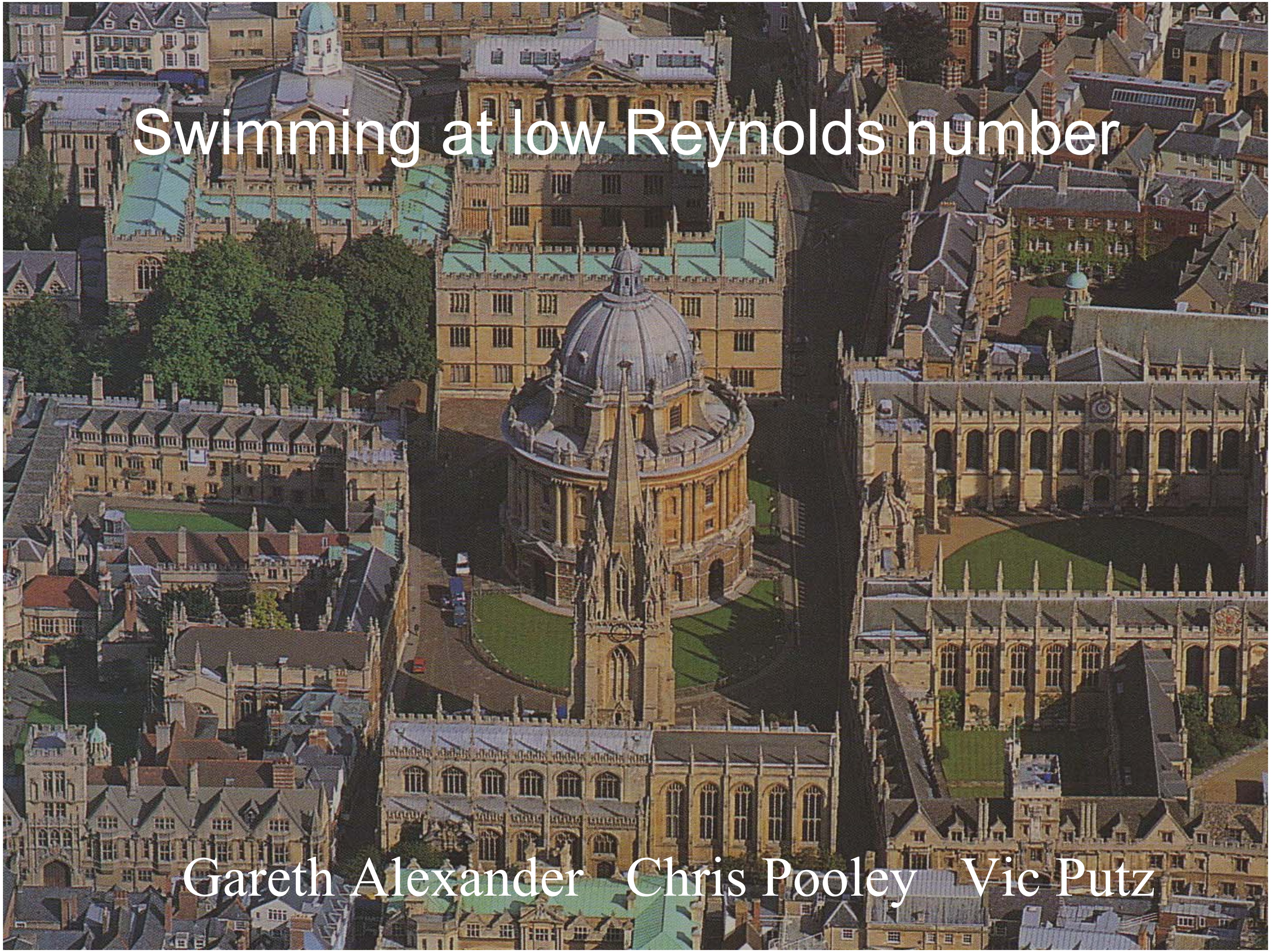
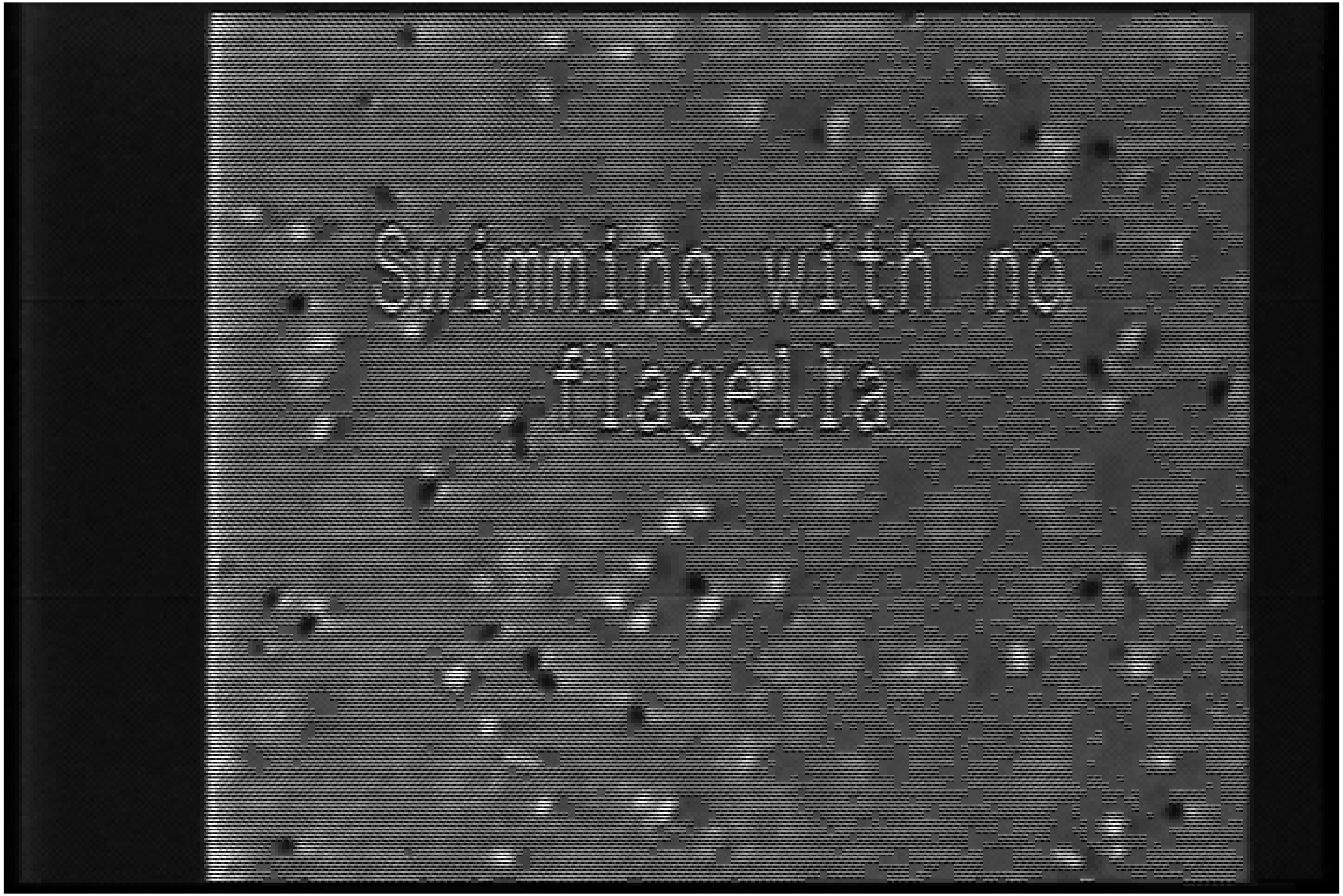


Swimming at low Reynolds number

Gareth Alexander Chris Pooley Vic Putz



synechococcus

http://mcb.harvard.edu/Faculty/Berg.html

Low Reynolds number swimming

inertial

$$\partial_t \left(n u_\gamma \right) + \partial_\alpha \left(n u_\gamma u_\alpha \right) =$$

viscous

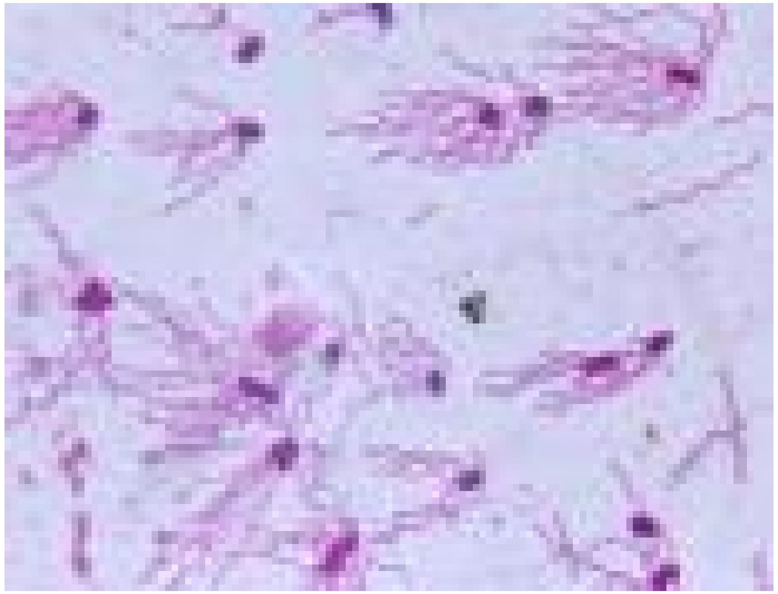
$$- \partial_\alpha p + \nu \partial_\beta \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha + \frac{1}{3} \partial_\gamma u_\gamma \delta_{\alpha\beta} \right)$$

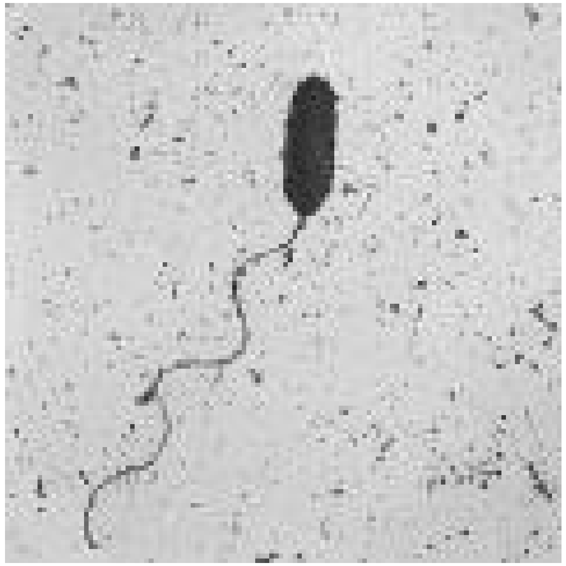
Re ~ inertial terms / viscous terms
~ length x velocity / viscosity ~10^{-6}

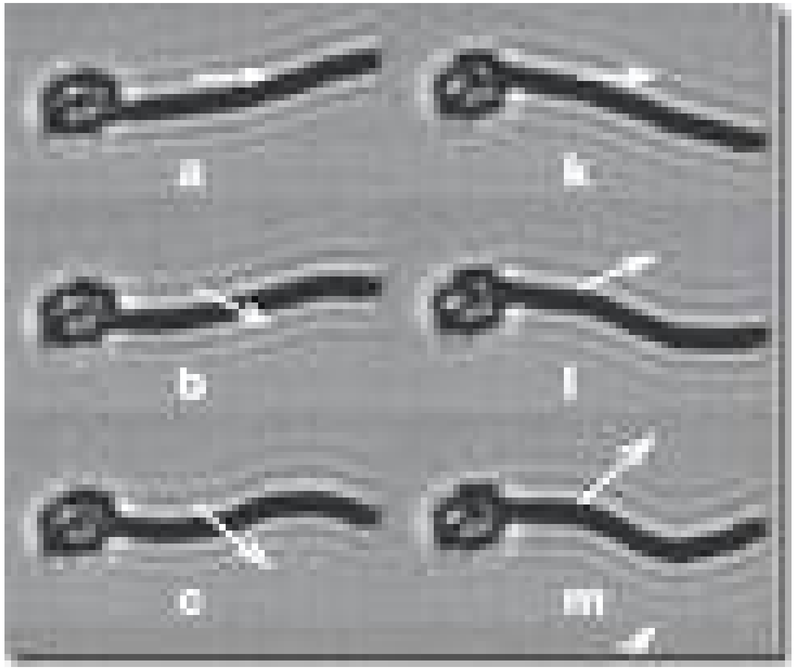
Navier-Stokes equations → Stokes equations which are time reversible

Need a non-reciprocal cycle for swimming
(Purcell, Shapere, Wilczek)

Micron scale swimmers





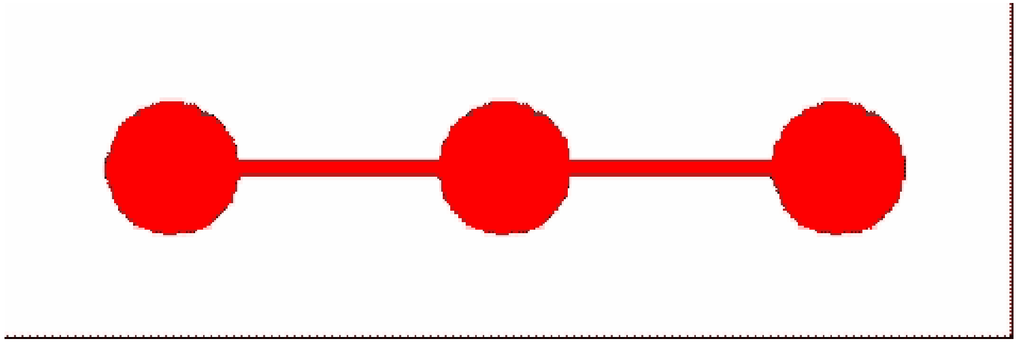


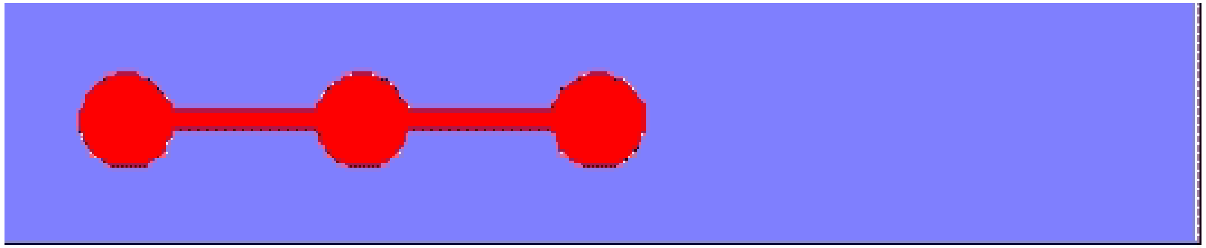
Dreyfus et al, Nature 2005

Low Reynolds number swimming

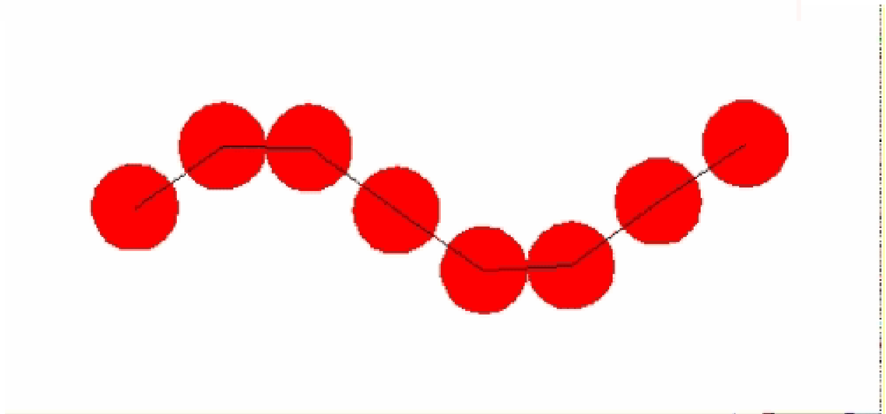
three sphere swimmer

Najafi and Golestanian, 2004





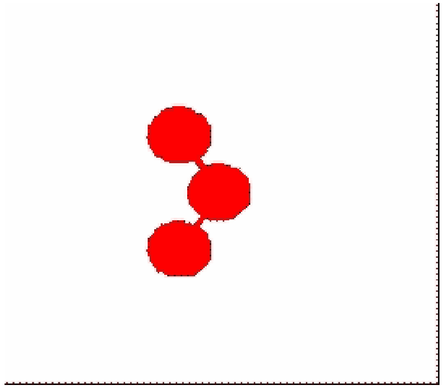
Snake swimmer

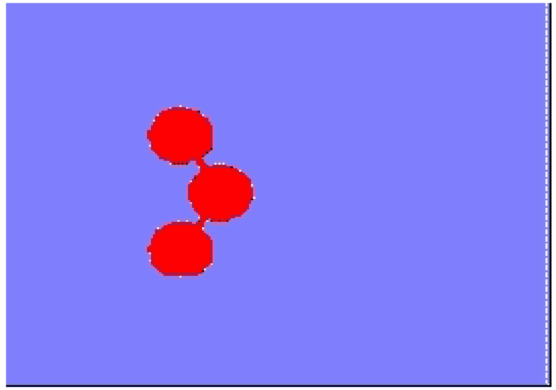


A generalised three sphere swimmer

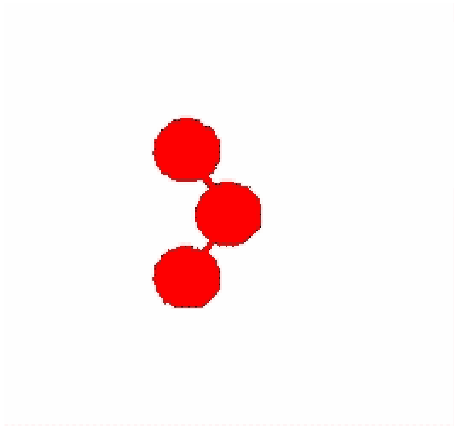
Radial and tangential motion

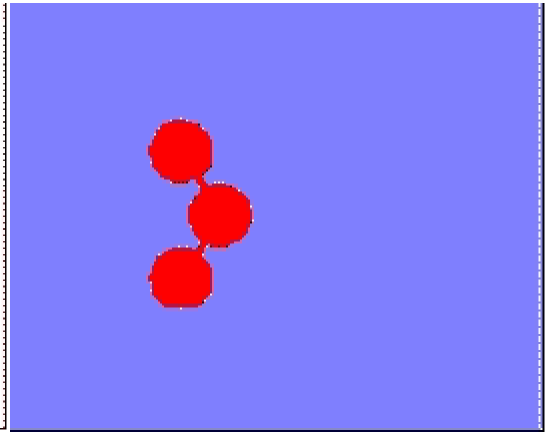
forward motion





turning motion





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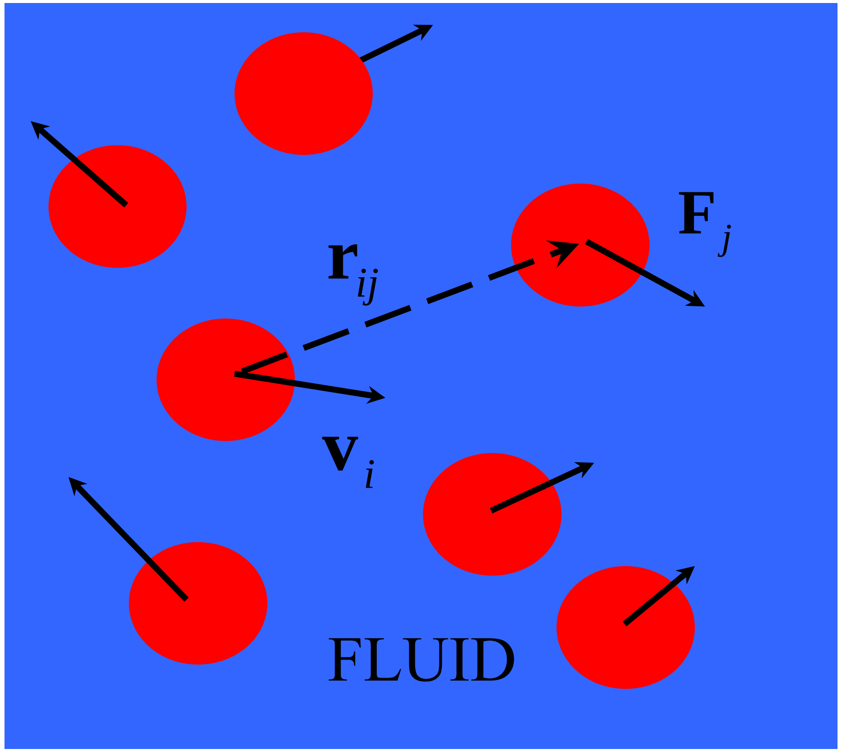
Oseen tensor hydrodynamics

$$v_i = H_{ij} F_j$$



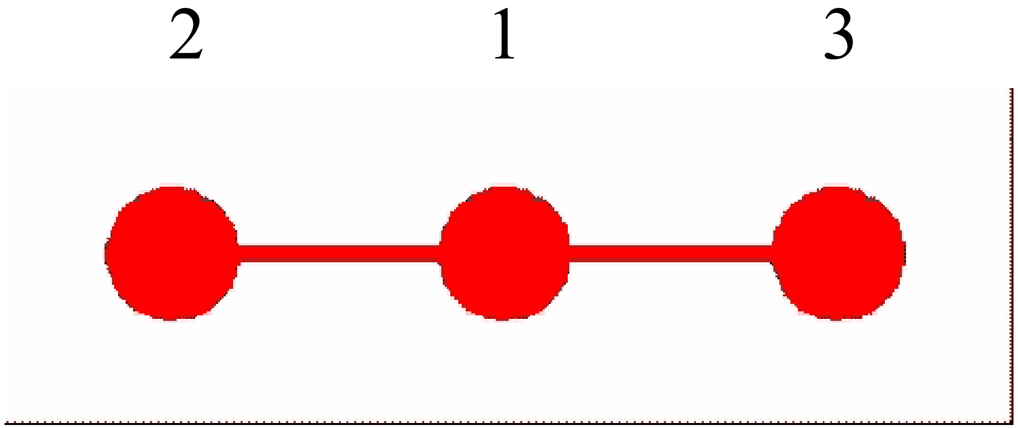
$$H_{ii} = \frac{1}{6\pi\eta R}, \qquad i = j$$

$$H_{ij} = \frac{1}{8\pi\eta r}\left(\delta_{ij} + \hat{r}_i \hat{r}_j\right), \qquad i \neq j$$

