QCD in an external magnetic field

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TIFR Mumbai, 20.2.12

Contents

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arXiv:1111.4956 [hep-lat], JHEP in print,
arXiv:1111.5155, PoS(Lattice 2011) 192.

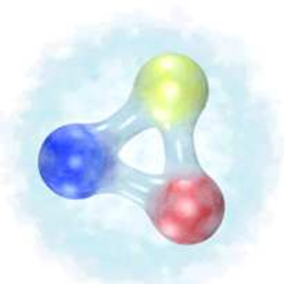
QCD (theory of strong interactions)

$$\mathcal{L}_{QCD} = -\frac{1}{16\pi\alpha_s}FF + \bar{\psi}_f(\not{D} + m_f)\psi_f$$

$\longrightarrow$ asymptotic freedom: $\alpha_s(q) \xrightarrow{q\to\infty} 0$

$\xrightarrow{?}$ confinement

$\xrightarrow{?}$ chiral symmetry breaking

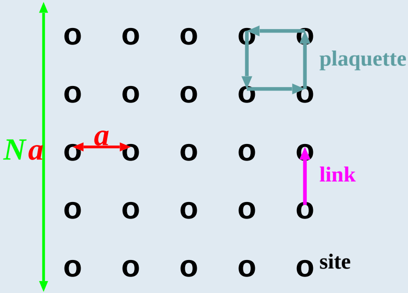


proton (artist's impression)

Theoretically beautiful but analytical quantitative predictions are very difficult in the region of small momentum transfers (*strong* QCD) !

$\Longrightarrow$ computer simulation

Lattice QCD



typical values:

$a^{-1} = 1.5$–4 GeV, $Na = 1.5$–6 fm

continuum limit: $a \to 0$, La fixed

infinite volume: $Na \to \infty$

$$\langle O \rangle = \frac{1}{Z} \int [dU]\,[d\psi][d\bar{\psi}]\; O[U] e^{-S[U,\psi,\bar{\psi}]}$$

"Measurement": average over a *representative* ensemble of gluon configurations $\{U_i\}$ with probability $P(U_i) \propto \int [d\psi][d\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$

$$\langle O \rangle = \frac{1}{n} \sum_{i=1}^{n} O(U_i) + \Delta O \qquad \Delta O \propto \frac{1}{\sqrt{n}} \xrightarrow{n\to\infty} 0$$

Input: $\mathcal{L}_{QCD} = -\frac{1}{16\pi\alpha_L} FF + \bar{\psi}_f (\not{D} + m_f)\psi_f$

$$m_p^{\text{lat}} = m_p^{\text{phys}} \longrightarrow a$$
$$m_{\text{PS}}^{\text{lat}}/m_p^{\text{lat}} = m_\pi^{\text{phys}}/m_p^{\text{phys}} \longrightarrow m_u \approx m_d$$
$$\ldots$$

Output: hadron masses, phase diagram, decay constants etc...

Extrapolations:

1. $a \rightarrow 0$: functional form known.
2. $N \rightarrow \infty$: harmless but computationally expensive.
3. $m_q^{\text{lat}} = m_q^{\text{phys}}$ has only very recently been realized.

Pure gauge theory

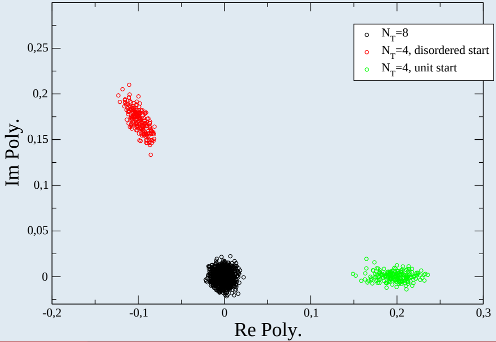
$\mathcal{L}_{YM} = -\frac{1}{16\pi\alpha_L} FF$

Consider lattice of time extent $N_t a = 1/T$ and spatial volume $V = (N_s a)^3$.

