Stability constraints in triplet extension of the MSSM

Sourov Roy

Indian Association for the Cultivation of Science (IACS), Kolkata, India

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Department of Theoretical Physics, TIFR

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Introduction

- Presence of scalar fields with color and electric charge in SUSY induces the possible existence of dangerous charge and color breaking minima, which would make the standard vacuum unstable

- There are potentially dangerous directions in the field space along which the potential becomes unbounded from below

- One can derive a set of necessary and sufficient conditions to avoid dangerous directions

- They are very strong and large regions of parameter space become forbidden

UFB and CCB minima

Appearance of UFB and CCB minima is not a shortcoming since

any SUSY models can be discarded on these grounds,

improving the predictive power of the theory

Scalar potential – Radiative corrections

The tree level scalar potential V_0 is strongly Q-dependent

The one-loop radiative corrections to it are crucial to make the potential stable against the variations of the Q scale

The complete one-loop potential $V_1 = V_0 + \Delta V_1$ has a very complicated structure

Minimization becomes an impossible task

In the region of Q where ΔV_1 is small, the predictions of V_0 and V_1 essentially coincide

Moreover, this corresponds to the maximal Q-invariance of V_1

Triplet extension of MSSM

Triplet extended SSM was introduced mainly to enhance the tree level Higgs boson mass

Espinosa, Quiros, 1991

With additional triplet it is possible to have spontaneous CP violation

The lightest Higgs boson mass has been calculated with extra triplet chiral superfields with hypercharge, $\mathrm{Y} = 0, \pm 1$

The one loop correction to the Higgs boson mass was calculated for MSSM with a $\mathrm{Y} = 0$ scalar triplet

Stefano di Chiara (2008)

TESSM

we are interested in studying the stability of the EW minimum of the TESSM scalar potential.

If the EW minimum is not a global minimum, correct EW symmetry breaking is not realized and its viability spoiled.

It is therefore important to determine the constraints on the parameter space that ensure that the EW minimum is stable.

For MSSM, there are a few directions possible along which the potential becomes Unbounded From Below (UFB)
Casas, Munoz, Lleyda (1995)

we would like to perform a full analysis of the UFB directions of the TESSM potential.

TESSM

The field content of TESSM is equal to that of MSSM extended by a $Y = 0$ $SU(2)$ triplet chiral superfield, whose scalar component can be written in matrix form as

$$T = \begin{pmatrix} \frac{1}{\sqrt{2}}T^0 & T^+ \\ T^- & -\frac{1}{\sqrt{2}}T^0 \end{pmatrix}$$

$$W_{\text{TESSM}} = \mu_T \text{Tr}(\hat{T}\hat{T}) + \mu_D \hat{H}_d \cdot \hat{H}_u + \lambda \hat{H}_d \cdot \hat{T} \hat{H}_u + y_t \hat{U} \hat{H}_u \cdot \hat{Q} - y_b \hat{D} \hat{H}_d \cdot \hat{Q} - y_\tau \hat{E} \hat{H}_d \cdot \hat{L}$$

TESSM– scalar potential

$$V_{\rm D} = \frac{g_3^2}{2}\left[\tilde{Q}_L^\dagger \mathcal{T}_3^a \tilde{Q}_L - \tilde{u}_R \mathcal{T}_3^{*a} \tilde{u}_R^* - \tilde{d}_R \mathcal{T}_3^{*a} \tilde{d}_R^*\right]^2 + \frac{g_Y^2}{2}\left[\phi_j^\dagger Y \phi_j\right]^2 +$$
$$\frac{g_L^2}{2}\left[H_u^\dagger \mathcal{T}_2^b H_u + H_d^\dagger \mathcal{T}_2^b H_d + \mathrm{Tr}\left(T^\dagger \mathcal{T}_2^b T - T \mathcal{T}_2^b T^\dagger\right) + \tilde{Q}_L^\dagger \mathcal{T}_2^b \tilde{Q}_L + \tilde{L}_L^\dagger \mathcal{T}_2^b \tilde{L}_L\right]^2$$

$$V_{\rm F} = \left|\frac{\partial W_{\rm TESSM}}{\partial \phi_j^c}\right|^2$$

$\mathcal{T}$ are group generators and ϕ_j^c runs over the scalar components of all the chiral superfields

The full potential is then given by:

$V = V_{\rm D} + V_{\rm F} + V_S$ ‘

Soft SUSY breaking terms

$$V_S =$$

$$\left[\mu_T B_T \mathrm{Tr}(TT) + \mu_D B_D H_d \cdot H_u + \lambda A_T H_d \cdot T H_u + y_t A_t \tilde{t}_R^* H_u \cdot \tilde{Q}_L + h.c.\right]$$

$$+ m_T^2 \mathrm{Tr}(T^\dagger T) + m_{H_u}^2 \left|H_u\right|^2 + m_{H_d}^2 \left|H_d\right|^2 + \ldots$$

