Modeling of Hadronic Interactions

Lecture 3

Ralph Engel

Karlsruhe Institute of Technology (KIT)

Outline

Lecture 1 – Low- and intermediate-energy interactions

- Particle production threshold: resonances
- Intermediate energies: two-string models
- Extension to nuclei and photons

Lecture 2 – Interactions at very high energy

- Jets and minijets, multiple interactions
- Unitarization and saturation scenarios
- Comparison of models and uncertainties of extrapolations

Lecture 3 – Air shower phenomenology and accelerator data

- Relation between hadronic interactions and air showers
- Accelerator experiments & discrimination potential of LHC
- Comparison of model predictions with accelerator data

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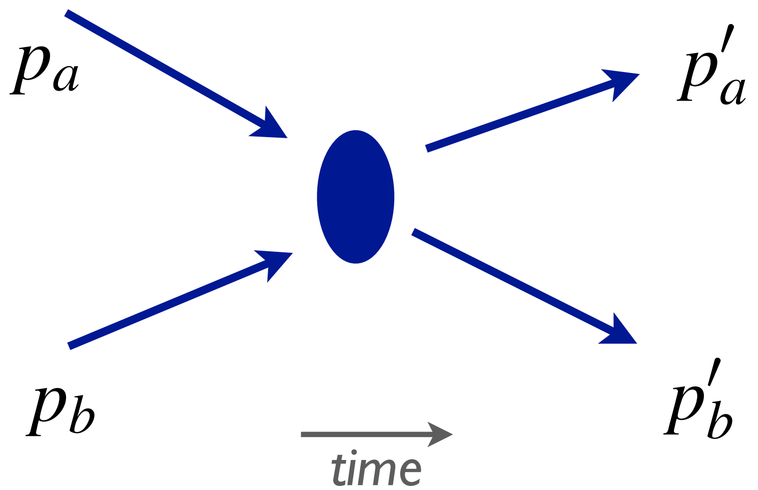
Pedestrian introduction to Reggeon and Pomeron

Basic relations

Theoretical results based on very general assumptions

- scattering amplitude exists
- maximum analyticity of scattering amplitude
- crossing allowed as result of analyticty
- unitarity (i.e. conservation of probability)



Lorentz-invariant description with Mandelstam variables

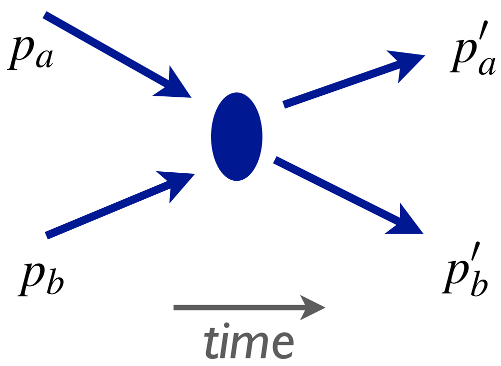
$$s = (p_a + p_b)^2$$

$$t = (p'_a - p_a)^2$$

In center-of-mass system: $t = -4p^2_{\mathrm{CMS}} \sin^2(\theta/2)$

Scattering amplitude and optical theorem

Scatering amplitude most generally defined as complex function through



$$\frac{d\sigma_{a,b\to a',b'}}{dt} = \frac{1}{16\pi s^2}\left|A_{a,b\to a',b'}(s,t)\right|^2$$

Special case: elastic scattering

$$a = a' \quad b = b'$$

$$\frac{d\sigma_{\mathrm{ela}}}{dt} = \frac{1}{16\pi s^2}\left|A(s,t)\right|^2$$

Optical theorem (unitarity, conservation of probability)

$$\sigma_{\mathrm{tot}} = \frac{1}{2s}\,\frac{1}{i}\lim_{\varepsilon\to 0}\left[A(s+i\varepsilon, t=0) - A(s-i\varepsilon, t=0)\right]$$

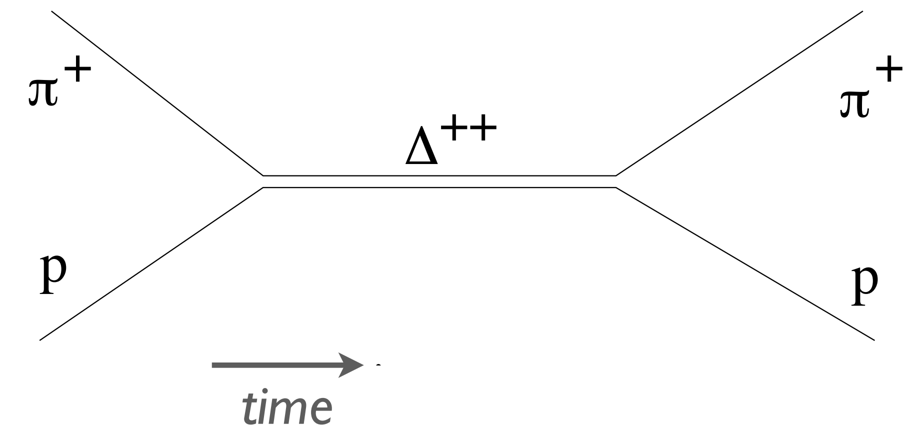
$$\sigma_{\mathrm{tot}} = \frac{1}{s}\Im m(A(s,t\to 0))$$

The classical Reggeon (and Pomeron)

Aim: calculation of A(s,t) at high energy

Partial wave expansion of scattering amplitude

Example: π-p scattering



$$A(s,t) = 16\pi \sum_{l=0}^{\infty} (2l+1) a_l(s) P_l(\cos\theta)$$

Angular momentum of resonance

Legendre polynomials

From Breit-Wigner resonance cross section follows

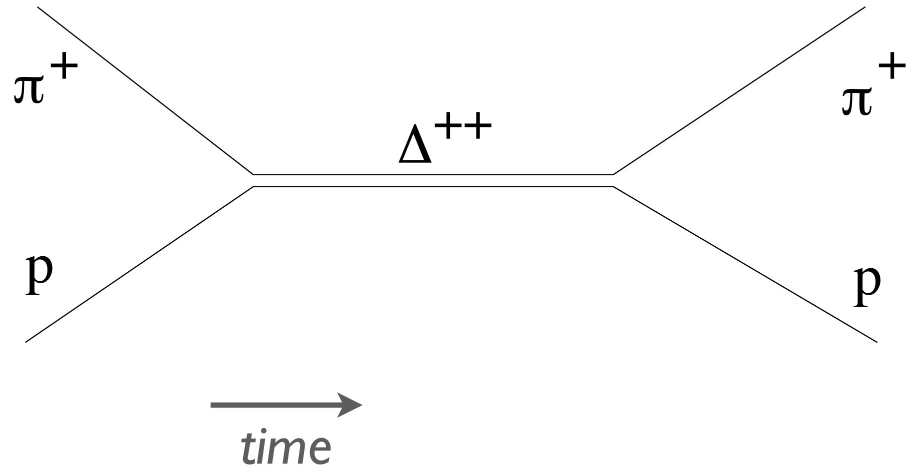
$$a_l \sim \frac{1}{s - m_l^2 + i m_l \Gamma_l}$$

High energy: more and more resonances contribute to sum since

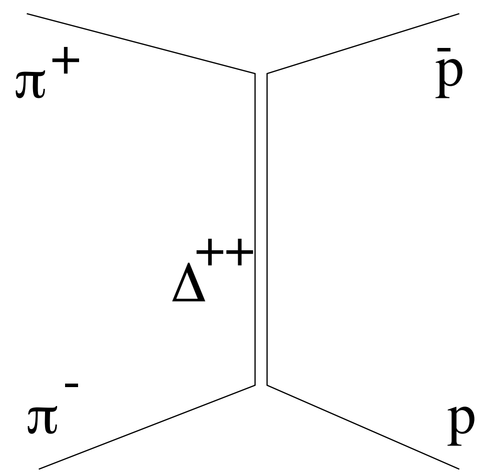
$$\sigma_l^{\mathrm{tot}}(s) = \frac{1}{s} \Im m \left(A(s, t \to 0)|_l\right) \leq (2l+1) \frac{4\pi}{k^2}$$

Analyticity: crossing from s- to t-channel

s-channel



t-channel



Exchange of s and t

$$A(s,t) = 16\pi \sum_l (2l+1) a_l(t) P_l(z_t)$$

Partial wave amplitude for given l after crossing

$$a_l(t) \sim \frac{1}{t - m_l^2 + i m_l \Gamma_l}$$

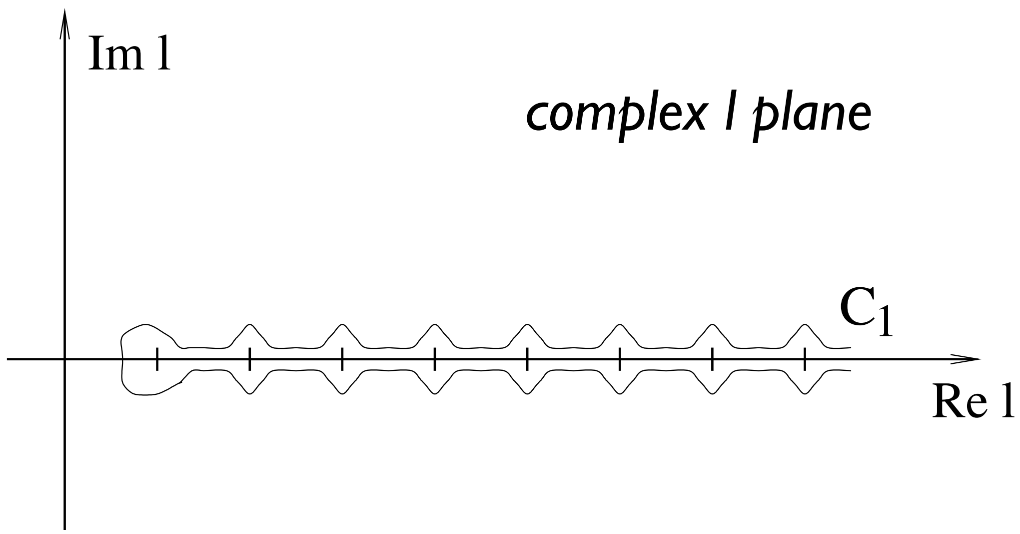
Scattering angle written in terms of Mandelstam variables, s and t exchanged

$$z_t = \cos\theta_t = \frac{2s}{t - s_0} + 1$$

Sommerfeld-Watson transformation

Aim: rewrite discrete sum as integral over angular momentum l in complex plane

Cauchy theorem: closed integral over analytic function equals sum of residuals (poles)