Order parameter: Polyakov line $\langle P \rangle \sim \exp(-F_q/T)$

Low temperature: $\langle P \rangle = 0$, confinement.

High temperature: $\langle P \rangle \propto z$, $z \in \mathbb{Z}_3$, deconfinement ($V \to \infty$).



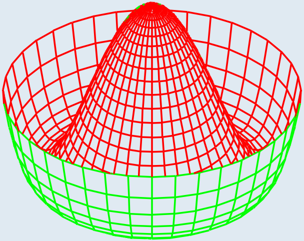
Chiral symmetry (breaking)

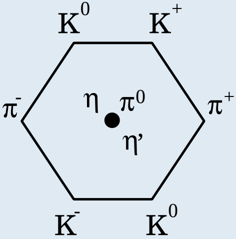
Global symmetry of the $m = 0$, $n_f = 3$ QCDlite™:

Ignore $U_V(1)$ symmetry (baryon number conservation)
$U_A(1)$ anomaly: $\partial_\mu j_\mu^5 = -\frac{1}{16\pi^2} F * F \longrightarrow$ heavy η'
$m = 0$ χ-symmetry *spontaneously* broken
at $T < T_c$ (order parameter $\langle\bar{\psi}\psi\rangle$):

$$SU_L(3) \otimes SU_R(3) \longrightarrow SU_V(3)$$

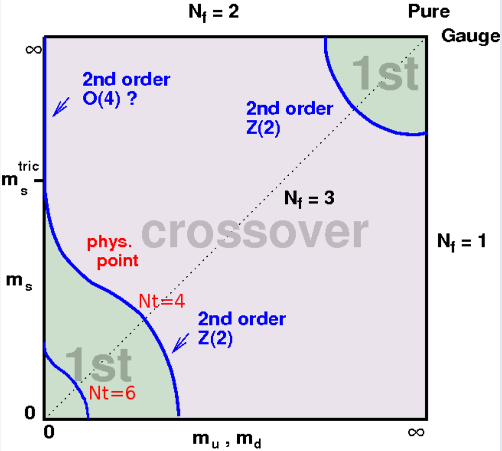
8 Nambu-Goldstone bosons!





QCD with masses

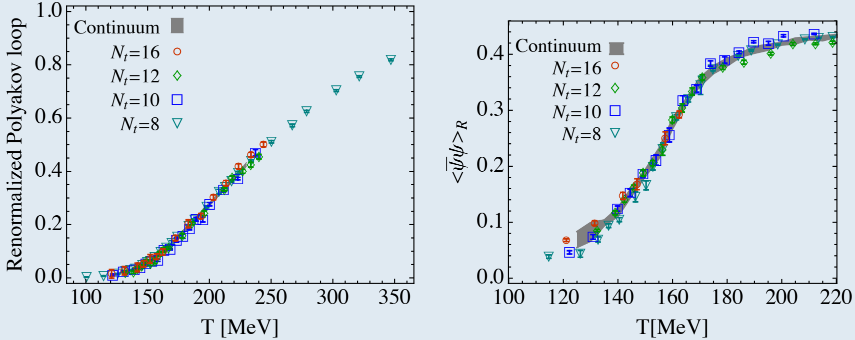
Columbia plot FR Brown et al, PRL 65 (90) 2491



(borrowed from O Philipsen arXiv:1111.5370)