EW symmetry breaking VEVs

$H_u^0 \equiv \frac{1}{\sqrt{2}}\left(a_u + i b_u\right)$, $\quad H_d^0 \equiv \frac{1}{\sqrt{2}}\left(a_d + i b_d\right)$, $\quad T^0 \equiv \frac{1}{\sqrt{2}}\left(a_T + i b_T\right)$; $\partial_{a_i} V|_{\rm vev} = 0$, $\quad \langle a_i \rangle = v_i$, $\quad i = u, d, T$

$$m_{H_u}^2 = -\mu_D^2 - \frac{g_Y^2+g_L^2}{8}\left(v_u^2 - v_d^2\right) + B_D\mu_D\frac{v_d}{v_u} - \frac{\lambda^2}{4}\left(v_d^2 + v_T^2\right) + \lambda v_T\left[\mu_D - \left(\frac{A_T}{2} + \mu_T\right)\frac{v_d}{v_u}\right] ,$$

$$m_{H_d}^2 = -\mu_D^2 + \frac{g_Y^2+g_L^2}{8}\left(v_u^2 - v_d^2\right) + B_D\mu_D\frac{v_u}{v_d} - \frac{\lambda^2}{4}\left(v_u^2 + v_T^2\right) + \lambda v_T\left[\mu_D - \left(\frac{A_T}{2} + \mu_T\right)\frac{v_u}{v_d}\right] ,$$

$$m_T^2 = -\frac{\lambda^2}{4}\left(v_d^2 + v_u^2\right) - 2\mu_T\left(B_T + 2\mu_T\right) + \lambda\left[\mu_D\frac{v_d^2+v_u^2}{2v_T} - \left(\frac{A_T}{2} + \mu_T\right)\frac{v_d v_u}{v_T}\right] .$$

Allow to determine three free prameters in terms of other ones

EW minimum

Using these equations one derives the expression for the potential at the EW minimum:

$$V_{\rm EW} = -\frac{g_Y^2+g_L^2}{32}\left(v_d^2 - v_u^2\right)^2 - \frac{\lambda^2}{8}\left[v_d^2 v_u^2 + v_T^2\left(v_d^2+v_u^2\right)\right] - \frac{\lambda v_T}{4}\left[v_d v_u\left(A_T + 2\mu_T\right) - \left(v_d^2+v_u^2\right)\mu_D\right]$$

We choose to simplify our analysis by evaluating $V_{\rm EW}$ at the EW scale $v_w = 246$ GeV

Stability condition

By taking all the vevs to be zero, the requirement that one of the eigenvalues of the neutral scalar squared mass matrix be negative is equivalent to imposing the condition

$$B_D^2 > \mu_D^2 \left(\frac{m_{H_d}^2}{\mu_D^2} + 1 \right) \left(\frac{m_{H_u}^2}{\mu_D^2} + 1 \right) .$$

When the condition above is satisfied, one can derive an important bound on the mass of the lightest neutral Higgs

$$m_{h_1^0}^2 \leq m_Z^2 \left(\cos^2 2\beta + \frac{\lambda^2}{g_Y^2 + g_L^2} \sin^2 2\beta \right) , \quad \tan\beta = \frac{v_u}{v_d}$$

Possible to generate correct light Higgs boson mass at the tree level (low $\tan\beta$, order one λ)

Unbounded from below directions

In SUSY models the potential is generally stable, the quartic terms are generated by the superpotential as well as by the gauge interactions (D-terms) and the SUSY tree-level potential is semidefinite positive.

If the quartic terms cancel, soft mass squared terms can eventually drive the potential to negative infinite values.

In general to find the deepest UFB direction of a supersymmetric theory in an N field subspace, one solves the minimization conditions with respect to $N-1$ fields, and then substitutes the solutions in the potential which turns out not to have quartic terms.

Non-zero charged real VEVs

The set of nonzero charged real vevs we work with is defined by:

$\langle \tilde{t}_L \rangle = \langle \tilde{t}_R \rangle = \frac{v_{\tilde{t}}}{\sqrt{2}}$, $\langle \tilde{b}_L \rangle = \alpha \langle \tilde{b}_R \rangle = \frac{v_{\tilde{b}}}{\sqrt{2}}$, $\langle H_u^+ \rangle = \langle H_d^- \rangle = \frac{v_{H^\pm}}{\sqrt{2}}$, $\langle T^+ \rangle = \langle T^- \rangle = \frac{v_{T^\pm}}{\sqrt{2}}$

For the neutral ones

$\langle \tilde{\nu}_L \rangle = \frac{v_{\tilde{\nu}}}{\sqrt{2}}$, $\langle H_u^0 \rangle = \frac{v_{H_u^0}}{\sqrt{2}}$, $\langle H_d^0 \rangle = \frac{v_{H_d^0}}{\sqrt{2}}$, $\langle T^0 \rangle = \frac{v_{T^0}}{\sqrt{2}}$

Relevant VEVs

To determine the sets of nonzero vevs which allow for UFB directions, we look for those vevs combinations that can cancel all the D and quartic F terms. Requiring the superpotential derivative with respect to the triplet components to cancel, we obtain:

$$\frac{\partial W_{\mathrm{TESSM}}}{\partial \phi_j^c} = 0 \,, \quad \phi_j^c = T^0, T^+, T^- \;\Rightarrow v_{H^\pm} = 0 \wedge \left(v_{H_d^0} = 0 \vee v_{H_u^0} = 0 \right)$$

Besides $v_{H^\pm}$, requires either $v_{H_u^0}$ or $v_{H_d^0}$ to be zero. Indeed it can be shown that there is no vevs combination canceling all the quartic terms for nonzero $v_{H_d^0}$, and therefore we impose: $v_{H_d^0} = 0$

