Transition to turbulence.

V. Kumaran

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Indian Institute of Science

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- Dr. V. Shankar (Professor & Chairman, IIT Kanpur),
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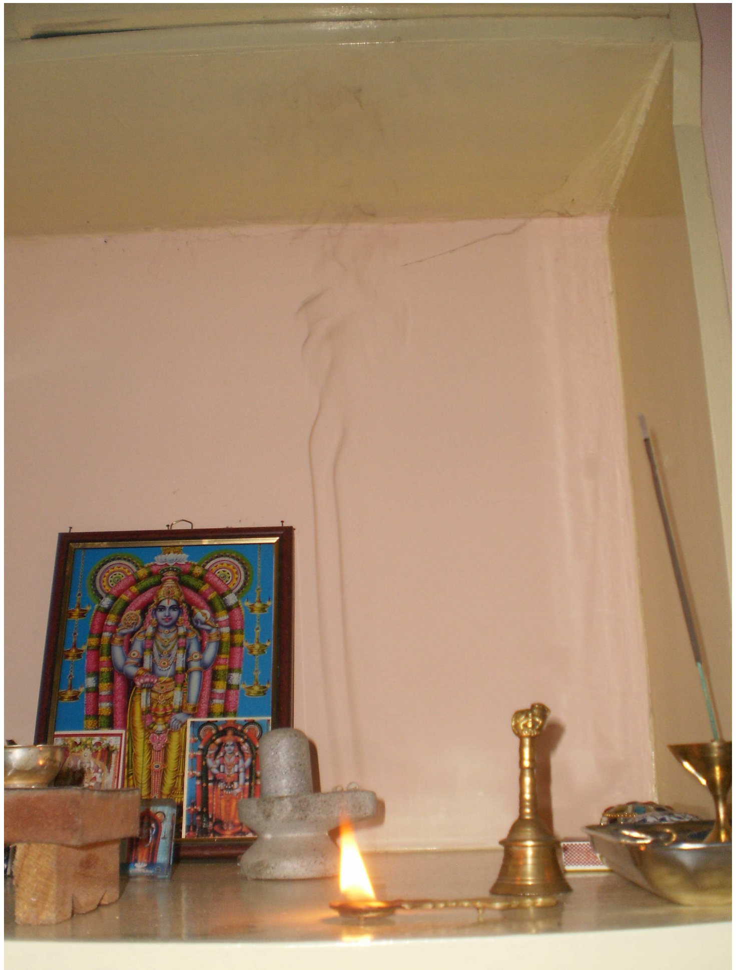
- Department of Science and Technology, Government of India.

Laminar & turbulent flows: Flow from a tap.

Laminar

Turbulent

Laminar & turbulent flows: Smoke rising.



Transition to turbulence:

- Laminar and turbulent flows.
- What is turbulence?
- Why turbulence?

Turbulence characteristics:

1. High velocity (Reynolds number).
2. Irregular.
3. Continuum. But requires statistical description?
4. Highly dissipative.
5. Highly diffusive.

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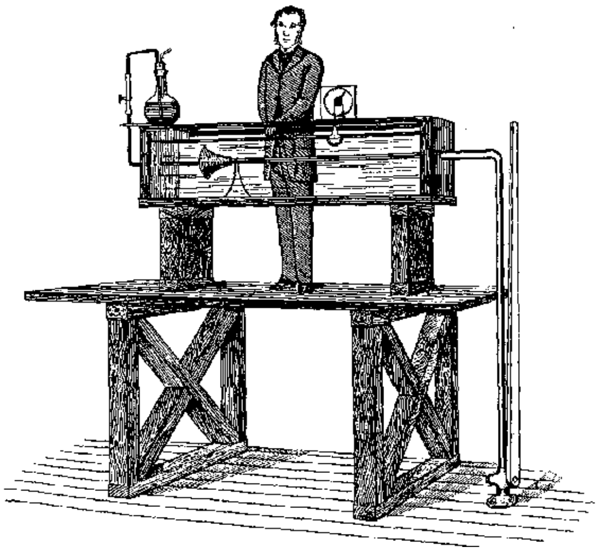
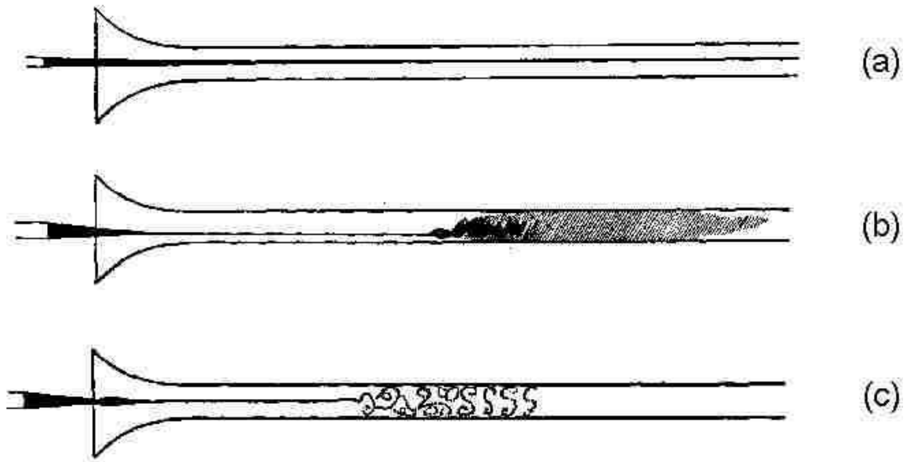
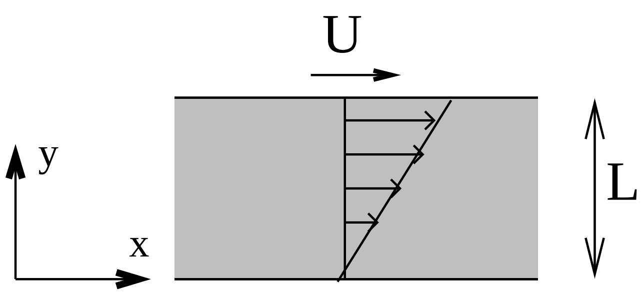
Turbulence: High velocity (Reynolds number)

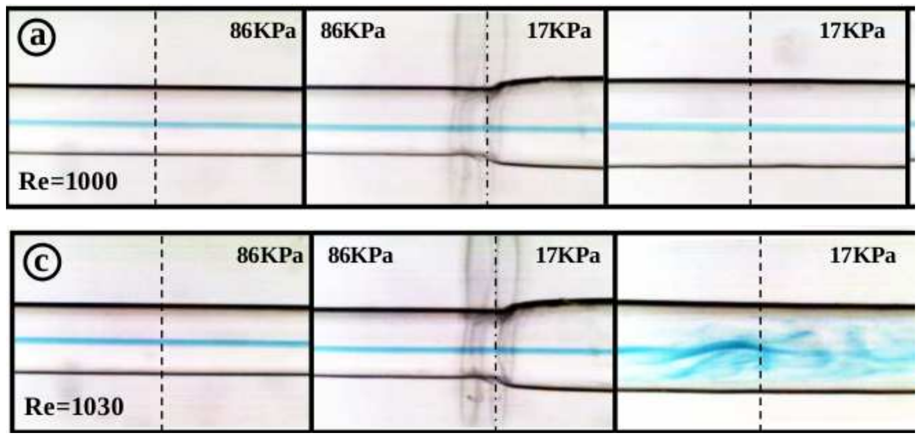




Reynolds number: $(\rho U L/\mu)$

*N. Rott, 1990 Ann. Rev. of Fluid Mech.





M. Verma & V. Kumaran, JFM, 2012.

$$\frac{\text{Force}}{\text{Area}} = \mu \frac{U}{L}$$

Turbulence: High Reynolds number

Transition Reynolds number $10^3 - 10^4$

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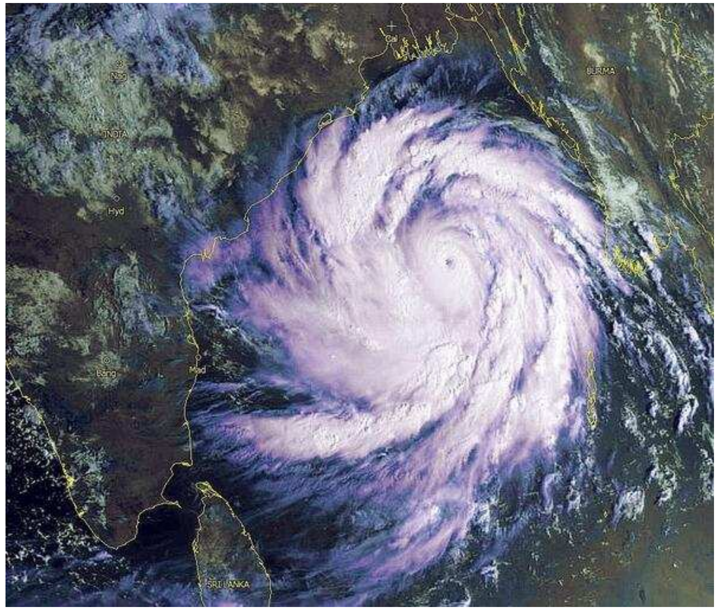
Birds (0.1-3m, 1-10m/s)	$10^4 - 10^5$	
Blood flow aorta	$10^3 - 10^4$	
Stirring sugar in a teacup	10^3	
Insects (1 cm, 0.1-1 m/s)	$10^2 - 10^3$	
Dust (10 - 100 μm, 0.01-0.1 m/s)	$1 - 10$	
Sedimenting sand (100 μm particles, 1 cm/s)	1	
Blood flow (smallest capillaries)	10^{-2}	
Cell migration (100 μm 0.6 μm/min)	10^{-4}	



http://en.wikipedia.org/wiki/File:Cyclone_Phailin_11_October_2013.jpg

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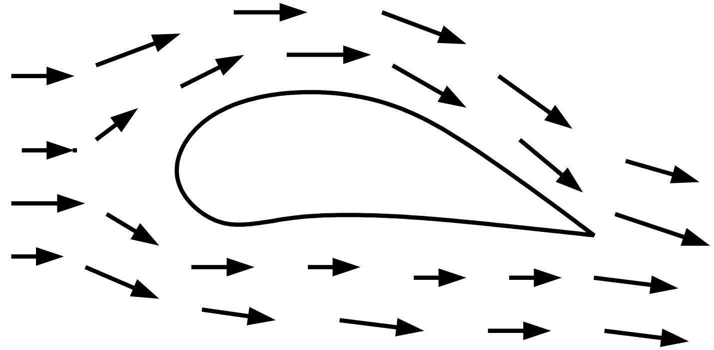

http://www.popularmechanics.com/technology/aviation/safety/4327148

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http://www.celsias.com/article/climate-war/

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http://www.thehindu.com/opinion/op-ed/the-great-dandi-march-eighty-years-after/

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http://en.wikipedia.org/wiki/Shy_Albatross, http://openingsacredspace.wordpress.com/hummingbird/

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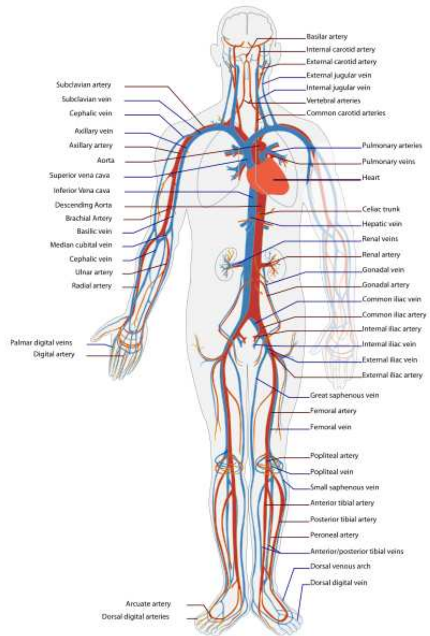

http://en.wikipedia.org/wiki/Respiratory_system, http://en.wikipedia.org/wiki/Circulatory_system

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http://commons.wikimedia.org/wiki/File:Mickeymox_-_Insect_Flight_%28by%29.jpg

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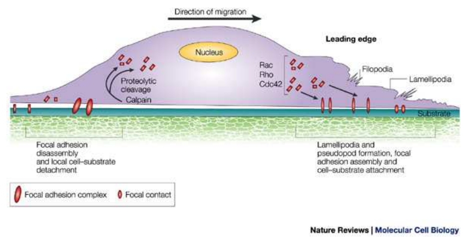

Frame et al, Nature Reviews 2002, http://www.nature.com/nrm/journal/v3/n4/fig_tab/nrm779_F3.html

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Turbulence characteristics: Irregular.

Swirling eddies of a range of sizes.

Turbulence characteristics: Swirling eddies of a range of sizes.

L. da Vinci, A deluge, c 1514-1517
www.leonardoda-vinci.org

L. da Vinci, Flowing stream, c 1507-1500

Turbulence: Irregular

Kolmogorov 1941: Smallest length scale $\eta \propto \mathrm{Re}^{-3/4}$ large scale.

	Re	L	η
Cyclone (10 km, 160 km/hr)	10^{10}	10 km	0.3 mm
Cloud (500 m, 5 m/s)	10^{8}	500 m	0.5 mm
River (10 m, 1 m/s)	10^{7}	10 m	0.05 mm
Airplane (0.1 m, 600 km/hr)	10^{6}	0.1 m	0.03 mm
Smokestack (1.5 m, 1 m/s)	10^{5}	1.5 m	0.3 mm
Human walking (2 m, 3 km/hr)	10^{5}	2 m	0.5 mm
Birds (0.1-3m, 1-10m/s)	$10^{4} - 10^{5}$	1 m	0.25 mm

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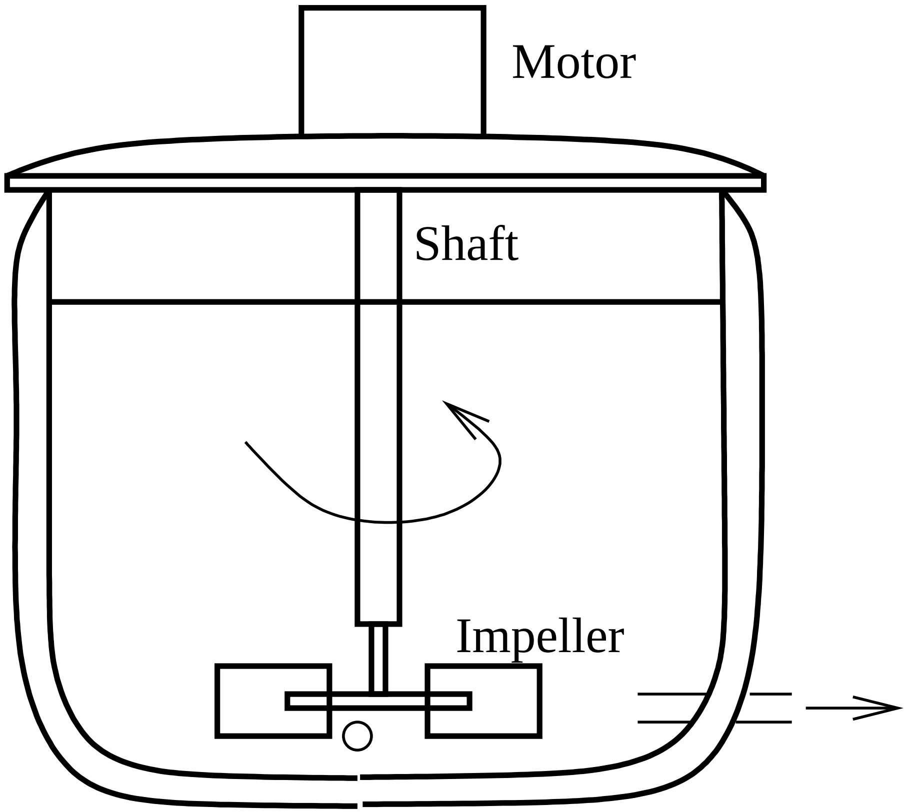
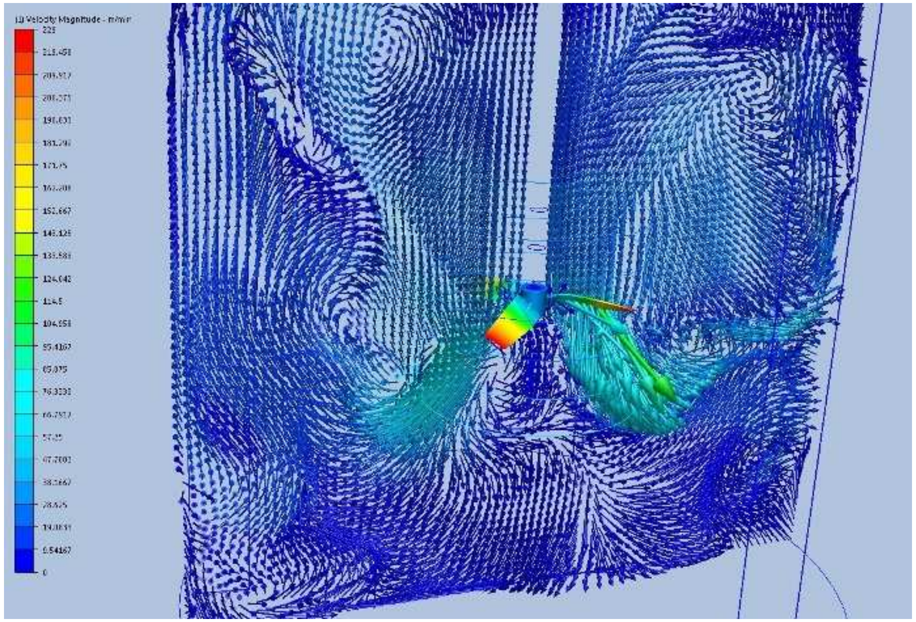
Turbulence: Continuum phenomenon.



$$\nabla \cdot \mathbf{u} = 0$$

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u}$$

Turbulence: Continuum phenomenon.