Polyakov line and chiral condensate for physical quarks

S Borsányi et al JHEP 1009 (10) 073

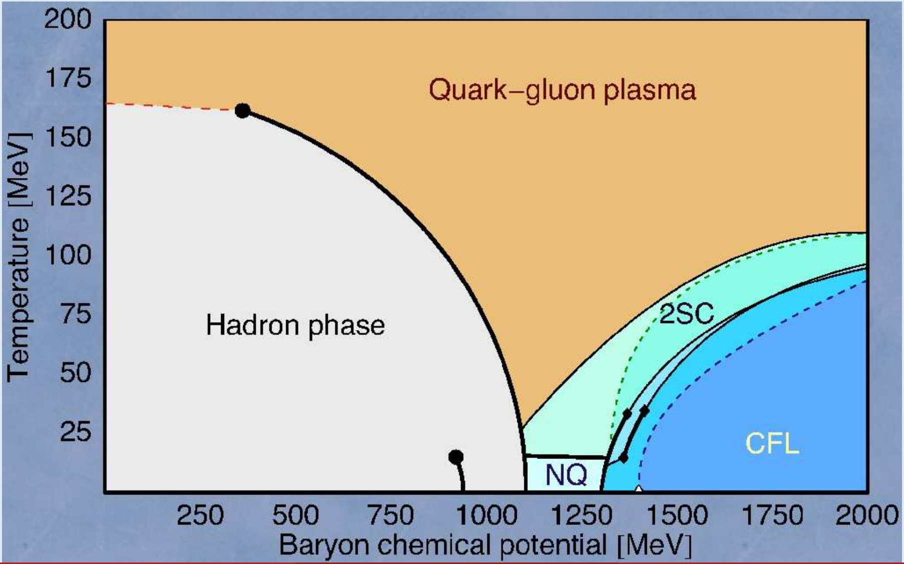


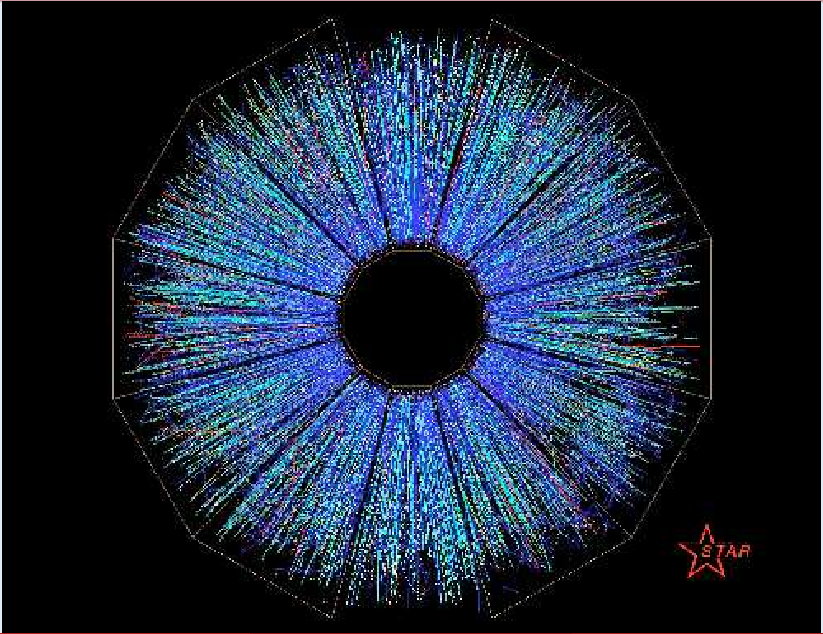
$T_c(\bar{\psi}\psi) = 155(3)(3)$ MeV$= 1.95(5) \cdot 10^{12}$ K.

(Cross-over: other quantities may have different pseudocritical temperatures.)

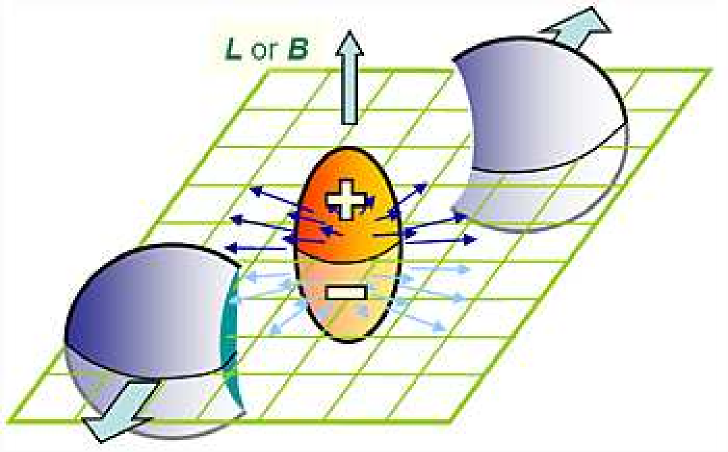
$\exists$ no order parameter but less than one in $2.1 \cdot 10^{21}$ electrically charged particles differ by more than $e/6$ from a multiple of e!

