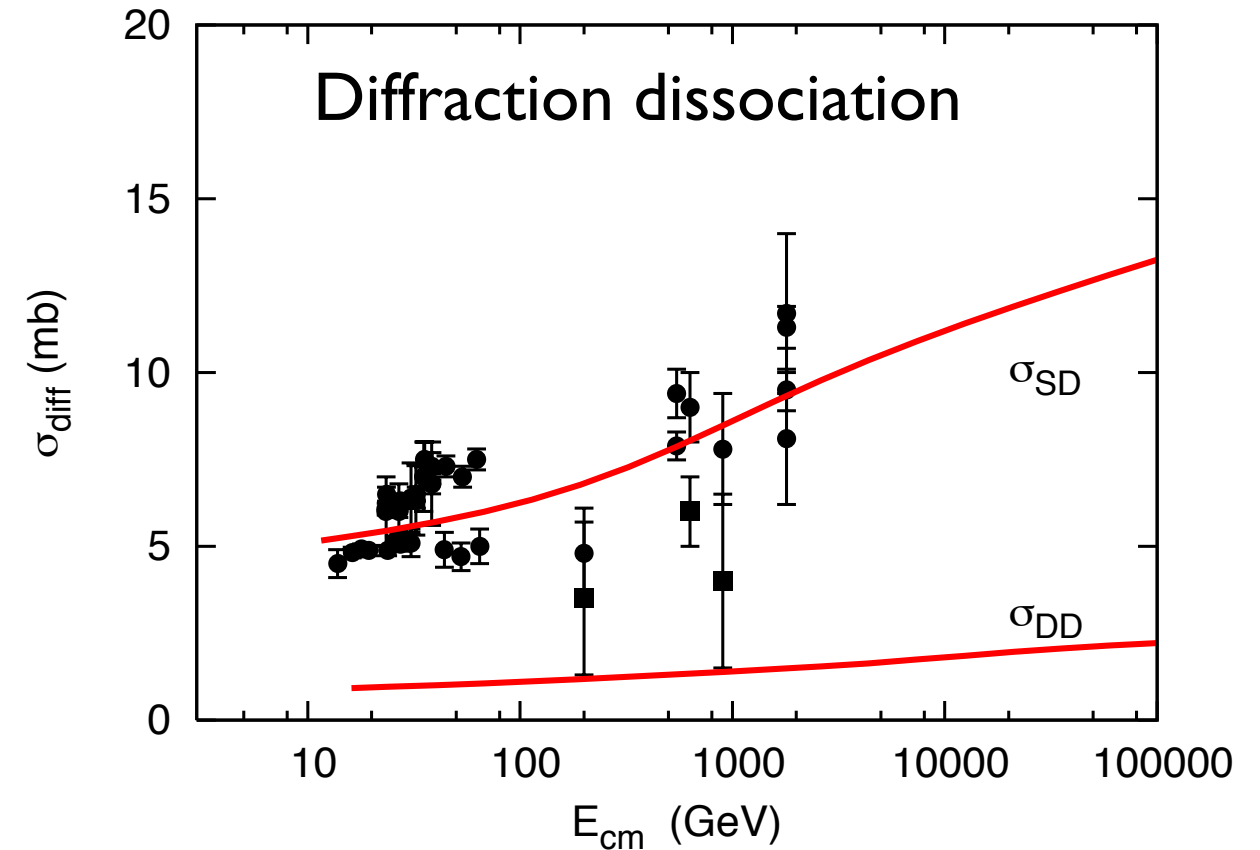
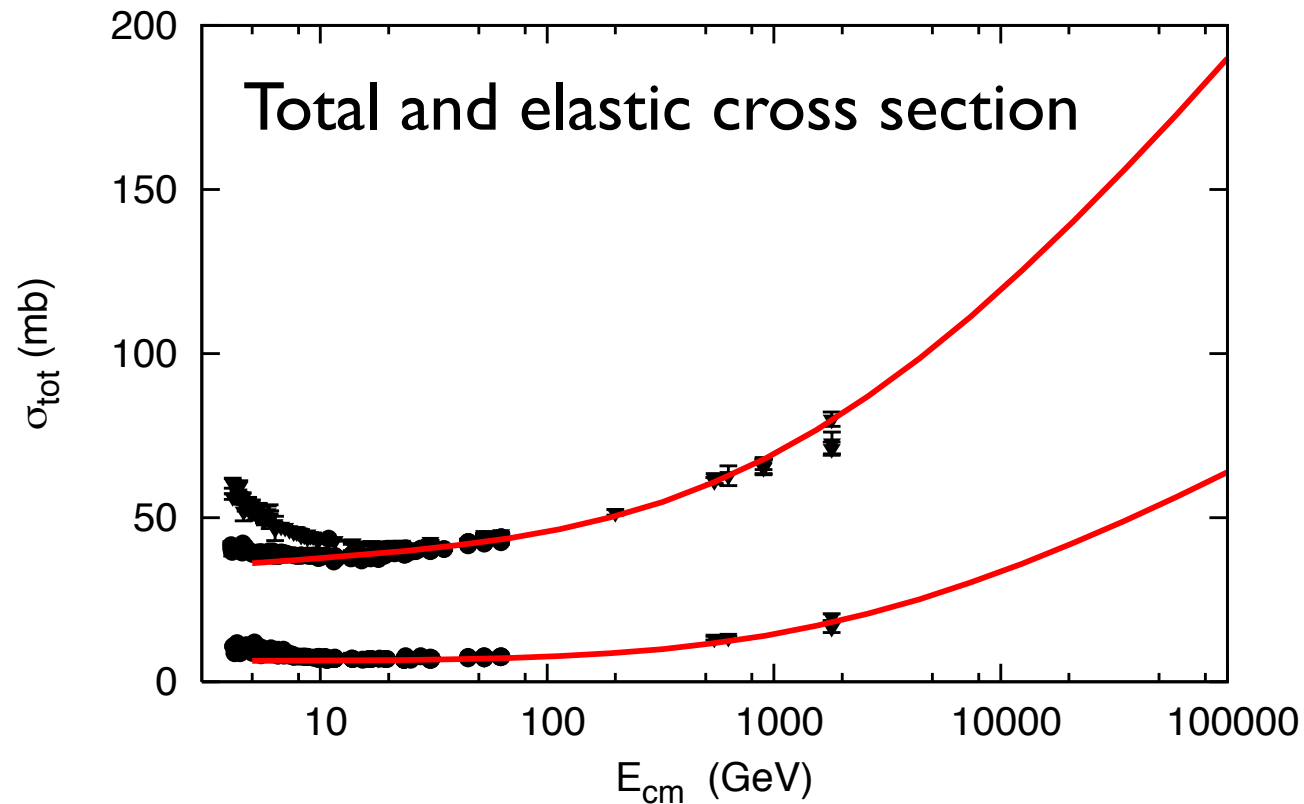
$$\pi R_0^2 \simeq \frac{\alpha_s(Q_s^2)}{Q_s^2} \cdot xg(x, Q_s^2)$$

size of proton

Size of  
one gluon

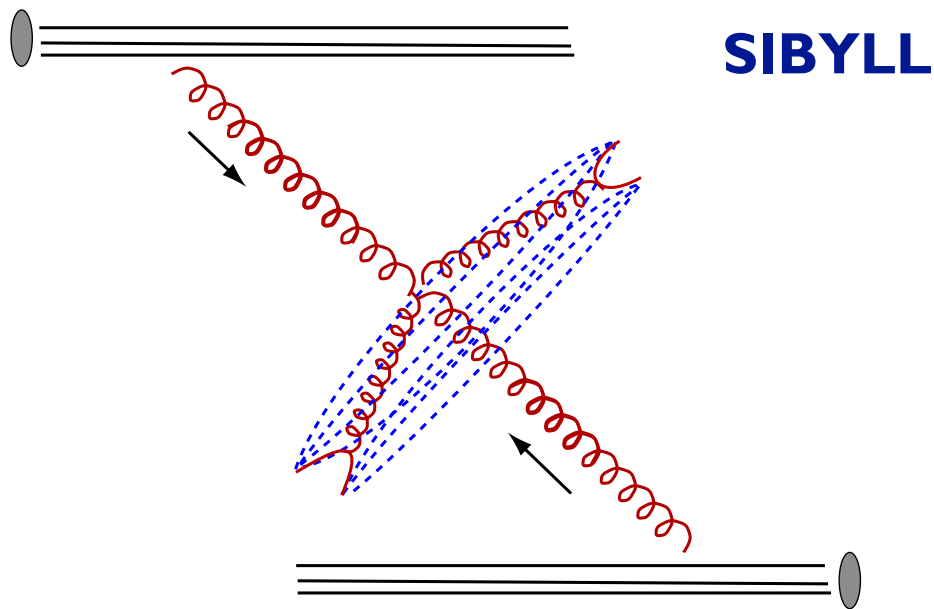
number of  
gluons

# SIBYLL 2.1 cross section fits



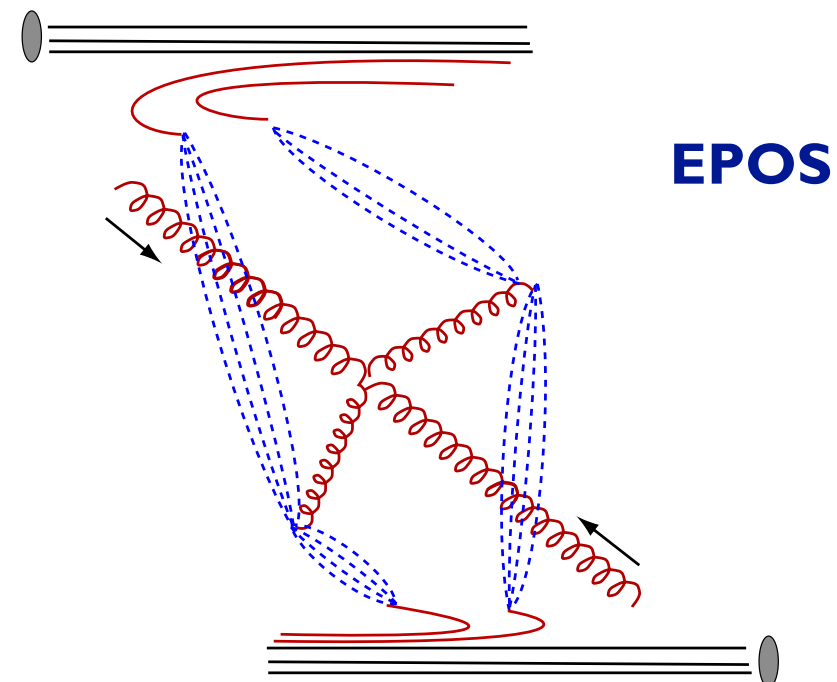
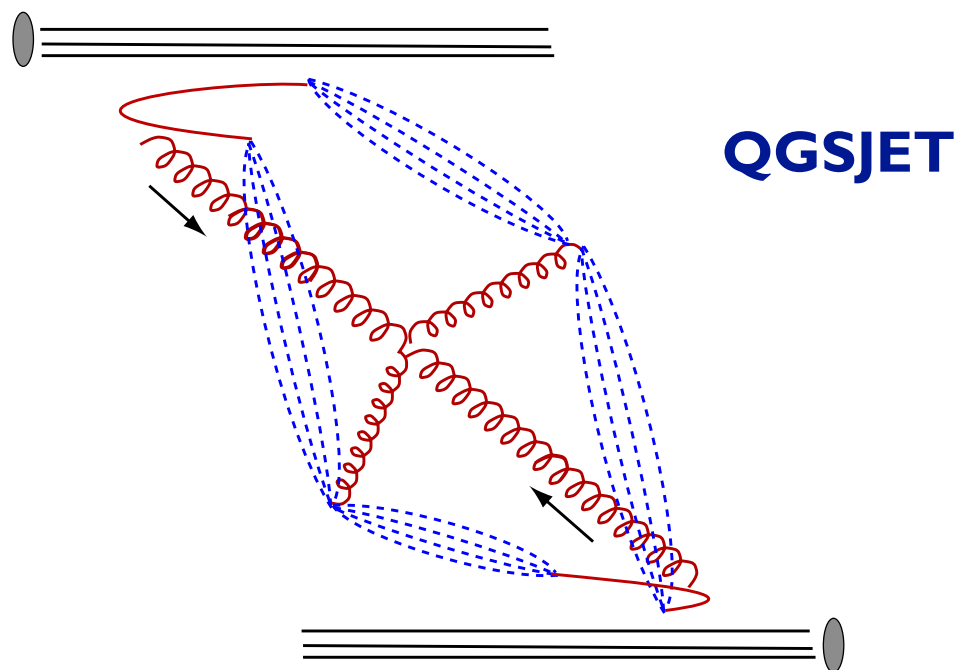
Low energy:  
parametrizations  
of data are used

