Surfing the cosmic web with tessellations

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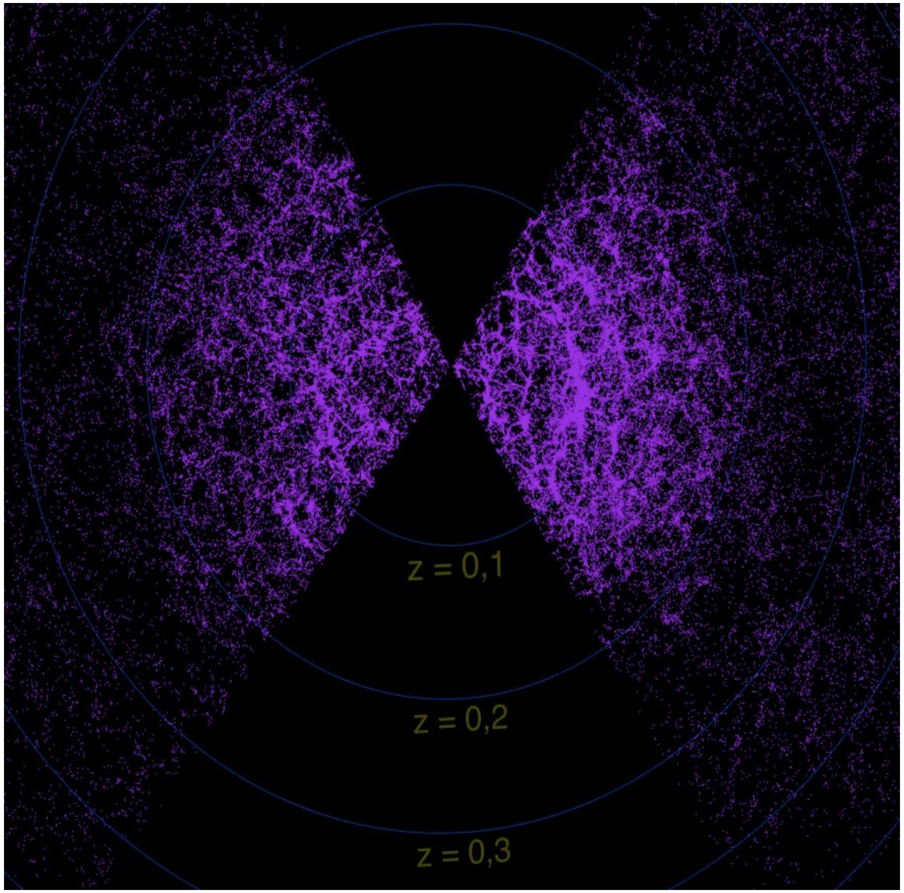
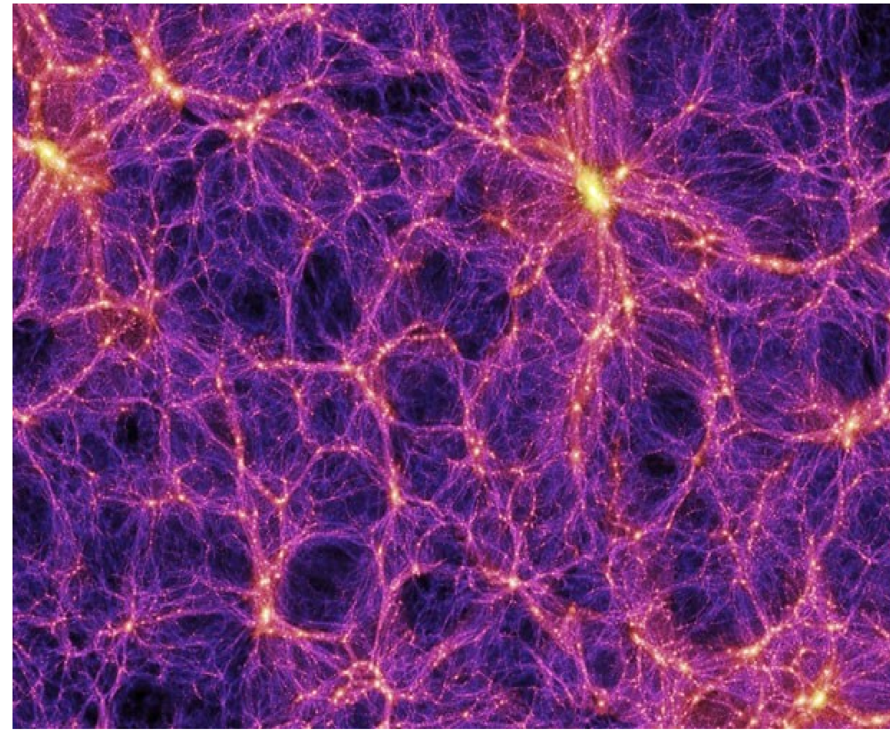
Cosmic Web : A weblike network of clusters, filaments and walls surrounded by voids

Observational



Sloan Digital Sky Survey

Theoretical



Millennium Simulation

Discretely sampled galaxy/particle distribution → Continuous density field

Density field reconstruction

Fixed smoothing kernel method:
At high resolution low density regions are severely under-sampled whereas at low resolution high density regions are poorly resolved.

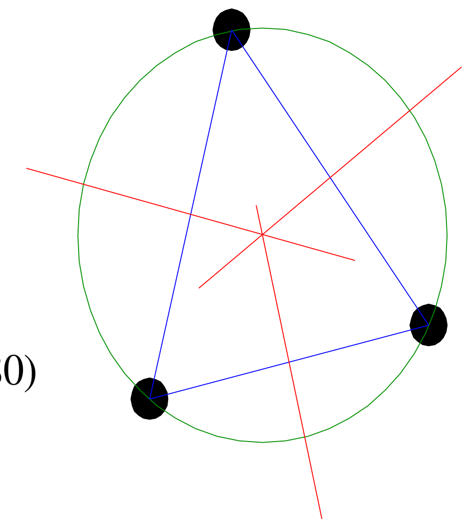
SPH method:
Self adaptive to the local density but still involves user defined parameter such as number of neighbours used for defining the adaptive kernel.

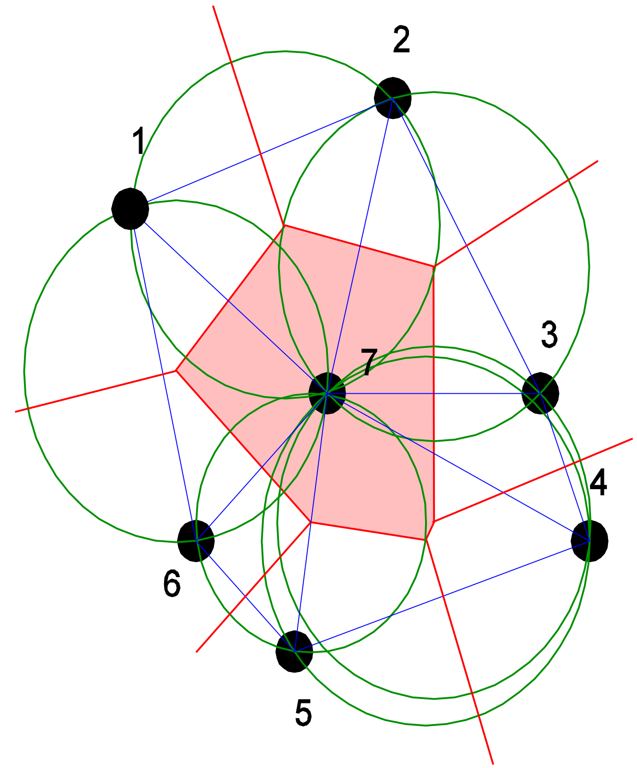
The rigid geometry of the filter is not appropriate for describing highly anisotropic features such as filaments and sheets in the cosmic web.

Ideally the appropriate density field reconstruction should be set solely and automatically by the point distribution itself.

Delaunay Tessellations

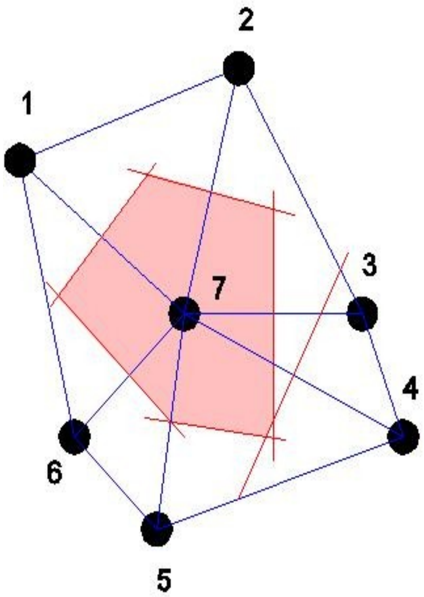
Boris Delaunay (1890-1980)