Relevant VEVs

After imposing $v_{H_d^0} = 0$ and requiring the cancellation of the quartic F terms corresponding to the H_u^0 and H_d^- fields, also the stop and charged triplet vevs turn out to be zero:

$$\frac{\partial W_{\rm TESSM}}{\partial \phi_j^c} = 0 \ , \quad \phi_j^c = H_u^0, H_d^- \ \Rightarrow v_{\tilde{t}} = v_{T^{\pm}} = 0$$

Relevant VEVs

Having set to zero the charged doublet and triplet Higgs vevs as well as the stop and the neutral down Higgs ones the only nonzero D and quartic F terms left are, respectively

$$V_D \supset \frac{g_Y^2+g_L^2}{32}\left(v_{\tilde{b}}^2 - v_{\tilde{\nu}}^2 + v_{H_u^0}^2\right)^2$$

$$V_F \supset \frac{1}{4}\left(y_b^2 v_{\tilde{b}}^4 + \sqrt{2} y_b \alpha \lambda v_{\tilde{b}}^2 v_{H_u^0} v_{T^0} + \frac{\lambda^2}{2} v_{H_u^0}^2 v_{T_0}^2\right)$$

Relevant VEVs

Assuming a nonzero neutral up Higgs vev, we find therefore that it is possible to cancel all the quartic terms for the following sets of nonzero vevs:

$$v_{H_u^0} \neq 0 \ \wedge \ v_{\tilde{\nu}} \neq 0 \ \wedge$$
$$\Big(\big(v_{T^0} = 0 \ \wedge \ v_{\tilde{b}} = 0\big) \vee \Big(v_{T^0} = 0 \ \wedge \ v_{\tilde{b}} \propto \sqrt{v_{H_u^0}}\Big) \vee \big(v_{T^0} \neq 0 \ \wedge \ v_{\tilde{b}} \neq 0\big) \Big)$$

with the other vevs being all identically zero. The only other possible set of non-trivial vevs canceling all the quartic terms is

$v_{T^0} \neq 0 \ \vee \ v_{T^\pm} \neq 0$

Stability Constraints – UFB-1

Case -1 – $v_{T^0} \neq 0$, all the other vevs 0

$$V_{\mathrm{UFB-1}} = \frac{v_{T0}^2}{2}\left[m_T^2 + 2\mu_T\left(B_T + 2\mu_T\right)\right]$$

We evaluate at a renormalization scale Λ of the order of the heaviest mass in the physical particle spectrum, so as to minimize the contribution of quantum corrections, which we neglect entirely

a given point in parameter space is stable against tunneling to UFB–1 if $V_{\mathrm{EW}}\left(v_w\right) < V_{\mathrm{UFB-1}}\left(\Lambda\right)\;, \quad v_w \leq \Lambda \leq \Lambda_{\mathrm{UV}}\;, \quad v_{T^0}^2 \sim 2\max\left[g_L^2, \lambda^2\right]^{-1}\Lambda^2$

Stability Constraints – UFB-2

Slightly more complicated case with only two nonzero vevs

$$V|_{v_{H_u^0}\neq 0, v_{\tilde{\nu}}\neq 0} =$$

$$\frac{1}{2}\left[m_L^2 v_{\tilde{\nu}}^2 + \left(m_{H_u^0}^2 + \mu_D^2\right) v_{H_u^0}^2 + \frac{g_Y^2+g_L^2}{16}\left(v_{\tilde{\nu}}^2 - v_{H_u^0}^2\right)^2\right]$$

m_L^2 is the soft mass squared of the slepton doublet L

By requiring the potential above to be flat along the $v_{\tilde{\nu}}$ direction we find

$$\partial_{v_{\tilde{\nu}}} V|_{v_{H_u^0}\neq 0, v_{\tilde{\nu}}\neq 0} = 0 \qquad \Rightarrow v_{\tilde{\nu}}^2 = v_{H_u^0}^2 - \frac{8m_L^2}{g_Y^2+g_L^2}$$

Stability Constraints – UFB-2

The deepest UFB direction corresponding to our choice of nonzero vacua:

$$V_{\mathrm{UFB-2}} = \frac{v^2_{H^0_u}}{2}\left(m_L^2 + m^2_{H^0_u} + \mu_D^2\right) - \frac{2m_L^4}{g_Y^2+g_L^2}$$

Analogously to the stability constraint derived from the UFB–1 direction, a point in the TESSM parameter space may feature a stable EW minimum only if

$$V_{\mathrm{EW}}\left(v_w\right) < V_{\mathrm{UFB-2}}\left(\Lambda\right)\ , \quad v_w \le \Lambda \le \Lambda_{\mathrm{UV}}\ , \quad v^2_{\tilde{\nu}} > 0\ , \quad v^2_{H^0_u} \sim 2\max\left[g_L^2, \lambda^2, y_t^2\right]^{-1}\Lambda^2$$

All the couplings and dimensionful parameters are evaluated at Λ.