Contour closed at +infinity

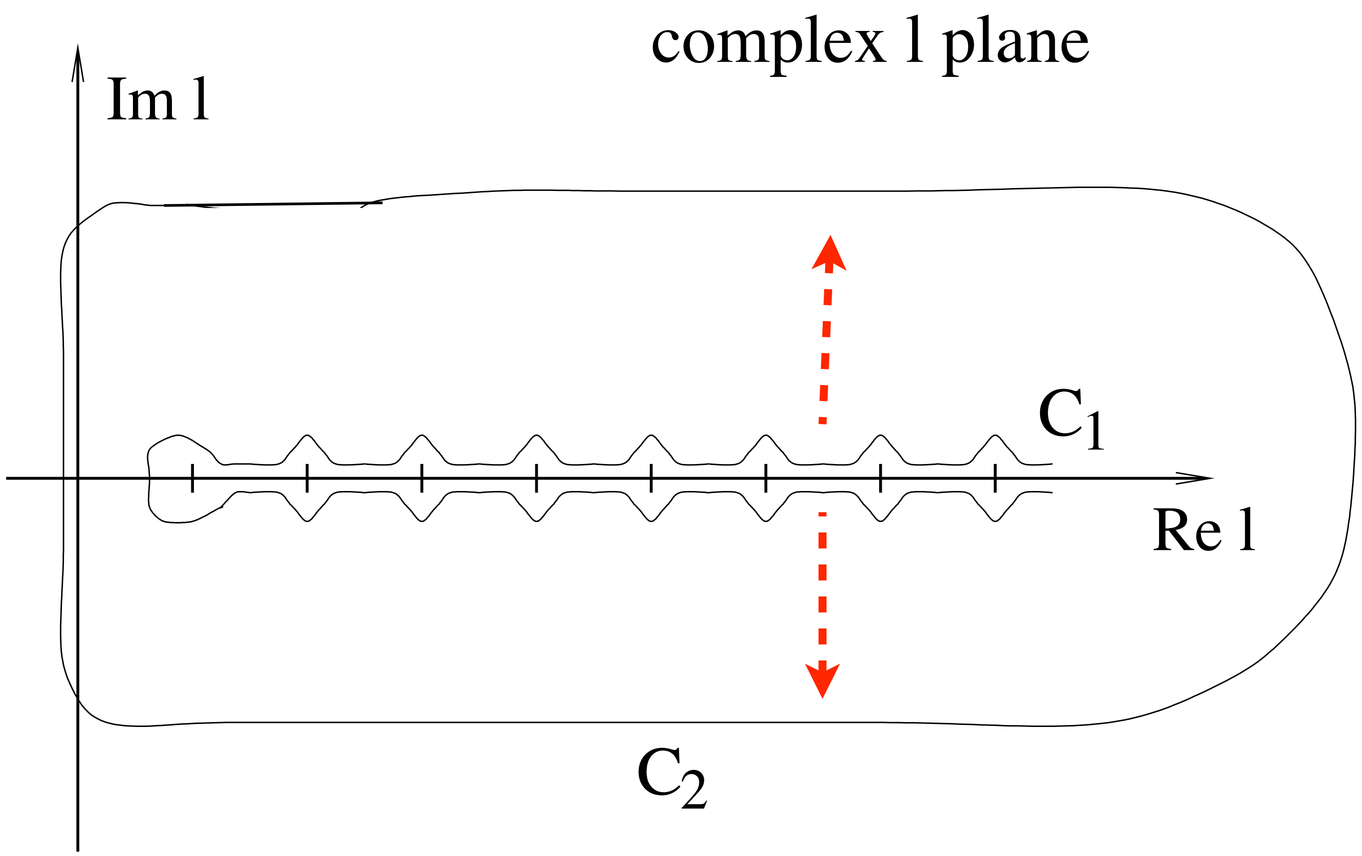
Result

$$A(s,t) = \sum_{\tau=\pm 1} \frac{16\pi}{2i} \int_{C_1} dl\,(2l+1) \left(\frac{1+\tau e^{-i\pi l}}{\sin(\pi l)} \right) a_l(t)\, P_l(-z_t), \qquad \tau = \pm 1$$

Signature

Poles introduced for even and odd values of l
(separation of even/odd needed for convergence of integral)

Deformation of integration contour from C1 to C2



Hopes:

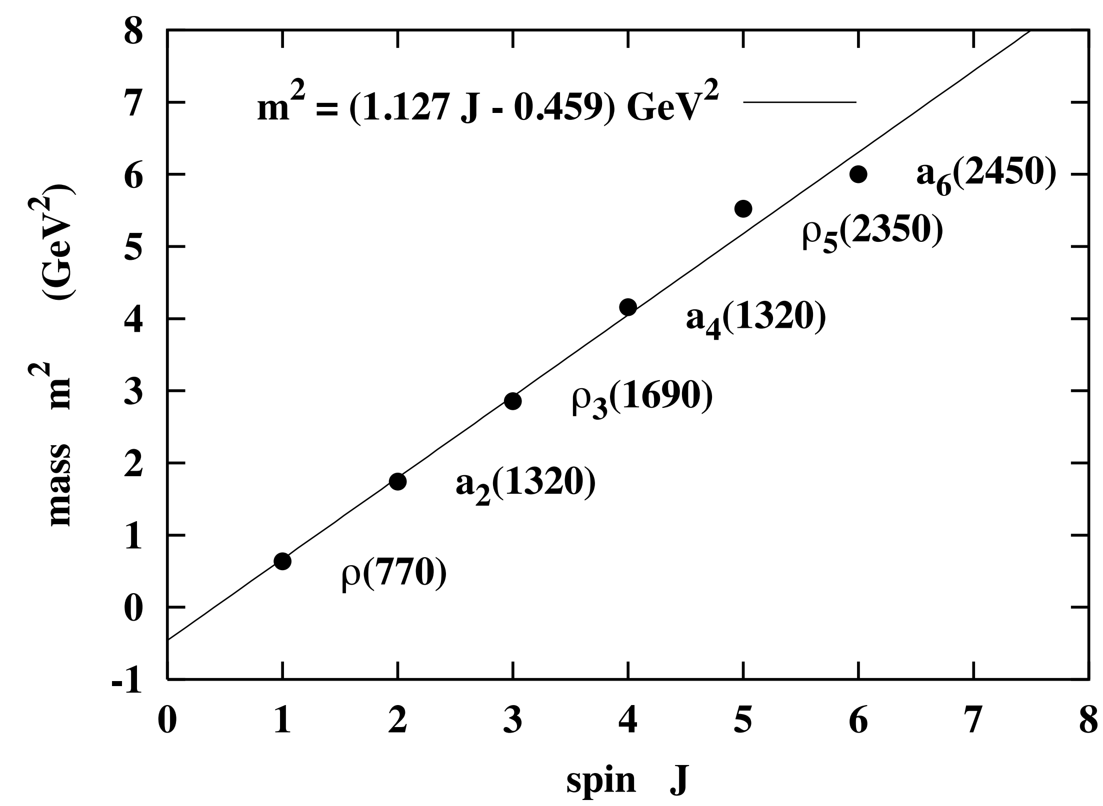
- integrand vanishes fast enough for large complex l to neglect contribution at infinity
  (this can be shown based on properties of Legendre polynomials and amplitudes)
- contribution from integration along imaginary l axis negligible (or const. term)
  (this is just a hope and cannot be proven)

But: if there were no additional poles or singularities, integral (= amplitude) would vanish!

