The dynamics of integer quantum Hall edge states far from equilibrium

John Chalker

Physics Department, Oxford University

Collaborators:

D. Kovrizhin: Phys. Rev. B 2009 - 2011 and arXiv

Y. Gefen and M. Veillette: Phys. Rev. B (2007)

Outline

Electron interference

In vacuum . . . . and in solids

Quantum Hall edge states

— as electron waveguides

Edge state interferometers

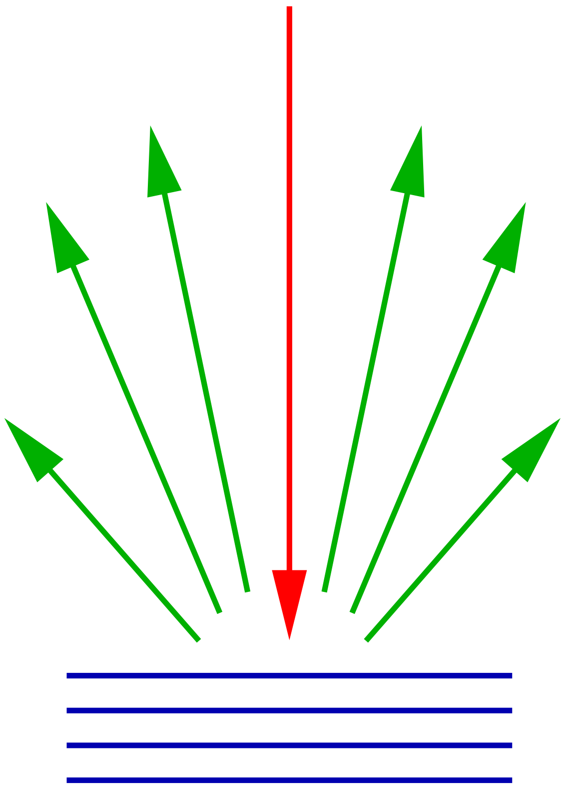
— evidence for coherence

Edge states out of equilibrium

— understanding quantum relaxation

Electron Diffraction in Vacuum

Davisson and Germer, 1927



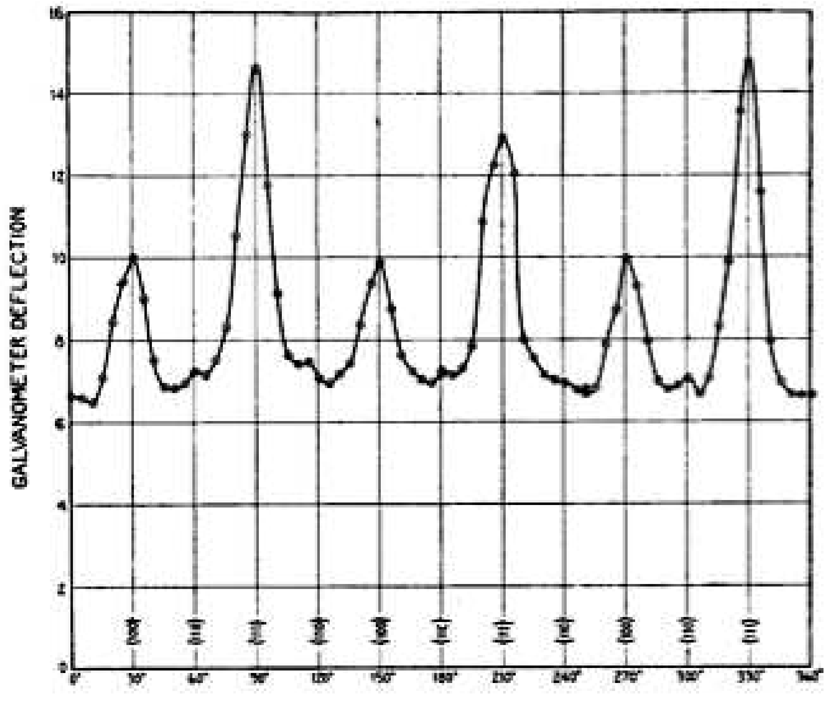


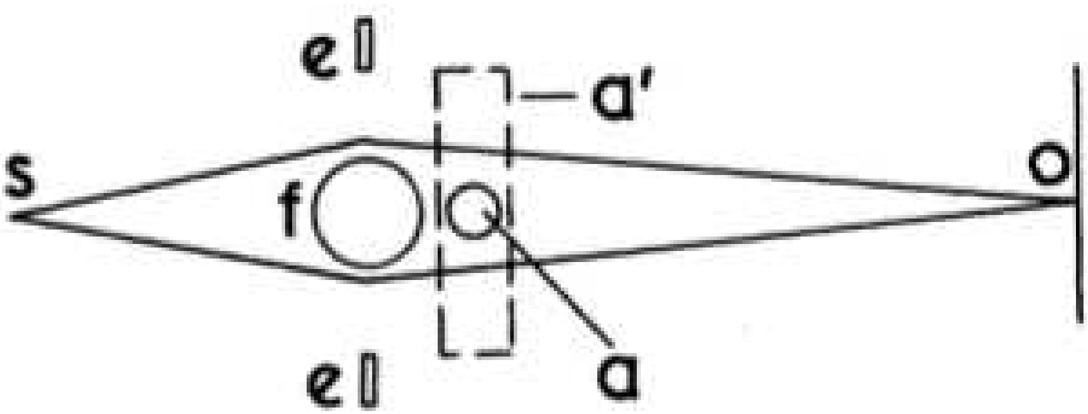
FIG. 2.—Intensity of electron scattering vs. azimuth angle—54 volts, co-latitude 50°.

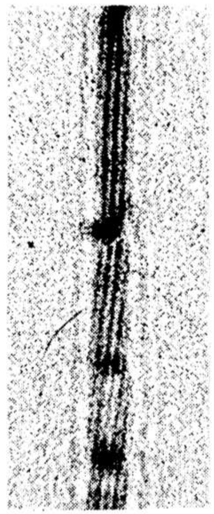
NATURE [APRIL 16, 1927

The Aharonov Bohm Effect

Chambers, 1960 **Phase Φ/Φ_0 from encircling flux Φ**

Flux quantum: $\Phi_0 = h/e$





SHIFT OF AN ELECTRON INTERFERENCE PATTERN BY ENCLOSED MAGNETIC FLUX

R. G. Chambers
H. H. Wills Physics Laboratory, University of Bristol, Bristol, England

(Received May 27, 1960)

