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THE ICTS SRINIVASA RAMANUJAN LECTURE SERIES



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The ICTS Srinivasa Ramanujan Lectures are delivered by eminent mathematicians and scientists on important developments in their area of speciality. The first lecture in any series is aimed at a general audience, while the remaining lectures are aimed at specialists.

THE GENERALIZED RAMANUJAN CONJECTURES & APPLICATIONS

- ① The Generalized Ramanujan Conjectures
- ② Thin Groups and Expansion
- ③ Mobius randomness and horocycle dynamics
- ④ Nodal lines of Maass forms and critical percolation

Peter Sarnak is a Eugene Higgins Professor of Mathematics at Princeton University, and is also a permanent faculty at the Institute for Advanced Study, Princeton. Peter Sarnak was awarded the *Polya Prize of Society of Industrial & Applied Mathematics* in 1998, the *Ostrowski Prize* in 2001, the *Levi L. Conant Prize* in 2003 and the *Frank Nelson Cole Prize in Number Theory* in 2005. He was also elected as member of the National Academy of Sciences (USA) and Fellow of the Royal Society (UK) in 2002. He was awarded an honorary doctorate by the Hebrew University of Jerusalem in 2010.

A JOINT PROGRAM OF ICTS AND THE SCHOOL OF MATHEMATICS, TIFR

LECTURE 1

THE GENERALIZED RAMANUJAN CONJECTURES

ABSTRACT: One of the central problems in the modern theory of automorphic forms is the Generalized Ramanujan Conjecture. We review the development and formulation of these conjectures as well as recent progress. While the general Conjecture is not known, even for $GL(2)$, strong approximations towards it have been established and we will illustrate how these suffice for various striking applications.

DATE: 21 May 2012 / **TIME:** 11:30 / **VENUE:** AG 66, TIFR, Mumbai

LECTURE 2

THIN GROUPS AND EXPANSION

ABSTRACT: Infinite index subgroups of matrix groups like $SL(n, \mathbb{Z})$ which are Zariski dense in $SL(n)$, arise in many geometric and diophantine problems (eg reflection groups, groups connected with elementary geometry such as integral apollonian packings, monodromy groups of families of algebraic varieties...). One of the key features needed for number theoretic applications is that these groups obey some form of the Ramanujan Conjectures. In this context this asserts that certain congruence graphs associated with these groups are expanders. We will introduce these ideas and review some of the many recent developments.

DATE: 22 May 2012 / **TIME:** 11:30 / **VENUE:** AG 66, TIFR, Mumbai

LECTURE 3

MOBIUS RANDOMNESS AND HOROCYCLE DYNAMICS

ABSTRACT: The Mobius function $\mu(n)$ is minus one to the number of distinct prime factors of n if n has no square factors and zero otherwise. Understanding the randomness (often referred to as the "Mobius randomness principle") in this function is a fundamental and very difficult problem. We will explain a precise dynamical formulation of the randomness and report on recent advances establishing it. In particular the disjointness of the resulting Mobius Flow from horocycle flows and related horocycle dynamics at "prime times".

DATE: 23 May 2012 / **TIME:** 11:30 / **VENUE:** AG 66, TIFR, Mumbai

LECTURE 4

NODAL LINES OF MAASS FORMS AND CRITICAL PERCOLATION

ABSTRACT: We describe some results concerning the number of connected components of nodal lines of high frequency Maass forms on the modular surface. Based on heuristics connecting these to an exactly solvable critical percolation model, Bogomolny and Schmit have conjectured, and numerics confirm, that this number follows an asymptotic law. While proving this appears to be very difficult, some approximations to it can be proved by developing number theoretic and analytic methods (Joint with A. Ghosh and A. Reznikov).

DATE: 24 May 2012 / **TIME:** 11:30 / **VENUE:** AG 66, TIFR, Mumbai

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