A possible phase diagram of QCD with chemical potential




STAR

Noncentral heavy ion collision



$$(100\ \text{MeV}^2) \approx 1.69 \cdot 10^{18} e\text{G} = 1.69 \cdot 10^{14} e\text{T}.$$

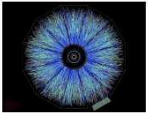
Comparison of magnetic fields



The Earths magnetic field	0.6 Gauss
A common, hand-held magnet	100 Gauss
The strongest steady magnetic fields achieved so far in the laboratory	4.5×10^5 Gauss
The strongest man-made fields ever achieved, if only briefly	10^7 Gauss
Typical surface, polar magnetic fields of radio pulsars	10^{13} Gauss
Surface field of Magnetars	10^{15} Gauss

http://solomon.as.utexas.edu/~duncan/magnetar.html

Slide of D. Kharzeev



At BNL we beat them all!

Off central Gold-Gold Collisions at 100 GeV per nucleon

$$e\,B(\tau=0.2\,\mathrm{fm}) = 10^3 \sim 10^4\ \mathrm{MeV}^2 \sim 10^{17}\ \mathrm{Gauss}$$

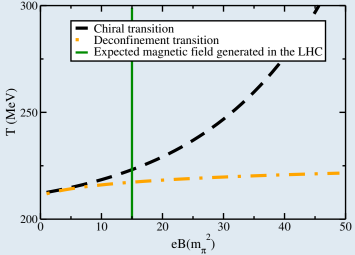
The QCD phase diagram in the B-T plane

Low energy effective models of QCD predict(ed):

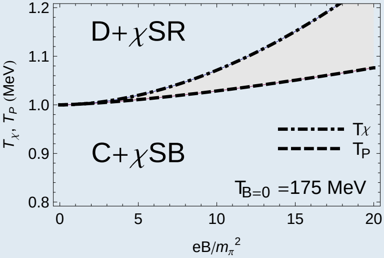
- increasing pseudocritical temperature $T_c(B)$
- increasing strength $1/W(B)$ Mizher et al 10

Supported by NJL models, large-N_c arguments, low-dimensional models, S-D equations

Gatto et al 11, Johnson et al 09, Alexandre et al 01, Klimenko et al 92, Kanemura et al 98



AJ Mizher et al PRD 82 (10) 105016

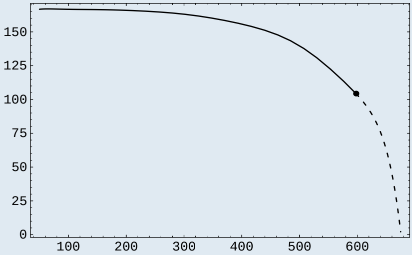


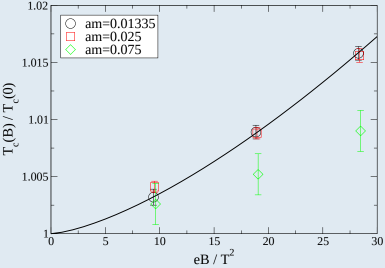
R Gatto, M Ruggieri PRD 83 (11)

Other results ($N_f = 2$):

Decreasing $T_c(B)$ NO Agasian, SM Fedorov PLB 663 (08) 445

Almost constant (Lattice) M D'Elia et al PRD 82 (10) 051501





Magnetic background field on the lattice

Vector potential $A_\nu = (0, Bx, 0, 0) \Longrightarrow \mathbf{B} = (0, 0, B)$

Lattice: multiply links U_ν with $u_\nu = e^{iaqA_\nu} \in \mathrm{U}(1)$

$$\begin{aligned} u_y(n) &= e^{ia^2 qBn_x} \\ u_x(n) &= 1 \qquad\qquad n \neq N_x - 1 \\ u_x(N_x - 1, n_y, n_z, n_t) &= e^{-ia^2 qBN_x n_y} \\ u_\nu(n) &= 1 \qquad\qquad \nu \neq x, y \end{aligned}$$