Electron Interference in Conductors

Obstacle: Scattering

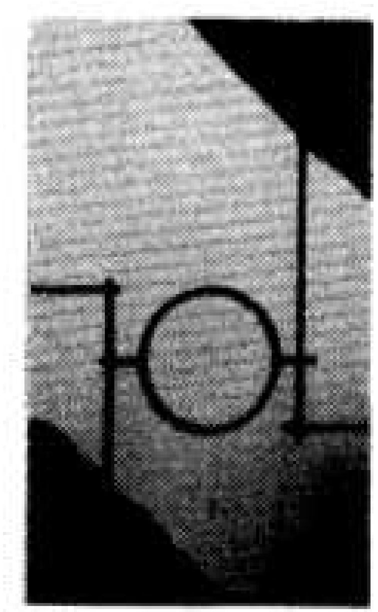
- by other electrons
- by impurities

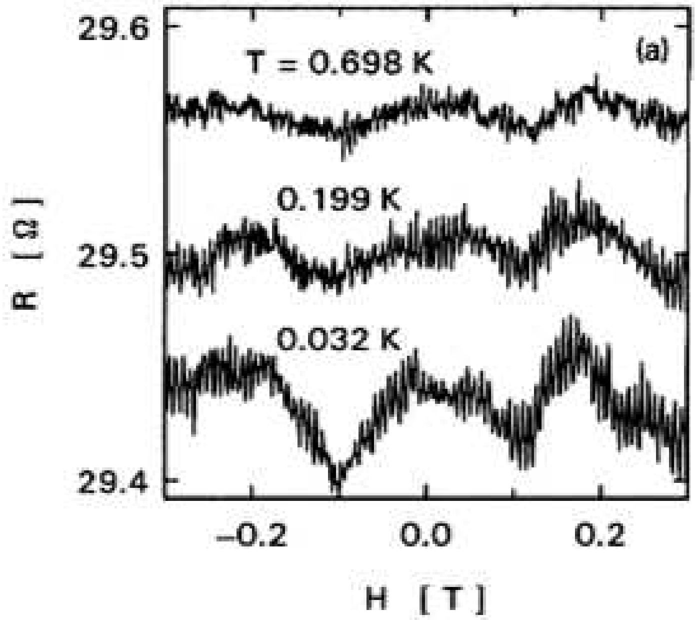
Solution: Work in the mesoscopic regime

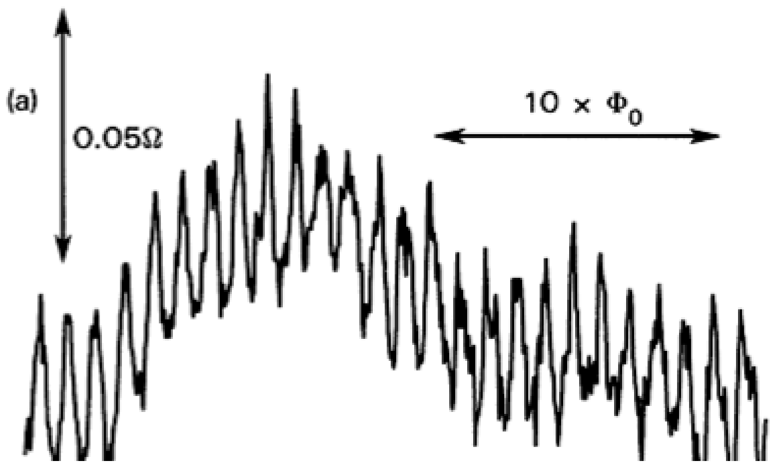
- small samples
- low temperatures

Aharonov Bohm Effect in Gold Rings

Measure resistance to probe interference







Diameter 800nm

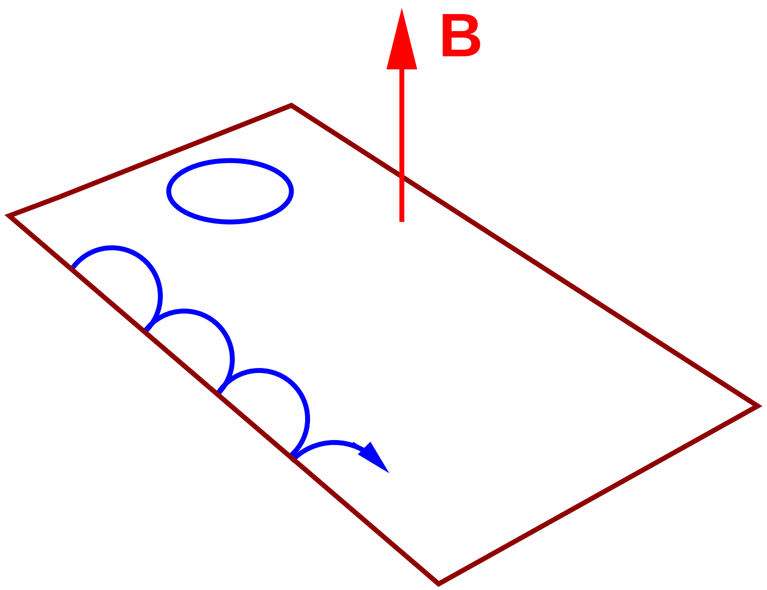
Webb *et al* Phys. Rev. Lett. (1985)

Many channels and impurities reduce fringe visibility

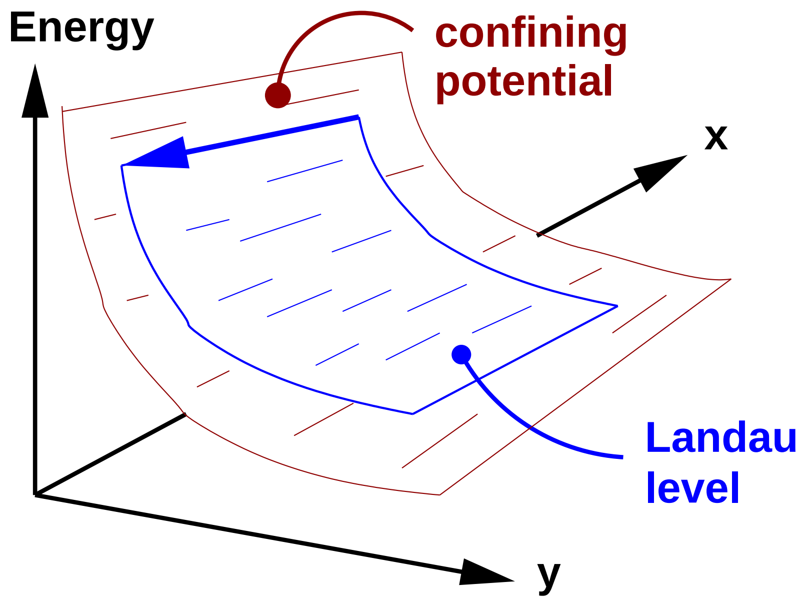
Quantum Hall Edge States

Two-dimensional electron gas in magnetic field

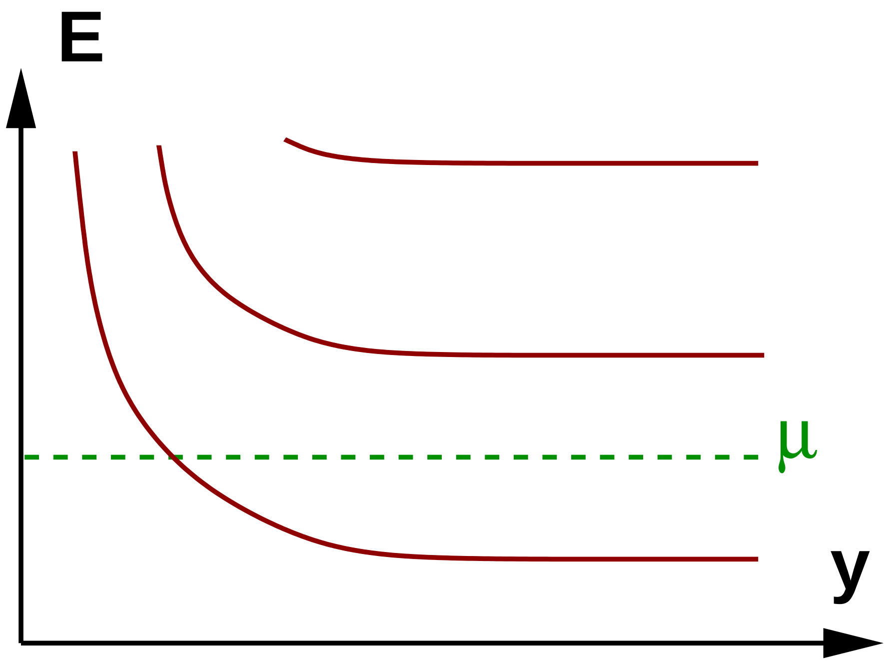
Classical skipping orbits



Quantum edge states



Edge states as Ideal Waveguides



Chiral motion

Only possible scattering is in forward direction

Theoretical Description of Edge States

Project from 2D to 1D

Classical Hamiltonian:

drift at constant speed

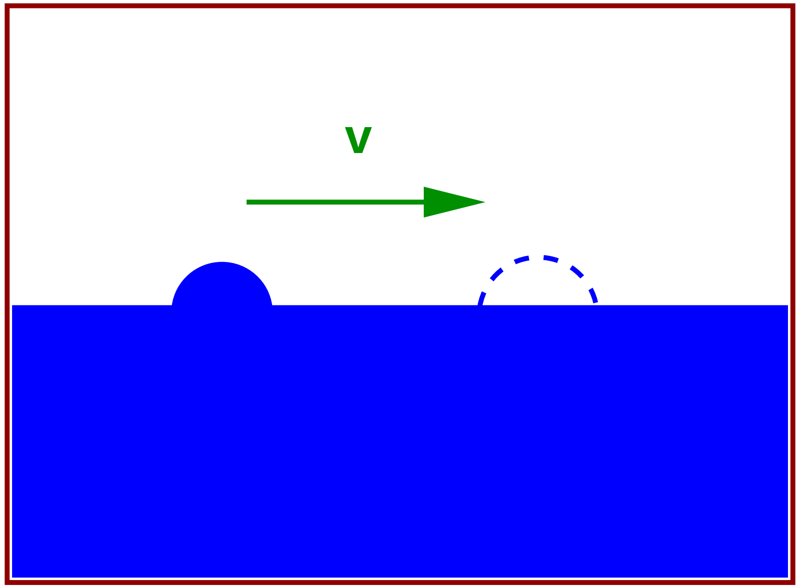
$$\mathcal{H} = vp_x \qquad \dot{x} = \partial_p \mathcal{H} = v$$