Shape change :

$$\mathbf{v}_2 - \mathbf{v}_1 = \mathbf{W}$$

$$\mathbf{v}_1 - \mathbf{v}_3 = \mathbf{0}$$

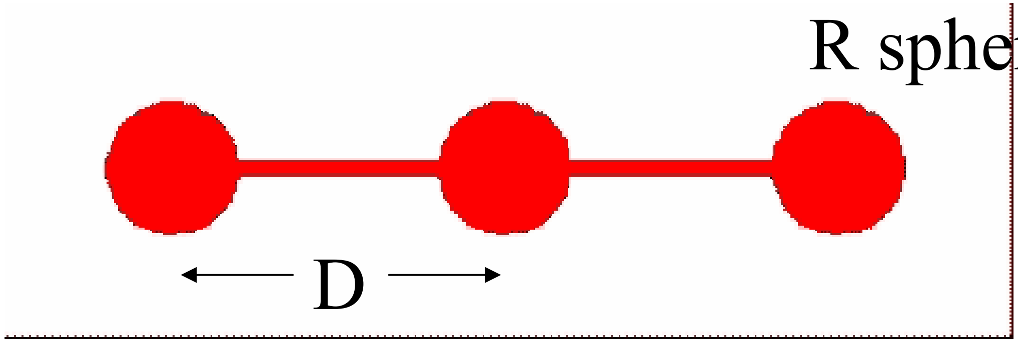


Enforce: no external forces or torques

$$\sum_i \mathbf{F}_i = \mathbf{0} \qquad \sum_i \mathbf{F}_i \times \mathbf{r}_i = \mathbf{0}$$

Oseen tensor (Stokes flow)

$$\mathbf{v}_i = \sum_{j=1}^{N} \mathbf{H}_{ij} \mathbf{F}_j$$



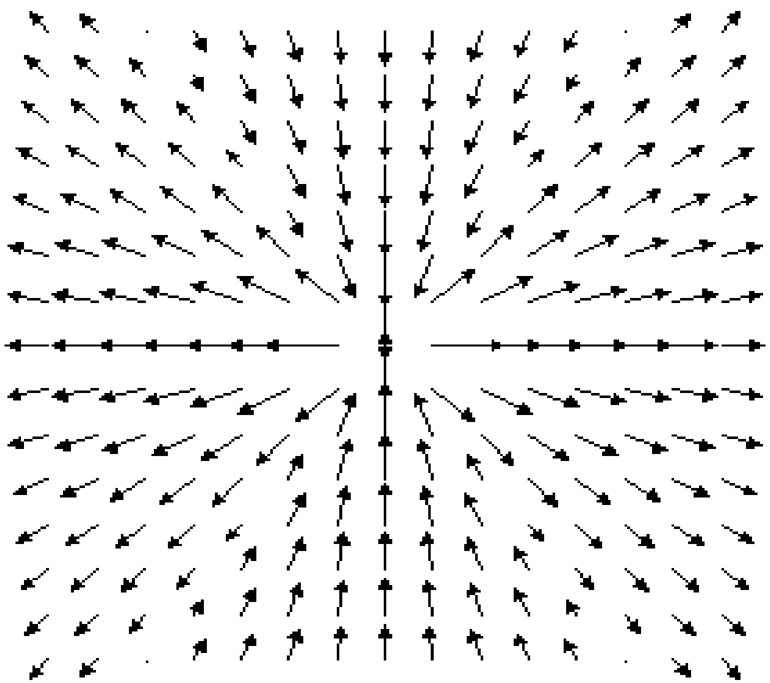
R sphere radius

ε length of swimming stroke

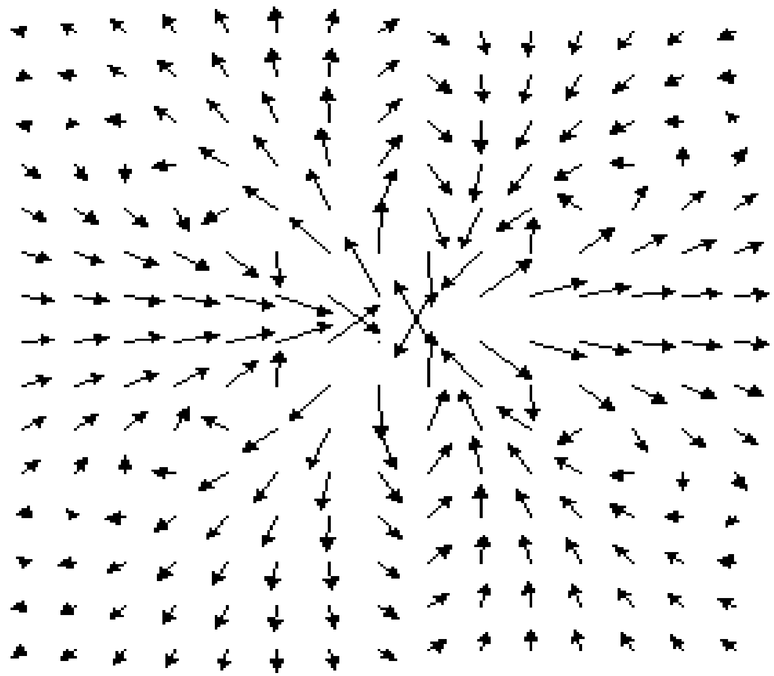
Displacement in a swimming stroke

$$\frac{7}{12} R \left(\frac{\varepsilon}{D} \right)^2$$

1. Time-averaged flow field – large r



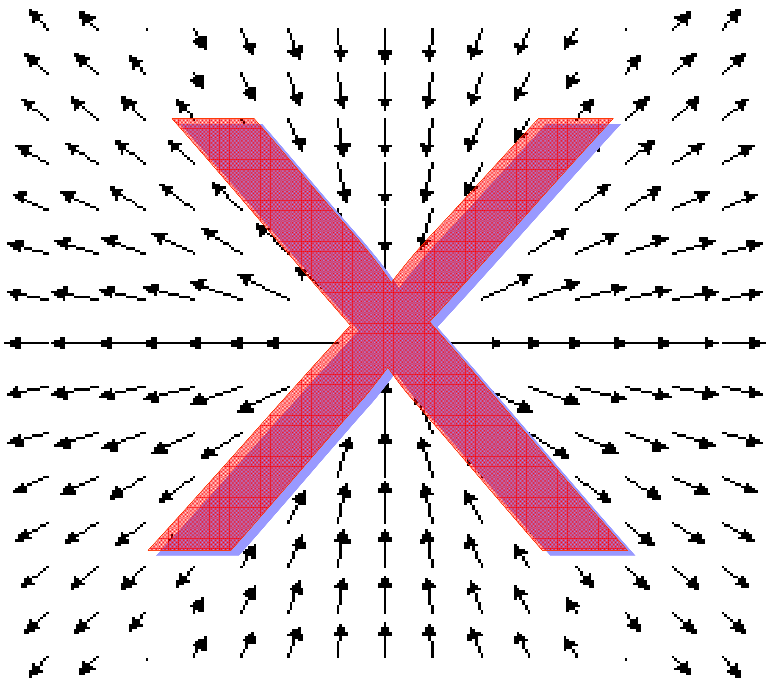
$$\left(\frac{1}{r}\right)^2$$

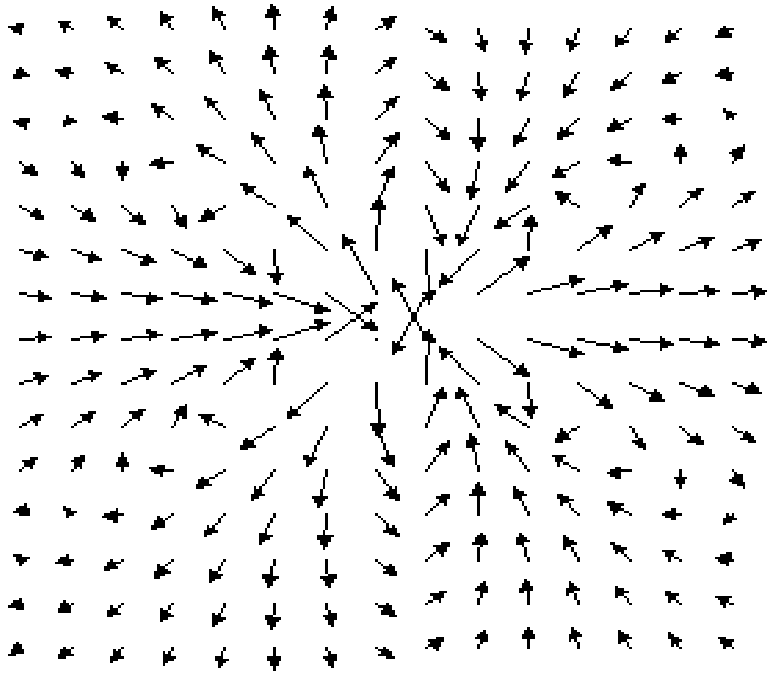


$$\left(\frac{1}{r}\right)^3$$

dipole + quadrupole

Time-averaged flow field – large r



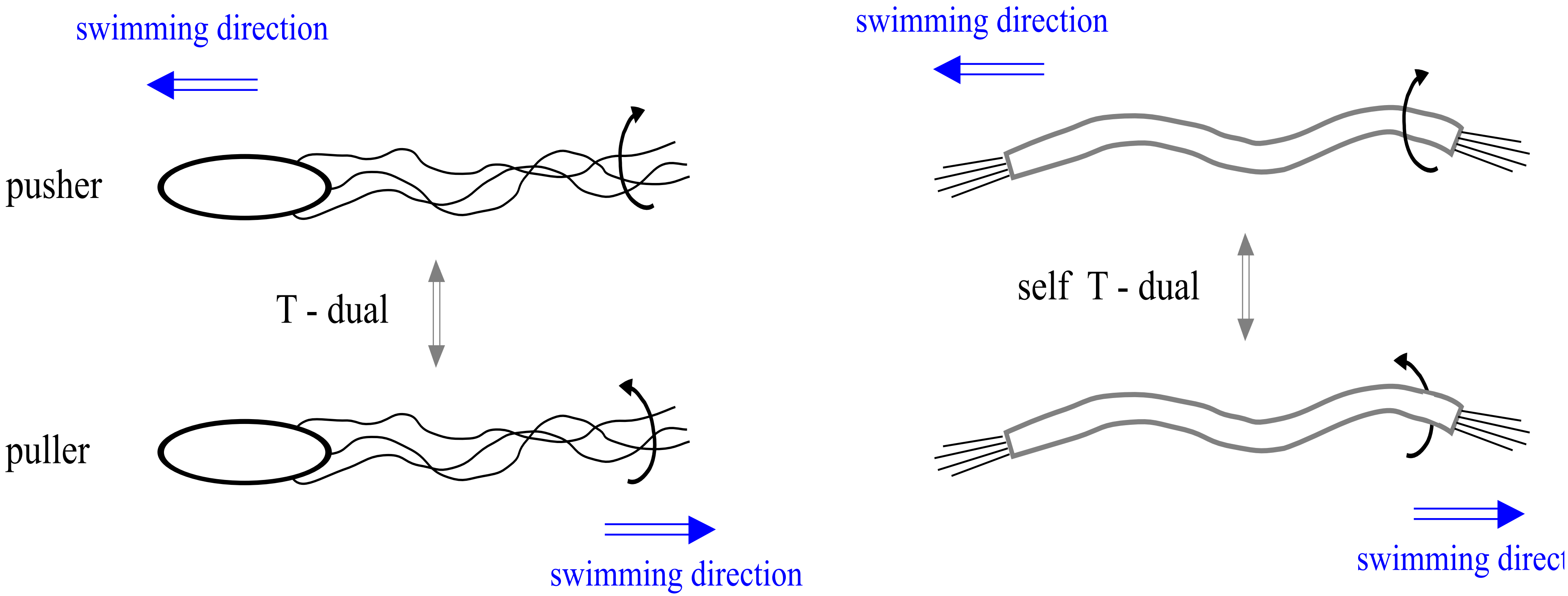


$$\left(\frac{1}{r}\right)^2$$

$$\left(\frac{1}{r}\right)^3$$

dipole + quadrupole

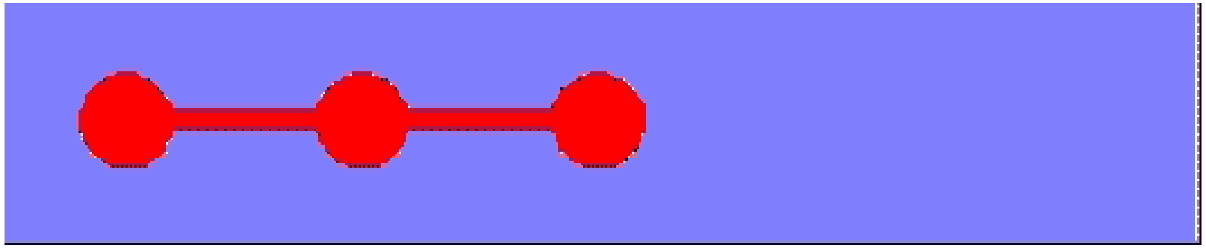
T-duality


swimming direction
pusher
T - dual
puller
swimming direction
swimming direction
self T - dual
swimming direct

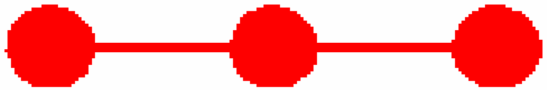
(a) **(b)**

self T-dual

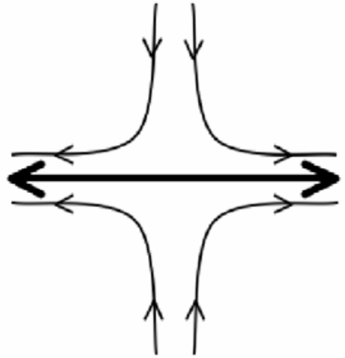
equal arm lengths

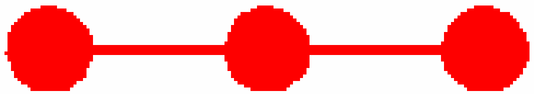


not self T-dual

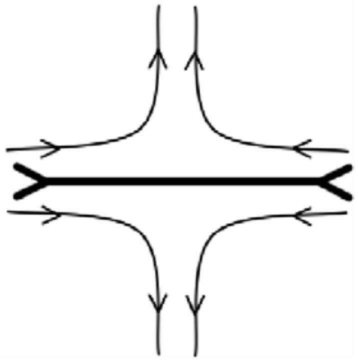


front arm longer
extensile





back arm longer
contractile



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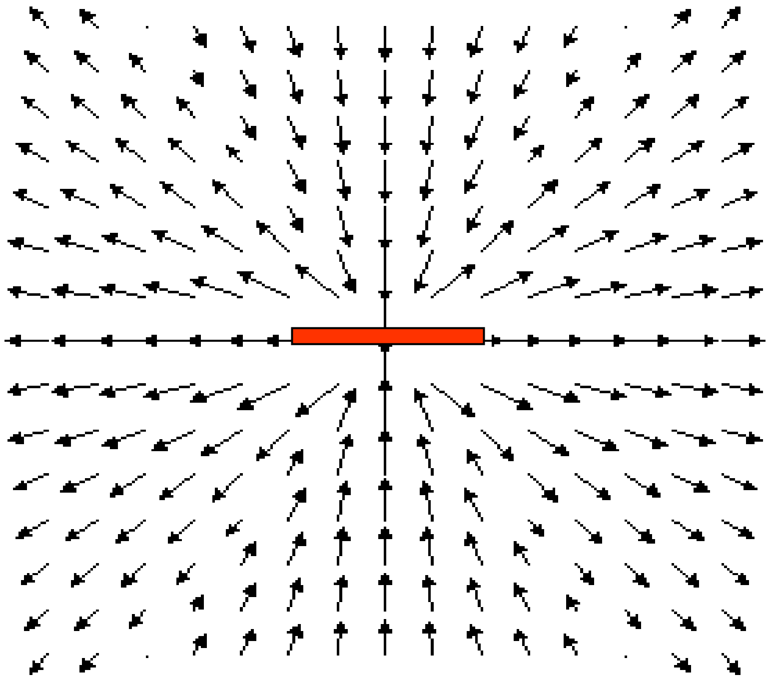
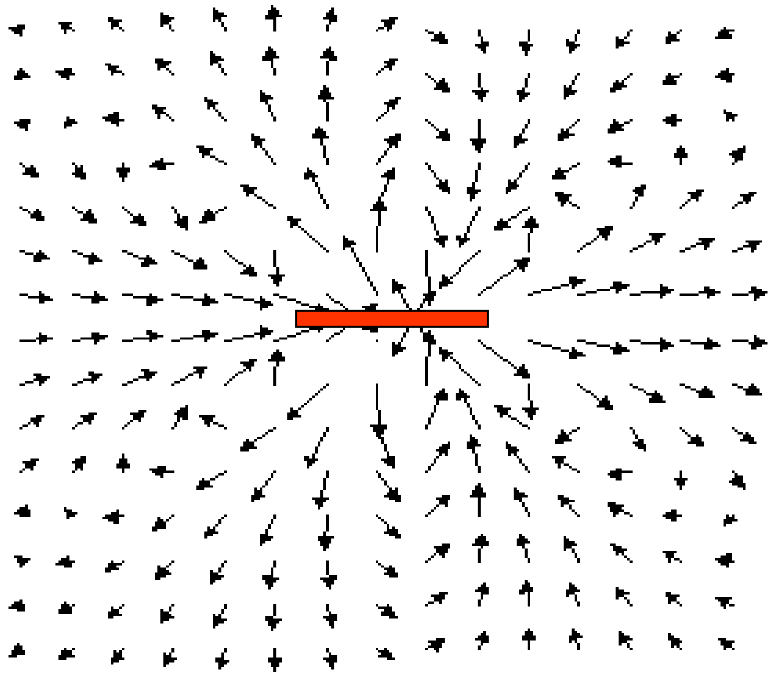


3-sphere, extensions different

$$\left(\frac{1}{r}\right)^2$$

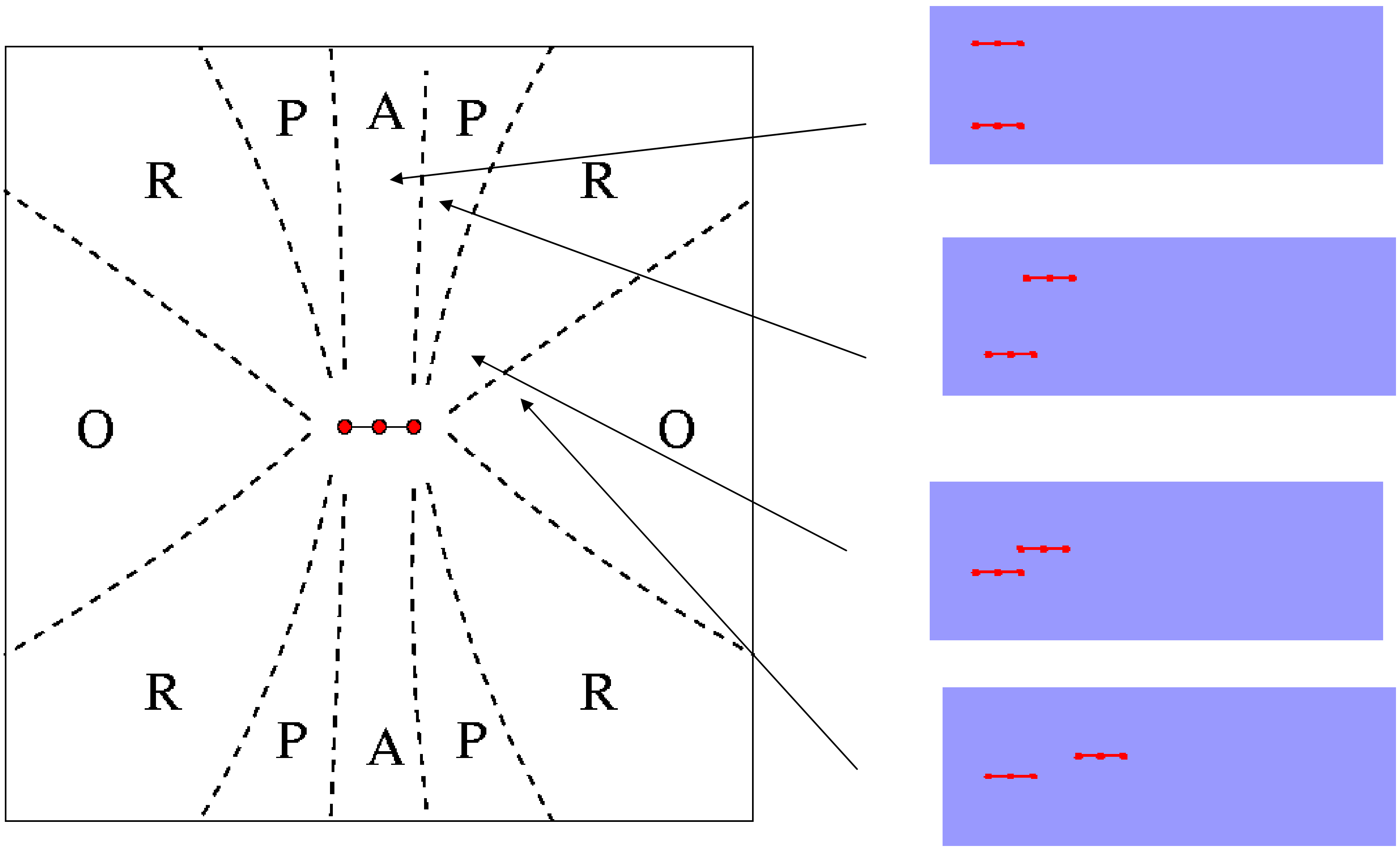


3-sphere, extensions same (self T-dual)