# Different implementations of two-gluon scattering



Kinematics etc. given by parton densities and perturbative QCD

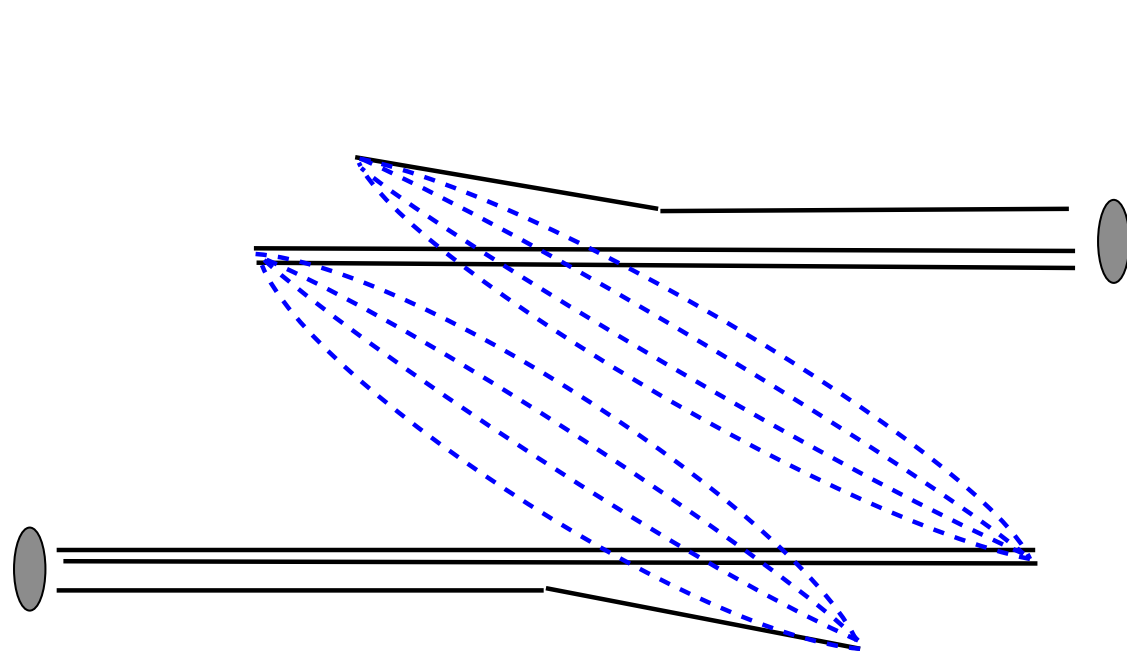
Two strings stretched between quark pairs from gluon fragmentation



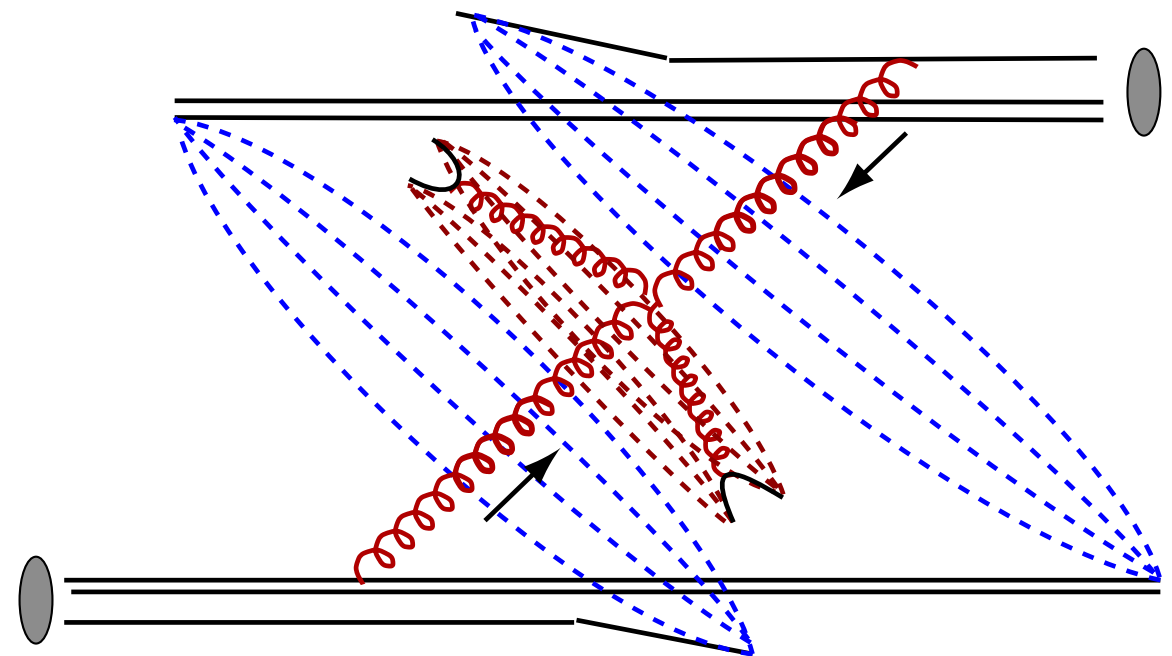
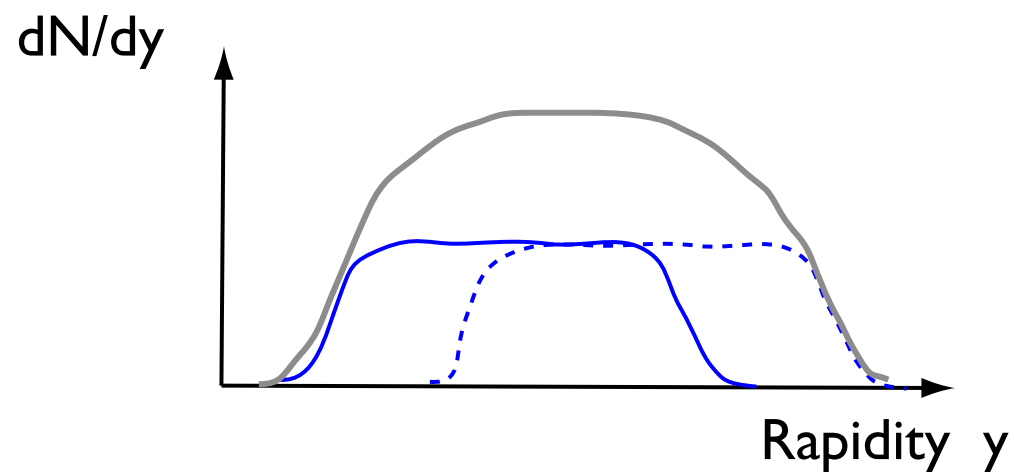


# Multiple soft and hard interactions

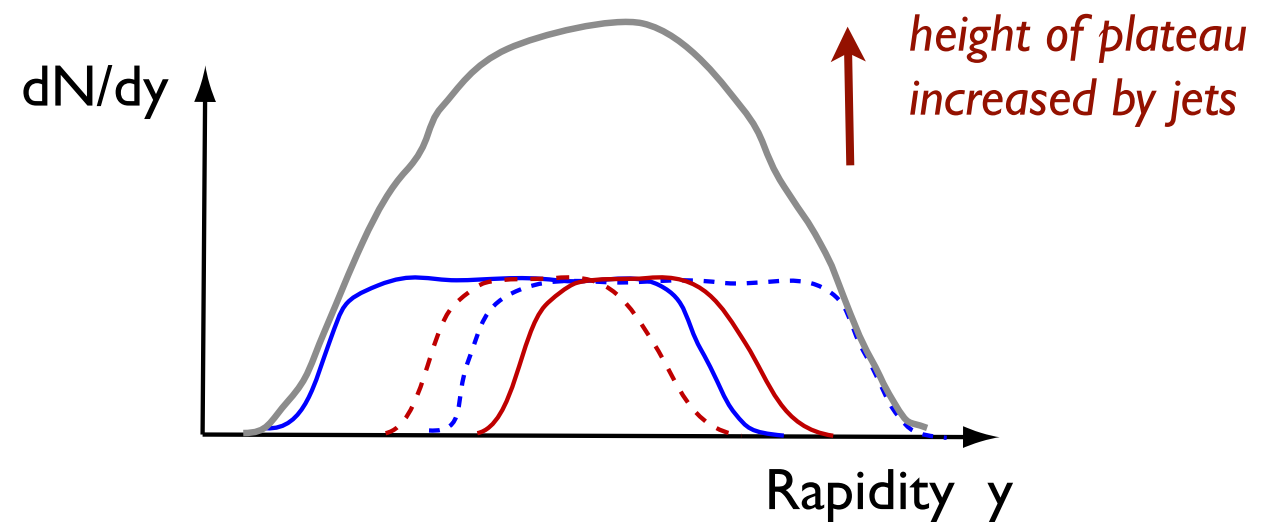
$$\sigma_{n_s, n_h} = \int d^2b \frac{[n_{\text{soft}}(b, s)]^{n_s}}{n_s!} \frac{[n_{\text{hard}}(b, s)]^{n_h}}{n_h!} e^{-n_{\text{hard}}(b, s) - n_{\text{soft}}(b, s)}$$



$n_s=1, n_h=0$



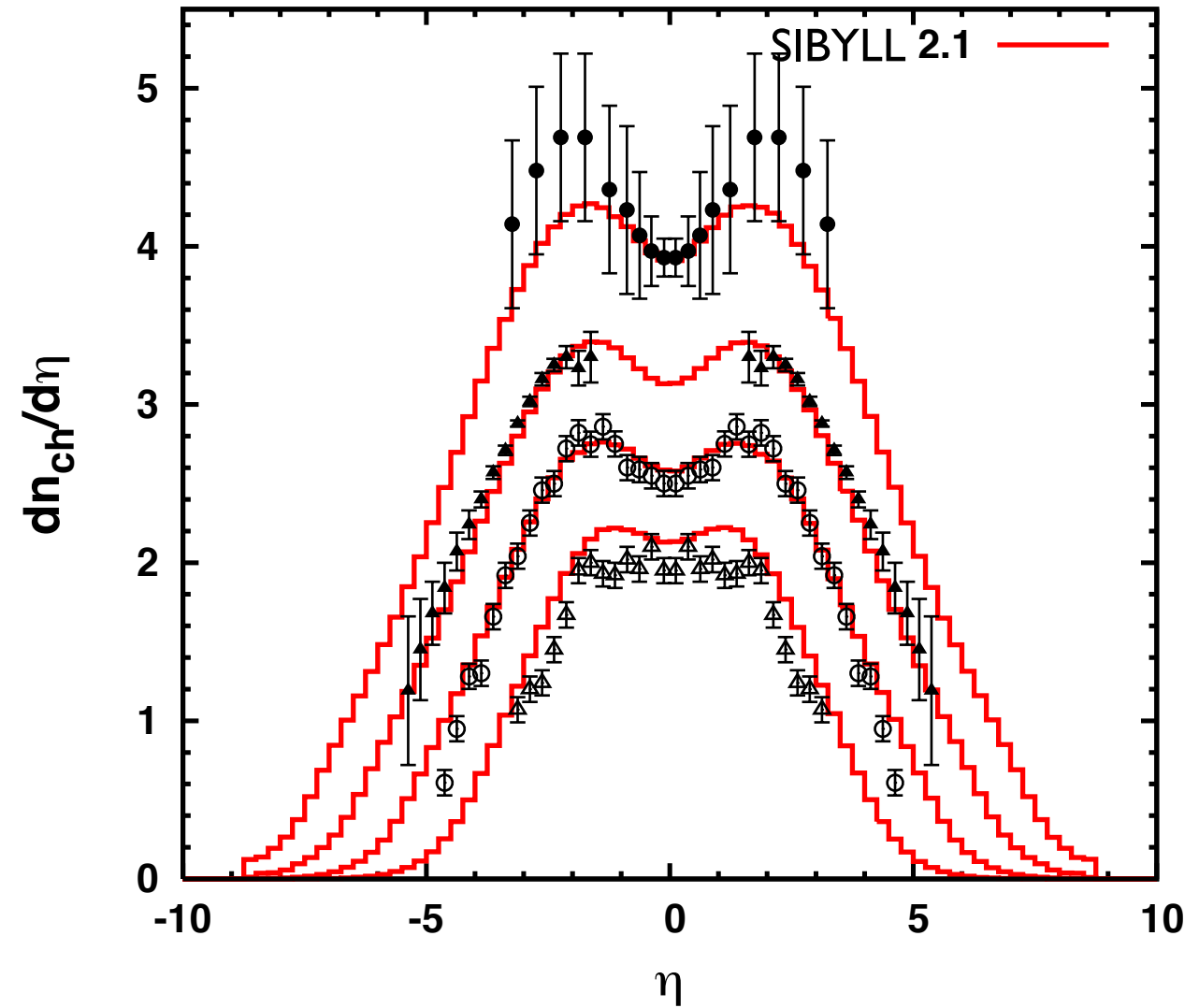
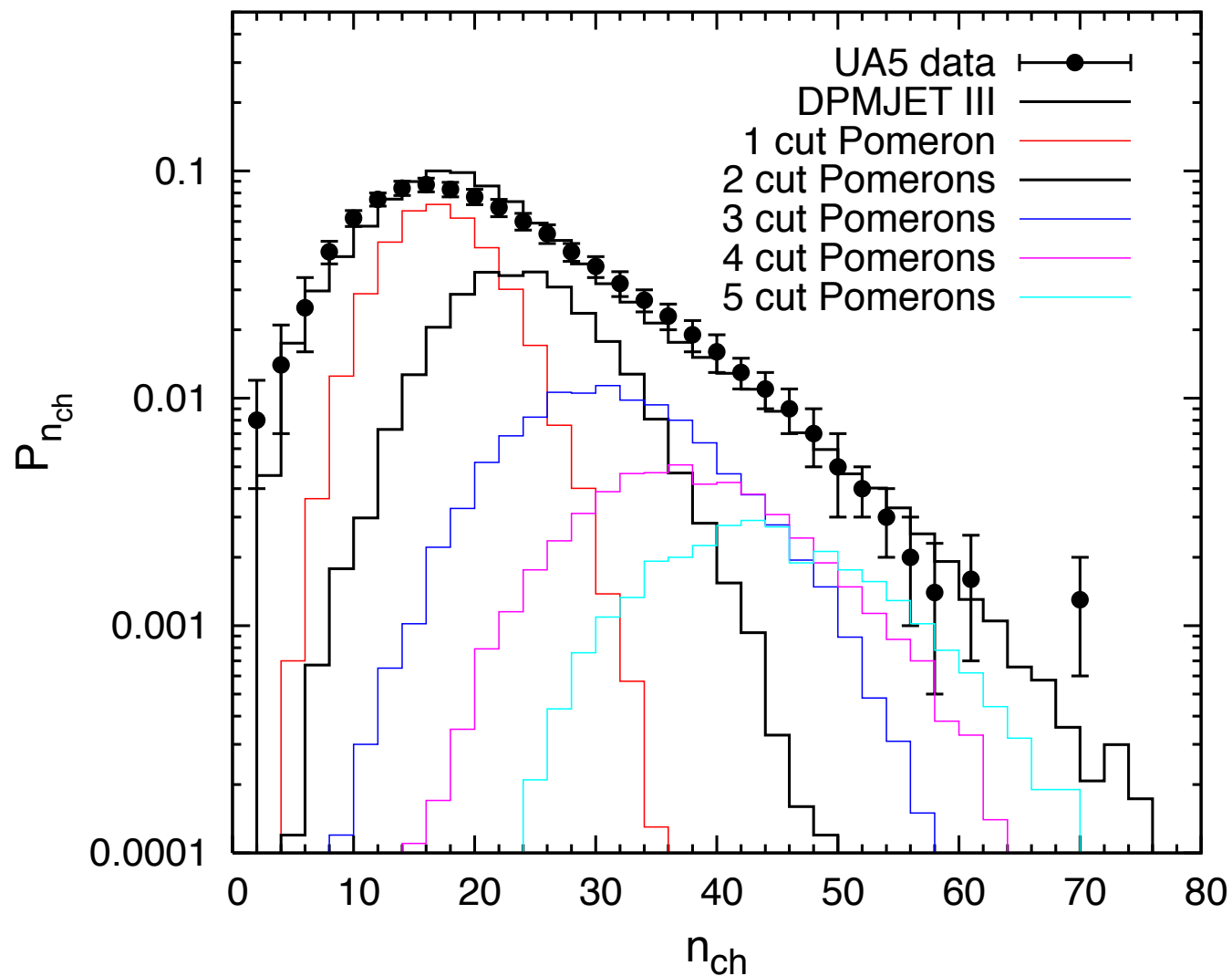
$n_s=1, n_h=1$