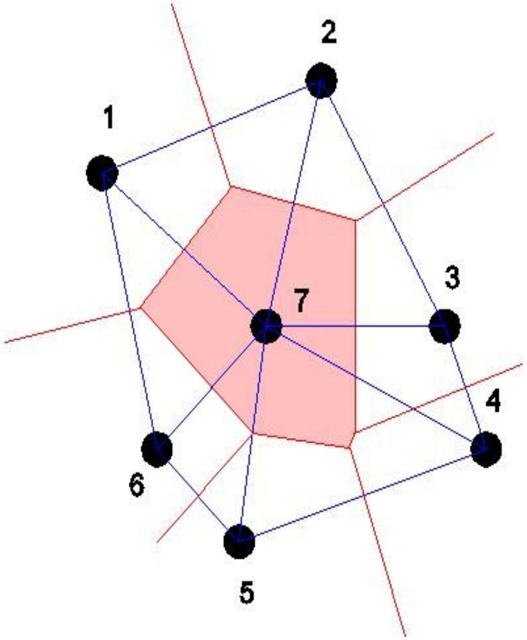


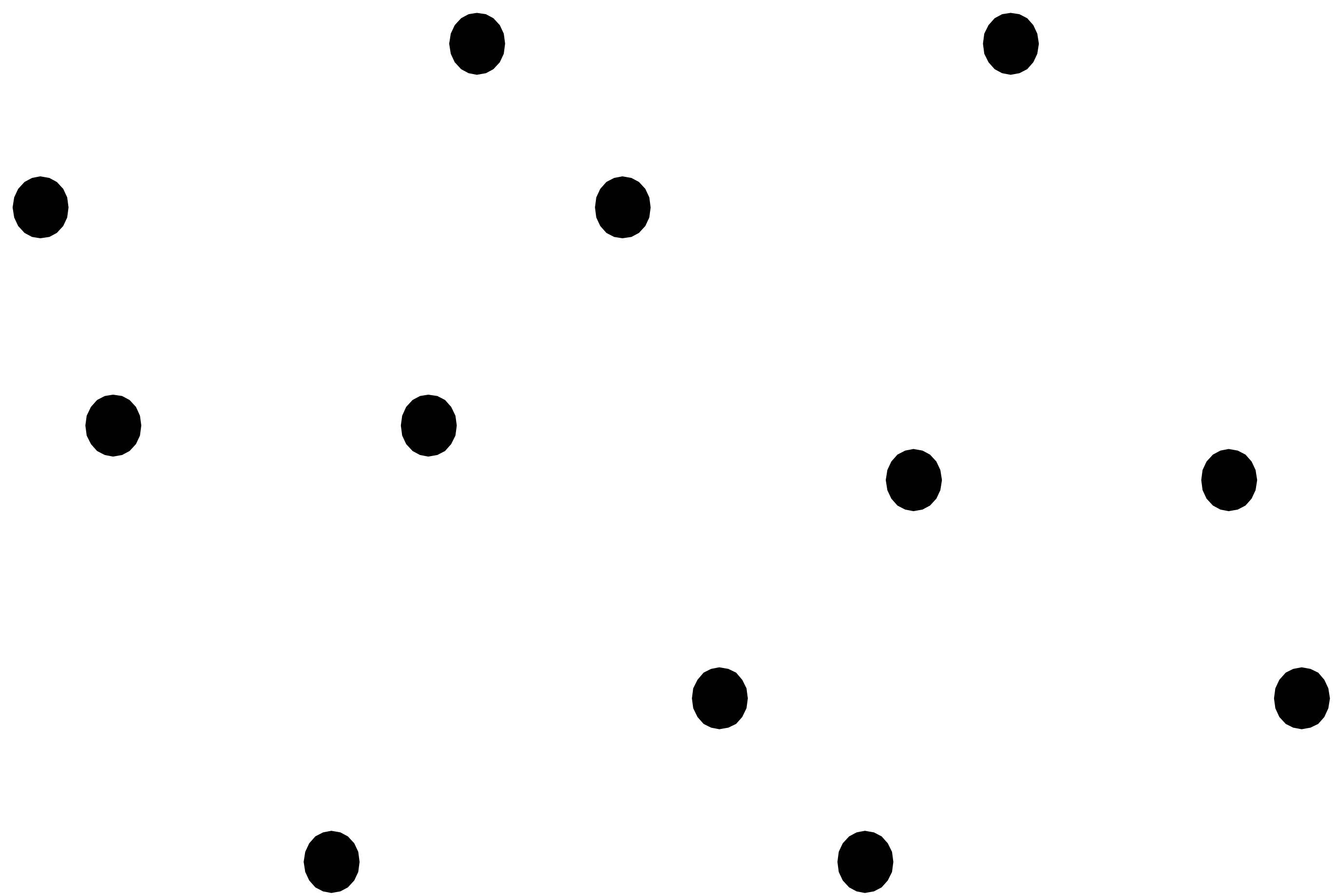


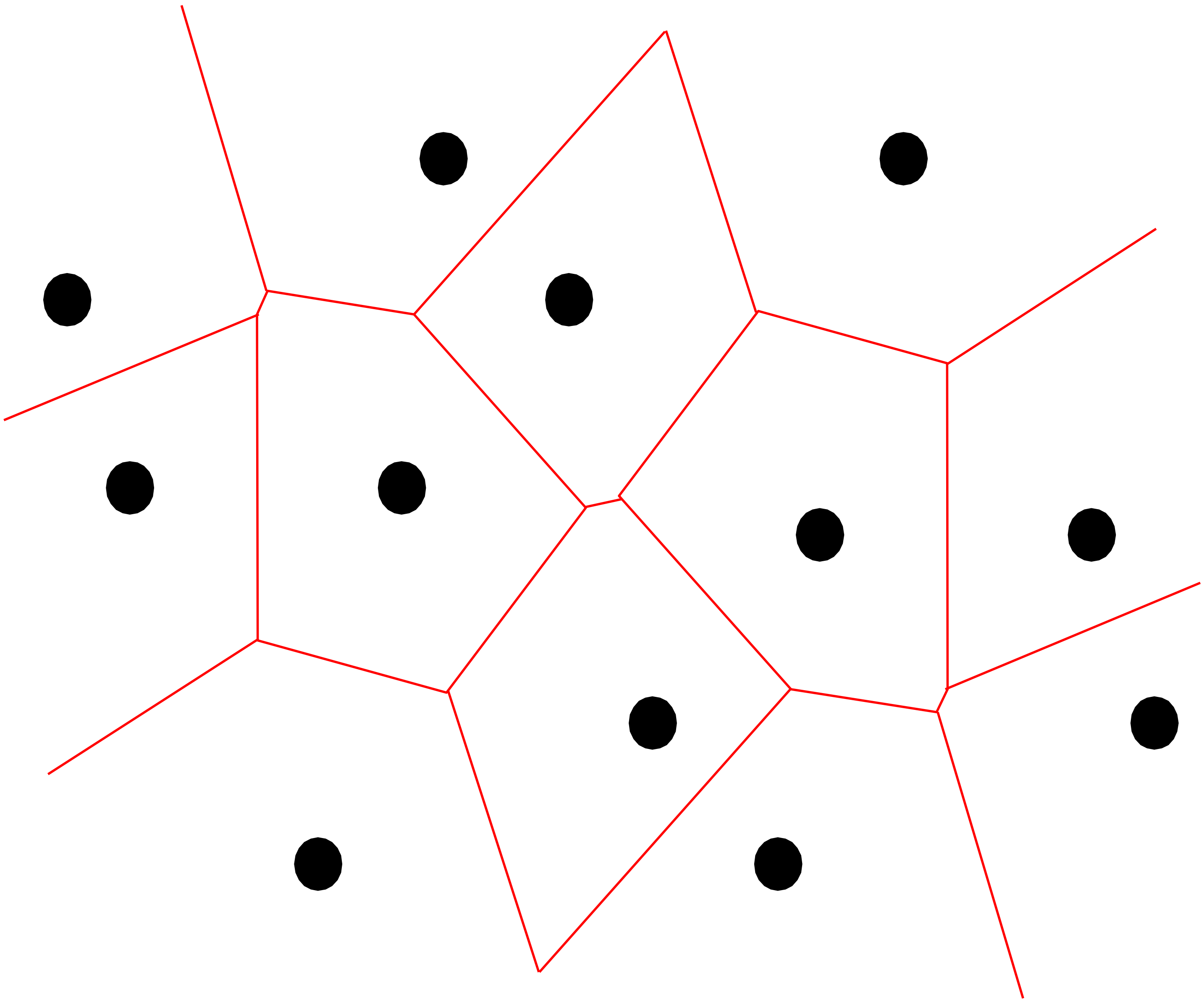
Voronoi Tessellations

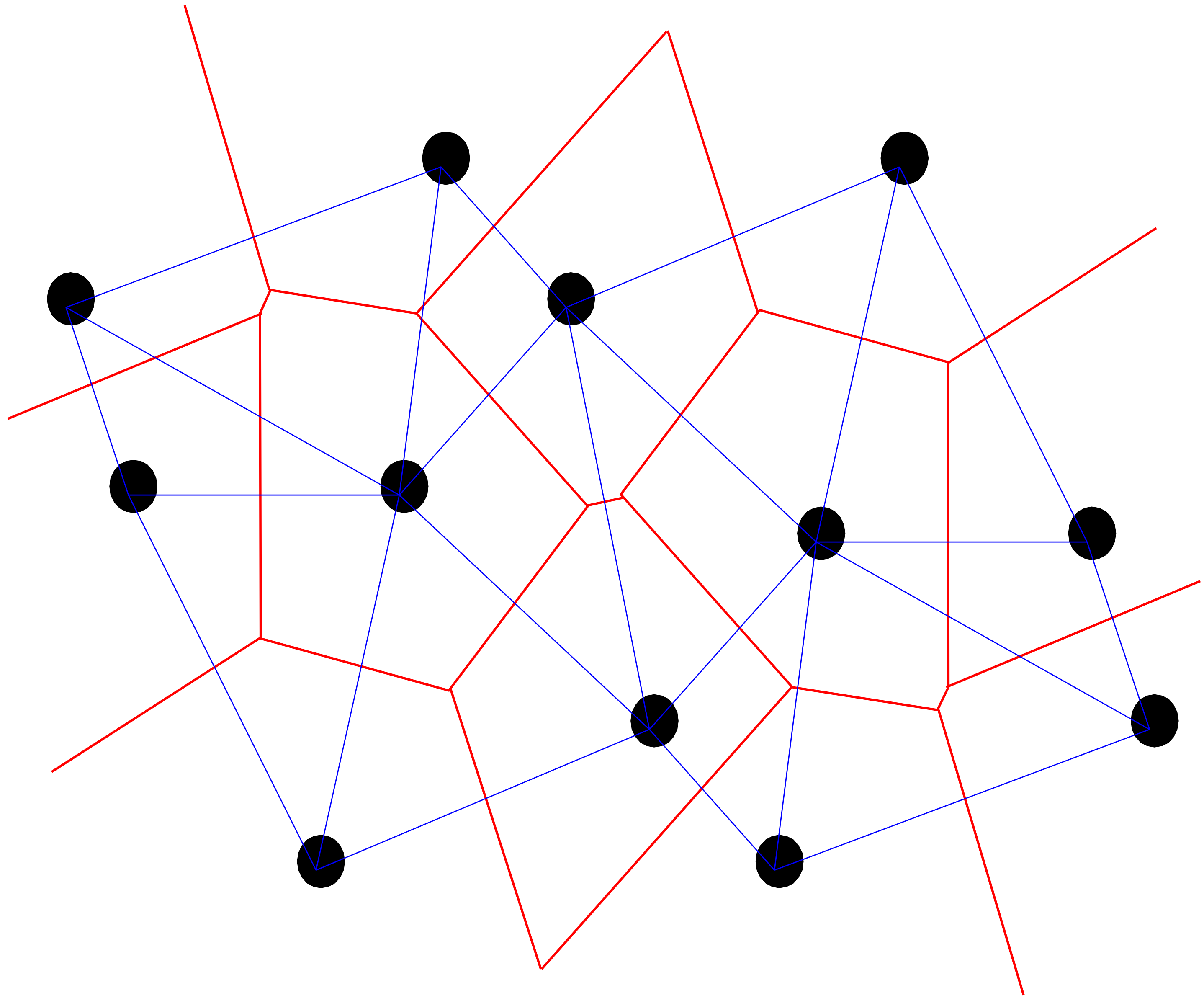
Georgy Voronoy (1868-1908)



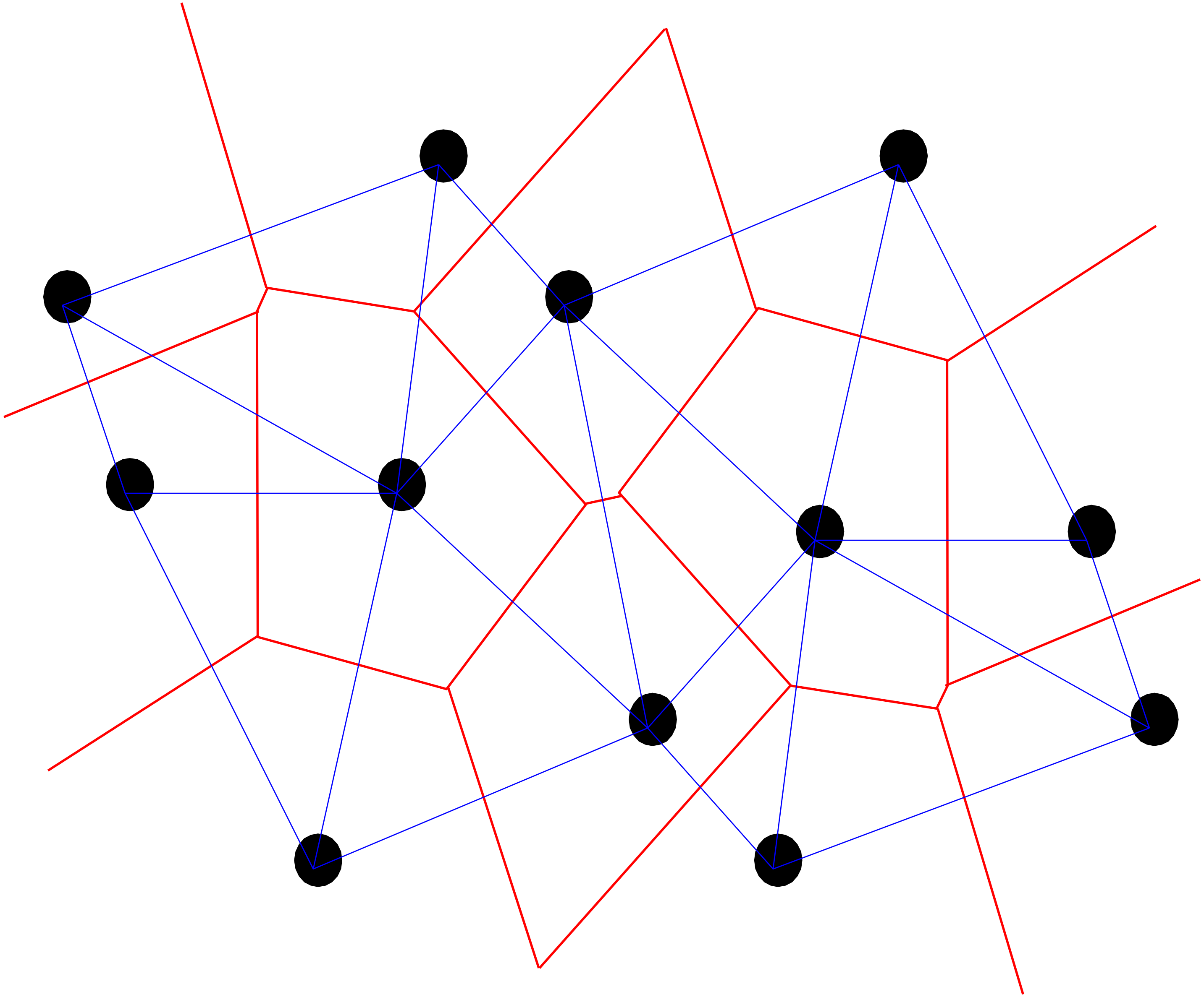


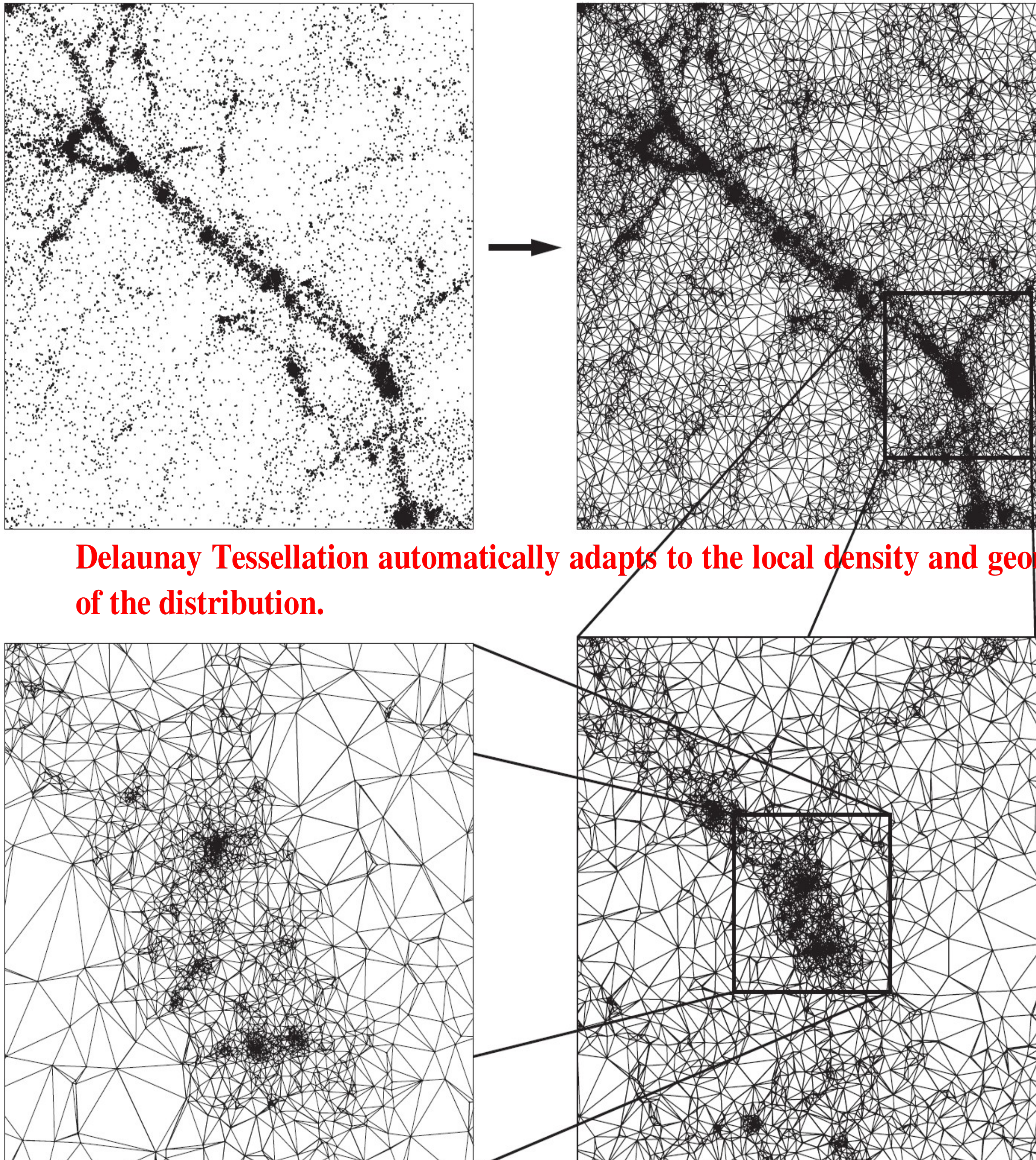






They are dual graphs and both hold the same information content.



