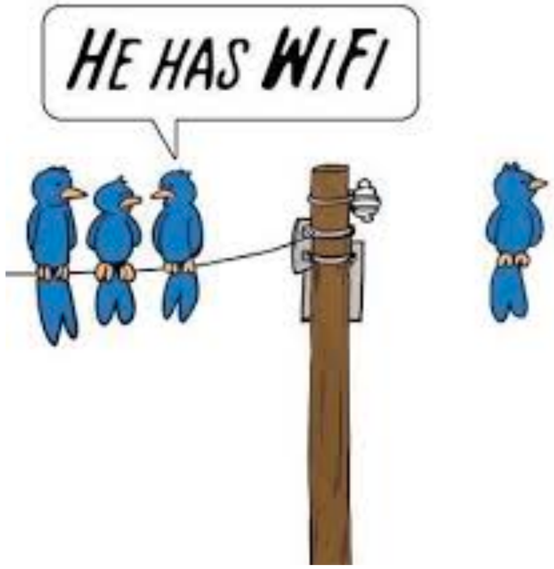
Graph Matchings and Wireless Communication

Rahul Vaze

Graph Matchings and Wireless Communication



Rahul Vaze

In this talk

In this talk

105 pictures 3 equations

In this talk

105 pictures 3 equations

color blind friendly

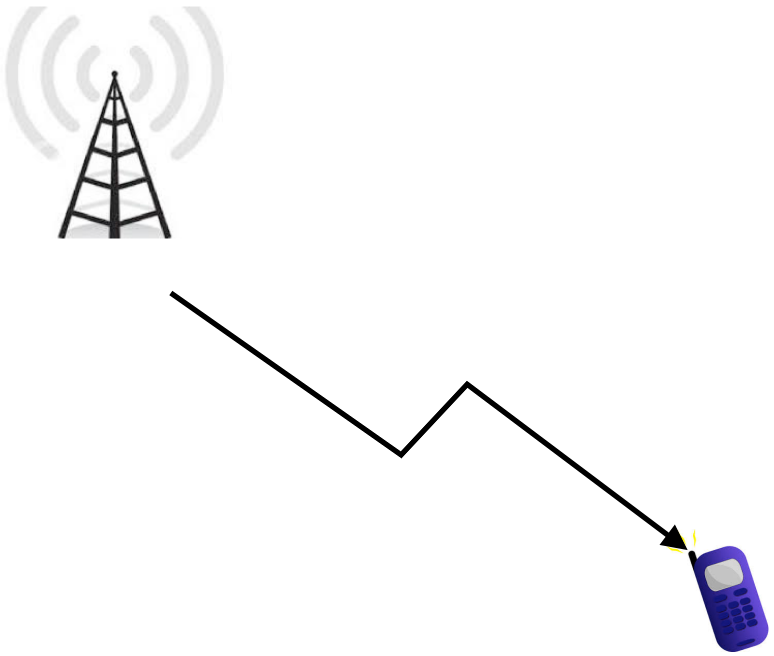
In this talk

105 pictures 3 equations

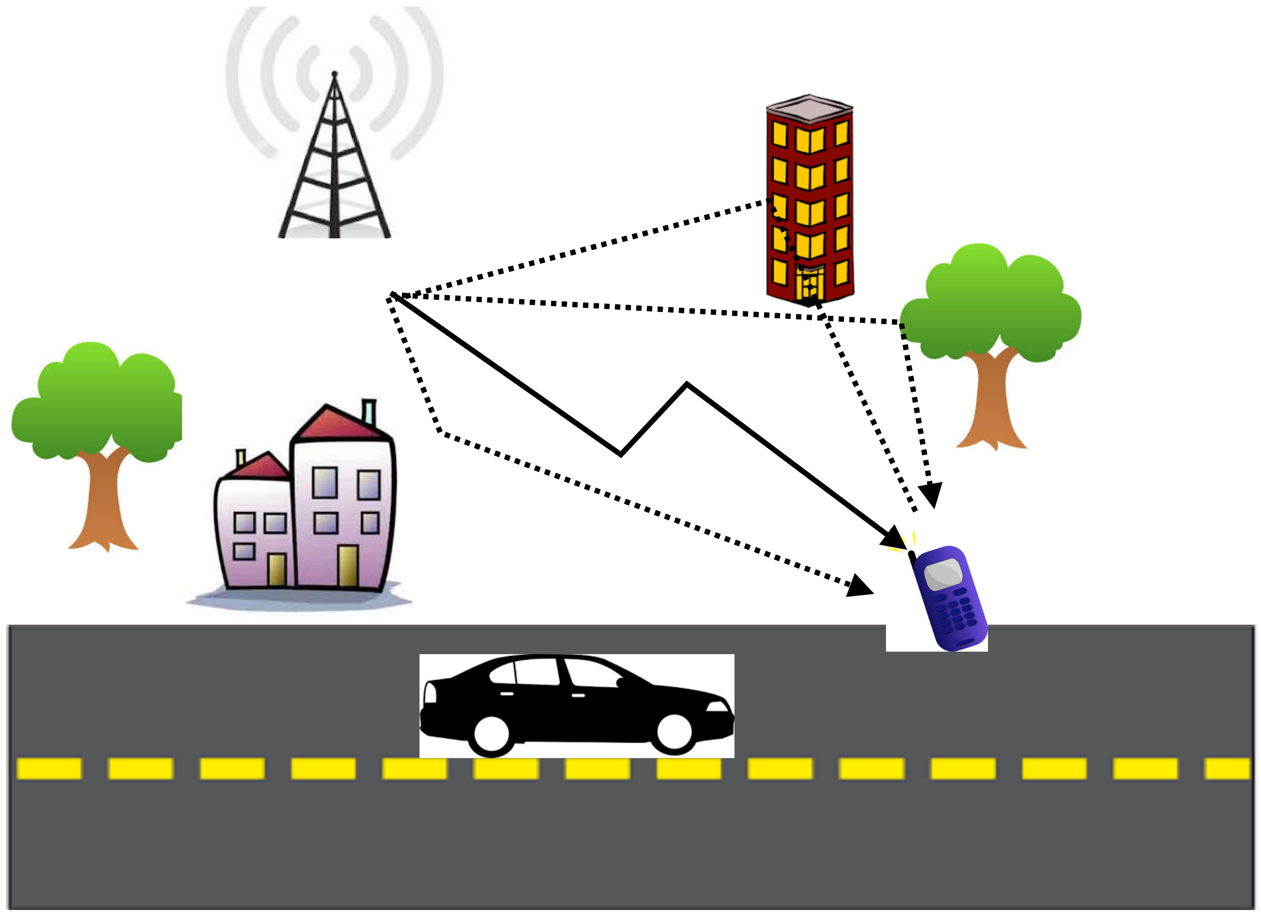
color blind friendly

NOT in this talk

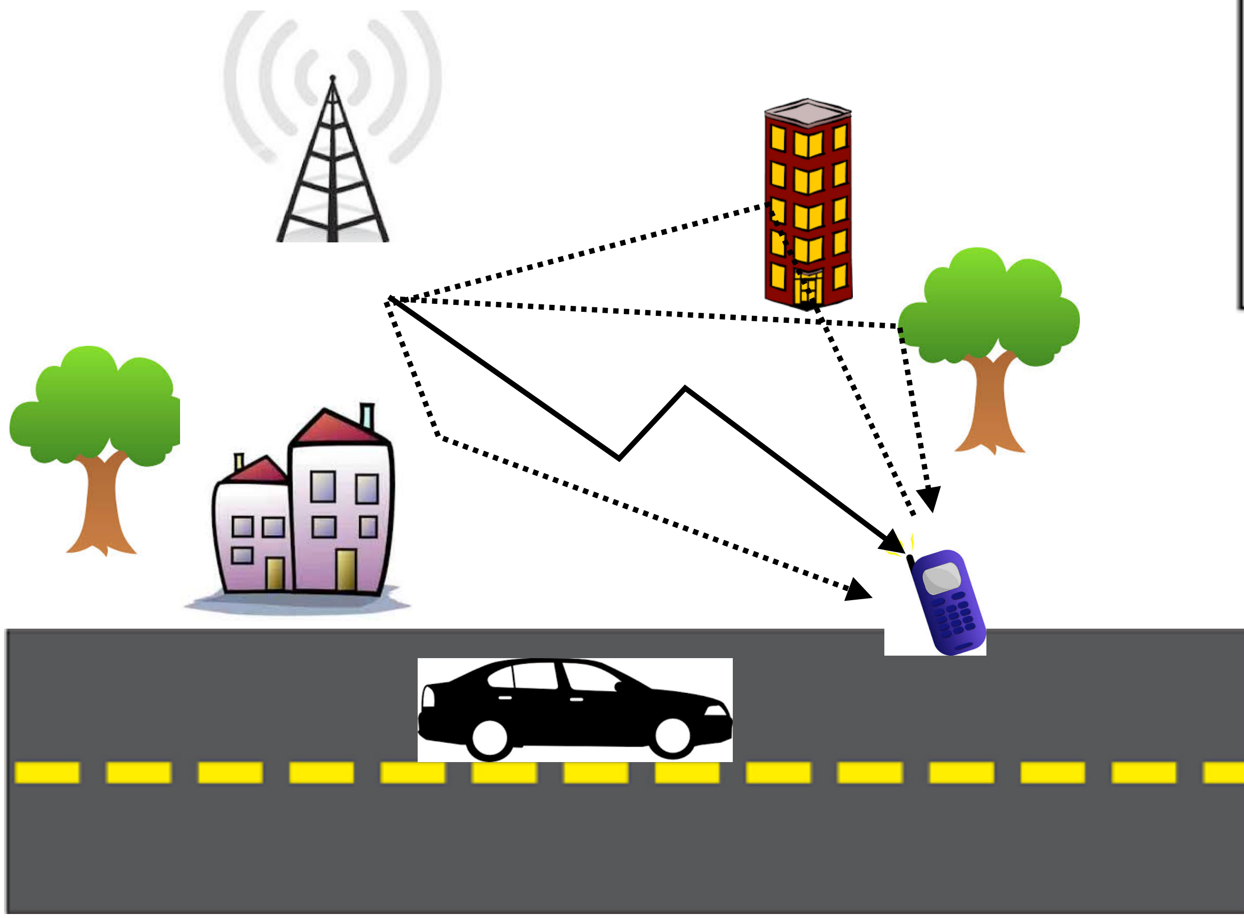
Wireless Channel

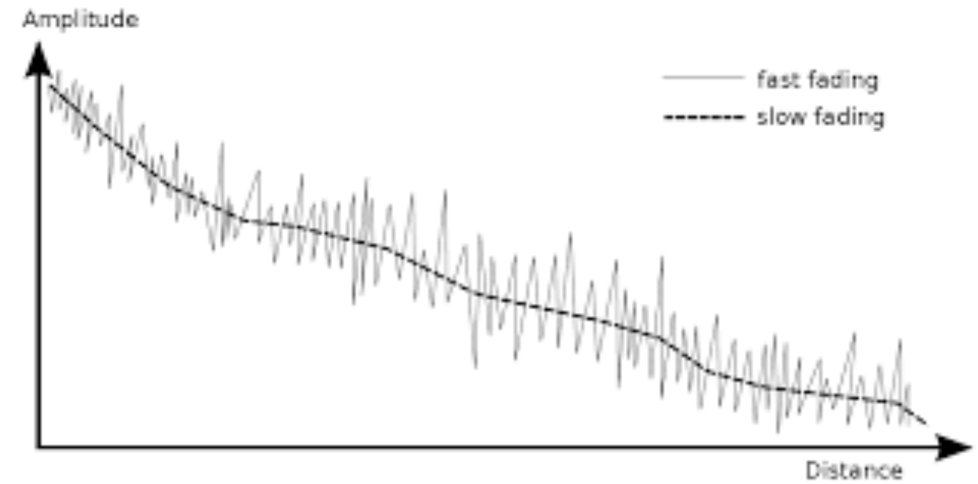


Wireless Channel

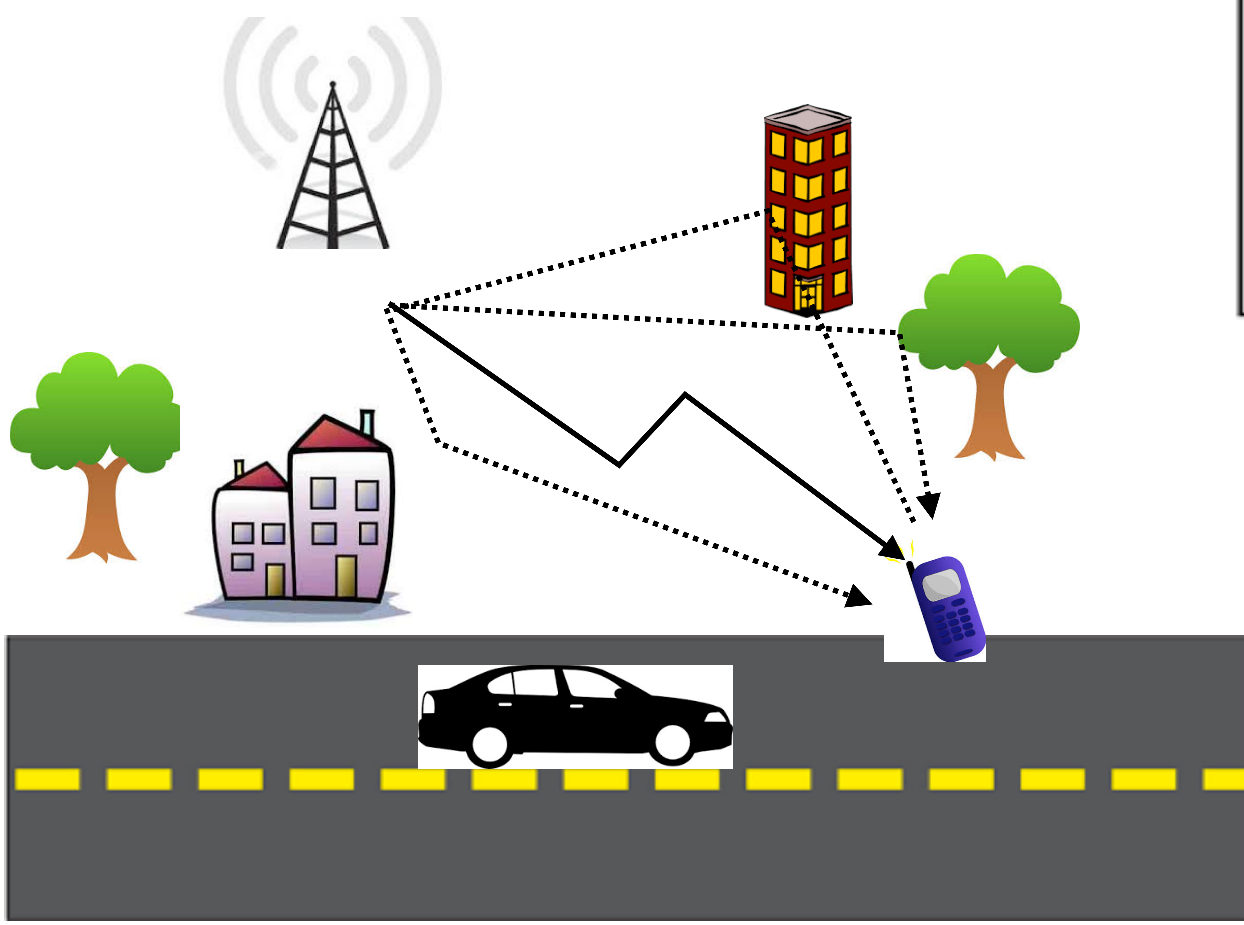


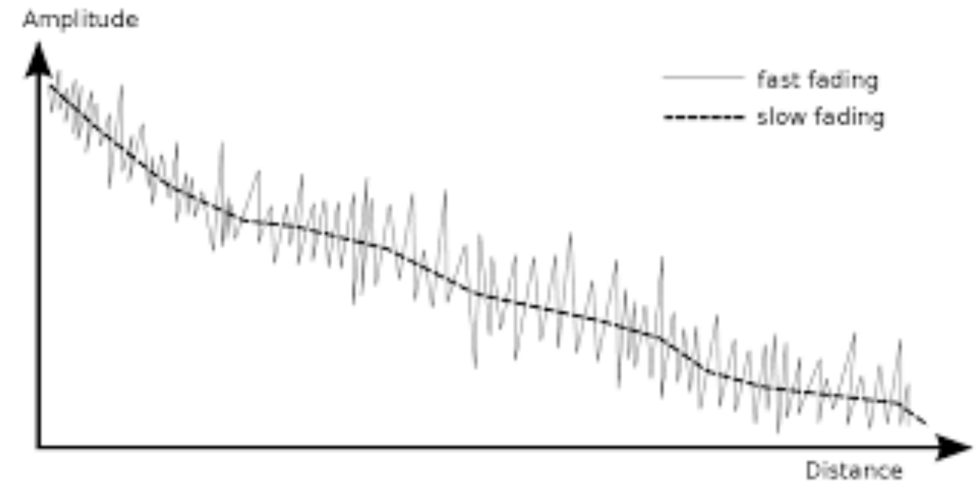