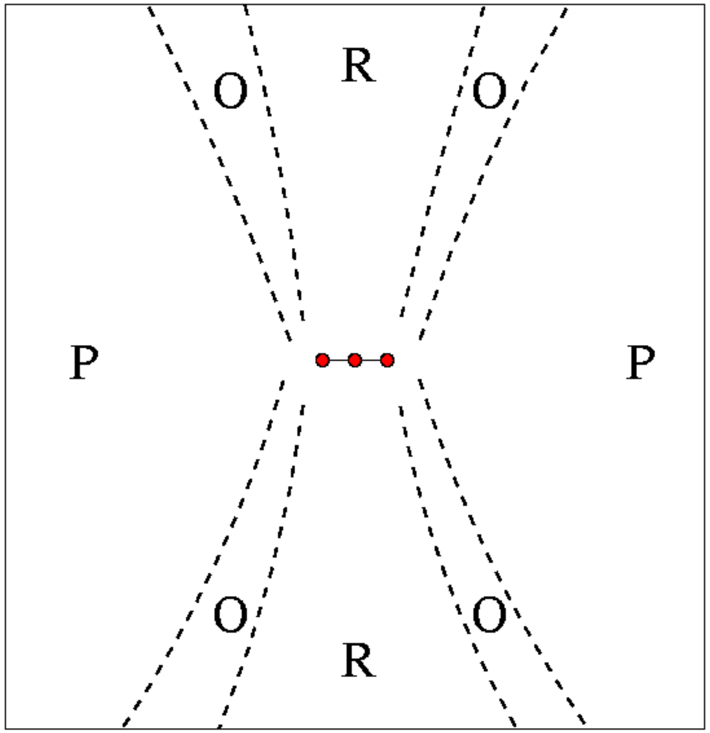
$$\left(\frac{1}{r}\right)^3$$

$\frac{1}{r}$

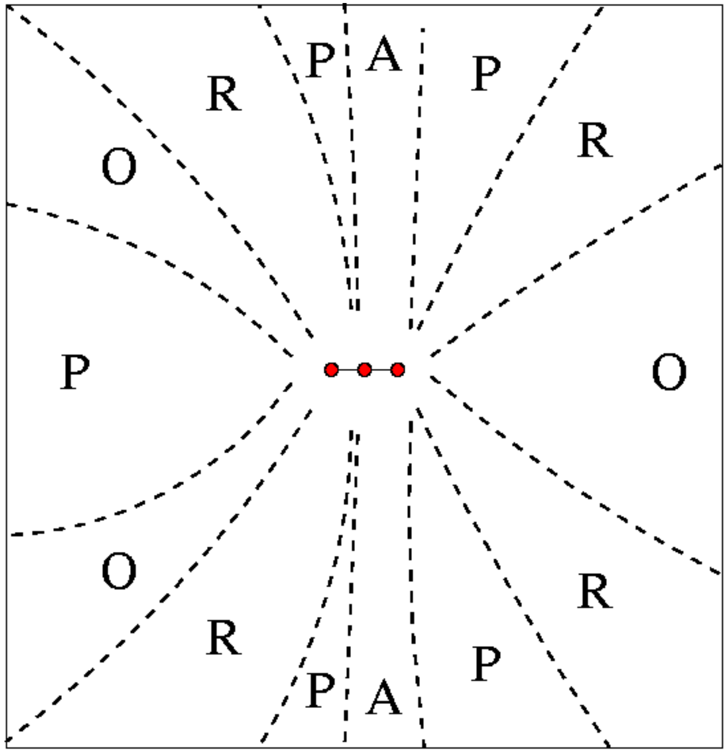
2. Two Golestanian swimmers in phase



Two Golestanian swimmers

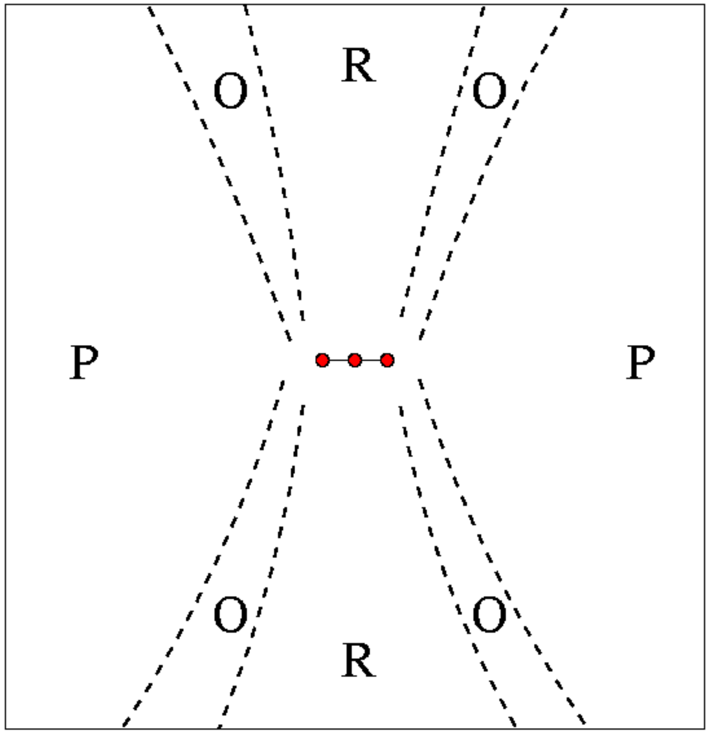


л out of phase

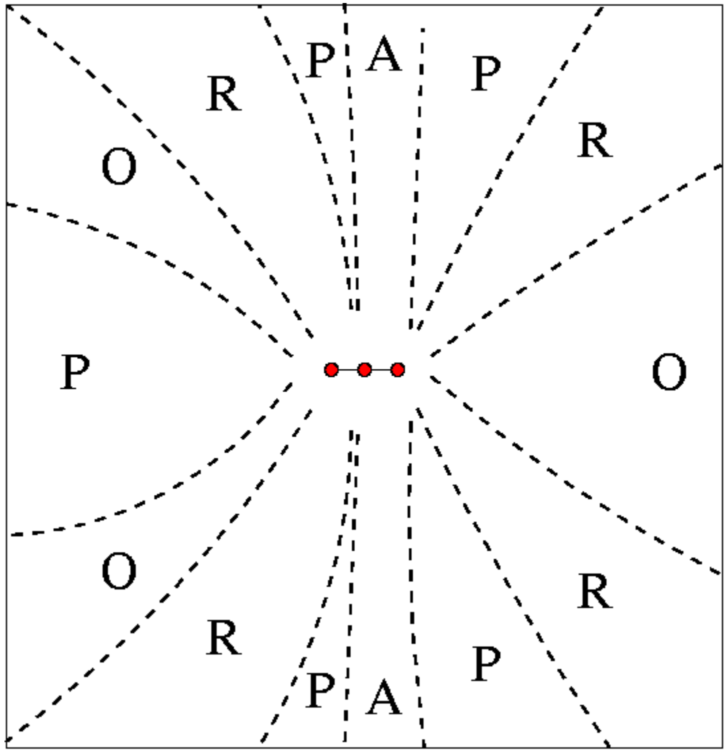


л/2 out of phase

Two Golestanian swimmers



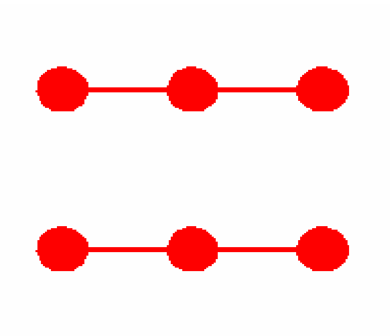
π out of phase

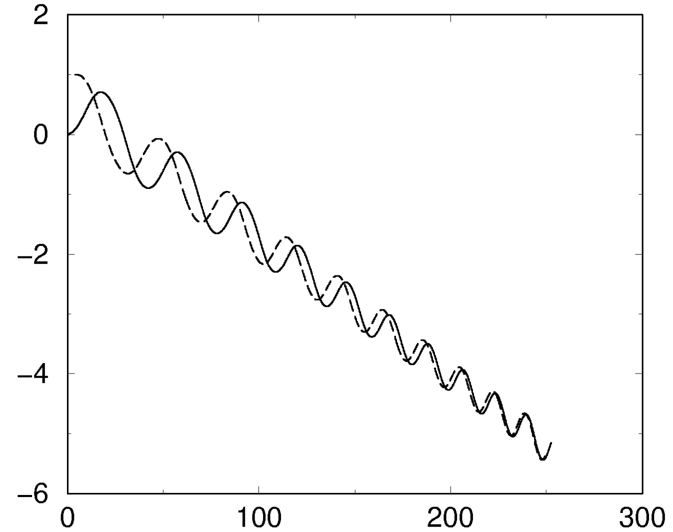


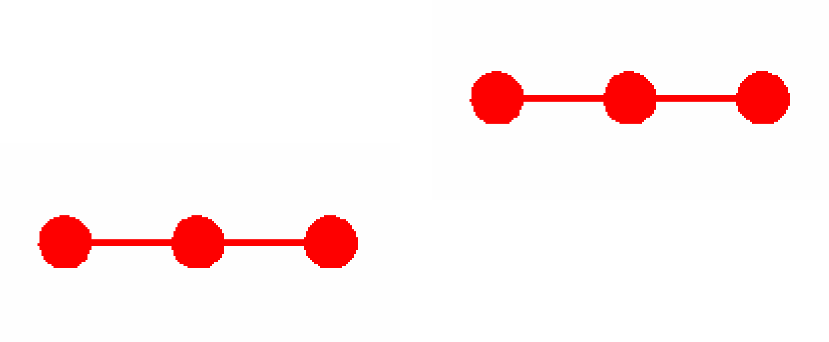
π/2 out of phase

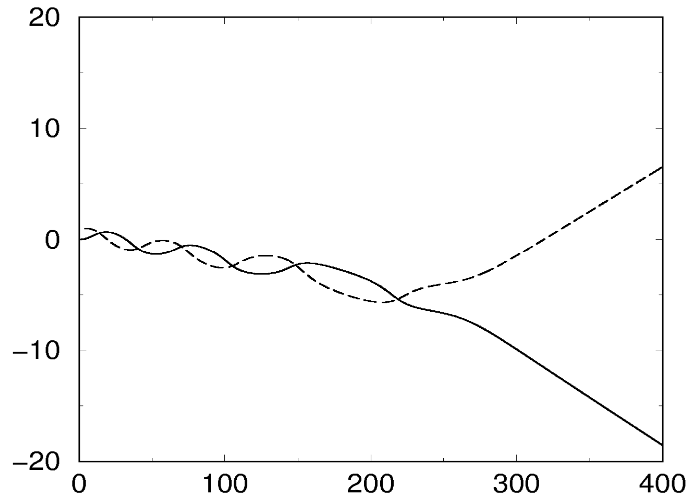
Relative phase matters – cannot treat a swimmer as a passive scalar

Non self T-dual swimmers









Swimming with friends at low Reynolds number

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consequence of time reversal invariance

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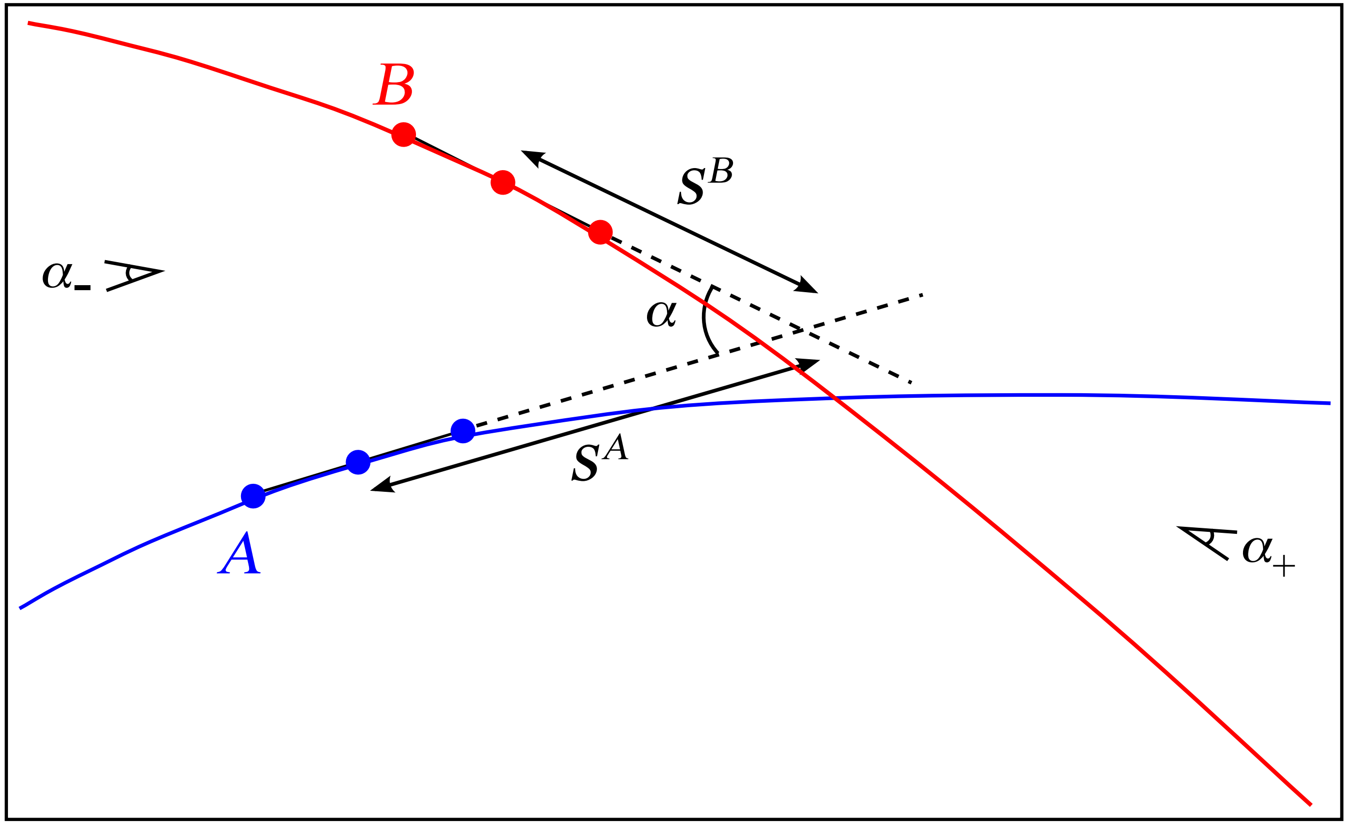
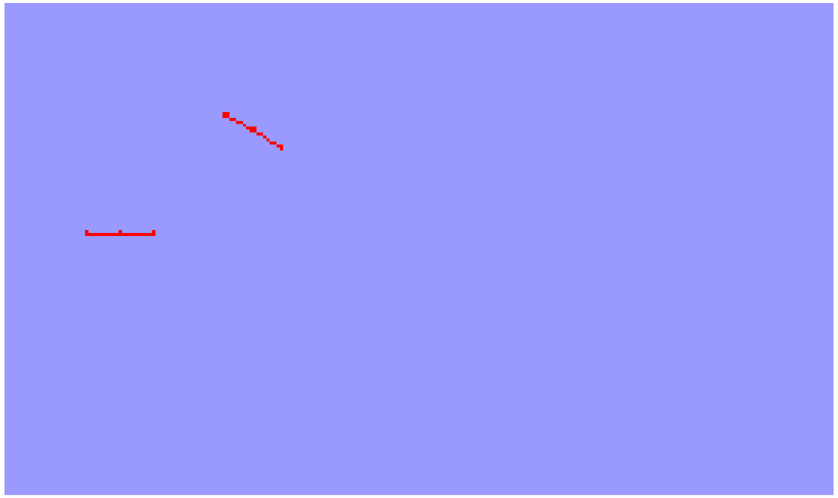
Swimmer-swimmer scattering