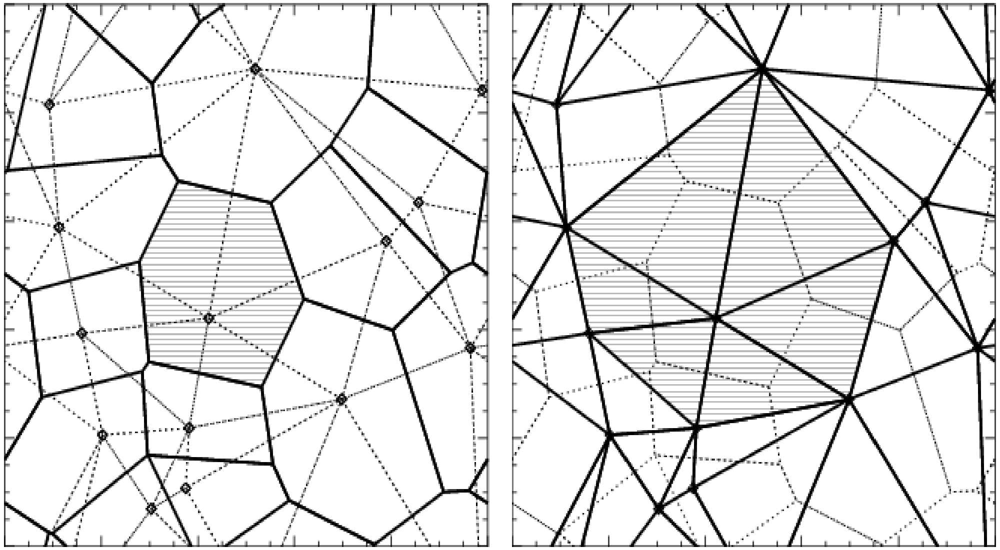


Delaunay Tessellation automatically adapts to the local density and geome of the distribution.

van de Weygaert
& Schaap 2007

Voronoi Tesselation Field Estimator (VTFE)

$$\rho(x_i) = \frac{m_i}{V_i}$$



Delaunay Tessellation Field Estimator (DTFE)

Schaap & van de Weygaert 2000

$$\rho(x_i) = \frac{(D+1)m_i}{W_i}$$

We use the tessellation engine of AREPO (Springel 2010) for constructing the Voronoi Tesselations and Delaunay Tessellations.

Geometry and Topology of the Cosmic Web

Minkowski Functionals

Mecke, Buchert & Wagner 1994

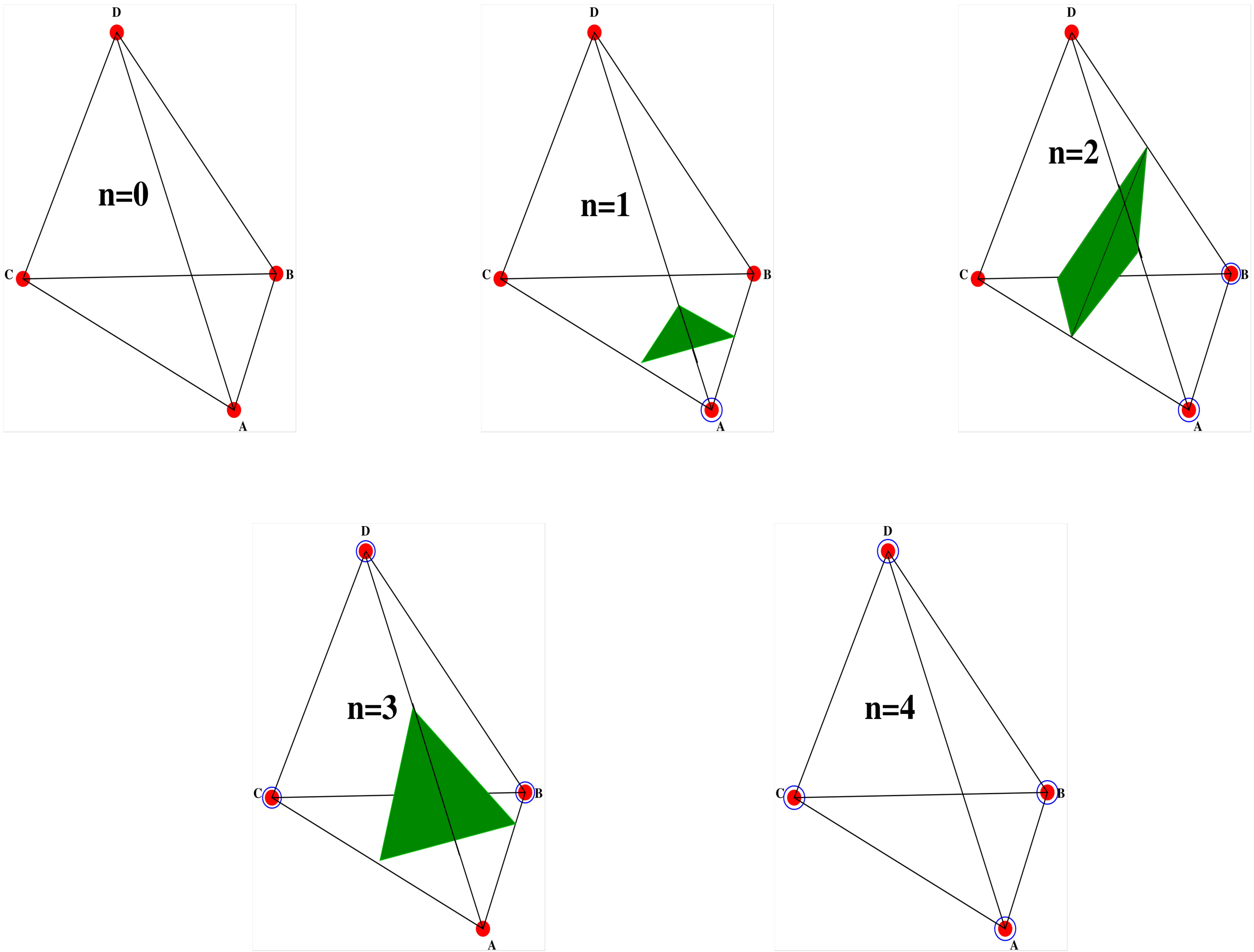
- Volume $\mathbf{V}$
- Surface area $\mathbf{A}$
- Integrated mean curvature $\mathbf{C} = \frac{1}{2}\oint_{S}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) dA$
- Integrated Gaussian curvature $\chi = \frac{1}{2\pi}\oint_{S}\left(\frac{1}{r_1 r_2}\right) dA$

Existing Methods for computing Minkowski functionals:

Boolean grains method (e.g. Mecke, Buchert and Wagner 1994, Kerscher et al. 1997)

The isodensity contours method of smoothed density field (e.g. Sheth et al. 2003, Shandarin, Sheth and Sahni 2004)

Possible configurations



All possibilities

Number of overdense vertex

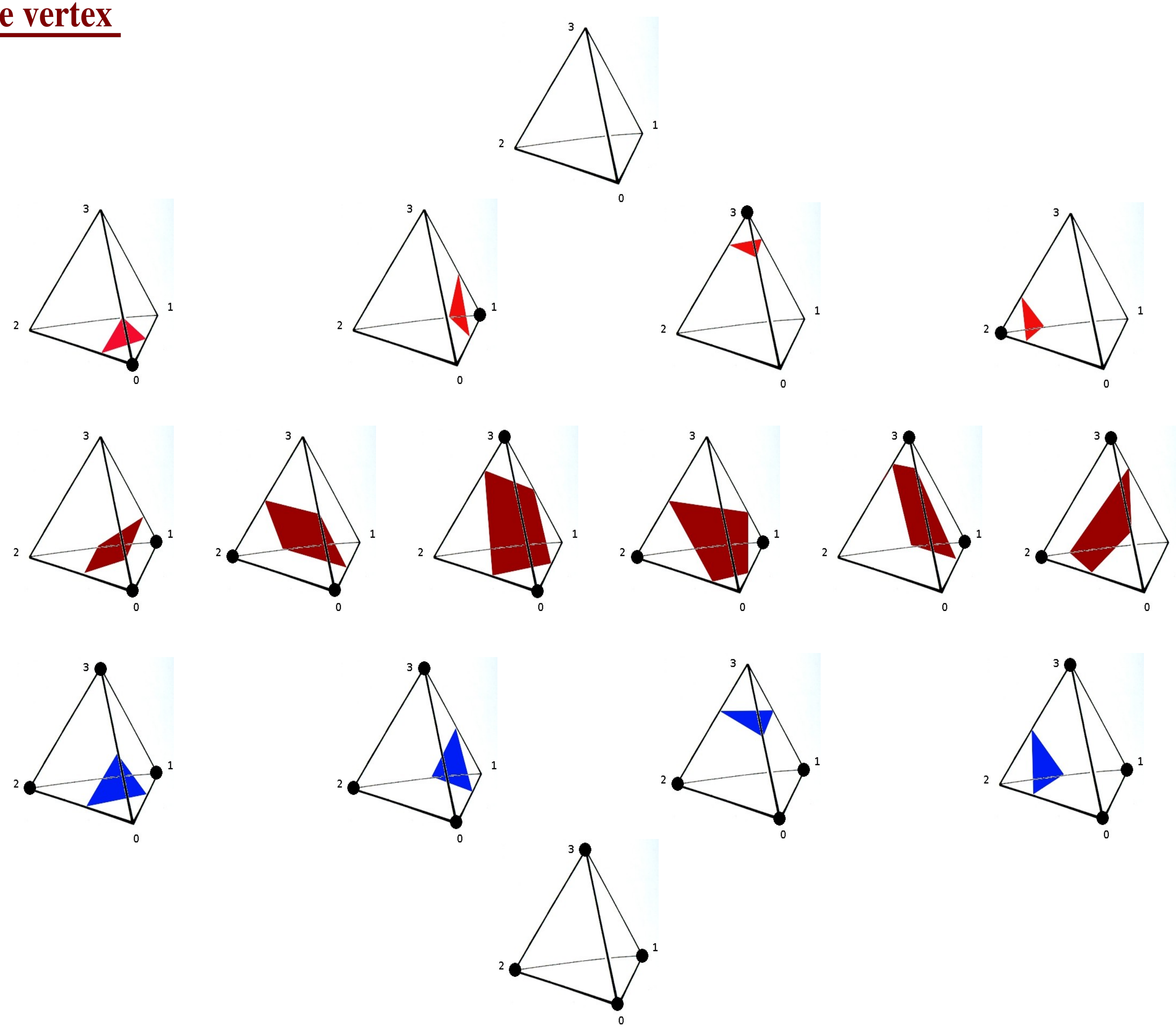
0

1

2

3

4



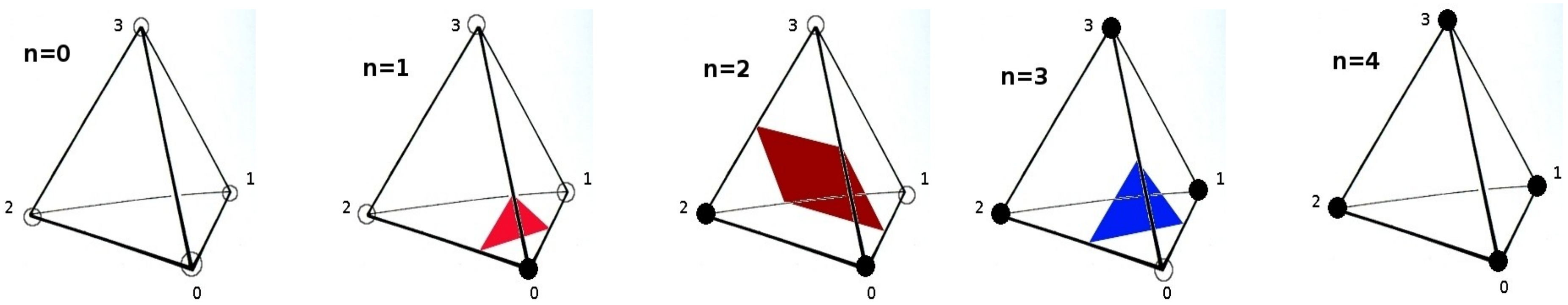
Computing Area and Volume

The area of the polygonal isodensity surface

$$A = \sum_i A_i$$

and the volume enclosed by the surface

$$V = \sum_i V_i$$



Integrated mean curvature

Integrated mean curvature of the surface

$$C = \frac{1}{2}\oint_S \left(\frac{1}{r_1} + \frac{1}{r_2}\right) dA$$

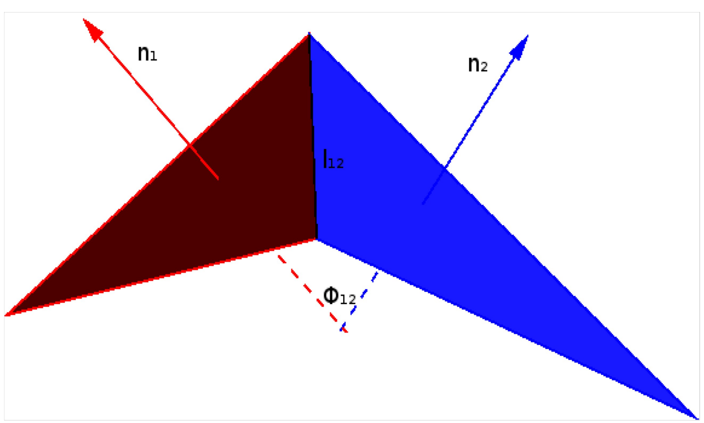
Computing Integrated Mean Curvature

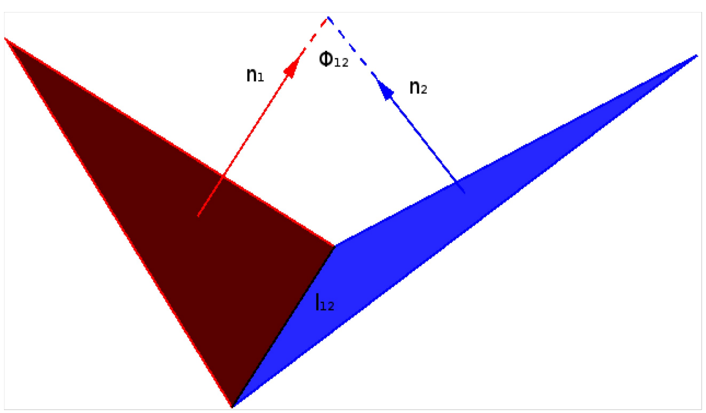
For the polygonal isodensity surface

$$C = \frac{1}{2}\sum_{i,j} l_{ij}\,\phi_{ij}\,\epsilon$$

Where, $\cos\phi_{ij} = \hat{n}_i \cdot \hat{n}_j$,

$\epsilon = 1$ if $\left(\hat{n}_i \times \hat{n}_j\right) \cdot \hat{l}_{ij} \geq 0$ and $\epsilon = -1$ otherwise