* http://www.mixtecna.com/products/research/computational-fluid-dynamics

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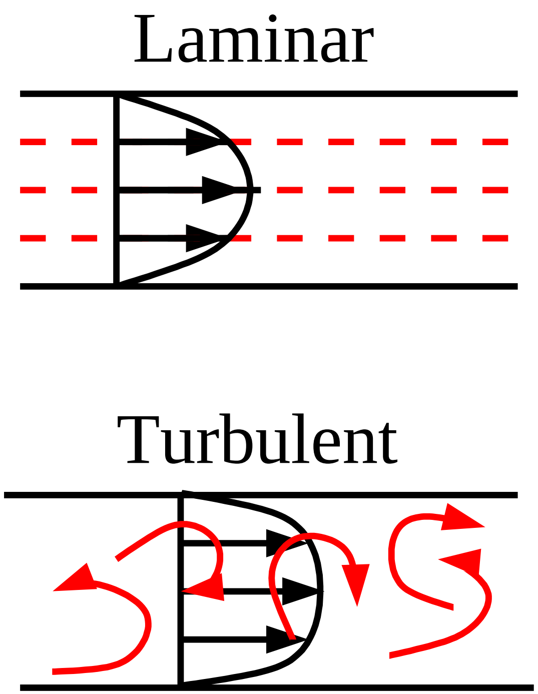

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Turbulence: Continuum phenomenon.

Stratified turbulence: G. Matheou and D. Chung, Phys. Fluids 24, 091106 (2012)

Turbulence: Statistical description?

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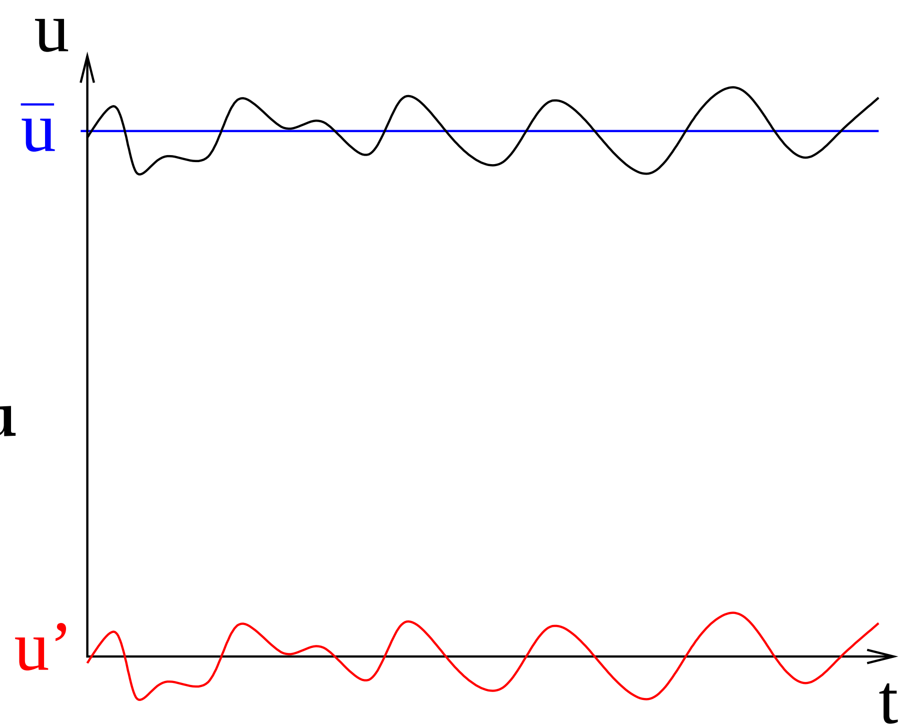

Mean + Fluctuation

Turbulence: Statistical description?

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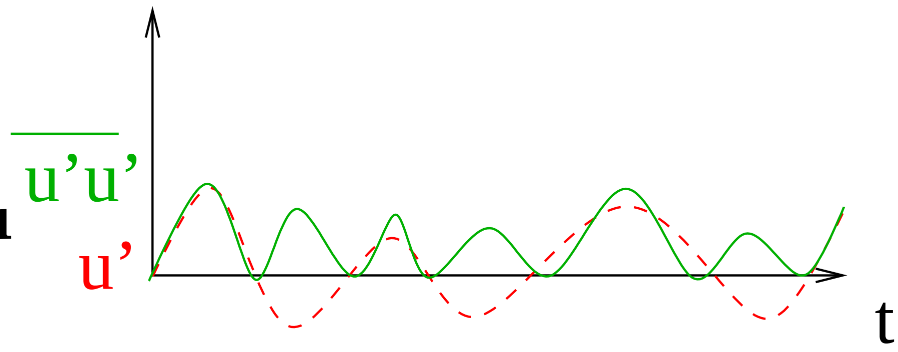


Mean + Fluctuation

$$\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'$$

$$\nabla\cdot\bar{\mathbf{u}} = 0$$

$$\rho\left(\frac{\partial\bar{\mathbf{u}}}{\partial t} + \bar{\mathbf{u}}\cdot\nabla\bar{\mathbf{u}} + \overline{\mathbf{u}'\cdot\nabla\mathbf{u}'}\right) = -\nabla\bar{p} + \mu\nabla^2\bar{\mathbf{u}}$$

Turbulence:

W. Heisenberg

When I meet God, I will ask him two questions: Why relativity? Why turbulence?
I really believe he will have an answer for the first.

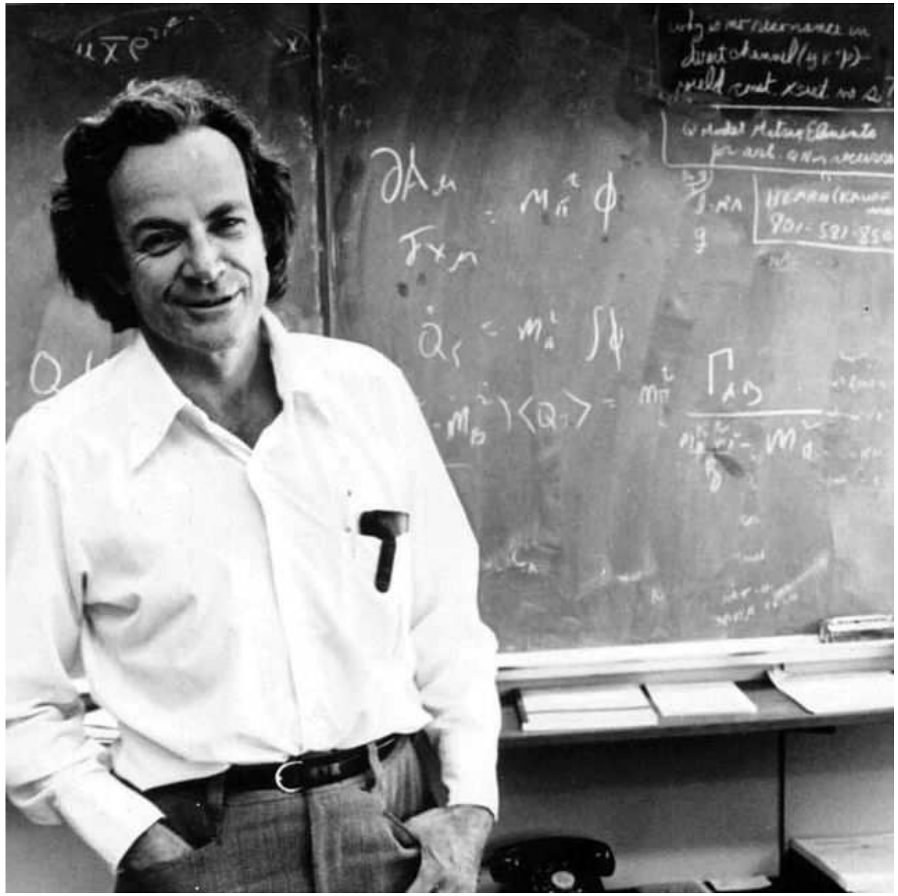
Turbulence:

H. Lamb

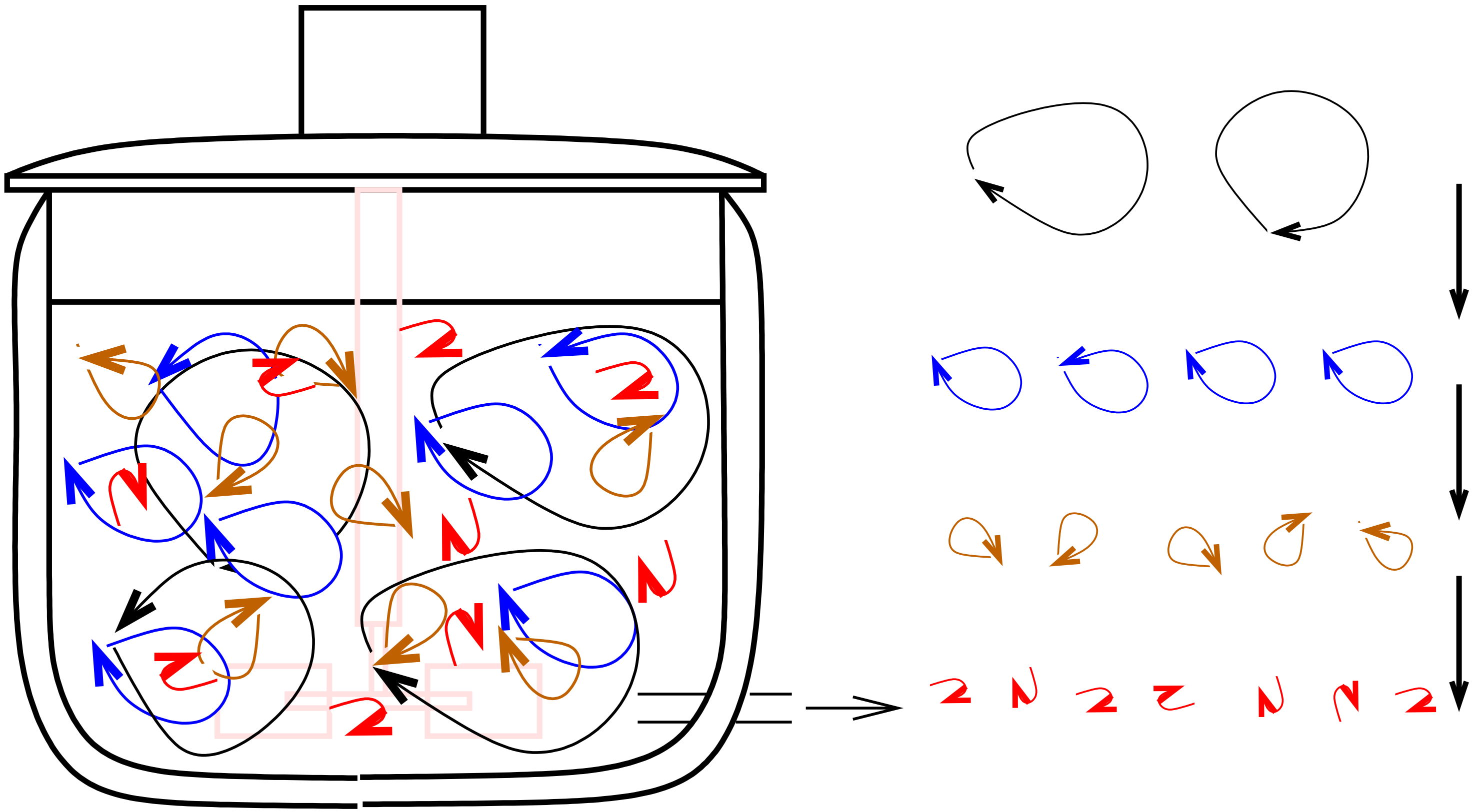
When I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids.
And about the former I am rather optimistic.

Turbulence

Feynman Lectures Vol 1. p 3-9
Finally, there is a physical problem that is common to many fields, that is very old, and that has not been solved. ... Nobody in physics has really been able to analyze it mathematically satisfactorily in spite of its importance to the sister sciences. It is the analysis of circulating or turbulent fluids.



Turbulence: Energy cascade



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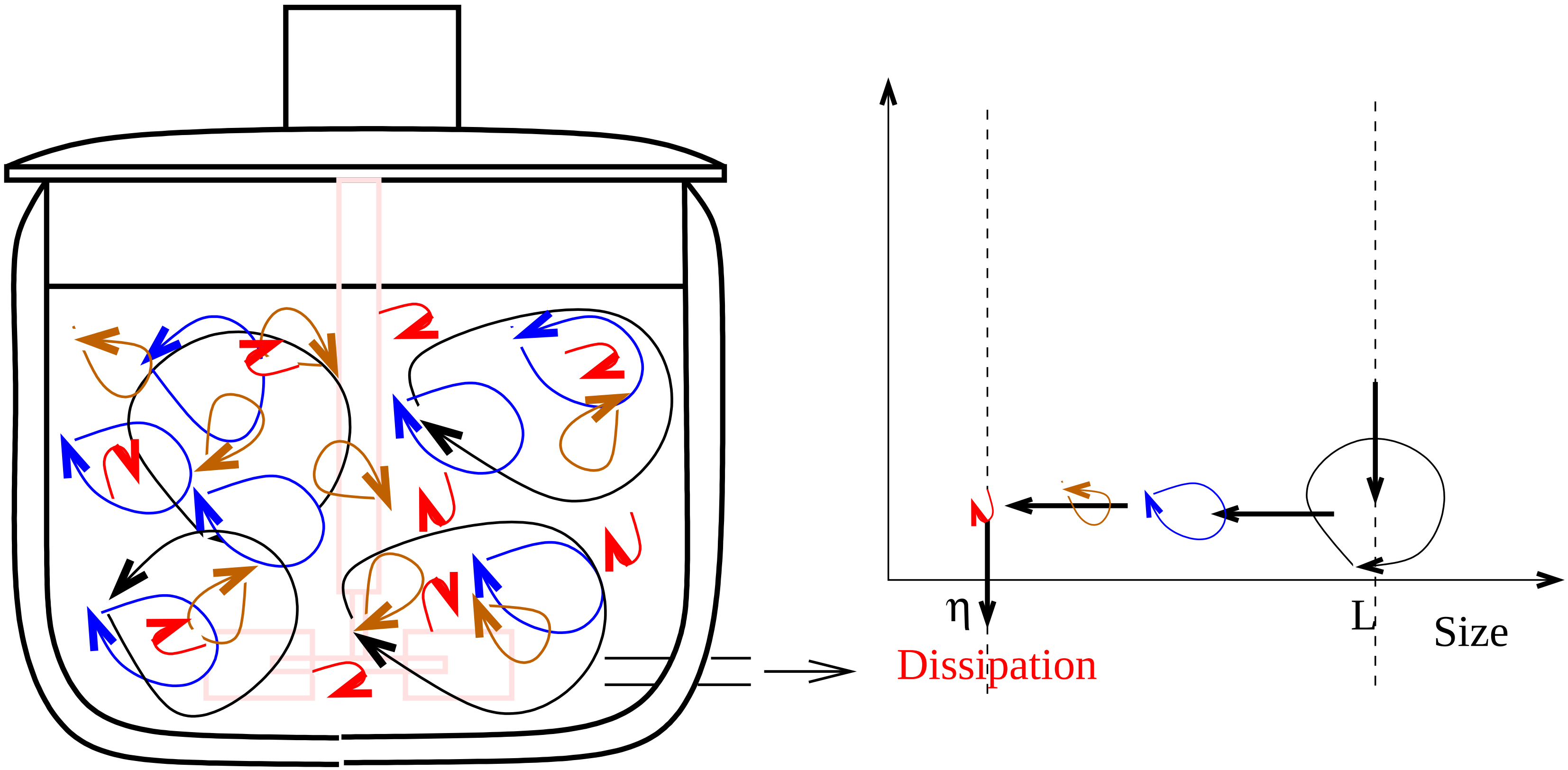