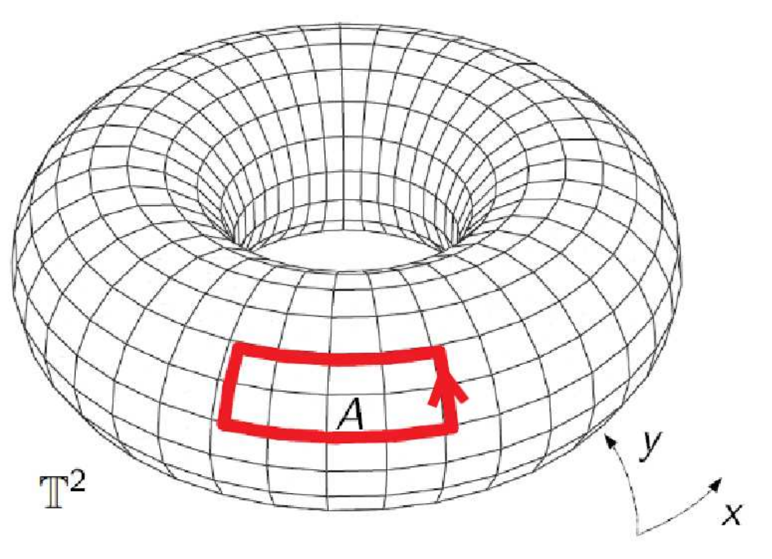
The magnetic flux through the x-y plane is constant:

$$\exp\left(iq \int_F \mathrm{d}\sigma\, \mathbf{B} \right) = \exp\left(iq \int_{\partial F} \mathrm{d}x_\nu\, A_\nu \right) = e^{ia^2 N_x N_y qB}$$

Flux quantization due to the finite volume + boundary conditions:

$$a^2 N_x N_y \cdot qB = 2\pi N_b \qquad N_b \in \mathbb{Z}$$

Flux quantization



Implementation and limitations

- B is invariant under $N_b \leftrightarrow N_b + N_x N_y$ (periodicity)
- Lattice field is unambiguous if $0 < N_b < N_x N_y / 4$
- Apply quantization for smallest charge $q = e/3$
- Typical lattice spacings:
  Maximal B: $qB^{\max} = \pi/(2a^2)$
  $\sqrt{eB} \approx 1\,\mathrm{GeV} \to 10^{20} e$ Gauss
- Typical aspect ratios:
  Minimal B: $qB^{\min} = 2\pi T^2 (N_t/N_s)^2$
  $\sqrt{eB} \approx 0.1\,\mathrm{GeV} \to 10^{18} e$ Gauss
  Phenomenologically interesting region!

Simulation and observables

- Partition function for three flavors ($\mu_f = 0$ in the simulation)

$$\mathcal{Z} = \int [dU]\, e^{-\beta S_g} \prod_{f=u,d,s} [\det M(q_f \cdot B, m_f, \mu_f)]^{1/4}$$

- Observables

$$\bar{\psi}\psi_f = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial m_f}, \qquad \chi_f = \frac{T}{V}\frac{\partial^2 \log \mathcal{Z}}{\partial m_f^2}, \qquad c_2^s = \frac{T}{V}\frac{\partial^2 \log \mathcal{Z}}{\partial \mu_s^2}$$

- Renormalization: cancel divergences by computing

$$\bar{\psi}\psi_f^r(B,T) = m_f \left[\bar{\psi}\psi_f(B,T) - \bar{\psi}\psi_f(B=0,T=0)\right]\frac{1}{m_\pi^4}$$

$$\chi_f^r(B,T) = m_f^2 \left[\chi_f(B,T) - \chi_f(B=0,T=0)\right]\frac{1}{m_\pi^4}$$

Does B get renormalized?