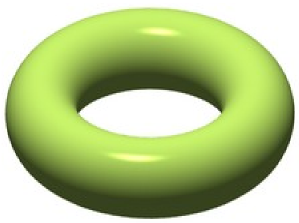




Genus

G= # holes - # isolated regions in the isodensity surface

Gott, Melott & Dickinson 1986

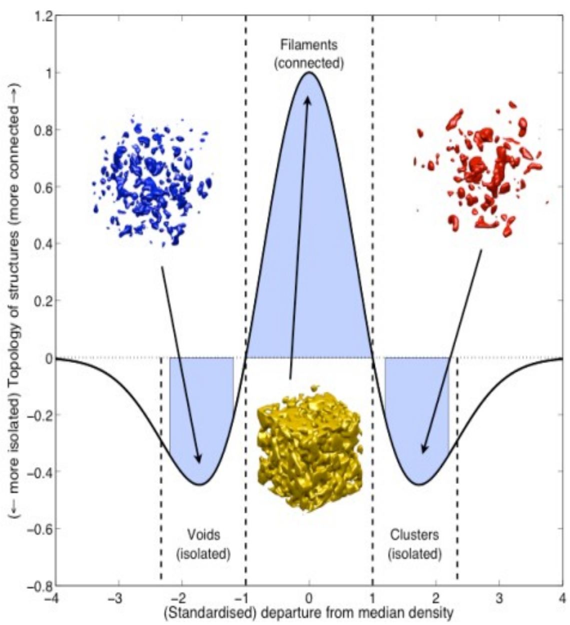




G=-1 **G=0** **G=1**

From Gauss-Bonnet Theorem

$$G = \frac{1}{4\pi} \oint_S \left(\frac{1}{r_1 r_2} \right) dA$$

Genus curve for a Gaussian random field



Genus for Gaussian Random Field (GRF)

Genus per unit volume for a GRF

$$g(\nu) = G(\nu)/V = A\,(1 - \nu^2)\exp(-\frac{\nu^2}{2})$$

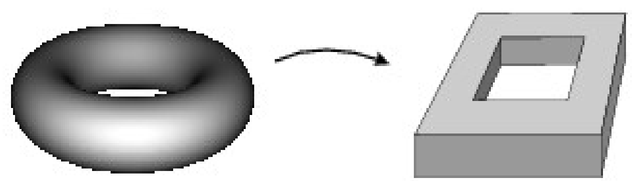
Where, $A = \frac{1}{2\pi^2}\left(\frac{\langle k^2 \rangle}{3}\right)^{\frac{3}{2}}$ and $\nu = \delta_t/\sigma$

Doroshkevich 1970

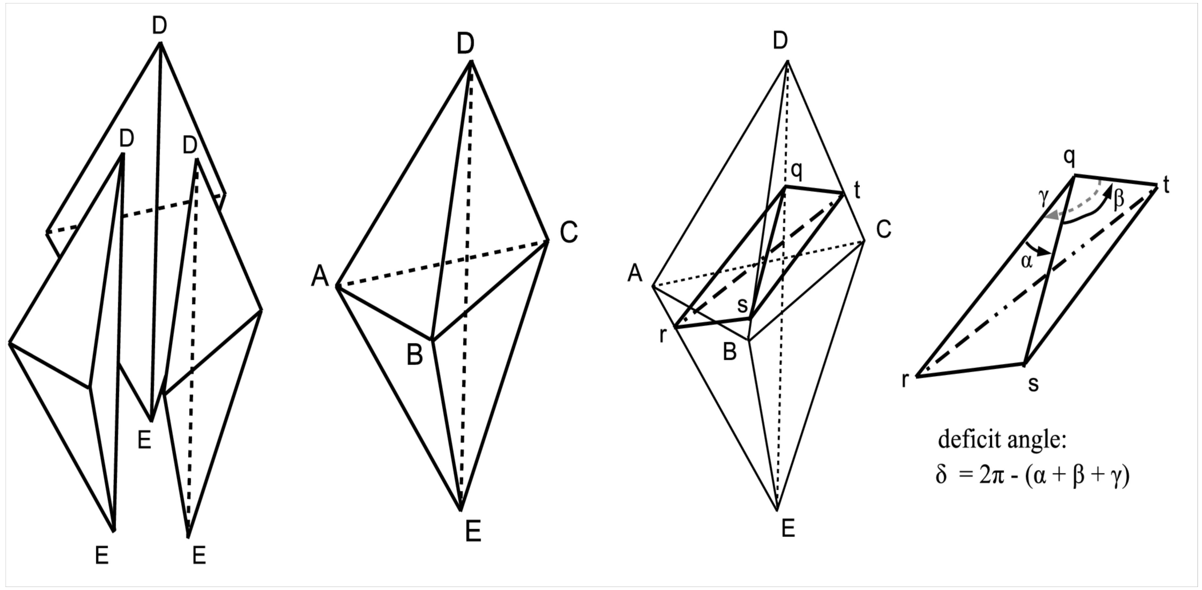
Computing Genus

Angle deficit = 360 degrees - sum of all angles between adjacent pair of edges that meet at that vertex

Genus = Sum of angle deficit at all the vertices in the surface

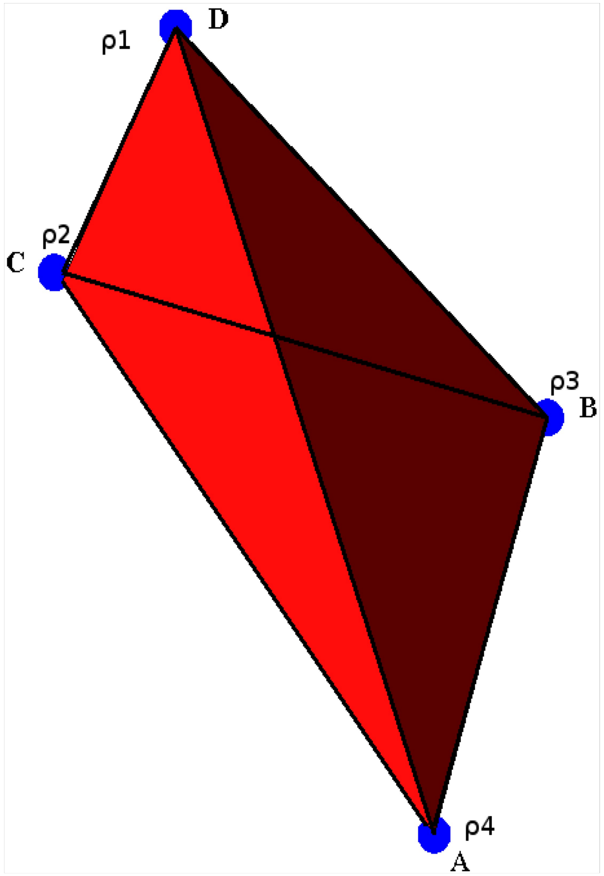


G = 0 **G = 0**



$\rho_1 > \rho_2 > \rho_3 > \rho_4$

Computing Mass

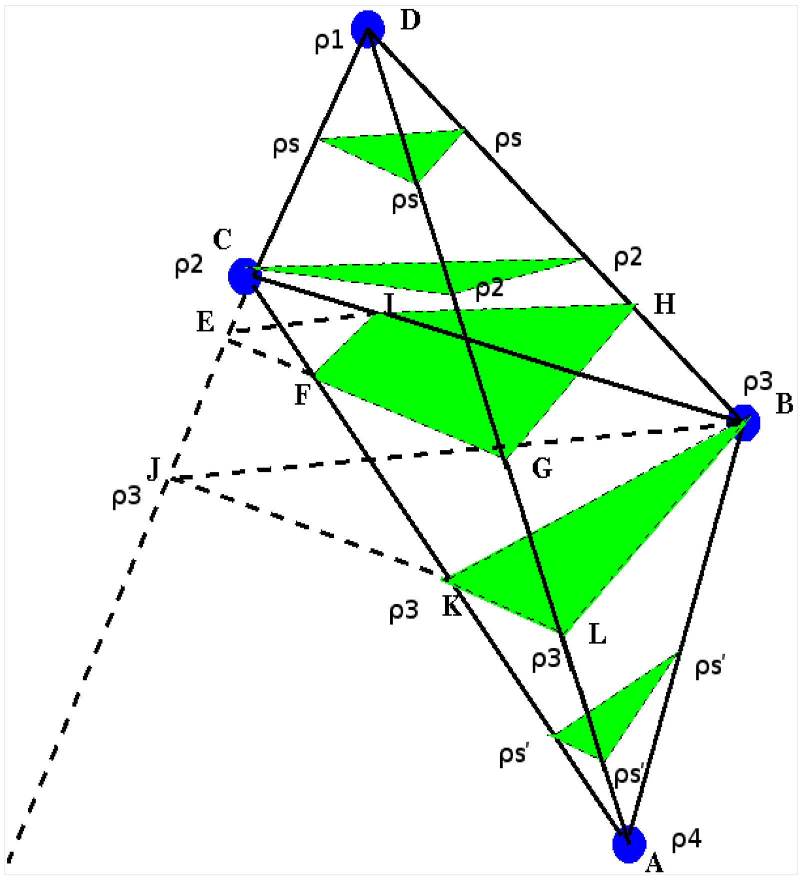


$\rho_1 > \rho_2 > \rho_3 > \rho_4$

Different situations:

- $\rho_s > \rho_1$
- $\rho_1 > \rho_s > \rho_2$
- $\rho_2 > \rho_s > \rho_3$
- $\rho_3 > \rho_s > \rho_4$
- $\rho_s < \rho_4$

Computing Mass



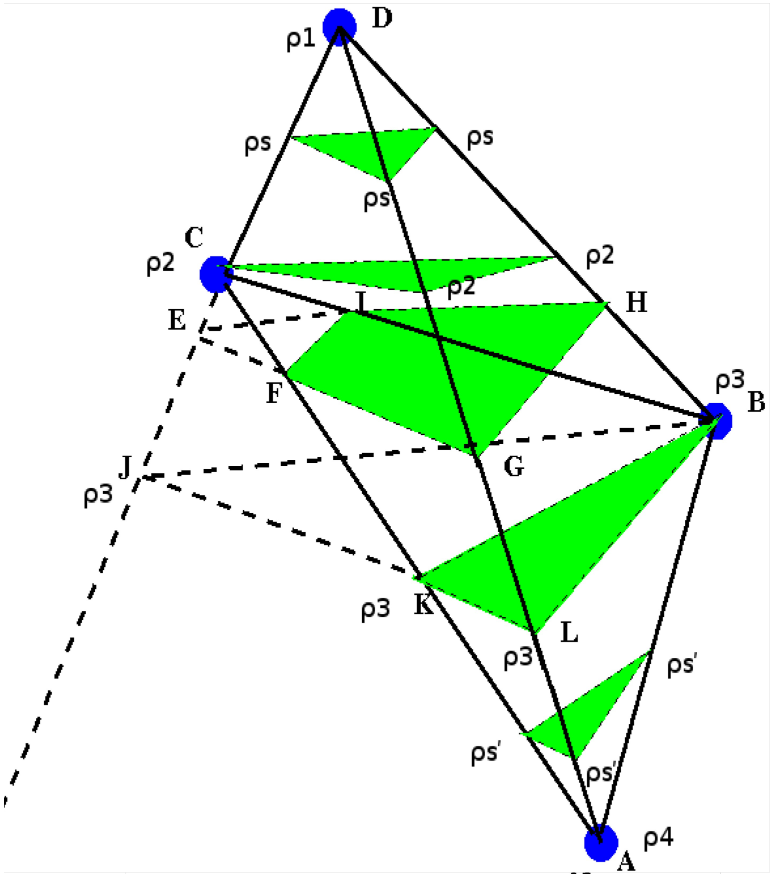
$$\mathrm{V_I}(\rho_\mathrm{s}) = 0, \mathrm{M_I}(\rho_\mathrm{s}) = 0$$

Computing Mass

$\rho_1 > \rho_2 > \rho_3 > \rho_4$

Different situations:

- $\rho_s > \rho_1$
- $\rho_1 > \rho_s > \rho_2$
- $\rho_2 > \rho_s > \rho_3$
- $\rho_3 > \rho_s > \rho_4$
- $\rho_s < \rho_4$



$$V_I(\rho_s) = V_0 \frac{(\rho_1-\rho_s)^3}{(\rho_1-\rho_2)(\rho_1-\rho_3)(\rho_1-\rho_4)}$$

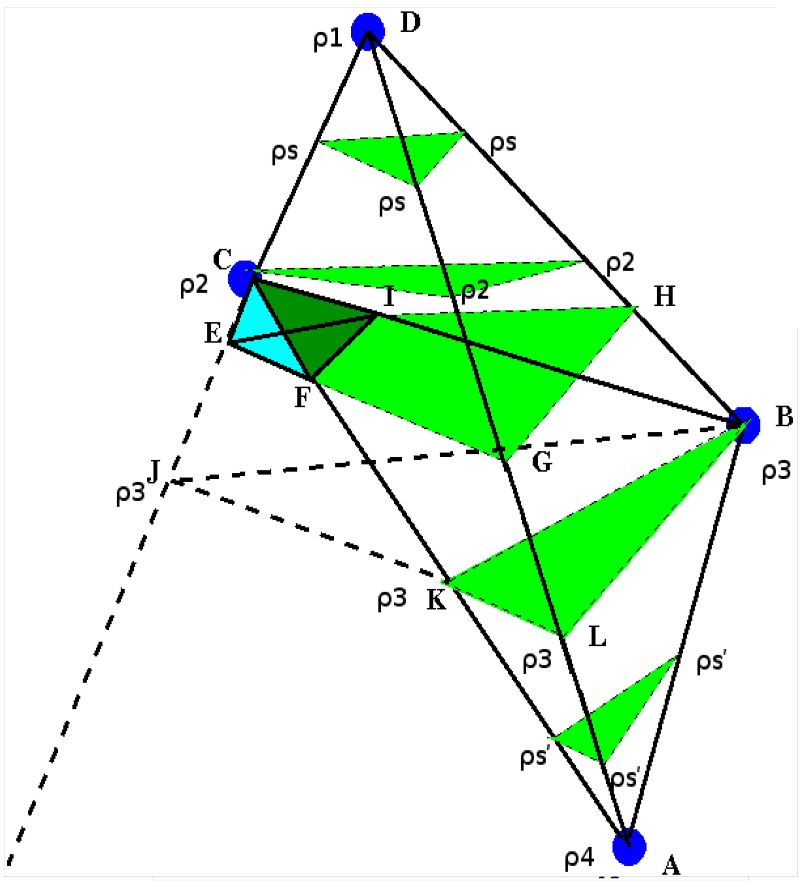
$$M_I(\rho_s) = \frac{1}{4}(\rho_1 + 3\rho_s)V_I(\rho_s)$$