Wireless Channel





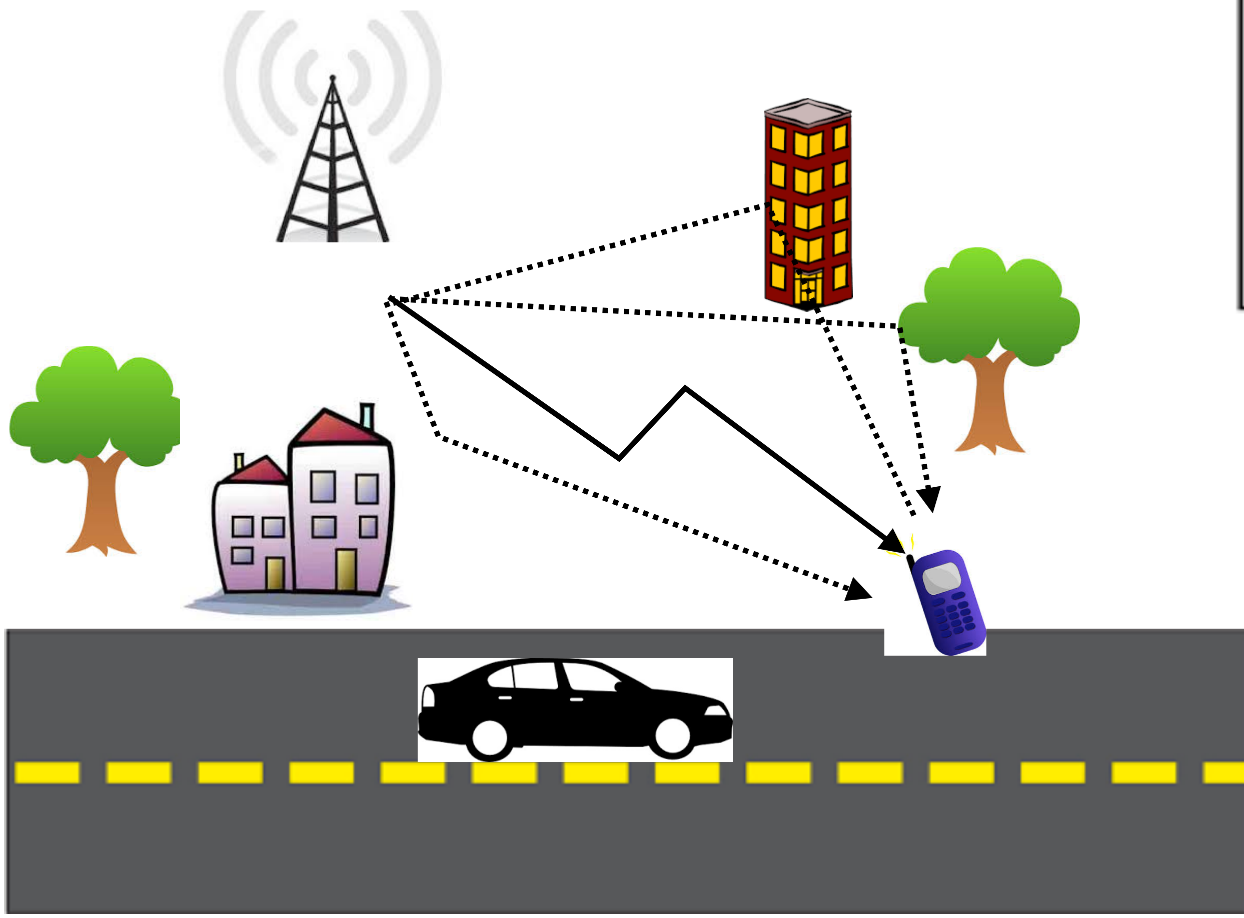
Wireless Channel

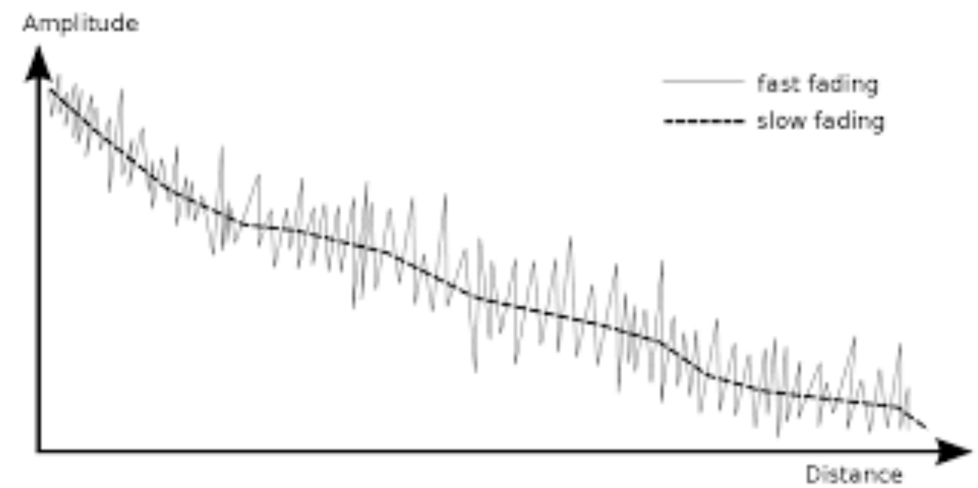




$$\text{Rate} = \log_2\left(1 + \frac{|h|^2 P}{N}\right) bits/sec/Hz$$

Wireless Channel

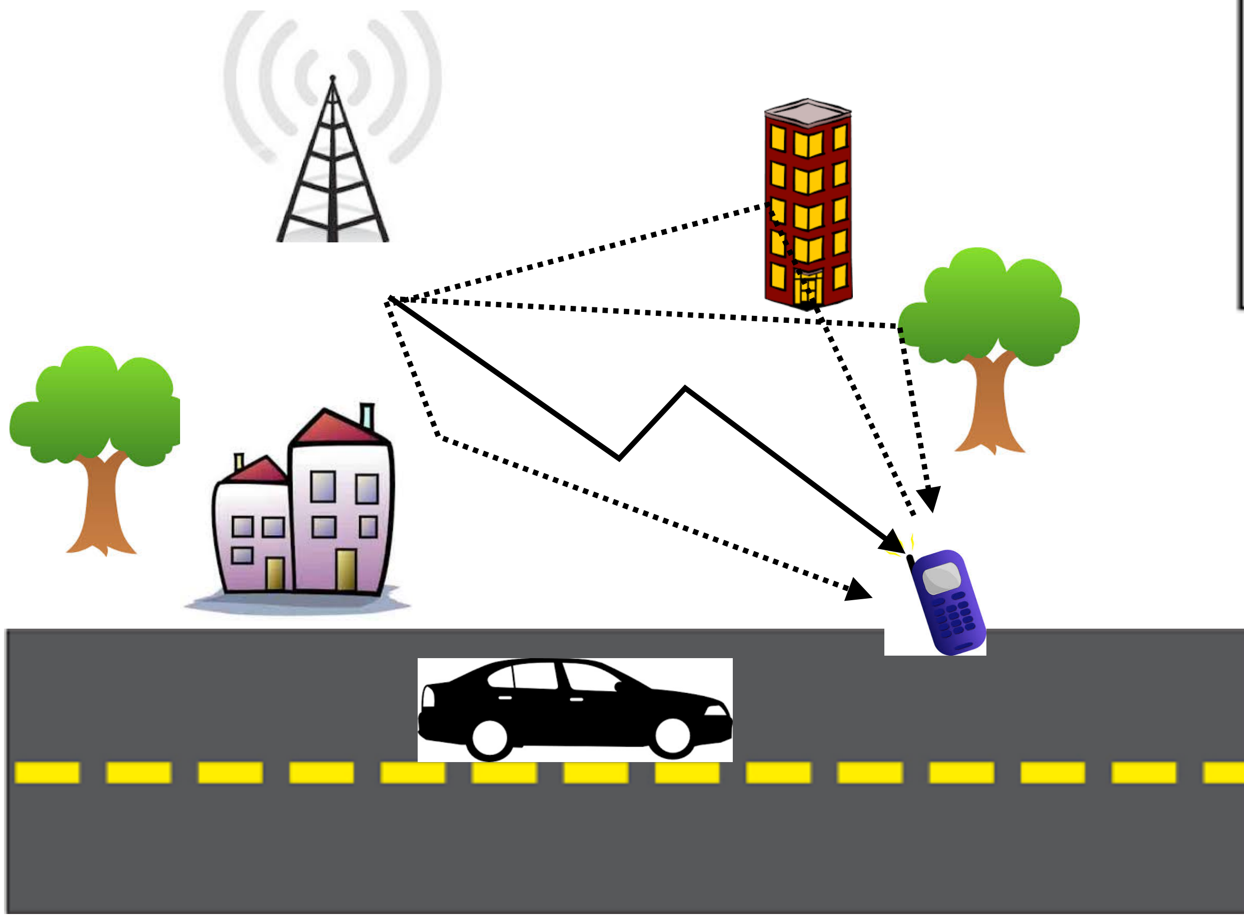


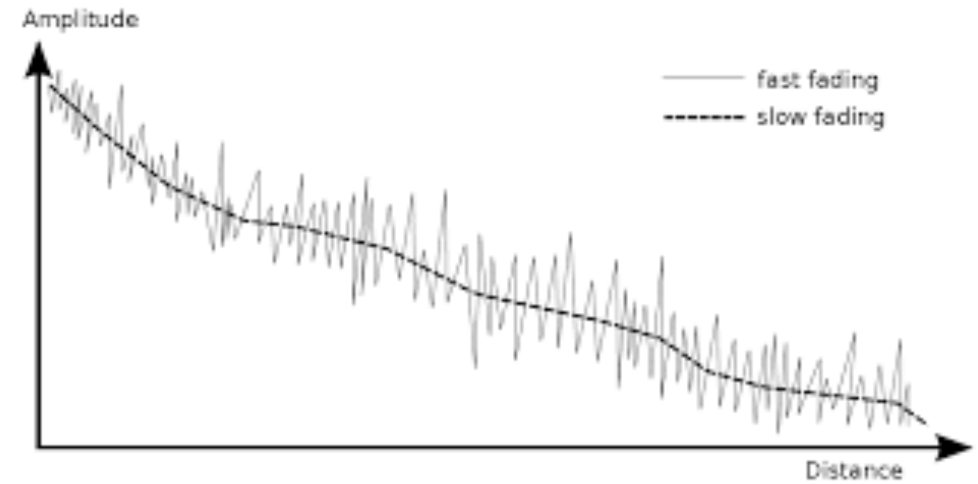


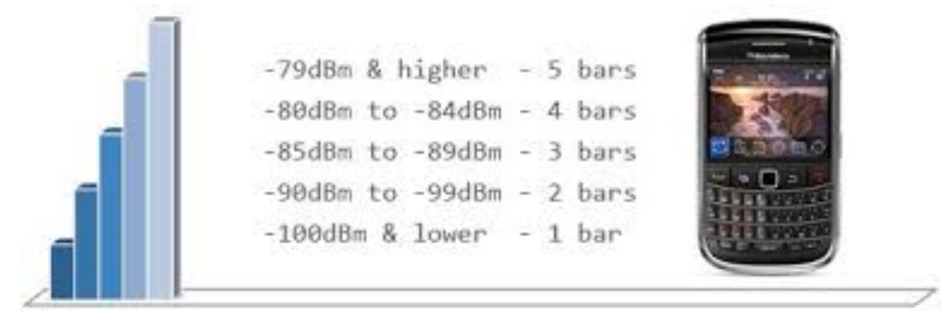
$$\text{Rate} = \log_2\left(1 + \boxed{\frac{|h|^2 P}{N}}\right) \; bits/sec/Hz$$

SNR

Wireless Channel





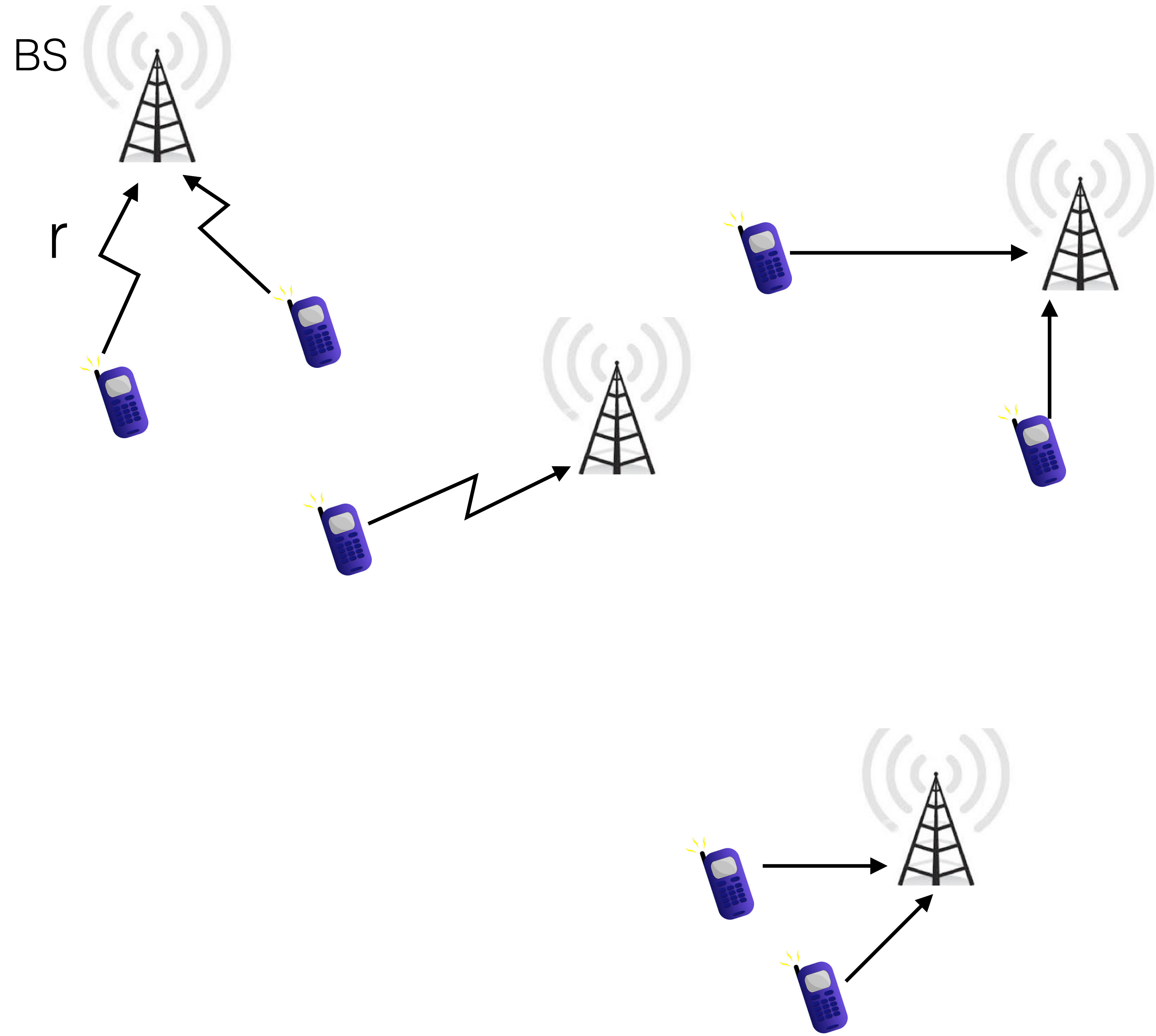


$$\text{Rate} = \log_2\left(1 + \underbrace{\boxed{\frac{|h|^2 P}{N}}}_{\text{SNR}}\right) bits/sec/Hz$$