Does B induce any new divergencies? If so it has to be renormalized.
Other renormalization factors may even become B-dependent.
Quark masses for instance get renormalized.
B breaks rotational symmetry and isospin symmetry.
B modifies the free dispersion relation:

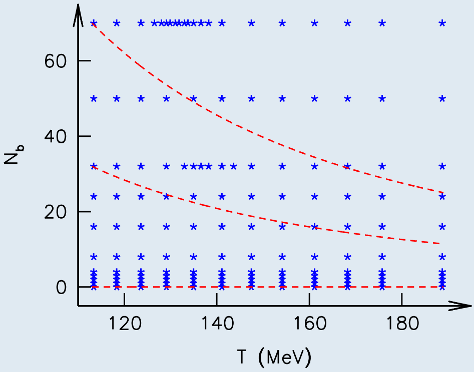
$$E(B) = \sqrt{p_z^2 + m^2 + 2n|qB|}$$

So maybe something sinister or complicated is happening?
Fortunately not:

- Protected by U(1) gauge invariance: $(e \cdot B)^r = e \cdot B$ due to Ward-Takahashi identity $Z_e\sqrt{Z_3} = 1$.
- B couples to a conserved current $A_\nu \bar{\psi}\gamma_\nu\psi$.
- For an external field there are no internal photon lines in Feynman diagrams $\rightarrow$ no new type of divergent diagram.

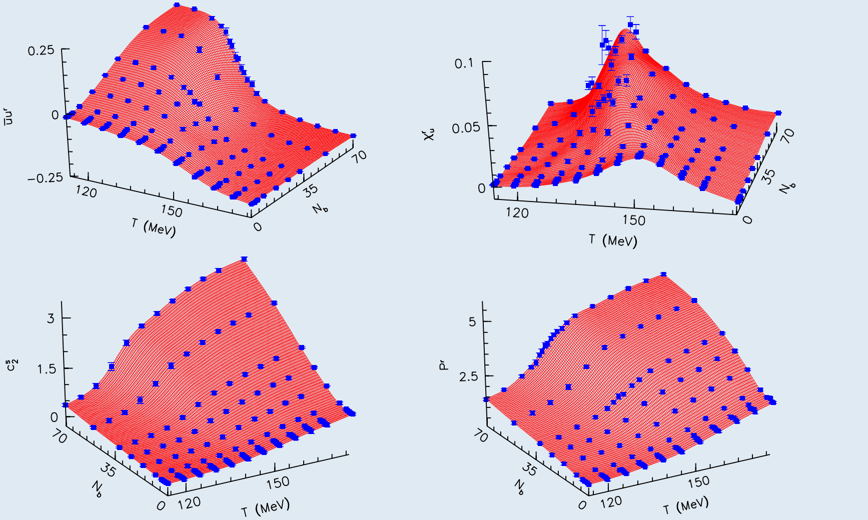
Simulation and analysis details

Symanzik improved gauge action, $N_f = 2 + 1$ stout smeared staggered quarks at physical masses (Budapest-Wuppertal) action

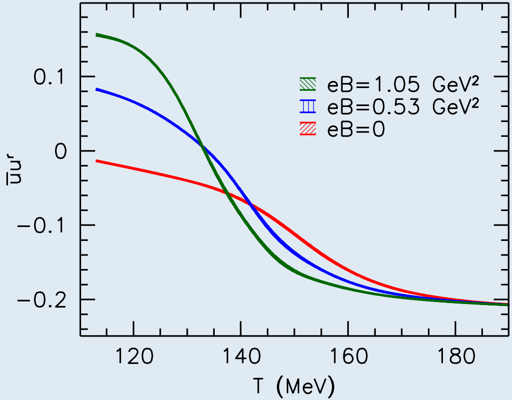


- Simulate at various T and N_b.
- Fit all points by a 2D spline function.
- Keep physical B fixed.
- Study finite volume effects with $N_s/N_t = 3, 4, 5$
- Extrapolate to the continuum limit with $N_t = 6, 8, 10$.