Analytic structure of partial wave amplitude

Chew-Frautschi plot

(initally found 1962)



Mass-angular momentum relation
for given set of quantum numbers

$$m_l^2 = m_0^2 + \Delta_m^2\, l$$

Partial wave amplitude re-written

$$a_l \sim \frac{1}{t - m_l^2} = \frac{1}{t - (m_0^2 + \Delta_m^2 l)} \sim \frac{1}{l - t/\Delta_m^2 + m_0^2/\Delta_m^2}$$

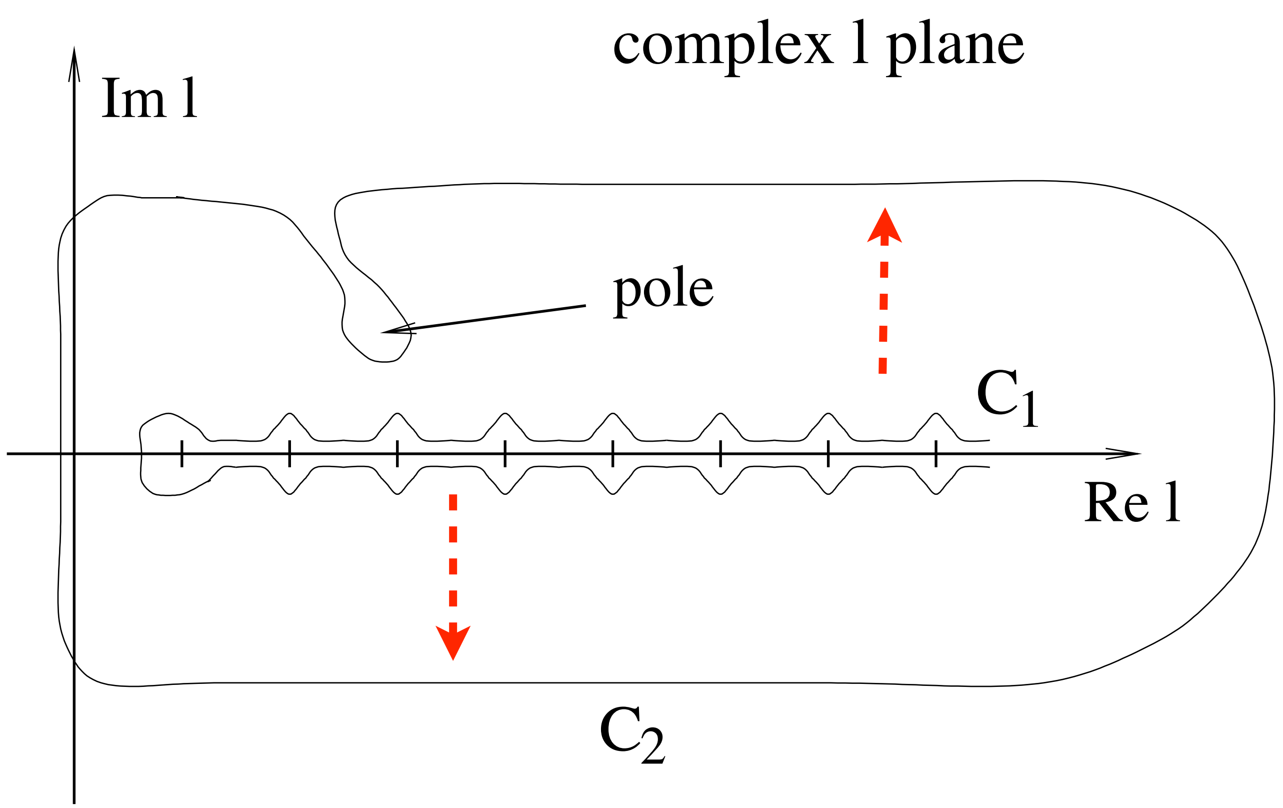
Pole in l

$$a_l \sim \frac{1}{l - \alpha(t)}$$

Regge trajectory

$$\alpha(t) = (t - m_0^2)/\Delta_m^2$$

Deformation of integration contour with Regge pole



Cauchy theorem: Summation over all poles along real l axis is equal to single pole contribution at complex l = α(t)

$$A(s,t) = \sum_{\tau=\pm 1} \frac{16\pi}{2i} \int_{C_1} dl\ (2l+1) \left(\frac{1+\tau e^{-i\pi l}}{\sin(\pi l)} \right) a_l(t)\ P_l(-z_t), \qquad \tau = \pm 1$$

$$= -\frac{1+\tau e^{-i\pi\alpha(t)}}{\sin(\pi\alpha(t))} \beta(t) P_{\alpha(t)}(-z_t)$$

High-energy limit: Regge amplitude

$$A(s,t) = -\frac{1+\tau e^{-i\pi\alpha(t)}}{\sin(\pi\alpha(t))}\beta(t)P_{\alpha(t)}(-z_t)$$

$$z_t = \cos\theta_t = \frac{2s}{t-s_0}+1$$

High-energy limit of Legendre polynomial

$$P_{\alpha(t)}\left(-\frac{2s}{t-s_0}-1\right) \xrightarrow{s\to\infty} \left(\frac{s}{s_0}\right)^{\alpha(t)}$$

Introduction of signature factor

$$\eta(\alpha(t)) = -\frac{1+\tau e^{-i\pi\alpha(t)}}{\sin(\pi\alpha(t))}$$

Regge amplitude

$$A(s,t) = \eta(\alpha(t))\;\beta(t)\;\left(\frac{s}{s_0}\right)^{\alpha(t)}$$

Cross section, Reggeon, Pomeron

Optical theorem

$$\sigma_{\text{tot}} = \frac{1}{s}\Im m(A(s,t \to 0))$$

Total cross section for one Regge pole

$$\sigma_{\text{tot}} = \frac{1}{s}\Im m\left\{\eta(\alpha(t))\,\beta(t)\,\left(\frac{s}{s_0}\right)^{\alpha(t)}\right\}_{t\to 0} = g^2\left(\frac{s}{s_0}\right)^{\alpha(0)-1}$$

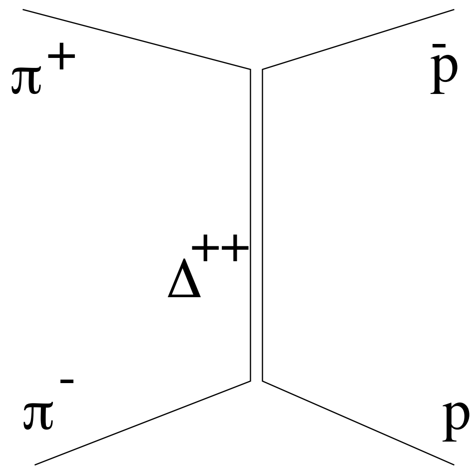
There could be several Regge poles
(Regge trajectories of different quantum numbers)

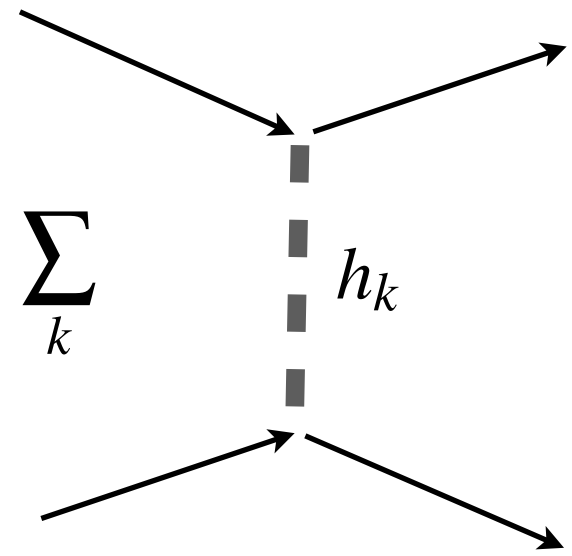
$$\sigma_{\text{tot}} = \sum_k g_k^2 \left(\frac{s}{s_0}\right)^{\alpha_k(0)-1}$$

Problem: all known Regge trajectories have α(0) < 1 but ctotal ross section rises with s

Pomeranchuk (1958): there must be a Reggeon with α(0) > 1, now called **Pomeron**

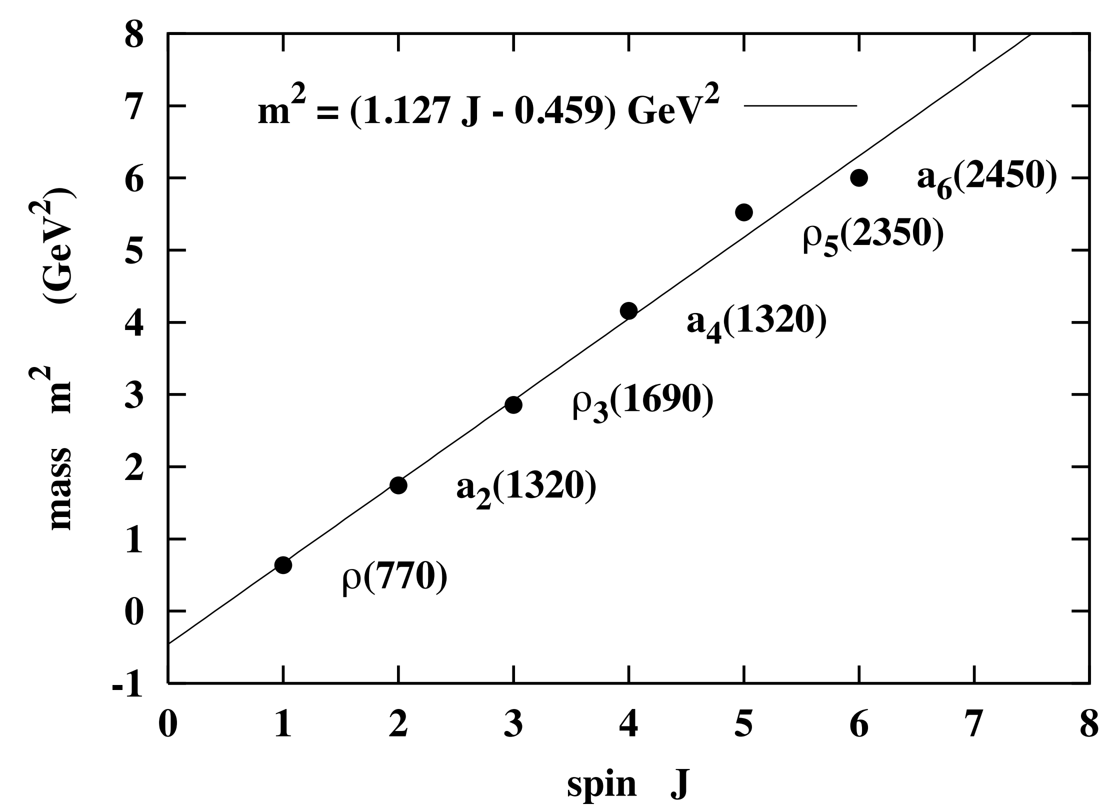
Reggeon: quasi-particle with fixed quantum numbers