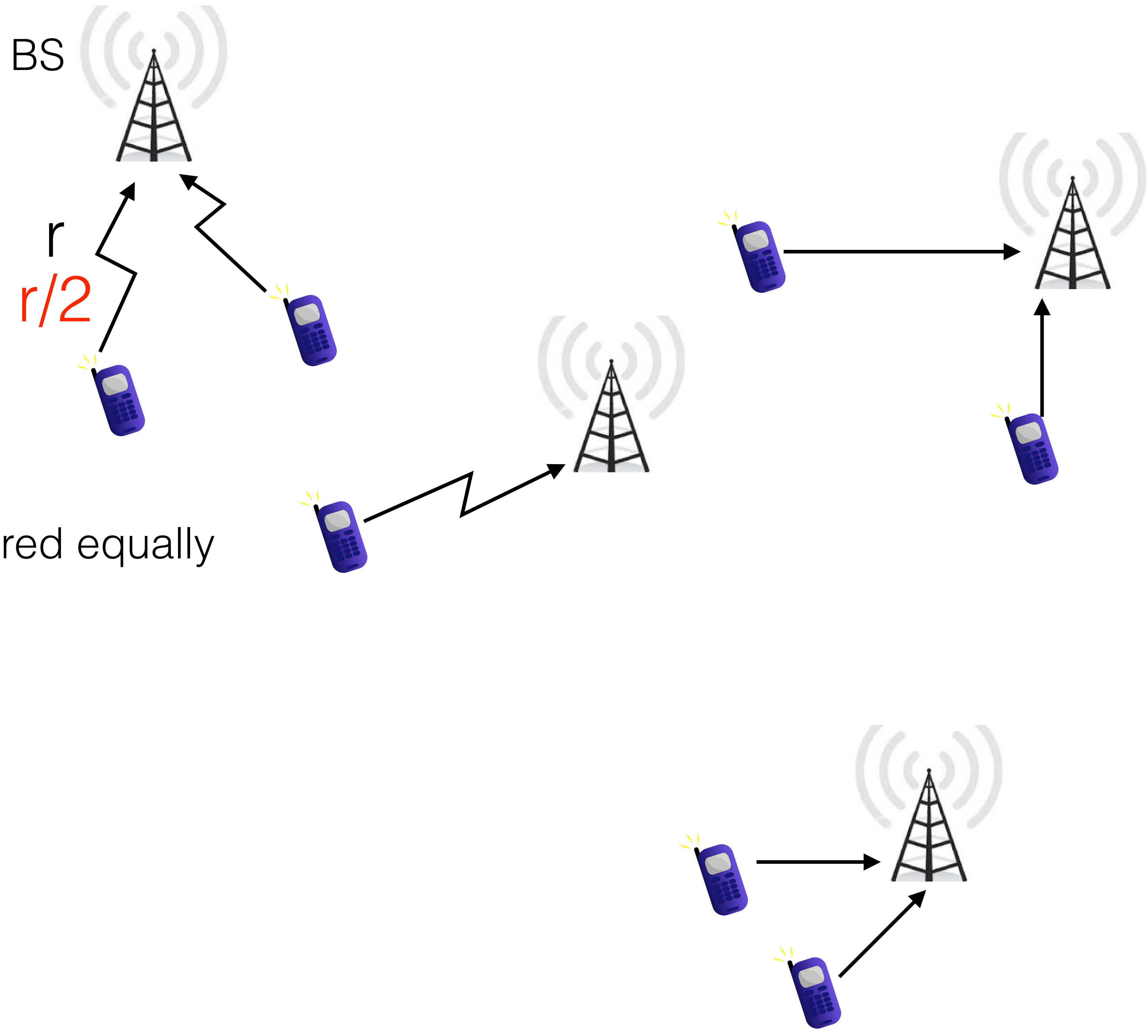
Legacy Problem

2G 3G 4G

Legacy Problem -Wireless Communication

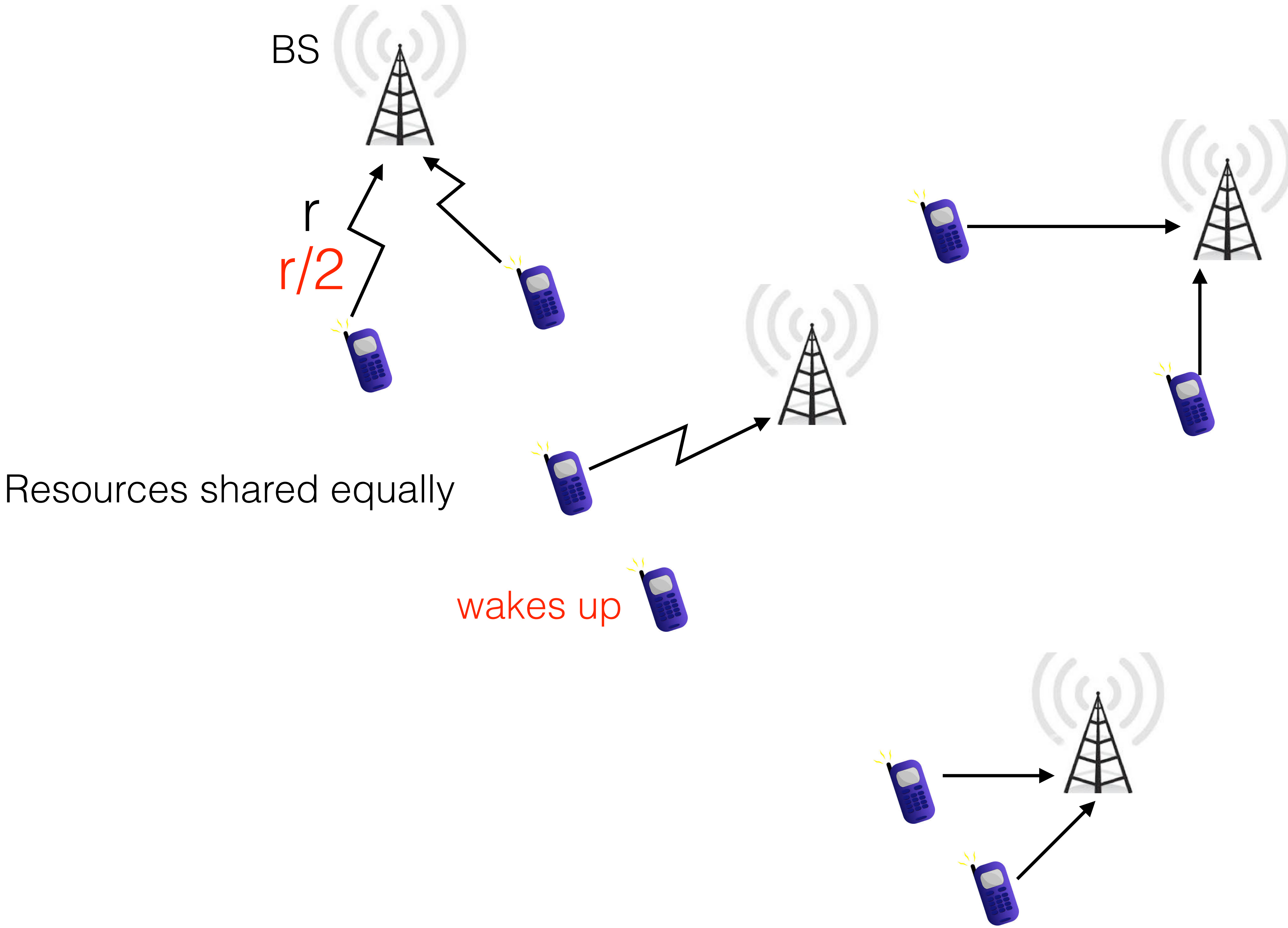


Legacy Problem -Wireless Communication

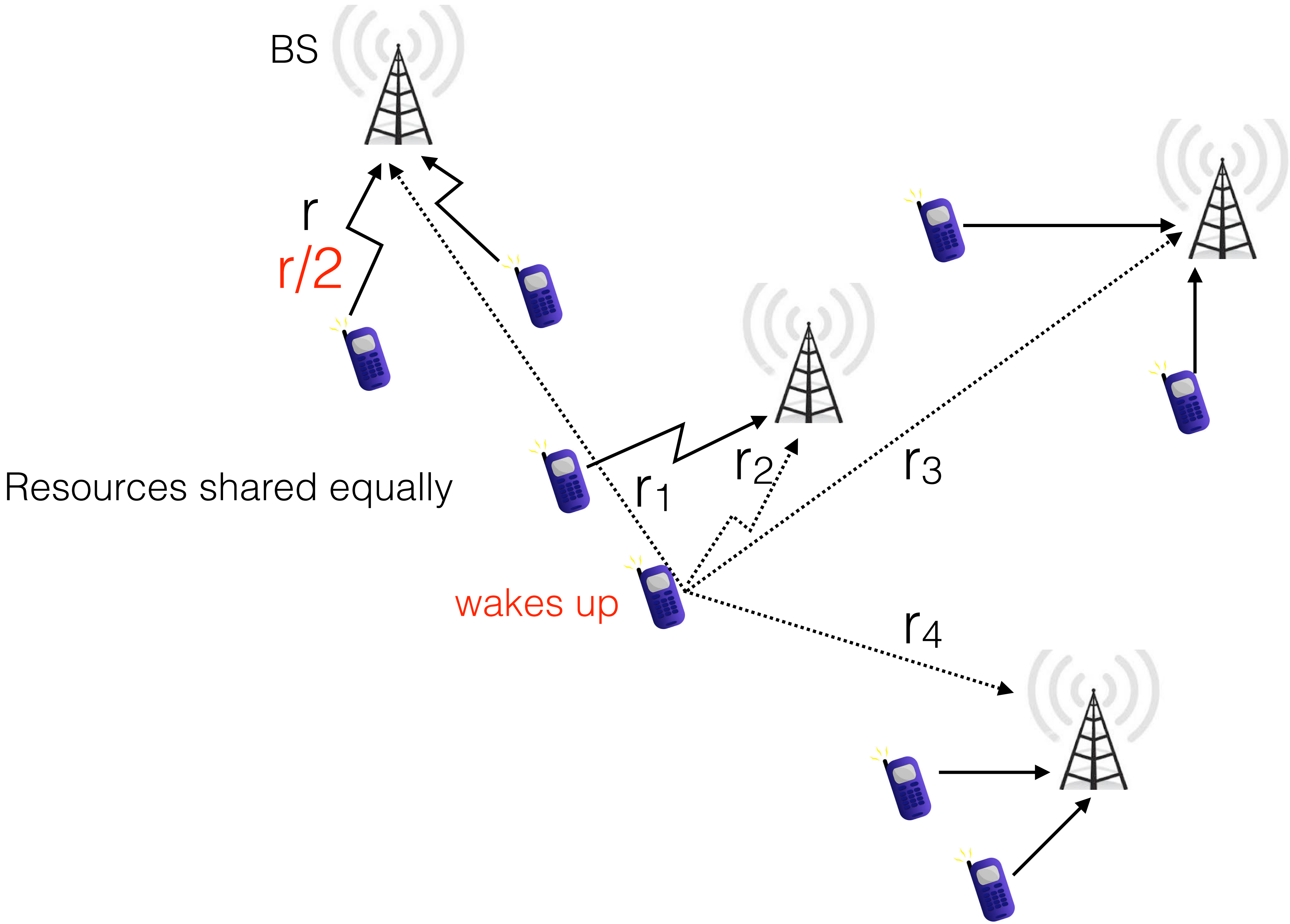


Resources shared equally

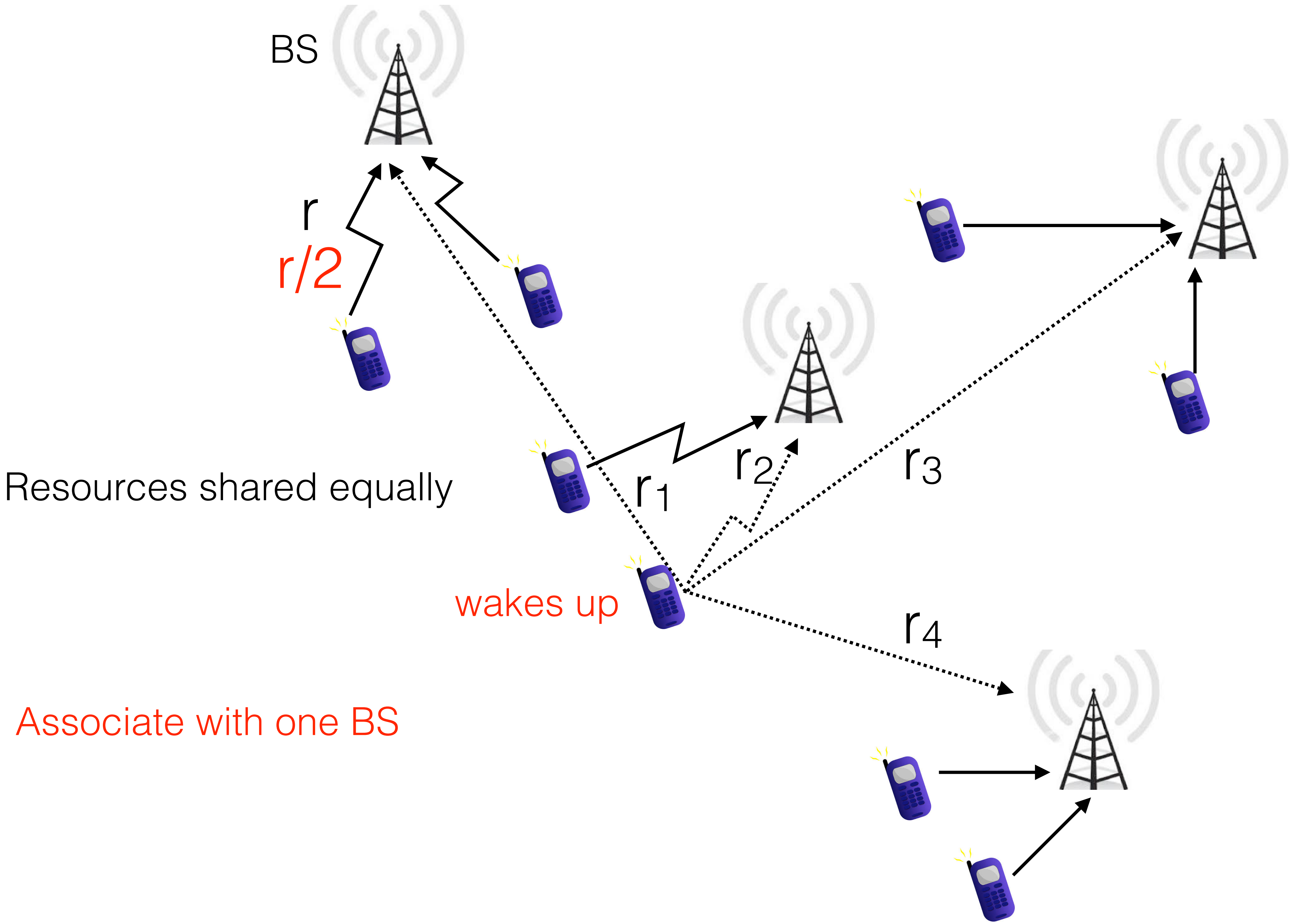
Legacy Problem -Wireless Communication



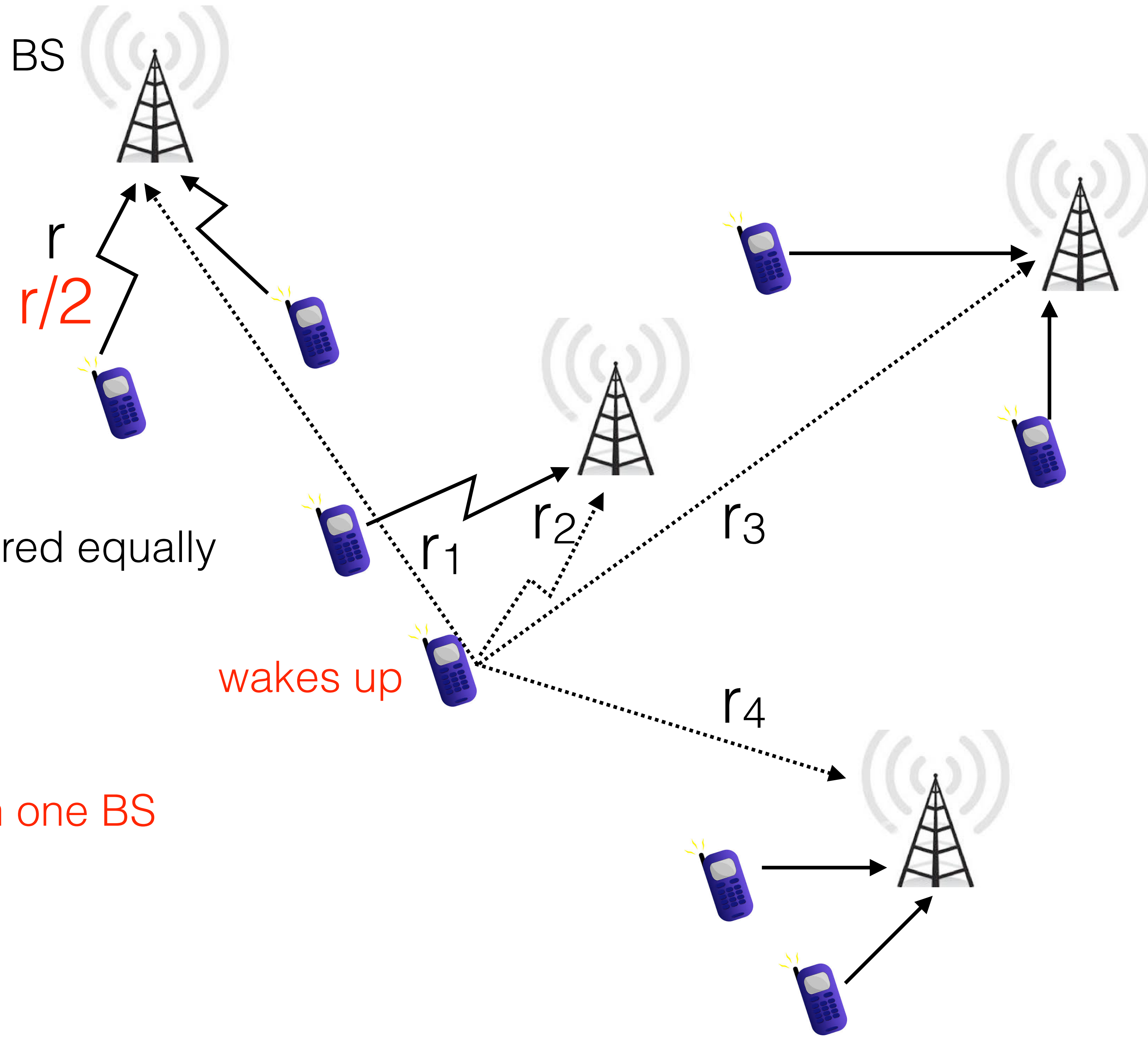