Stability Constraints – UFB-3

The case with three nonzero vevs, is a little more complicated, but the potential can be readily simplified by imposing its derivative with respect to $v_{\tilde{\nu}}$ to be zero:

$$\partial_{v_{\tilde{\nu}}} V|_{v_{H_u^0}\neq 0, v_{\tilde{b}}\neq 0, v_{\tilde{\nu}}\neq 0} = 0 \quad \Rightarrow \quad v_{\tilde{\nu}}^2 = v_{H_u^0}^2 + v_{\tilde{b}}^2 - \frac{8m_L^2}{g_Y^2+g_L^2}$$

$$V|_{v_{H_u^0}\neq 0, v_{\tilde{b}}\neq 0, v_{\tilde{\nu}}\neq 0} =$$
$$\frac{y_b^2}{4} v_{\tilde{b}}^4 + \frac{v_{\tilde{b}}^2}{2}\left(m_L^2 + m_Q^2 + m_{\tilde{b}}^2 - \sqrt{2} y_b \alpha v_{H_u^0} \mu_D\right) +$$
$$\frac{v_{H_u^0}^2}{2}\left(m_L^2 + m_{H_u^0}^2 + \mu_D^2\right) - \frac{2m_L^4}{g_Y^2+g_L^2}$$

Stability Constraints – UFB-3

By requiring the potential above to be flat along the $v_{\tilde{b}}$ direction and solving for $v_{\tilde{b}}$ we find

$$v_{\tilde{b}}^2 = \frac{\sqrt{2} v_{H_u^0} y_b |\mu_D| - m_L^2 - m_Q^2 - m_{\tilde{b}}^2}{y_b^2}$$

The deepest UFB direction corresponding to our choice of nonzero vacua

$$V_{\mathrm{UFB-3}} = \left(m_L^2 + m_{H_u^0}^2\right) \frac{v_{H_u^0}^2}{2} + \frac{|\mu_D| \left(m_L^2 + m_Q^2 + m_{\tilde{b}}^2\right) v_{H_u^0}}{\sqrt{2} y_b} - \frac{\left(m_L^2 + m_Q^2 + m_{\tilde{b}}^2\right)^2}{4 y_b^2} - \frac{2 m_L^4}{g_Y^2 + g_L^2}$$

where m_Q^2 is the soft mass squared of the squark doublet Q

Stability Constraints – UFB-3

The corresponding stability constraint that any point in the TESSM parameter space has to satisfy is

$$V_{\rm EW}\left(v_w\right) < V_{\rm UFB-3}\left(\Lambda\right)\ , \quad v_w \leq \Lambda \leq \Lambda_{\rm UV}\ , \quad v_{\tilde{\nu}}^2, v_{\tilde{b}}^2 > 0\ , \quad v_{H_u^0}^2 \sim 2\,{\rm max}\left[g_L^2, \lambda^2, y_t^2\right]^{-1}\Lambda^2$$

Require the potential to be flat along the $v_{H_u^0}$ direction instead

$$v_{\tilde{b}}^2 = \frac{\sqrt{2}v_{H_u^0}}{y_b}\left|\frac{m_L^2+m_{H_u^0}^2+\mu_D^2}{\mu_D}\right|$$

Stability Constraints – UFB-3$'$

In this case the potential along the deepest UFB direction is

$$V'_{\mathrm{UFB-3}} = \frac{\left(m_L^2+m_{H_u^0}^2\right)\left(m_L^2+m_{H_u^0}^2+\mu_D^2\right)}{2\mu_D^2} v_{H_u^0}^2 +$$
$$\left|\frac{m_L^2+m_{H_u^0}^2+\mu_D^2}{\mu_D}\right| \frac{m_L^2+m_Q^2+m_{\tilde{b}}^2}{\sqrt{2}y_b} v_{H_u^0} - \frac{2m_L^4}{g_Y^2+g_L^2}$$

Stability Constraints – UFB-4

Finally, we take up the last scenario:

$v_{H_u^0} \neq 0 \wedge v_{\tilde{\nu}} \neq 0 \wedge v_{T^0} \neq 0 \wedge v_{\tilde{b}} \neq 0,$

The requirement for the potential to be flat along the ν direction determines $v_{\tilde{\nu}}$ as earlier

The remaining minimization conditions produce rather involved solutions, which turn out to be complex on a large and disconnected region of field space.

For this reason we choose simply to cancel the quartic F terms

$$v_{\tilde{b}}^2 = \frac{|\lambda v_{T^0}| v_{H_u^0}}{\sqrt{2} y_b}$$

Stability Constraints – UFB-4

The potential along the plane expressed in terms of the remaining two vevs

$$V_{\mathrm{UFB-4}} = \frac{v_{T^0}^2}{2}\left[m_T^2 + 2\mu_T\left(B_T + 2\mu_T\right)\right] + \frac{v_{H_u^0}^2}{2}\left(m_L^2 + m_{H_u^0}^2 + \mu_D^2\right) - \frac{2m_L^4}{g_Y^2+g_L^2} + \frac{|\lambda v_T| v_{H_u^0}}{2\sqrt{2}y_b}\left(m_L^2 + m_Q^2 + m_{\tilde{b}}^2\right)$$

Viable parameter space

The first relevant phenomenological constraint is given by the T parameter, which in TESSM receives a nonzero contribution already at tree level

$$\alpha_e T = \frac{\delta m_W^2}{m_W^2} = \frac{4v_T^2}{v^2} \le 0.2 \,, \quad v^2 = v_u^2 + v_d^2 \,, \quad v_w^2 = v^2 + 4v_T^2 = 246^2 \text{ GeV}^2$$