# Comparison with collider data

Note: one cut pomeron means  
one soft or hard interaction

Charged particle multiplicity  
distribution at 200 GeV cms.



Charged particle  
pseudorapidity distributions

# Comparison of high energy interaction models

## **DPMJET II.5 and III**

*(Ranft / Roesler, RE, Ranft, Bopp)*

- universal model
- saturation for hard partons via geometry criterion
- HERA parton densities

## **EPOS**

*(Pierog, Werner)*

- universal model
- saturation by RHIC data parametrizations
- custom-developed parton densities

## **QGSJET 01**

*(Kalmykov, Ostapchenko)*

- no saturation corrections
- old pre-HERA parton densities
- replaced by QGSJET II

## **QGSJET II.03**

*(Ostapchenko)*

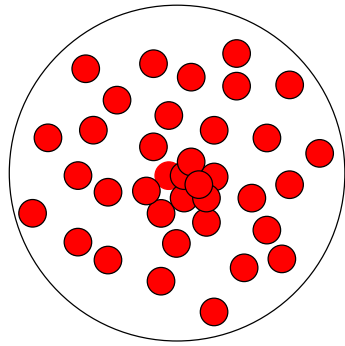
- saturation correction for soft partons via pomeron-resummation
- custom-developed parton densities

## **SIBYLL 2.1**

*(Engel, RE, Fletcher, Gaisser, Lipari, Stanev)*

- saturation for hard partons via geometry criterion
- HERA parton densities

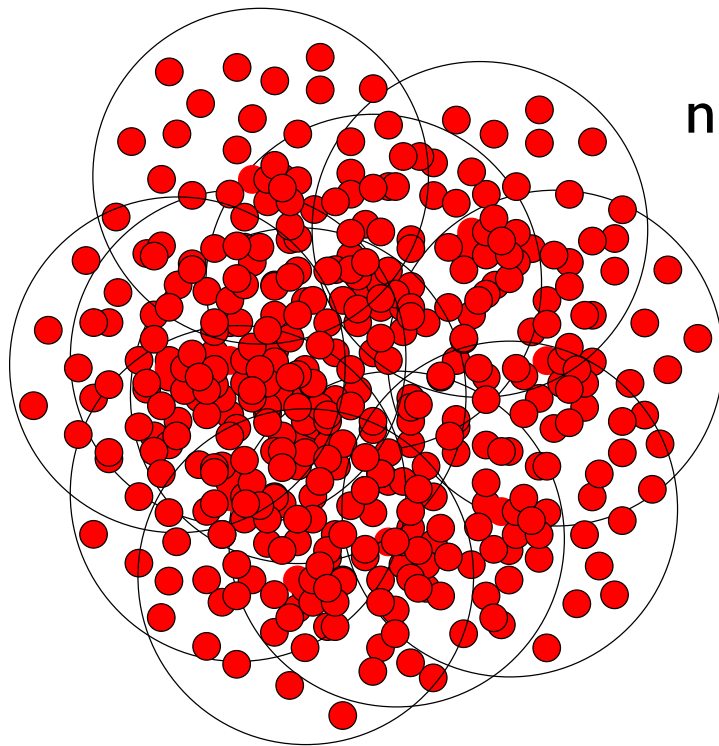
# High parton densities: modification of minijet threshold



nucleon

SIBYLL: simple geometric criterion

$$\pi R_0^2 \simeq \frac{\alpha_s(Q_s^2)}{Q_s^2} \cdot xg(x, Q_s^2)$$



nucleus

$$xg(x, Q^2) \sim \exp \left[ \frac{48}{11 - \frac{2}{3}n_f} \ln \frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{Q_0^2}{\Lambda^2}} \ln \frac{1}{x} \right]^{\frac{1}{2}}$$

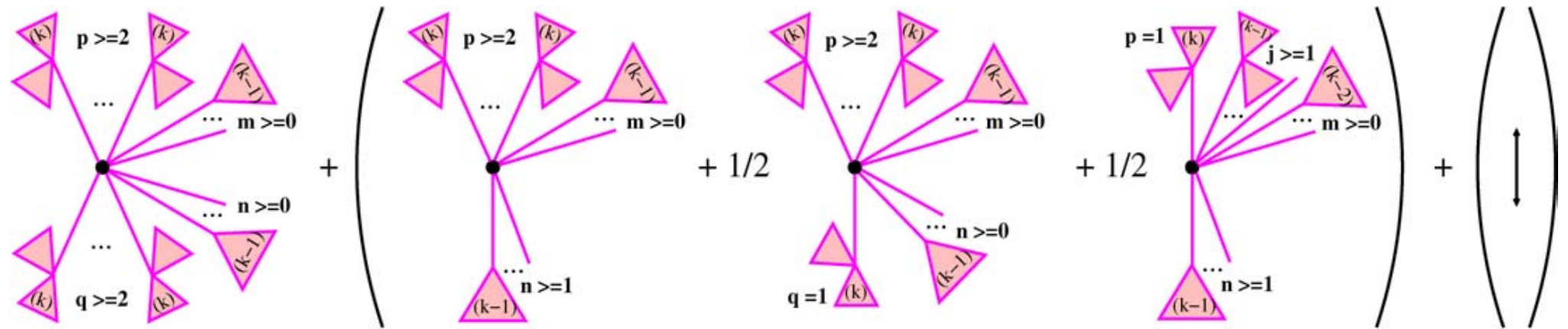
No dependence on  
impact parameter !

SIBYLL:  $p_{\perp}(s) = p_{\perp}^0 + 0.065 \text{GeV} \exp \left\{ 0.9 \sqrt{\ln s} \right\}$

DPMJET:  $p_{\perp}(s) = p_{\perp}^0 + 0.12 \text{GeV} \left( \log_{10} \frac{\sqrt{s}}{50 \text{GeV}} \right)^3$

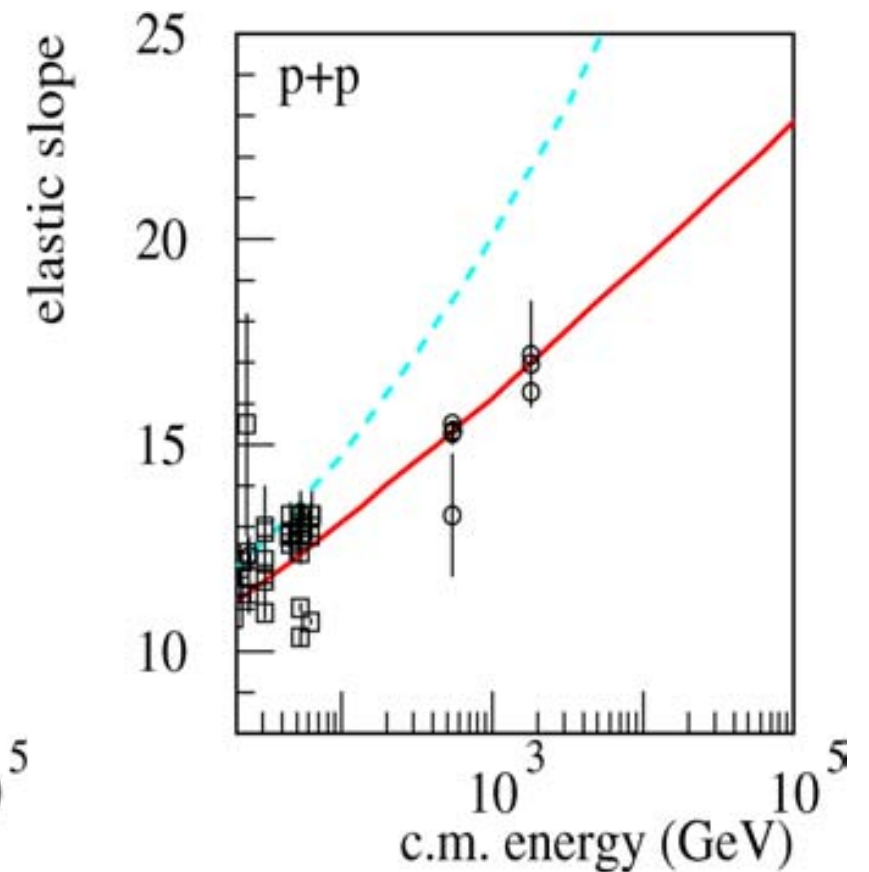
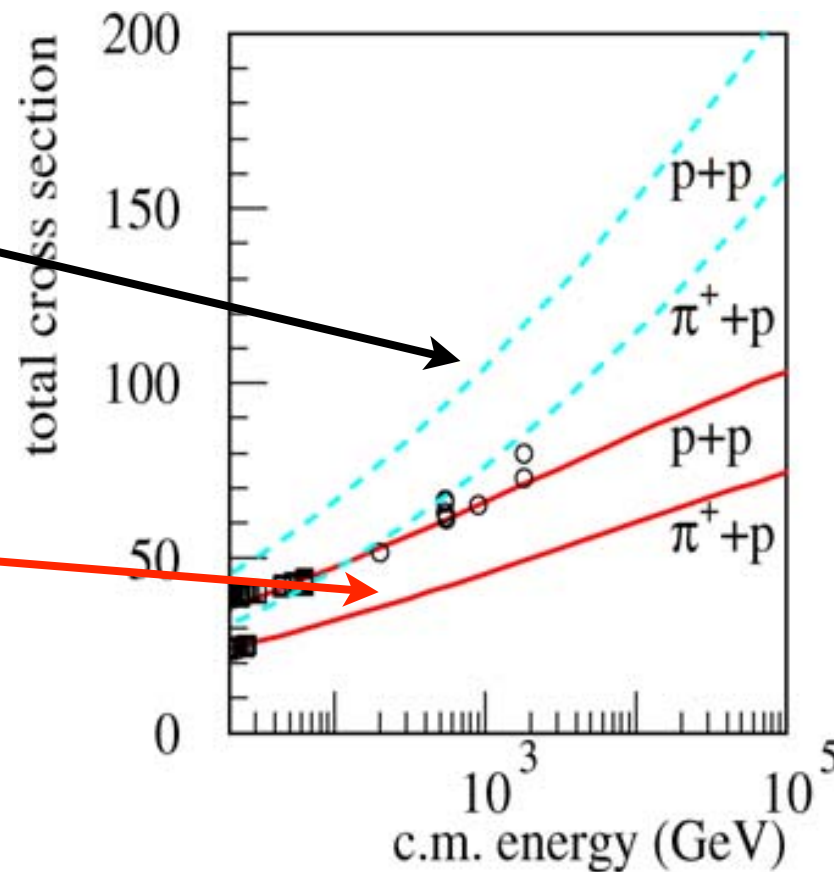
# QGSJET II: high parton density effects