Legacy Problem -Wireless Communication



Legacy Problem -Wireless Communication



Legacy Problem -Wireless Communication



Resources shared equally

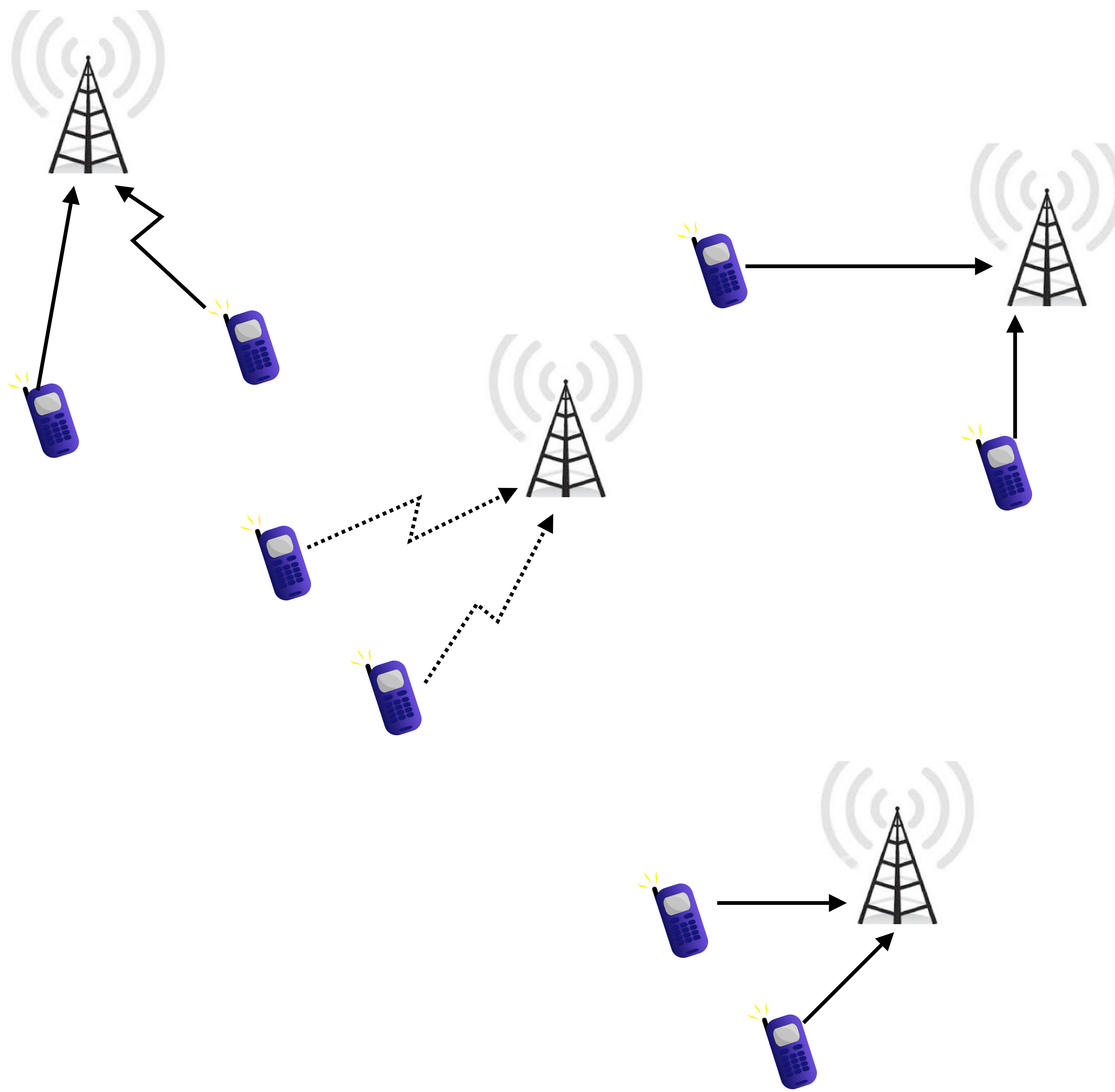
Associate with one BS

Find optimal BS allocation to maximize sum-rate

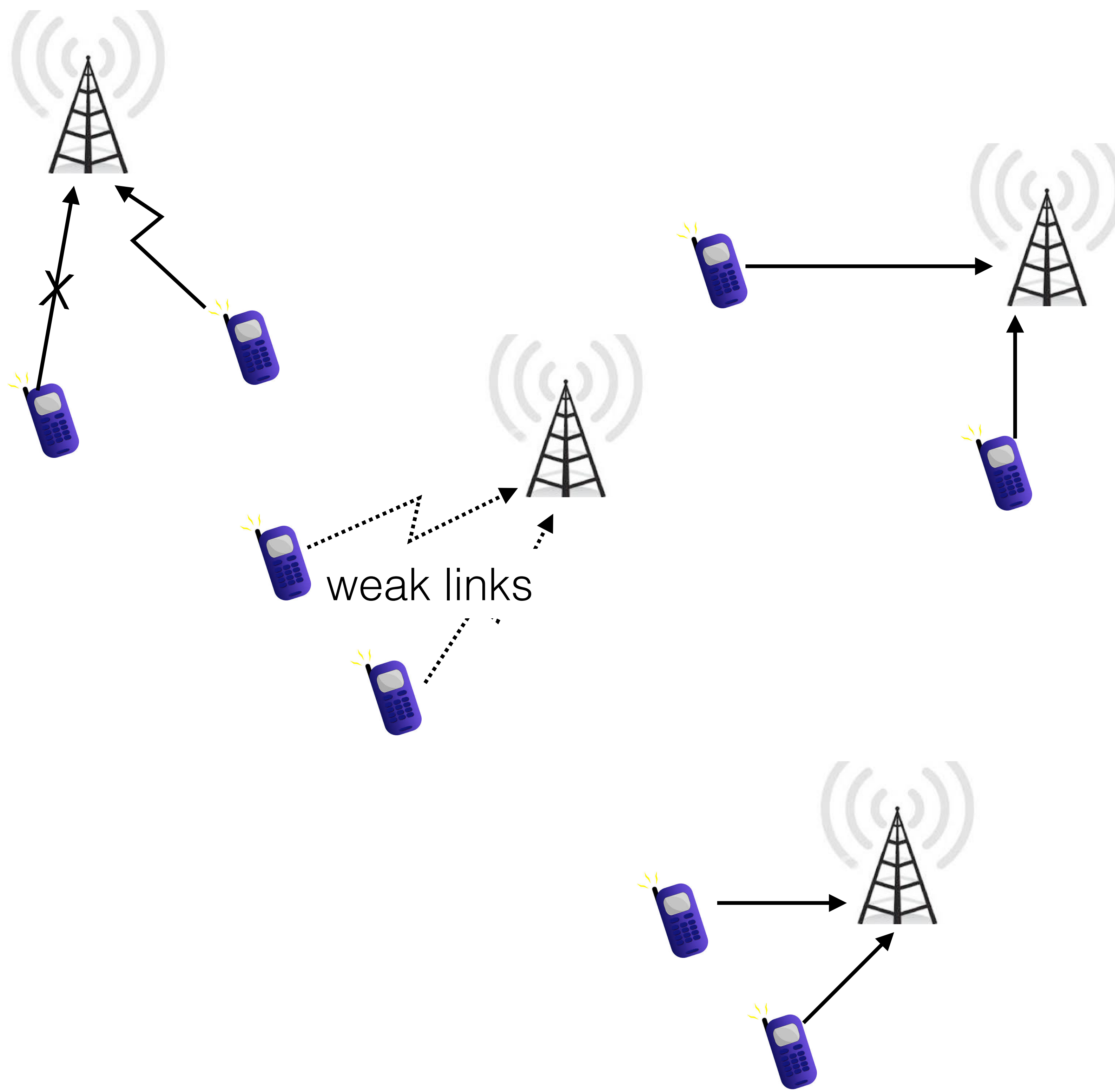
Modern Problem

5G

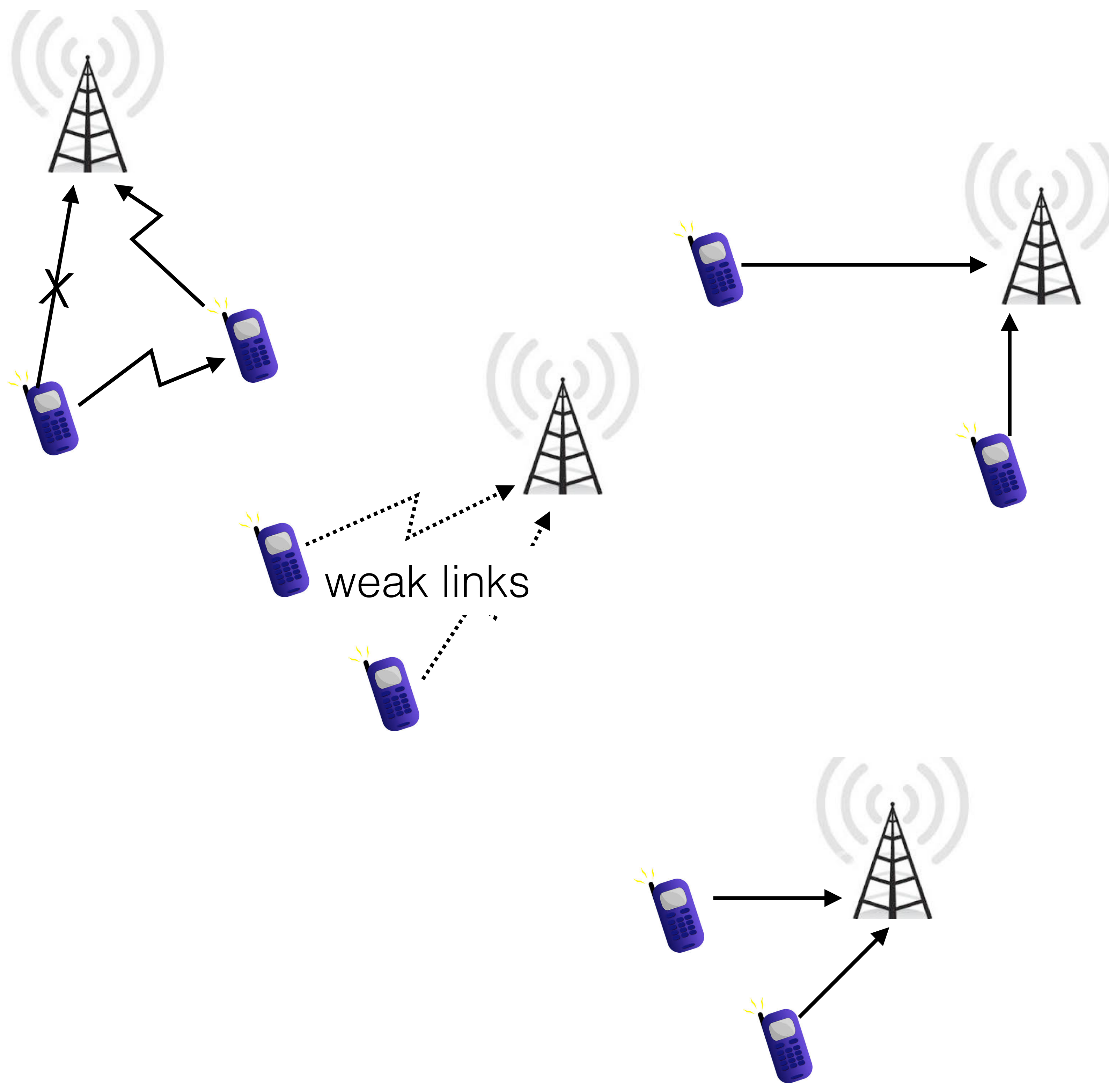
Modern Problem Device-2-Device Communication