Single-particle quantum Hamiltonian:

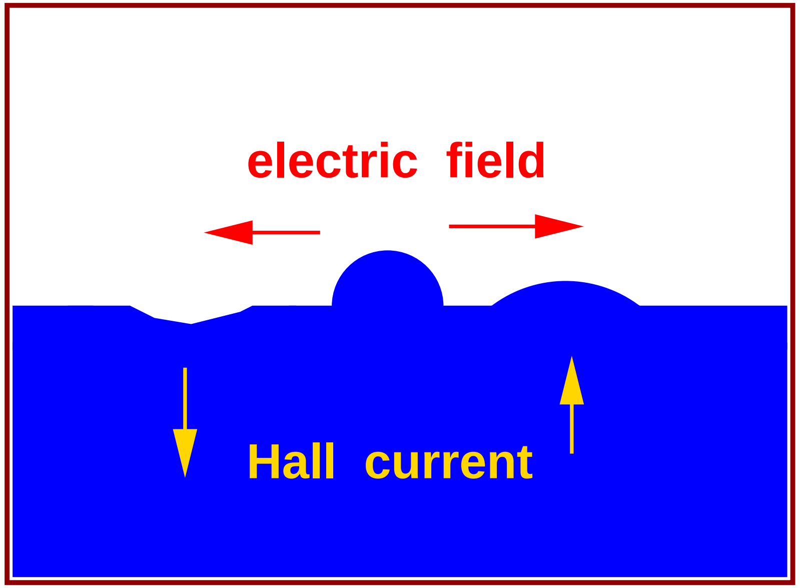
$$\mathcal{H} = \int \psi^\dagger(x)(-i\hbar v \partial_x)\psi(x)dx$$

Edge state dynamics with interactions

Free propagation



Charge flow in and out of bulk



Interactions make collective modes dispersive

Two alternative descriptions

— related via bosonization

As electrons:

$$H = -i\hbar v \int dx \psi^\dagger(x) \partial_x \psi(x) + \int dx \int dx' \, U(x - x') \rho(x) \rho(x')$$

$$\rho(x) = \psi^\dagger(x) \psi(x)$$

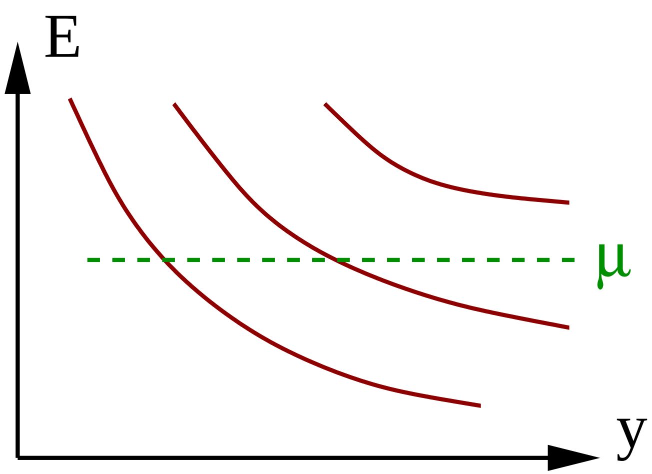
As collective modes:

$$H = \sum_q \hbar\omega(q) b_q^\dagger b_q$$

$$\omega(q) = [v + u(q)]\, q \qquad u(q) = (2\pi\hbar)^{-1} \int dx \; e^{iqx} U(x)$$