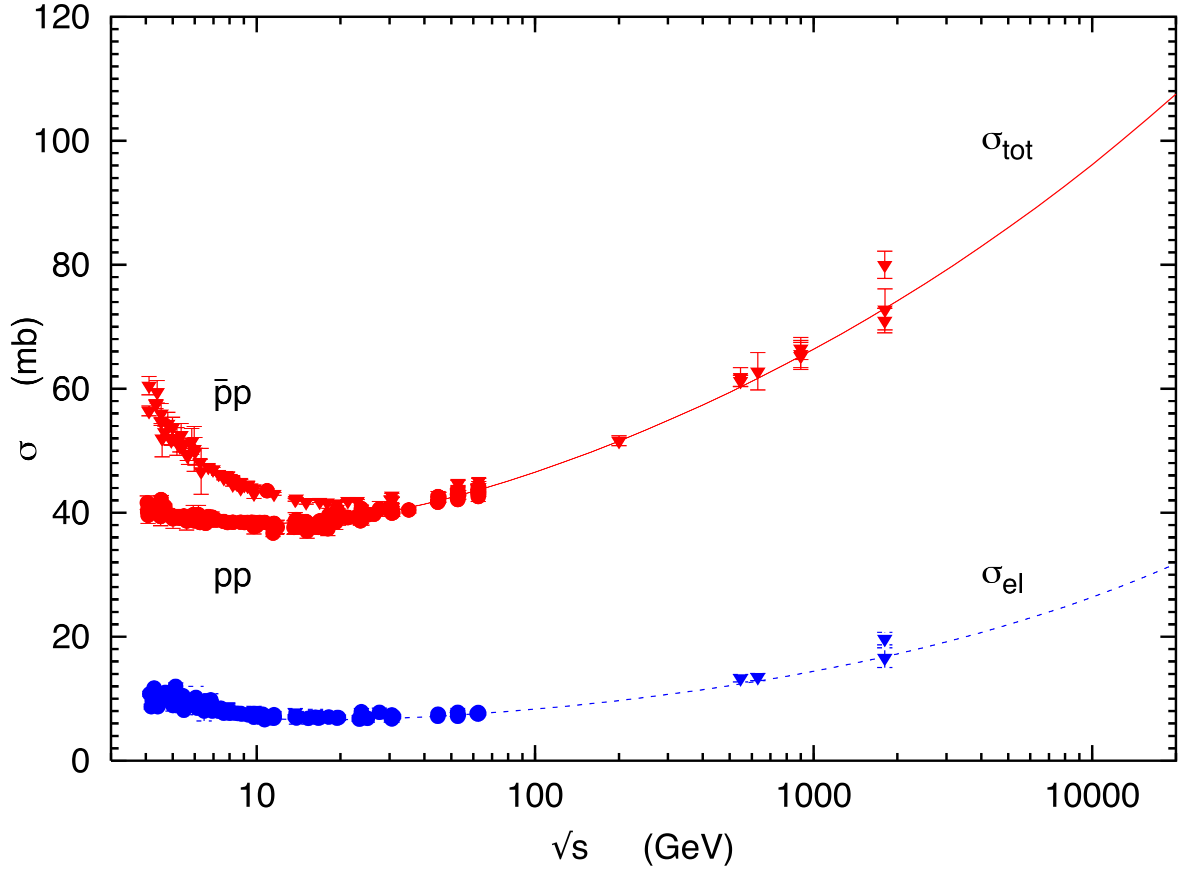
Summation over all possible particles exchanged in t-channel can be represented by one or several quasi-particles

Reggeon have quantum numbers of exchanged particles but non-integer spin given by $\alpha(t)$



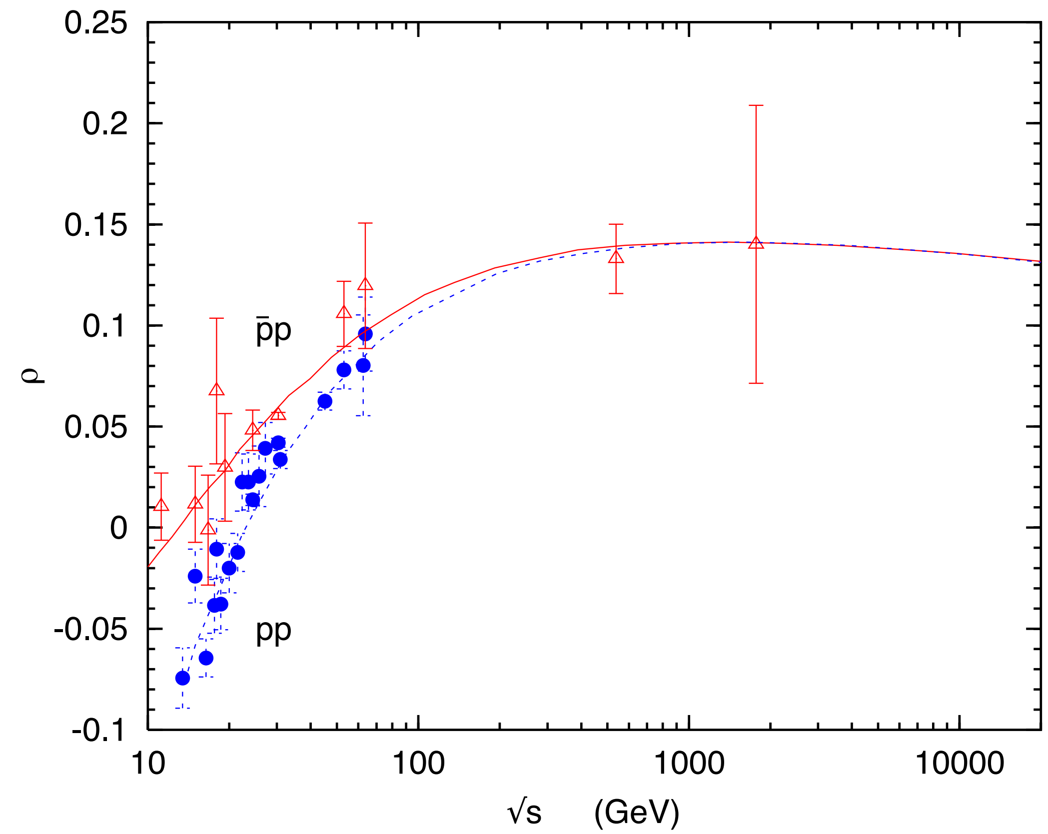
Pomeron is postulated ´special´ and universal Reggeon with $\alpha(0) > 1$ to describe rise of cross sections
(glue ball exchange?)

Example: Donnachie-Landshoff fit to cross sections



Signature factor (analyticity) determines ratio of real to imaginary part of amplitude, also well described !

$$\sigma_{\rm tot} = g_P^2 \left(\frac{s}{s_0}\right)^{0.08} + g_R^2 \left(\frac{s}{s_0}\right)^{-0.45}$$



Pomeron term (universal)

Reggeon term (non-universal)

(Donnachie & Landshoff, PLB 1992)

Pomeron and Reggeon in non-perturbative QCD

Topological expansion of QCD

Large N_c-N_f expansion of QCD

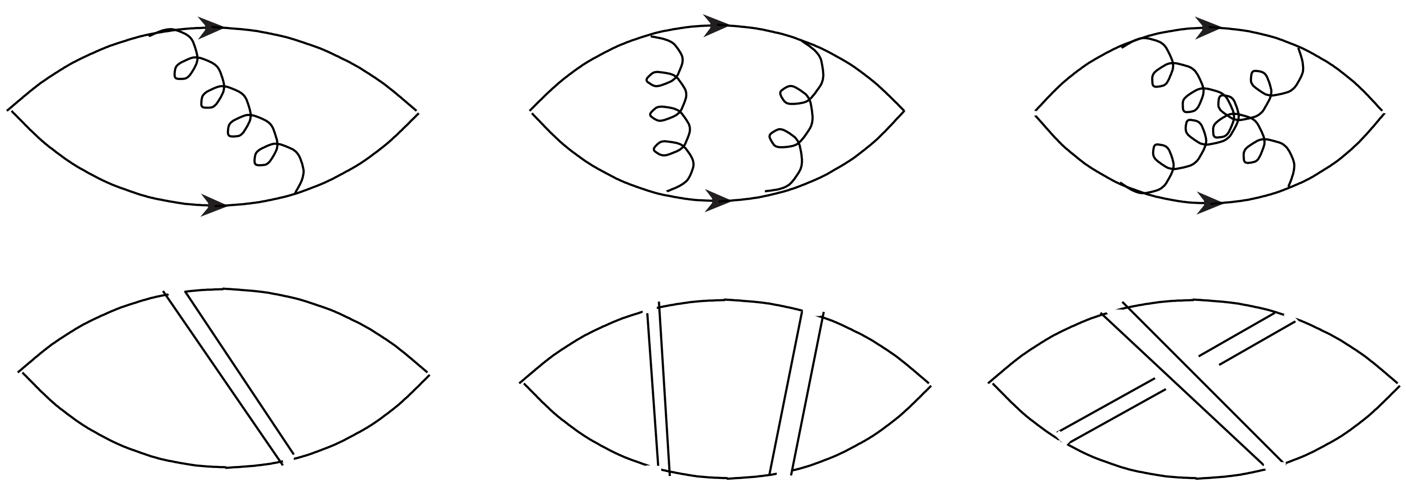
Problem: no small coupling constant for perturbative expansion in soft physics