Computing Mass

$\rho_1 > \rho_2 > \rho_3 > \rho_4$

Different situations:

- $\rho_s > \rho_1$
- $\rho_1 > \rho_s > \rho_2$
- $\rho_2 > \rho_s > \rho_3$
- $\rho_3 > \rho_s > \rho_4$
- $\rho_s < \rho_4$



$$V_I(\rho_3) = V_0 - V_0\frac{(\rho_3-\rho_4)^2}{(\rho_2-\rho_4)(\rho_1-\rho_4)}$$

$$= V_0\frac{(\rho_1-\rho_3)^2}{(\rho_1-\rho_2)(\rho_1-\rho_4)} - V_{ext}(\rho_3)$$

$$V_{ext}(\rho_3) = [\frac{(\rho_1-\rho_3)^2}{(\rho_1-\rho_2)(\rho_1-\rho_4)} + \frac{(\rho_3-\rho_4)^2}{(\rho_2-\rho_4)(\rho_1-\rho_4)} - 1]V_0$$

$$V_{ext}(\rho_s) = \frac{(\rho_2-\rho_s)^3}{(\rho_2-\rho_3)^3}V_{ext}(\rho_3)$$

$$V_I(\rho_s) = V_0\frac{(\rho_1-\rho_s)^3}{(\rho_1-\rho_2)(\rho_1-\rho_3)(\rho_1-\rho_4)} - V_{ext}(\rho_s)$$

$$M_I(\rho_s) = \frac{1}{4}(\rho_1+3\rho_s)(V_I(\rho_s)+V_{ext}(\rho_s))$$

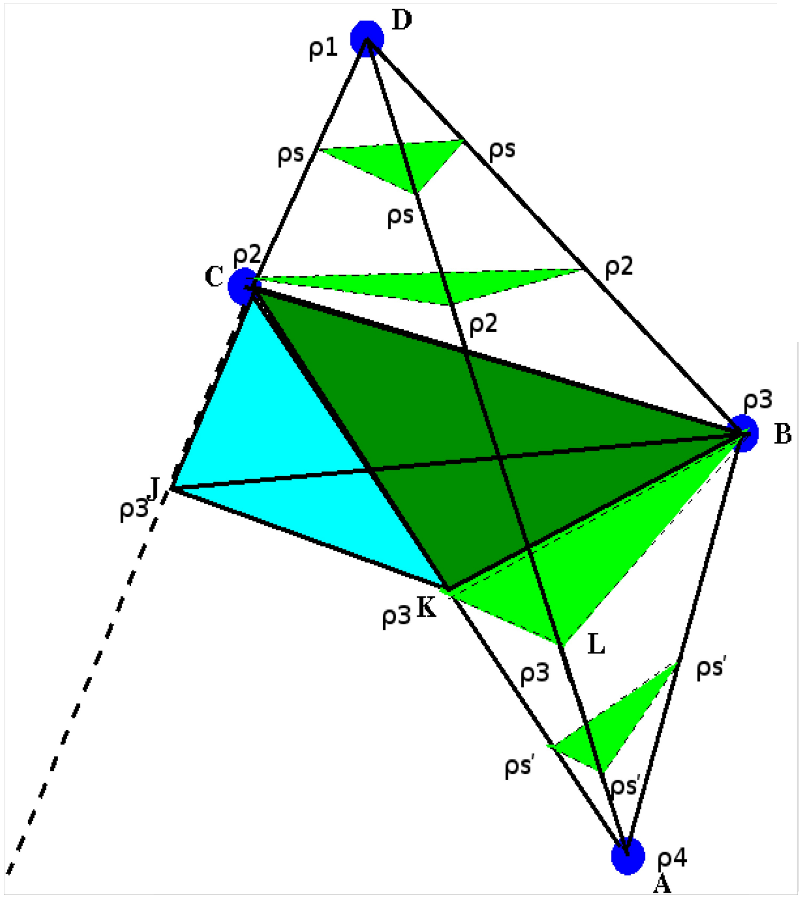
$$-\frac{1}{4}(\rho_2+3\rho_s)V_{ext}(\rho_s))$$

Computing Mass

$\rho_1 > \rho_2 > \rho_3 > \rho_4$

Different situations:

- $\rho_s > \rho_1$
- $\rho_1 > \rho_s > \rho_2$
- $\rho_2 > \rho_s > \rho_3$
- $\rho_3 > \rho_s > \rho_4$
- $\rho_s < \rho_4$



$$V_I(\rho_3) = V_0 - V_0\frac{(\rho_3-\rho_4)^2}{(\rho_2-\rho_4)(\rho_1-\rho_4)}$$

$$= V_0\frac{(\rho_1-\rho_3)^2}{(\rho_1-\rho_2)(\rho_1-\rho_4)} - V_{ext}(\rho_3)$$

$$V_{ext}(\rho_3) = [\frac{(\rho_1-\rho_3)^2}{(\rho_1-\rho_2)(\rho_1-\rho_4)} + \frac{(\rho_3-\rho_4)^2}{(\rho_2-\rho_4)(\rho_1-\rho_4)} - 1]V_0$$

$$V_{ext}(\rho_s) = \frac{(\rho_2-\rho_s)^3}{(\rho_2-\rho_3)^3}V_{ext}(\rho_3)$$

$$V_I(\rho_s) = V_0\frac{(\rho_1-\rho_s)^3}{(\rho_1-\rho_2)(\rho_1-\rho_3)(\rho_1-\rho_4)} - V_{ext}(\rho_s)$$

$$M_I(\rho_s) = \frac{1}{4}(\rho_1 + 3\rho_s)(V_I(\rho_s) + V_{ext}(\rho_s))$$

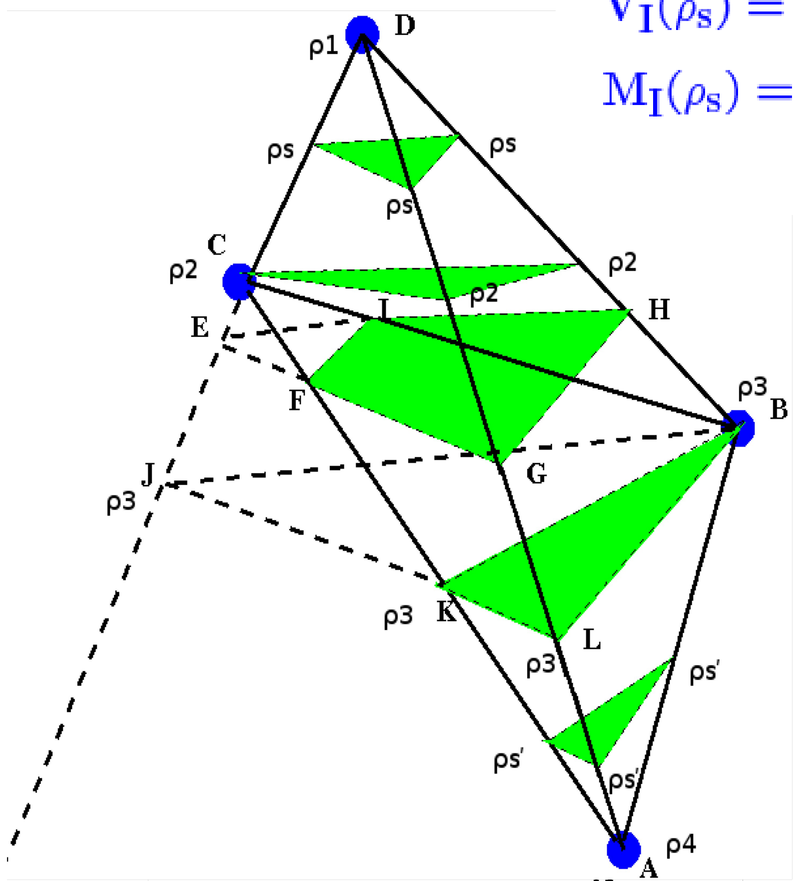
$$-\frac{1}{4}(\rho_2 + 3\rho_s)V_{ext}(\rho_s))$$

Computing Mass

$\rho_1 > \rho_2 > \rho_3 > \rho_4$

Different situations:

- $\rho_s > \rho_1$
- $\rho_1 > \rho_s > \rho_2$
- $\rho_2 > \rho_s > \rho_3$
- $\rho_3 > \rho_s > \rho_4$
- $\rho_s < \rho_4$



$$V_I(\rho_s) = V_0 - V_0\frac{(\rho_s-\rho_4)^3}{(\rho_1-\rho_4)(\rho_2-\rho_4)(\rho_3-\rho_4)}$$

$$M_I(\rho_s) = M_0 - \frac{1}{4}(\rho_4 + 3\rho_s)(V_0 - V_I(\rho_s))$$

Computing Mass

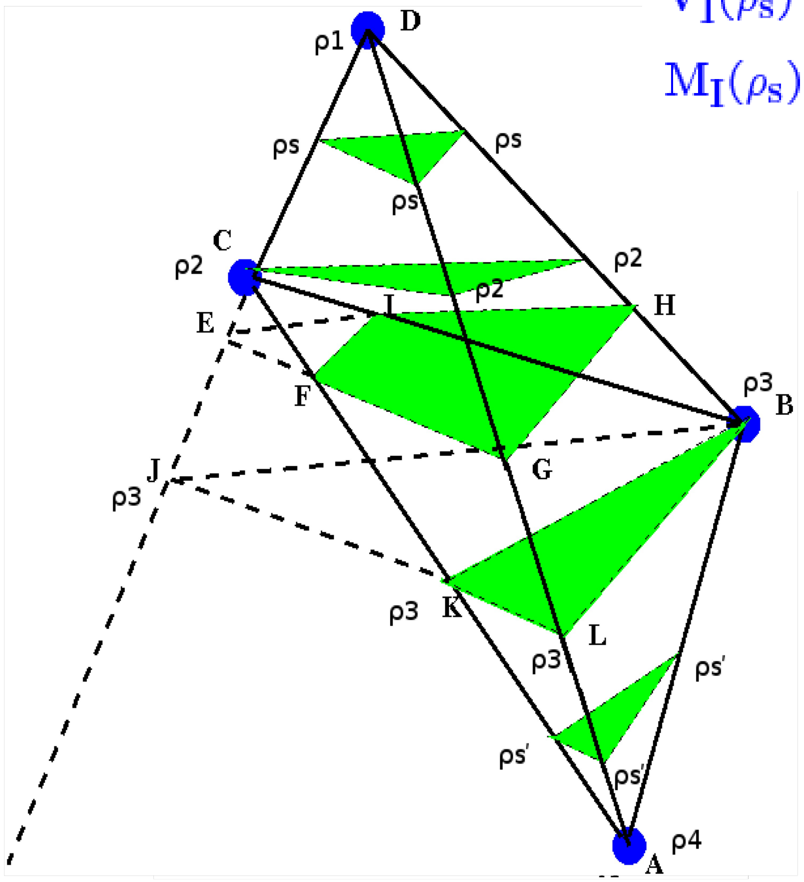
$\rho_1 > \rho_2 > \rho_3 > \rho_4$

Different situations:

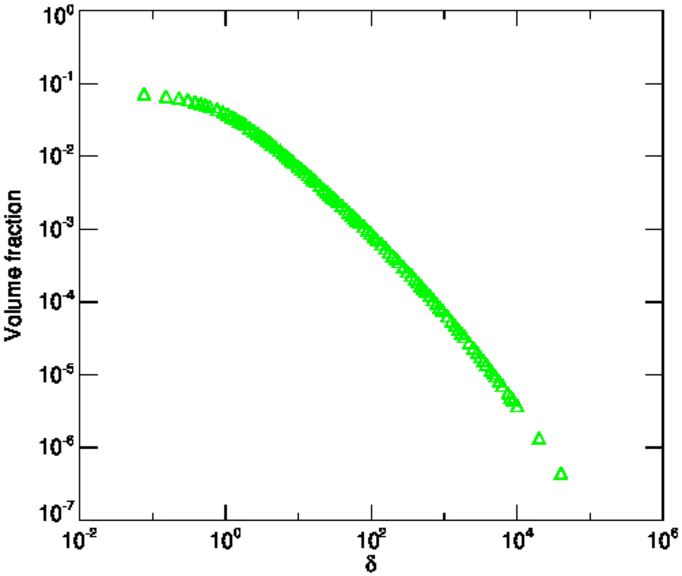
- $\rho_s > \rho_1$
- $\rho_1 > \rho_s > \rho_2$
- $\rho_2 > \rho_s > \rho_3$
- $\rho_3 > \rho_s > \rho_4$
- $\rho_s < \rho_4$

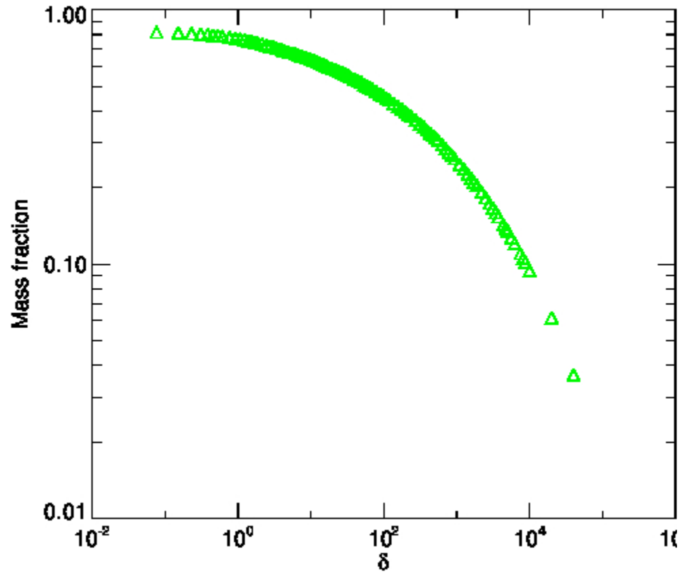
$$V_I(\rho_s) = V_0$$

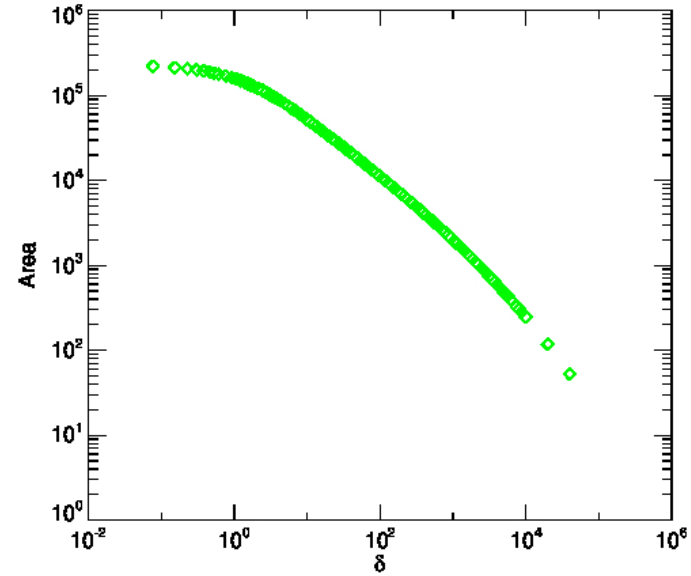
$$M_I(\rho_s) = M_0 = \frac{1}{4}(\rho_1 + \rho_2 + \rho_3 + \rho_4)V_0$$