To satisfy the constraint above we take a small but non-zero fixed value for v_T

$$v_T = 3\sqrt{2}\ \text{GeV}$$

Scanning the parameter space

We then scan a large region of the TESSM parameter space, defined by

$$1 \leq t_\beta \leq 10\ ,\ 5\,\text{GeV} \leq |\mu_D, \mu_T| \leq 2\,\text{TeV}\ ,\ 50\,\text{GeV} \leq |M_1, M_2| \leq 1\,\text{TeV}$$

$$|A_t, A_T, B_D, B_T| \leq 2\,\text{TeV}\ ,\ 500\,\text{GeV} \leq m_Q, m_{\tilde{t}}, m_{\tilde{b}} \leq 2\,\text{TeV}$$

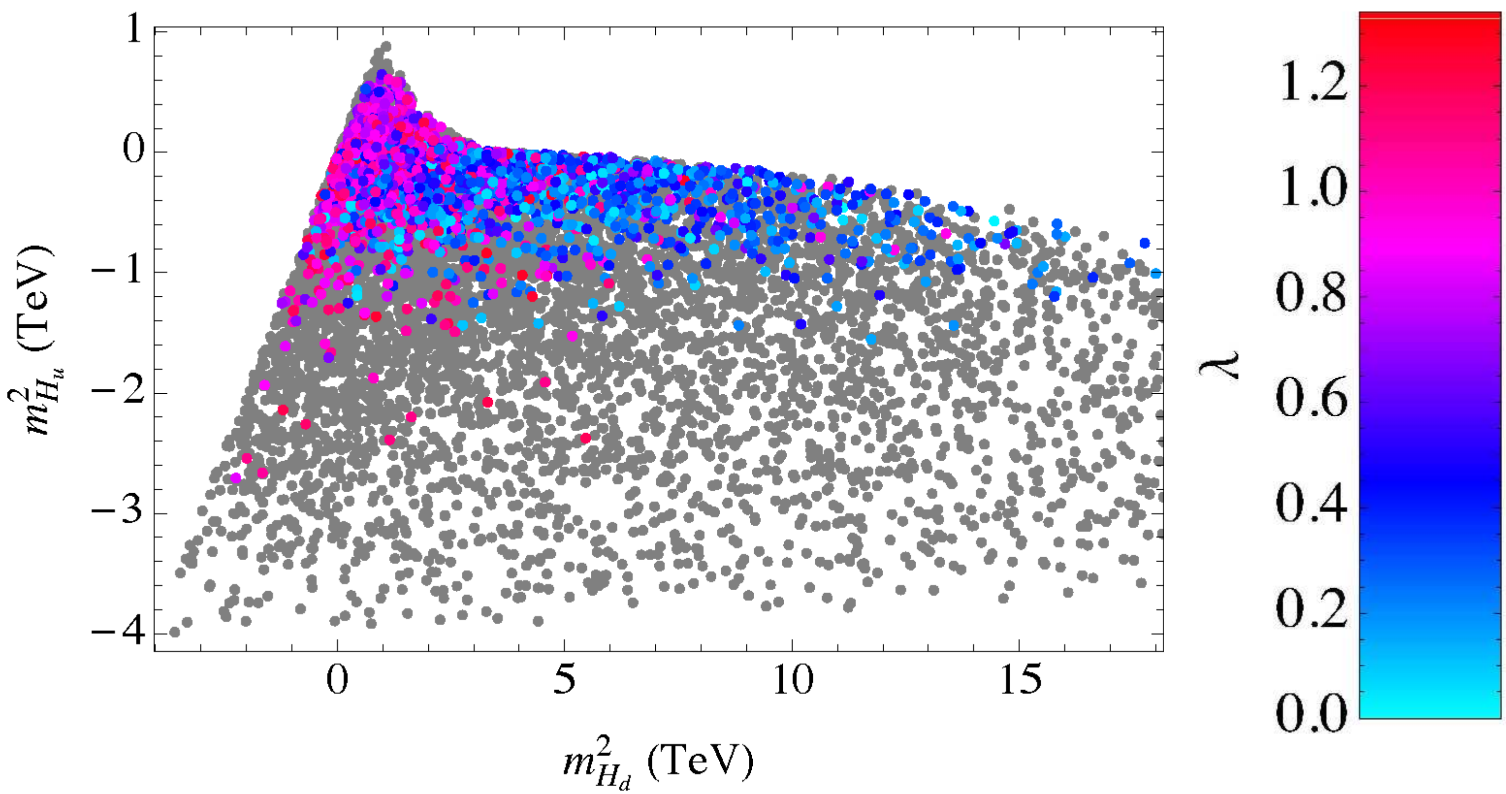
for data points producing the observed mass spectrum for SM fermions and gauge bosons and satisfying the direct search constraints defined below

$$m_{h_1^0} = 125.5 \pm 0.1\,\mathrm{GeV}\ ;\ m_{A_{1,2}},\ m_{\chi^0_{1,2,3,4,5}} \geq 65\,\mathrm{GeV}\ ;$$

$$m_{h^0_{2,3}}, m_{h^\pm_{1,2,3}}, m_{\chi^\pm_{1,2,3}} \geq 100\,\mathrm{GeV}\ ;\ m_{\tilde{t}_{1,2}}, m_{\tilde{b}_{1,2}} \geq 650\,\mathrm{GeV}$$