´t Hooft, Veneziano, Witten (1974)

$$N_c \to \infty$$

$$N_c/n_f = \text{const}$$

$$g^2 N_c^2 \simeq 1$$



Graphs can be sorted according to number of colors and power of coupling constant

Topology of graph: surface on which it can be drawn without crossing color lines

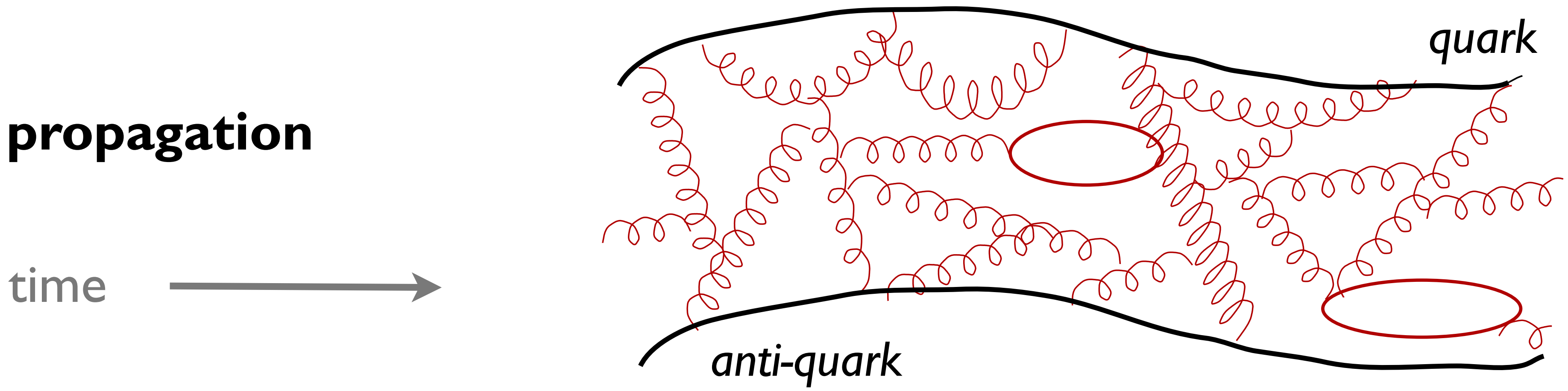
Planar diagrams preferred: planar diagram theory of QCD

Color flow topologies in large-N_c/n_f QCD (i)

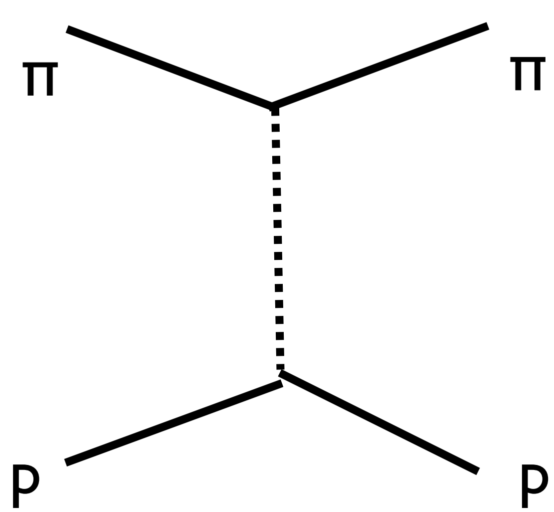
Partons only asymptotically free, work with ´strings´ instead

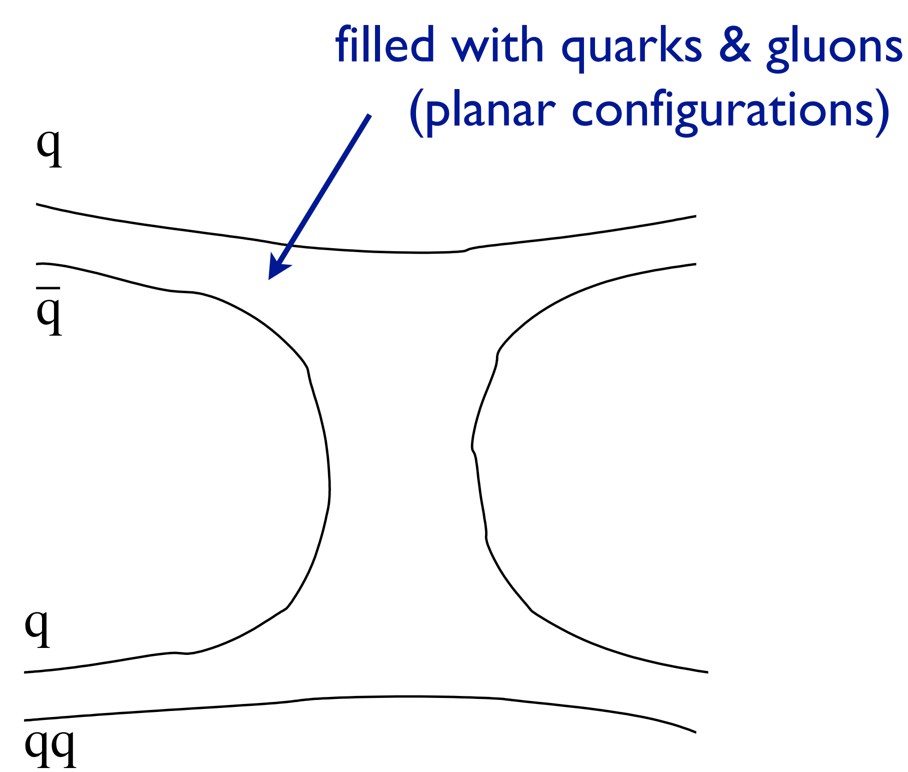
Example:

meson propagation



Scattering process:

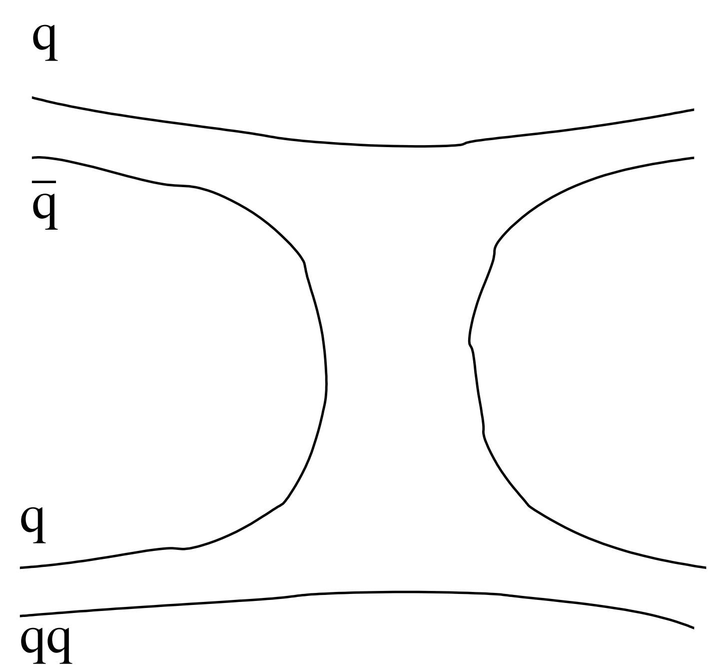




Color flow topologies in large-N_c/n_f QCD (ii)

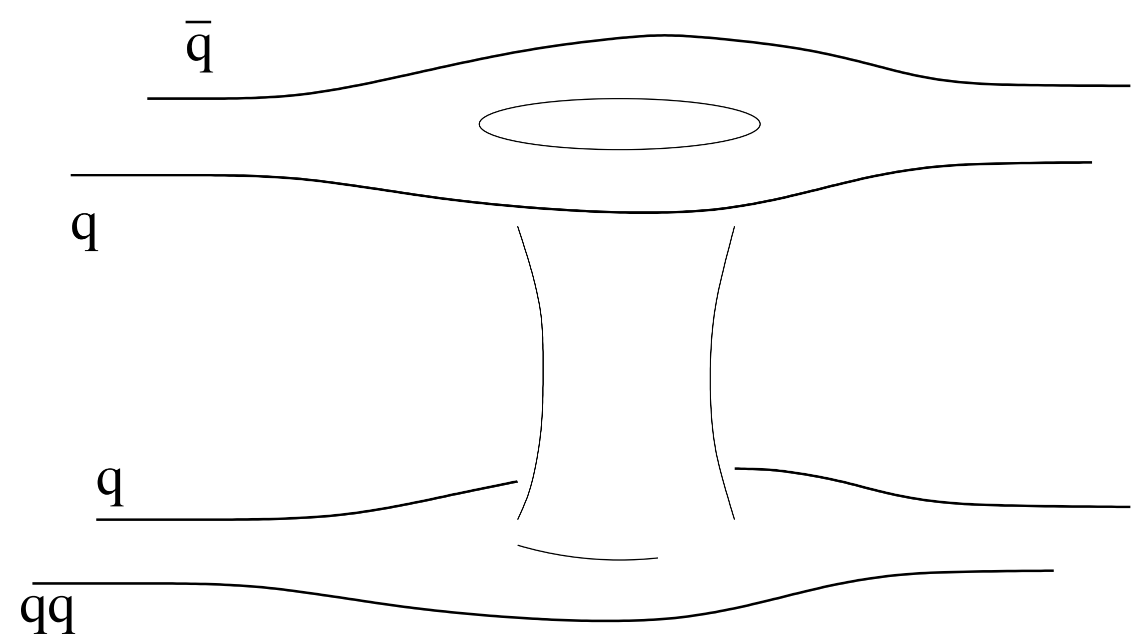
Reggeon exchange

flat topology (dependence on valence quark combinatorics)