Re-summation of enhanced pomeron graphs

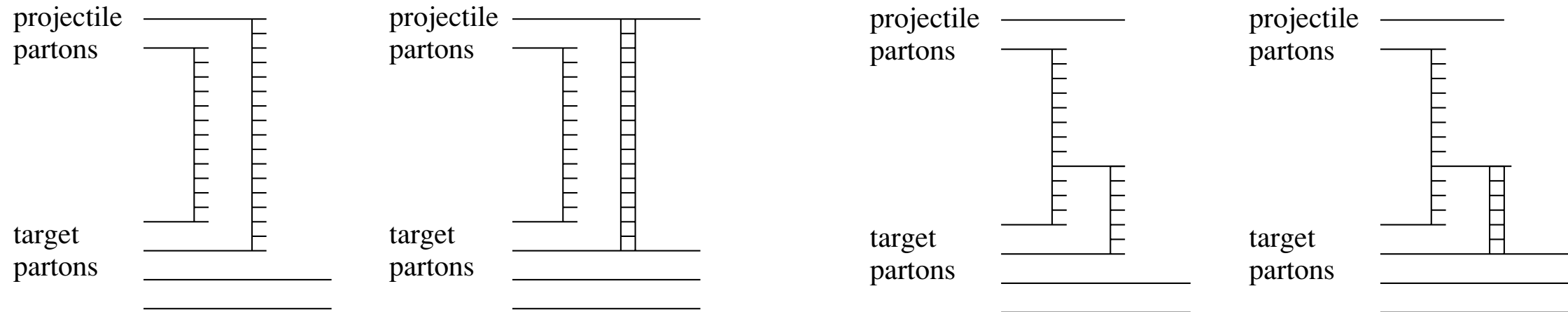


Without enhanced graphs

With enhanced graphs



# EPOS 1.6x – high parton density effects (i)



No effective coupling

$$A_{\text{pom}} \sim (x_1 x_2)^\beta$$

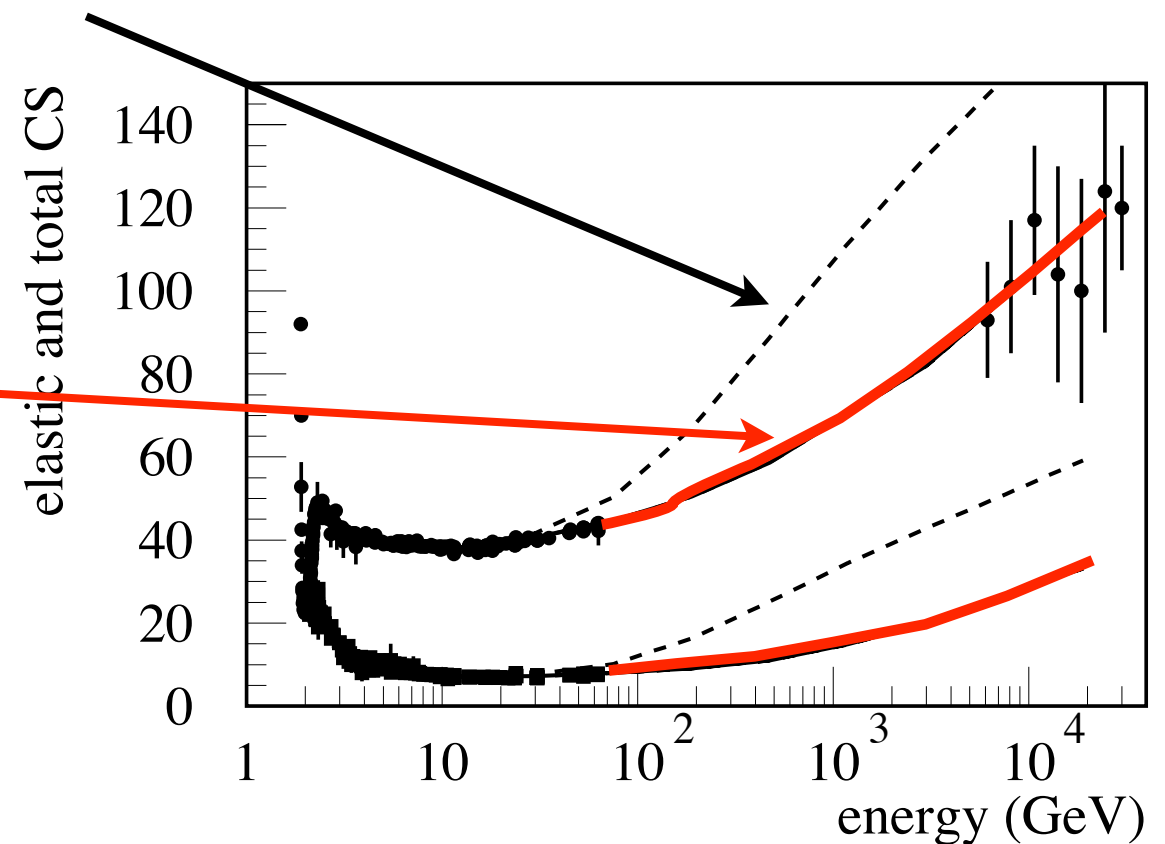
With effective coupling

$$A_{\text{pom}} \sim x_1^\beta x_2^{\beta - \varepsilon}$$

Parametrization

$$\varepsilon_S = a_S \beta_S Z,$$

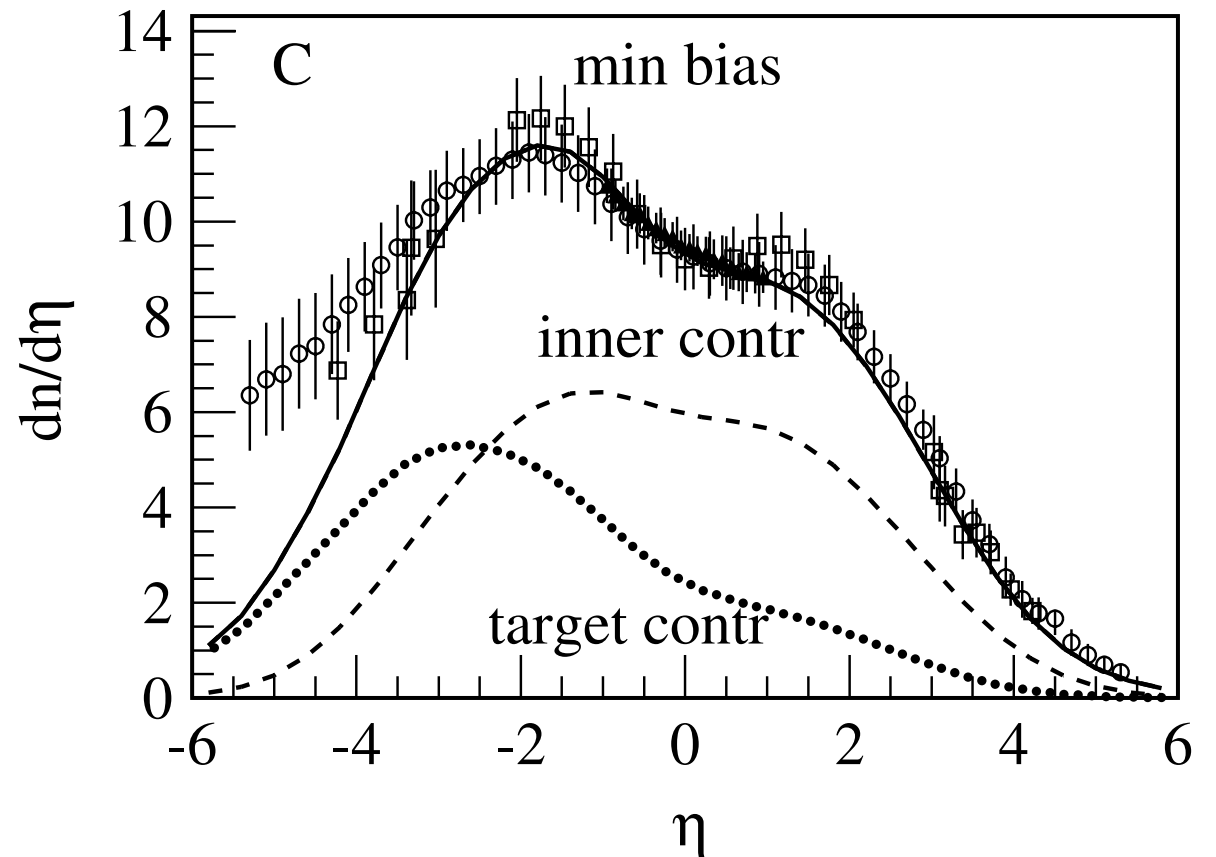
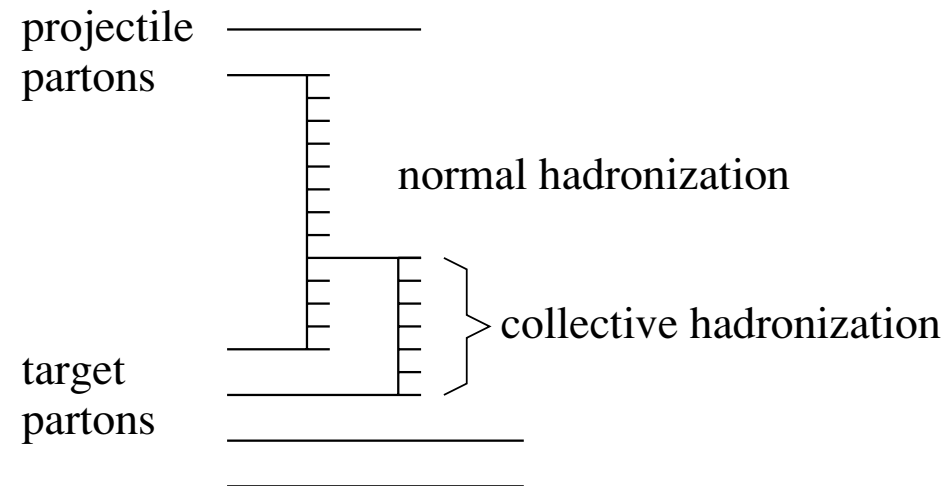
$$\varepsilon_H = a_H \beta_H Z,$$



(Werner et al., PRC 2006)

# EPOS 1.6x – high parton density effects (ii)

(Werner et al., PRC 2006)



| Coefficient | Corresponding variable             | Value                |
|-------------|------------------------------------|----------------------|
| $s_M$       | Minimum squared screening energy   | $(25 \text{ GeV})^2$ |
| $w_M$       | Defines minimum for $z'_0$         | 6.000                |
| $w_Z$       | Global Z coefficient               | 0.080                |
| $w_B$       | Impact parameter width coefficient | 1.160                |
| $a_S$       | Soft screening exponent            | 2.000                |
| $a_H$       | Hard screening exponent            | 1.000                |
| $a_T$       | Transverse momentum transport      | 0.025                |
| $a_B$       | Break parameter                    | 0.070                |
| $a_D$       | Diquark break probability          | 0.110                |
| $a_S$       | Strange break probability          | 0.140                |
| $a_P$       | Average break transverse momentum  | 0.150                |

$$Z_T(i, j) = z_0 \exp(-b_{ij}^2/2b_0^2) + \sum_{\substack{\text{target nucleons} \\ j' \neq j}} z'_0 \exp(-b_{ij'}^2/2b_0^2),$$