Results: overview

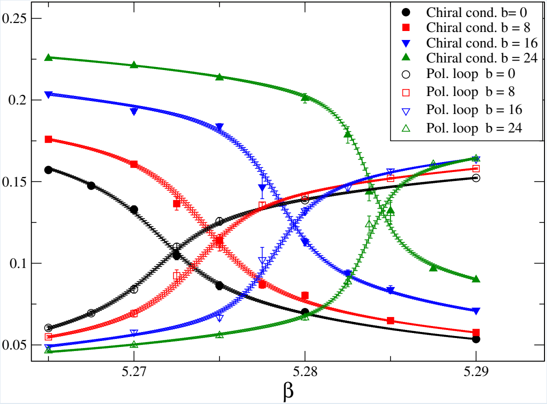


Chiral condensate



$\bar{\psi}\psi$ decreases with B in the transition region.
$T_c(B)$ decreases with B

Previous study by D'Elia et al



$\bar{\psi}\psi$ always grows with B.

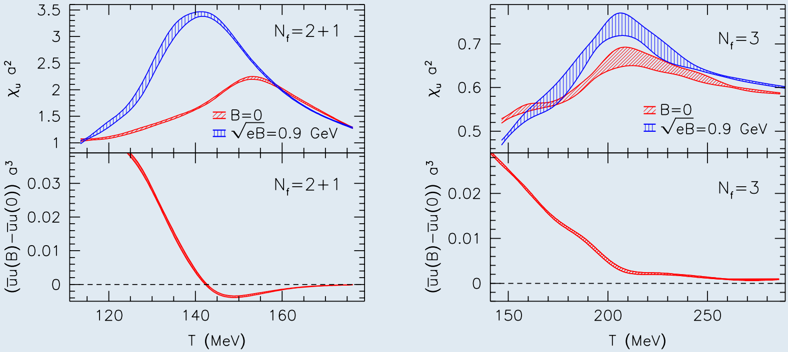
$T_c(B)$ increases with B (larger $\beta = 3/(2\pi\alpha)$: smaller $N_t a$, higher T).

The transition becomes narrower with bigger B.

Comparison with D'Elia et al

	D'Elia et al	Present study
discretization errors	$N_t = 4$	$N_t = 6, 8, 10$
	naive staggered + Wilson	stout + Symanzik
quark flavours	$N_f = 2$	$N_f = 2 + 1$
light quark mass	$m_\pi = 195$ MeV	$m_\pi = 135$ MeV

Quark mass dependence



$\bar{u}u(B, T) - \bar{u}u(0, T)$: condensates are significantly different!

Shift in the χ_u peak positions!

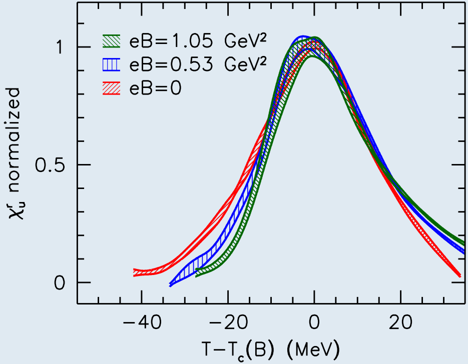
$\Rightarrow$ dramatic dependence on the light quark mass!

Width of the transition

At $B = 0$: broad crossover. What happens at $B > 0$?

Height of the peak increases.

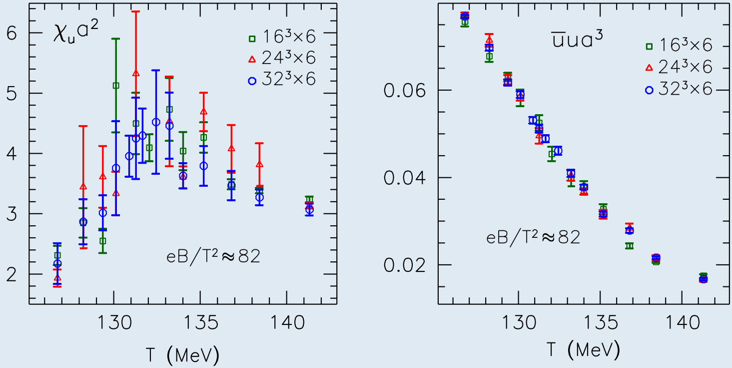
However when normalized to the same height...