Allowed region of parameter space

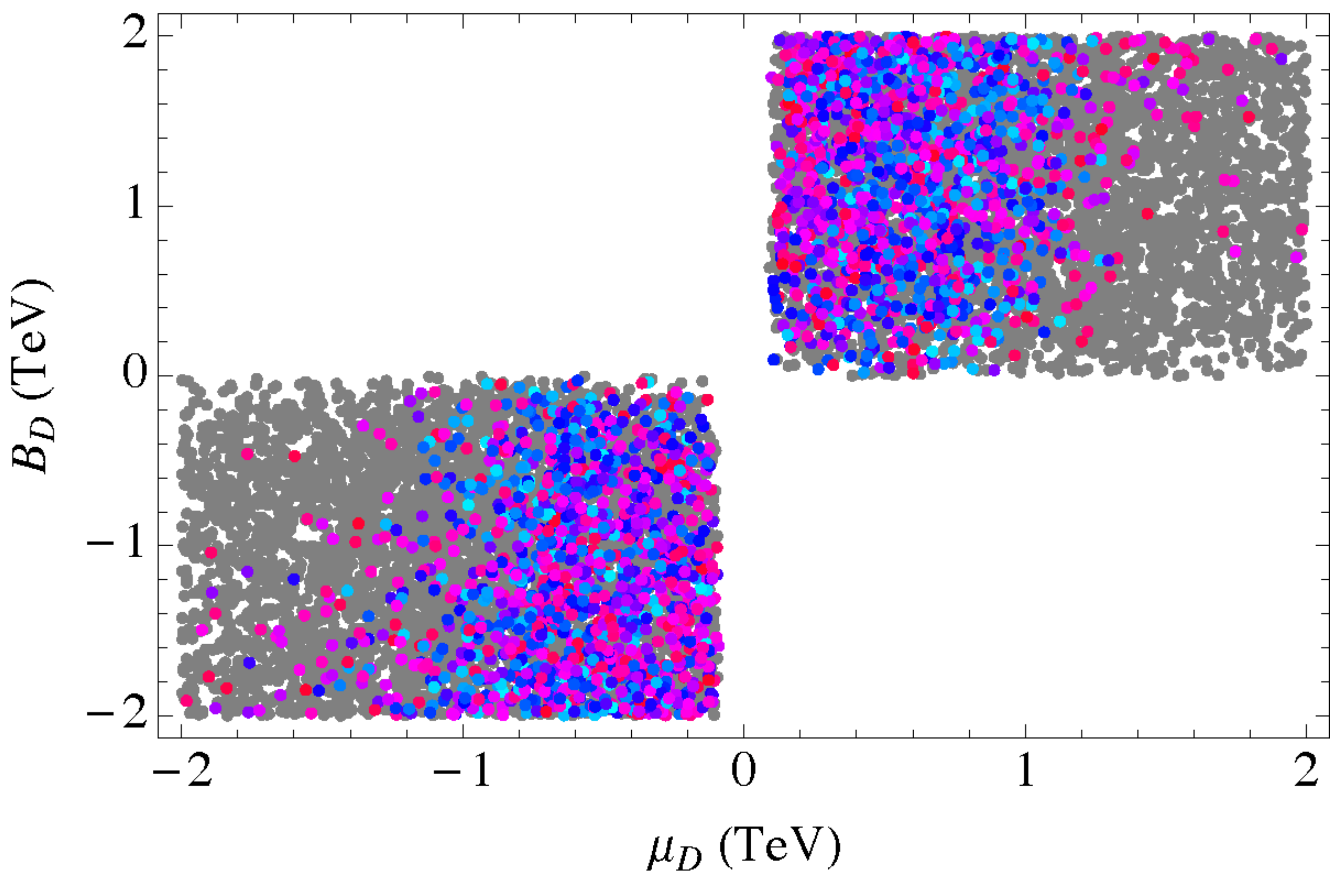
(M.Das, S. Di Chiara, SR, PRD (2015))



Grey points are unstable and colored points are stable.