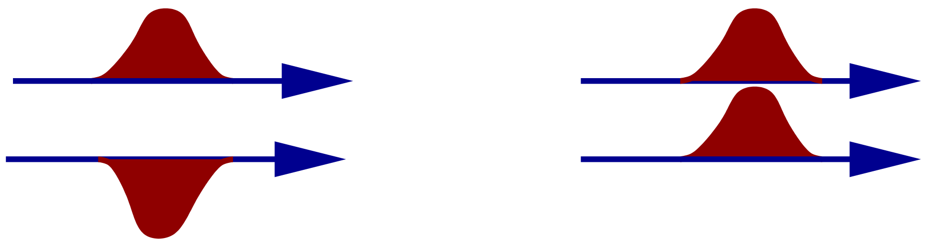
Consequences of interactions

Example: two filled Landau levels



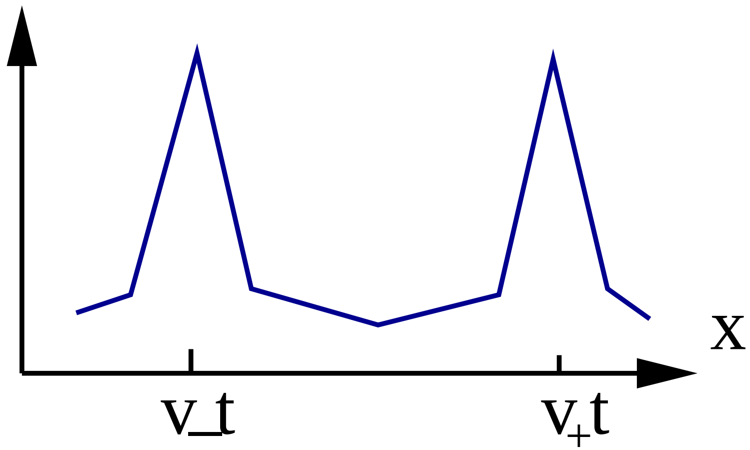
Modes mixed by interaction g

$$v_{\pm} = v \pm g$$



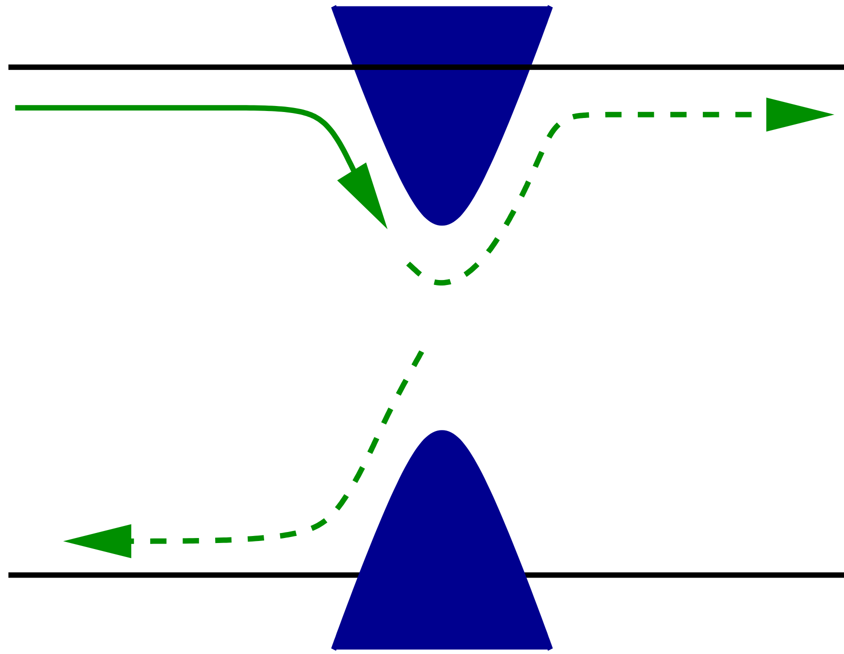
Injected electron fractionalises

$$|\langle \psi_1^{\dagger}(x,t)\psi_1(0,0)\rangle|^2 \sim$$



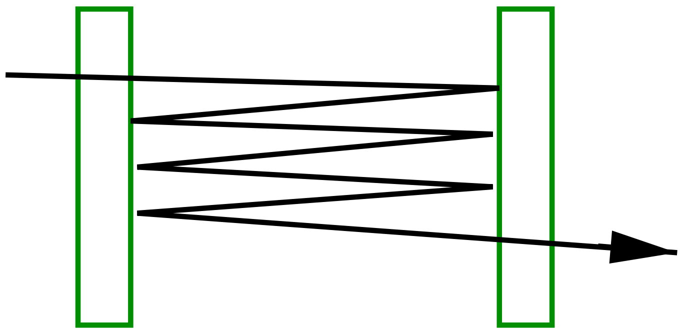
Manipulating Edge States

Quantum point contacts as beam splitters

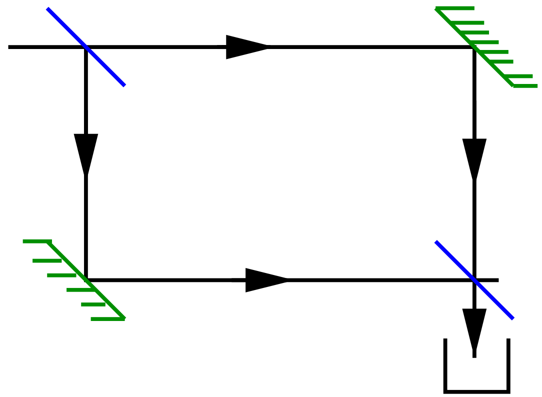


Edge State Interferometer Design

Fabry-Perot