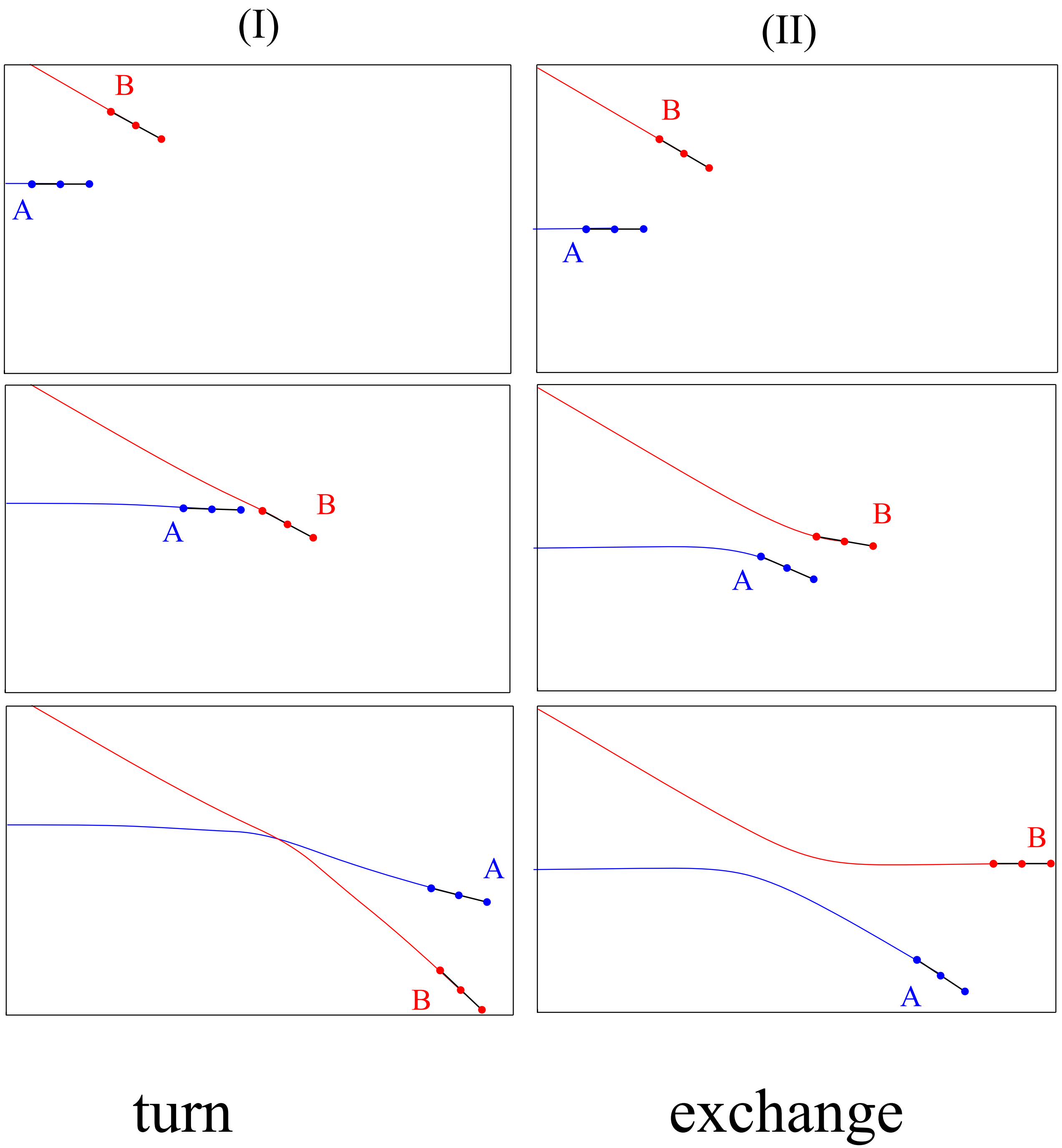
Hydrodynamic scattering

Oseen tensor method breaks down

EXCHANGE trajectories

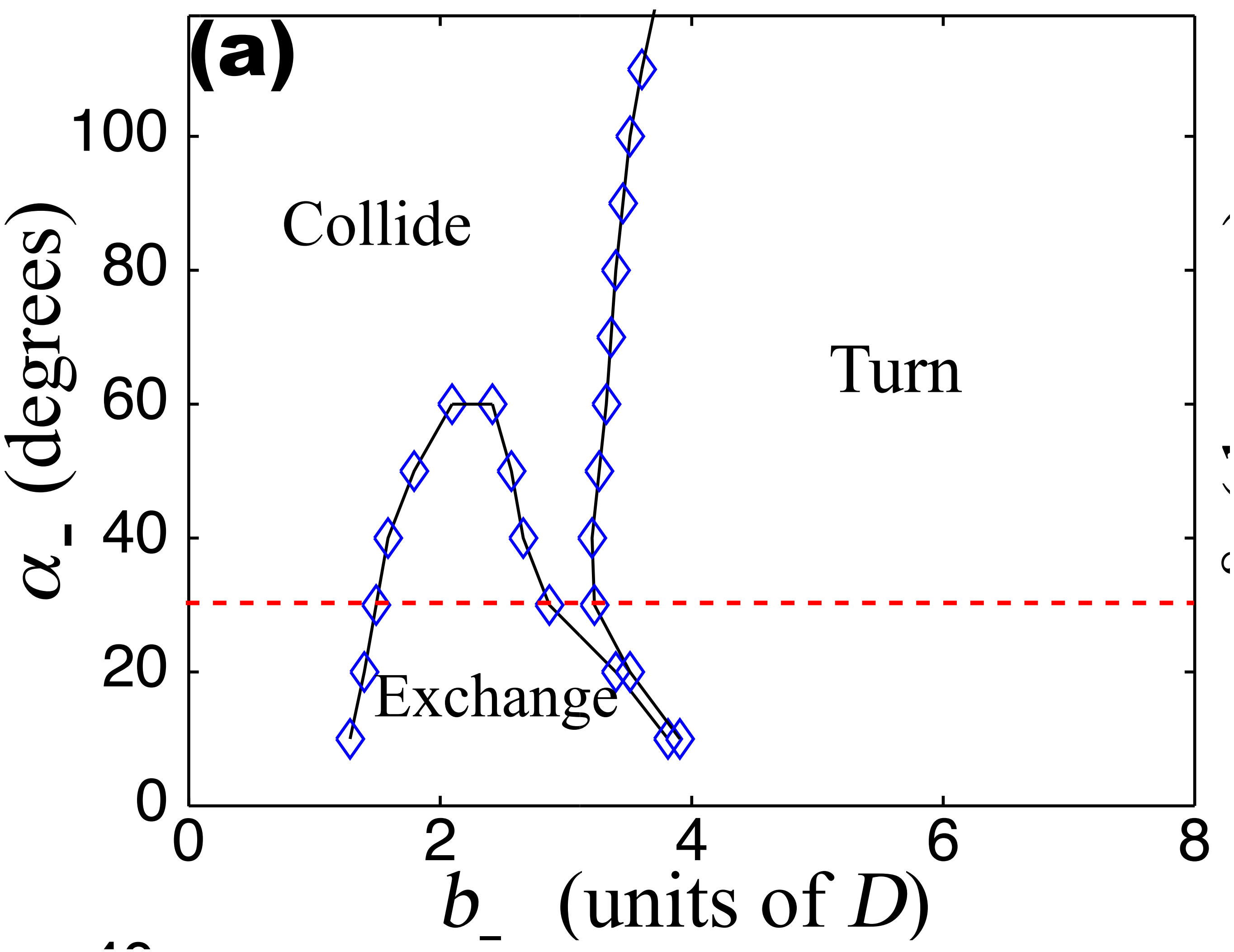


Both swimmers TURN in same direction

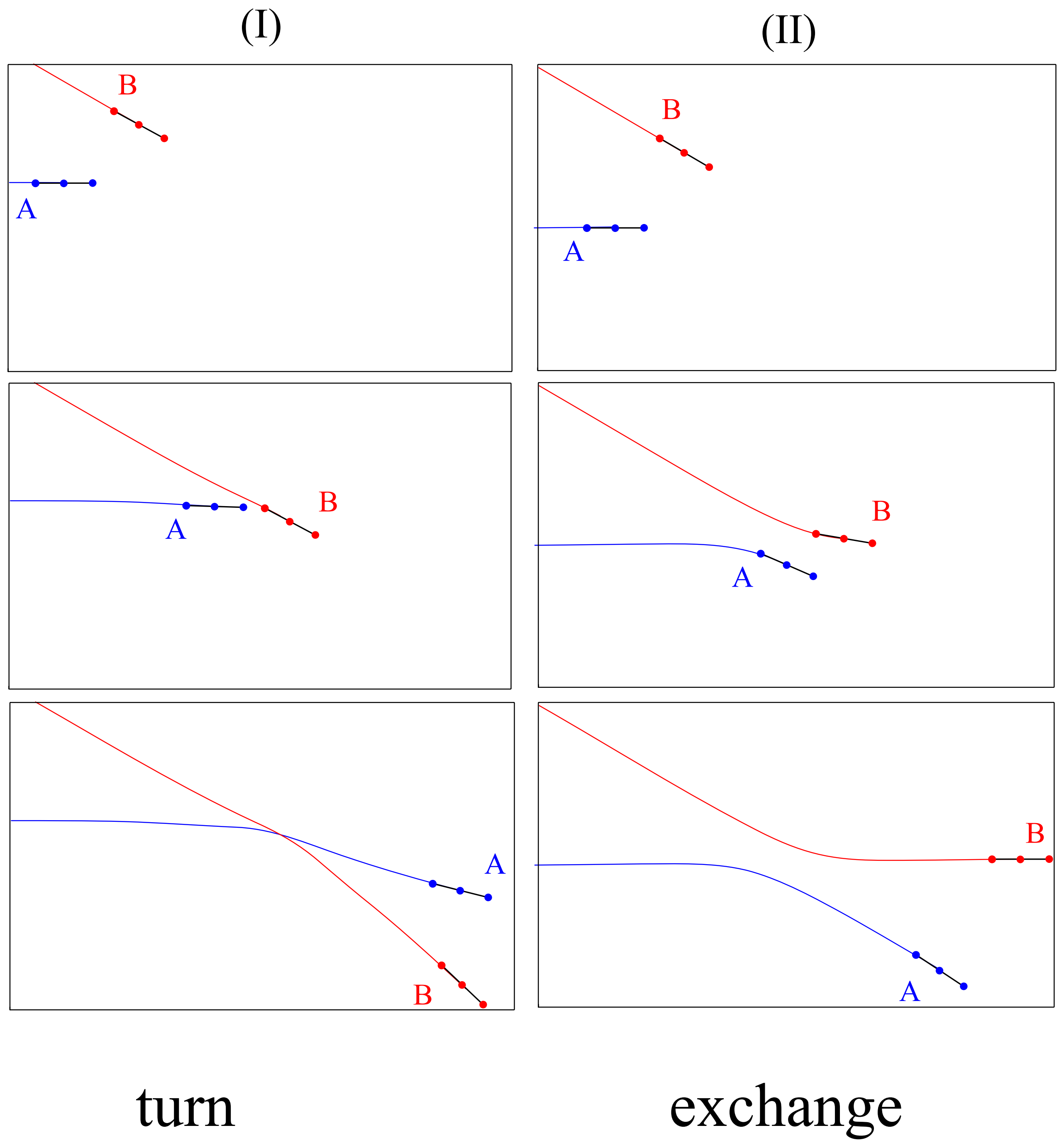


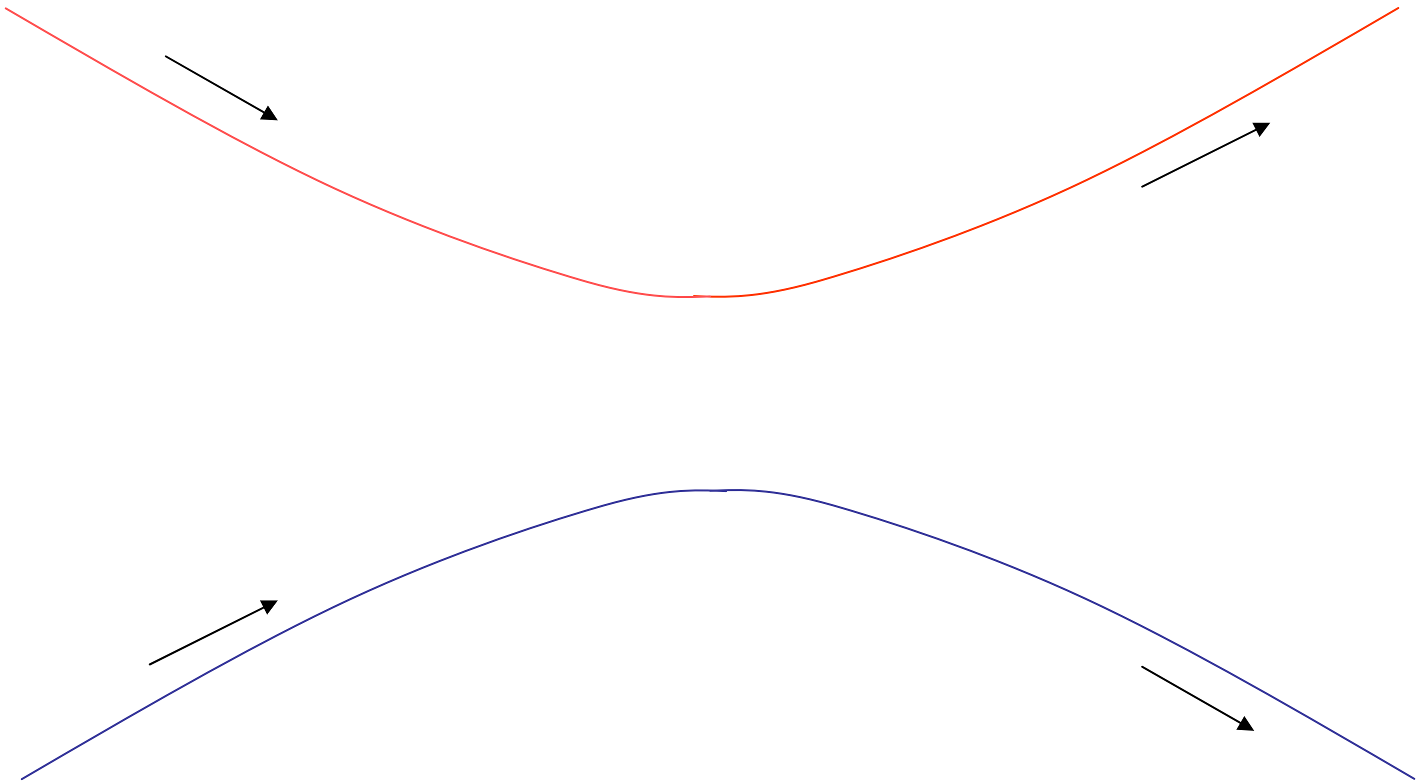
(I)
(II)
B
A
B
A
B
A
B
A
A
B
B
A
turn
exchange