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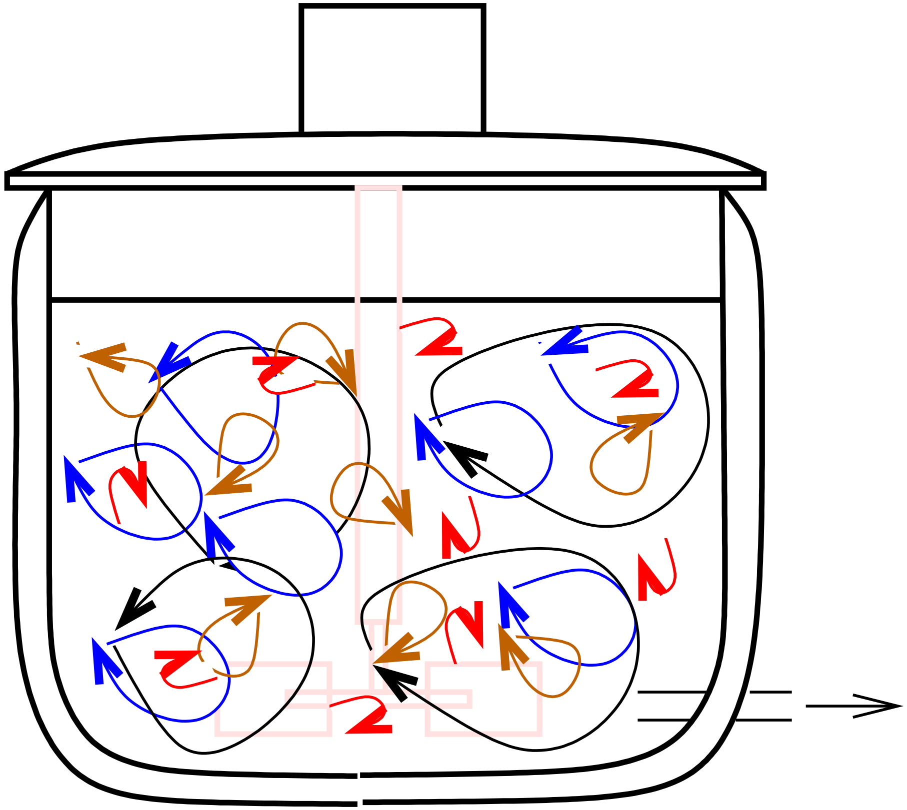

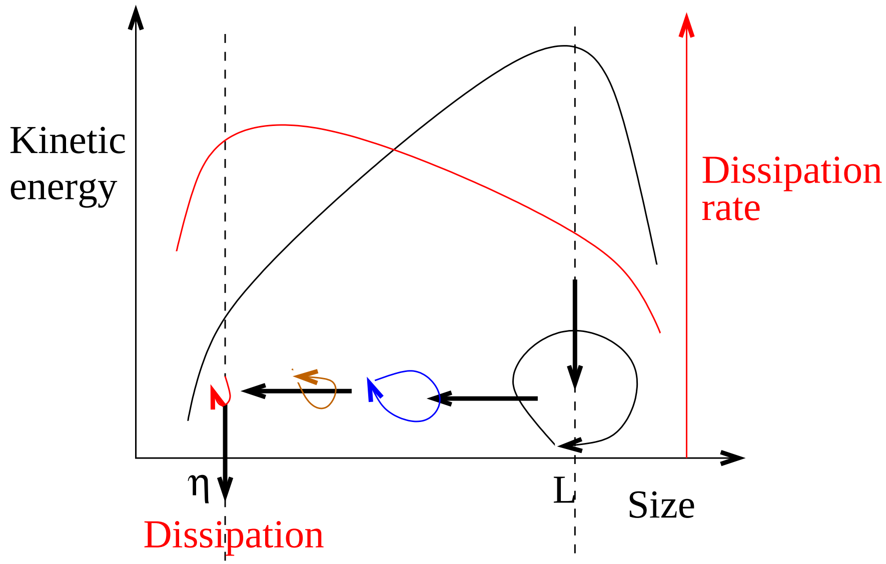

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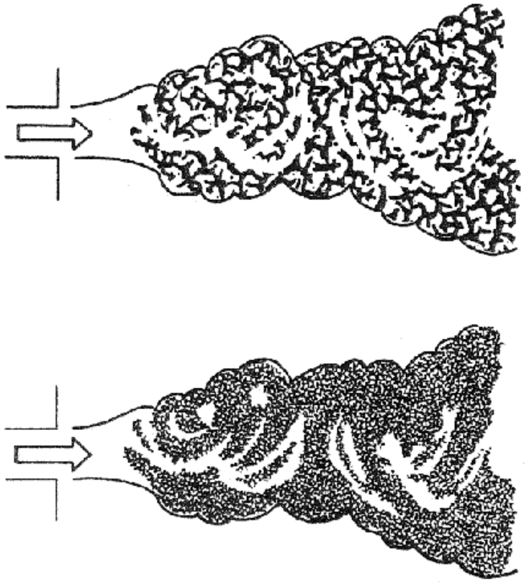
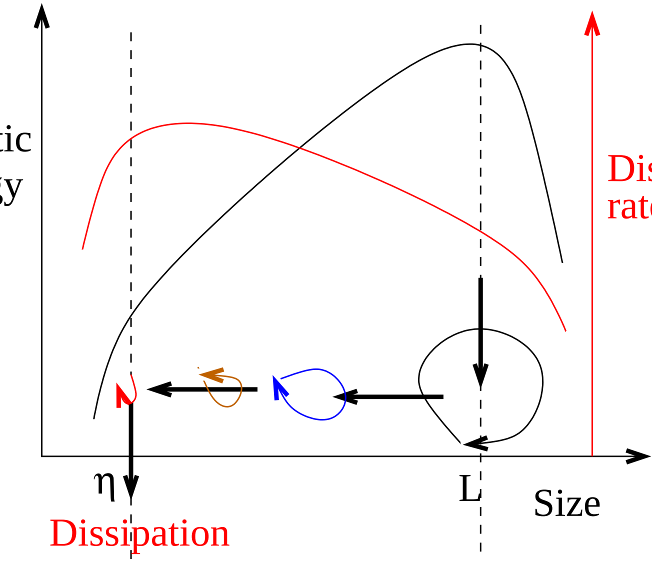


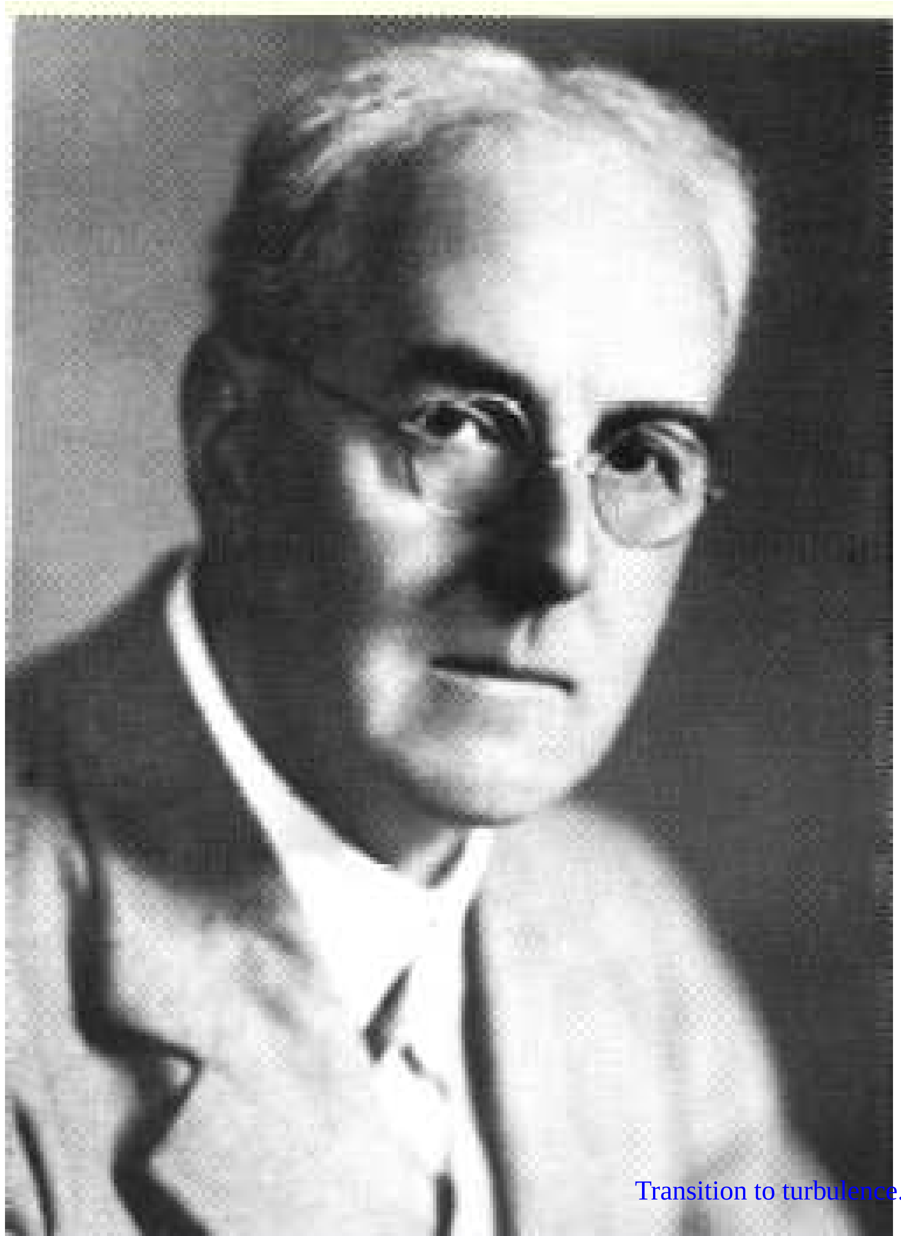


Kolmogorov Universal Equilibrium Theory: The smallest turbulence length & velocity scales do not depend flow configuration or velocity at the largest scales, but only on the energy dissipation rate and kinematic viscosity (μ/ρ).

An introduction to fluid turbulence, Tennekes and Lumley

Turbulence: Energy cascade

Big whorls have little whorls
That feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity.
– Lewis F. Richardson



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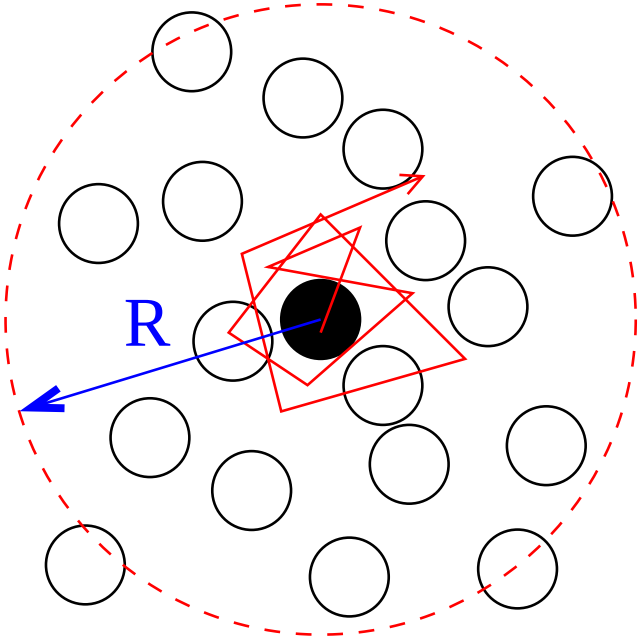
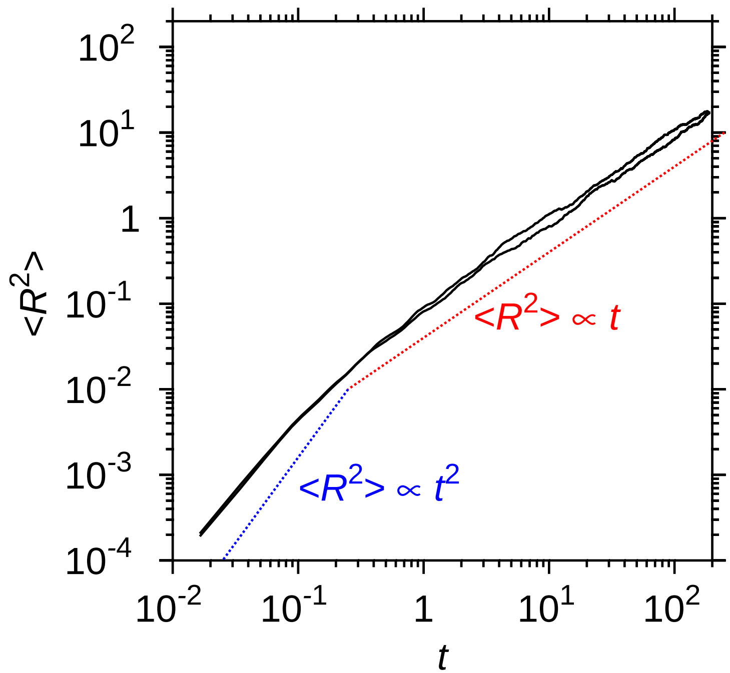


R~t$^{1/2}$

Diffusion coefficient
$D = (\langle R^2 \rangle / t) \sim c\lambda$.
Molecular speed c.
Persistance length λ.



	c (m/s)	λ (m)	D (m^2/s)
Air	330	7×10^{-8}	2×10^{-5}
Water	1500	10^{-10}	$10^{-9}*$

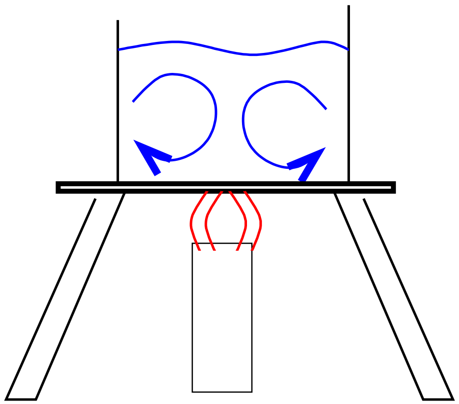
Molecular diffusion

Mixing:

	c (m/s)	λ (m)	D (m^2/s)
Air	330	7×10^{-8}	2×10^{-5}
Water	1500	10^{-10}	10^{-9*}

Cup diameter $d = 4cm$.

Diffusion time $t_D \propto (d^2/D) \sim 10^6 s$

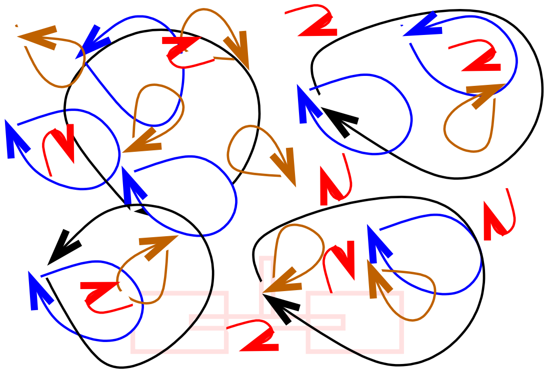


Heating

Thermal diffusion $\alpha \sim 10^{-7} m^2/s$

Time $t_D \propto (d^2/\alpha) \sim 10^4 s$.

Turbulent diffusion



Turbulent diffusion:
$t_T \sim (L/U)$

	Re	L	t_D	η	t_T
Cloud	10^8	500 m	$2.5 \times 10^{10} s$	0.5 mm	$10^2 s$
River	10^7	10 m	$10^{10} s$	0.05 mm	$10 s$
Smokestack	10^5	1.5 m	$10^5 s$	0.3 mm	$1.5 s$
Human movement	10^4	10 cm	$10^3 s$	0.5 mm	$10^{-1} s$

Turbulence:

W. Heisenberg

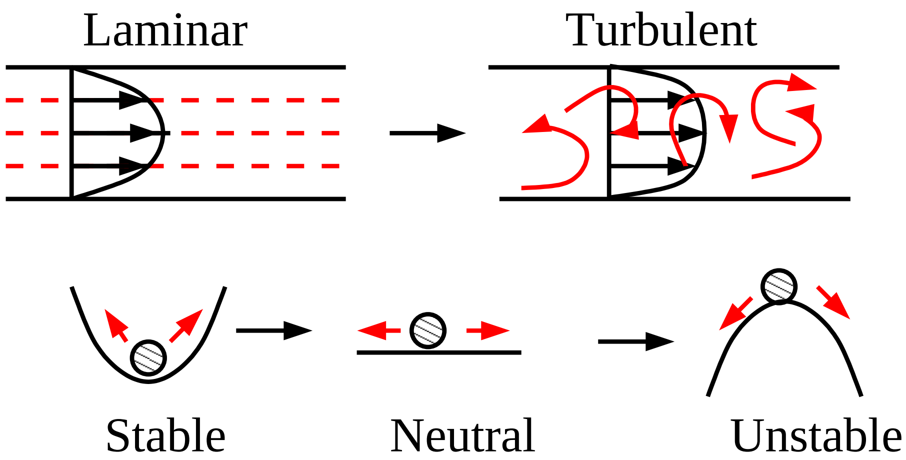
When I meet God, I will ask him two questions: Why relativity? Why turbulence?
I really believe he will have an answer for the first.