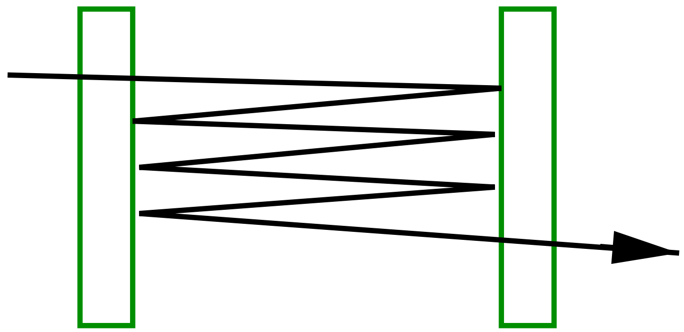


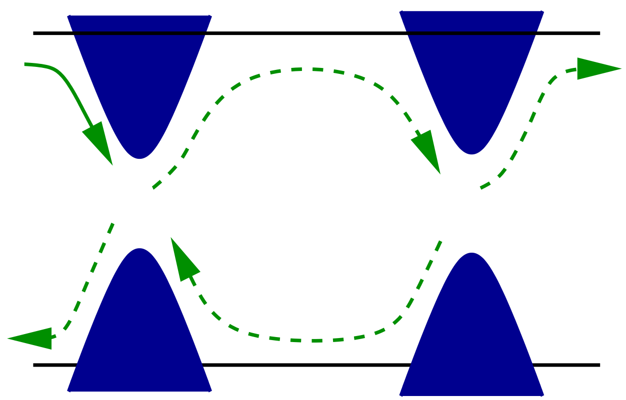
Mach-Zehnder



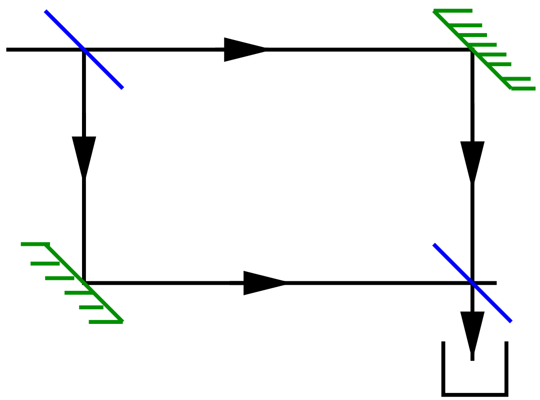
Edge State Interferometer Design

Fabry-Perot



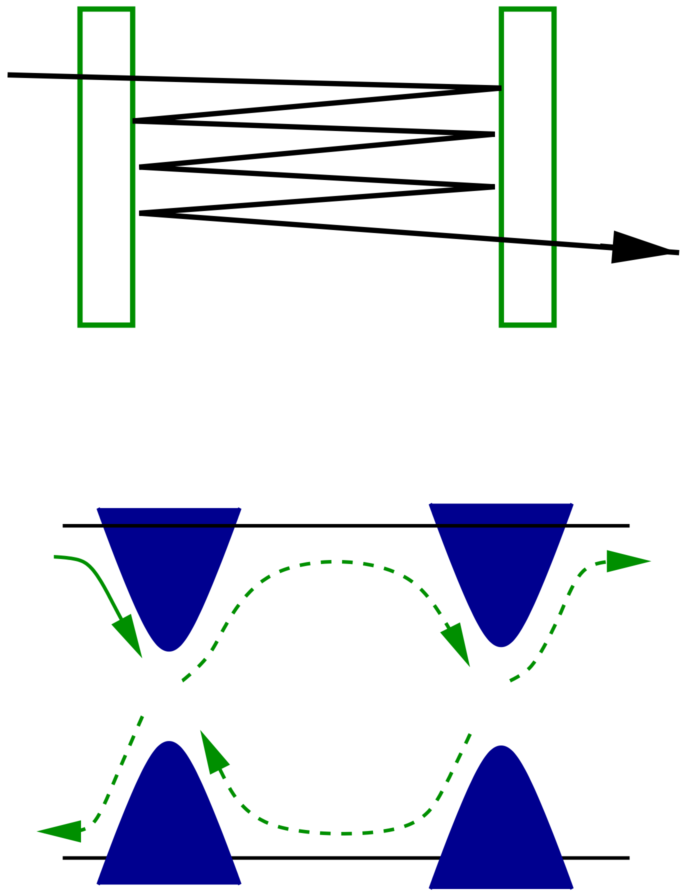


Mach-Zehnder

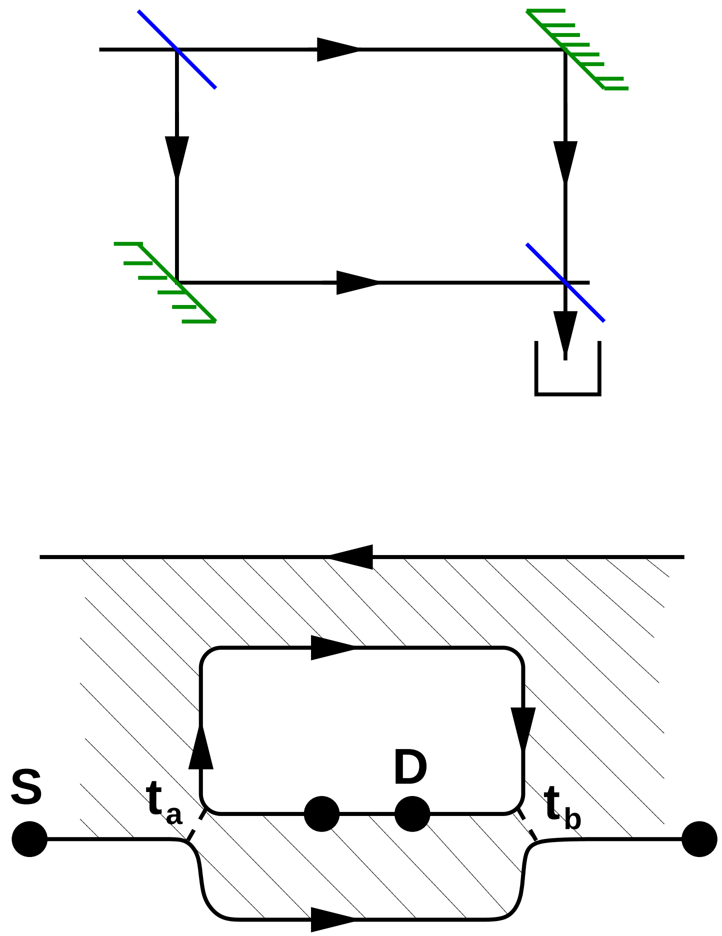


Edge State Interferometer Design

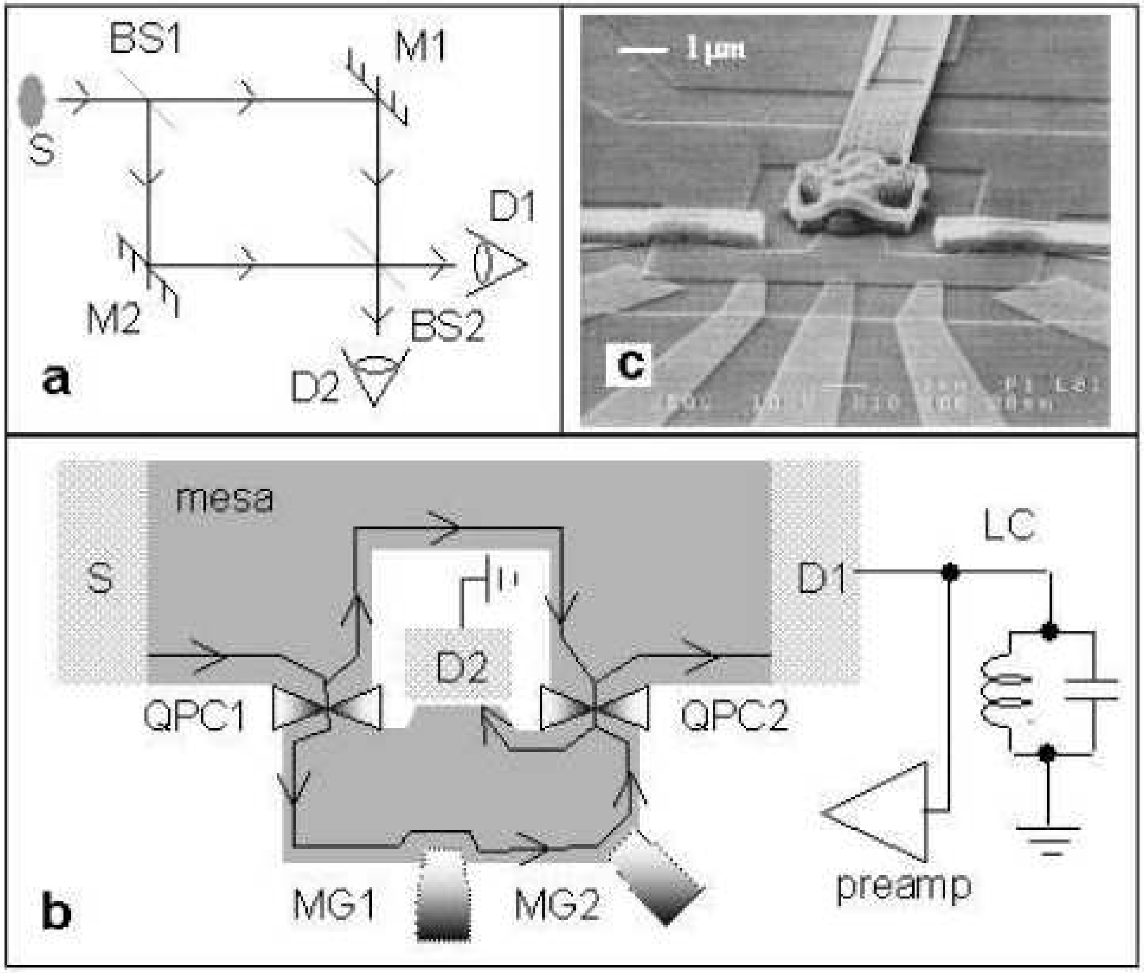
Fabry-Perot



Mach-Zehnder

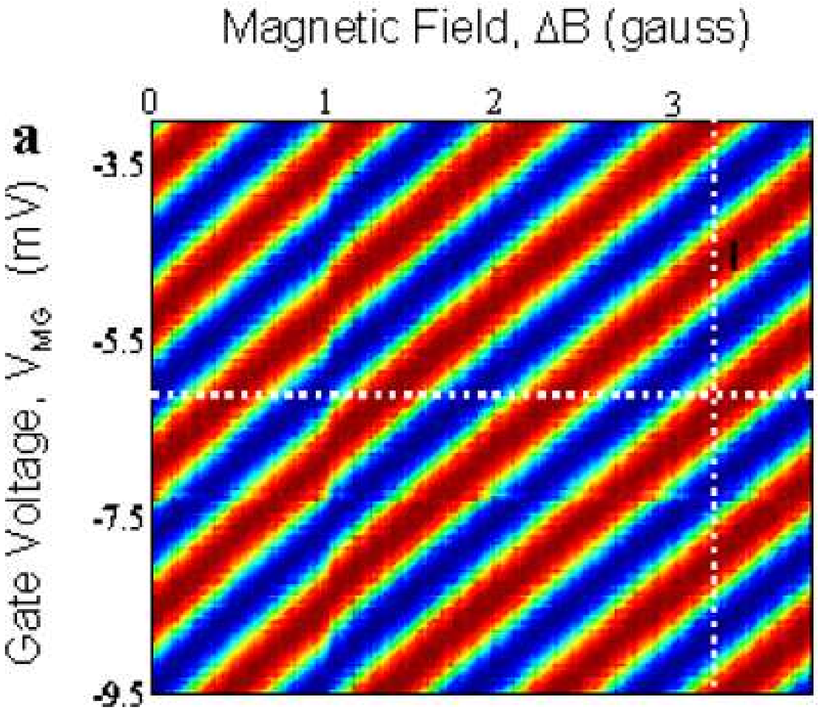


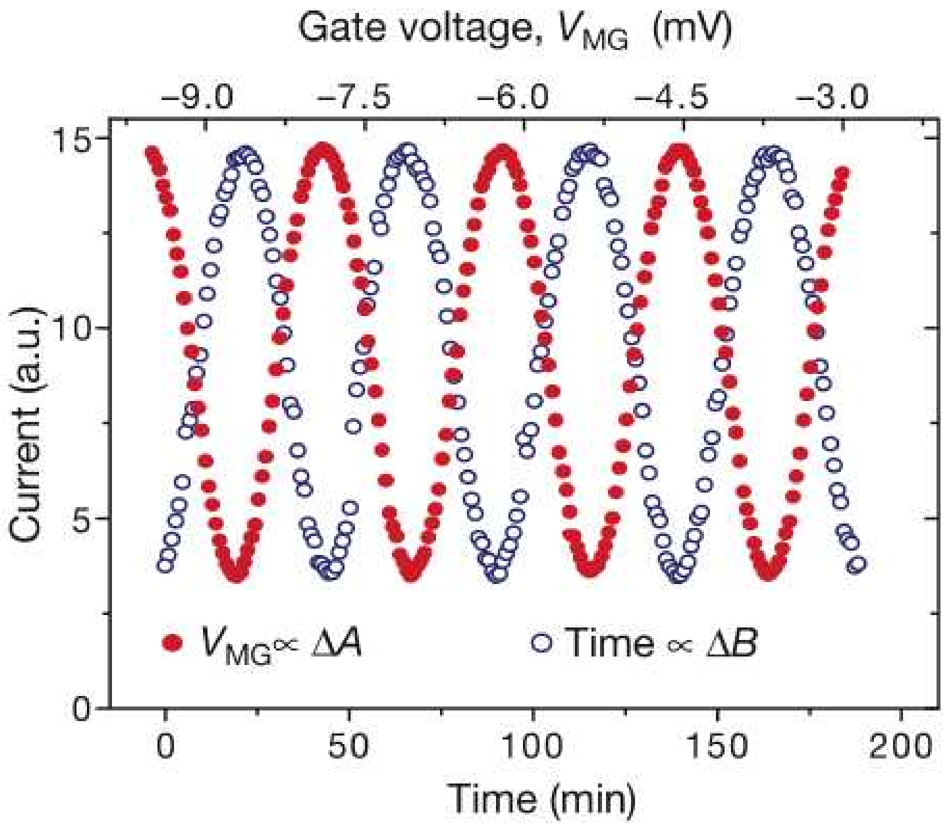
Experimental system



Heiblum Group, Weizmann Institute