... not much changes.

Finite size scaling

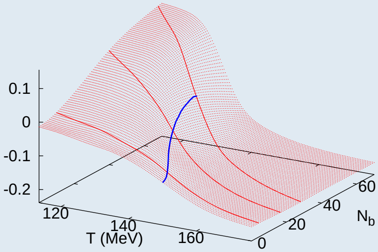
Comparison between $N_s = 16, 24, 32$ on $N_t = 6$ at $eB/T^2 \approx 82$
(Largest volume: $V \approx 7\mathrm{fm}^3$)



The crossover persists up to $\sqrt{eB} = 1$ GeV!

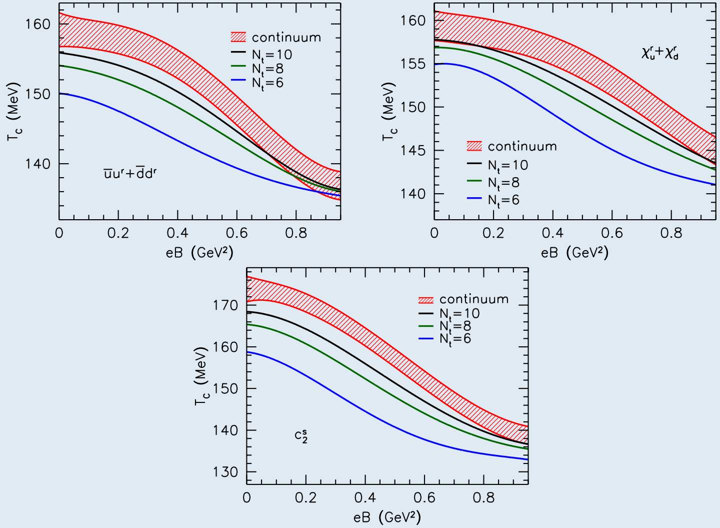
Transition temperature

- Analyze $B = \mathrm{const}$ slices of the 2D surfaces
- Define transition temperatures by
  - inflection point for $\bar{\psi}\psi^r$ and c_2^s
  - peak position for χ^r
- E.g. for the condensate:

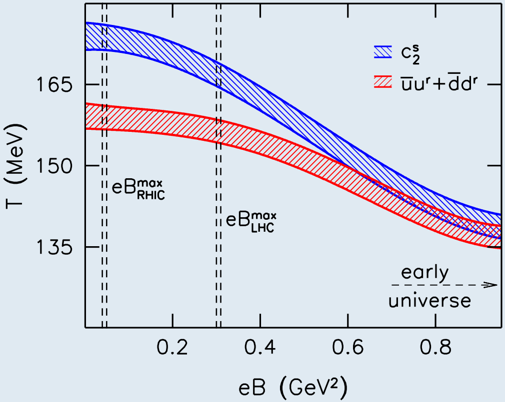


- Then fit various N_t results to $T_c(B, N_t) = T_c(B) + b(B)/N_t^2$.

Phase diagram I



Phase diagram II



The effect is negligible for RHIC.
The temperature reduces by less than $5-10\%$ for LHC.
The effect may be significant in the early universe.

Summary and Outlook

- QCD with and external (electro-)magnetic field is interesting
- Lattice discretization & finite size effects are under control
- Phase diagram: **decreasing** $T_c(B)$
  - complex, non-monotonic dependence in $\bar{\psi}\psi(B, T)$
  - the crossover persists for large magnetic fields
  - no critical endpoint below $\sqrt{eB} \approx 1$ GeV
- Other questions are under investigation
  - magnetic susceptibility of the QCD vacuum
  - effects of magnetic fields on instanton shapes
  - the hadronic Zeemann/Paschen-Back/cigar-shape effect