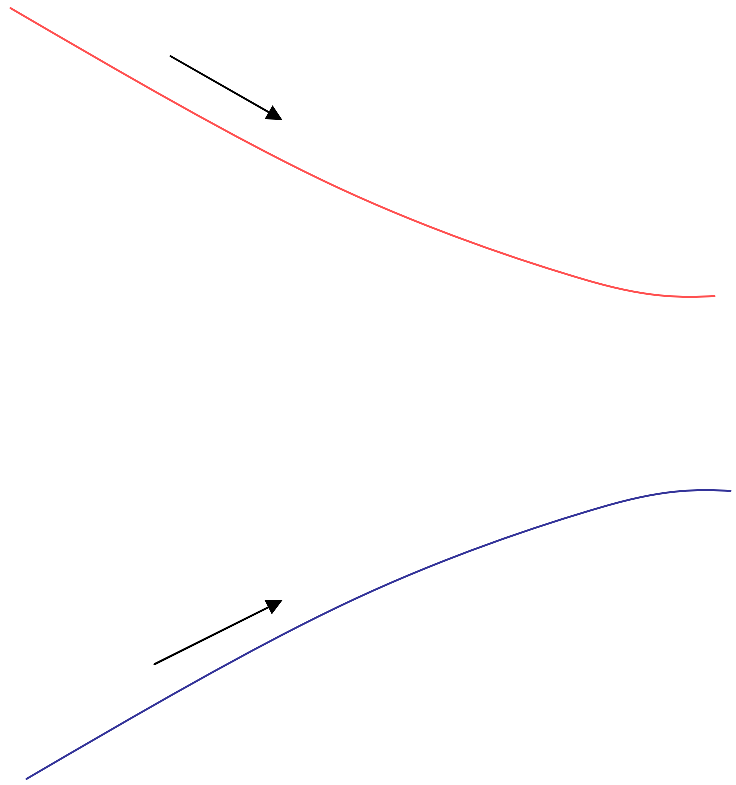
angle between incoming trajectories

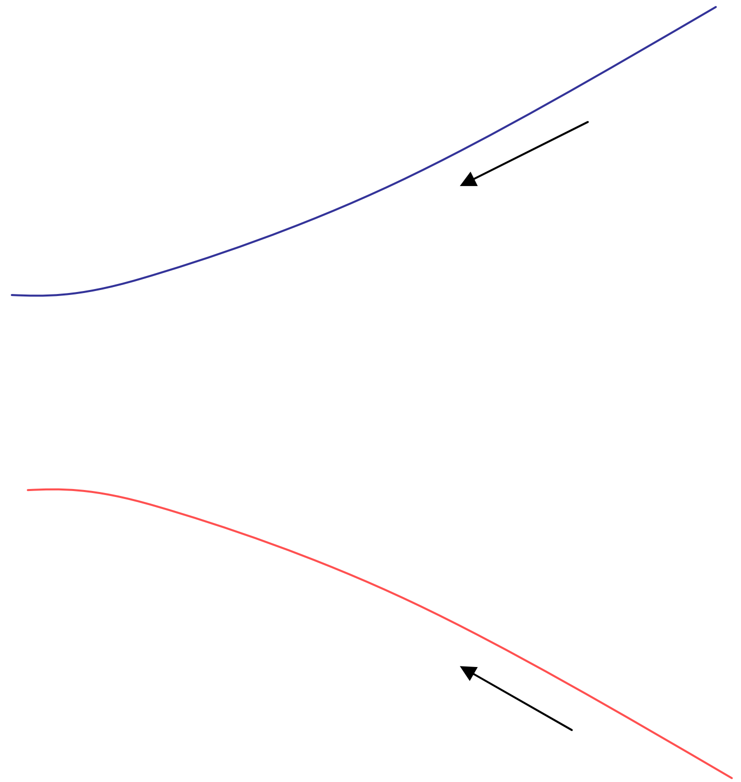


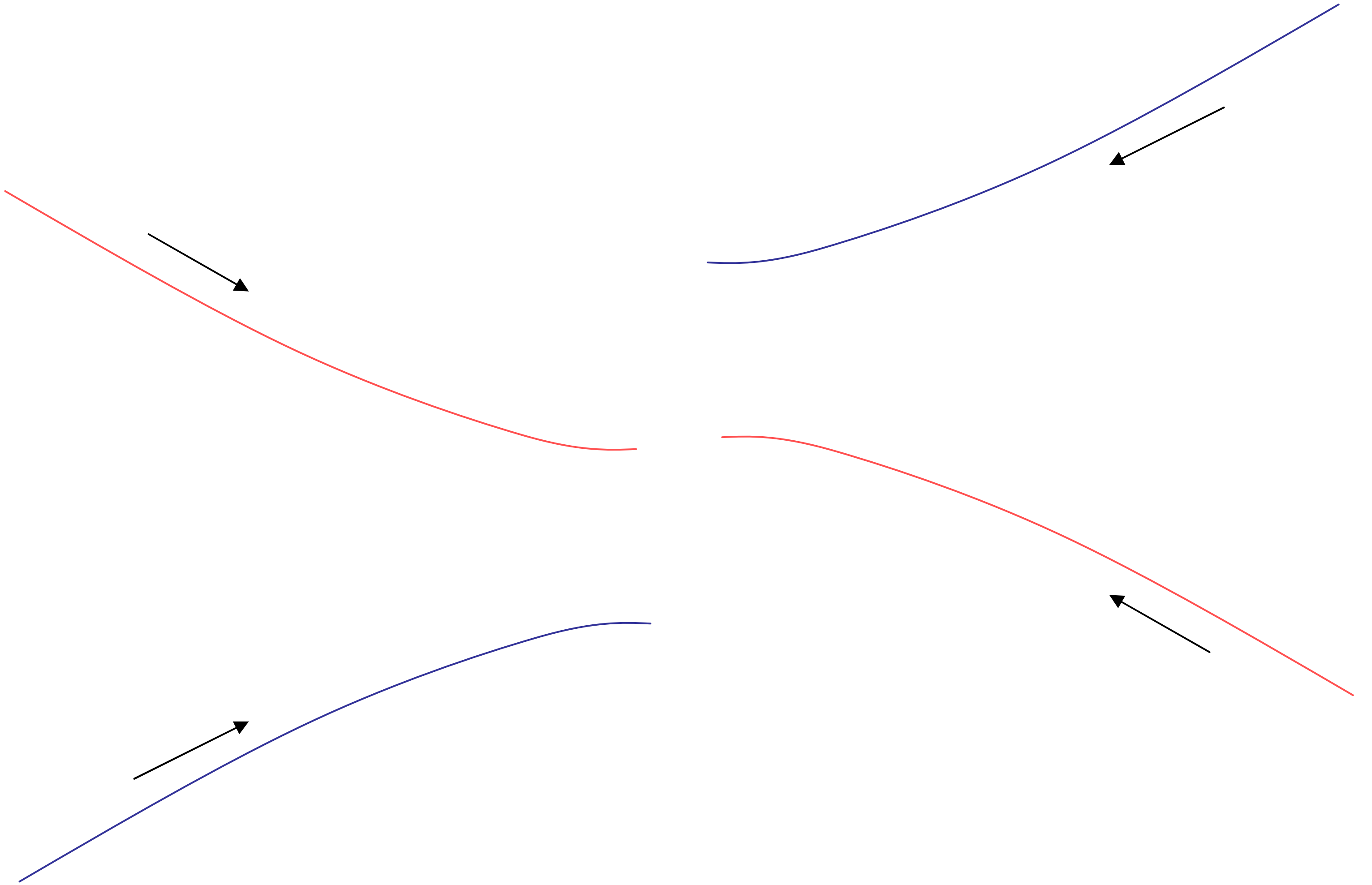
impact parameter

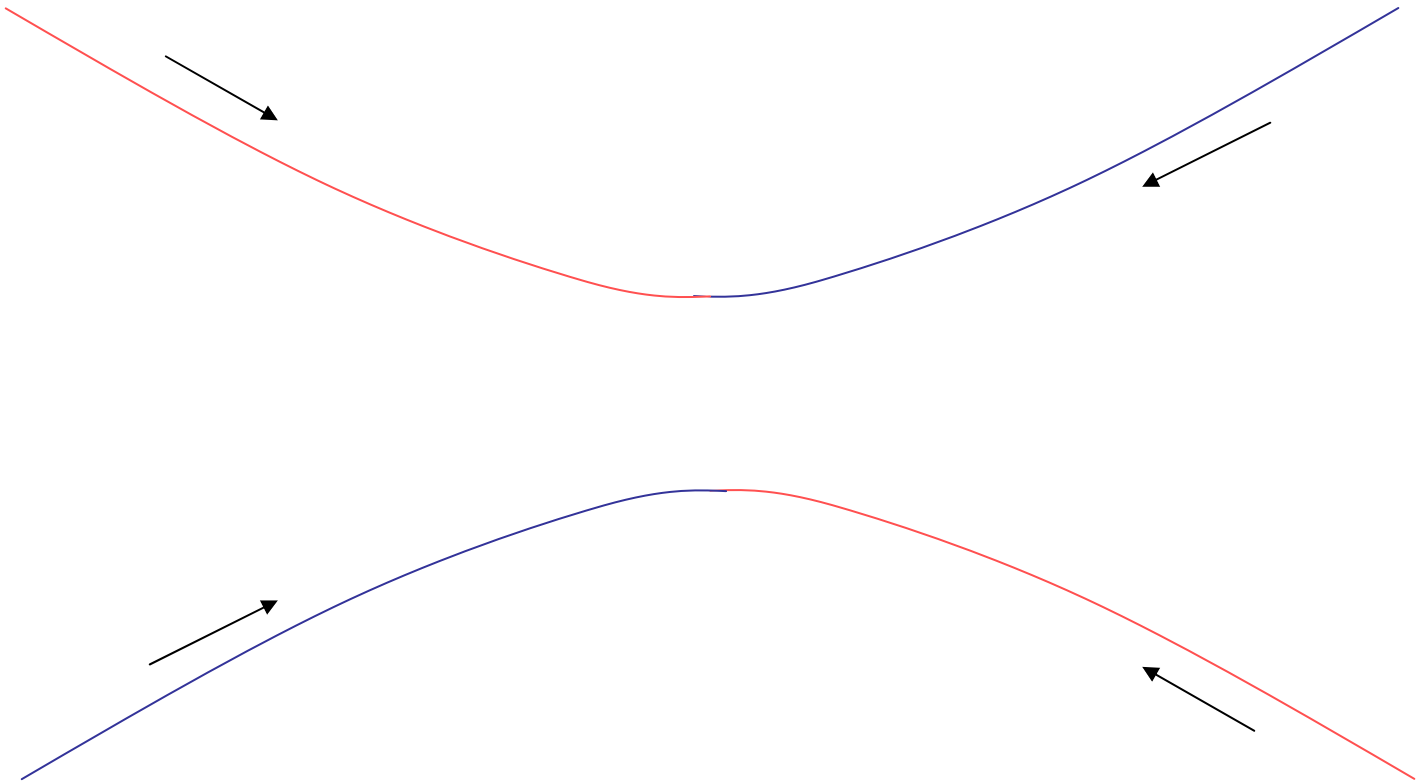

(I)
(II)
B
A
B
A
B
A
B
A
A
B
B
A
turn
exchange

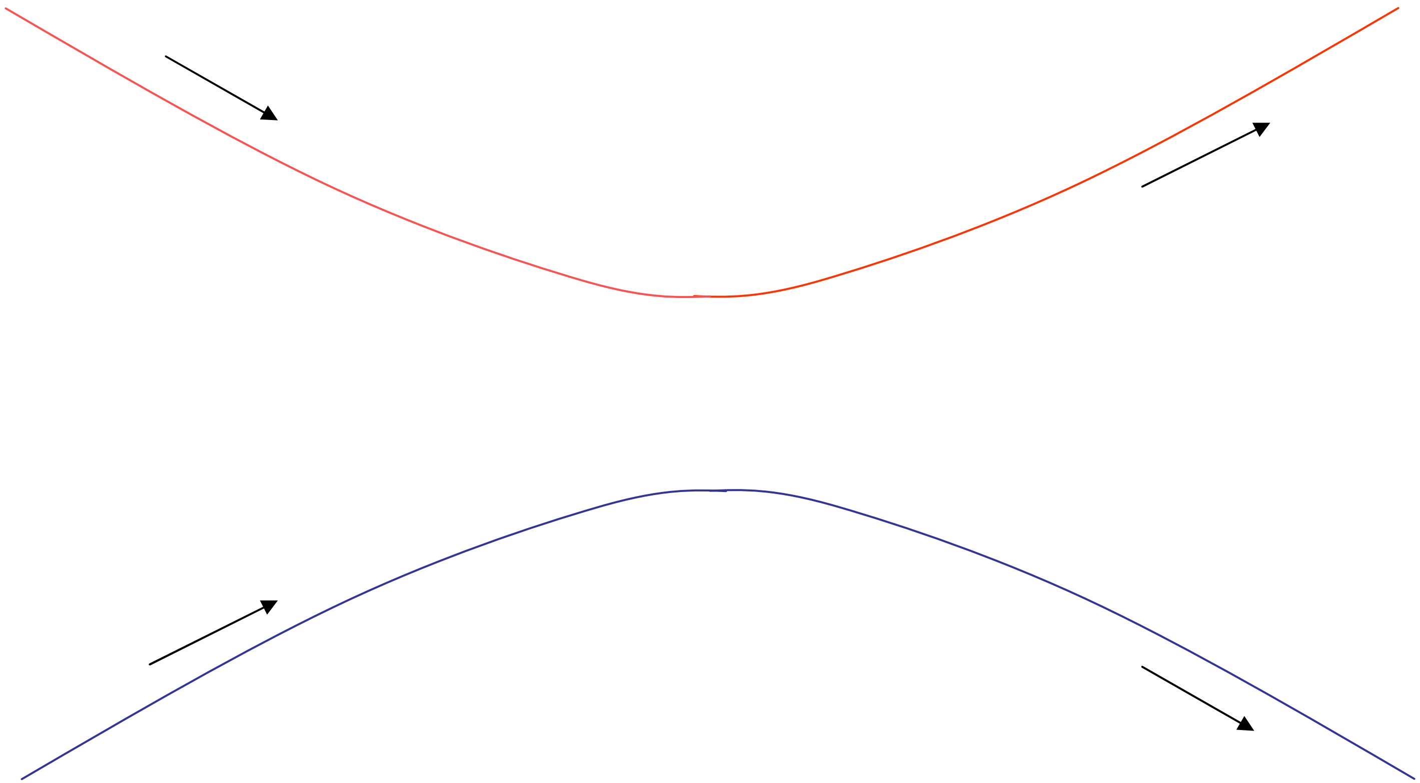


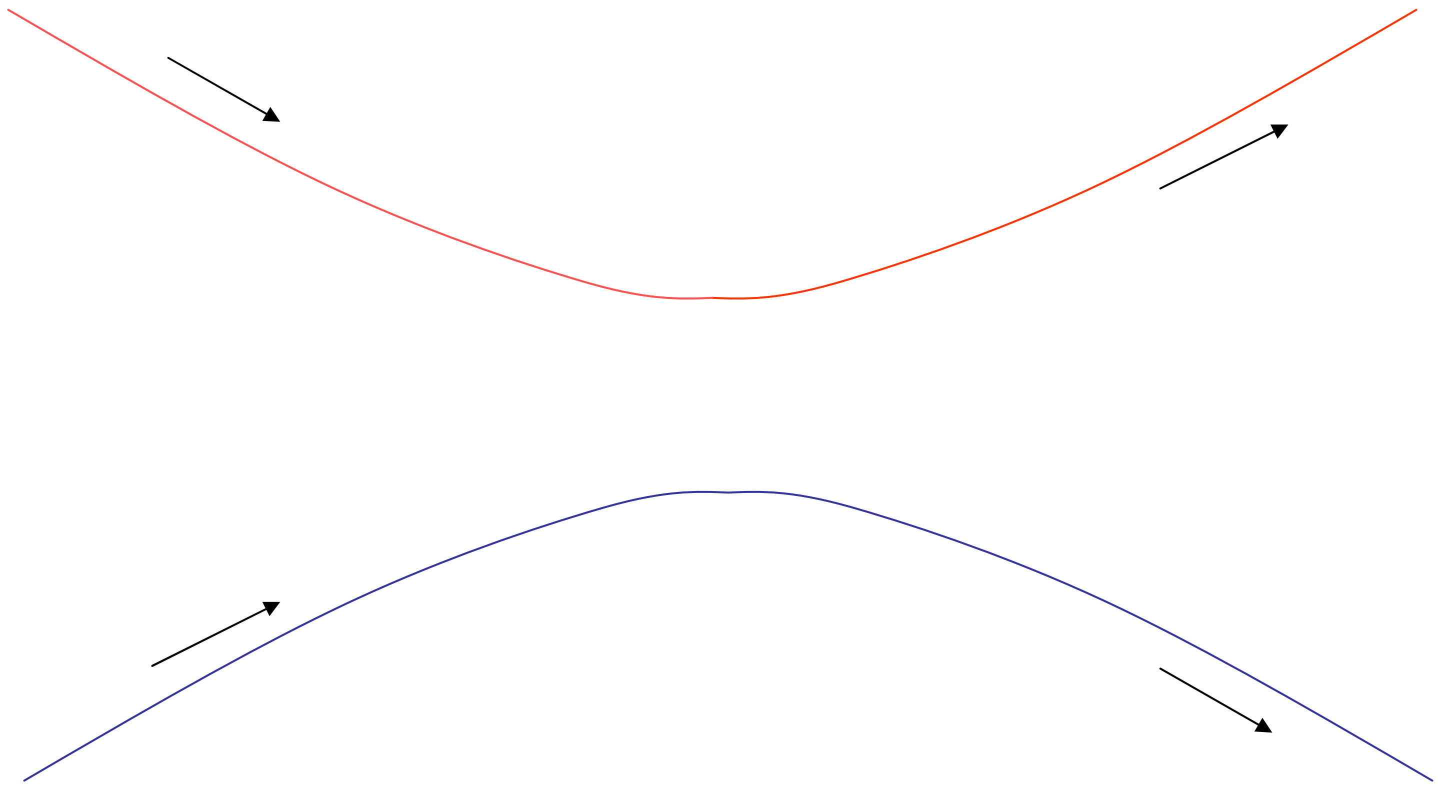


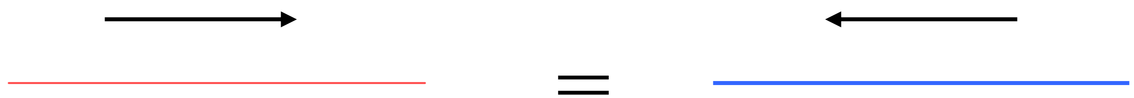








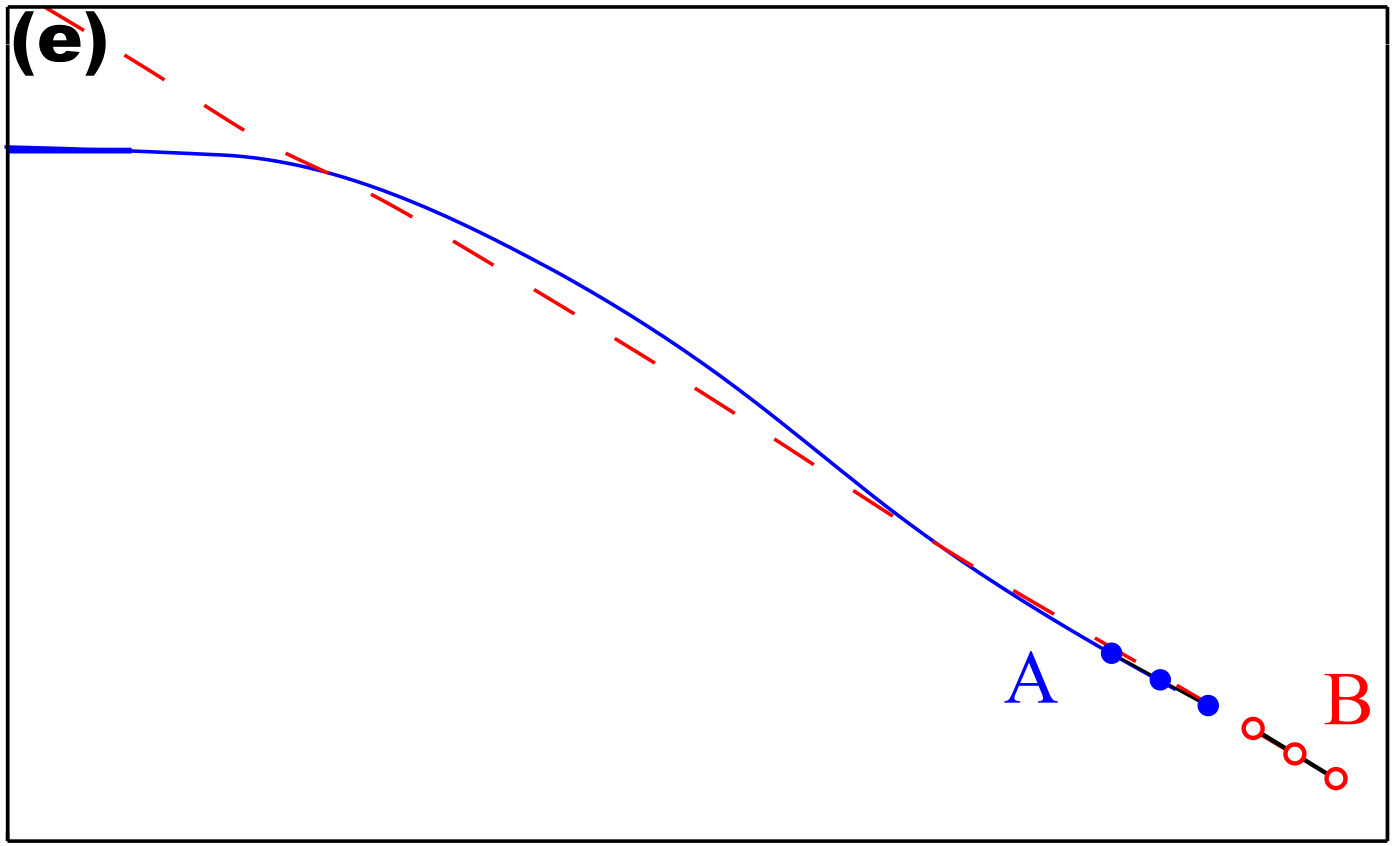


OK if 

ie swimmers are duals under time reversal

If A,B are mutually T-dual swimmers the final state after scattering is the same as the initial state before scattering