Life, as we know it, requires turbulence at large scales.

Transition to turbulence:

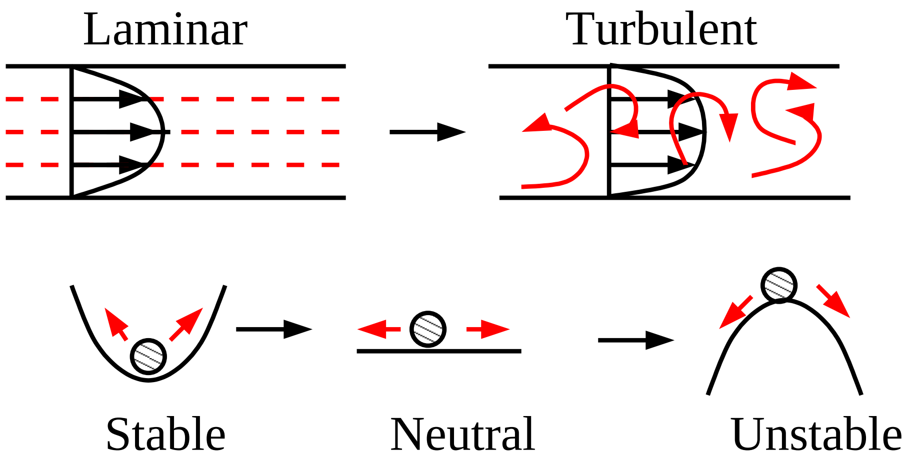
- Hydrodynamic stability.
- Transition in rotating flows & heated fluids.
- Transition in pipe & channel flows.

Transition to turbulence:

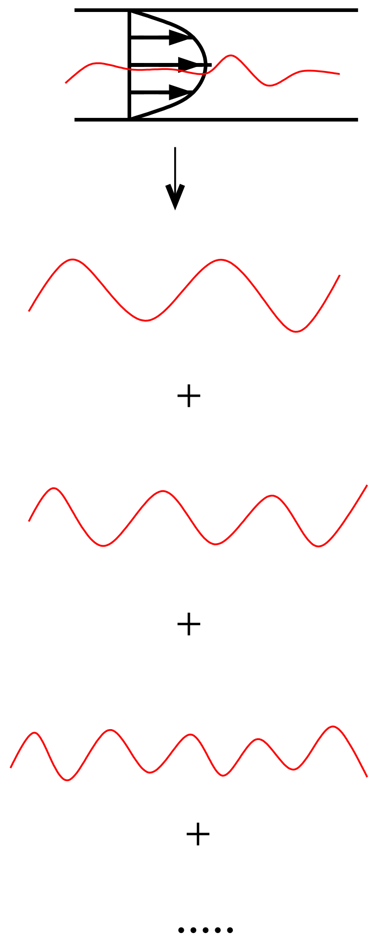


- Laminar flow — solution of equations of motion.
- *But*, disturbances grow when the flow becomes unstable, take the flow to another (turbulent) state.

Transition to turbulence:

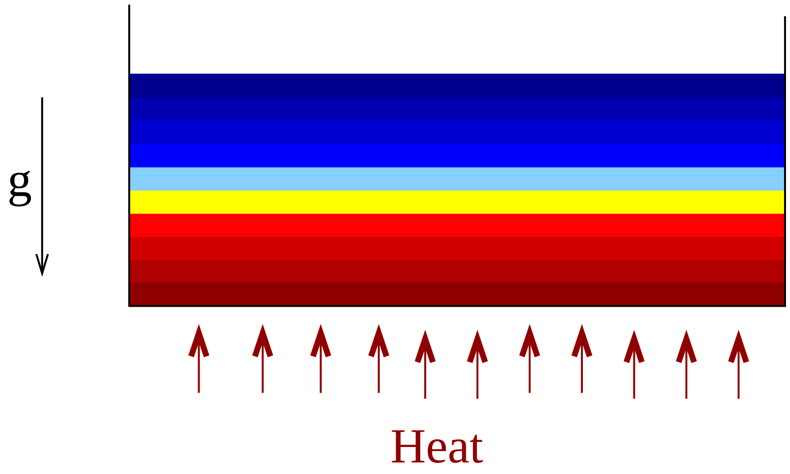


Hydrodynamic stability:



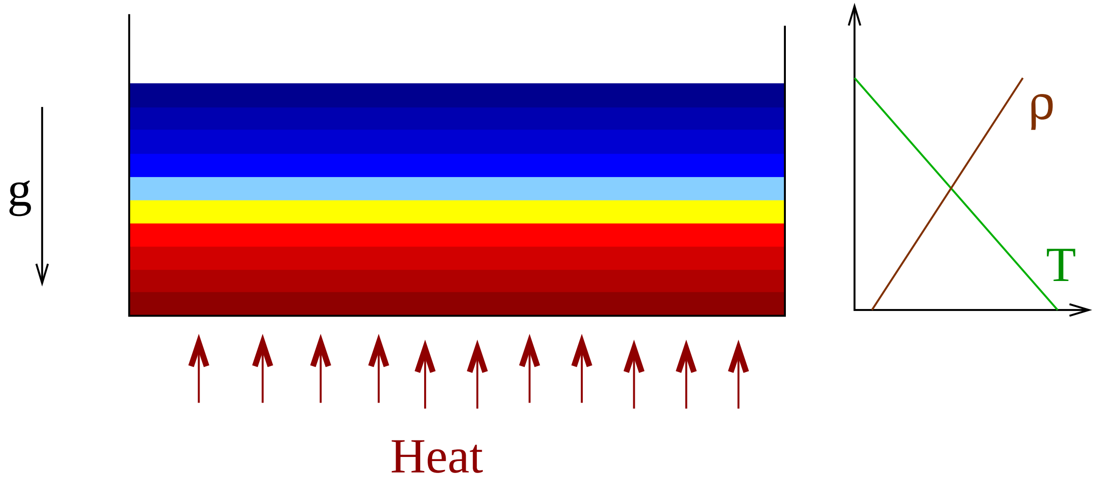
Transition to turbulence:

Fluid heated from below: Rayleigh-Benard instability.



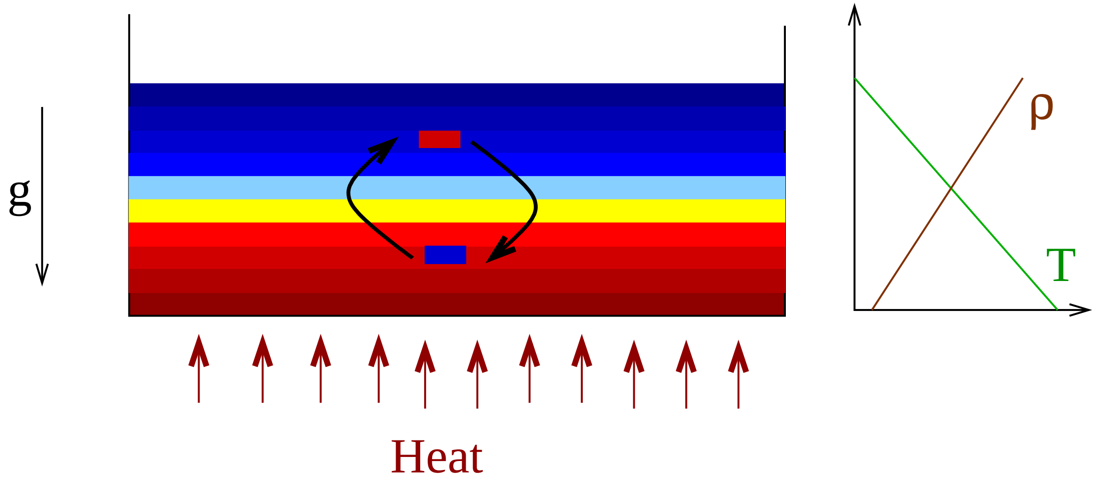
Transition to turbulence:

Fluid heated from below: Rayleigh-Benard instability.



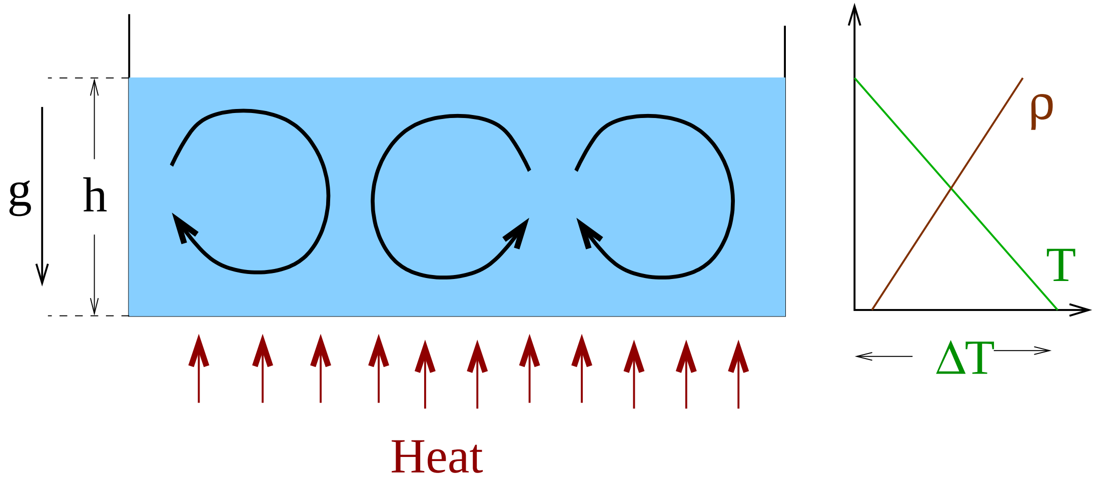
Transition to turbulence:

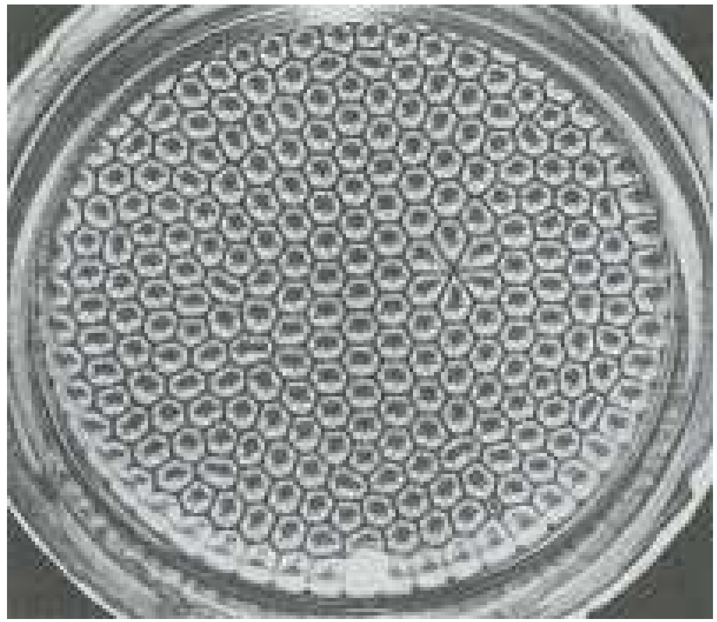
Fluid heated from below: Rayleigh-Benard instability.



Transition to turbulence:

Fluid heated from below: Rayleigh-Benard instability.





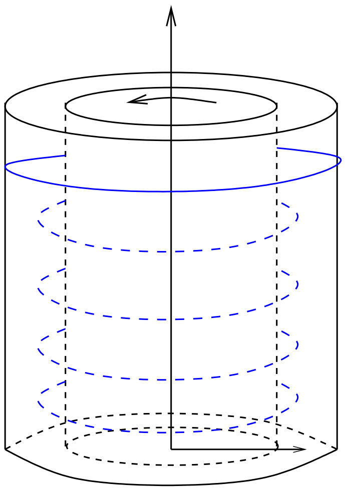
$$\text{Ra} = \frac{\rho g \beta \Delta T h^3}{\kappa \mu} > 1100$$

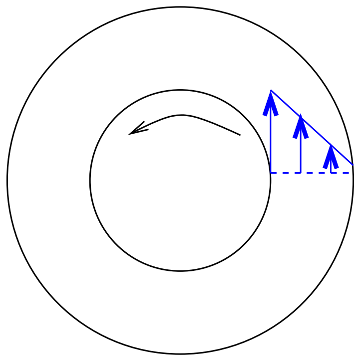
Thermal conductivity κ, thermal expansion coefficient β.

http://permaculturetokyo.blogspot.in/2012_08_01_archive.html

Transition to turbulence:

Rotating fluids: Taylor-Couette instability.





Angular momentum (constant):

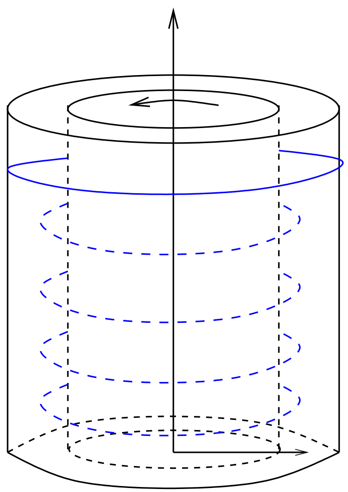
$L = \rho r^2 \Omega$

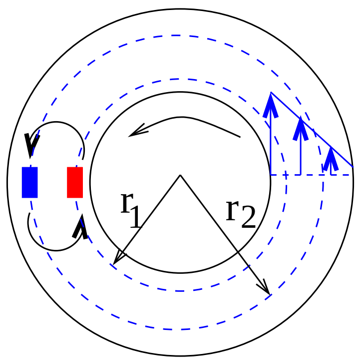
Kinetic energy per unit mass:

$$\mathrm{KE} = \frac{\rho u_\theta^2}{2} = \frac{\rho \Omega^2 r^2}{2} = \frac{\rho L^2}{2 r^2}$$

Transition to turbulence:

Rotating fluids: Taylor-Couette instability.





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Kinetic energy per unit mass:

$$\mathrm{KE} = \frac{\rho u_\theta^2}{2} = \frac{\rho \Omega^2 r^2}{2} = \frac{\rho L^2}{2r^2}$$

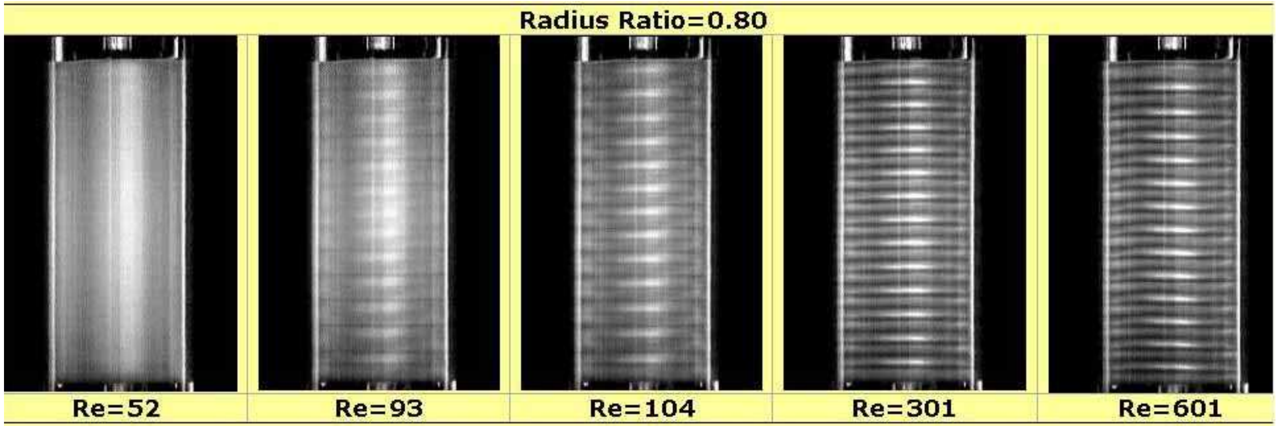
Interchange fluid parcels at r_1, r_2.

$$\Delta\mathrm{KE} = (L_2^2 - L_1^2)\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$

Unstable for $L_2^2 < L_1^2$, or $(r^2\Omega) \downarrow$ as $r \uparrow$.