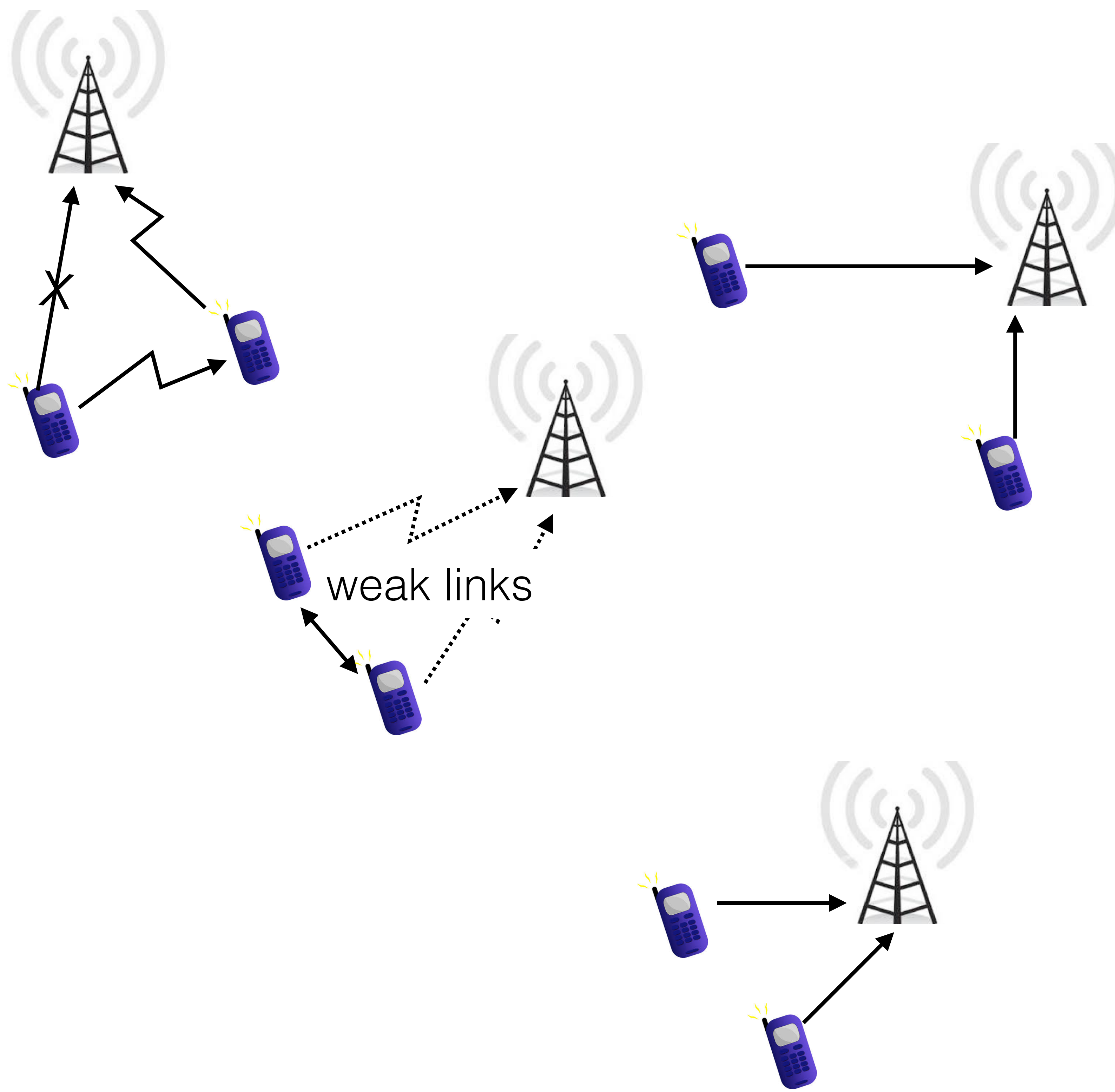
Modern Problem Device-2-Device Communication



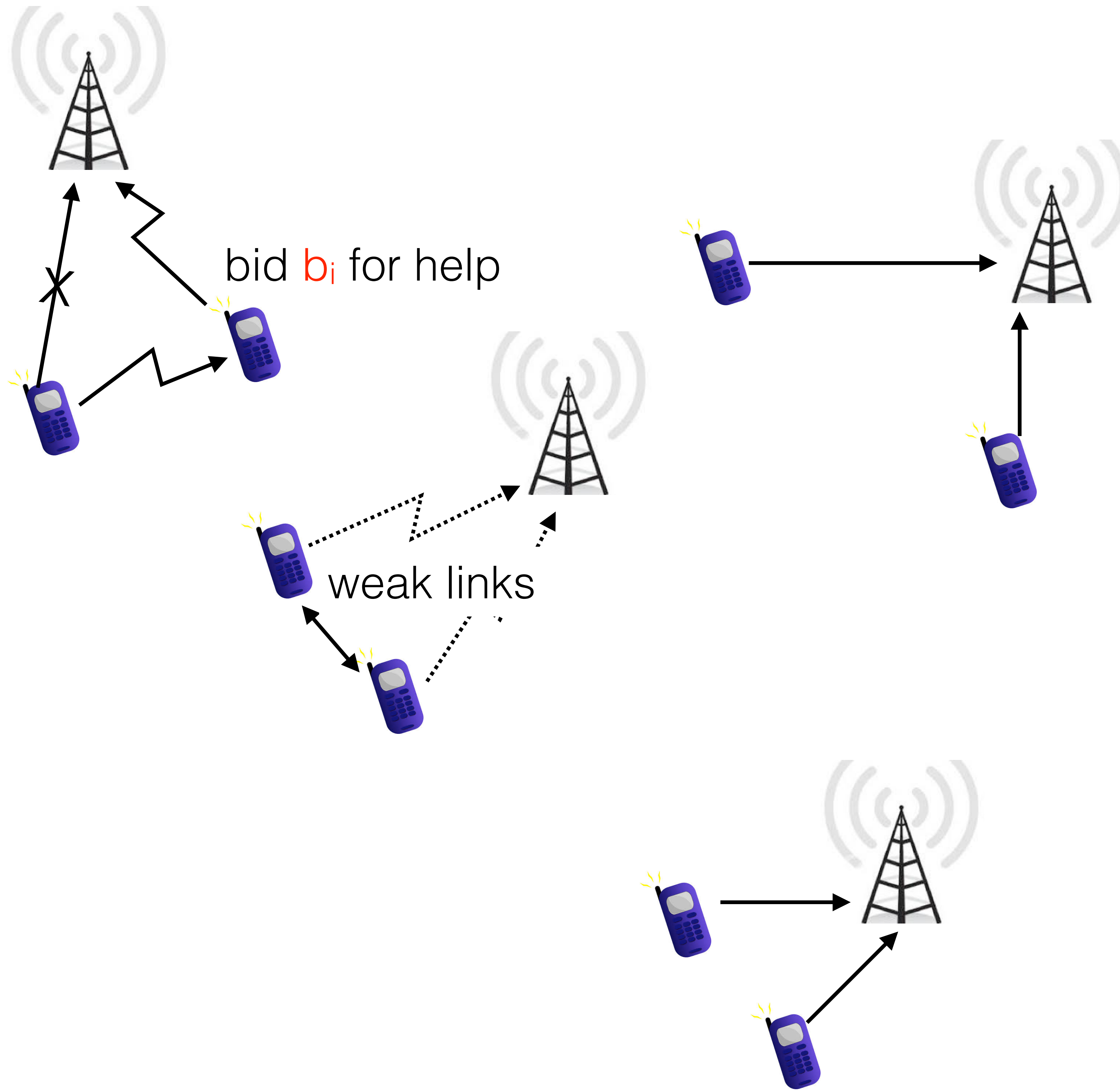
Modern Problem Device-2-Device Communication



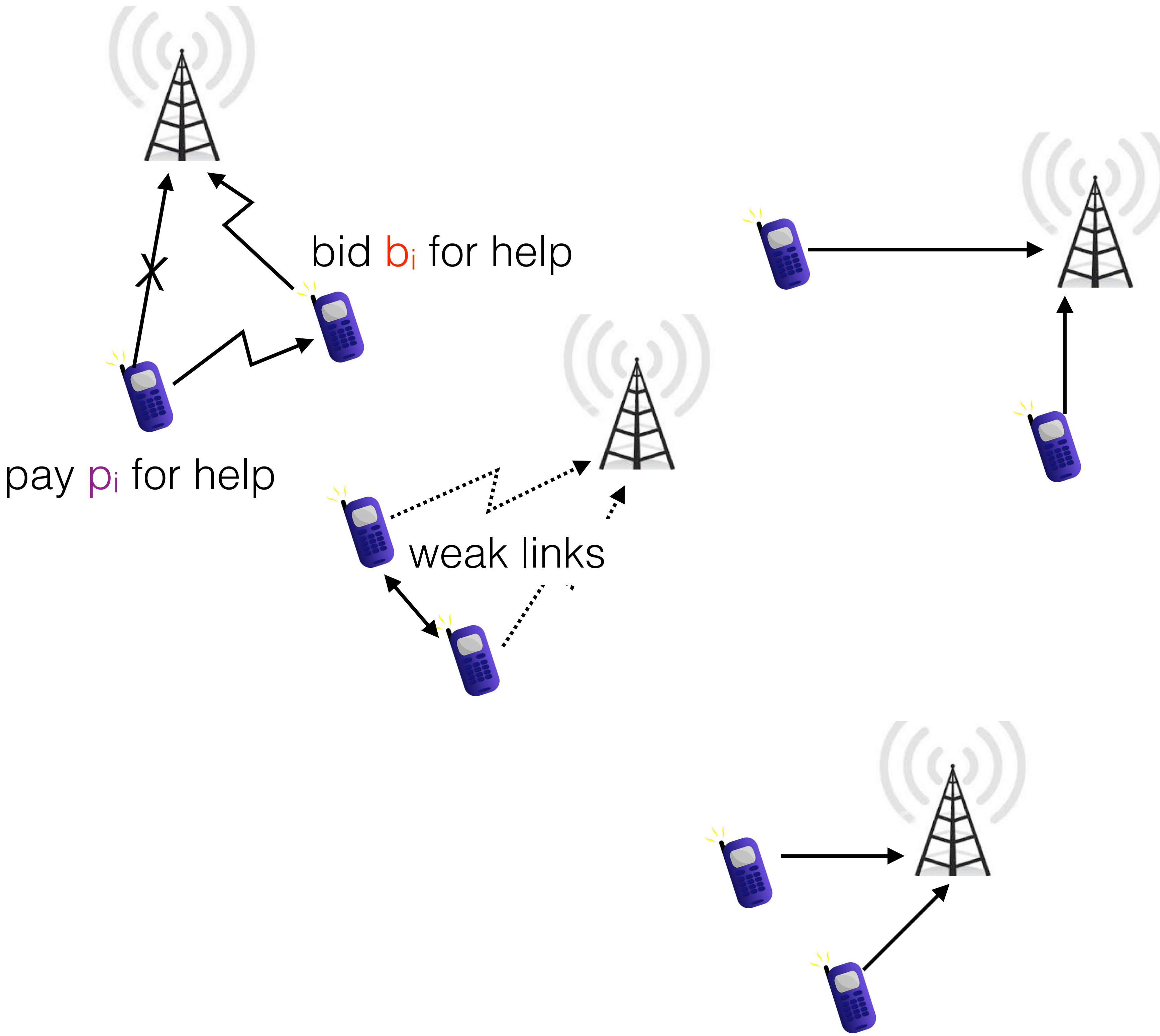
Modern Problem Device-2-Device Communication



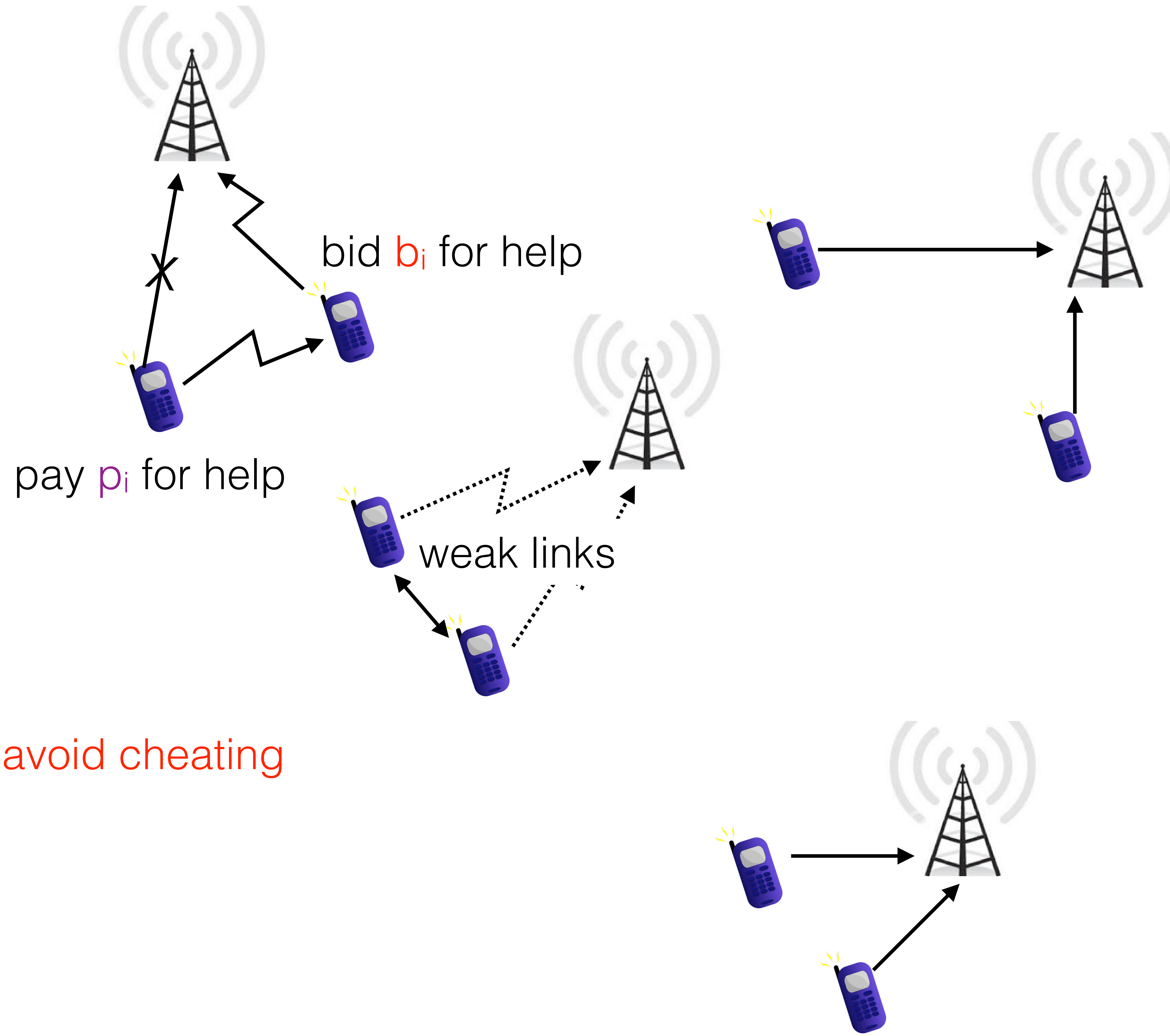
Modern Problem Device-2-Device Communication