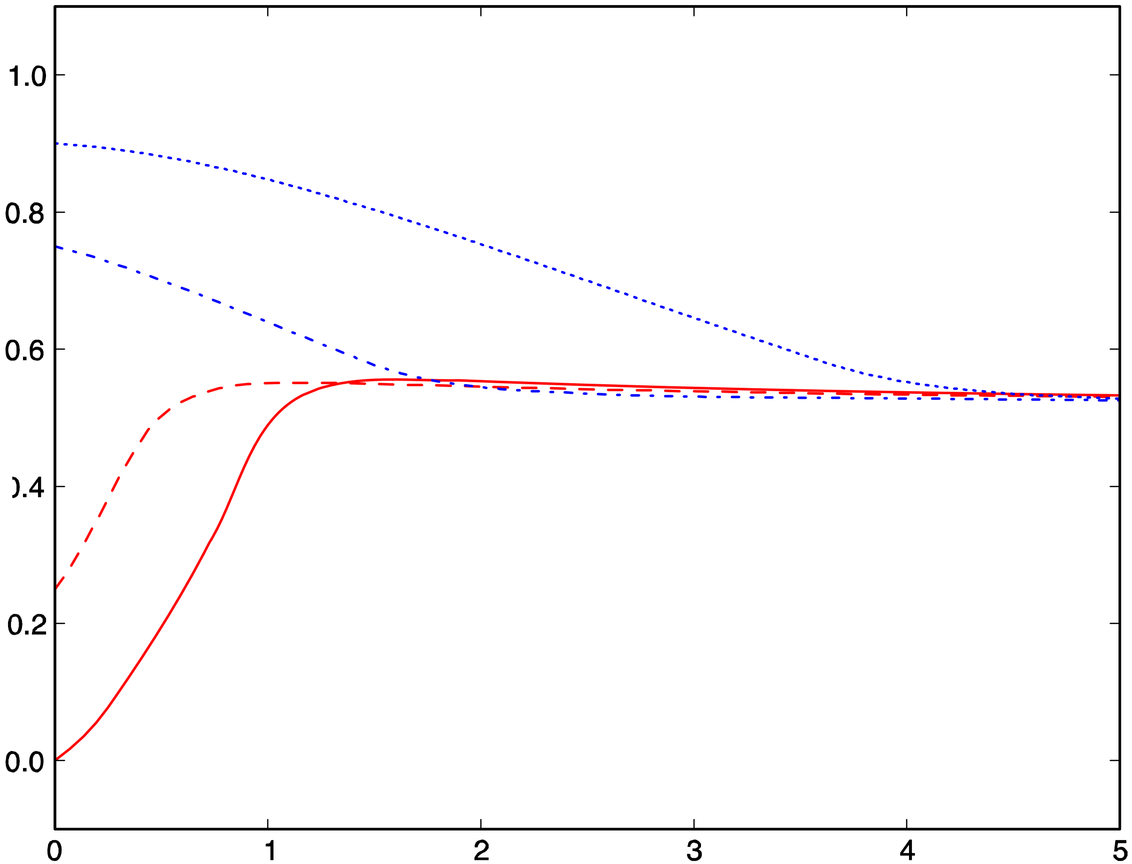
Swimmers that are not T-dual



phase locking: two collinear swimmers

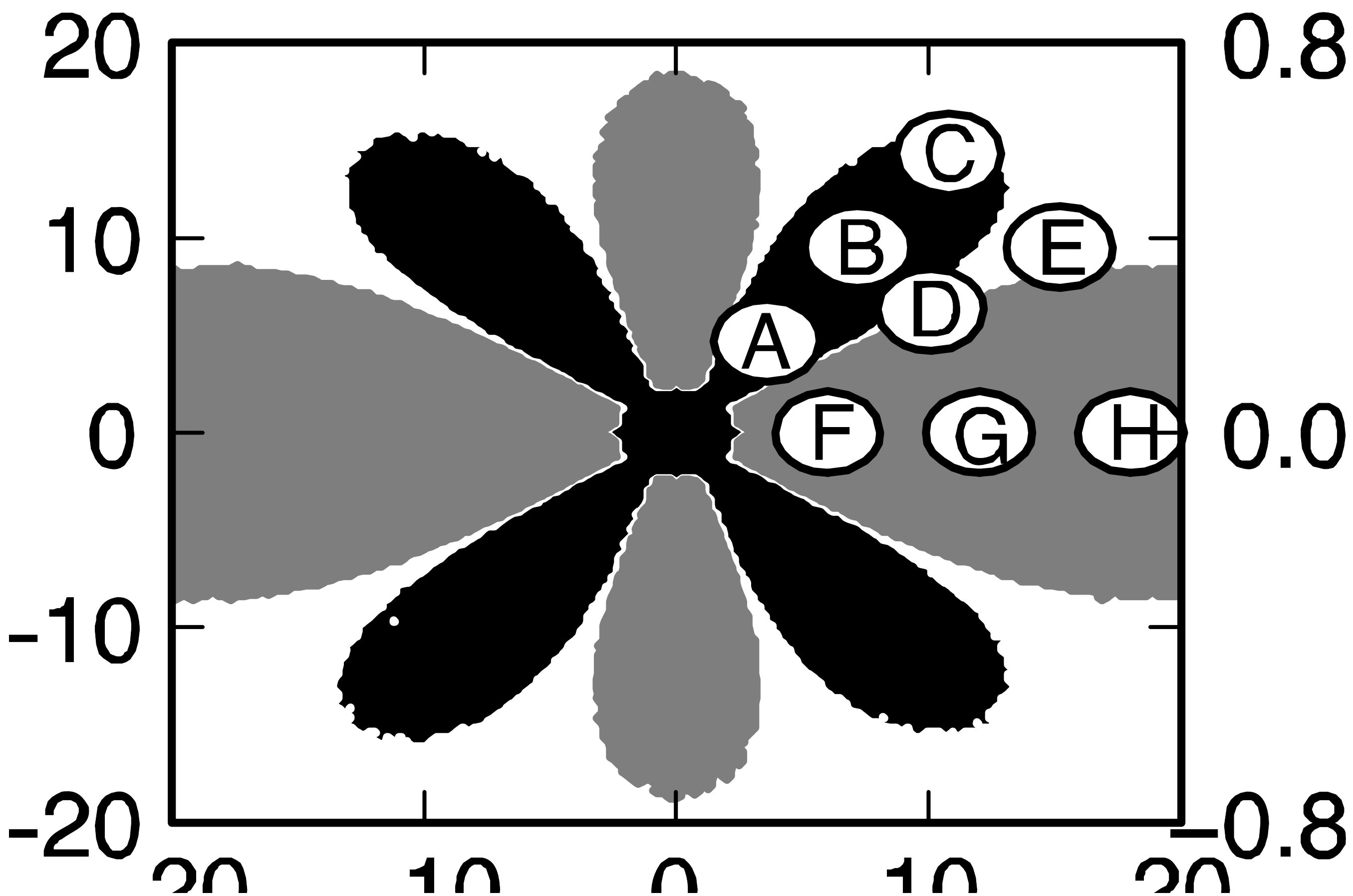
extensile

phase difference between the swimmers



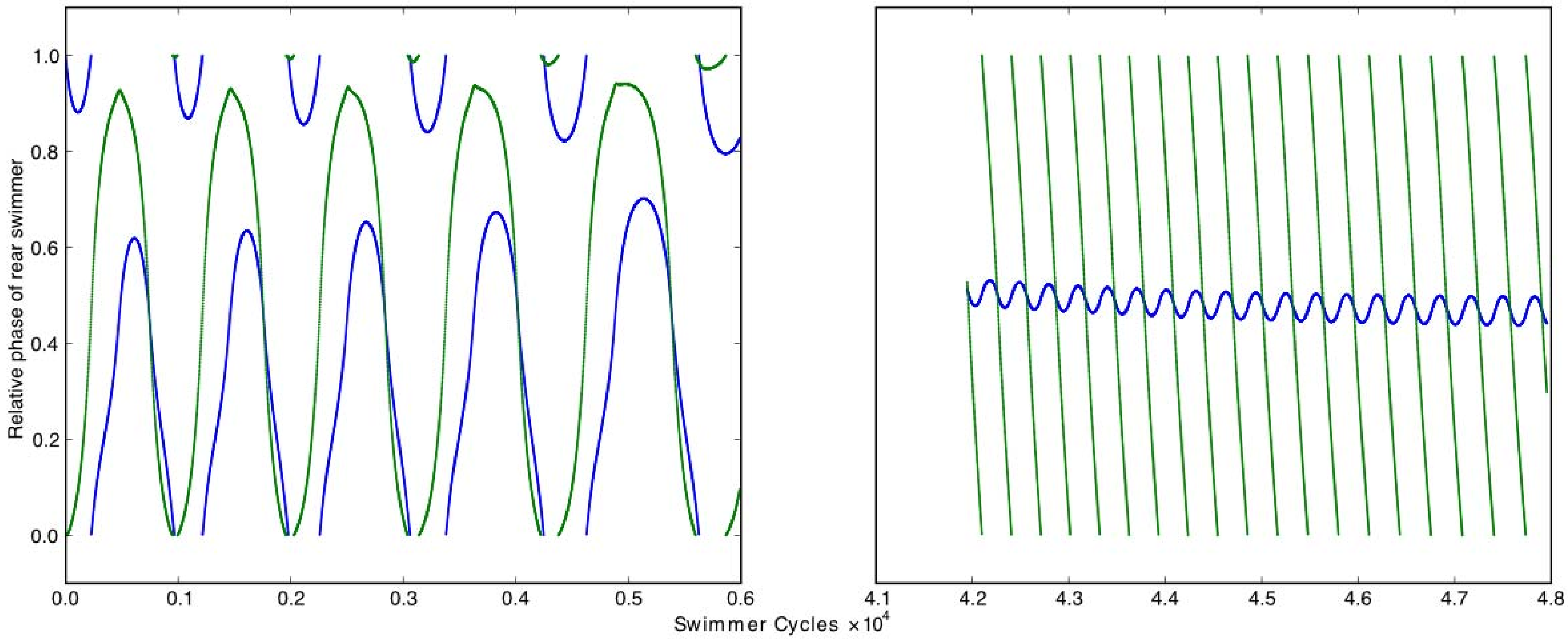
time

Phase locking: two dimensions



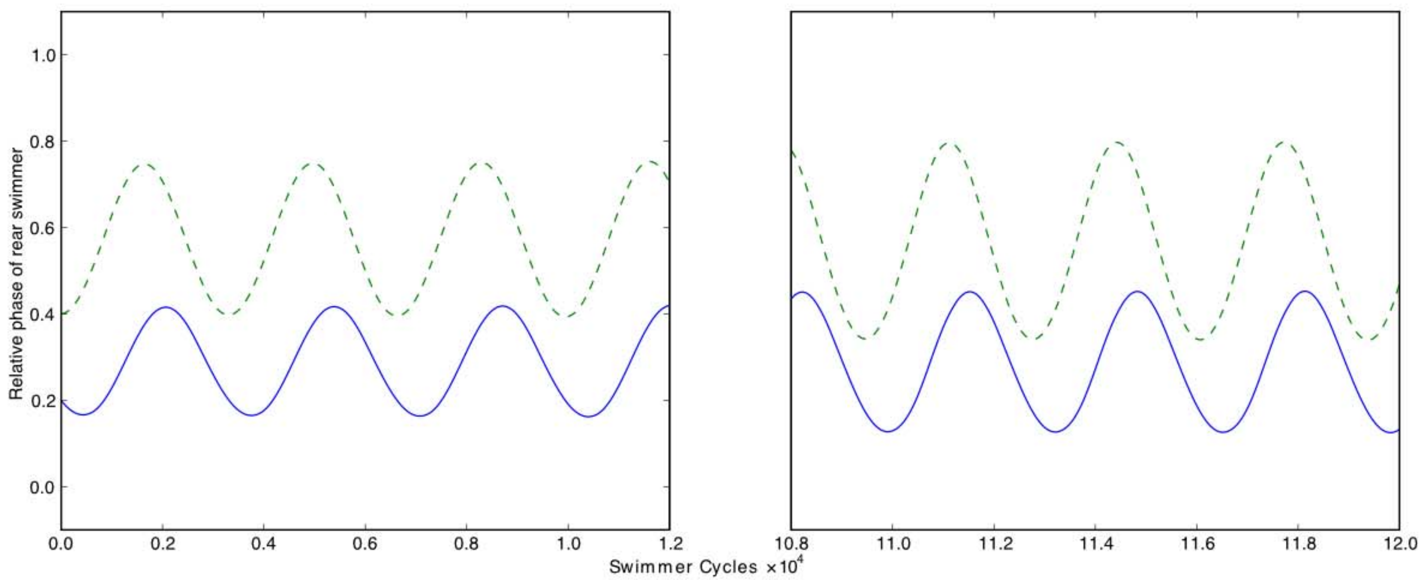
phase locking: three co-linear swimmers

relative phase

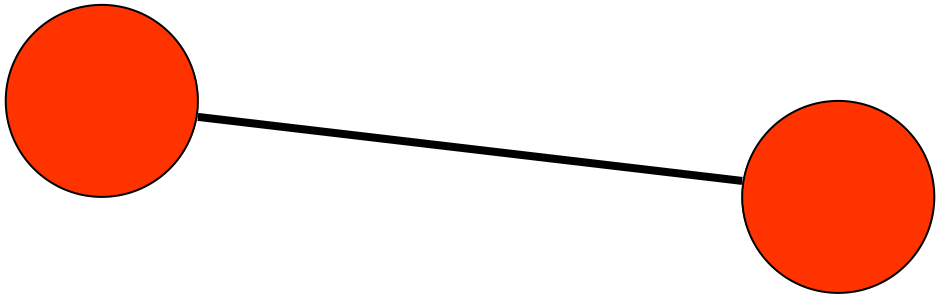


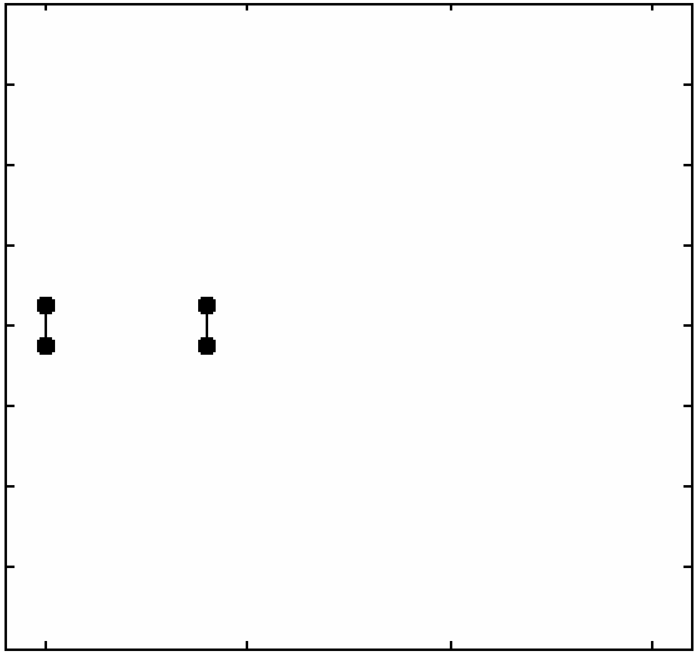
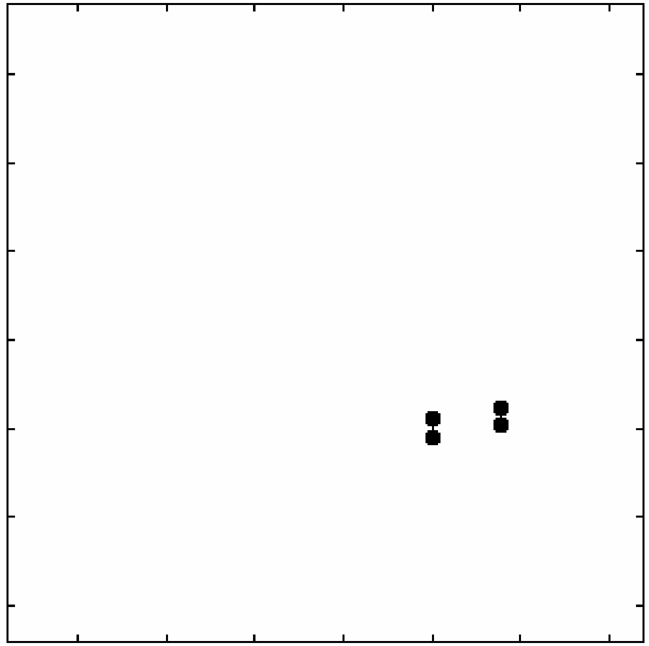
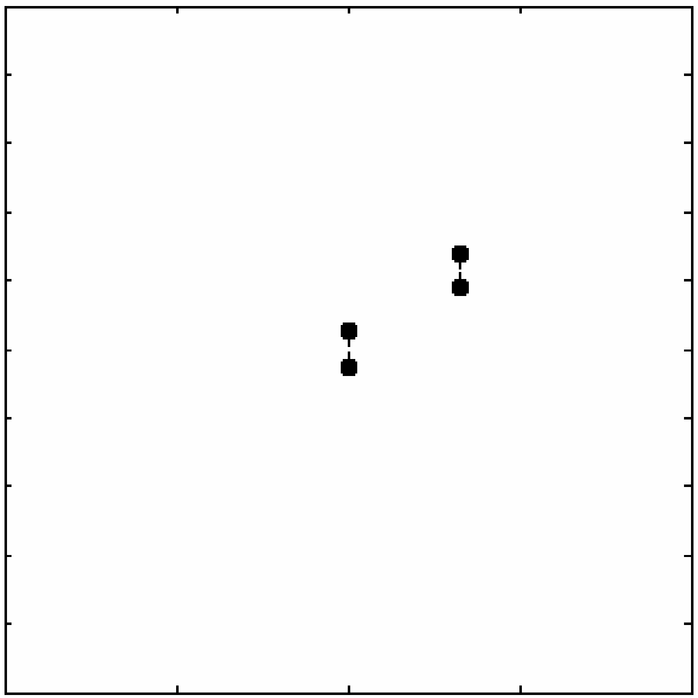
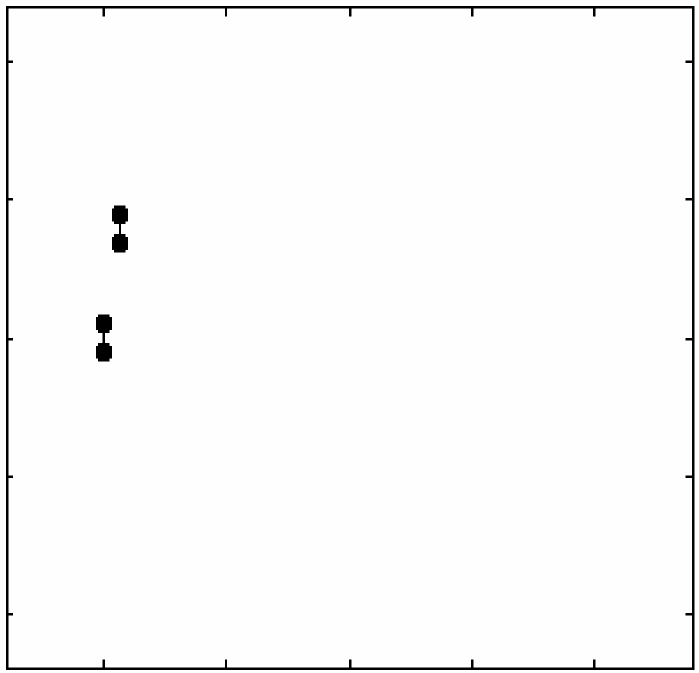
time