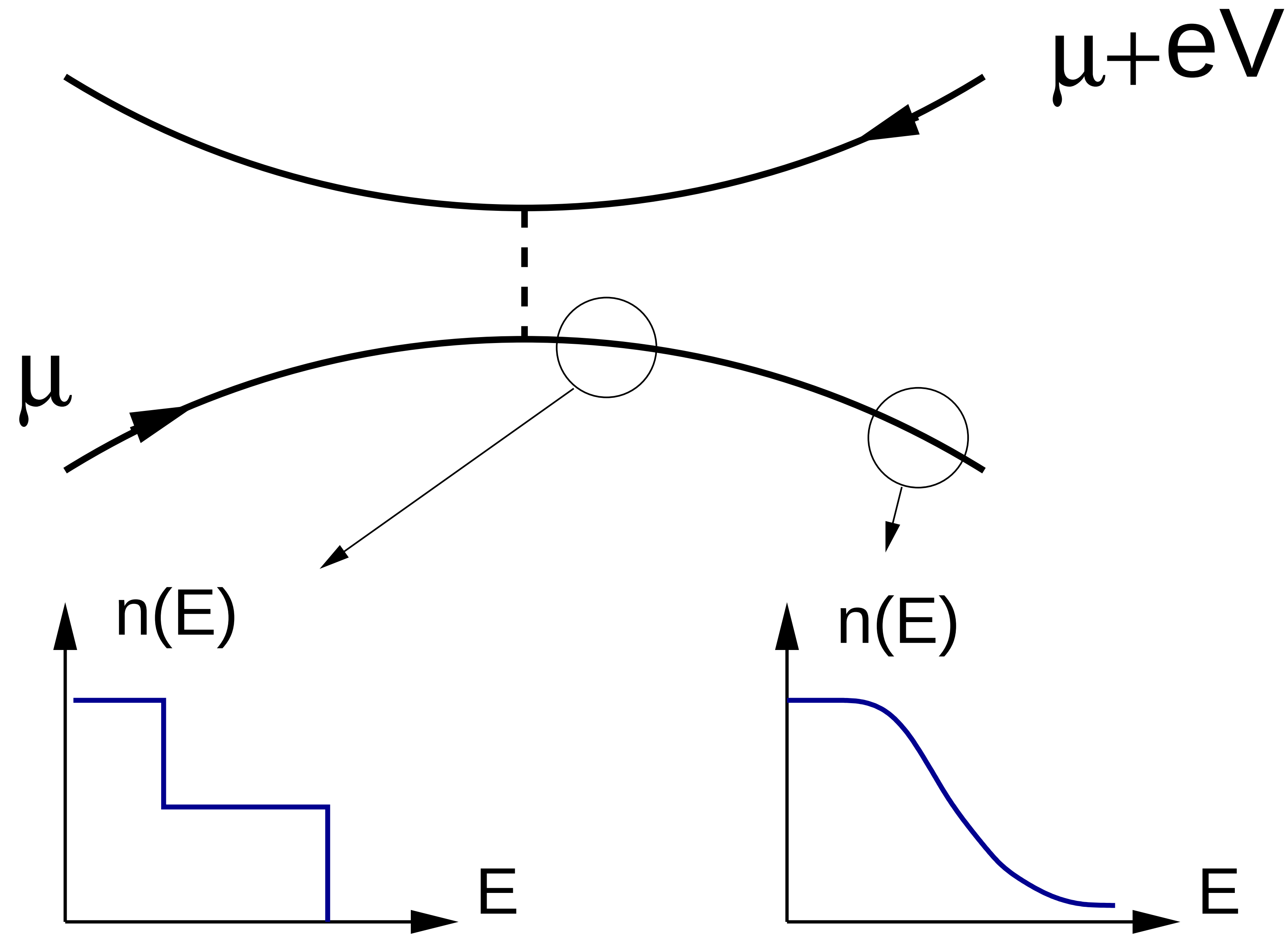
Fringes in Edge State Interferometer





G_{SD} **vs** $Flux\ density$ **and** $Area$

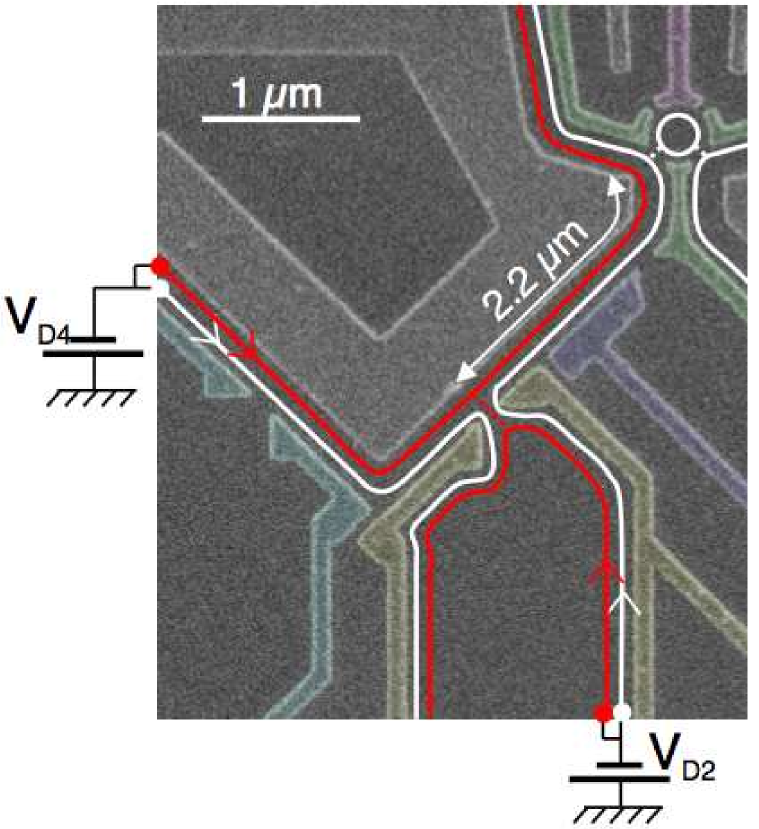
Edge states far from equilibrium



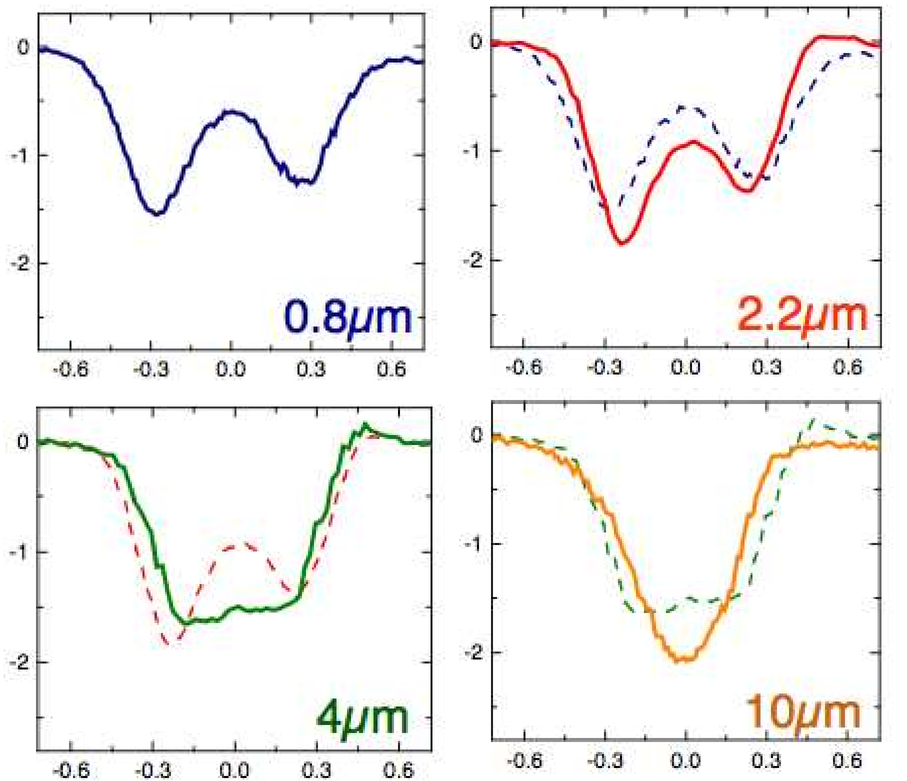
Experiment

le Sueur, Altimiras, Gennser, Cavanna, Mailly & Pierre, PRL (2010)

Sample Design



Evolution of Distribution



$\partial n(E)/\partial E$ **vs.** E

Theoretical Issues

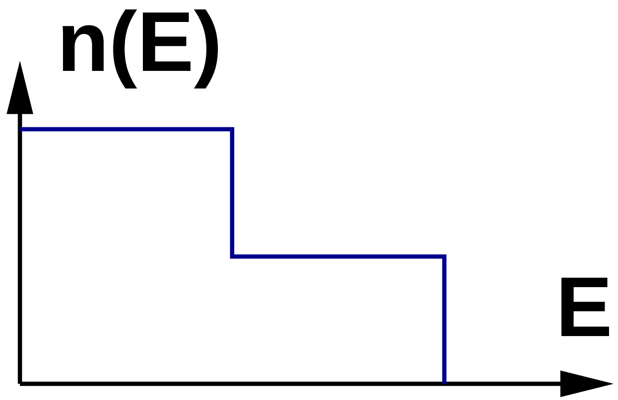
How does equilibration occur in isolated quantum system?