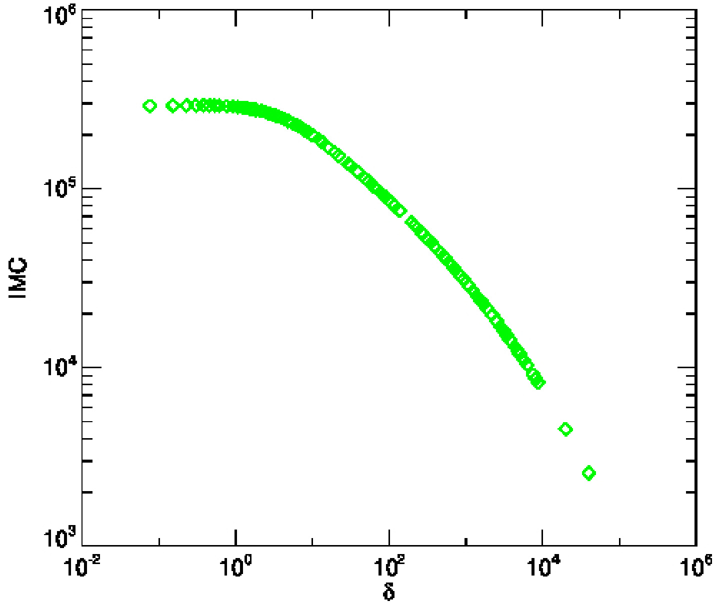


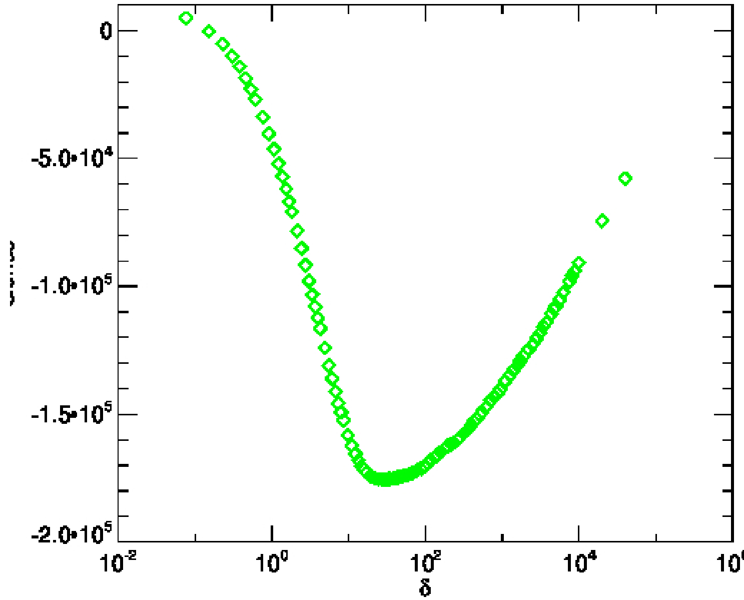
Results for the whole milli-Millennium simulation