EPOS 1.99: see Tanguy Pierog's lecture

$$b_0 = w_B \sqrt{\sigma_{\text{inel}pp}/\pi} \quad z_0 = w_Z \log s/s_M,$$

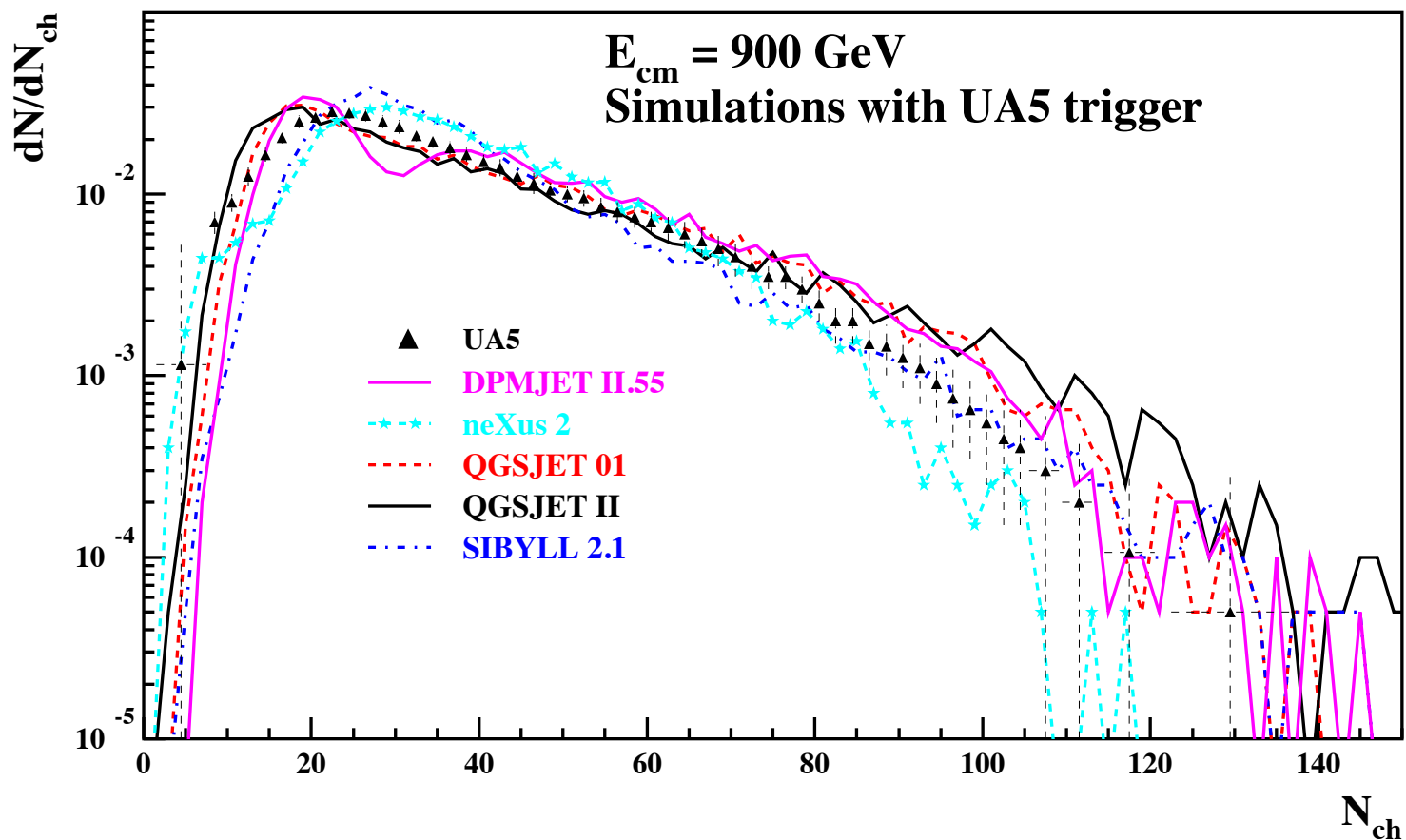
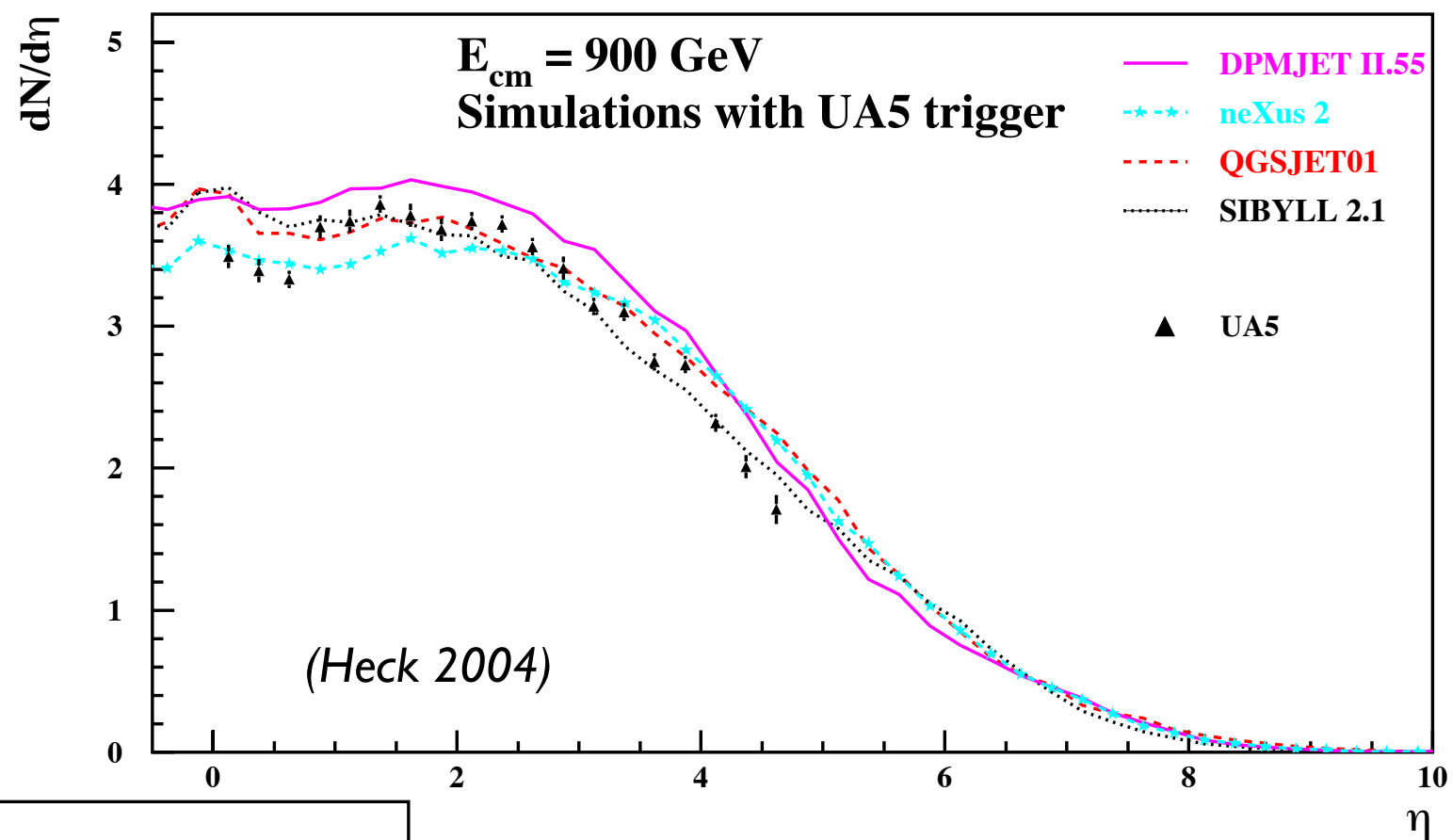
$$z'_0 = w_Z \sqrt{(\log s/s_M)^2 + w_M^2},$$

# Comparison of model predictions

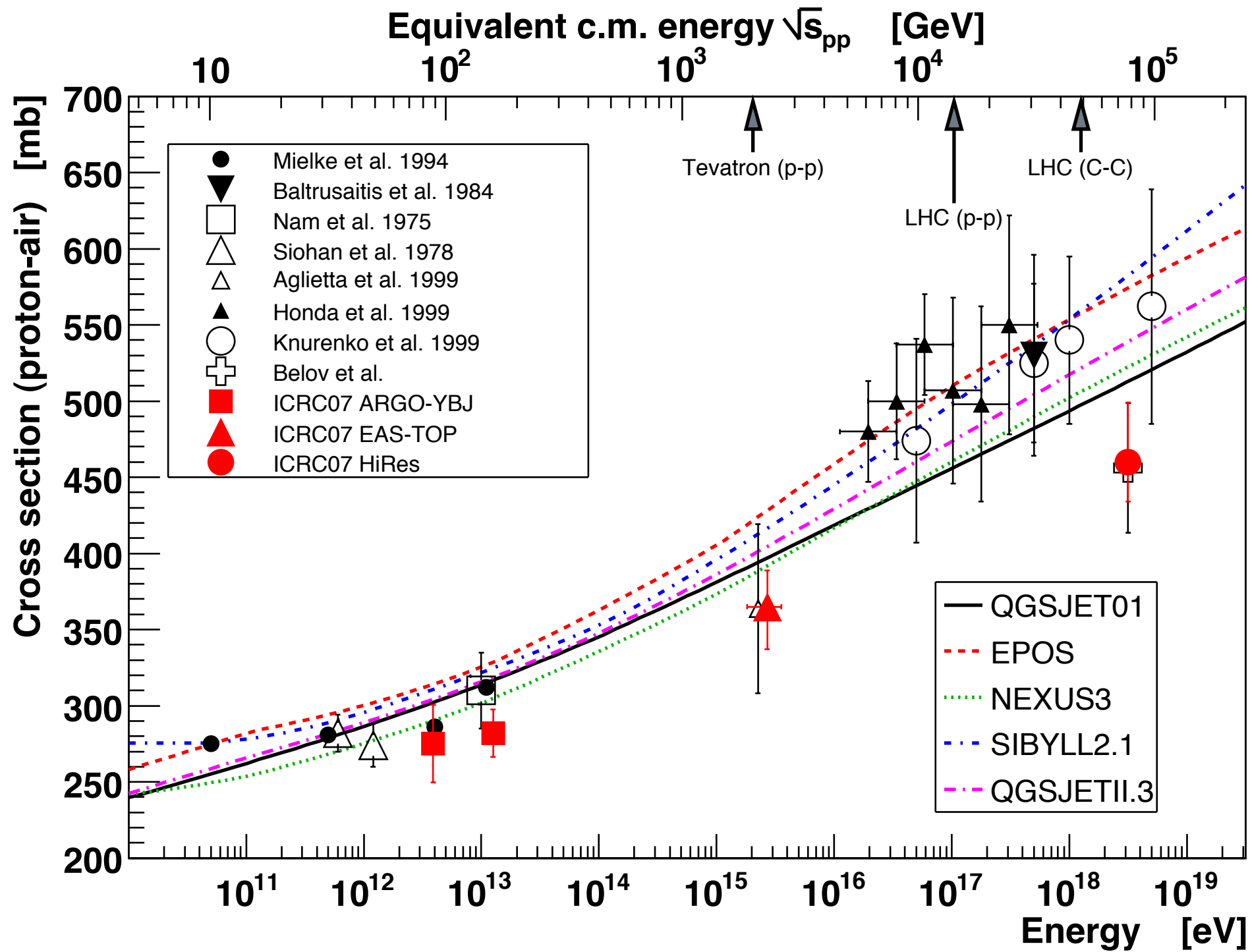


# Collider distributions: reasonable agreement

Many updated comparisons of models and data available, see talk by Tanguy Pierog



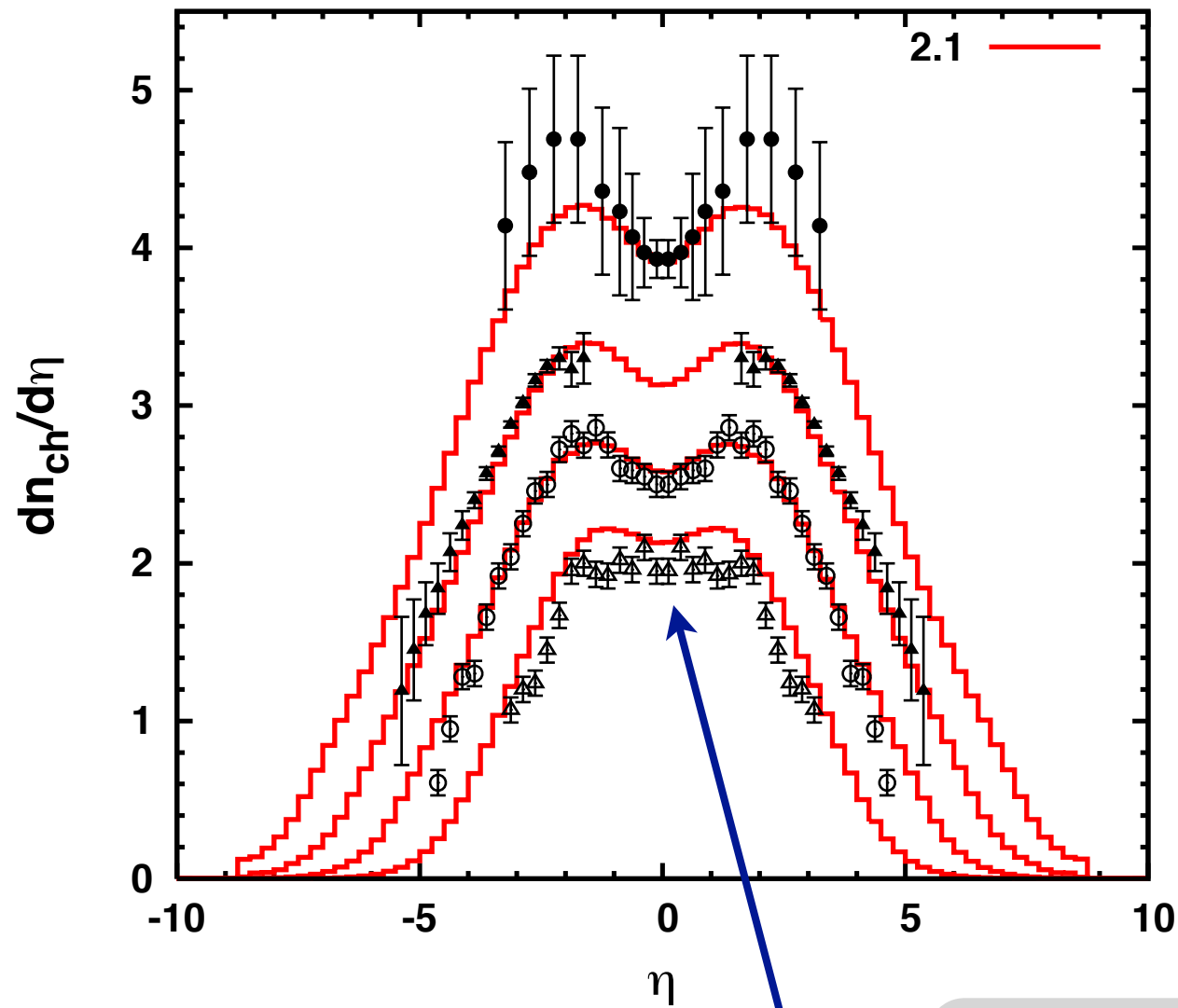
# Particle production cross section (p-air)