For approx theory with QPC see: Lunde *et al*, (2010) & Degiovanni *et al* (2010)

Analogies with quantum quench problems

Initial state

$|\Psi_0\rangle$ **with**



Time evolution

$$|\Psi(t)\rangle = \mathrm{e}^{\mathrm{i}\mathcal{H}t}|\Psi_0\rangle$$

Properties of $|\Psi(t)\rangle$ **?**

Energies of collective modes conserved

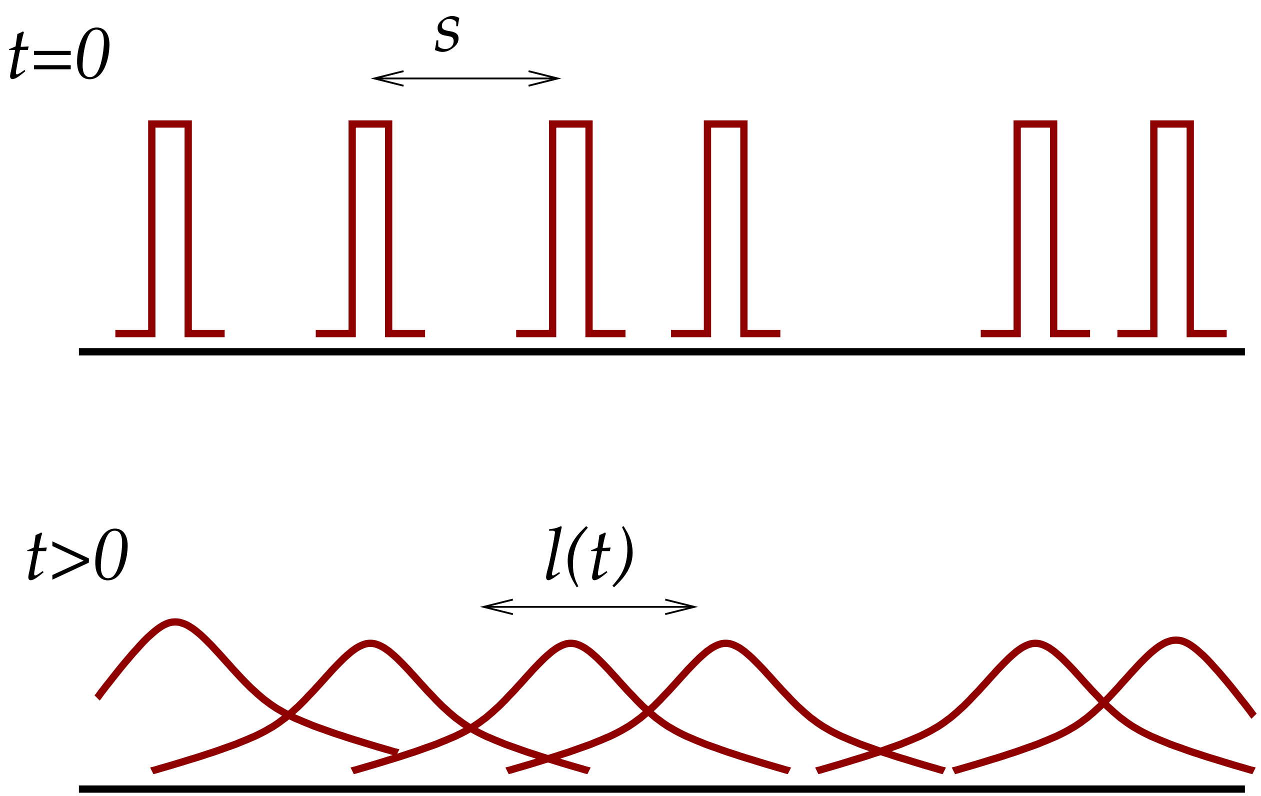
— consequences for equilibration?

Physical picture of equilibration

Collective mode Hamiltonian $\mathcal{H} = \sum_{nq} \hbar\omega_n(q) b^\dagger_{nq} b_{nq}$

Edge magnetoplasmon dispersion $\longrightarrow$ electron equilibration?

Initial quasi-particle separation $s = \hbar v / eV$



Equilibration when wavepacket spread $l(t) \gtrsim s$

Equilibration from two mode velocities

Two edge modes with short-range interactions

Two linearly dispersing modes $\omega_1(q) = v_+ q$ **&** $\omega_2(q) = v_- q$

Initial quasi-particle separation $s = \hbar v / eV$

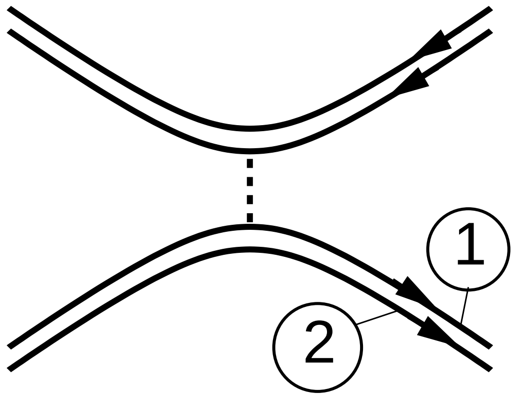
Equilibration when wavepacket spread $l(t) \gtrsim s$

Spread $l(t) = [v_+ - v_-]t$

Equilibration time: $t_{\mathrm{eq}} \sim \frac{\hbar}{eV} \cdot \frac{v_+ + v_-}{v_+ - v_-}$

Equilibration distance: $t_{\mathrm{eq}} \times v_\pm$

Exact solution



Each edge

$$\mathcal{H} = \mathcal{H}_{\text{kin}} + \mathcal{H}_{\text{int}}$$

$$\mathcal{H}_{\text{kin}} = -\mathrm{i}\hbar v \int \left(\Psi_1^\dagger(x)\partial_x\Psi_1(x) + \Psi_2^\dagger(x)\partial_x\Psi_2(x) \right) \mathrm{d}x$$

$$\mathcal{H}_{\text{int}} = 2\pi\hbar g \int \rho_1(x)\rho_2(x)\mathrm{d}x\,, \qquad \rho_n(x) = \Psi_n^\dagger(x)\Psi_n(x)$$

Bosonize and diagonalise

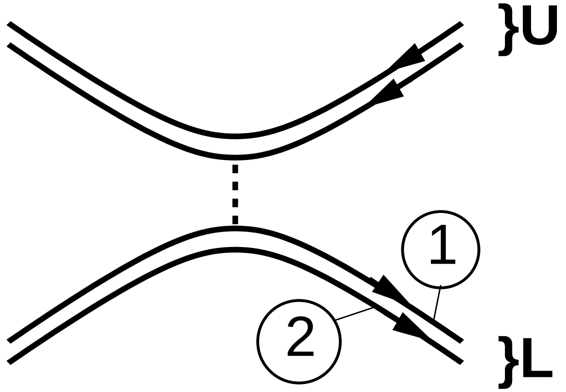
$$\Psi_{1,2}(x) \sim \mathrm{e}^{-\mathrm{i}\varphi_{1,2}(x)}$$

$$\mathcal{H} = \frac{\hbar v_+}{2} \int [\partial_x\varphi_+(x)]^2 \frac{\mathrm{d}x}{2\pi} + \frac{\hbar v_-}{2} \int [\partial_x\varphi_-(x)]^2 \frac{\mathrm{d}x}{2\pi}$$