Modern Problem Device-2-Device Communication

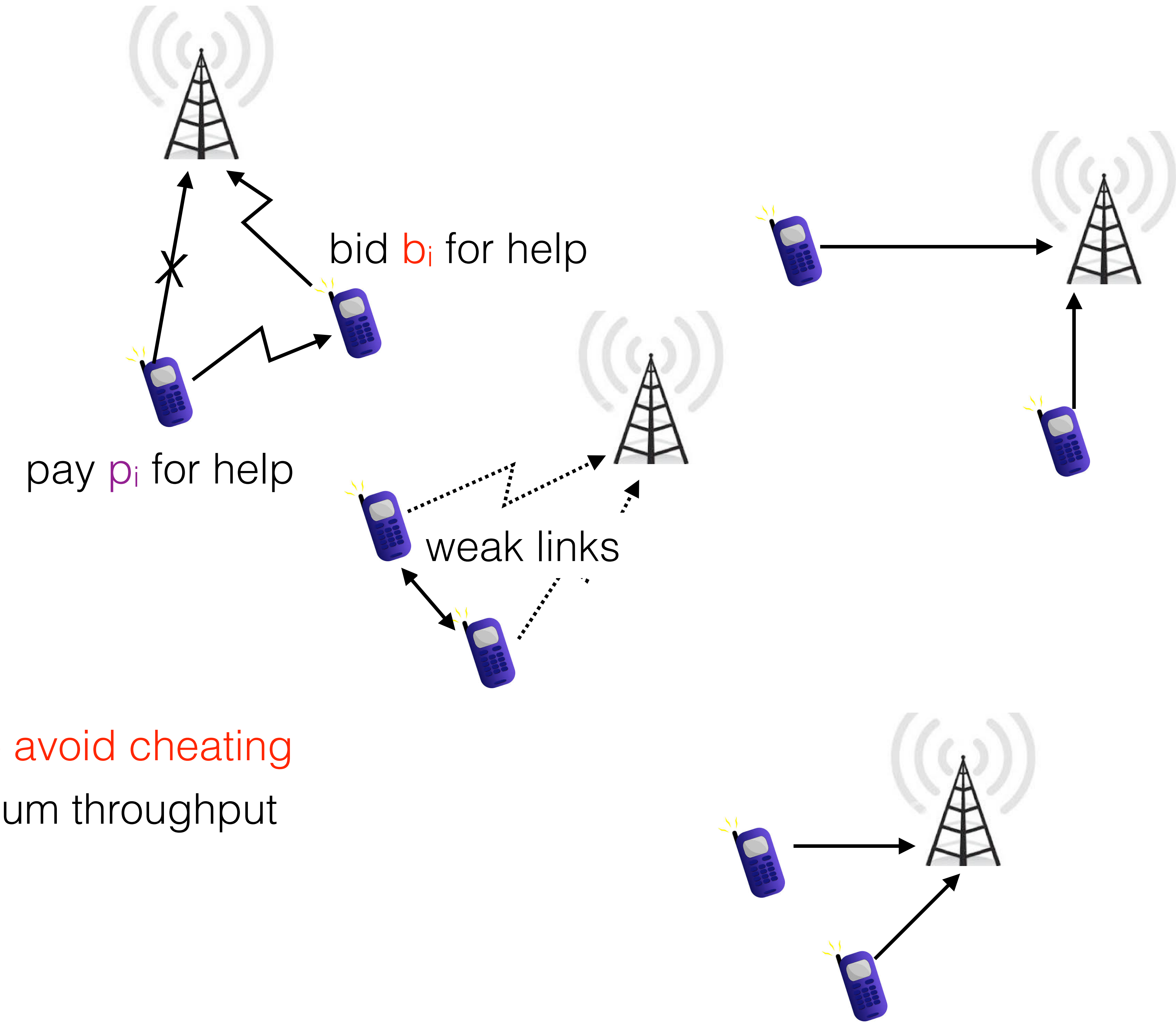


Modern Problem Device-2-Device Communication



mechanism to avoid cheating

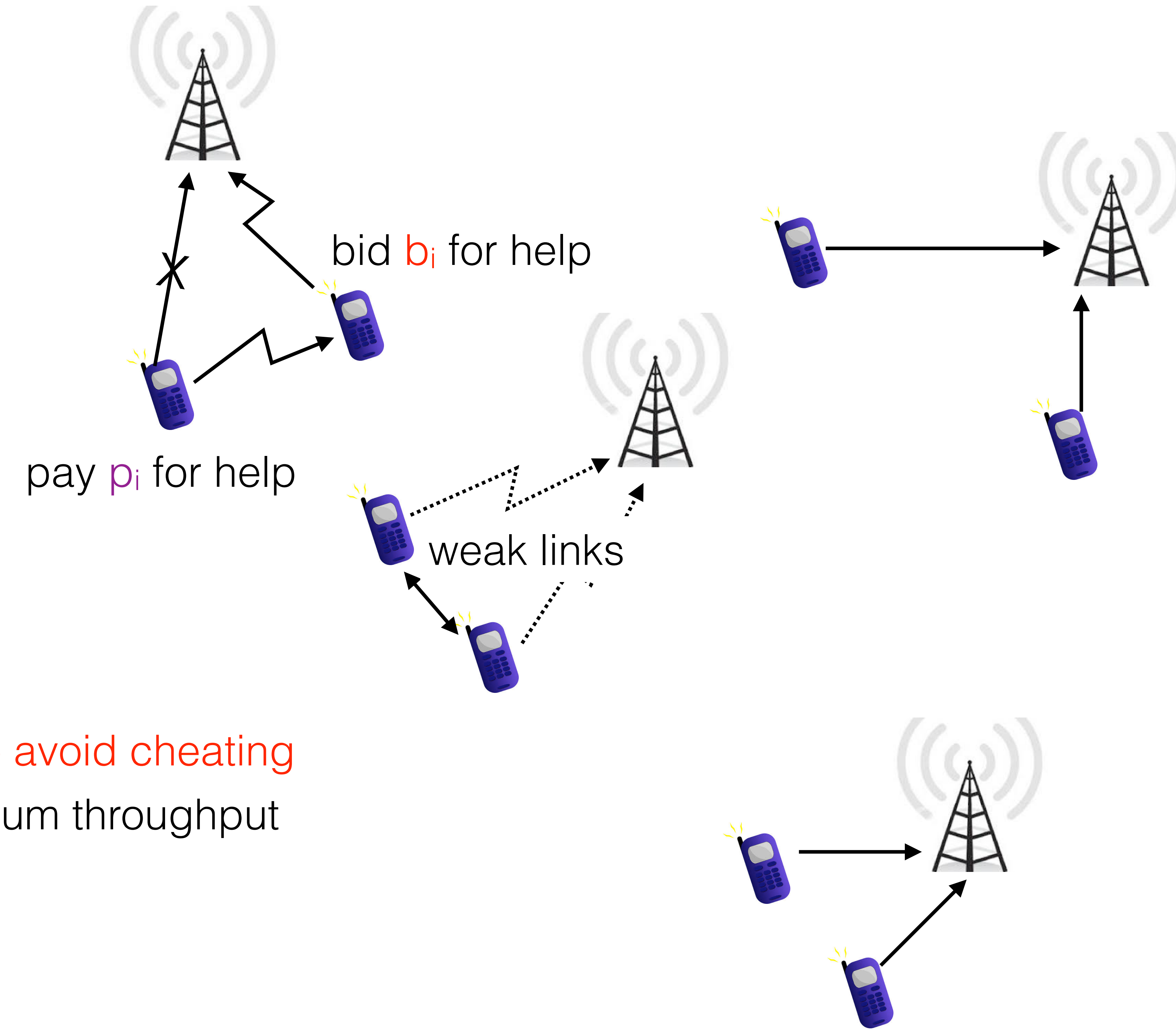
Modern Problem Device-2-Device Communication



mechanism to avoid cheating

ensure maximum throughput

Modern Problem Device-2-Device Communication



mechanism to avoid cheating

ensure maximum throughput

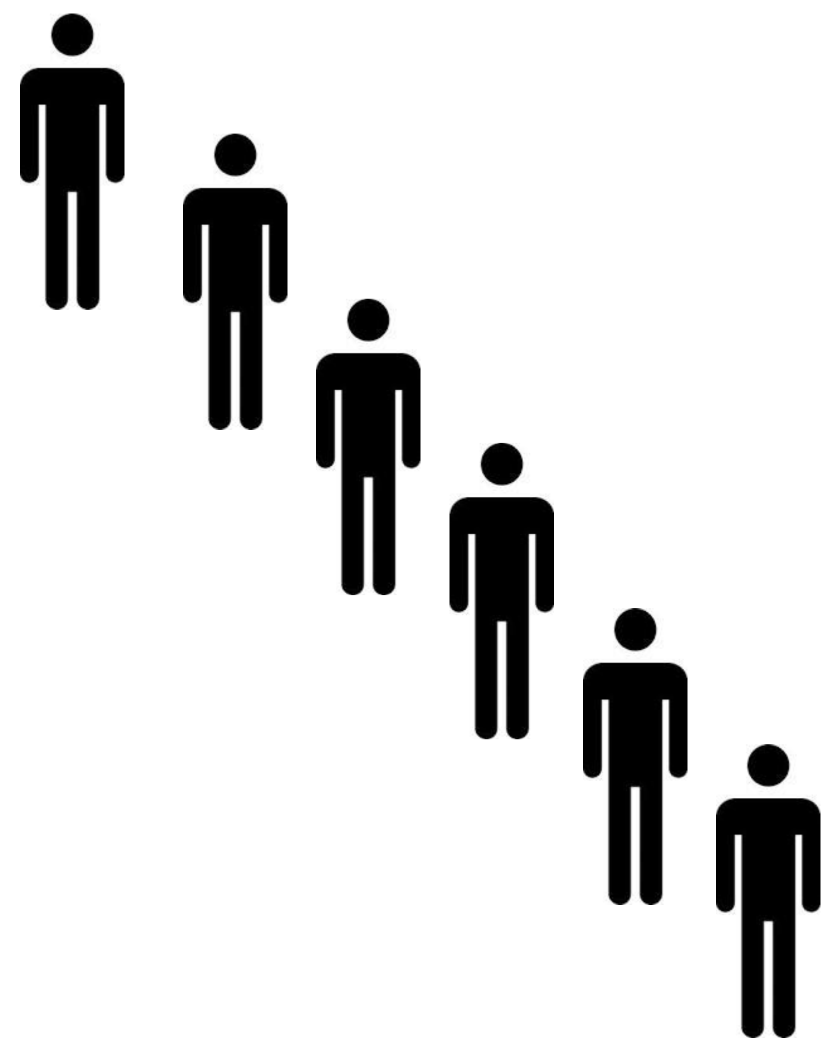
Find optimal helper association and incentive rule that is truthful

small detour

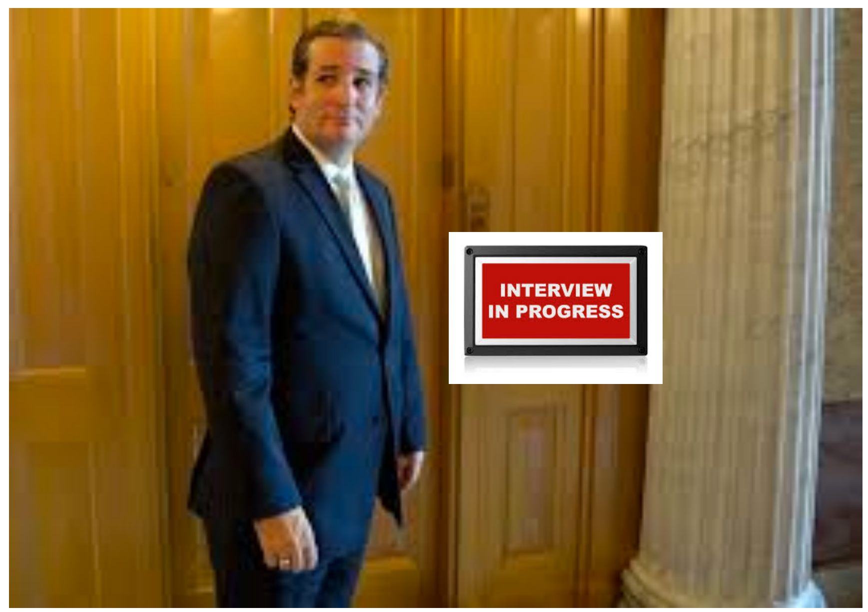
The mathematics behind a perfect match

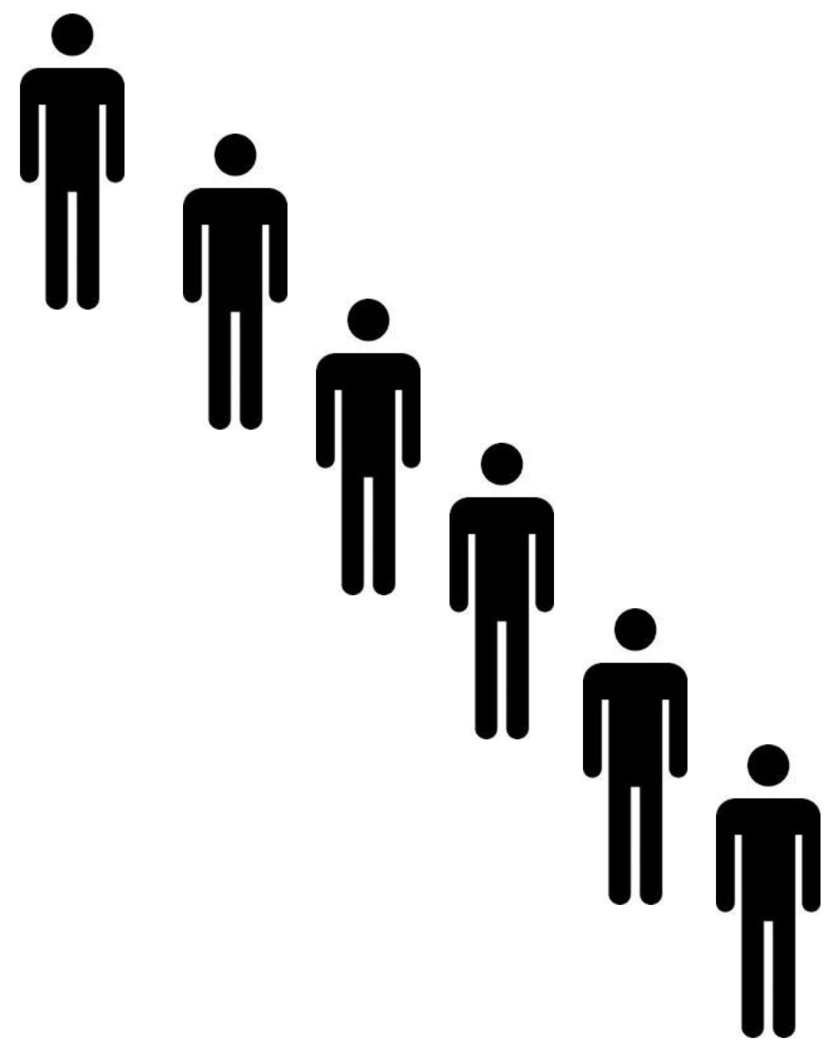
how many to date before committing !