Differences stem from different effective profile functions and saturation models

Cross section extrapolations very similar

# Violation of Feynman scaling



Feynman scaling

$$2E \frac{dN}{d^3 p} = \frac{dN}{dy d^2 p_{\perp}} \longrightarrow f(x_F, p_{\perp})$$

With Feynman scaling:

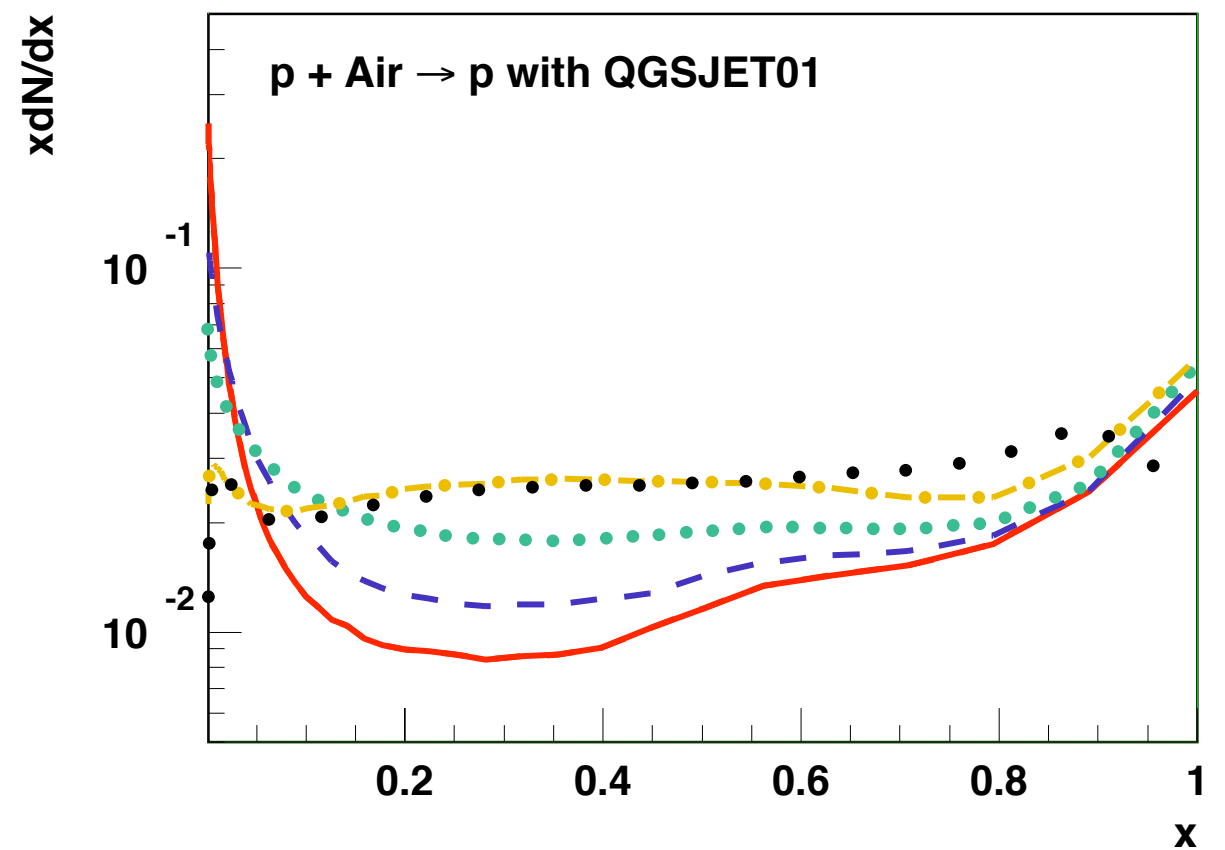
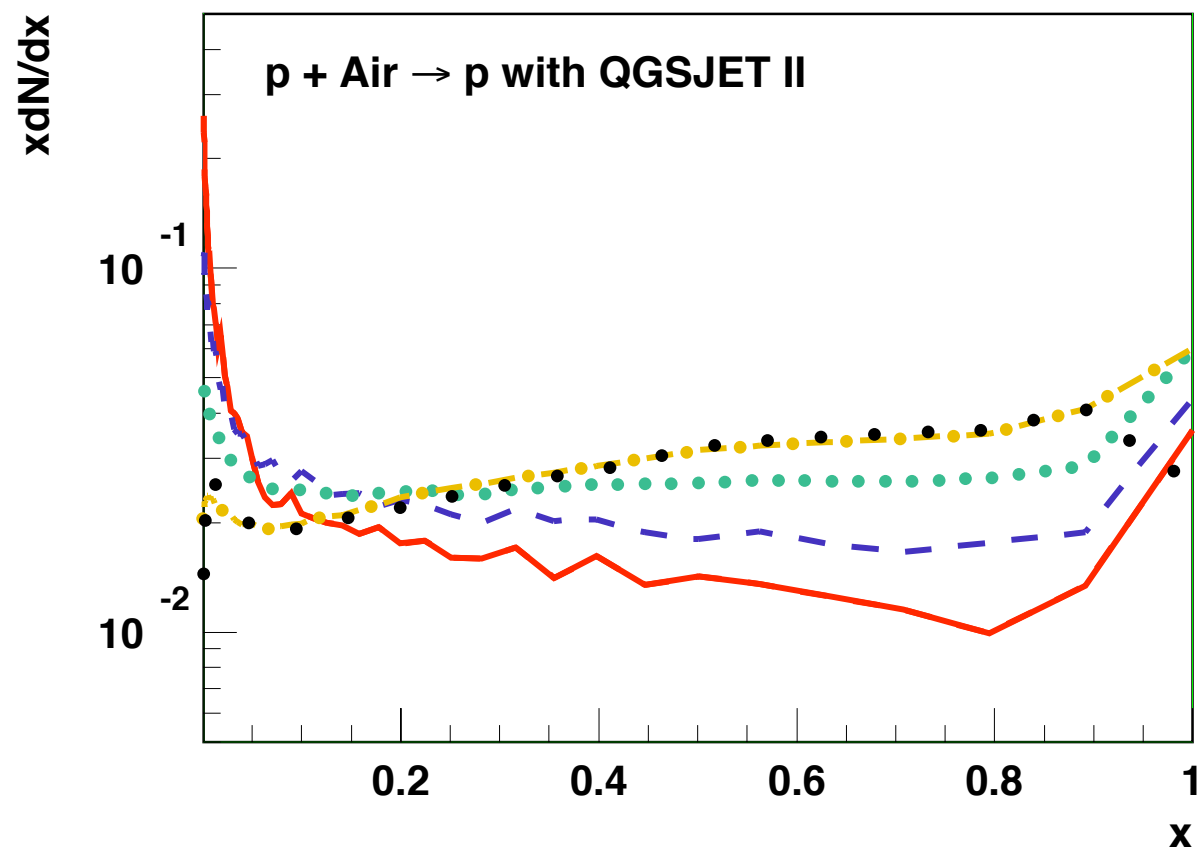
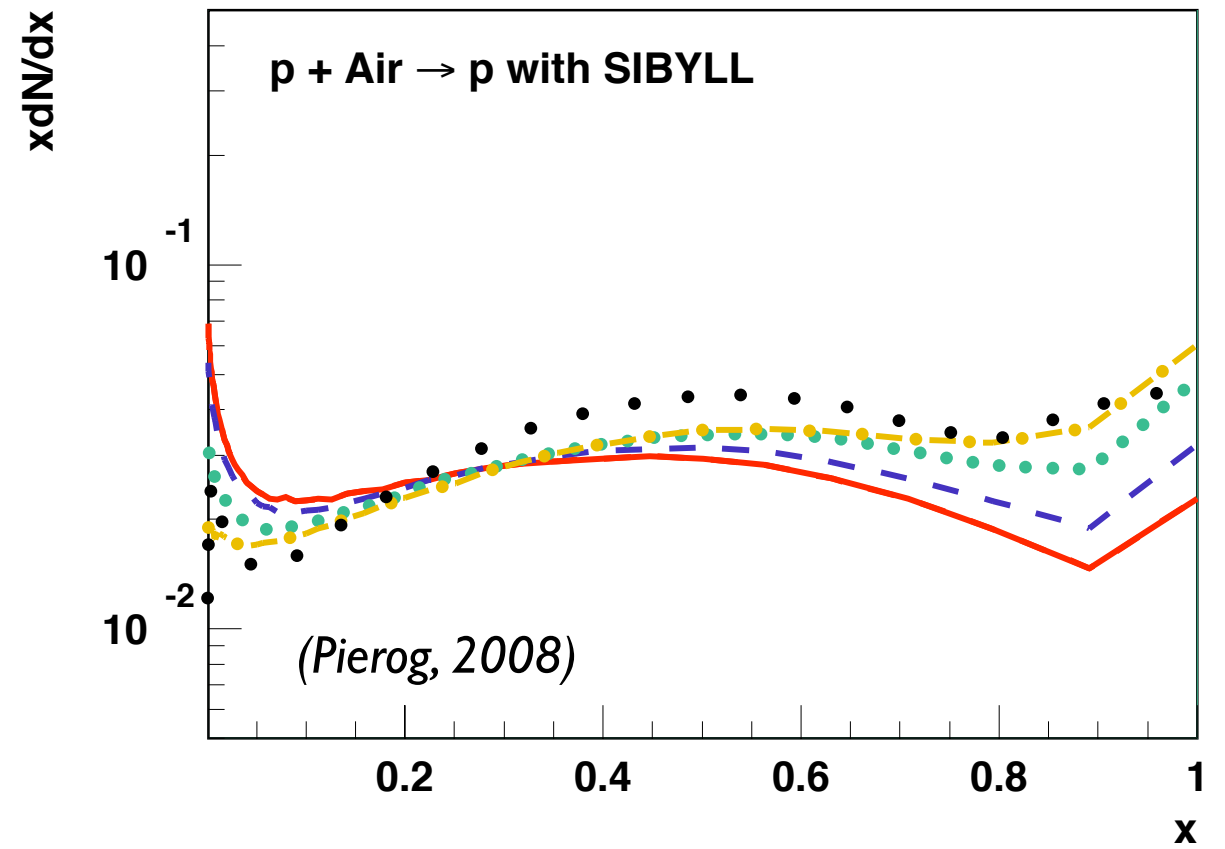
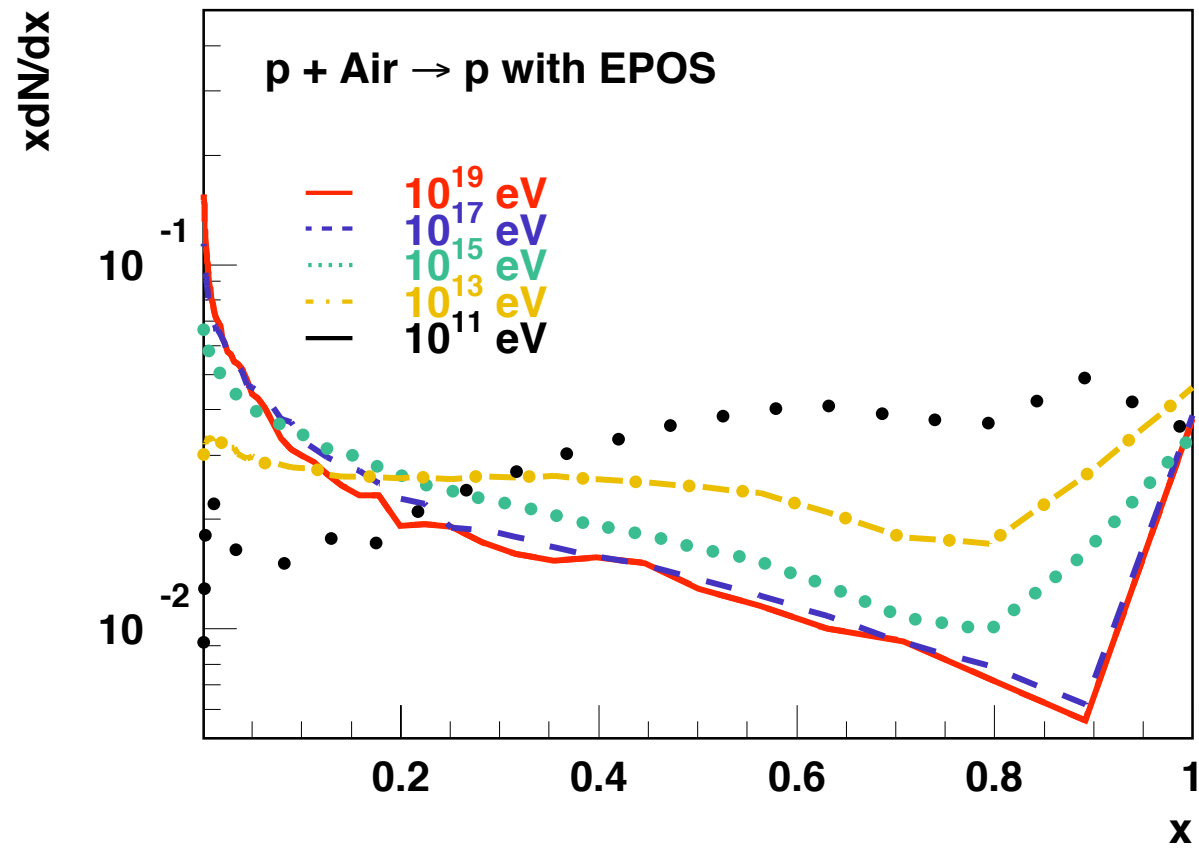
distribution independent of energy