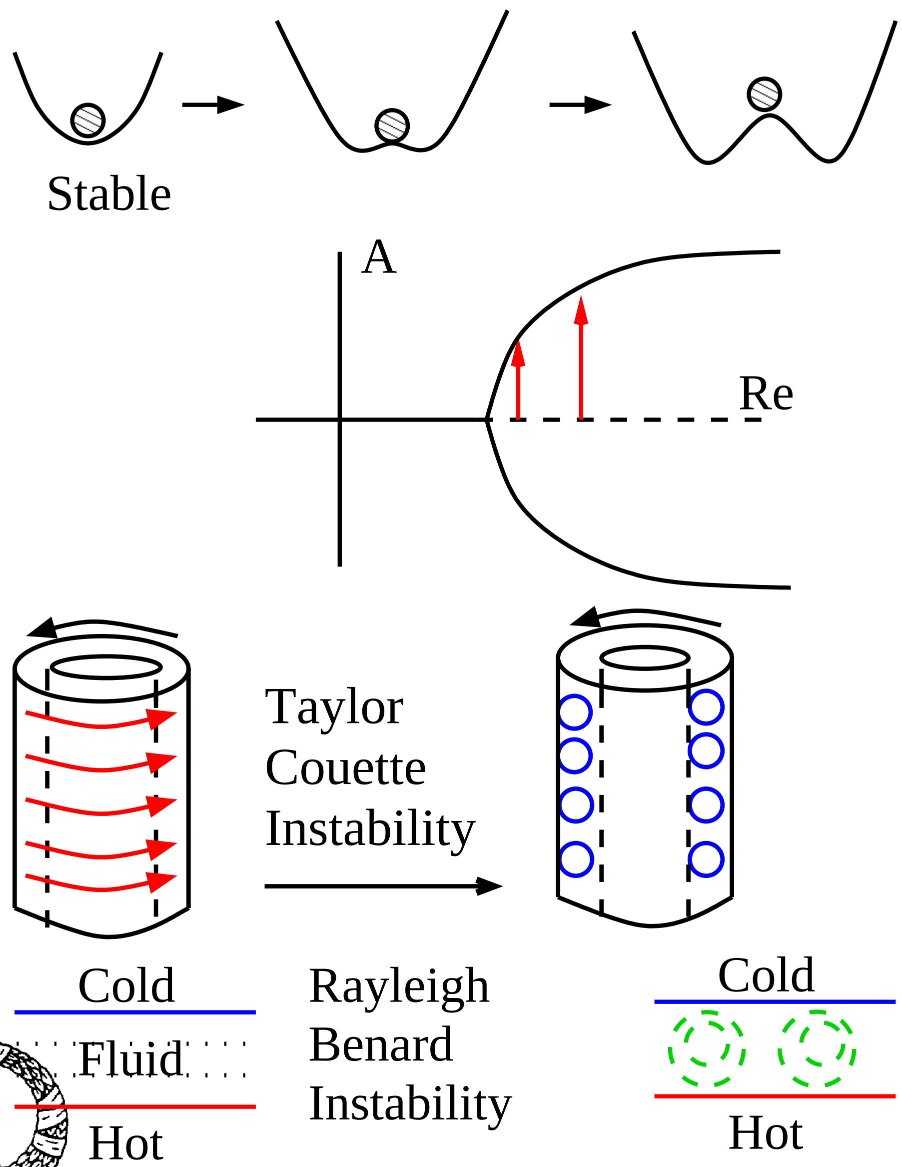
Transition to turbulence

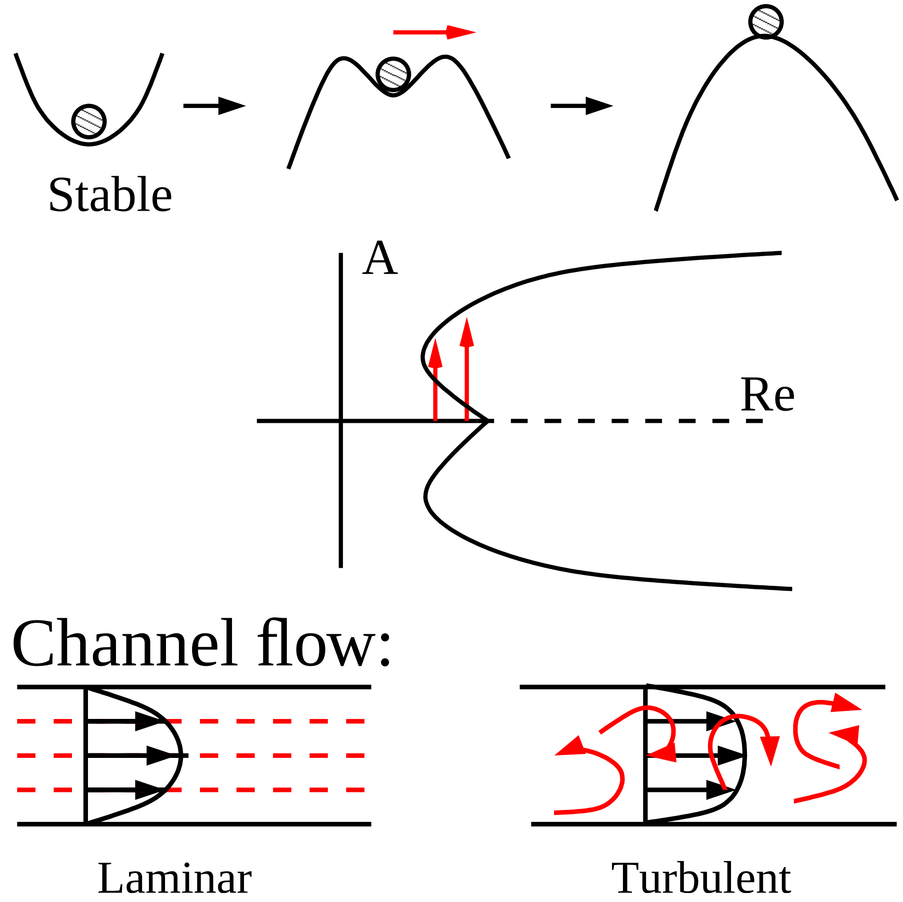
Rotating fluids: Taylor-Couette instability.



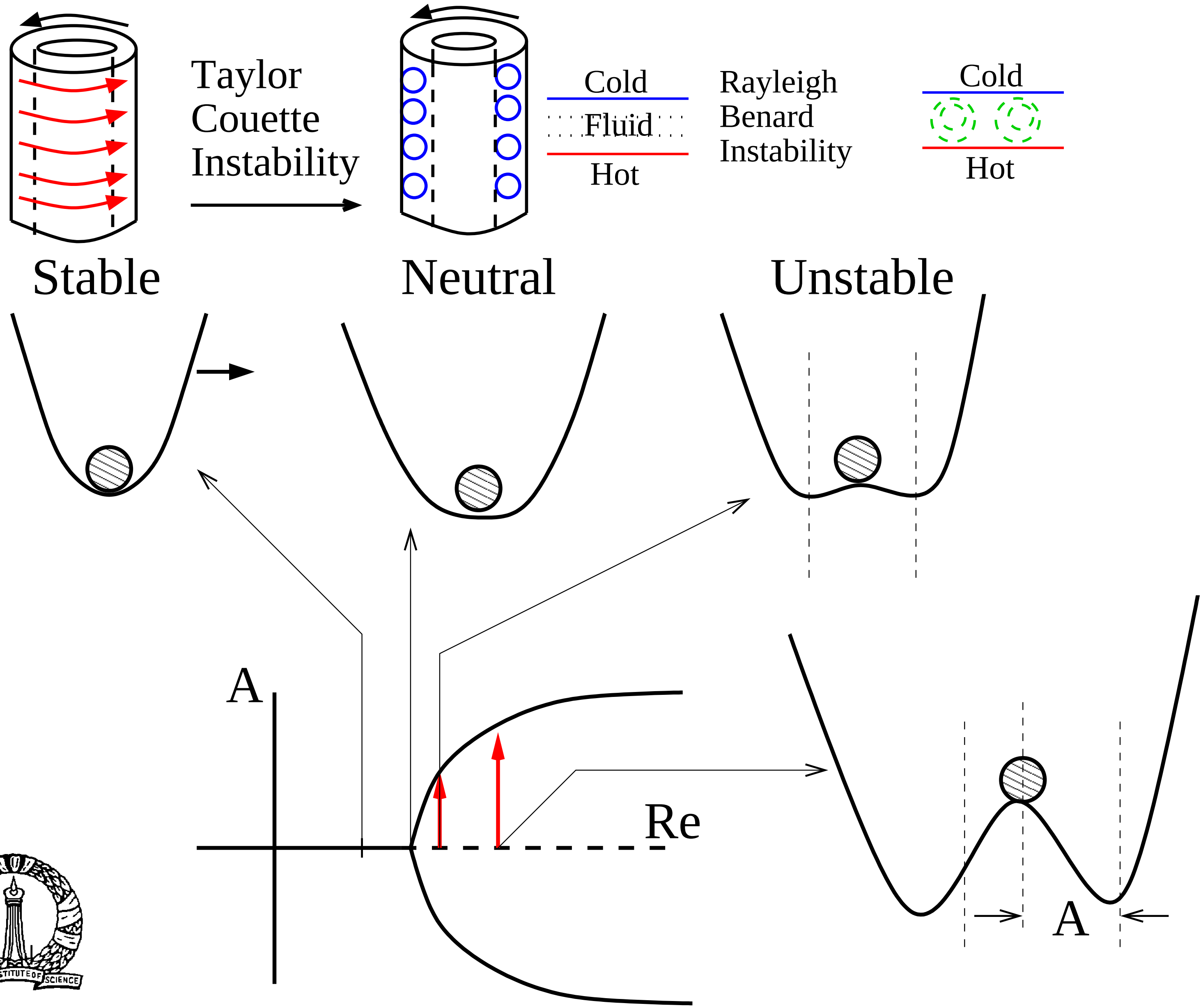
http://serve.me.nus.edu.sg/limtt/

Transition to turbulence

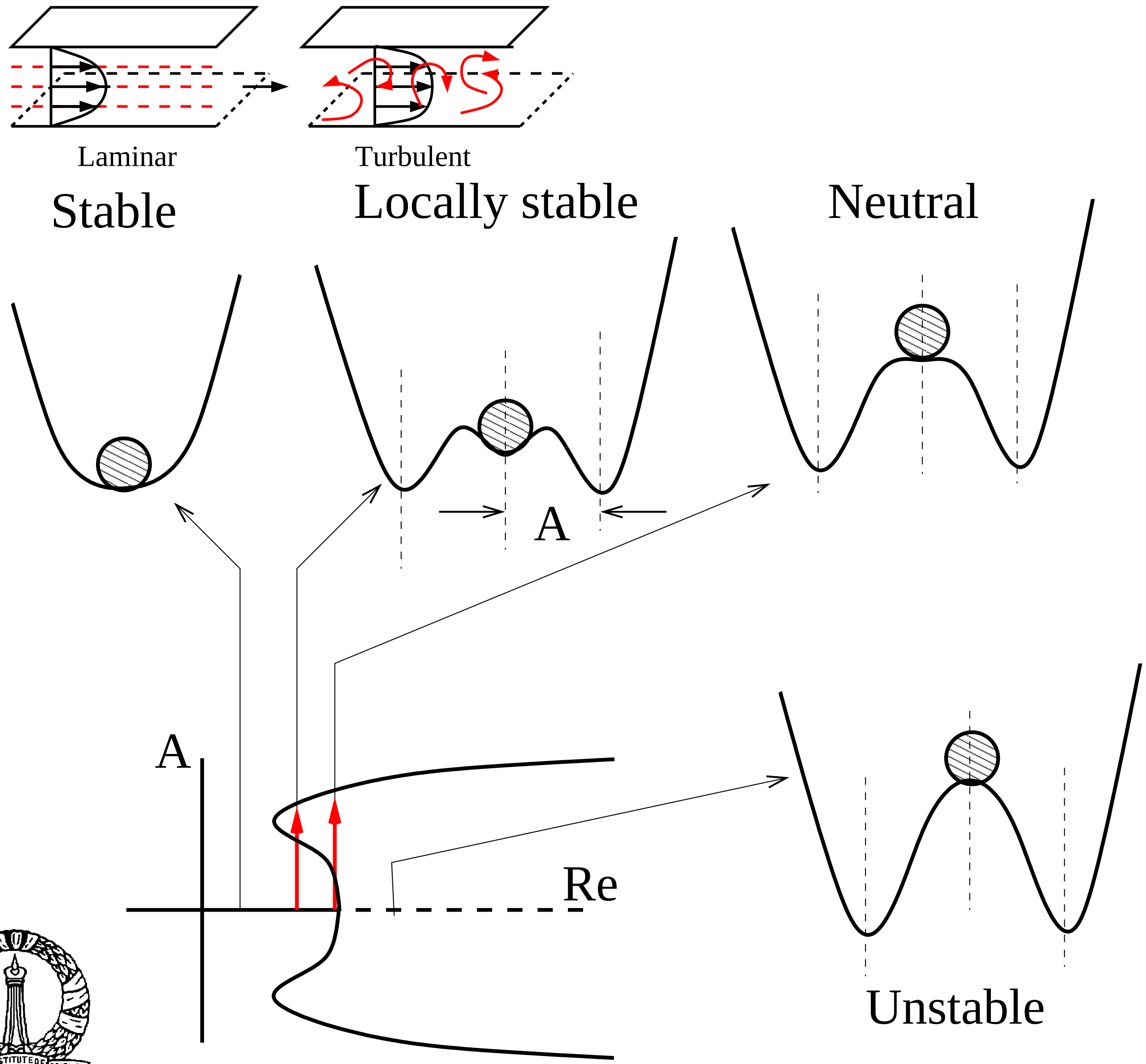




Super-critical bifurcation



Sub-critical bifurcation



Transition in a channel

- High Reynolds number $(\rho UL/\mu)$, neglect viscosity.
- *But*, theorems due to Rayleigh and Fjortoft show that the pressure-driven flow can never become unstable.
- Viscous effects important in an ‘internal critical layer’ of thickness decreases as $\mathrm{Re}^{-1/3}$.
- When viscous effects are included, a linear instability is predicted at $\mathrm{Re} = 5772$.
- Experiments — unstable at $\mathrm{Re} \sim 1200$.

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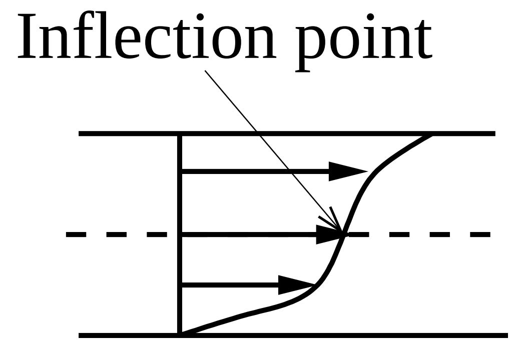
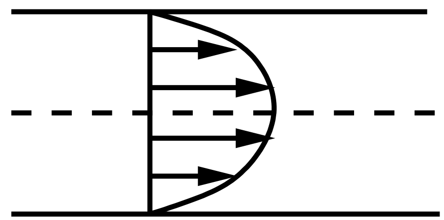


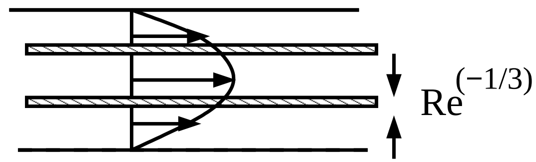
Possibly Unstable



Always Stable

Transition in a channel

- High Reynolds number $(\rho UL/\mu)$, neglect viscosity.
- *But*, theorems due to Rayleigh and Fjortoft show that the pressure-driven flow can never become unstable.
- Viscous effects important in an 'internal critical layer' of thickness $\mathrm{Re}^{-1/3}$.*
- When viscous effects are included, a linear instability is predicted at $\mathrm{Re} = 5772$.
- Experiments — unstable at $\mathrm{Re} \sim 1200$.



* Wasow 1954.