Pomeron exchange

cylinder topology (does not depend on flavour of scattering particles)



time →

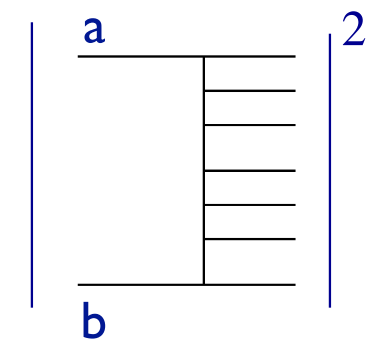
Graphical representation of optical theorem (i)

Standard method of calculating cross sections

$$\sigma_{\text{tot}} = \frac{1}{\Phi} \sum_X \int dP_X \, |M_{pp \to X}|^2$$

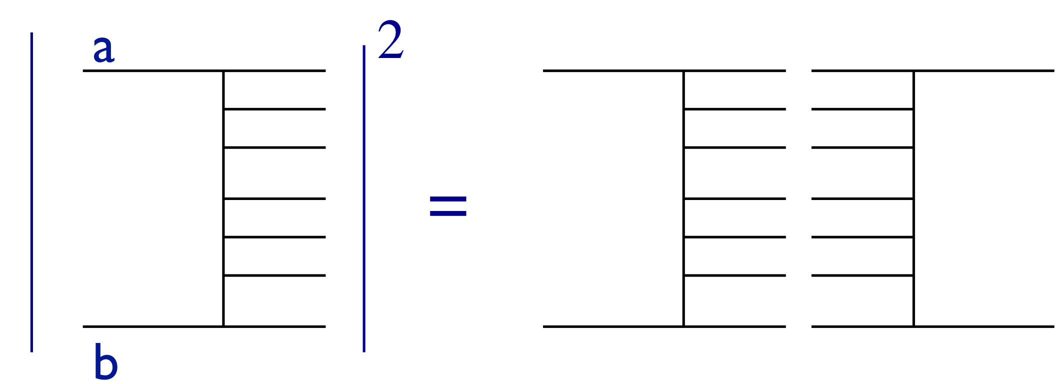
sum over all final states

integration over phase space of final state particles



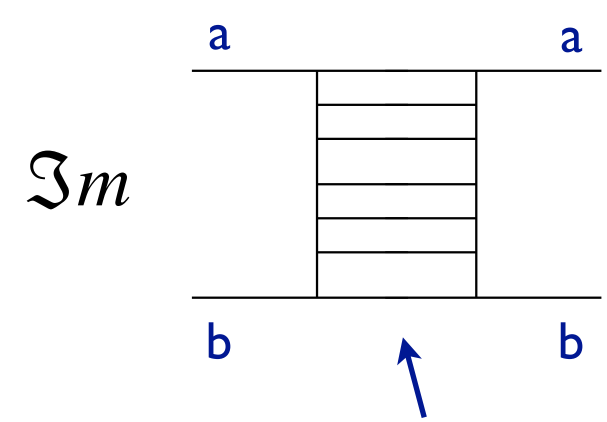
$$dP_X = \prod_j \frac{d^3 k_j}{2E_j}$$

$$= \frac{1}{\Phi} \sum_X \int dP_X \, M^+_{pp \to X} M_{pp \to X}$$



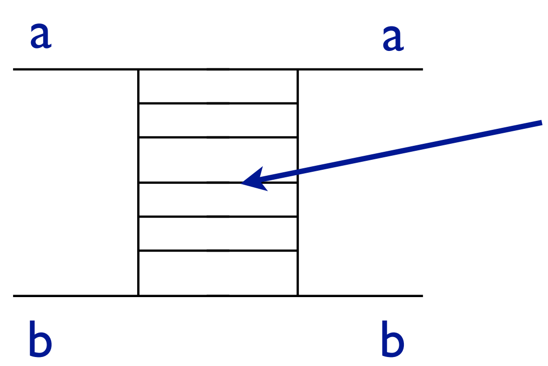
Optical theorem (elastic scattering)

$$= \frac{1}{s} \Im m (A_{\text{ela}}(s, t = 0))$$



sum over all intermediate states

Graphical representation of optical theorem (ii)

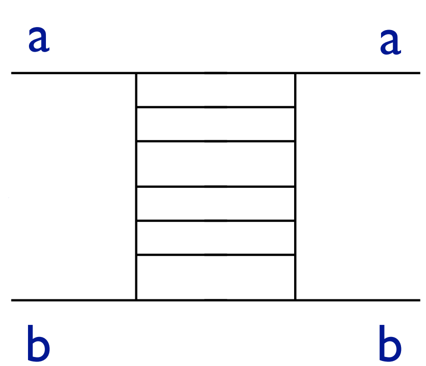
$\Im m$ 

Field theory: particle propagator $\frac{d^4k}{k^2 - m^2 + i\varepsilon}$

Imaginary part of particle propagator

particle put on mass shell

$$\Im m \left(\frac{d^4k}{k^2 - m^2 + i\varepsilon} \right) = \delta(k^2 - m^2) d^4k = \frac{d^3k}{2E}$$

$\Im m$ 

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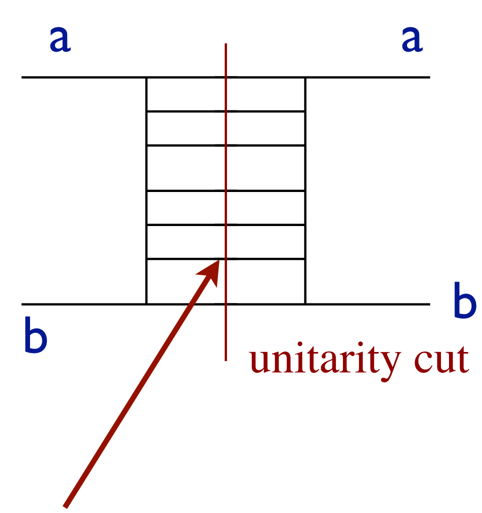
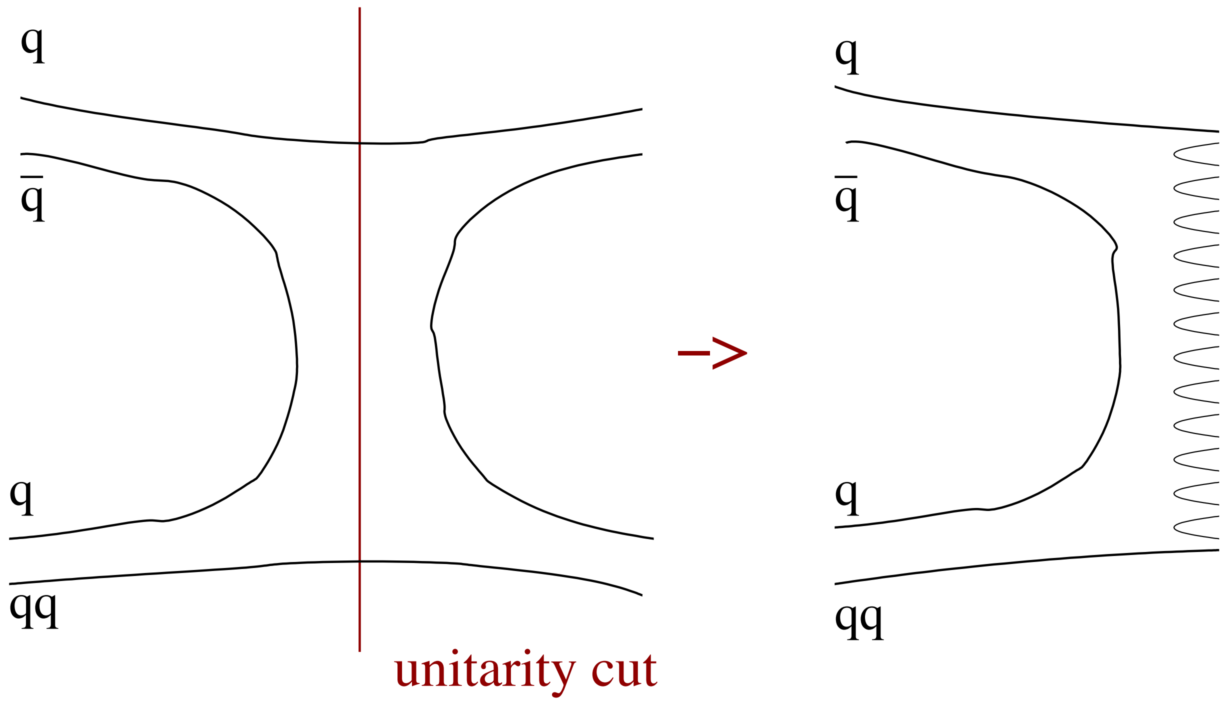