$$\frac{dN}{dx} \approx \tilde{f}(x) \quad x = E/E_{\text{prim}}$$

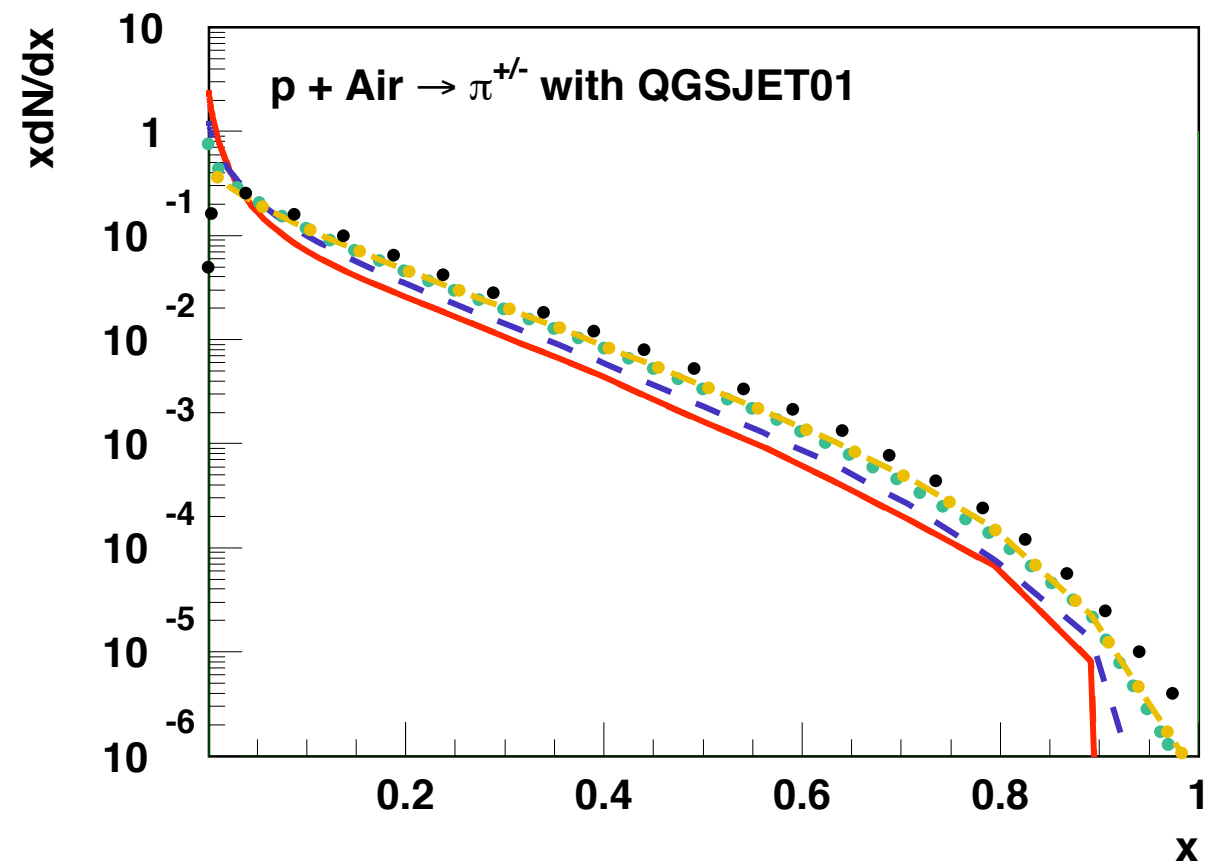
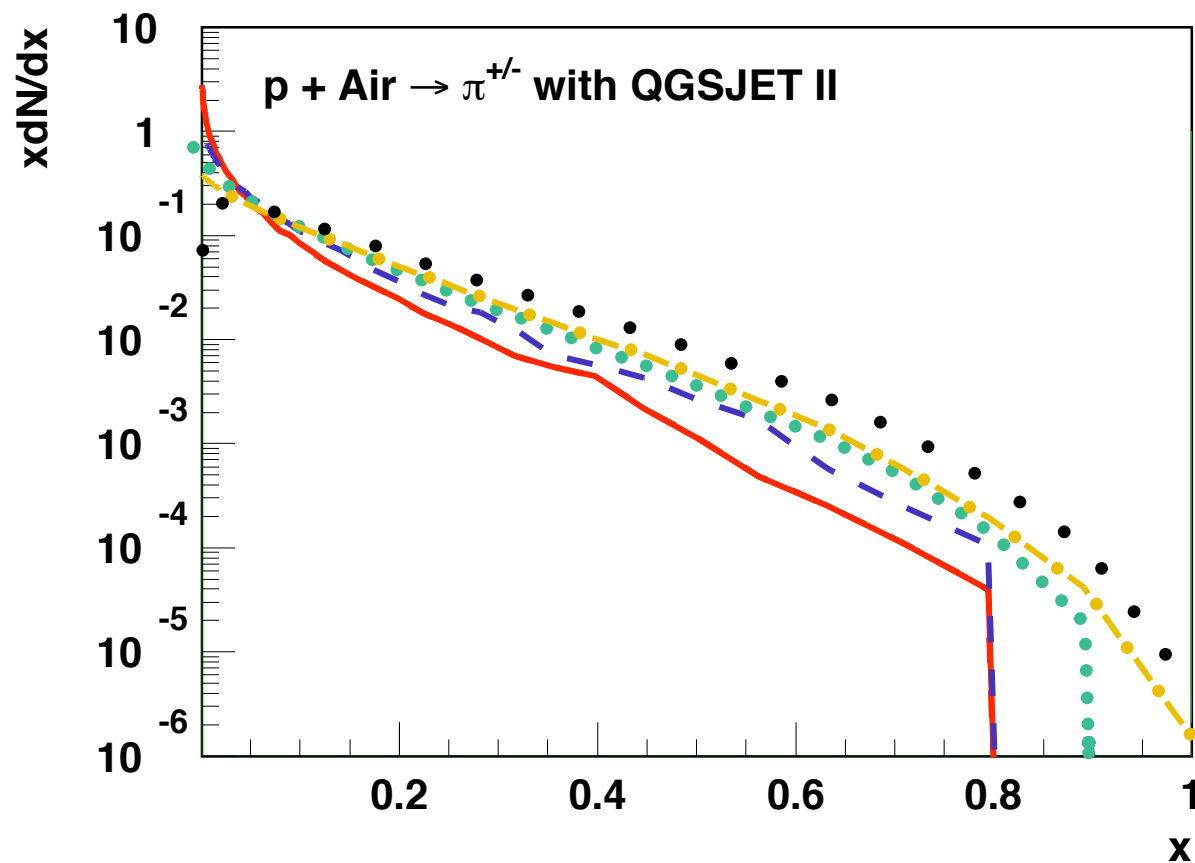
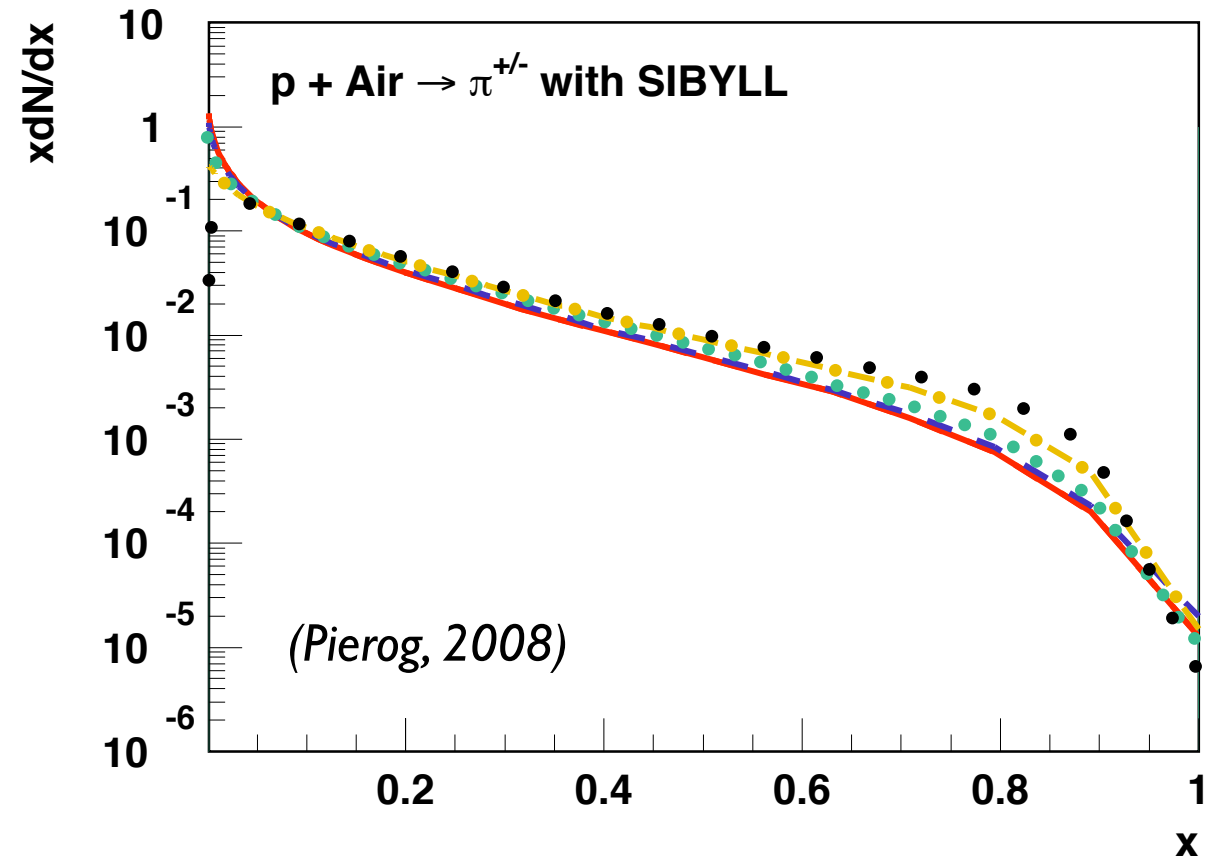
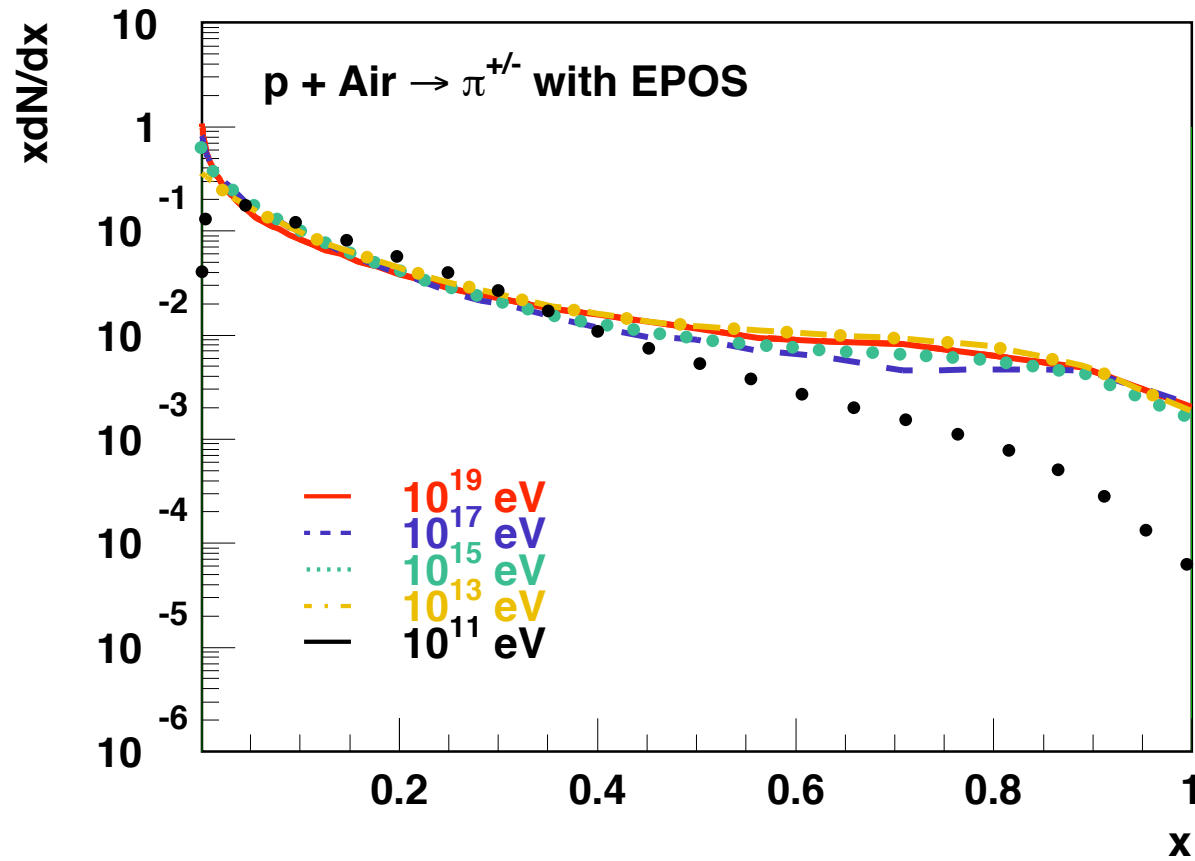
Feynman scaling violated for small  $|x_F|$