Transition in a channel

- High Reynolds number $(\rho UL/\mu)$, neglect viscosity.
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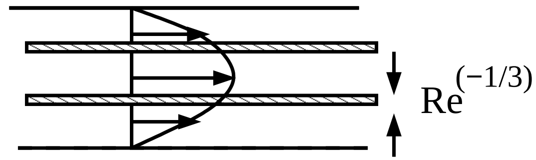

Transition in a channel

- High Reynolds number $(\rho U L/\mu)$, neglect viscosity.
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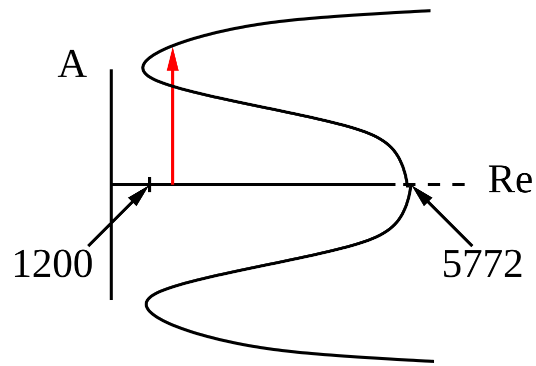
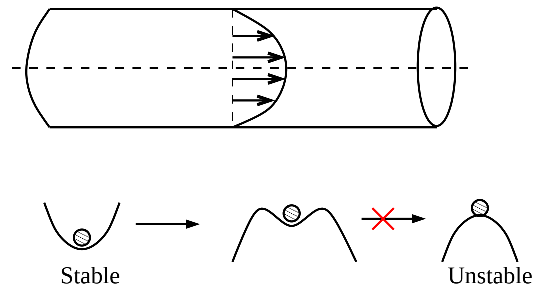
- When viscous effects are included, a linear instability is predicted at Re $= 5772$.
- Experiments — unstable at Re ~ 1200.



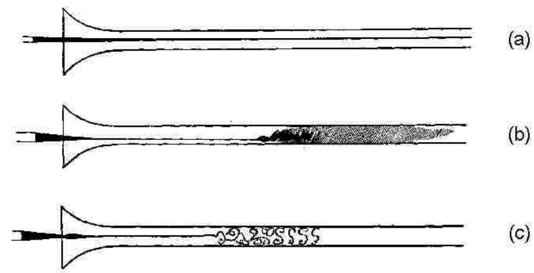
Transition in a tube:

- Linear stability — flow always stable.
- Experiments — transition at Re $= 2100$.
- Transition due to instability to finite-amplitude disturbances.
- Can maintain laminar flow even at Re $\sim 10^5$ if care is taken to dampen disturbances.



Transition in a tube:

- Linear stability — flow always stable.
- Experiments — transition at around $\mathrm{Re} = 2100$.
- Transition due to instability to finite-amplitude disturbances.
- Can maintain laminar flow even at $\mathrm{Re} \sim 10^5$ if care is taken to dampen disturbances.



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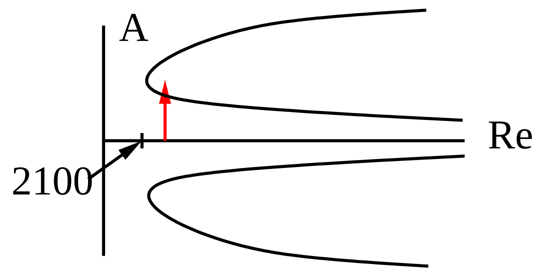

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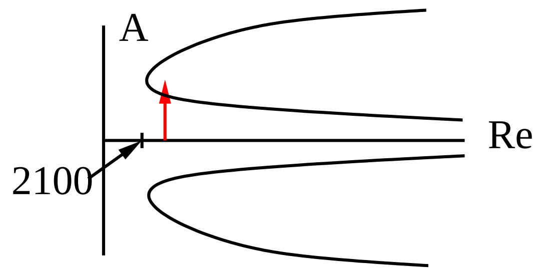
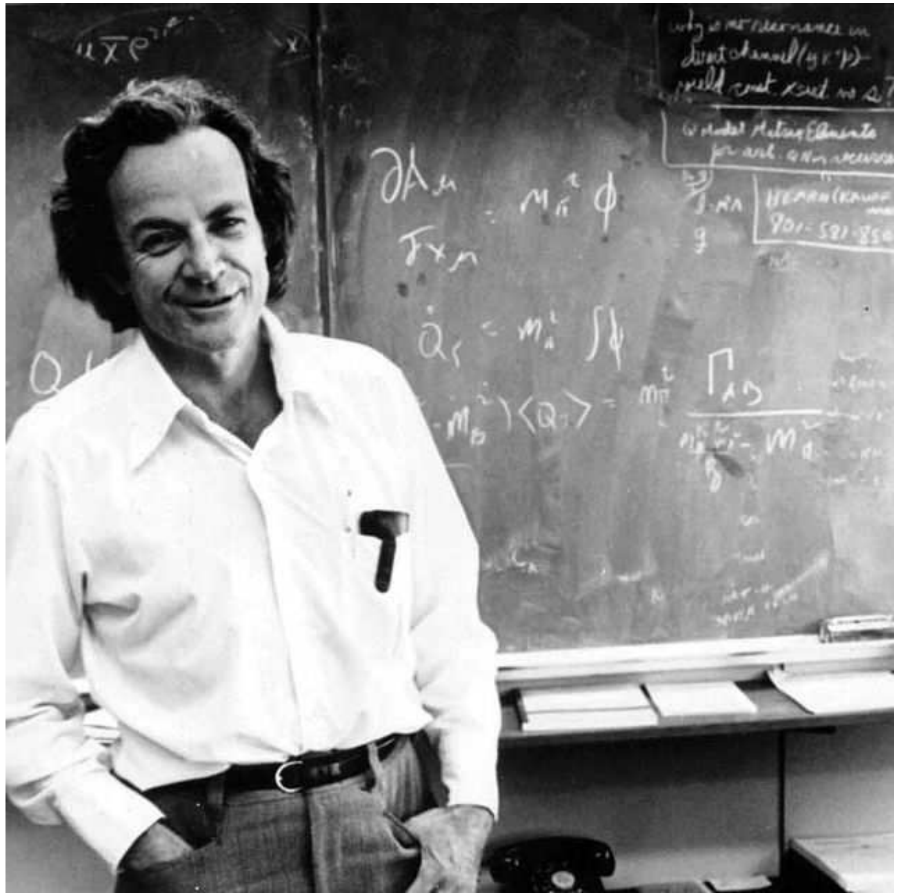


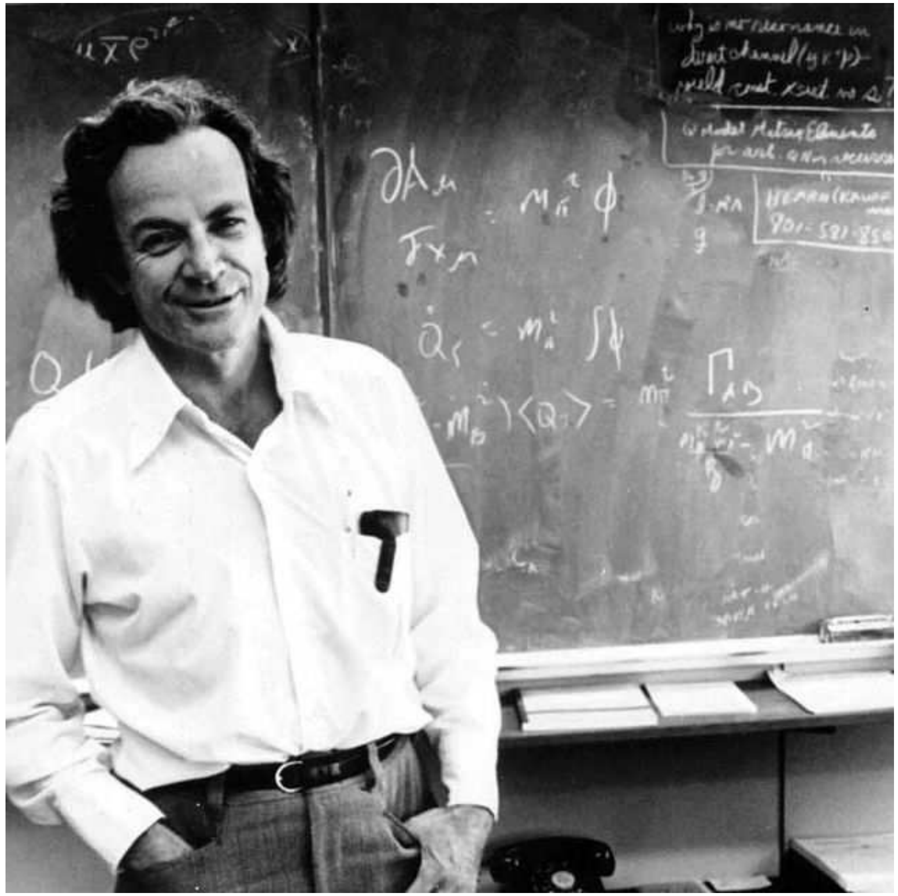
Turbulence

Feynman Lectures Vol 1. p 3-9
The simplest form on the problem is to take a pipe that is very long and push water through it at high speed. We ask: to push a given amount of water through that pipe, how much pressure is needed?

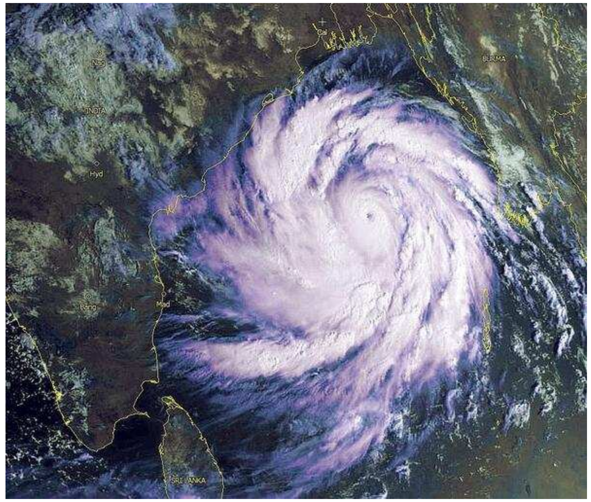


Turbulence

Feynman Lectures Vol 1. p 3-9
No one can analyze it from first principles and the properties of water. ... What we really cannot do is deal with actual, wet water running through a pipe. That is the central problem which we ought to solve some day, and we have not.



Turbulence



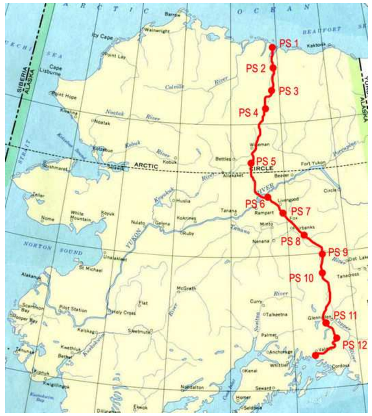
http://en.wikipedia.org/wiki/Fluidized_bed_reactor http://en.wikipedia.org/wiki/Oil_platform

http://en.wikipedia.org/wiki/File:1er_vol_de_l%27_A380.jpg

Turbulence: Highly dissipative

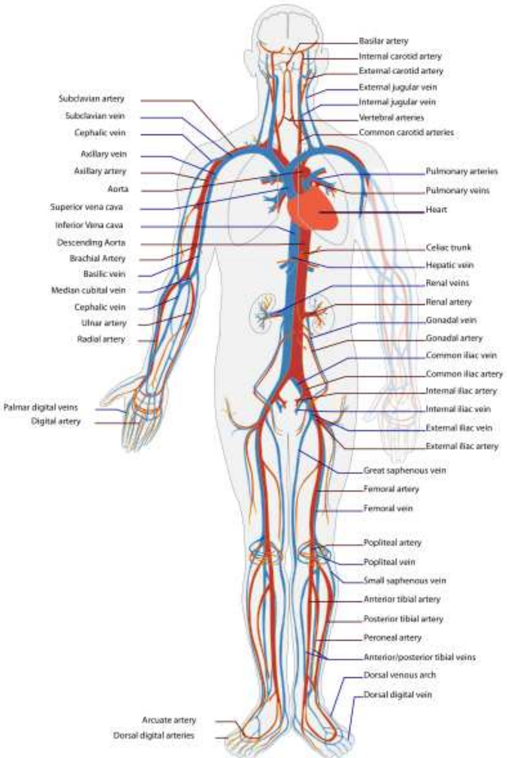
Drag reduction: Trans-Alaska pipeline.

- 1200 km length, 4 ft diameter.
- > 2 million barrels of oil per day.
- Small amounts of specialised polymers added in a few parts per million, reduce drag by 40%.
- Principle of drag reduction not clear.



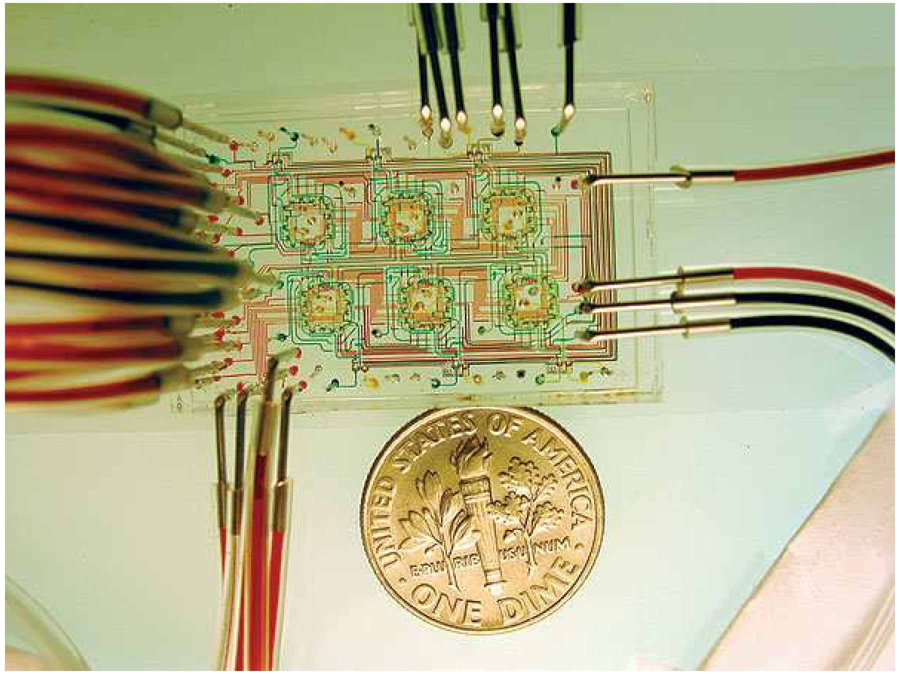
Transition in soft channels and tubes

Circulatory system:



en.wikipedia.org/wiki/Circulatory_system

Microfluidic mixing:

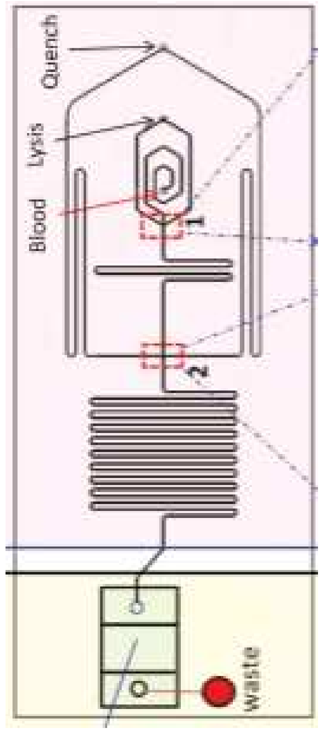


Jakubek, Popular Science, 2009.

INDIAN INSTITUTE OF SCIENCE

Turbulence at small scales

Kenis et al, Science, 1999.



van Berkel et al, Lab Chip, 2011.

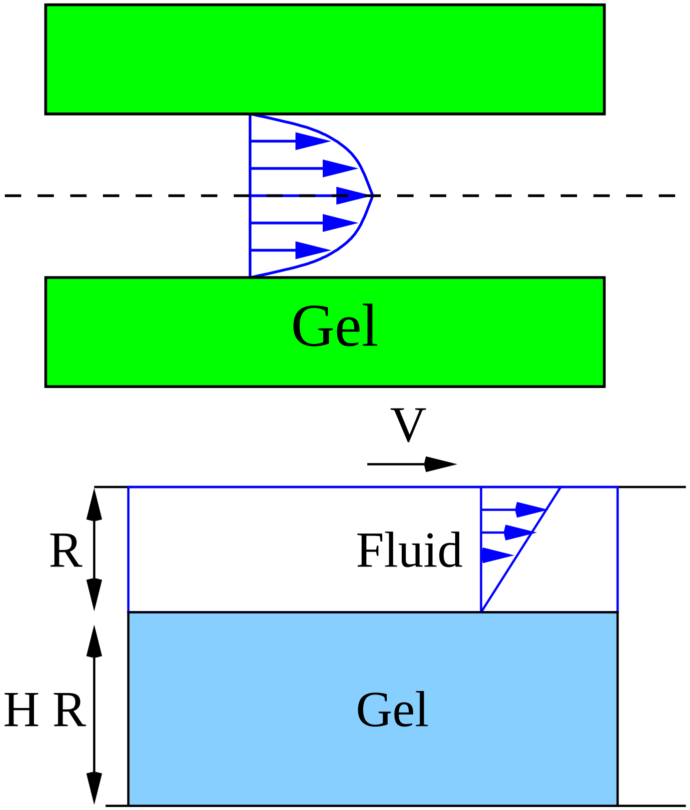
Laminar flows due to small dimensions.