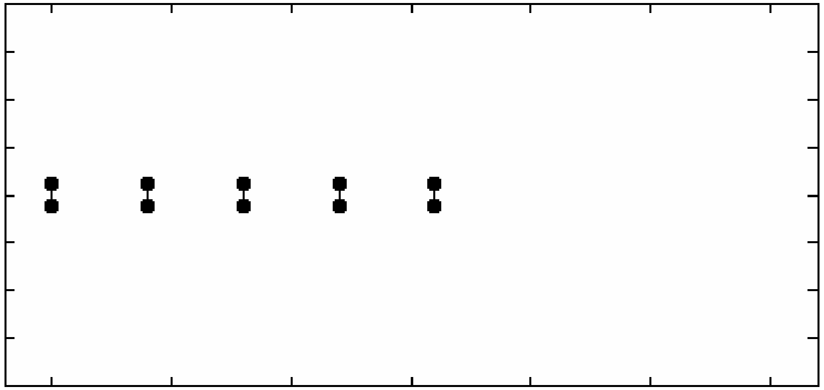
Phase locking: three co-linear pumps



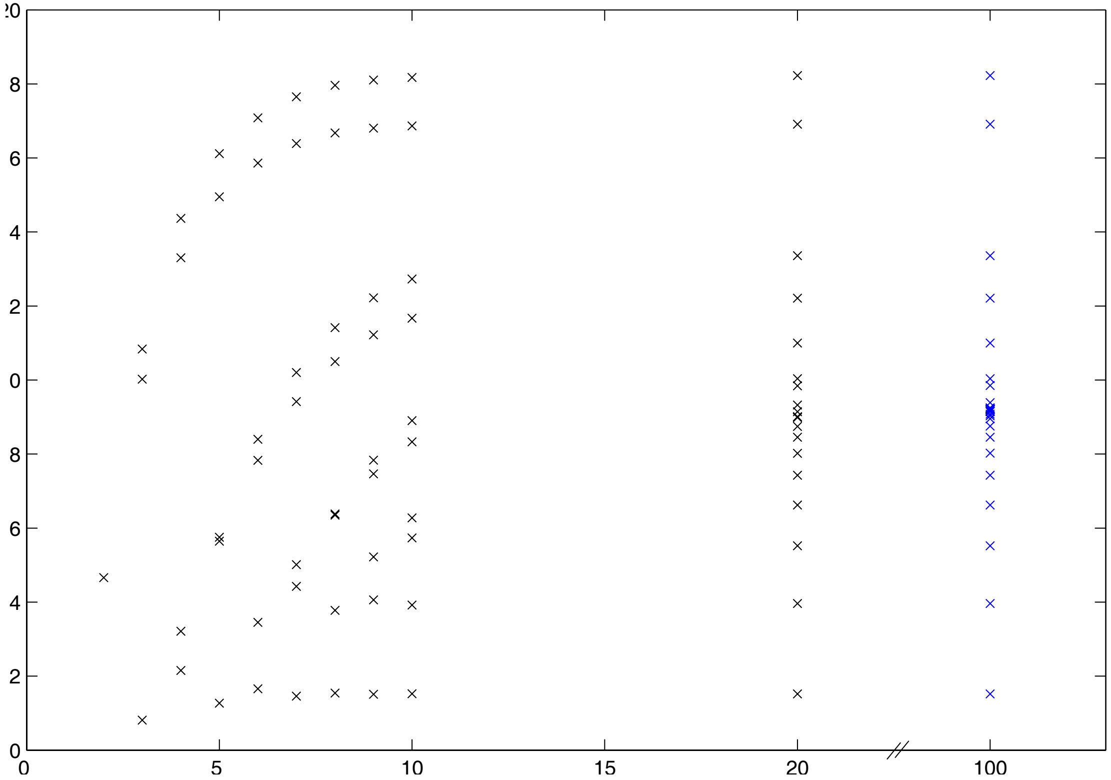
Dumbbell swimmers



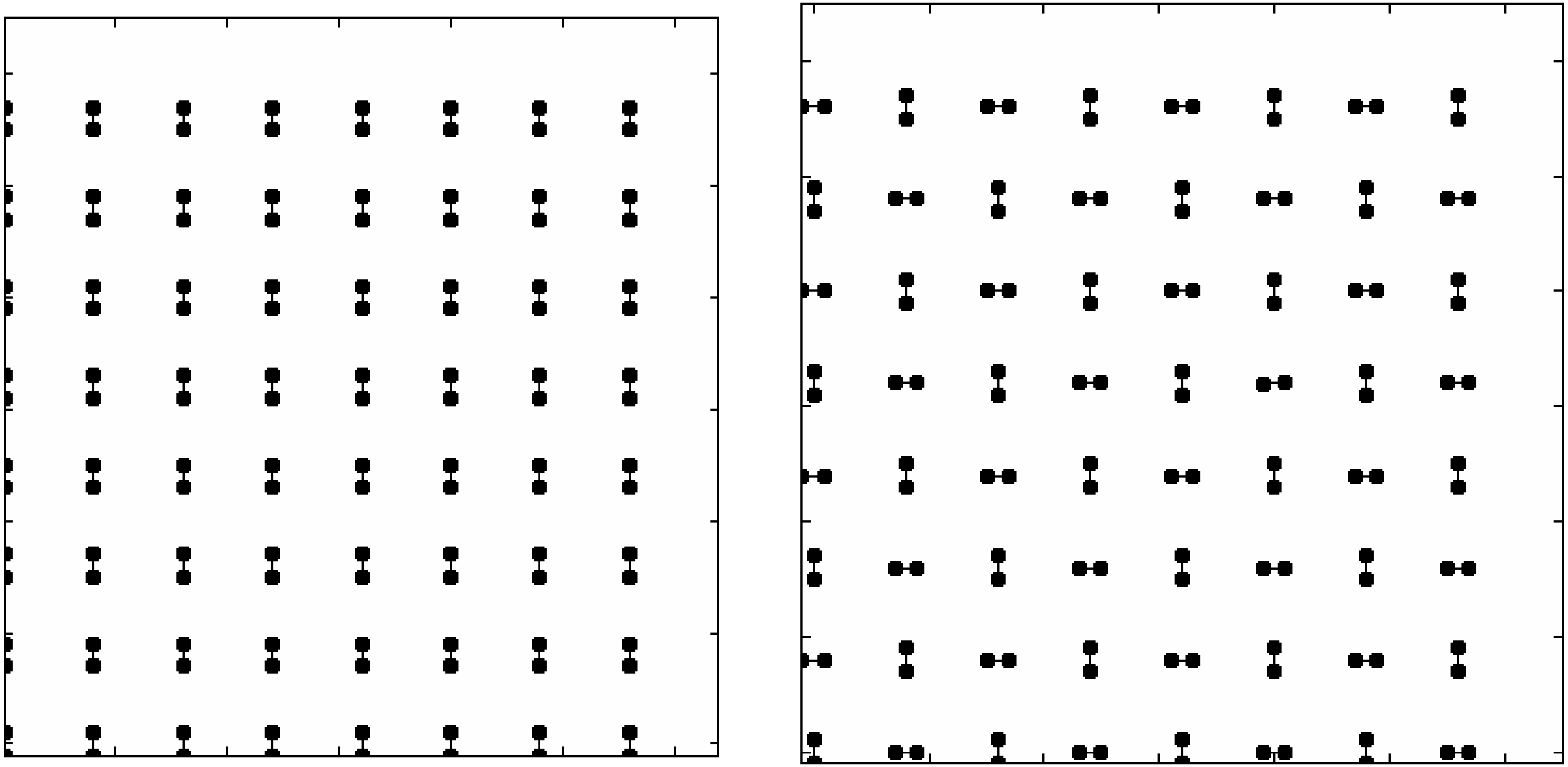





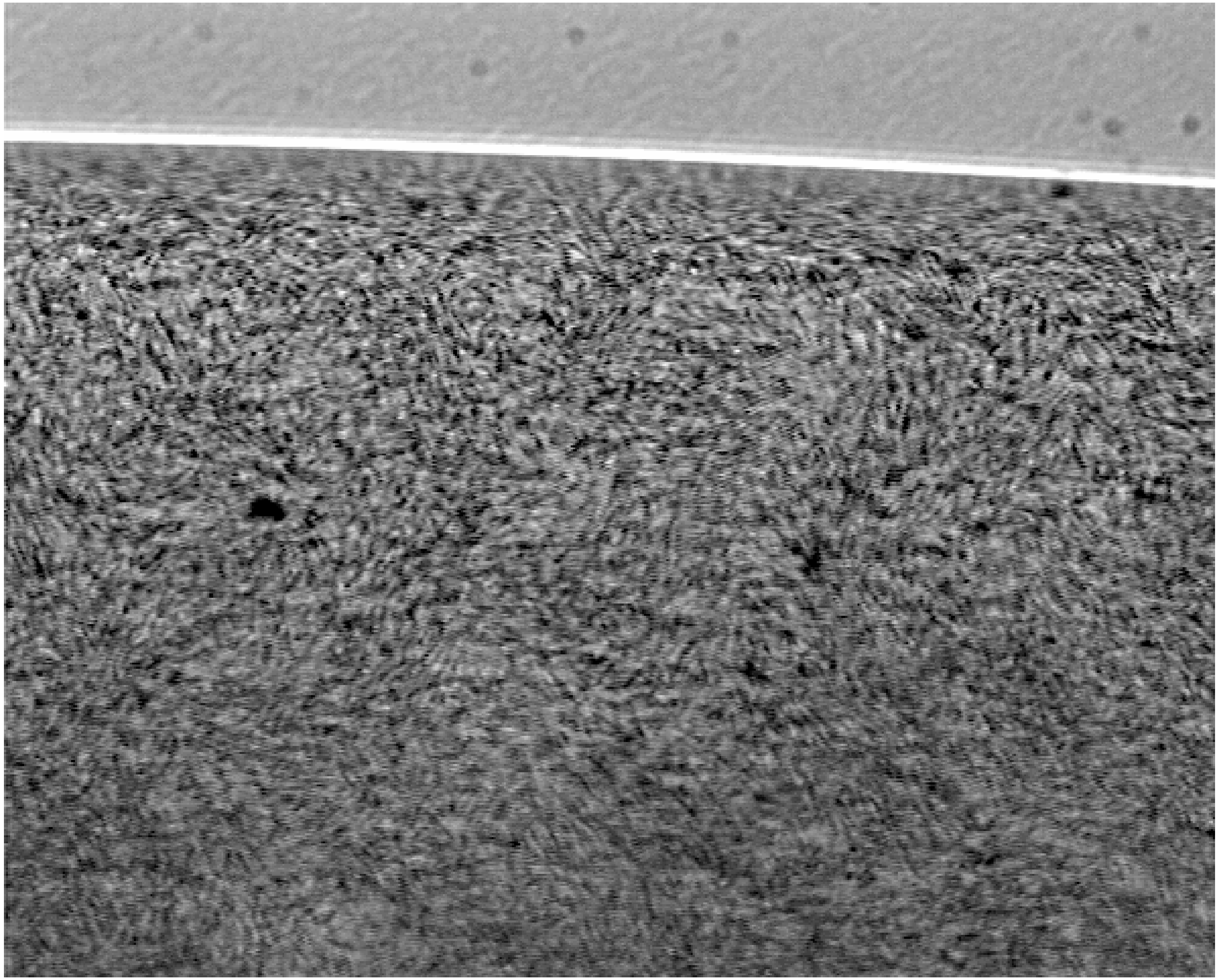


displacement after
a long time



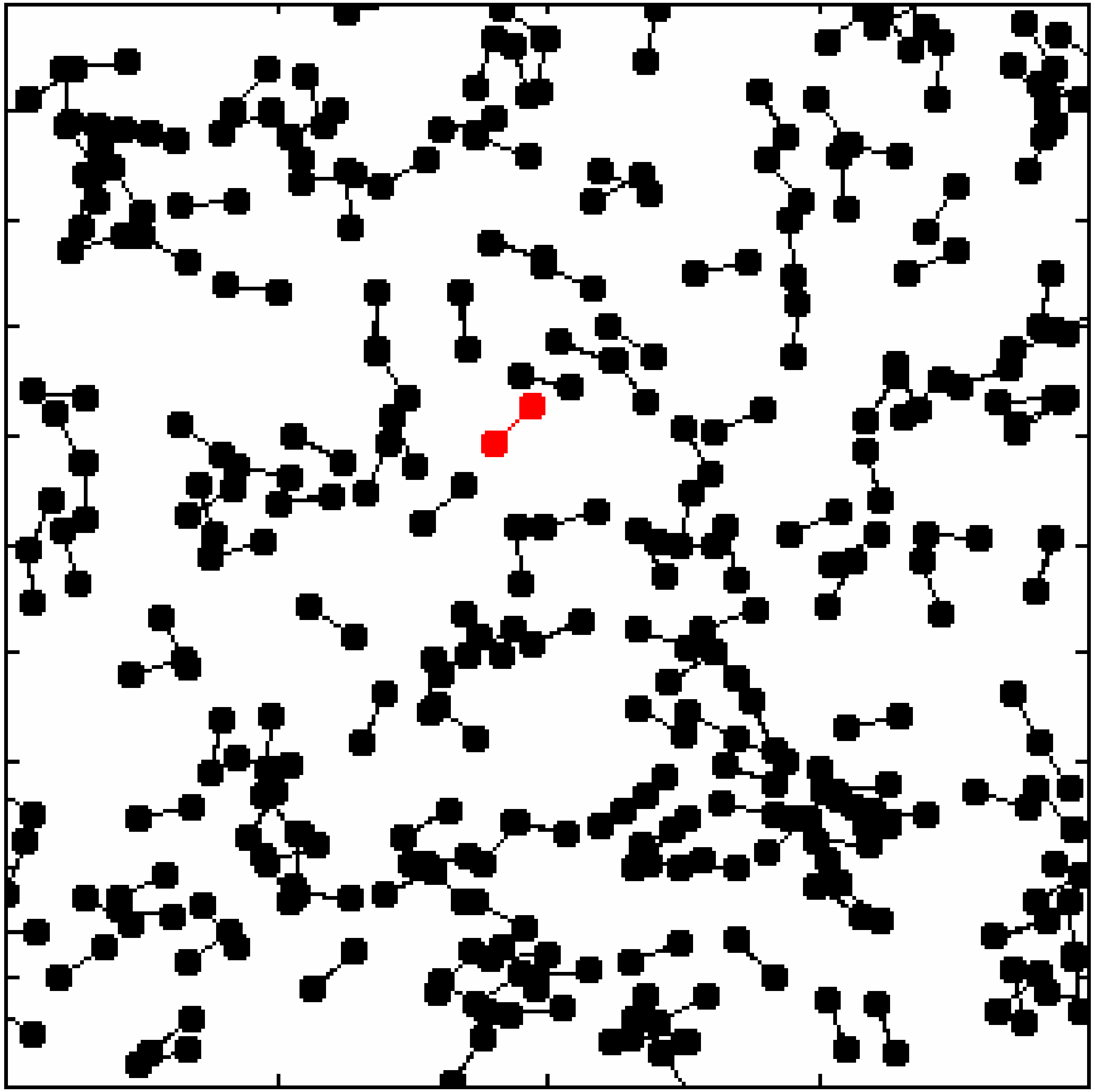
number of dipoles



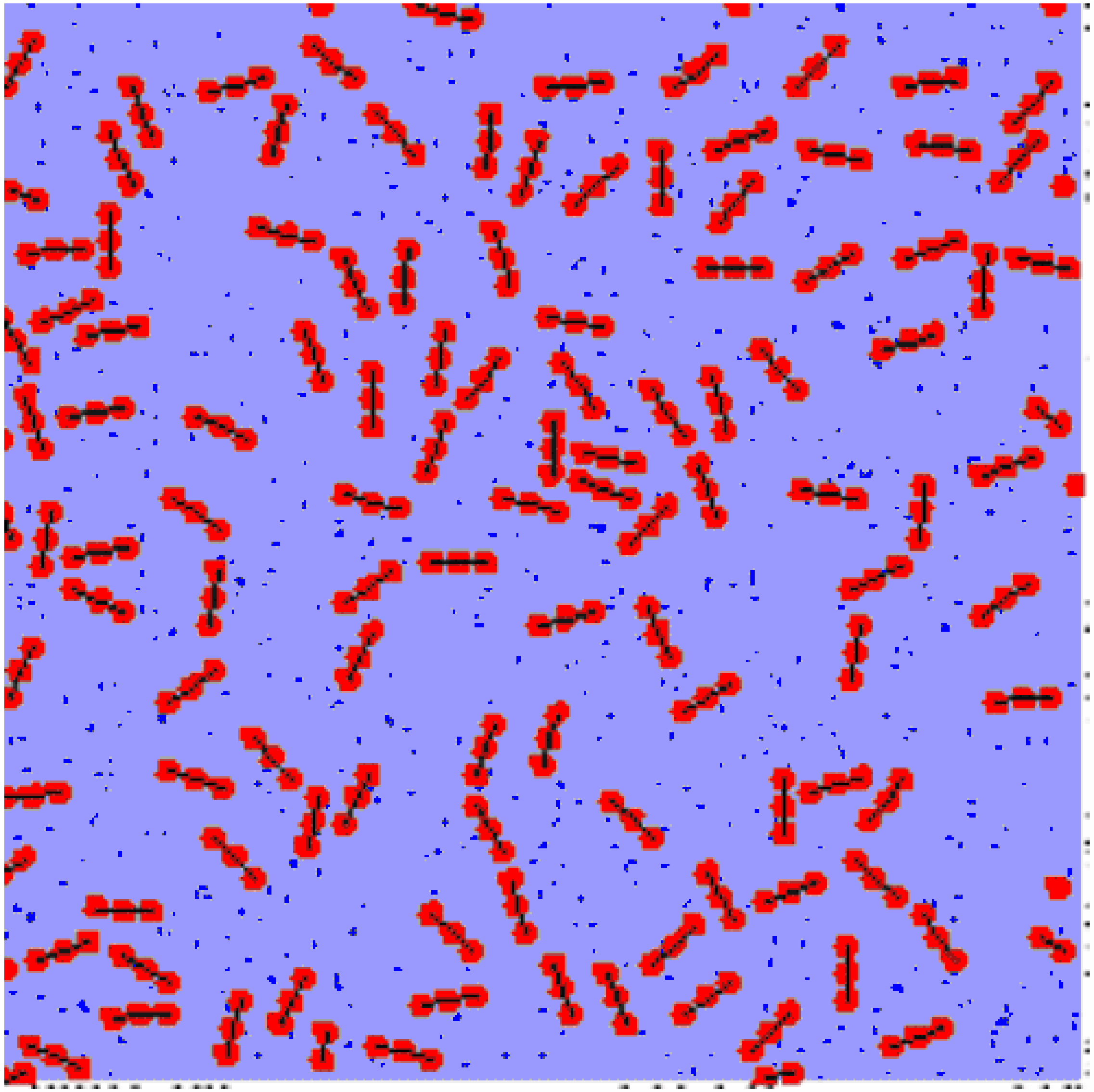


Ray Goldstein
http://www.dampt.cam.ac.uk/user/gold/movies.html

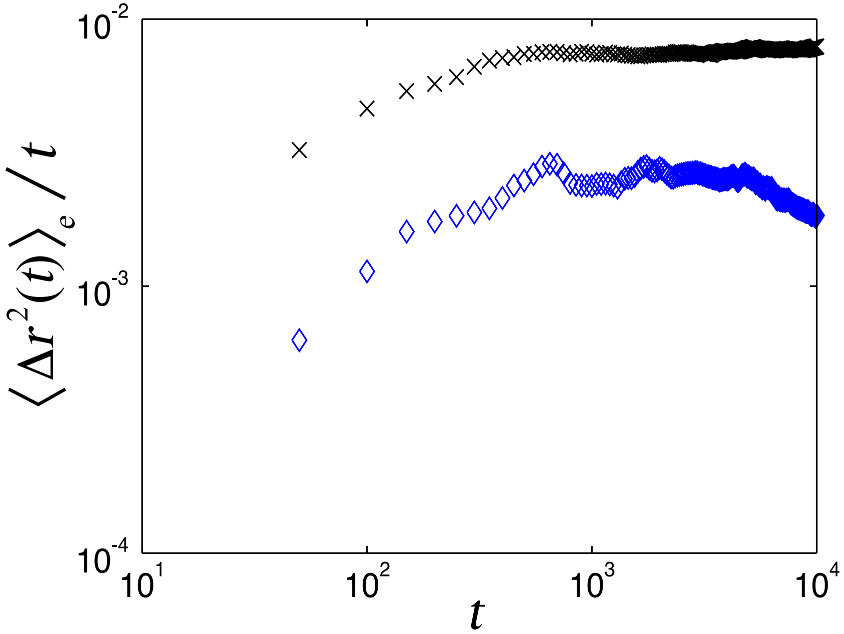
Many dumbbell swimmers



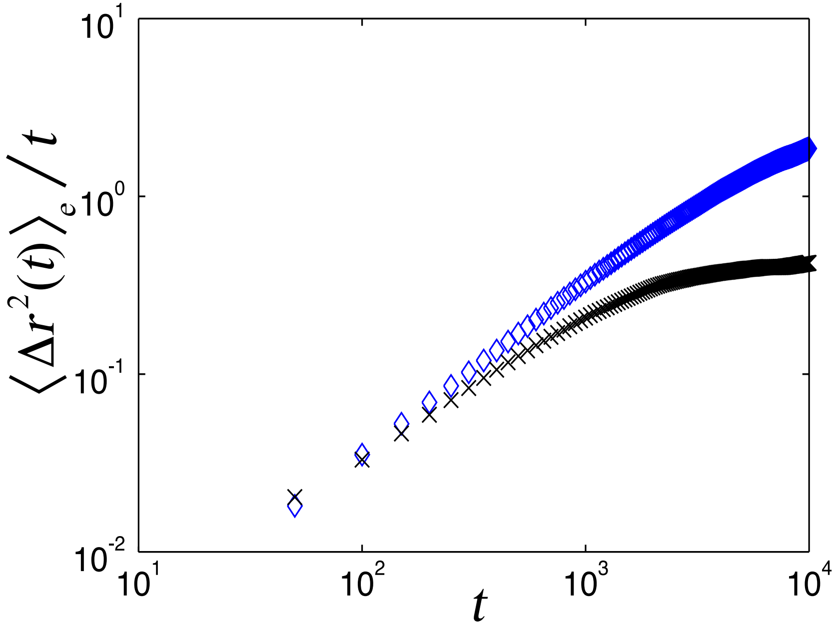
4. Many Golestanian swimmers



Comparing apolar and polar swimmers

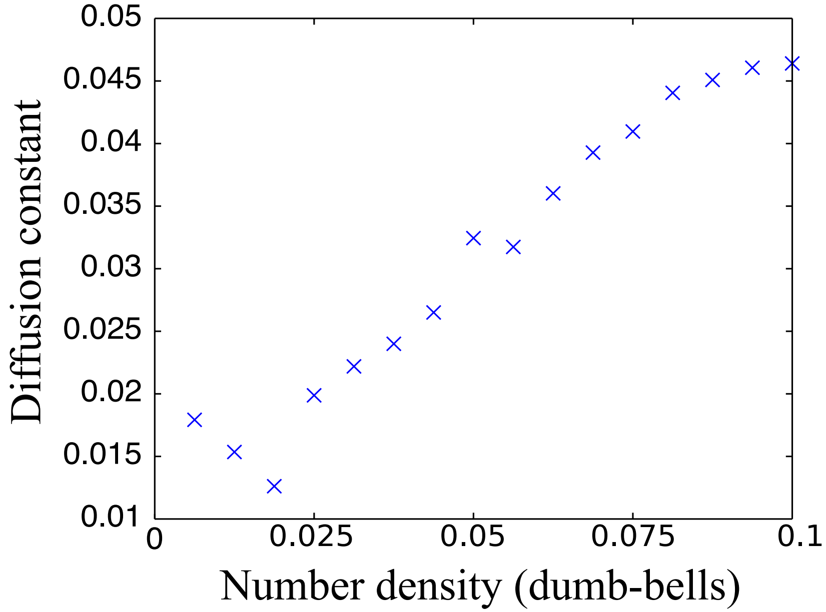


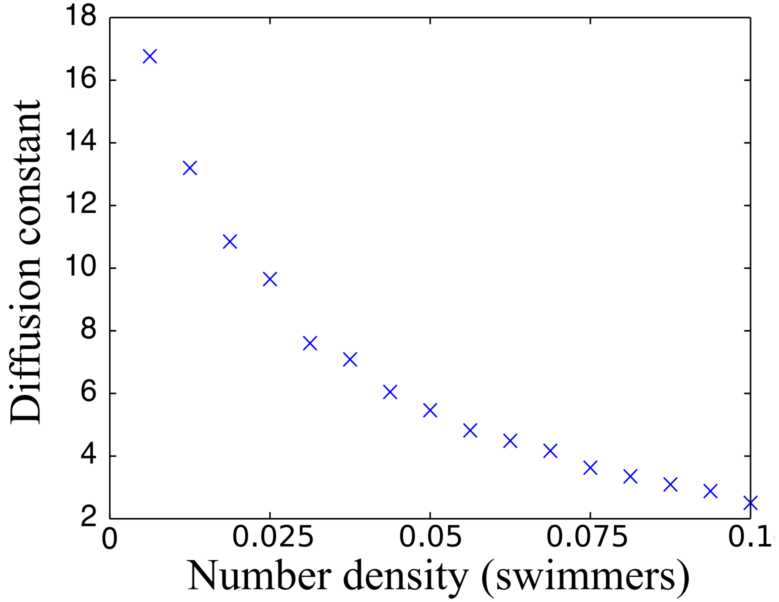
dumbbells



polar swimmers

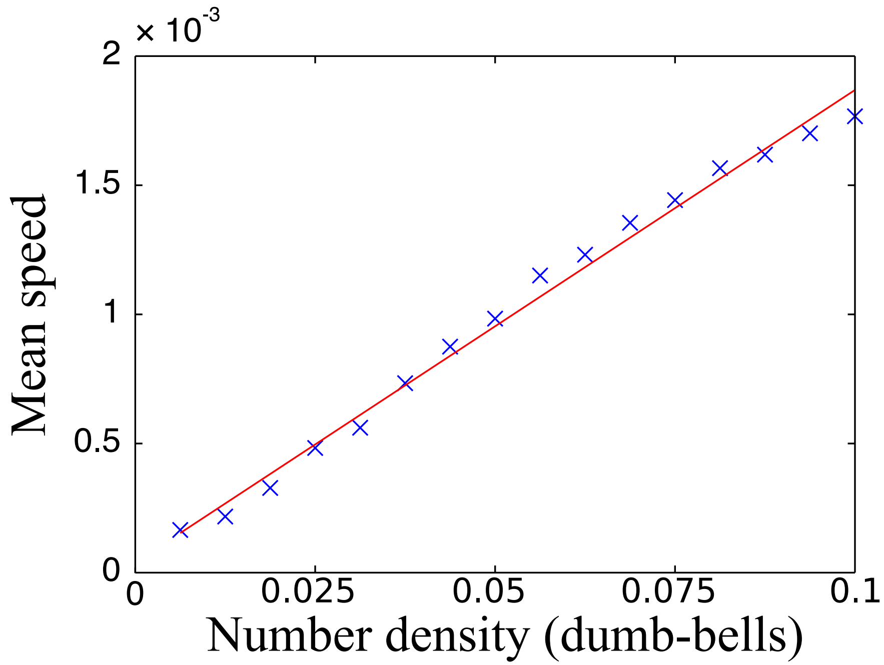
Comparing apolar and polar swimmers



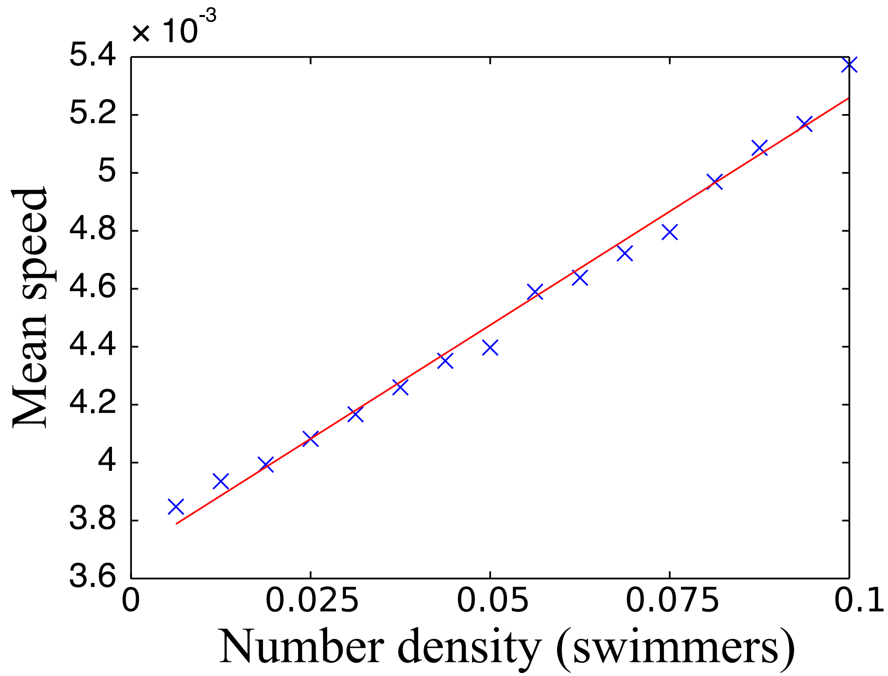


Comparing apolar and polar swimmers

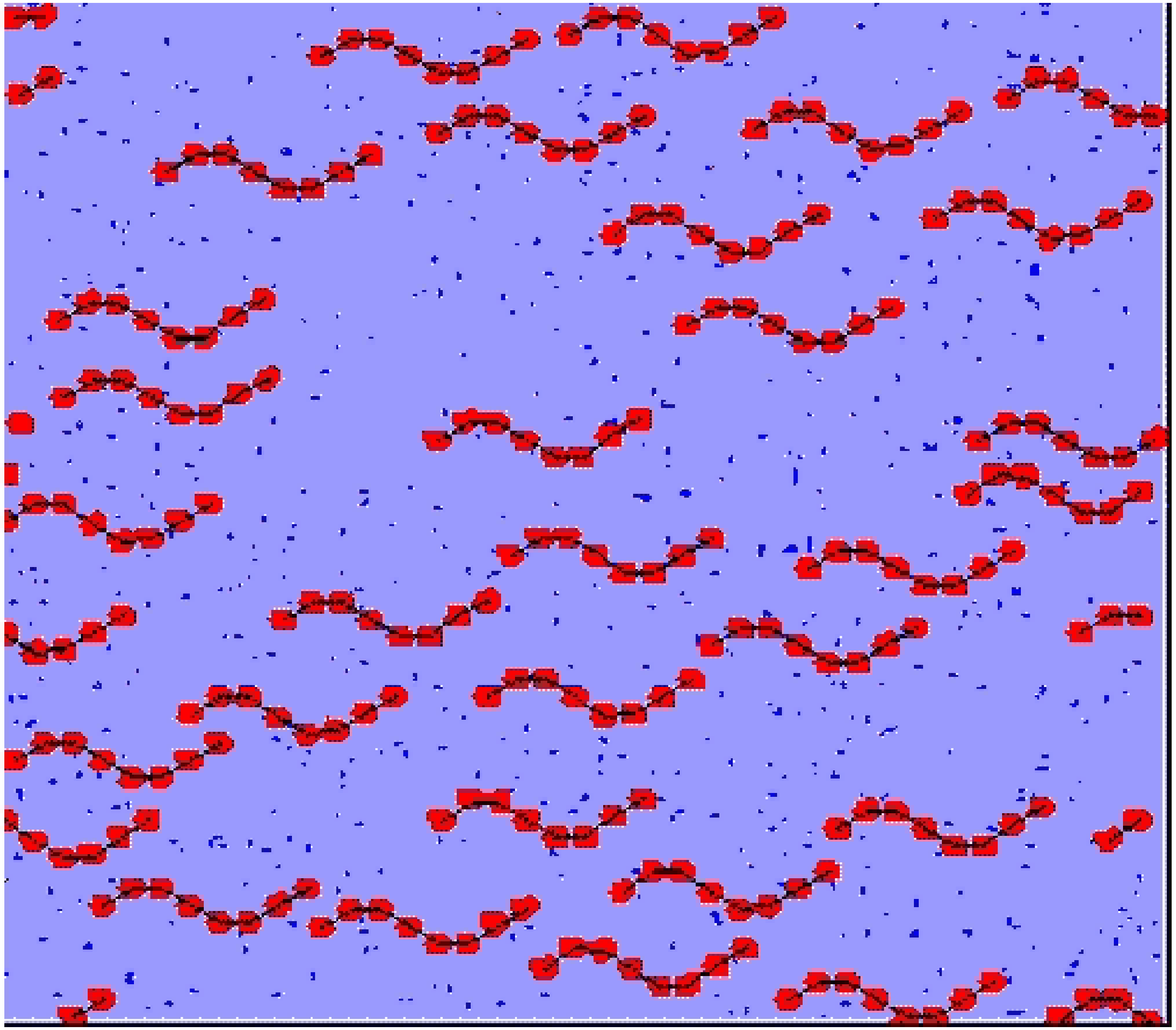
(I)



(II)



Many snake swimmers



What next?

1. Matching simulations and the continuum theories

2. What is the continuum theory for self T-dual swimmers?

3. Noise

4. When does the zero Re approximation fail?

5. Real swimmers: which results are generic?

6. Shear flow, obstacles, swimmer rheology

Summary

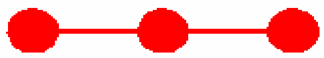
1. Two swimmer hydrodynamic interactions beautiful and complex

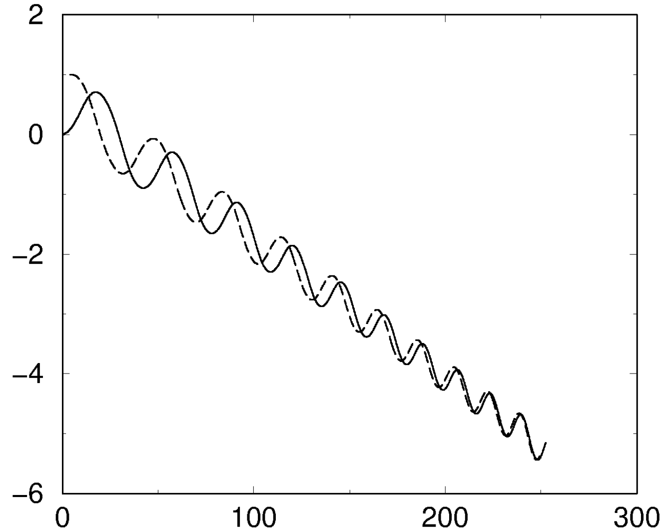
2. Relative swimmer phase is important

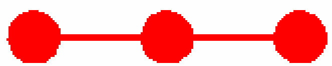
3. Scattering parameters are preserved if the swimmers are mutually T-dual

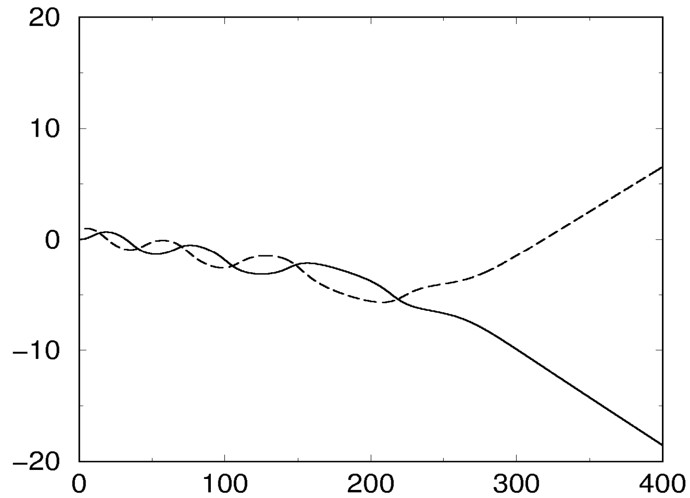
With many thanks to Gareth Alexander, Chris Pooley, Vic Putz