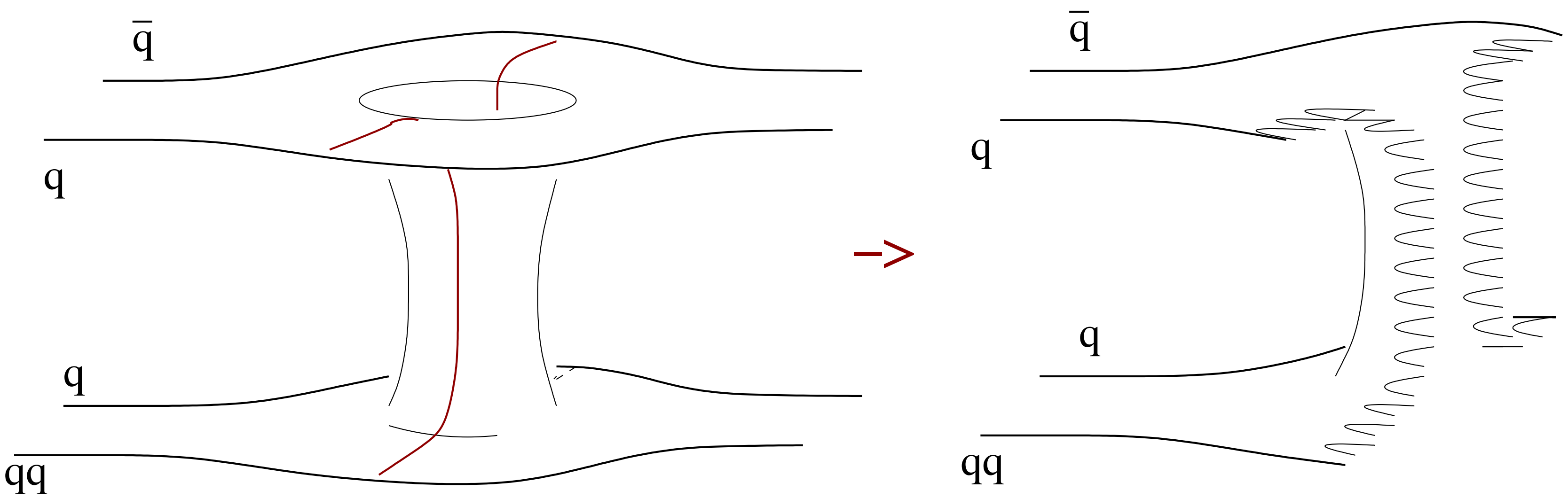
Calculating imaginary part shows particle configurations of final state for total cross section

cut particle lines correspond to particles in final state

Unitarity cuts (optical theorem): final state particles



Unitarity cut of Reggeon exchange: chain of hadrons



elastic scattering

inelastic scattering

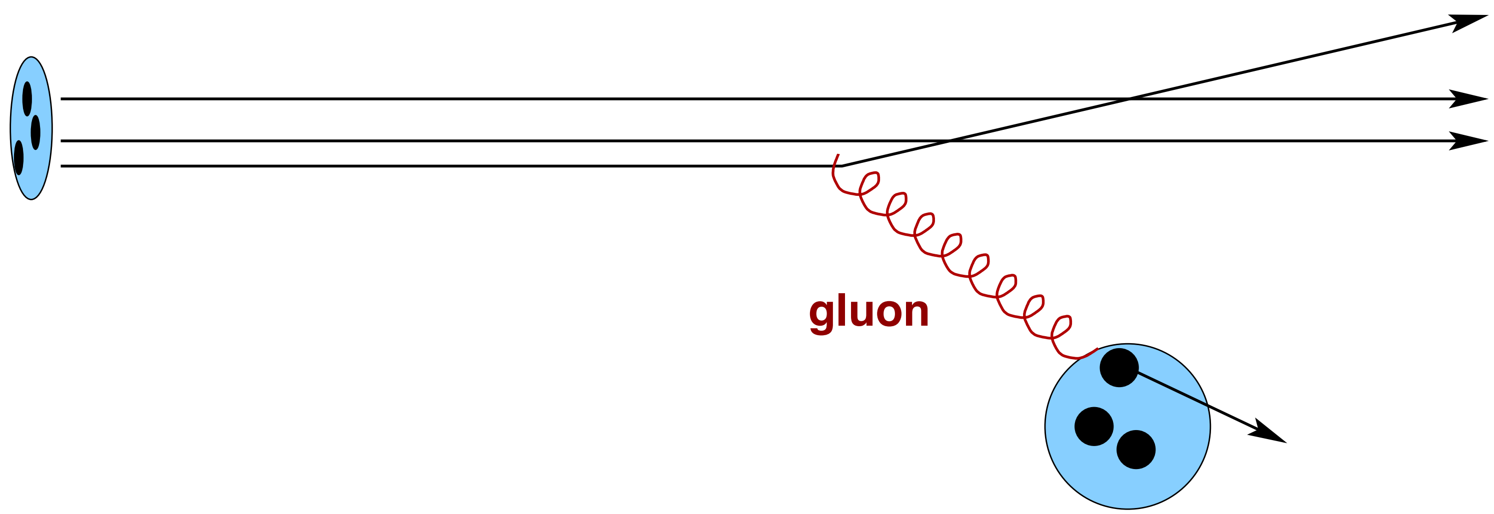
Pomeron exchange: two chains of hadrons

Pomeron and Reggeon in perturbative QCD

Scattering by gluon exchange

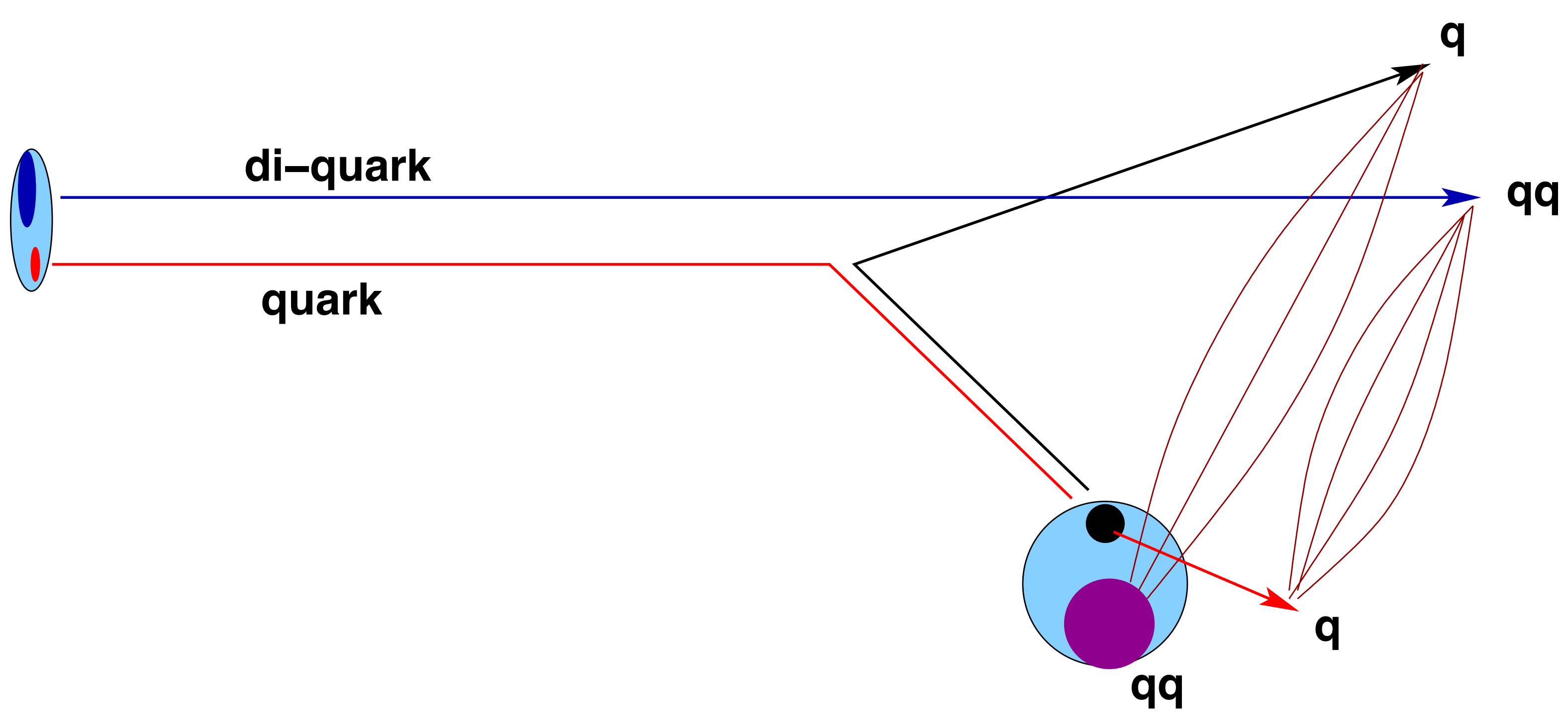
QCD color flow configurations (i)

Partonic view:



Simplest scenario:
one-gluon exchange

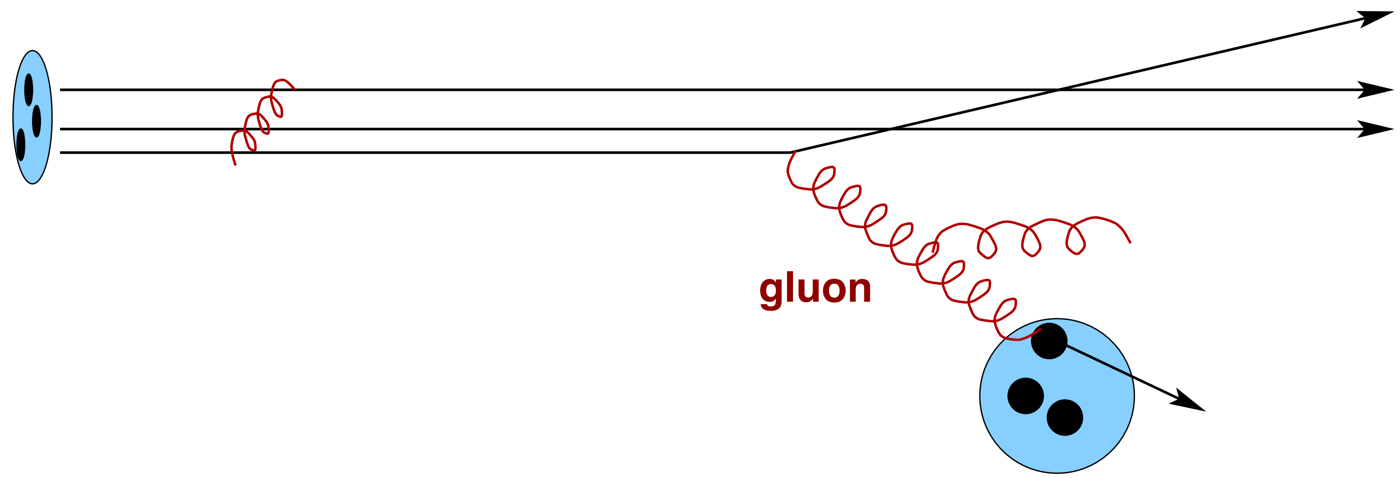
Color flow:



One-gluon exchange:
two color fields (strings)

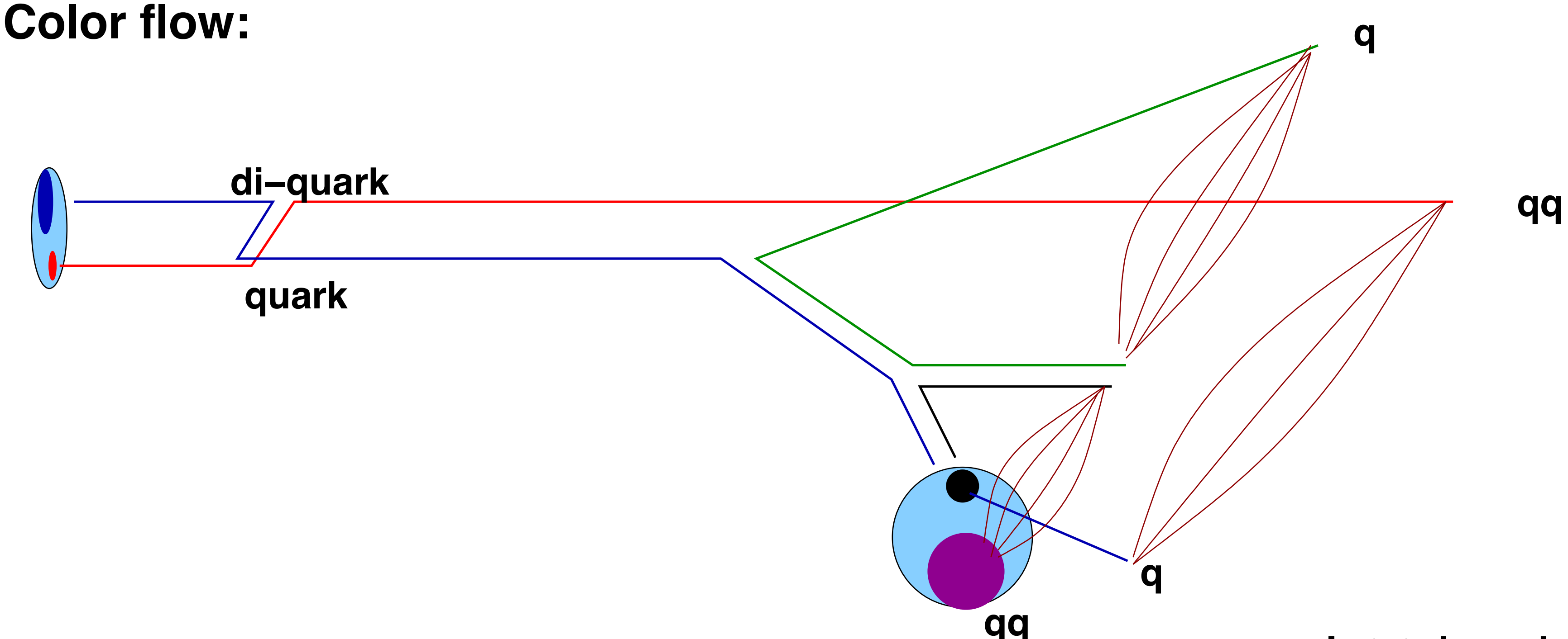
QCD color flow configurations (ii)

Partonic view:



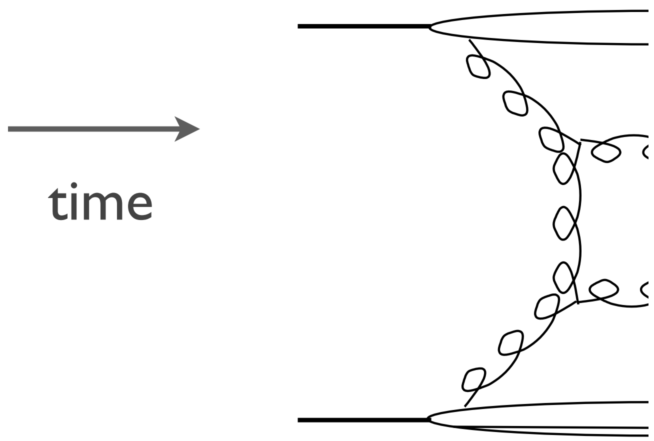
One-gluon exchange with additional radiations

Color flow:



Initial and final state radiation does not change topology

Gluon-gluon scattering and cylinder topology

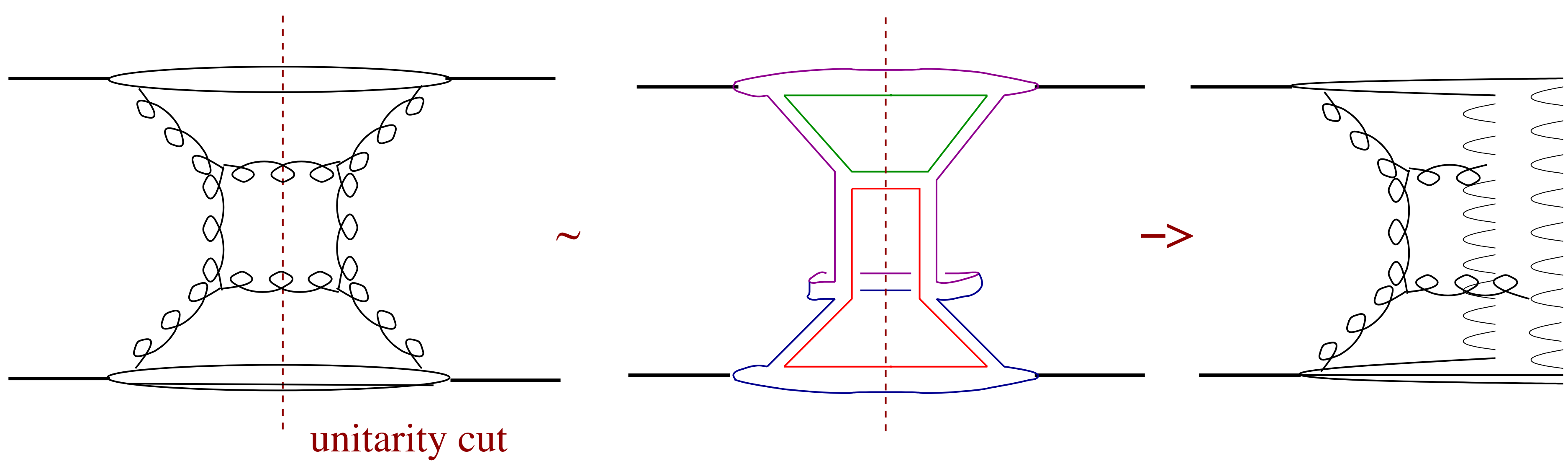


Generic diagram of hard scattering

$$\sigma_{QCD} = \sum_{i,j,k,l} \frac{1}{1+\delta_{kl}} \int dx_1\, dx_2 \int_{p_\perp^{\text{cutoff}}} dp_\perp^2\, f_i(x_1, Q^2)\, f_j(x_2, Q^2)\, \frac{d\sigma_{i,j \to k,l}}{dp_\perp}$$

Standard procedure: total gluon-gluon cross section obtained by squaring matrix element

Same calculation using optical theorem: need to cut graph for elastic scattering



leading contribution: cylinder topology

Modern understanding of Pomeron

$$A(s,t) = \eta(\alpha(t))\ \beta(t)\ \left(\frac{s}{s_0}\right)^{\alpha(t)}$$

- Quasi-particle that effectively accounts for all exchanged hadronic states
- Amplitude exhibits power-law dependence on energy
- Regge trajectory of Pomeron: exchanged particles might be glue balls
- Pomeron trajectory only phenomenologically known
- Large N_c-n_f approximation of QCD: Pomeron corresponds to cylinder topology
- Final state configuration: leading contribution is two chains (strings) of hadrons
- Gluon-gluon scattering in pQCD corresponds to ´hard´ contribution to Pomeron

Multiple exchanges & interaction of quasi-particles (Pomerons):
Gribov´s Reggeon Field Theory