Mixing time $t_m \sim (L^2/\mathcal{D})$, $\mathcal{D} \sim 10^{-9}$ m^2/s.

For $L \sim 1mm$, $t_m \sim 1000s$.

Transition in soft channels & tubes

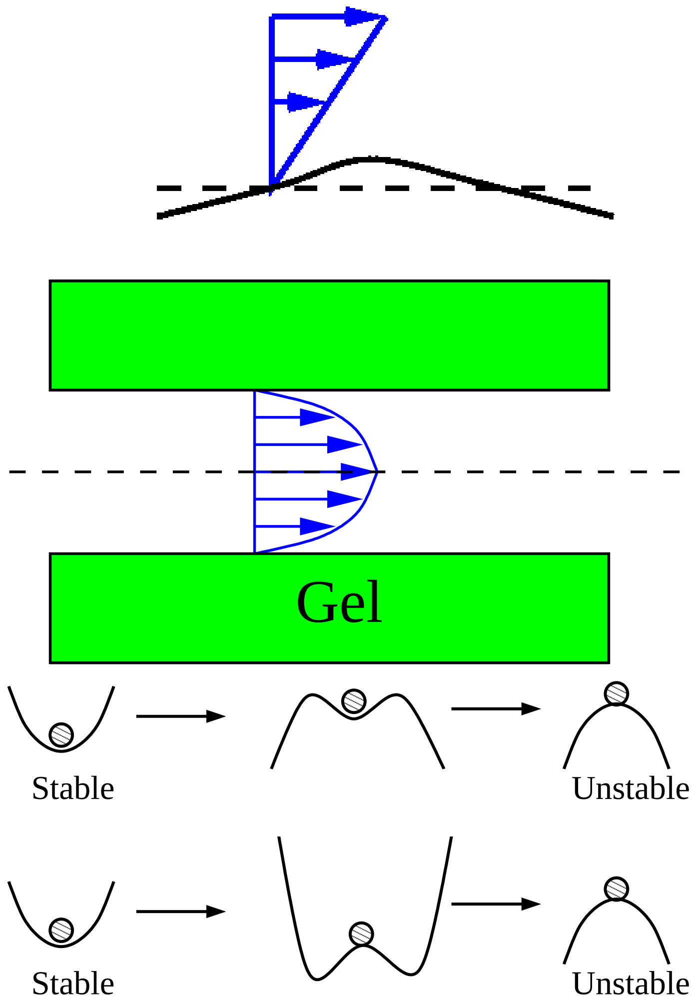
Coupling between fluid and wall material:



- Elasticity of wall material — $G(\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-2})$ (shear modulus).
- Dimensionless parameter with fluid viscosity and wall elasticity — $\Sigma = (\rho G R^2/\eta^2)$.
- Transition a function of two parameters Re & Σ.
- Soft interfaces —

$$G \sim 10^4 - 10^5 Pa \sim 10^{-6} G(\text{steel})$$

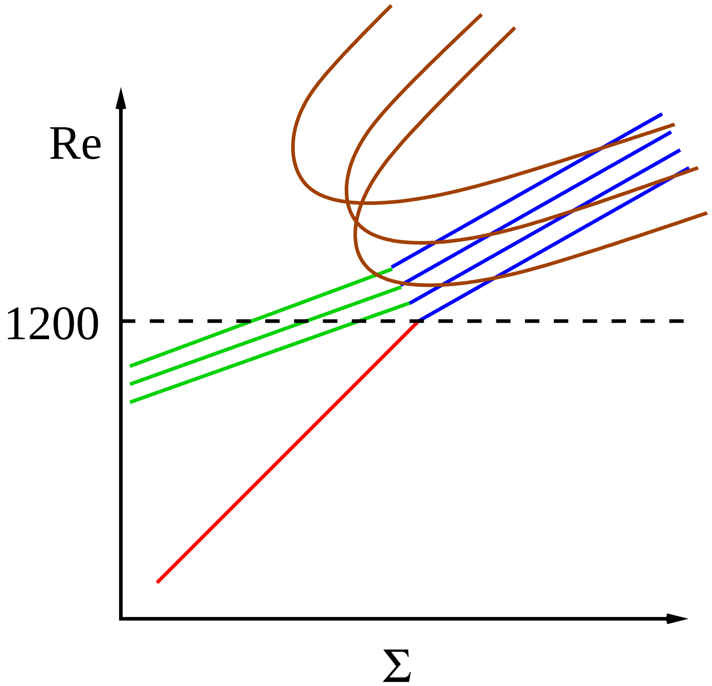
Transition in soft tubes & channels



- Fluid flow — wall deformation — flow modification.
- Linear instability — *qualitatively different from rigid tubes.*
- Strategy — Obtain analytical results in well defined limits of low and high Reynolds number, and numerical results in intermediate regimes.

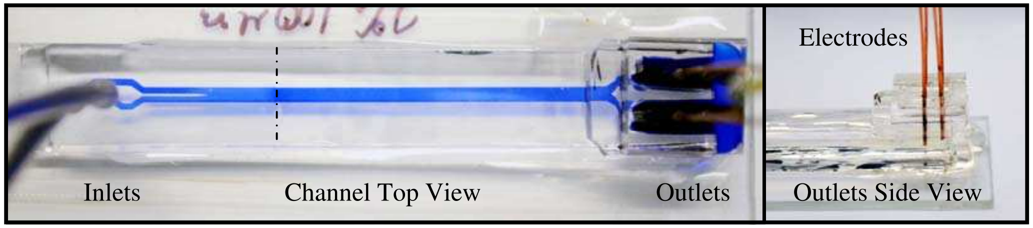
Transition in soft channels & tubes:

Summary:

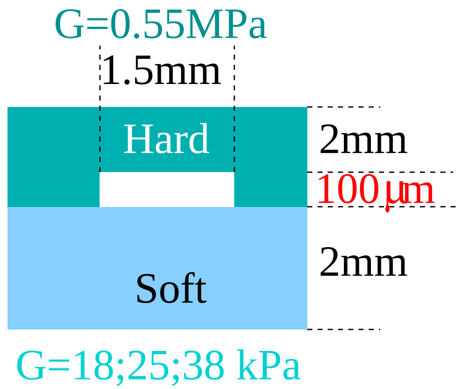


	Regime	Mechanism	Flow structure	Non-linear
Viscous	$Re \ll 1$ $Re \propto \Sigma$	Shear work at interface		Sub-critical
Low Re inertial	$Re \ll 1$ $Re \propto \Sigma^{1/2}$	Reynolds stress		
Inviscid	$Re \gg 1$ $Re \propto \Sigma^{1/2}$	Reynolds stress	Internal viscous layer $Re^{-1/3}$	
Wall	$Re \gg 1$ $Re \propto \Sigma^{3/4}$	Shear work at interface	Wall viscous layer $Re^{-1/3}$	Super-critical

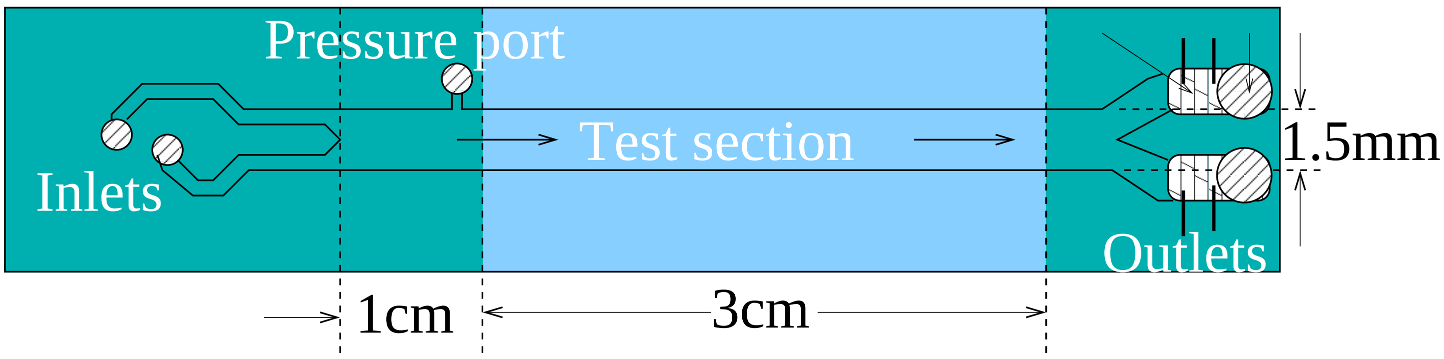
Soft-walled microchannel



Cross-section:

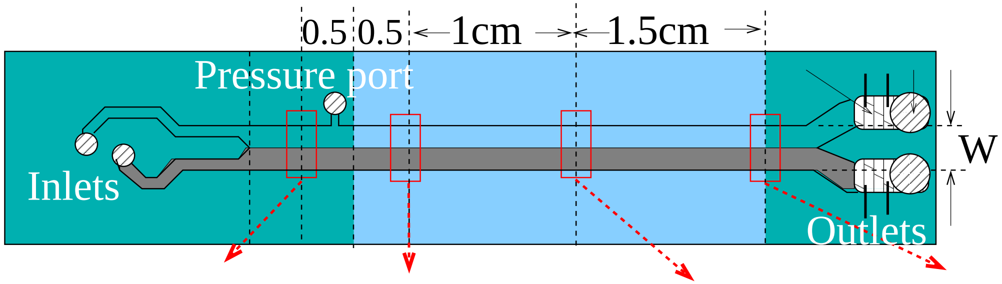


Top view:



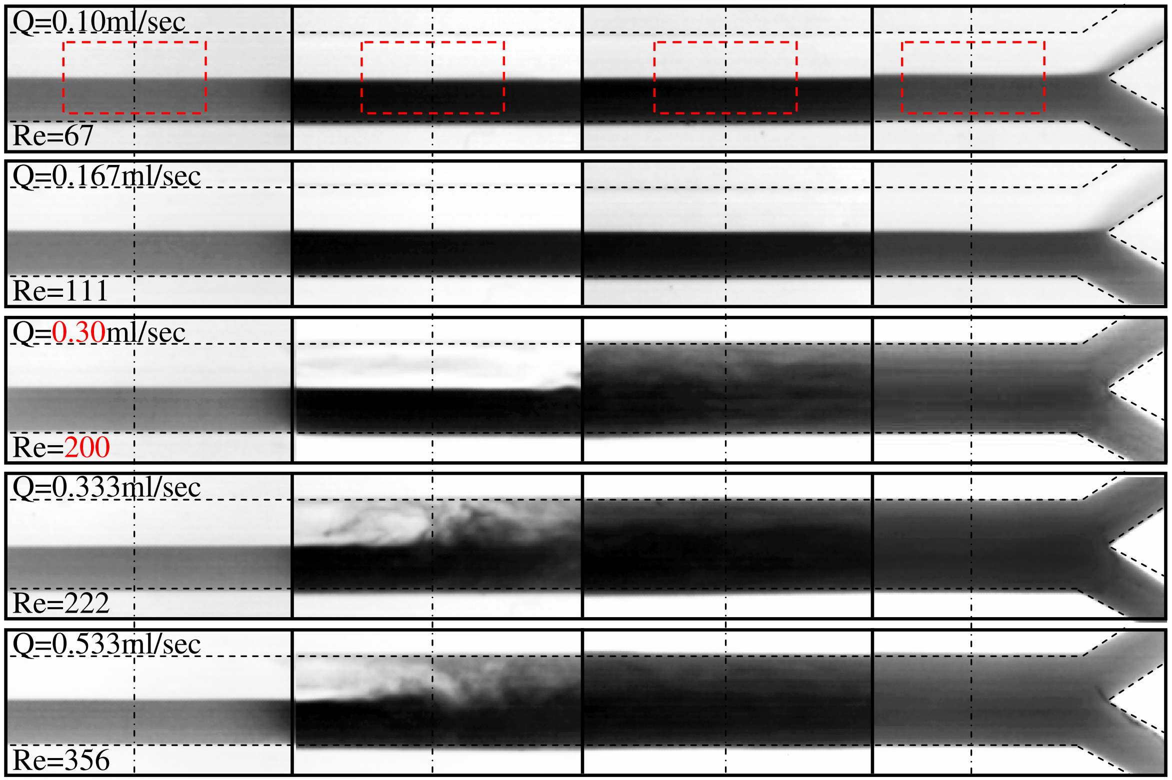
Verma and Kumaran, J. Fluid Mech.**705**, *322-347,2012;* J. Fluid Mech.**727**, *407-455, 2013.*

Soft-walled microchannel



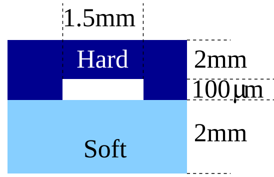
Re = $(\rho Q/W\eta)$

$G = 18kPa$

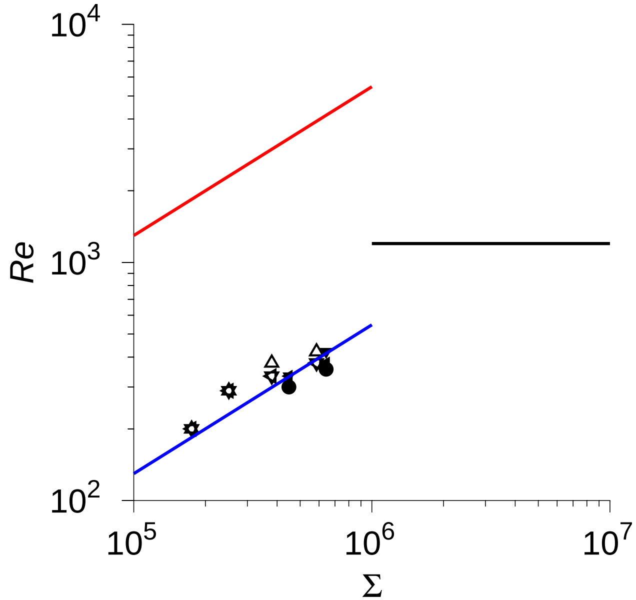


Re=67

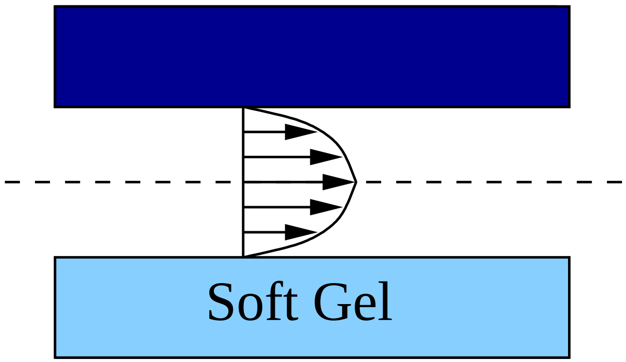
Microchannel flow:



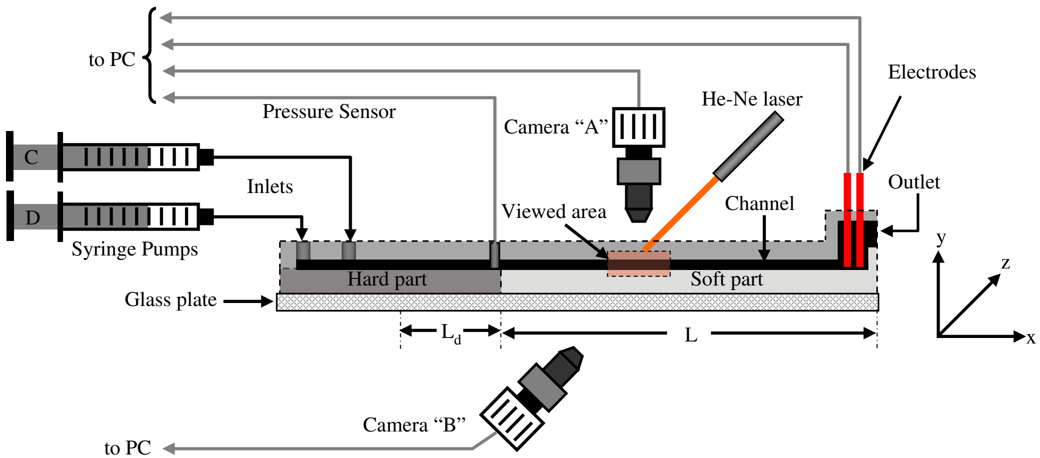
- Close to $Re \propto \Sigma^{3/4}$ (wall mode).
- Super-critical; continuous transition.
- Transition Re 0.1 times theoretical prediction!



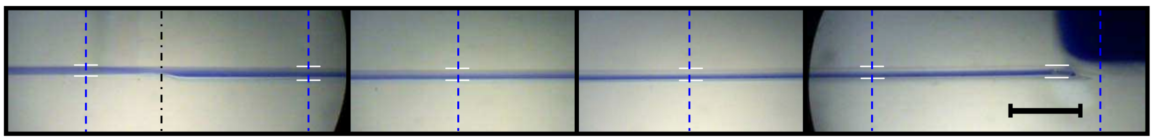
Theory, Experiment.