Hiring impatient staff



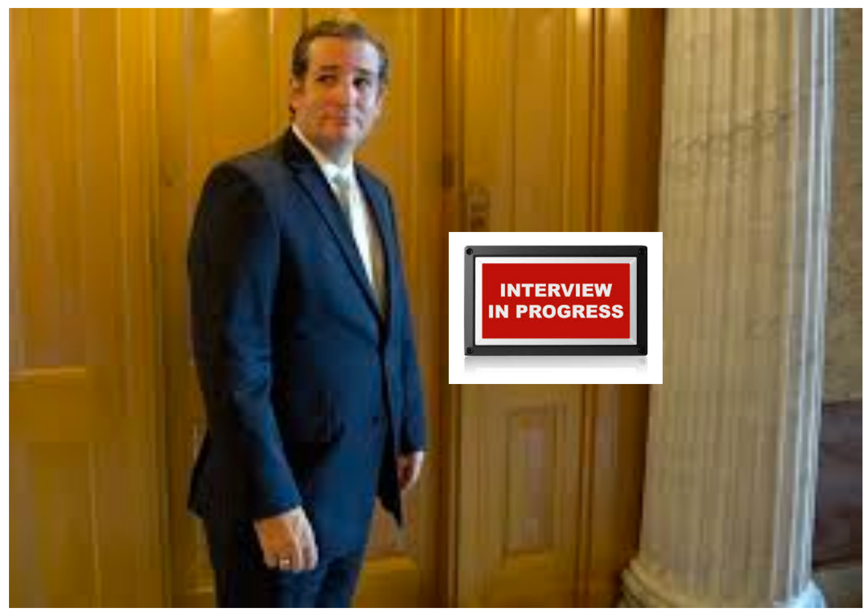
Hiring impatient staff

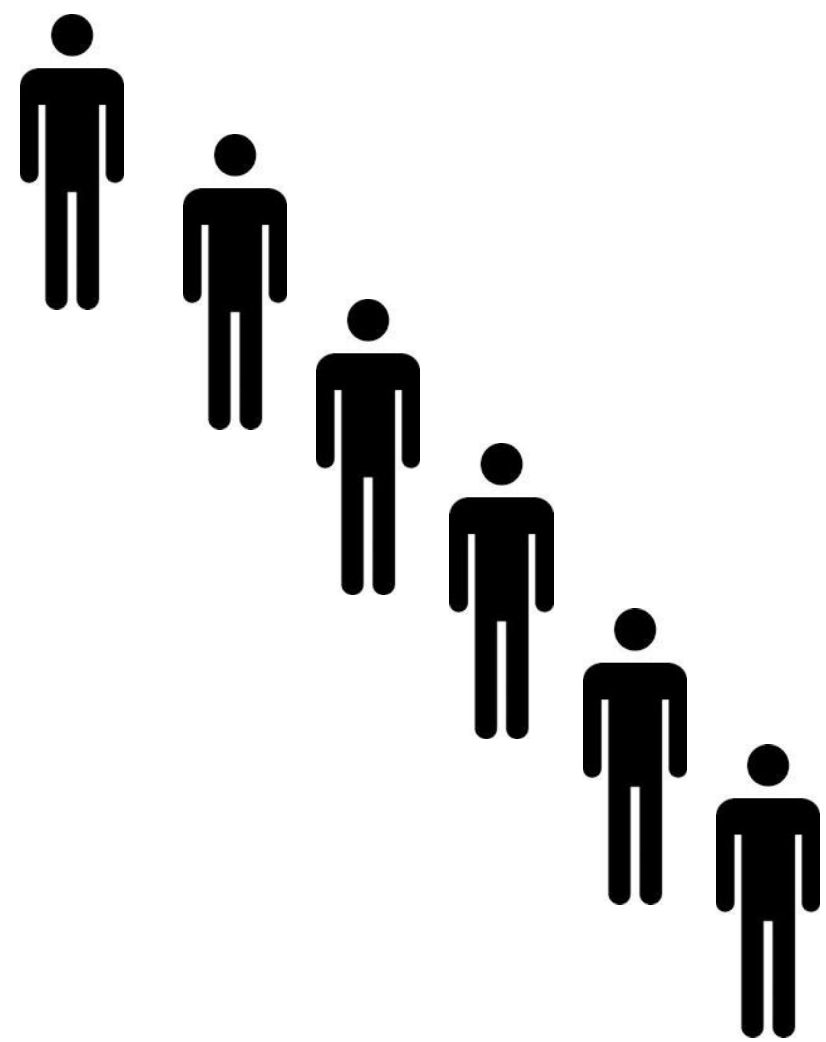




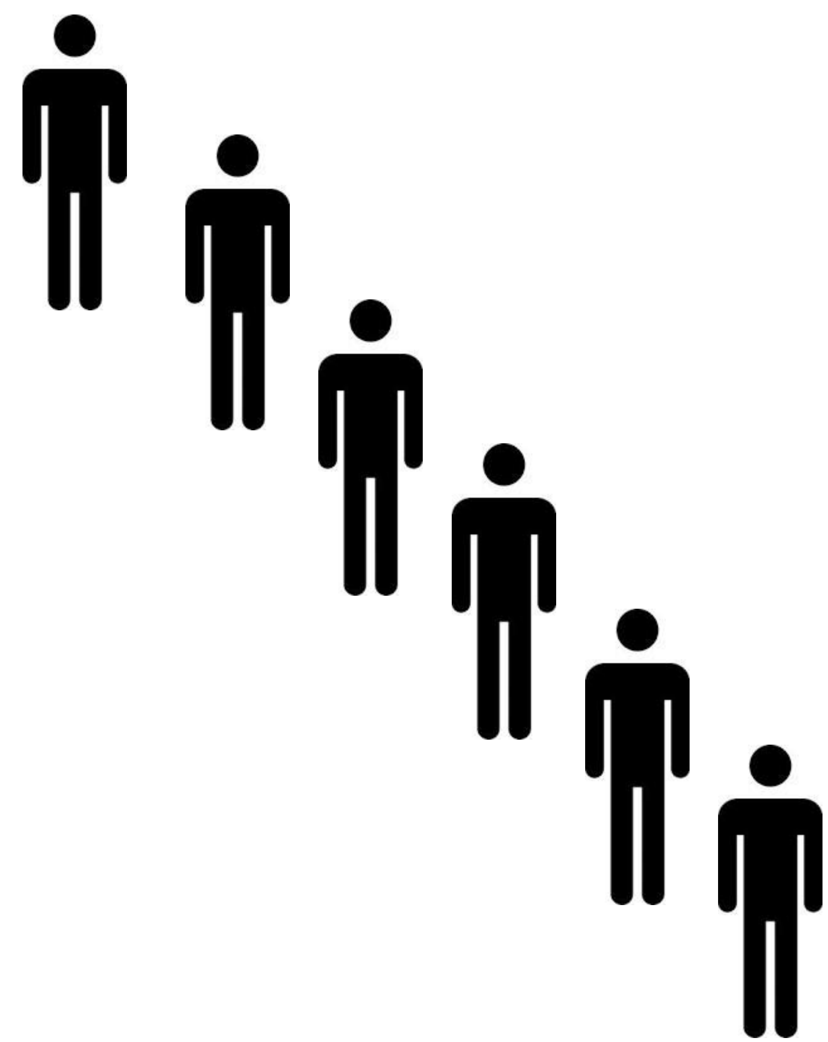
Hiring impatient staff

prob. of choosing best candidate is 1/n

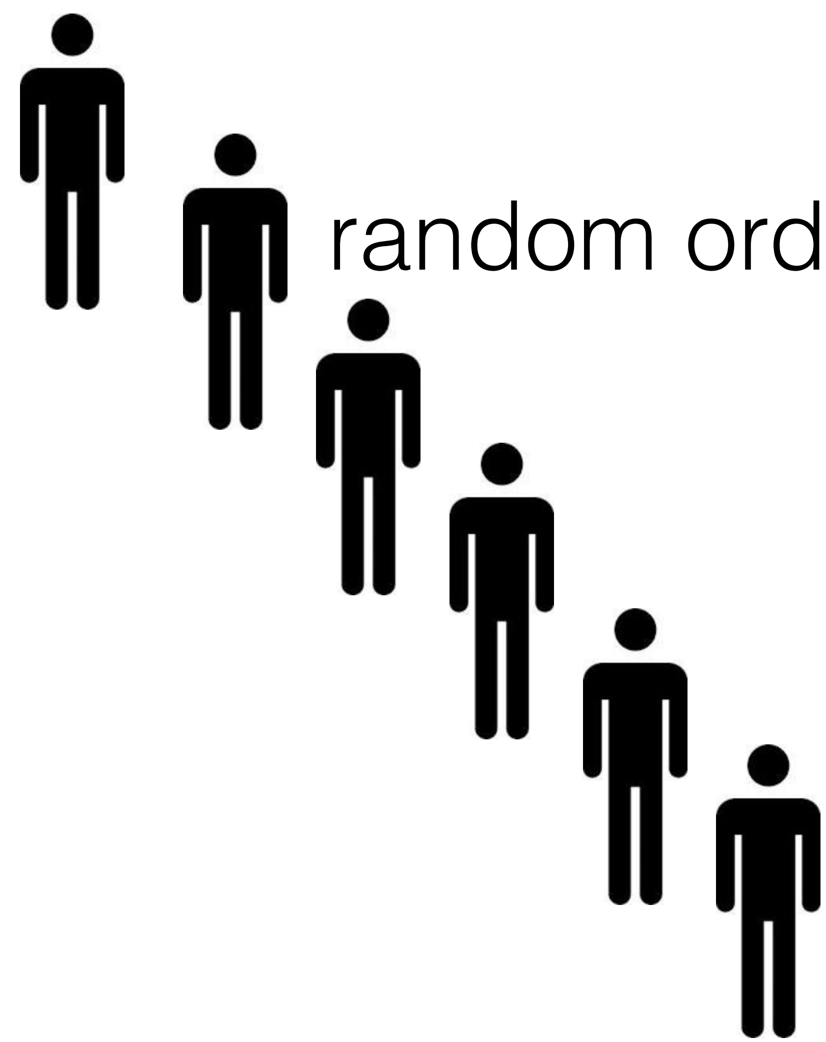




Hiring staff - not adversarial

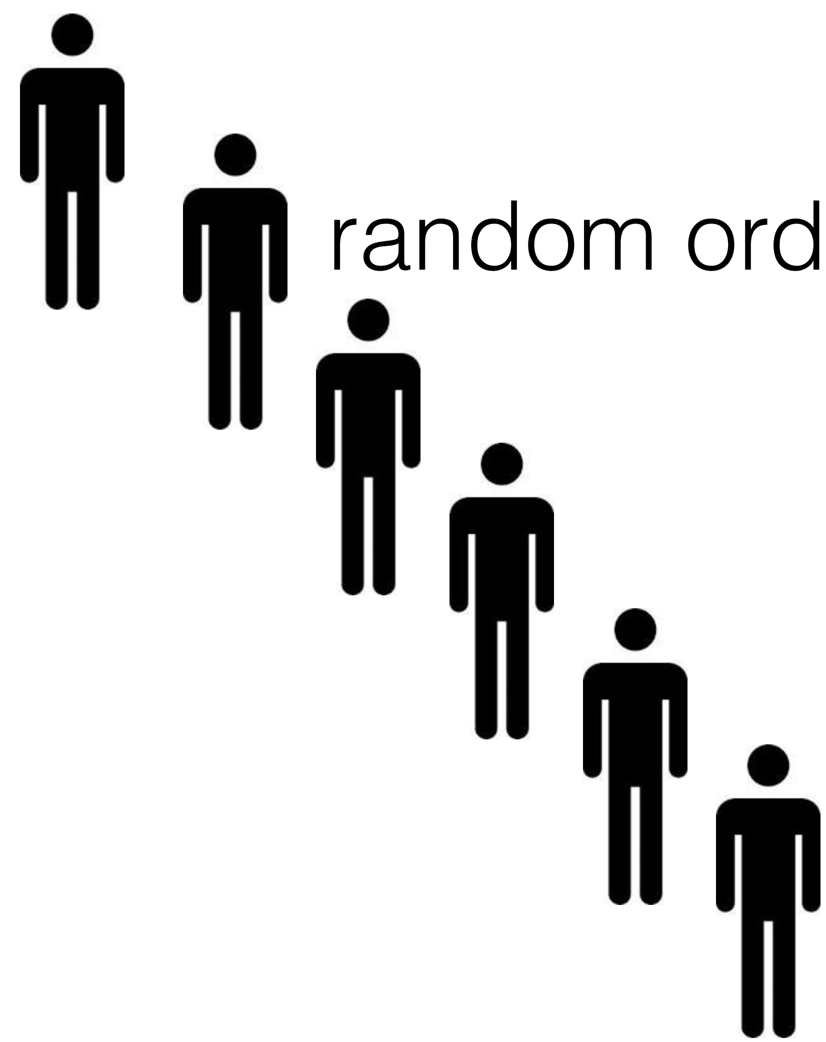


Hiring staff - not adversarial



random order

Hiring staff - not adversarial

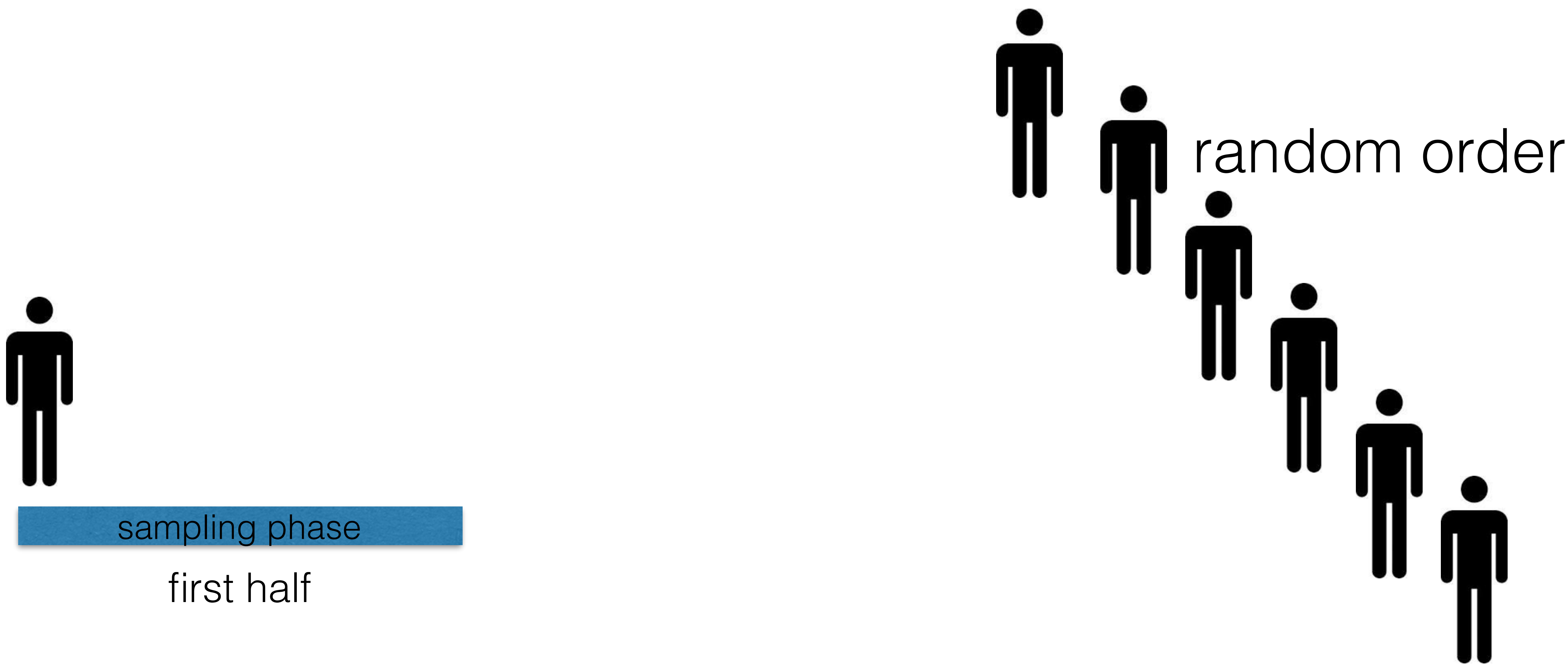


random order

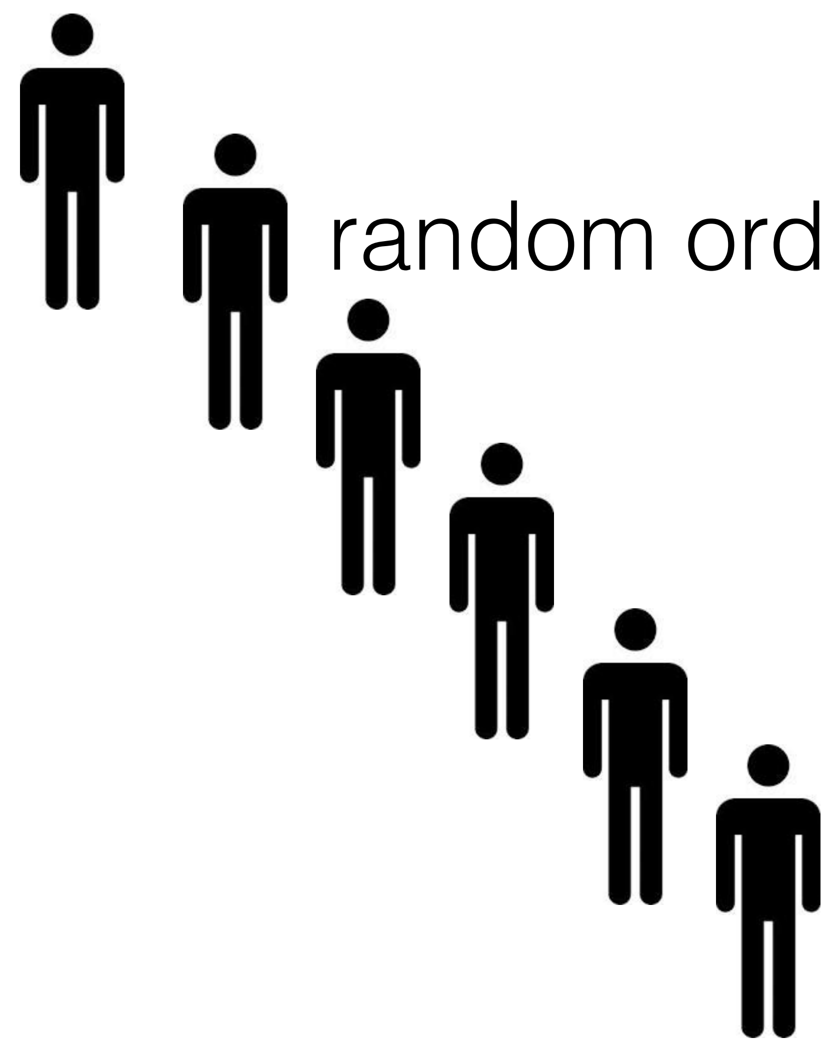
sampling phase

first half