Non self T-dual swimmers

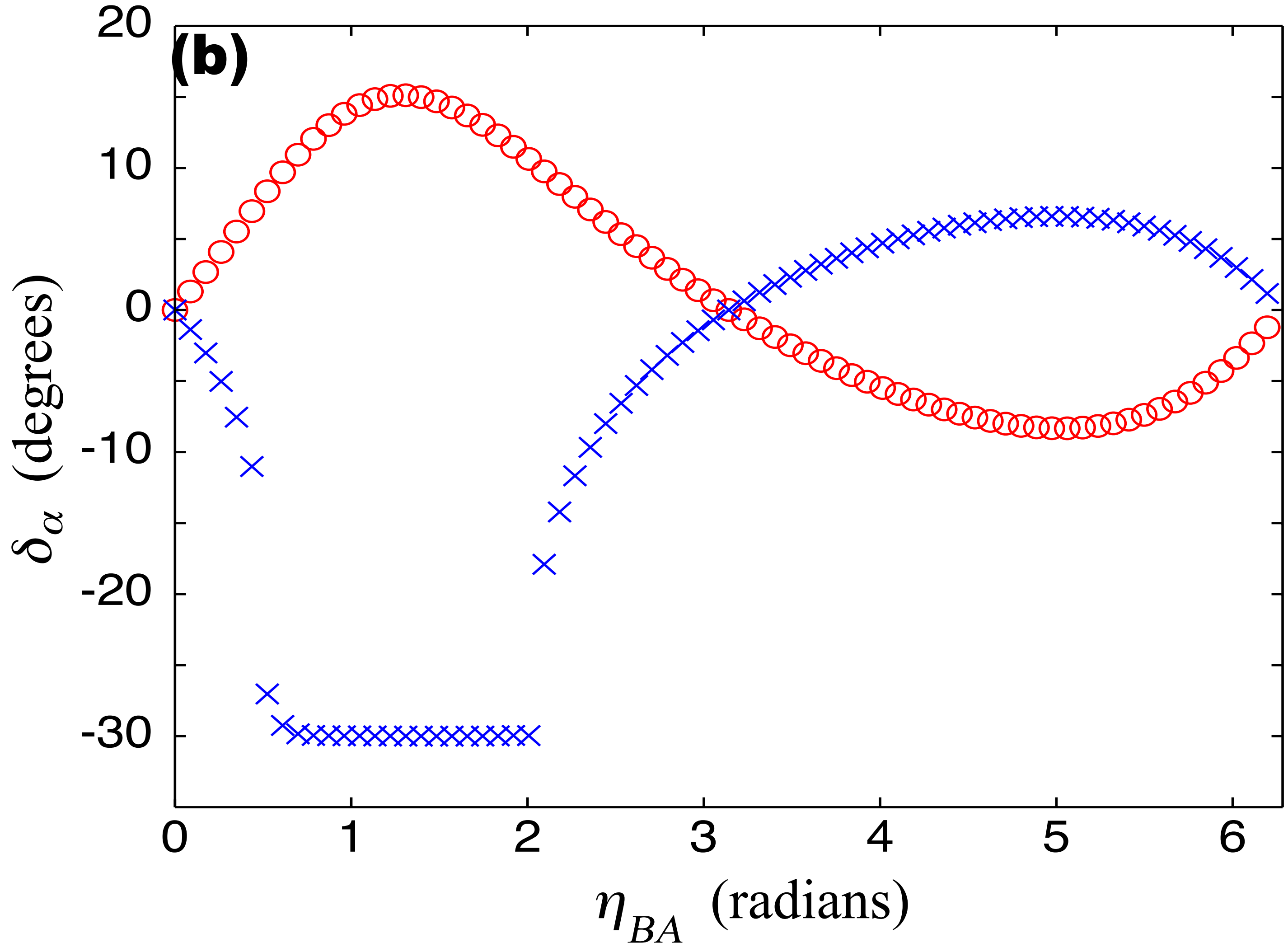




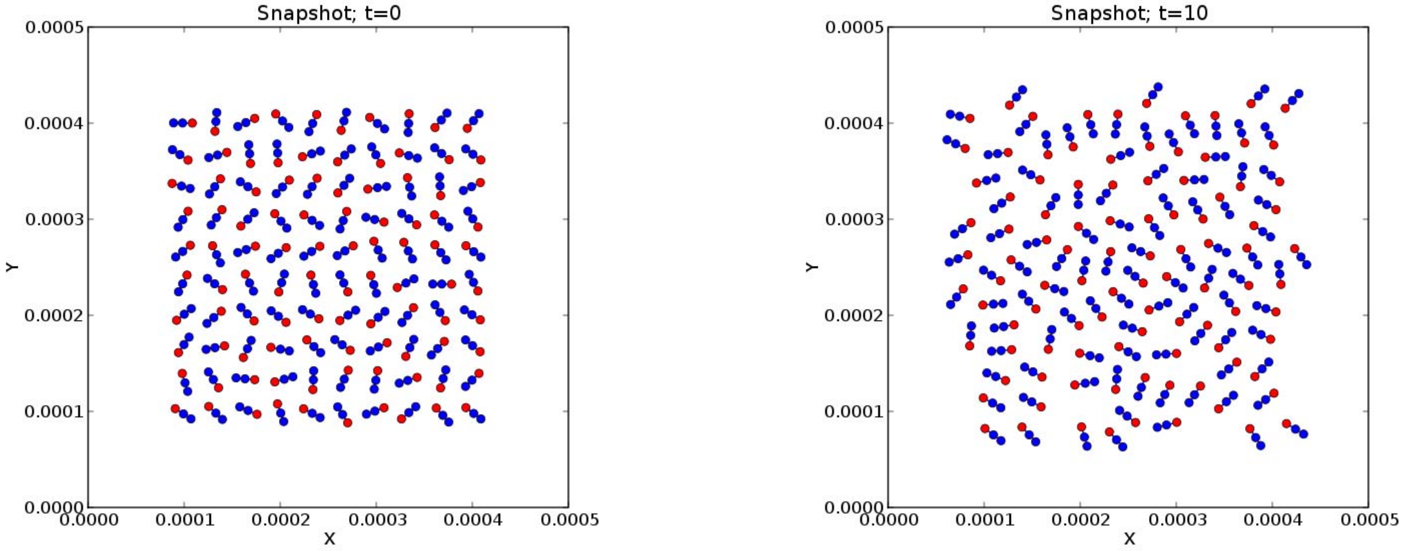


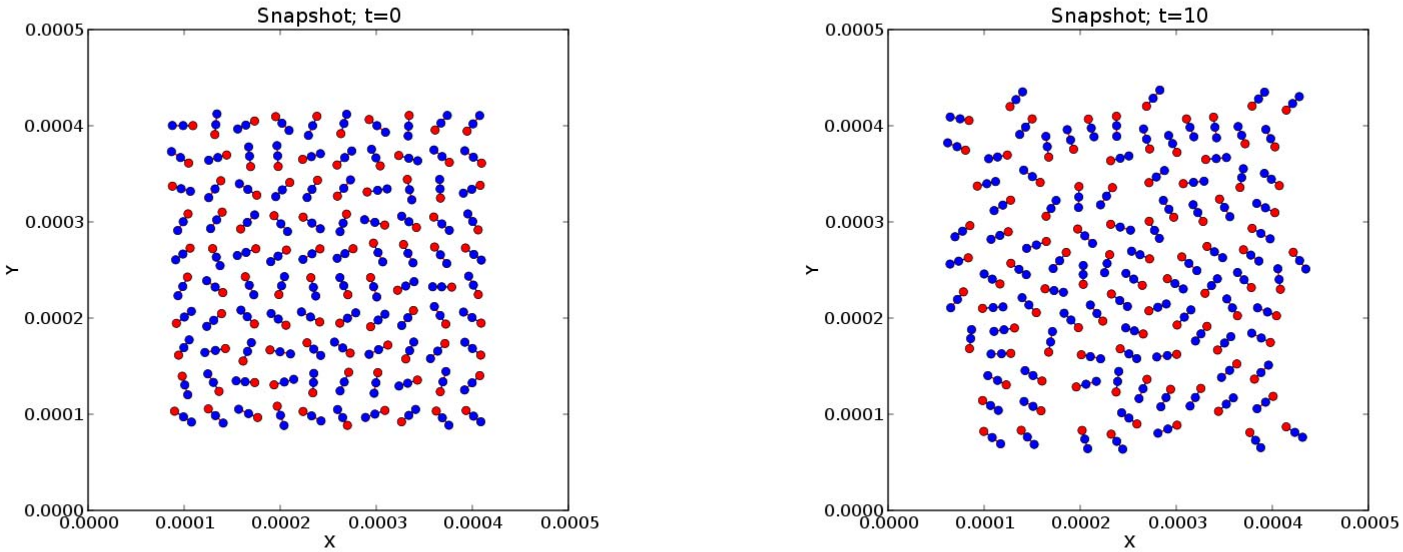


Scattering: swimmers that are not T-dual



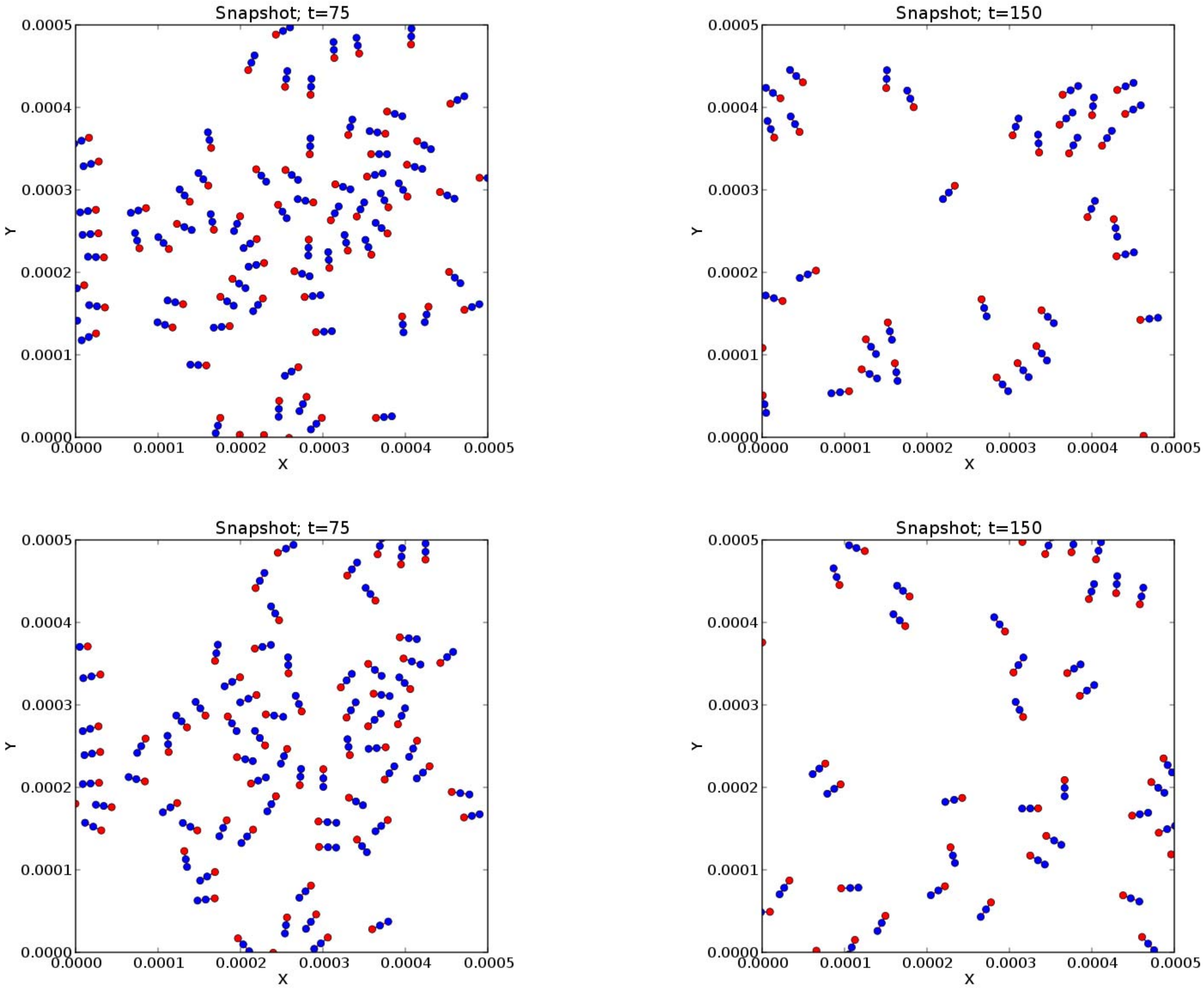
In Phase



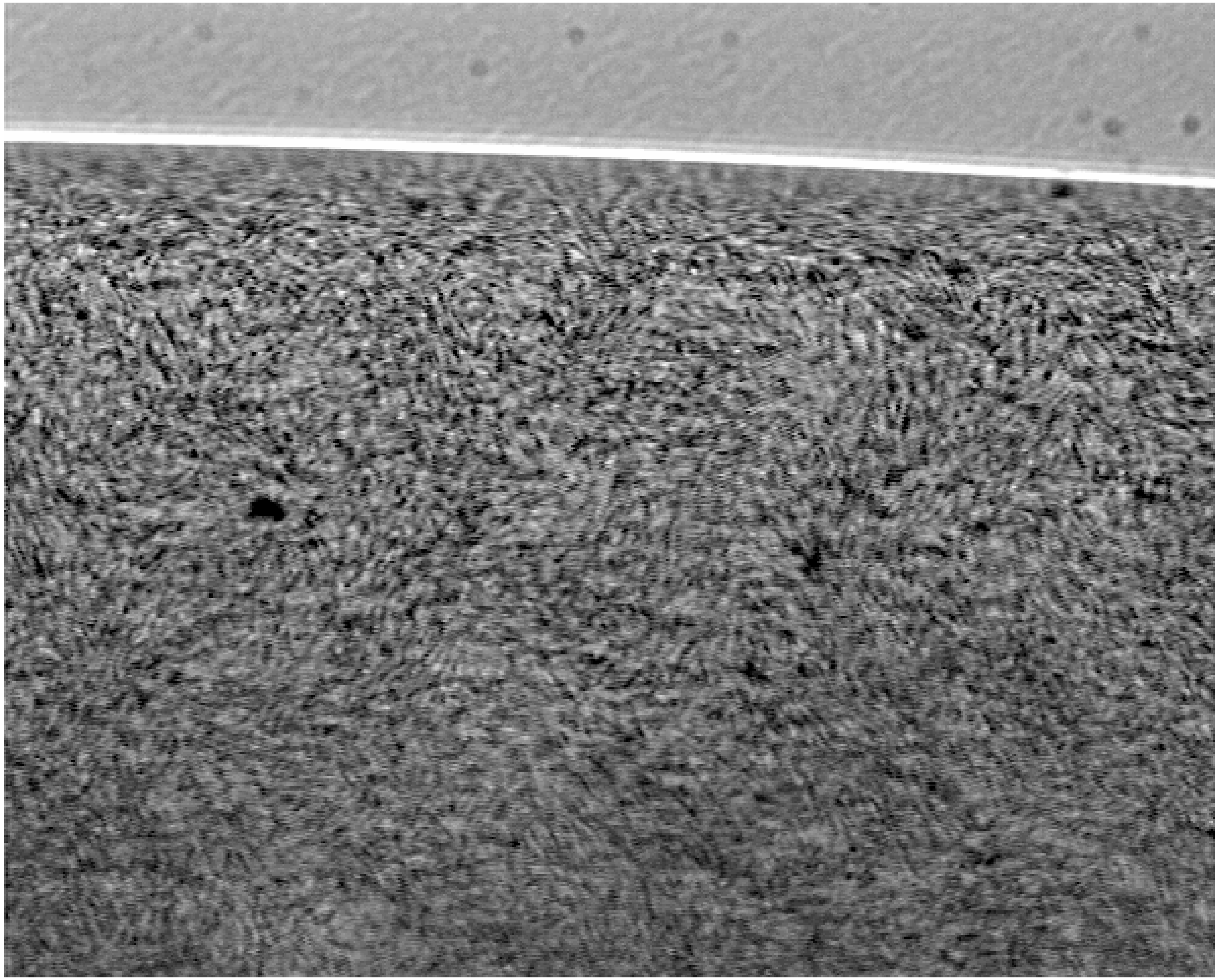


Random Phase

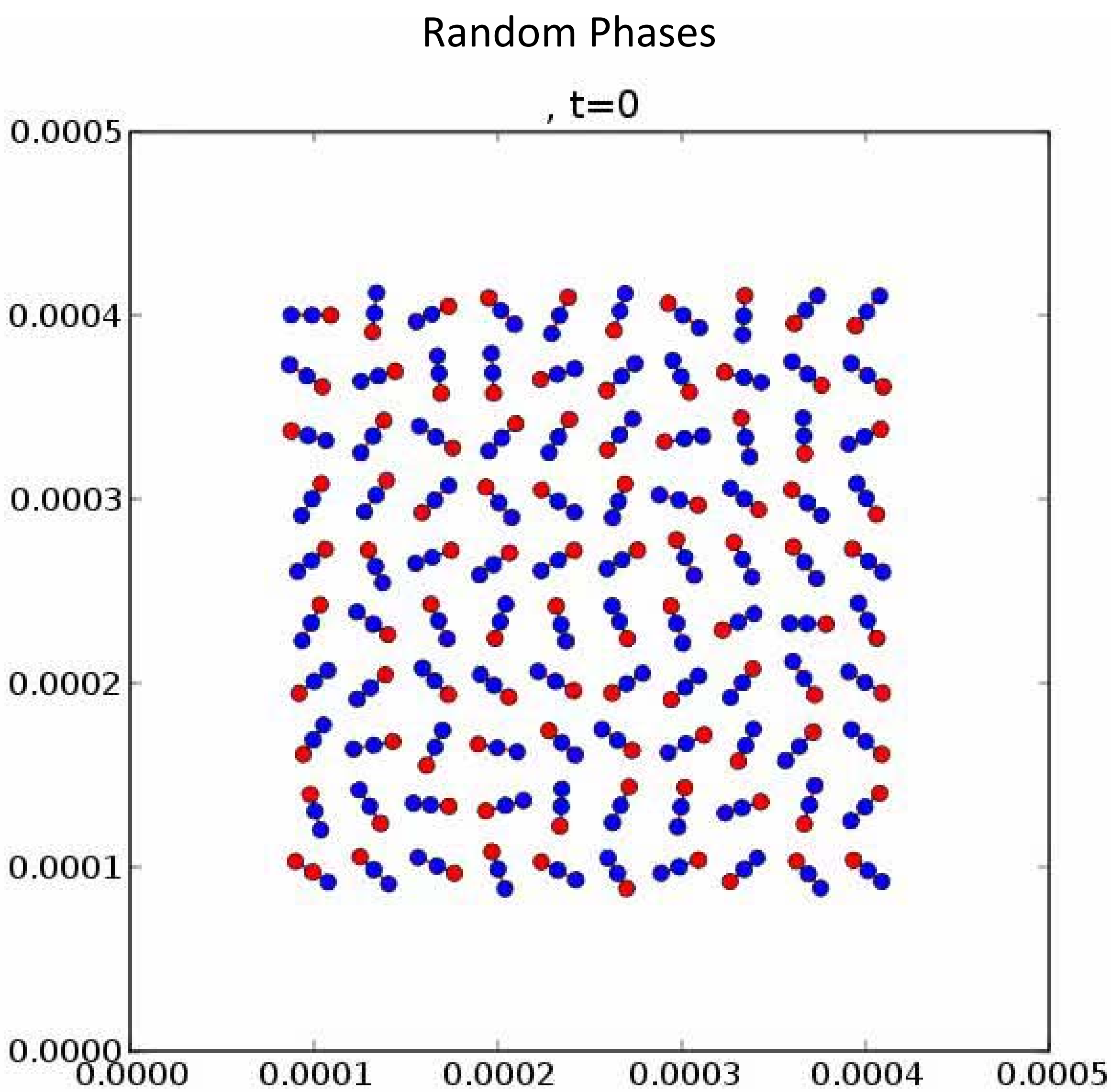
In Phase



Random Phase



Ray Goldstein
http://www.dampt.cam.ac.uk/user/gold/movies.html


Random Phases
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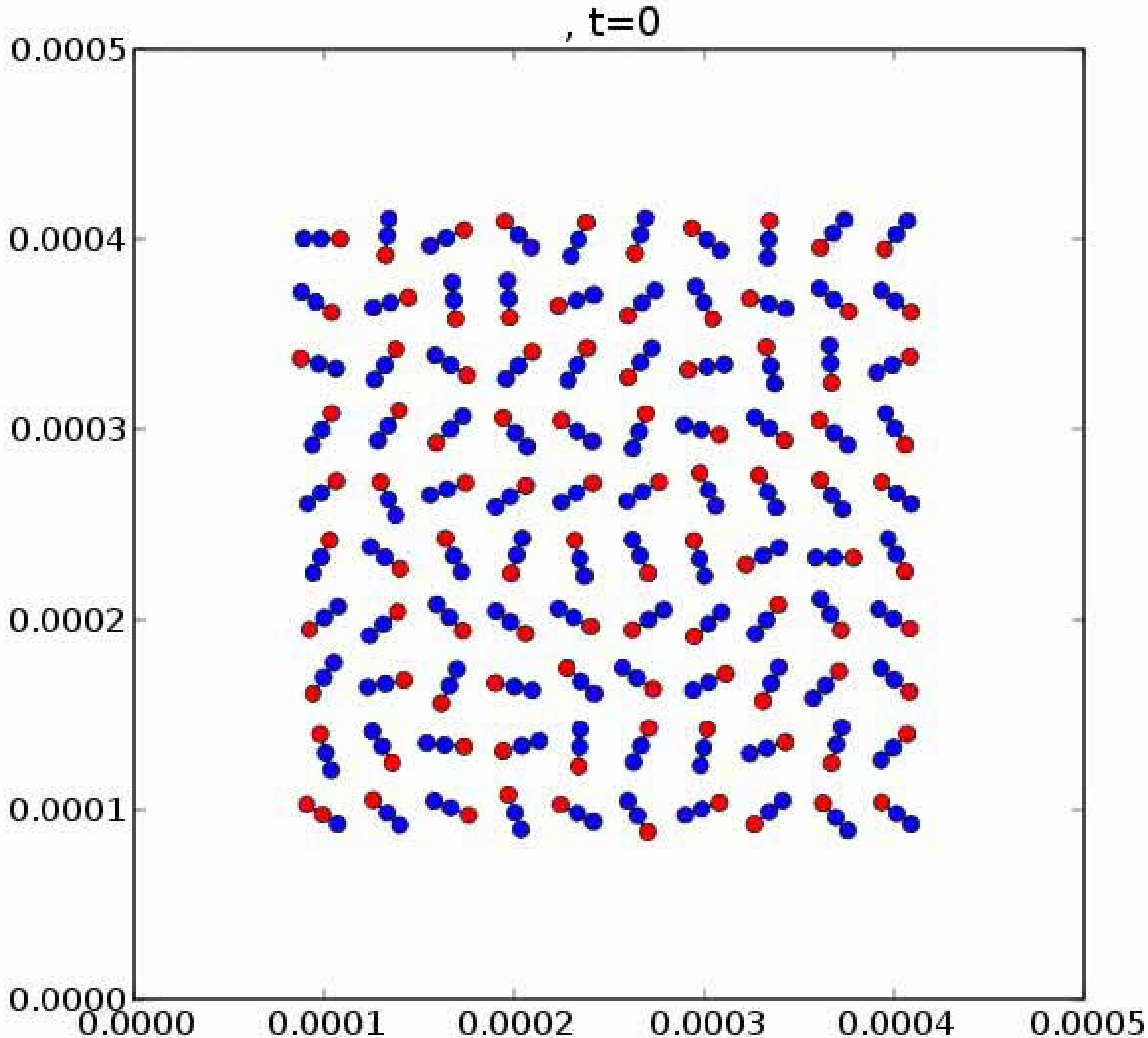
0.0002
0.0001
0.0000
0.0000
0.0001
0.0002
0.0003
0.0004
0.0005

In Phase


, t=0
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