Microchannel flow:

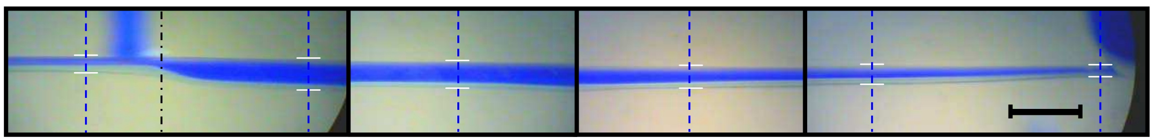


Undeformed:



x=-0.1cm x=0cm x=0.2cm x=1.0cm x=2.0cm x=2.7cm x=3.0cm

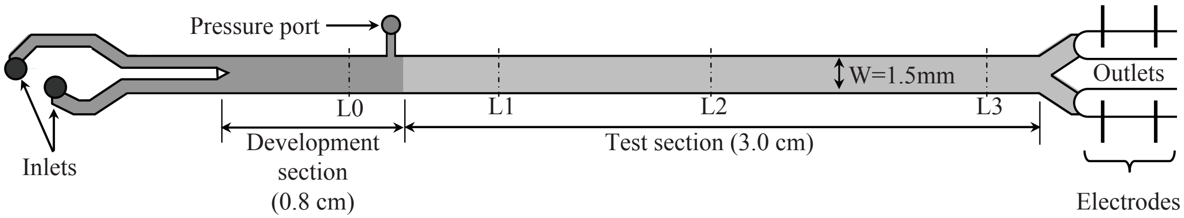
Deformed (Re=244, G = 25 kPa)



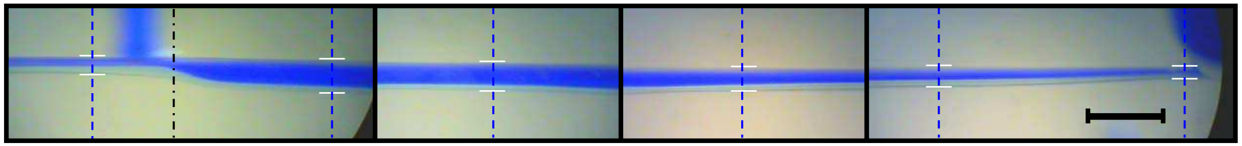
x=-0.1cm x=0cm x=0.2cm x=1.0cm x=2.0cm x=2.7cm x=3.0cm

Microchannel flow:

Top view:

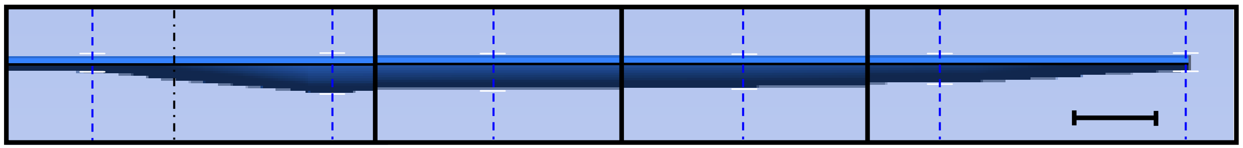


Side view: Deformed (Re=244, G = 25 kPa)

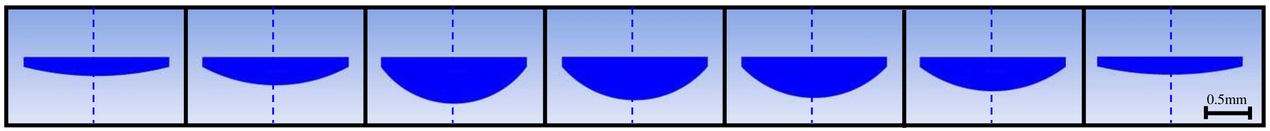


x=-0.1cm x=0cm x=0.2cm x=1.0cm x=2.0cm x=2.7cm x=3.0cm

Side view Simulation reconstruction:

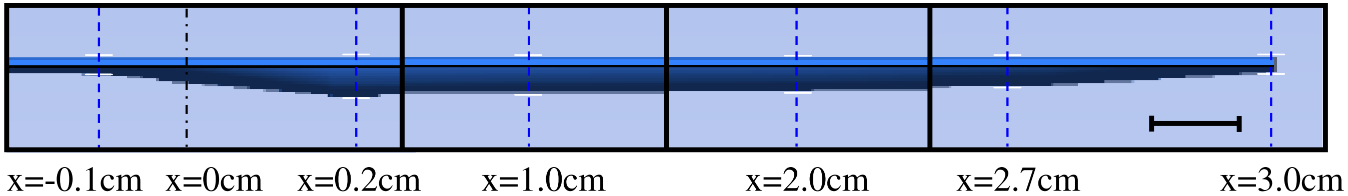


x=-0.1cm x=0cm x=0.2cm x=1.0cm x=2.0cm x=2.7cm x=3.0cm



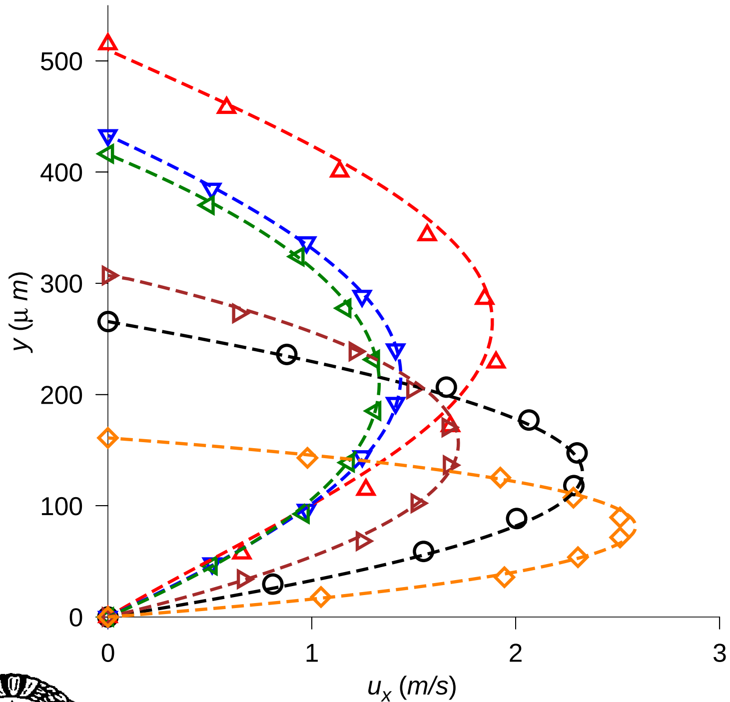
x=-0.1cm x=0cm x=0.2cm x=1.0cm x=2.0cm x=2.7cm x=3.0cm

Microchannel flow:

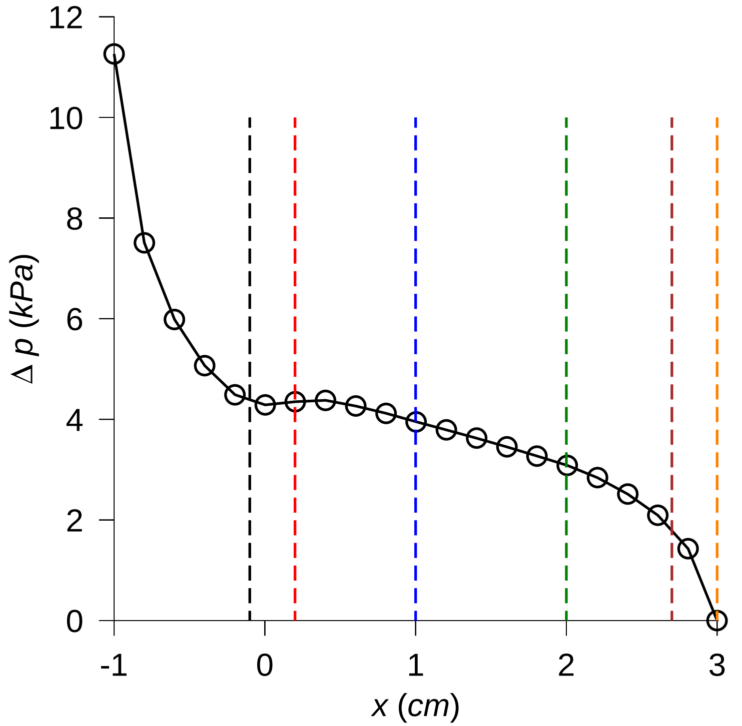


x = -0.1 cm 0.2 cm 1 cm 2 cm 2.7 cm 3 cm

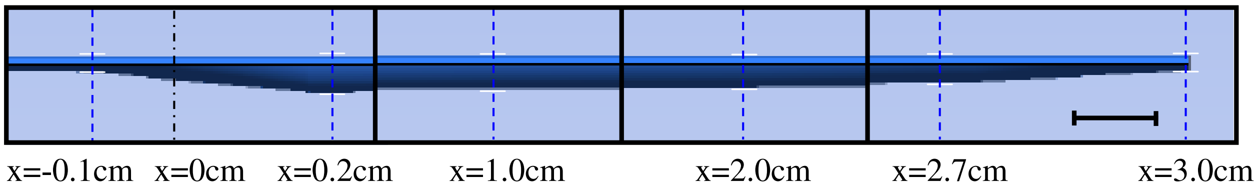
Velocity profiles:



Pressure:



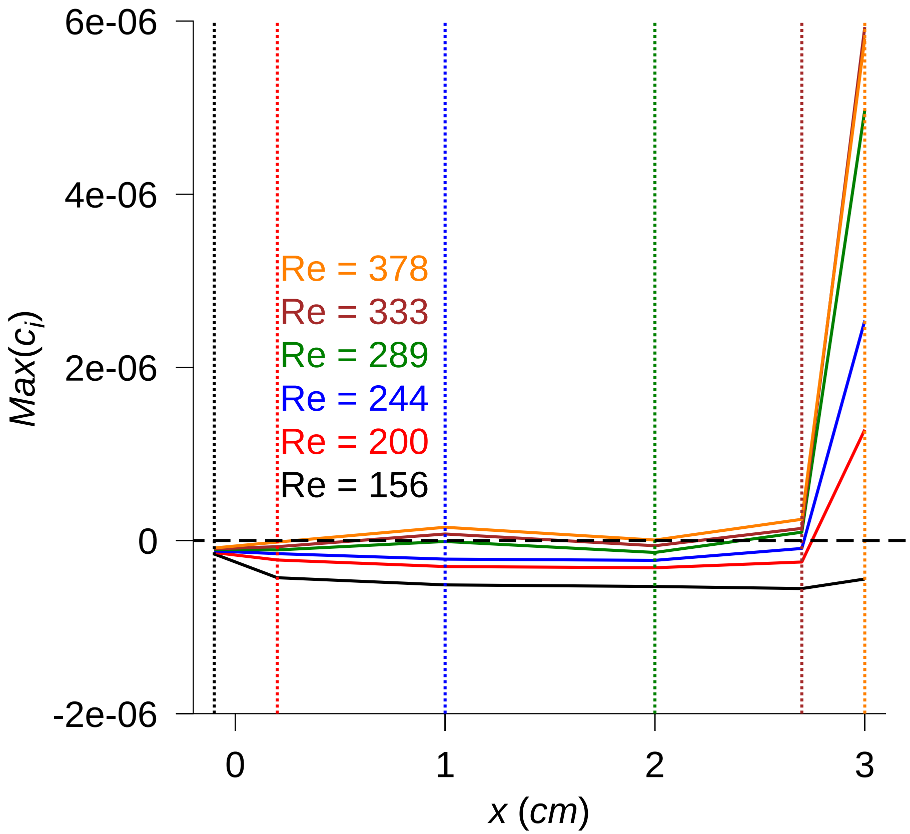
Microchannel flow:



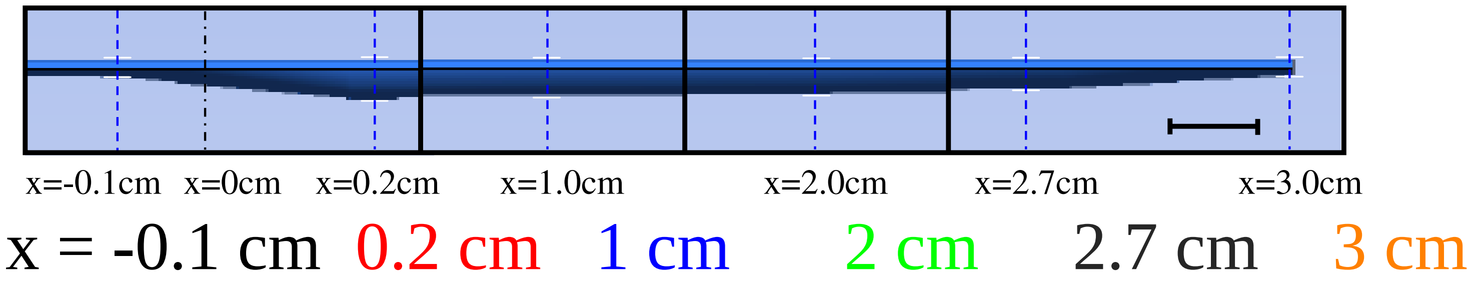
x = -0.1 cm 0.2 cm 1 cm 2 cm 2.7 cm 3 cm

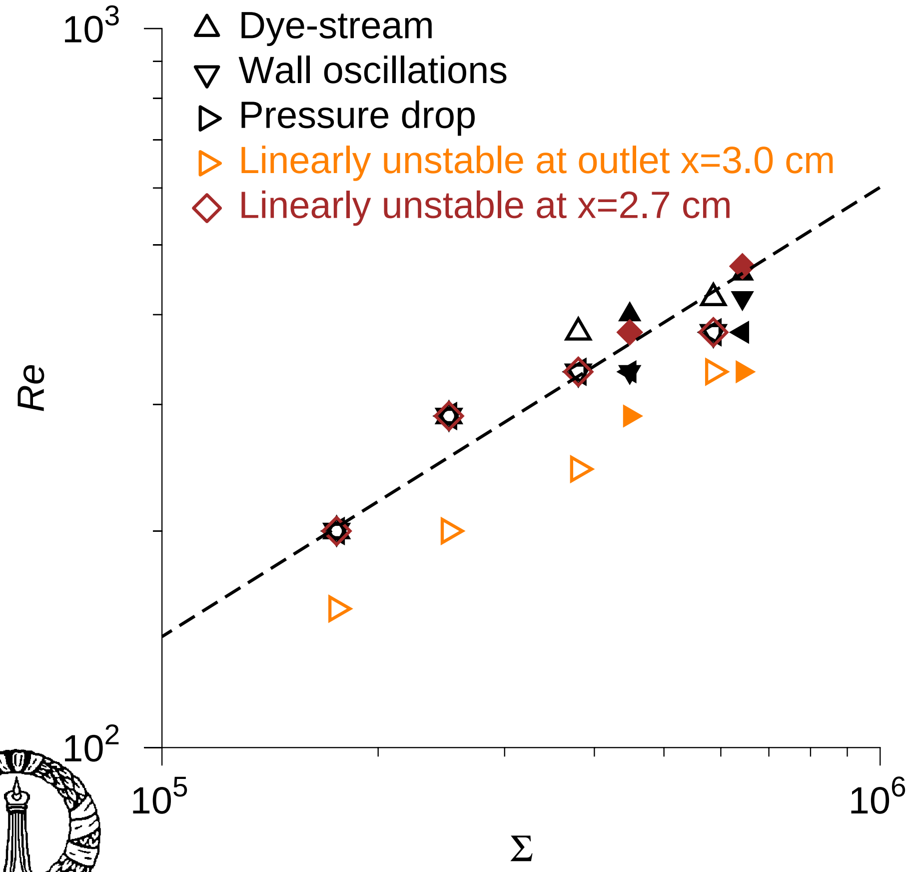
Linear stability analysis:
Use local velocity profile and pressure gradient, along with parallel flow approximation, go calculate the growth rates.

Instability first predicted downstream in converging section, not in upstream diverging section where deformation is maximum.



Microchannel flow:





Summary:

Fluid turbulence, and the transition from a laminar to a turbulent flow, is probably the most important unsolved problems in physics, more than a century after it was first discovered. The distinction between laminar and turbulent flows, and the characteristics of turbulent flows, will first be reviewed. This will be followed by a discussion about why fluid turbulence is difficult to understand, but also essential for life as we know it. The transition to turbulence due to hydrodynamic instabilities is well understood in some flows, such as rotating flow and fluids heated from below, and the routes to turbulence in these kinds of flows will be discussed. The transition is poorly understood in simple flows such as pipe flows; the reasons for these will be highlighted. Some of our work in understanding turbulence at small scales will be discussed. The talk will be at a basic level accessible to all???