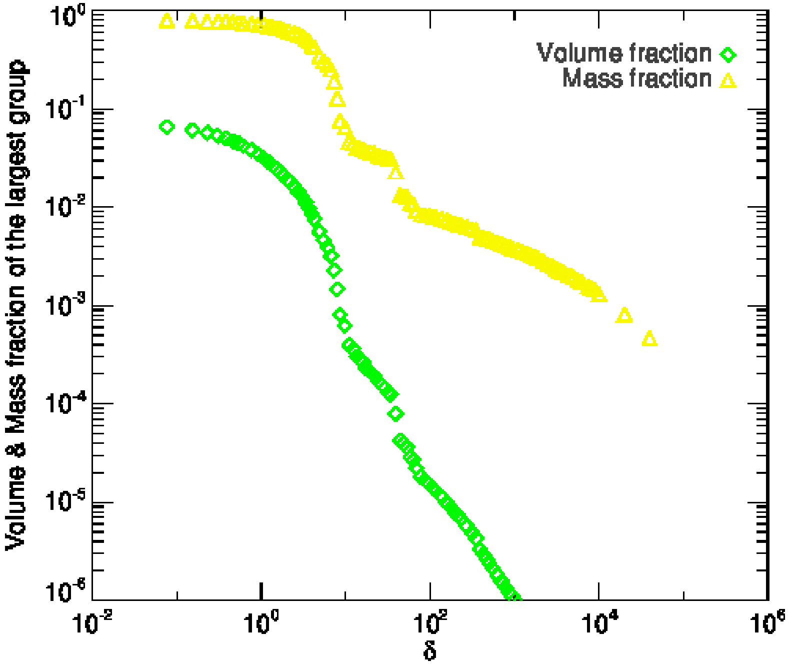


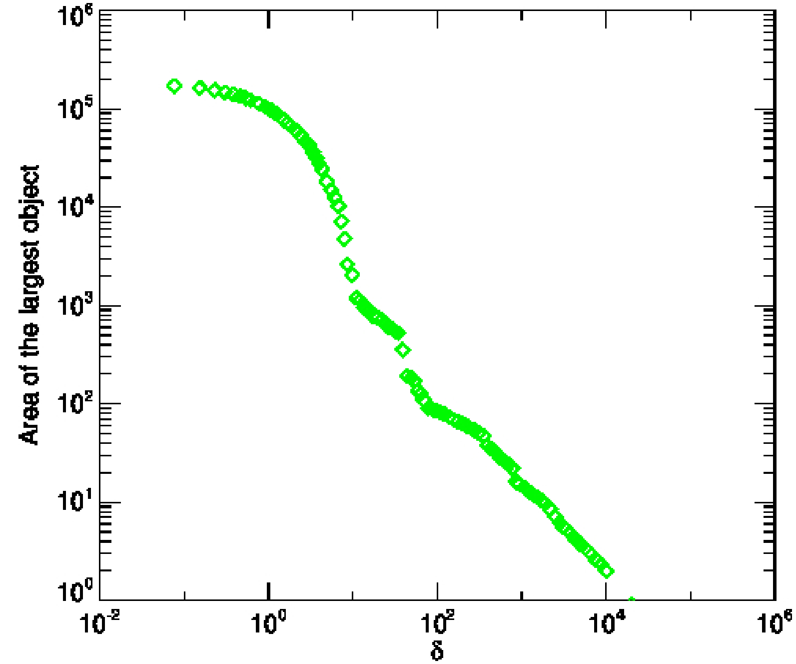


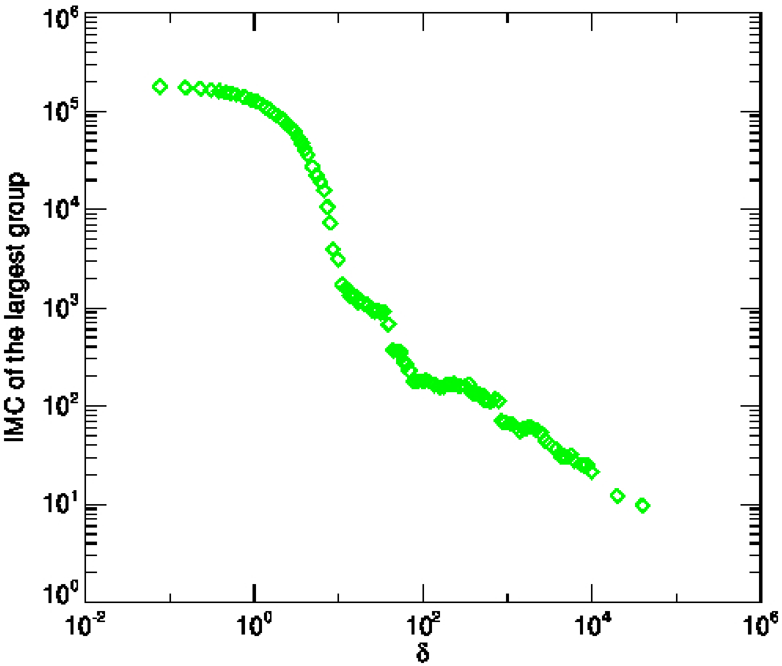


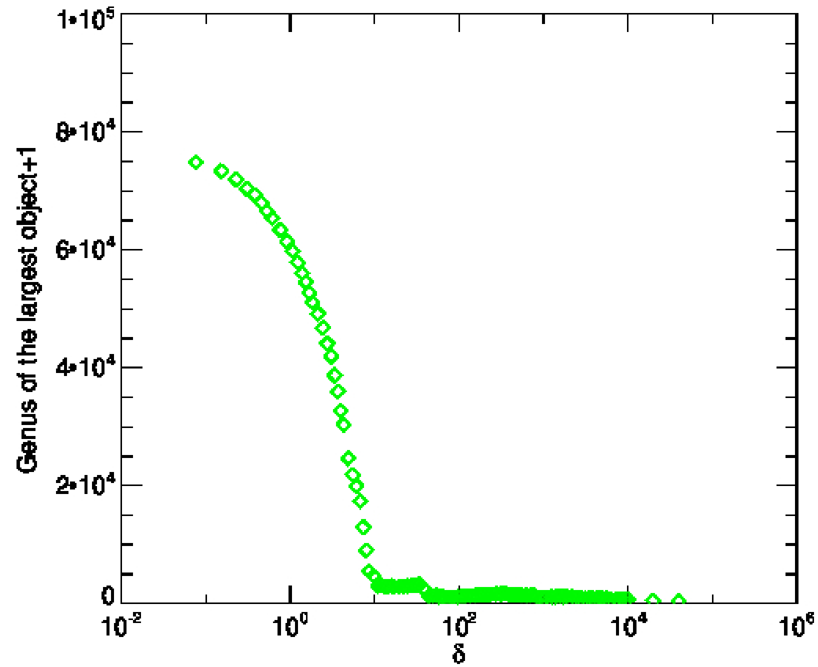


Results for the largest object in milli-Millennium simulation

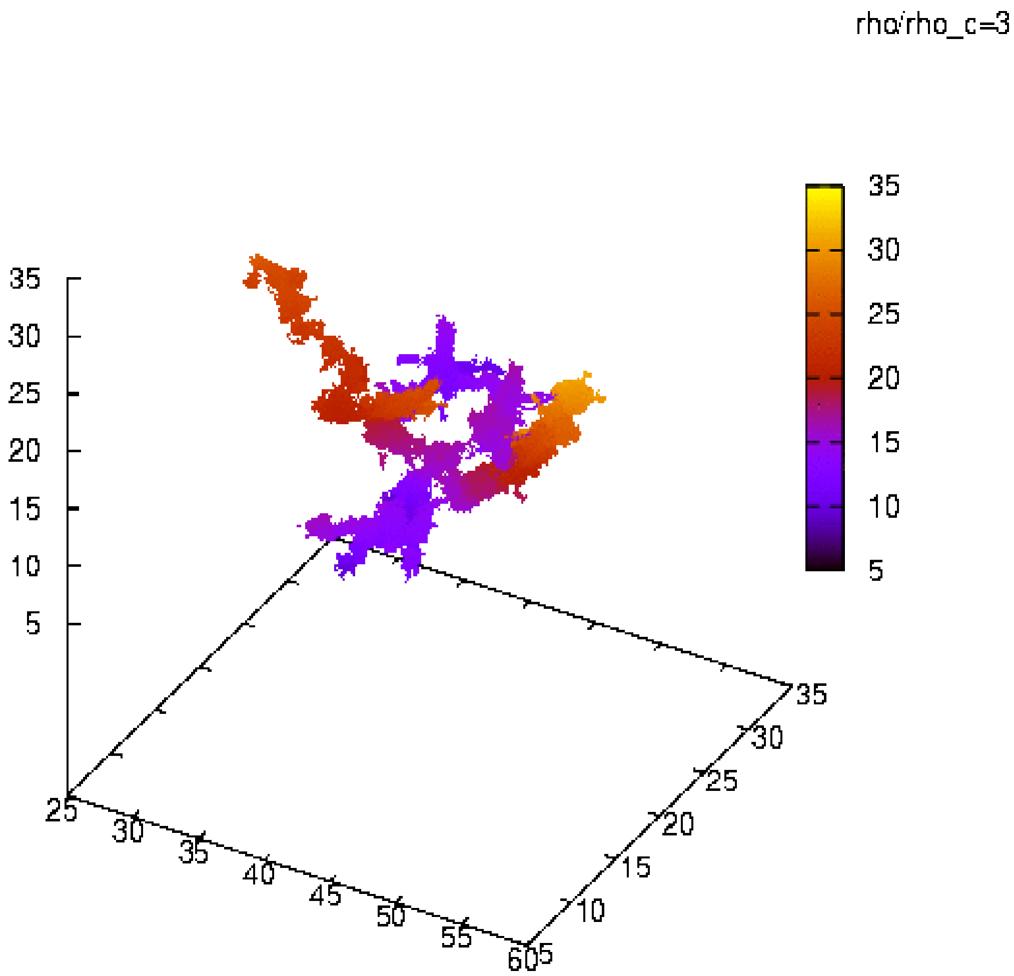


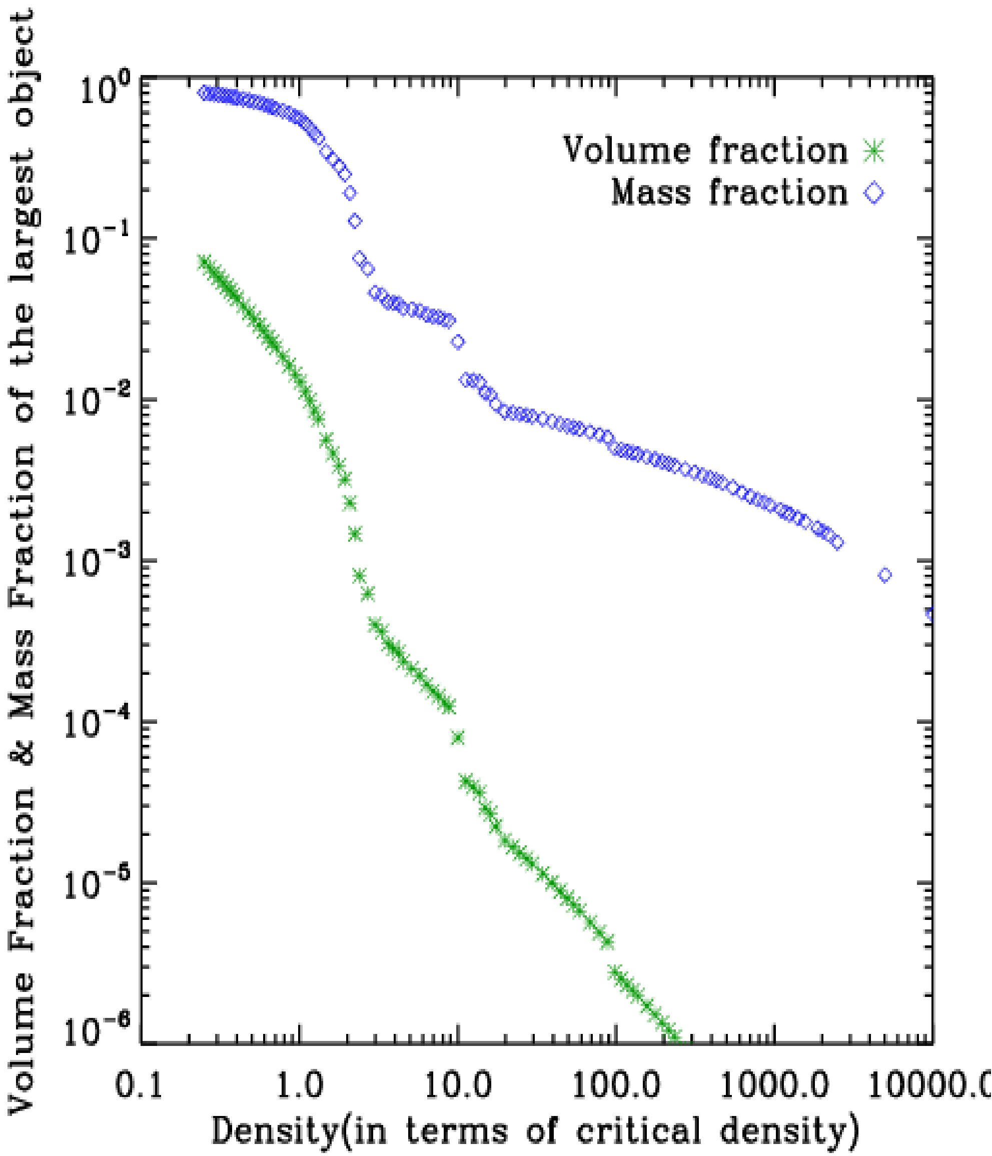




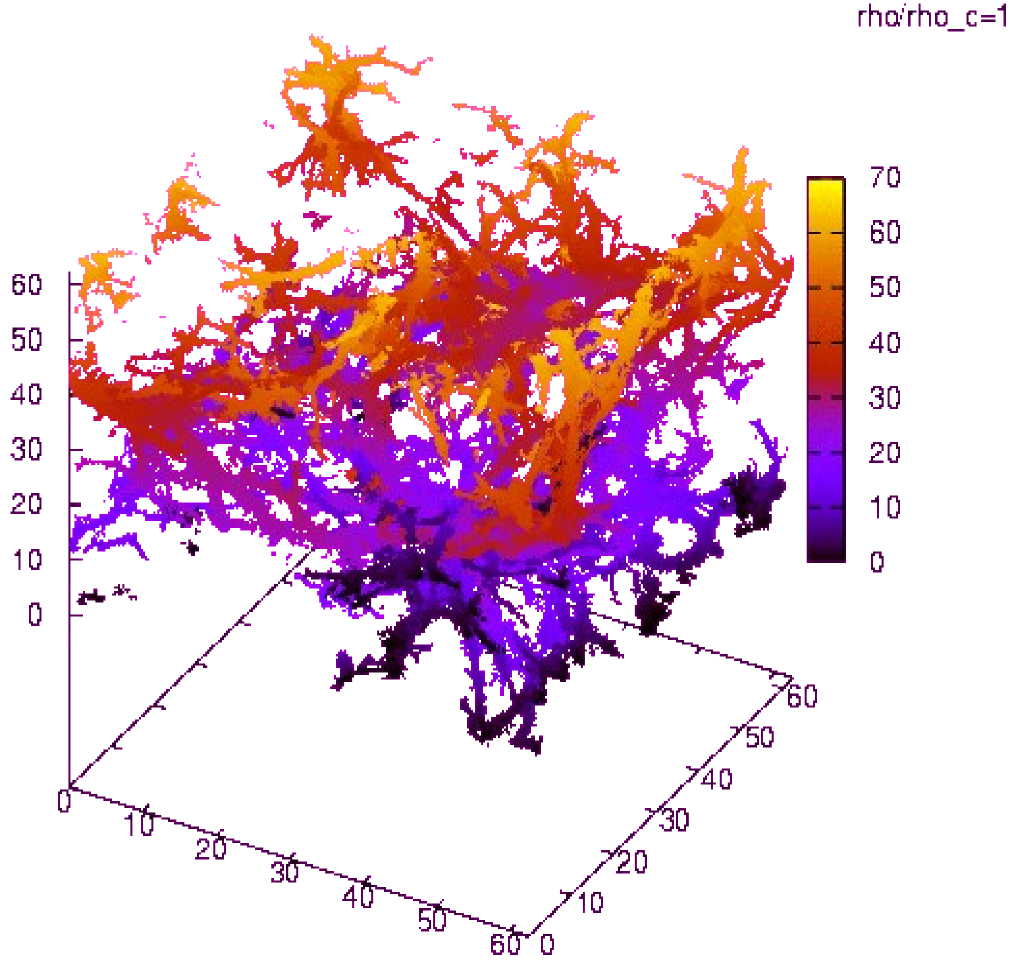


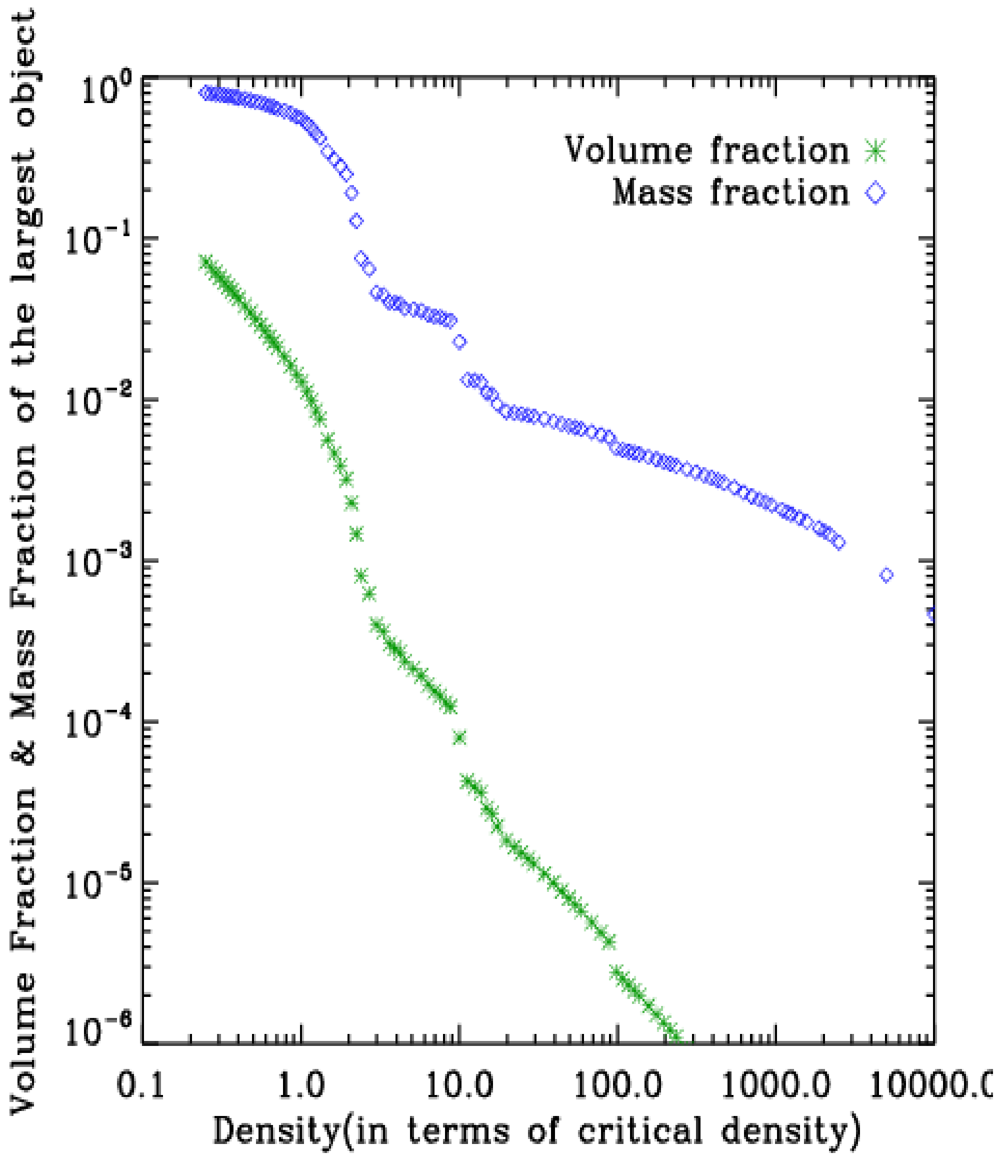
Volume fraction and mass fraction of the largest object





Volume fraction and mass fraction of the largest object

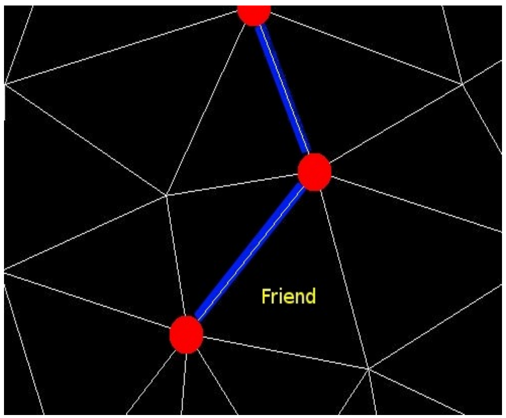


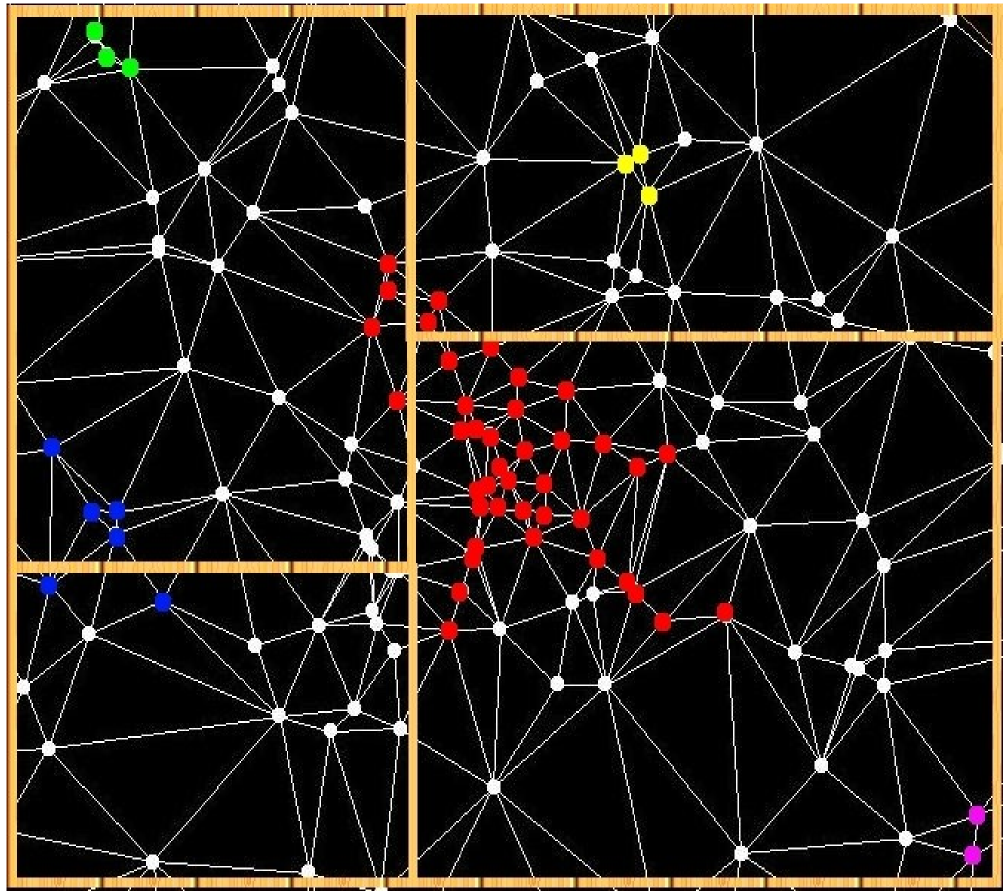


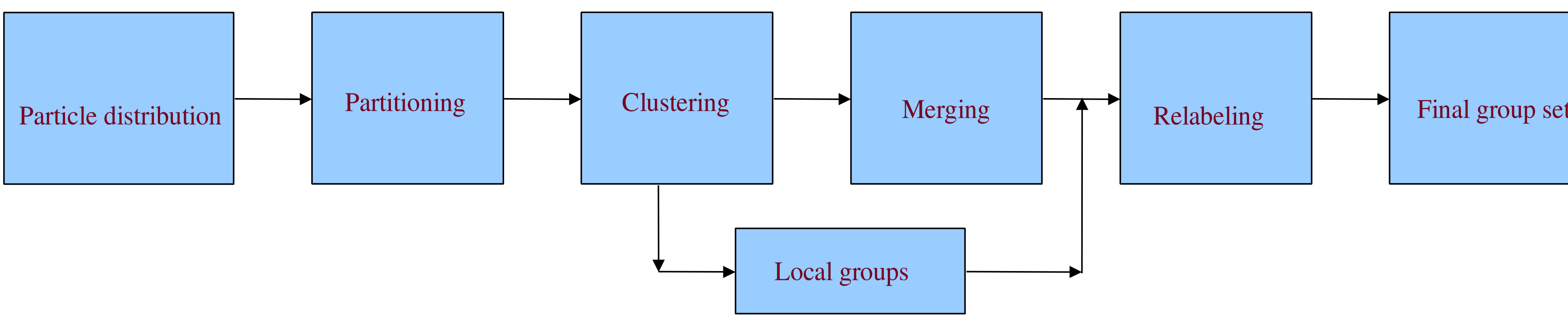
At 3 times the critical density: 0.04 % of Volume and 5% of Mass