Rule out large negative soft squared mass $m^2_{H_u}$

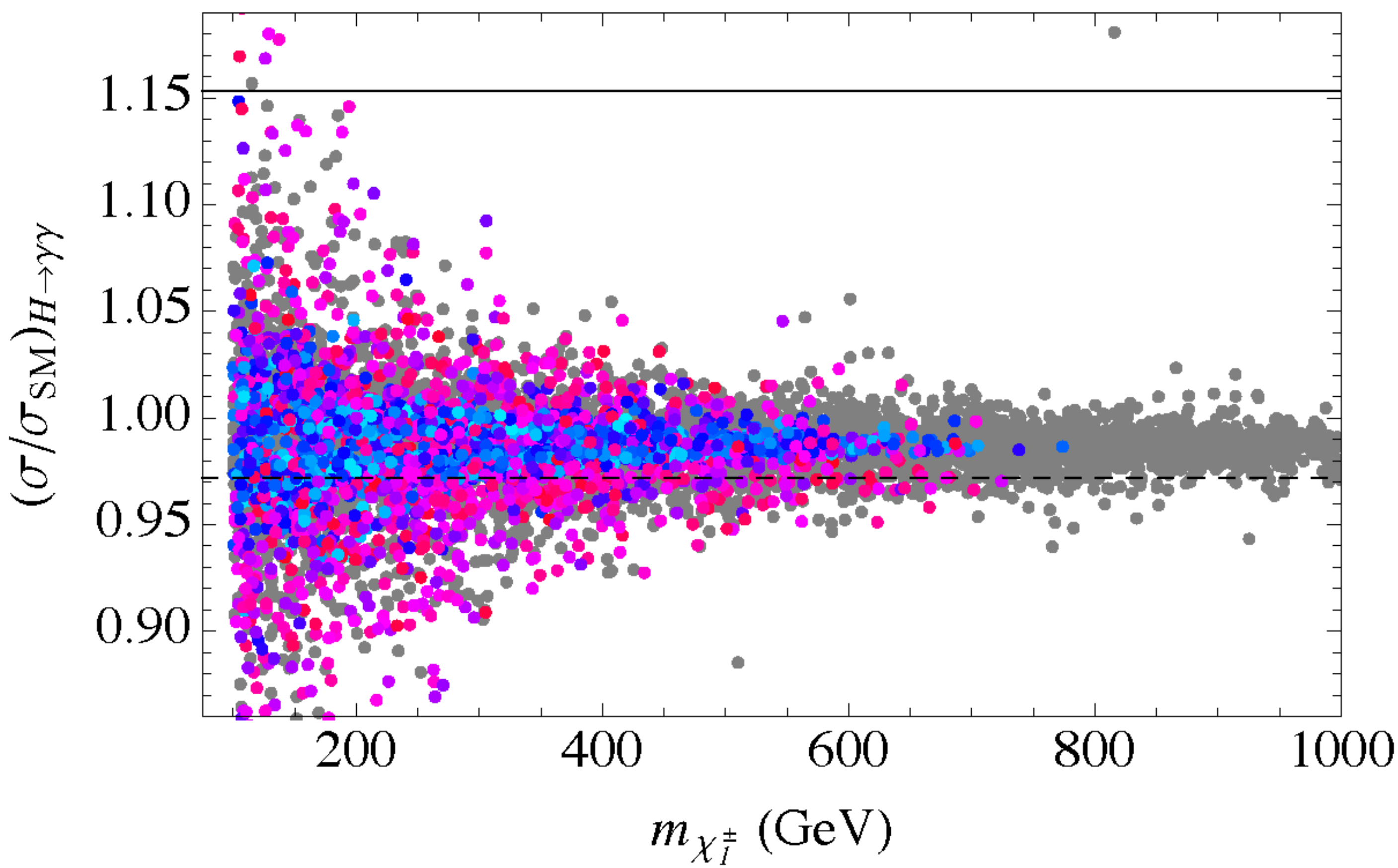
Allowed region of parameter space



Grey points are unstable and colored points are stable.

Rule out a large $|\mu_D|$

Higgs boson diphoton cross section



Stability constraints require the mass of the lightest chargino (and neutralino) to be lighter than about 700 GeV

Fine tuning

A simple estimate of FT in supersymmetry is given by the logarithmic derivative of the EW vev v_w with respect to a given model parameter μ_p

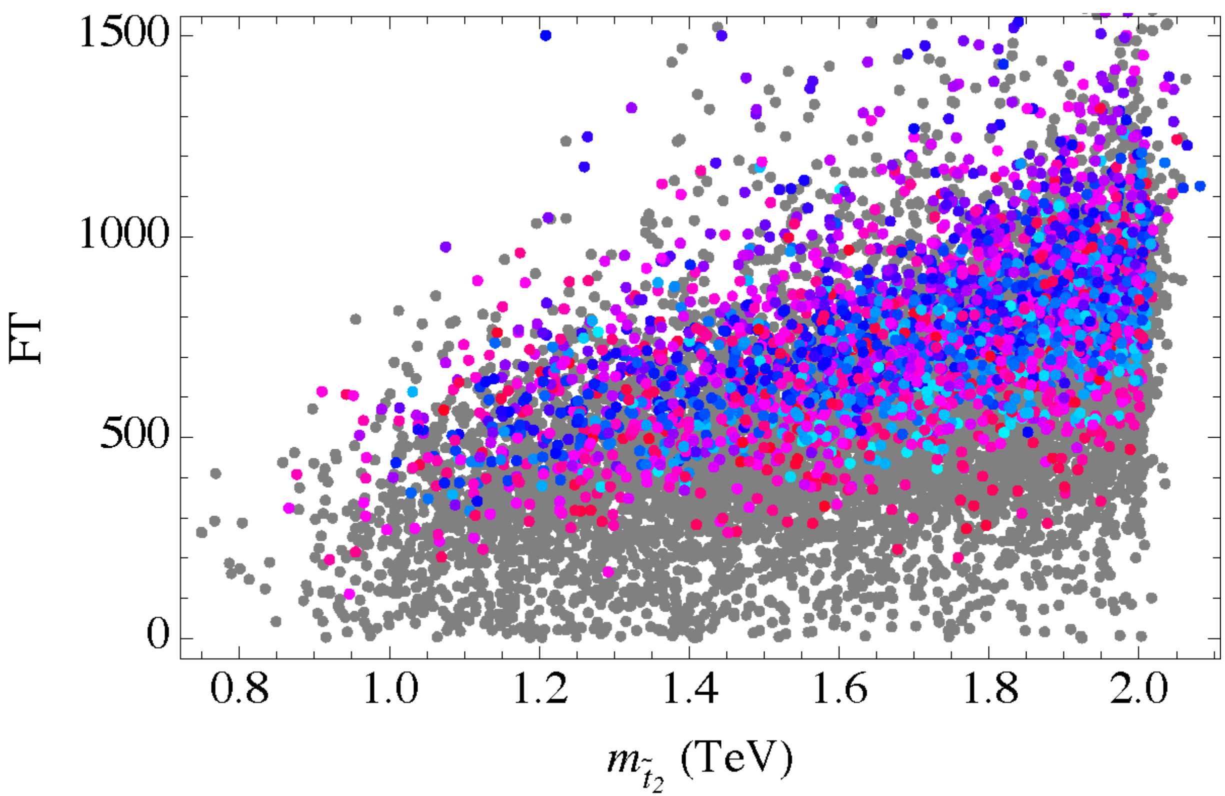
This represents the change of v_w for a 100% change in the given parameter