$$\frac{dN}{dy d^2 p_{\perp}} \approx \frac{dN}{dy} g(p_{\perp}^2)$$

# Scaling: model predictions (i)

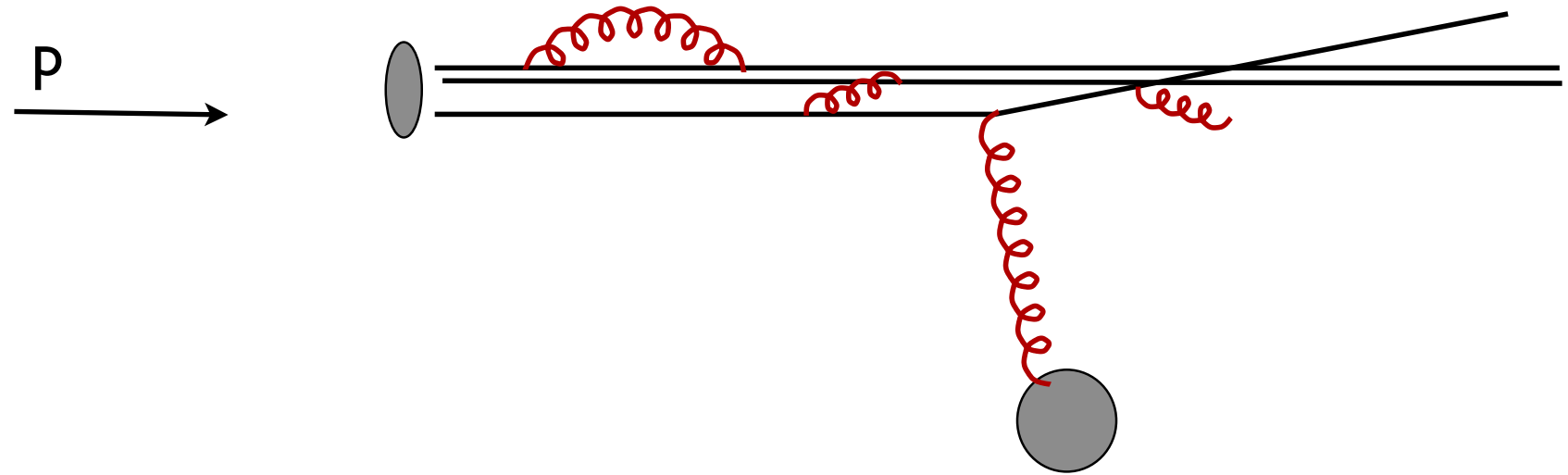


# Scaling: model predictions (ii)



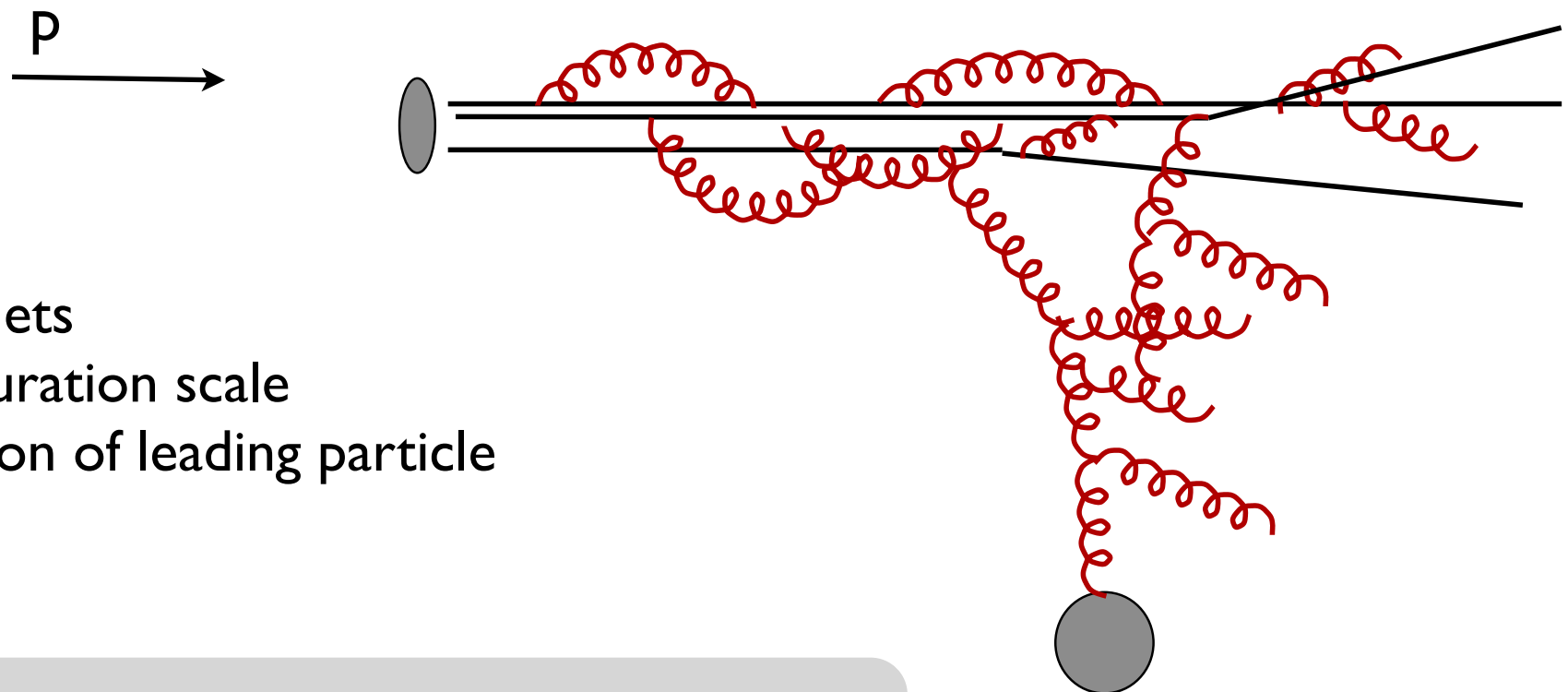
# Black disk scenario of high energy scattering ?

High energy scattering



## Black Disk Model

- large number of minijets
- high perturbative saturation scale
- complete disintegration of leading particle



Not implemented as dominating process in current models

(Drescher et al. PRL 94, 2005)

# Some conclusions

## Minijet production changes characteristics of interactions

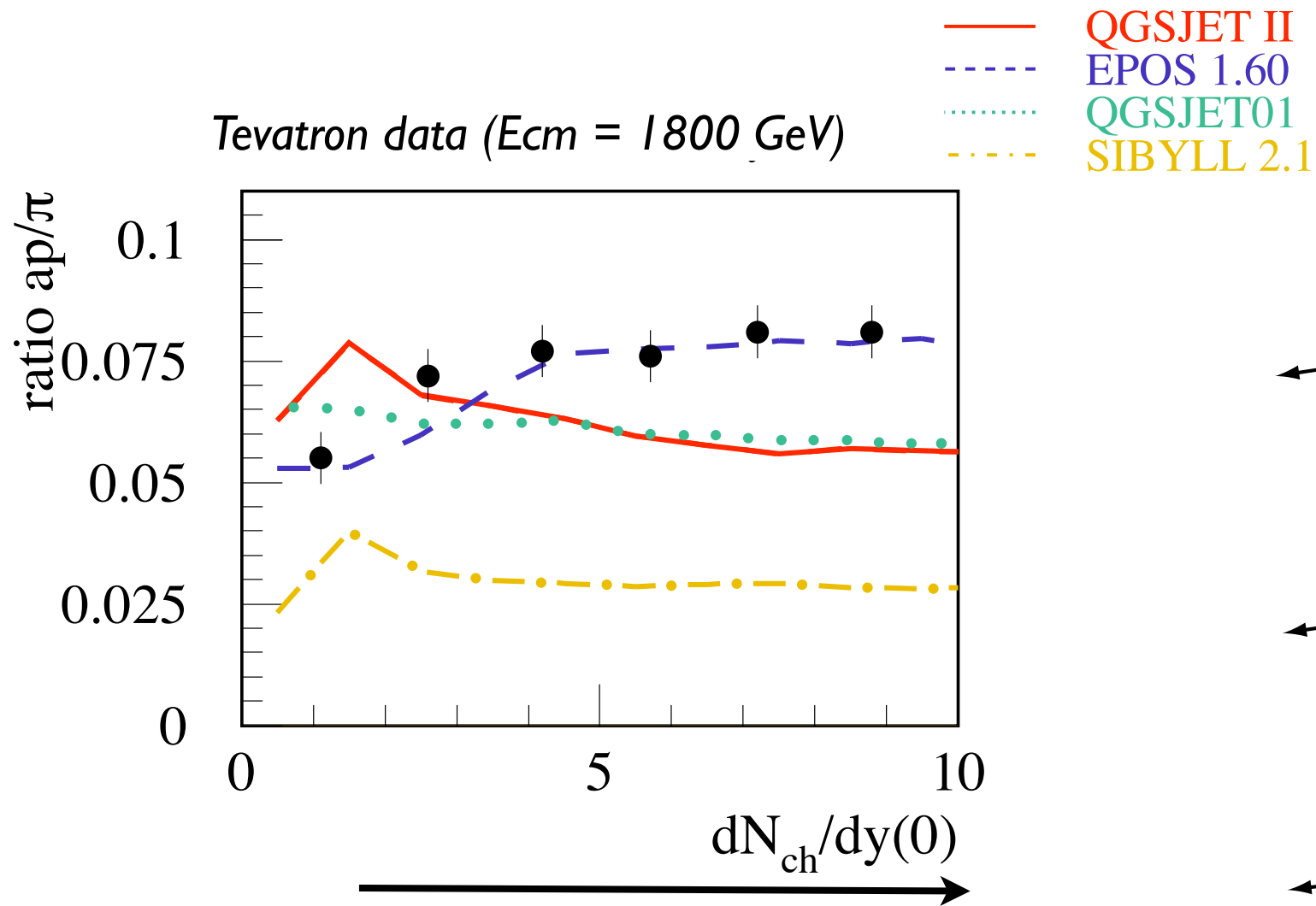
- Predicted within perturbative QCD
- Natural source of scaling violations
- Parameters for calculation very uncertain
- Saturation effects very important

## Models construction

- Construction elements very similar
- Model philosophies complementary
- Tuned to data from fixed target and collider experiments
- Differences in treatment of key questions for high-energy extrapolation
- Predicted particle production still very similar
- Models do not cover full range of uncertainty

# Baryon pair-production not understood

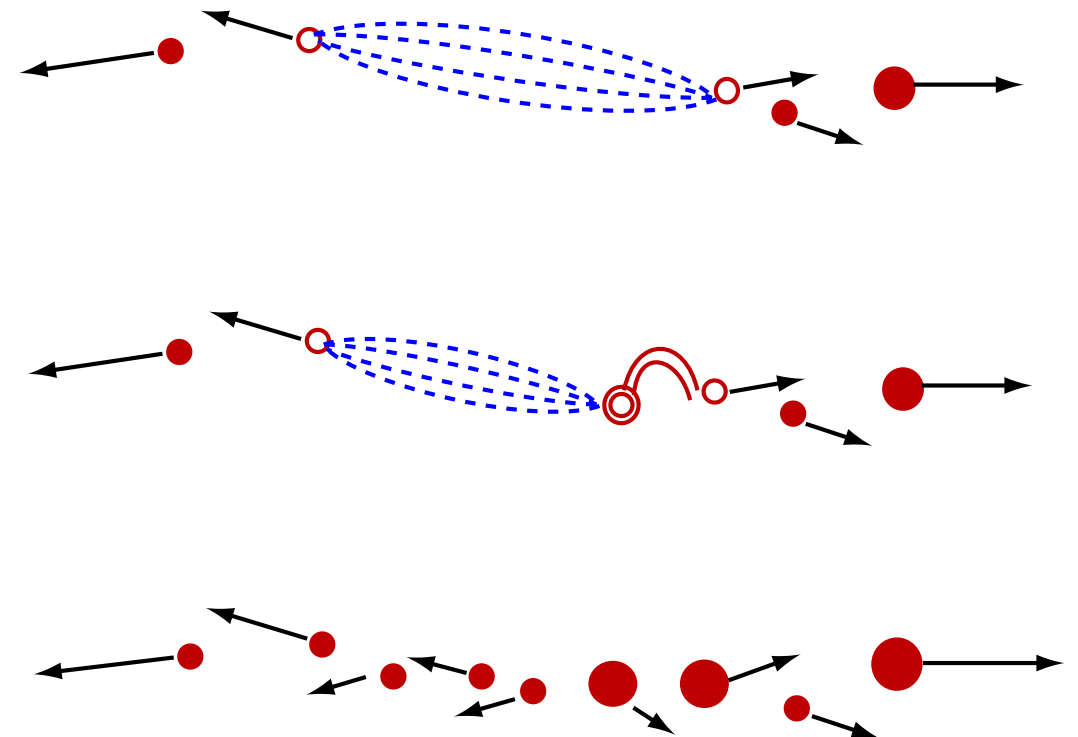
Tevatron data ( $E_{cm} = 1800 \text{ GeV}$ )



Two strings of high mass

Many strings of low mass

String fragmentation and baryon pairs



EPOS: modification of fragmentation parameters as function of string density (RHIC data)