At critical density: 1% of Volume and 60% of Mass

A parallel FOF group finder in Tessellation







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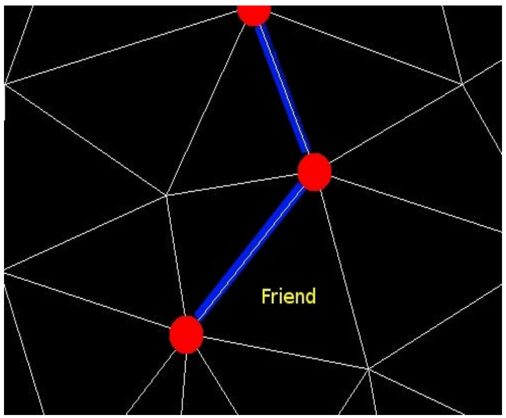
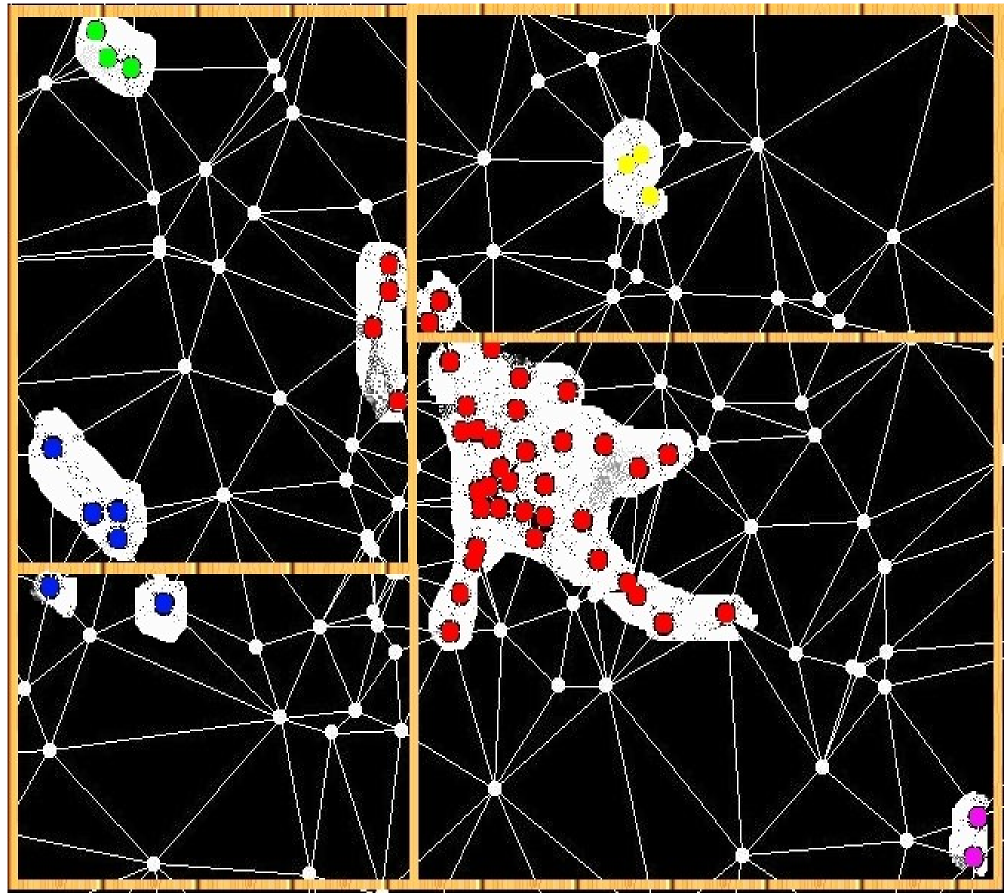
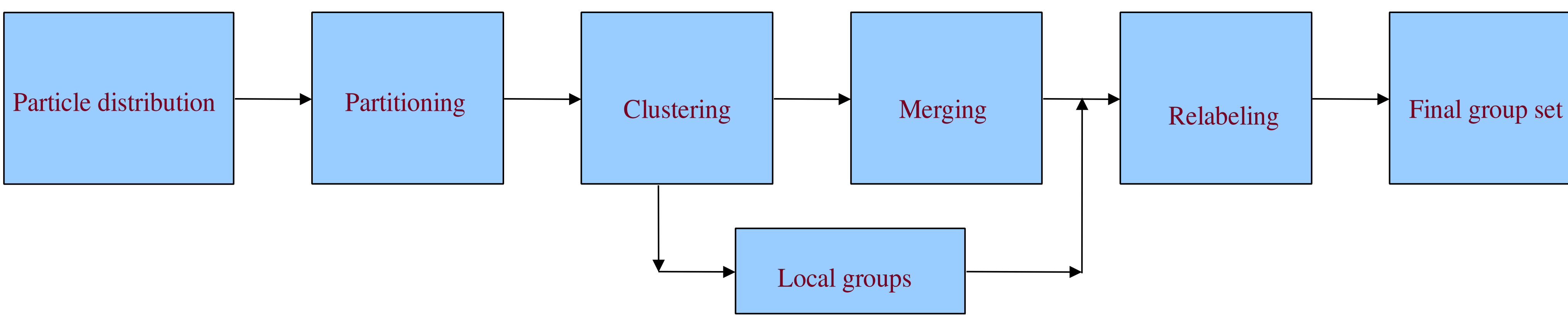
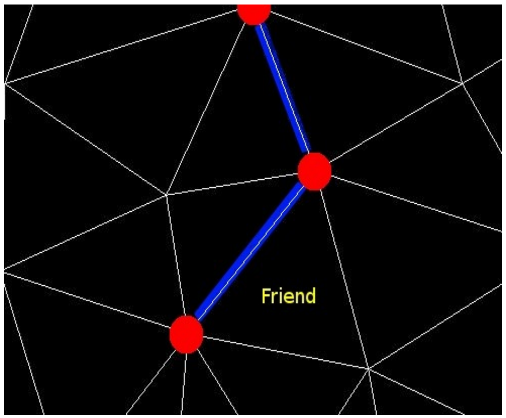
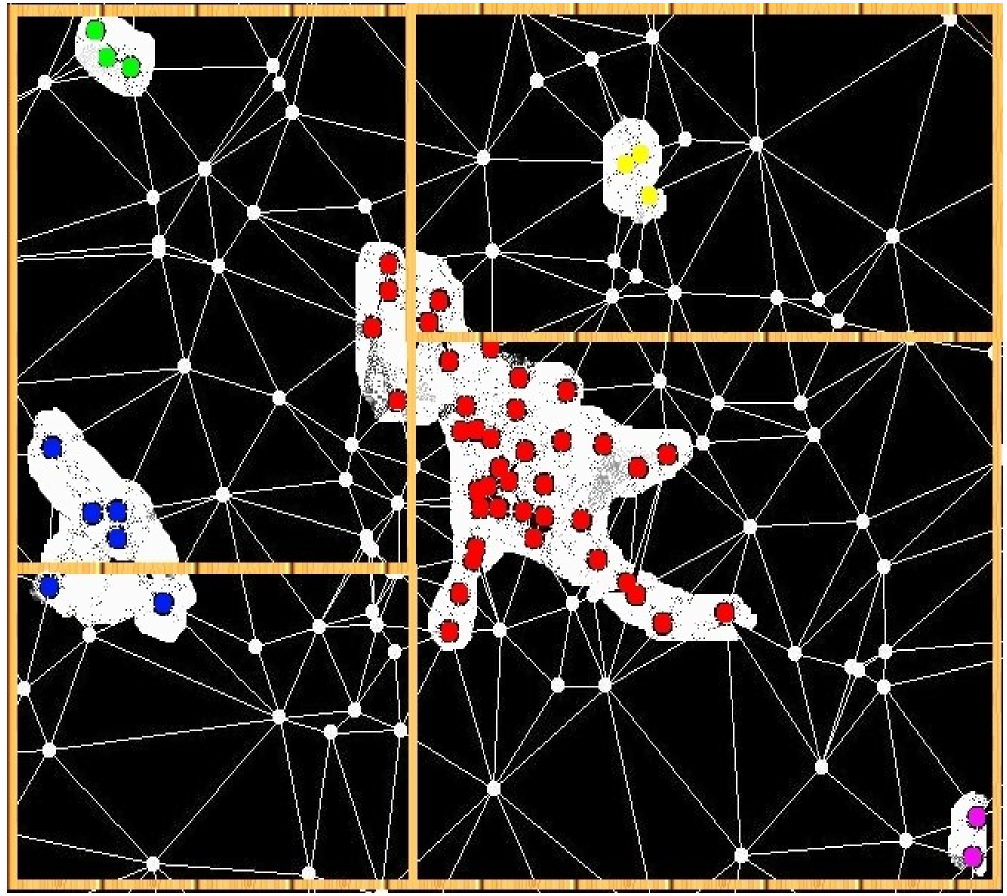


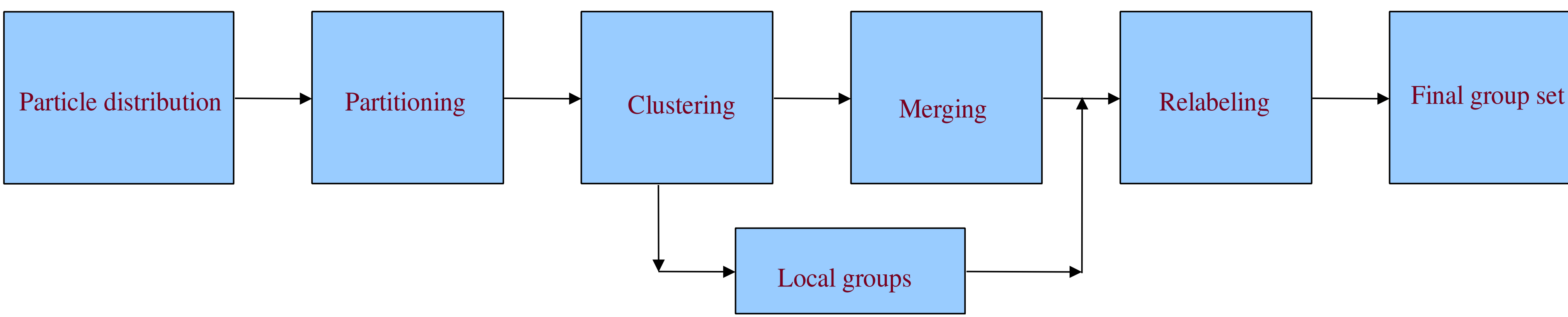


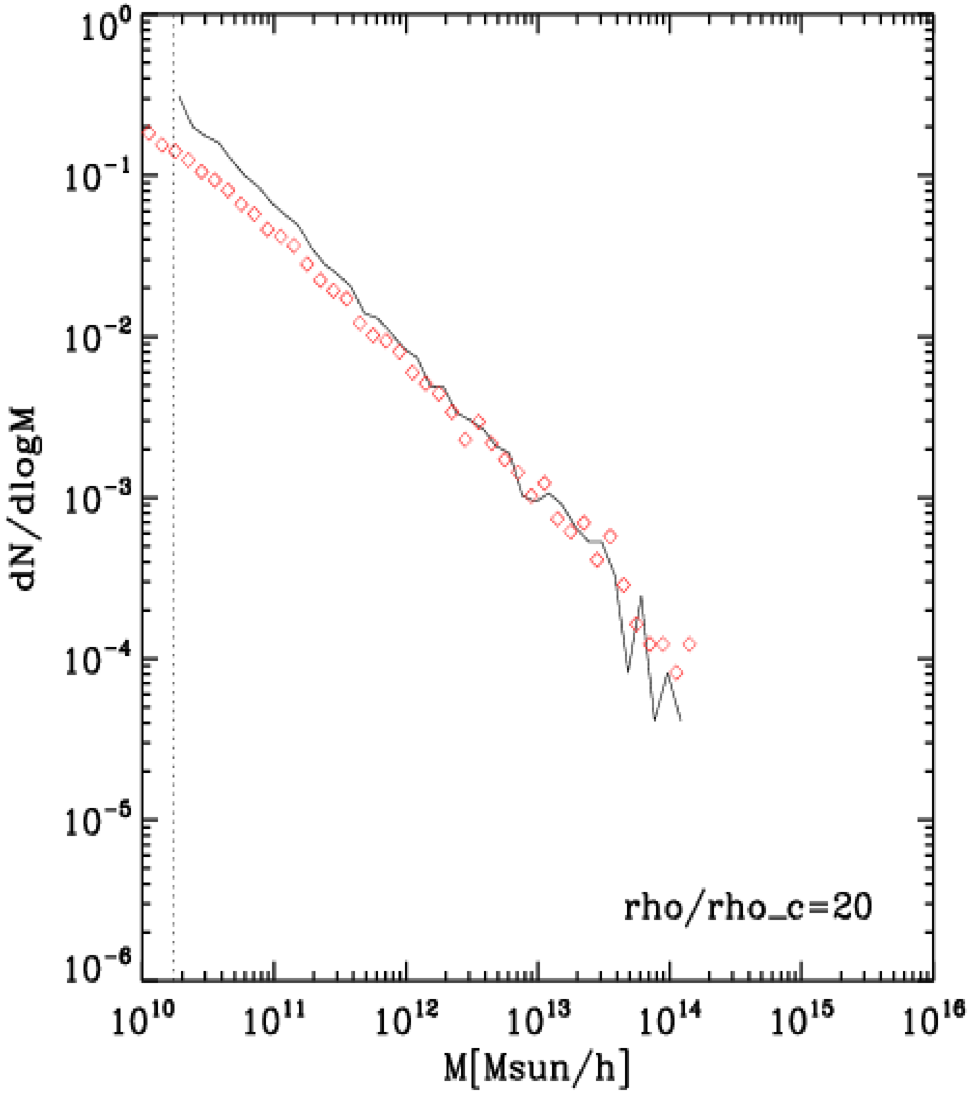


A parallel FOF group finder in Tessellation








dN/dlogM
M[Msun/h]
rho/rho_c=20
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
10^{16}

THANKS FOR YOUR KIND ATTENTION