$$\mathbf{f}_{\mu_p} \equiv \frac{\partial \log v_w^2}{\partial \log \mu_p^2(\Lambda)}$$

$$\mu_p^2\,(\Lambda) = \mu_p^2\,(M_Z) + \frac{\beta_{\mu_p^2}}{16\pi^2} \log\left(\frac{\Lambda}{M_Z}\right)$$

$$\beta_{\mu_p^2} = 16\pi^2 \frac{d\mu_p^2}{d\text{logQ}}$$

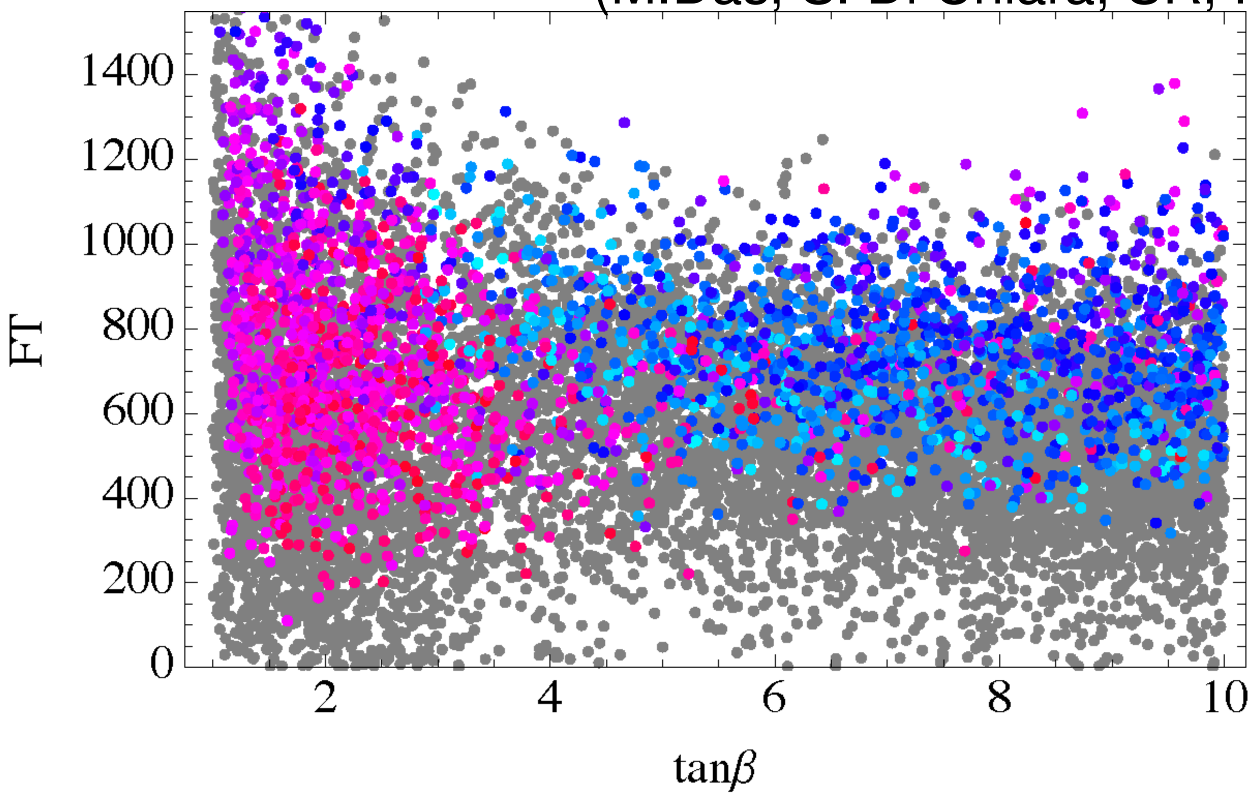
Fine tuning in $m^2_{H_u}$



Grey points are unstable and colored points are stable. FT is about 26% higher after imposing the stability constraints

Fine tuning in $m^2_{H_u}$

(M.Das, S. Di Chiara, SR, PRD (2015))



Data points featuring low amount of fine tuning are unstable

Conclusions

- We studied the Unbounded From Below (UFB) directions in the potential of a $Y = 0$ SU(2) triplet chiral superfield extension of MSSM
- Looked for sets of nonzero vevs that can cancel the quartic terms belonging to the D and F sectors of the TESSM tree level potential
- We found four inequivalent sets of vevs that allow for UFB directions in the tree level potential
- Soft up-type Higgs mass M_{H_u} and μ_D are generally smaller than about 1 TeV
- Lightest chargino lighter than about 700 GeV
- Fine tuning is about 26$\%$ higher
- Stability constraints should be taken into account