Modes

$$v_\pm = v \pm g \qquad \varphi_\pm(x) = \frac{1}{\sqrt{2}}[\varphi_1(x) \pm \varphi_2(x)]$$

Refermionize



Combine bosons from opposite edges

$$\varphi_{S\pm} = \frac{1}{\sqrt{2}}[\varphi_{U\pm}(x) + \varphi_{L\pm}(x)]$$

$$\varphi_{A\pm} = \pm\frac{1}{\sqrt{2}}[\varphi_{U\pm}(x) - \varphi_{L\pm}(x)]$$

Tunneling

$$\mathcal{H}_{\mathrm{tun}} = t_{\mathrm{QPC}}[\Psi^{\dagger}_{U1}(0)\Psi_{L1}(0) + \Psi^{\dagger}_{L1}(0)\Psi_{U1}(0)]$$

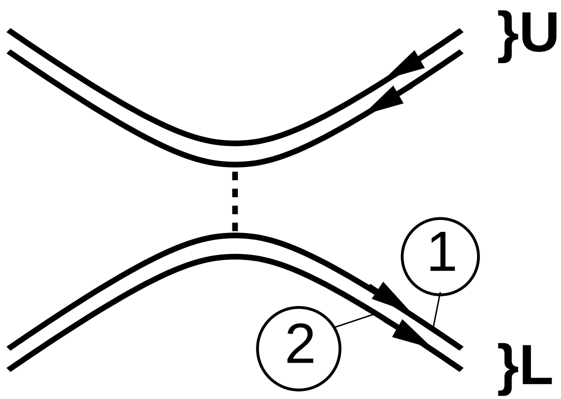
$$\Psi^{\dagger}_{1}(0)\Psi_{2}(0) \sim \mathrm{e}^{\mathrm{i}[\varphi_{U1}(0)-\varphi_{L1}(0)]} \sim \mathrm{e}^{\mathrm{i}[\varphi_{A+}(0)-\varphi_{A-}(0)]} \sim \Psi^{\dagger}_{A+}(0)\Psi_{A-}(0)$$

Edges

$$\mathcal{H}_{\mathrm{kin}}+\mathcal{H}_{\mathrm{int}} = -\mathrm{i}\hbar\int[v_{+}\Psi^{\dagger}_{A+}(x)\partial_x\Psi_{A+}(x)+v_{-}\Psi^{\dagger}_{A-}(x)\partial_x\Psi_{A-}(x)]\mathrm{d}x$$

$$+[\mathrm{A}\leftrightarrow\mathrm{S}]$$

Observables



Tunneling density of states at x from

$$\langle \Psi^\dagger_{L1}(x,t)\Psi_{L1}(x,0)\rangle$$

or

$$\langle \Psi^\dagger_{L2}(x,t)\Psi_{L2}(x,0)\rangle$$

Transforms to $\langle \mathrm{e}^{\mathrm{i}\pi(n_- - n_+)}\rangle$ **or** $\langle \mathrm{e}^{\mathrm{i}\pi(n_- + n_+)}\rangle$

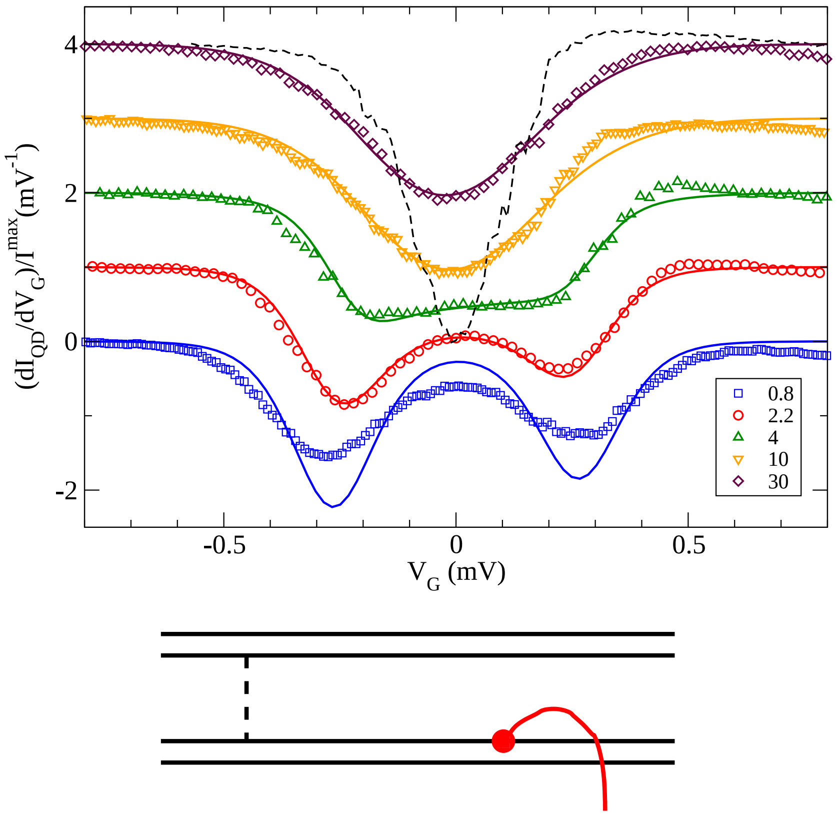
with $n_\pm = \int_x^{x+v_\pm t} :\Psi^\dagger_{A\pm}(y)\Psi_{A\pm}(y): \, \mathrm{d}y$

Large x – recover results from quantum quench

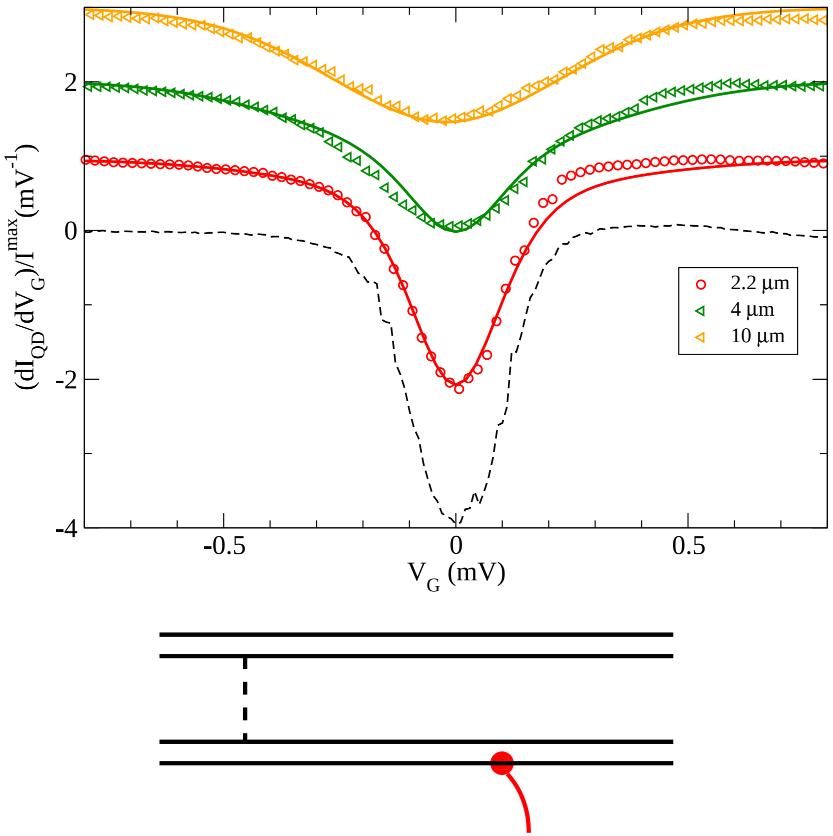
General x – evaluate free fermion averages numerically

Comparison with experiment

Measurement in channel coupled at QPC



Measurement in channel coupled by interactions



Summary

Coherent many-body quantum dynamics

observed in QH edge states

Interferometer demonstrates

coherence in QH edge states

Quantum evolution far from equilibrium

probed in experiments with point contacts