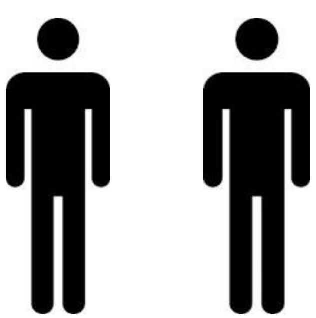
Hiring staff - not adversarial



Hiring staff - not adversarial



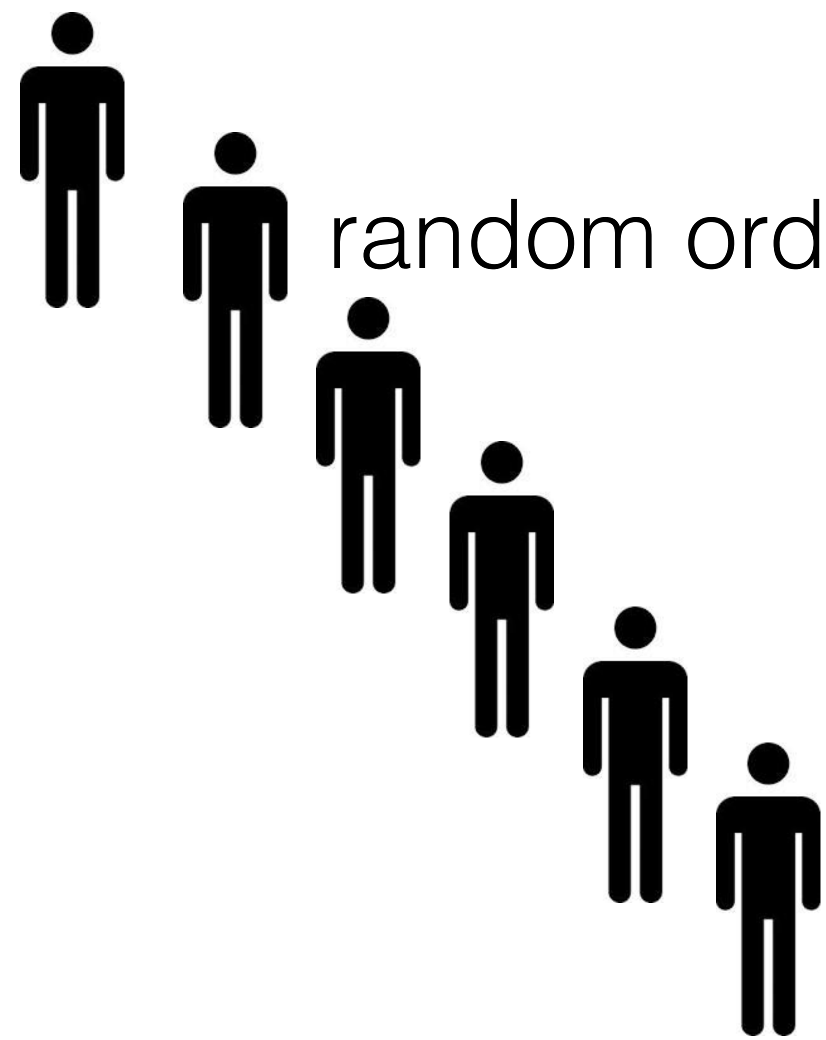
random order



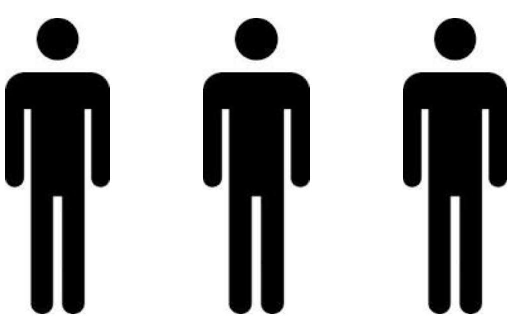
sampling phase

first half

Hiring staff - not adversarial



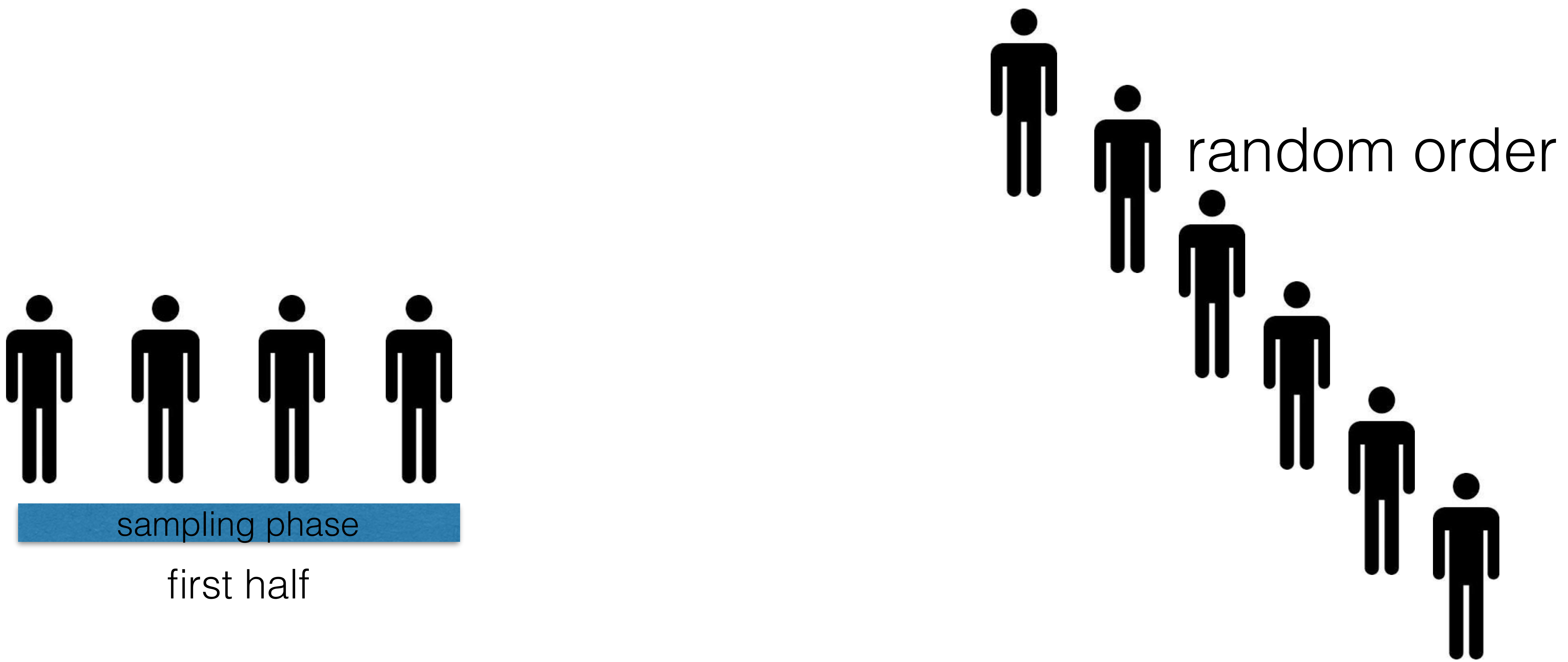
random order



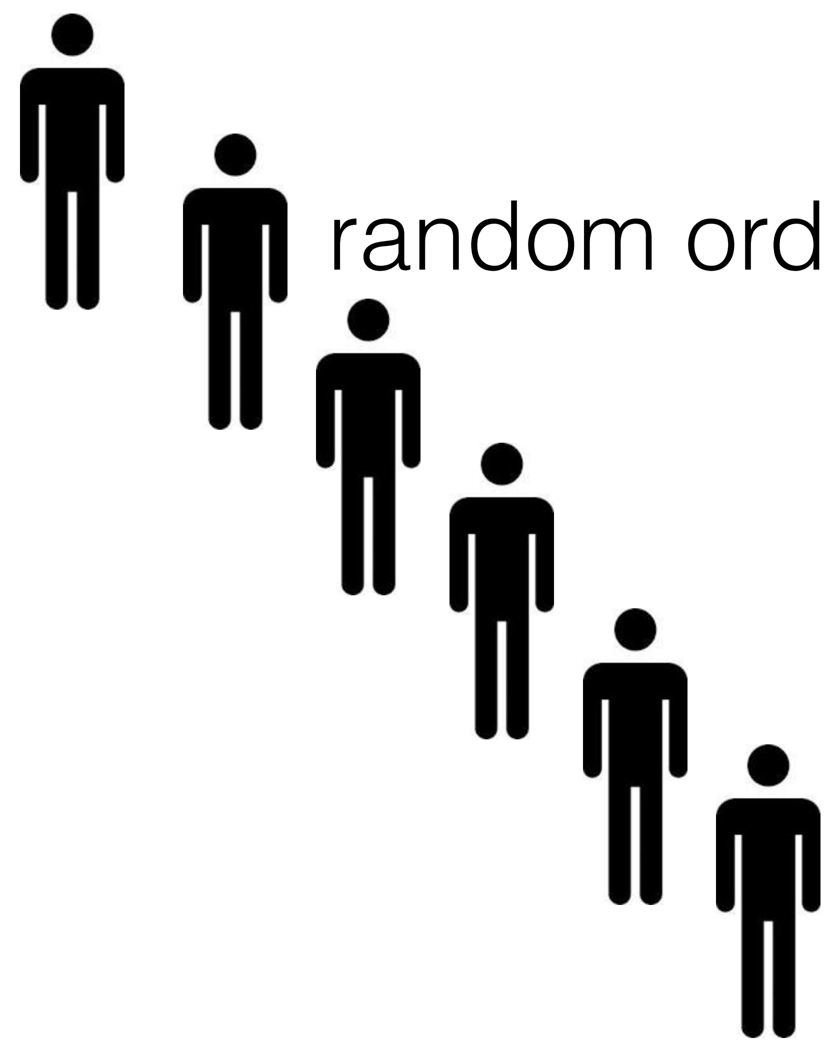
sampling phase

first half

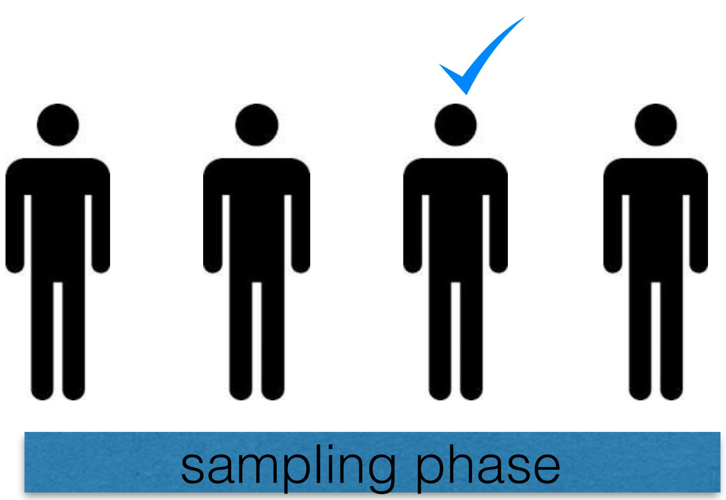
Hiring staff - not adversarial



Hiring staff - not adversarial



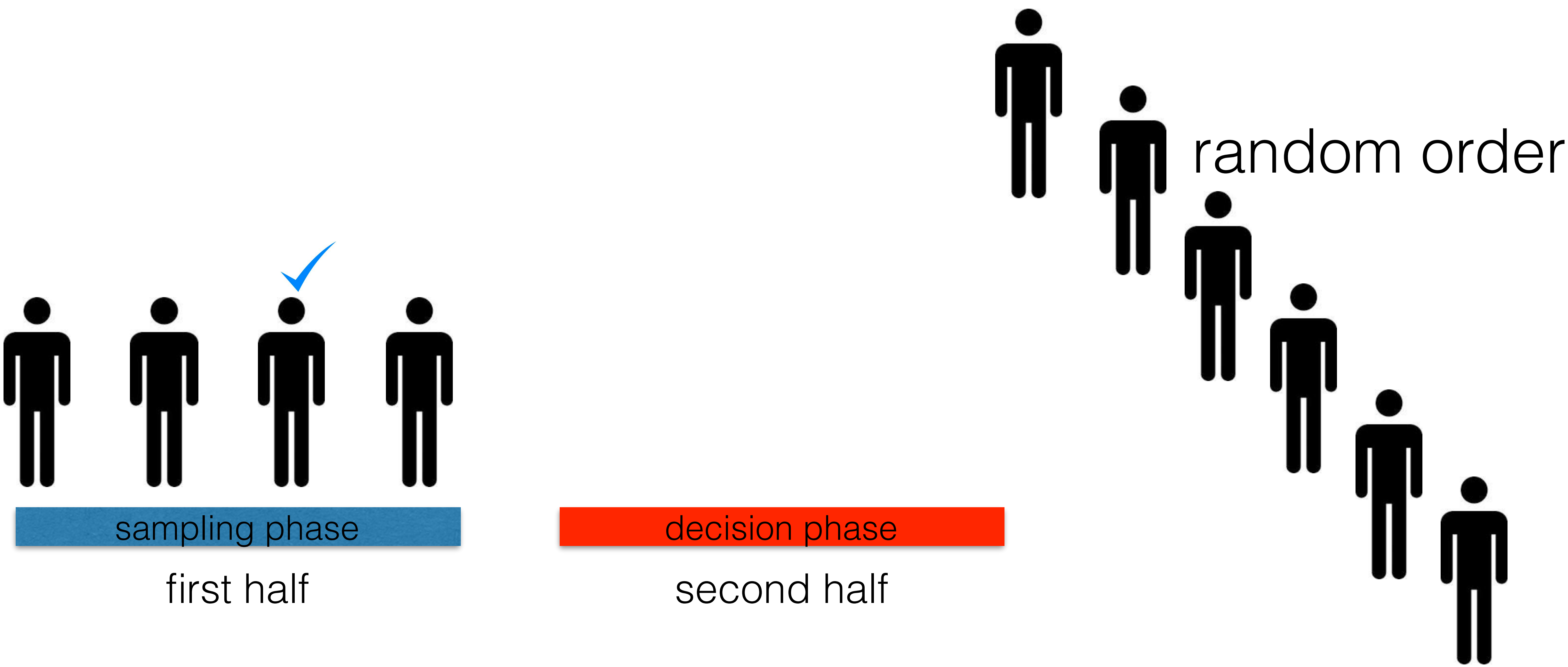
random order



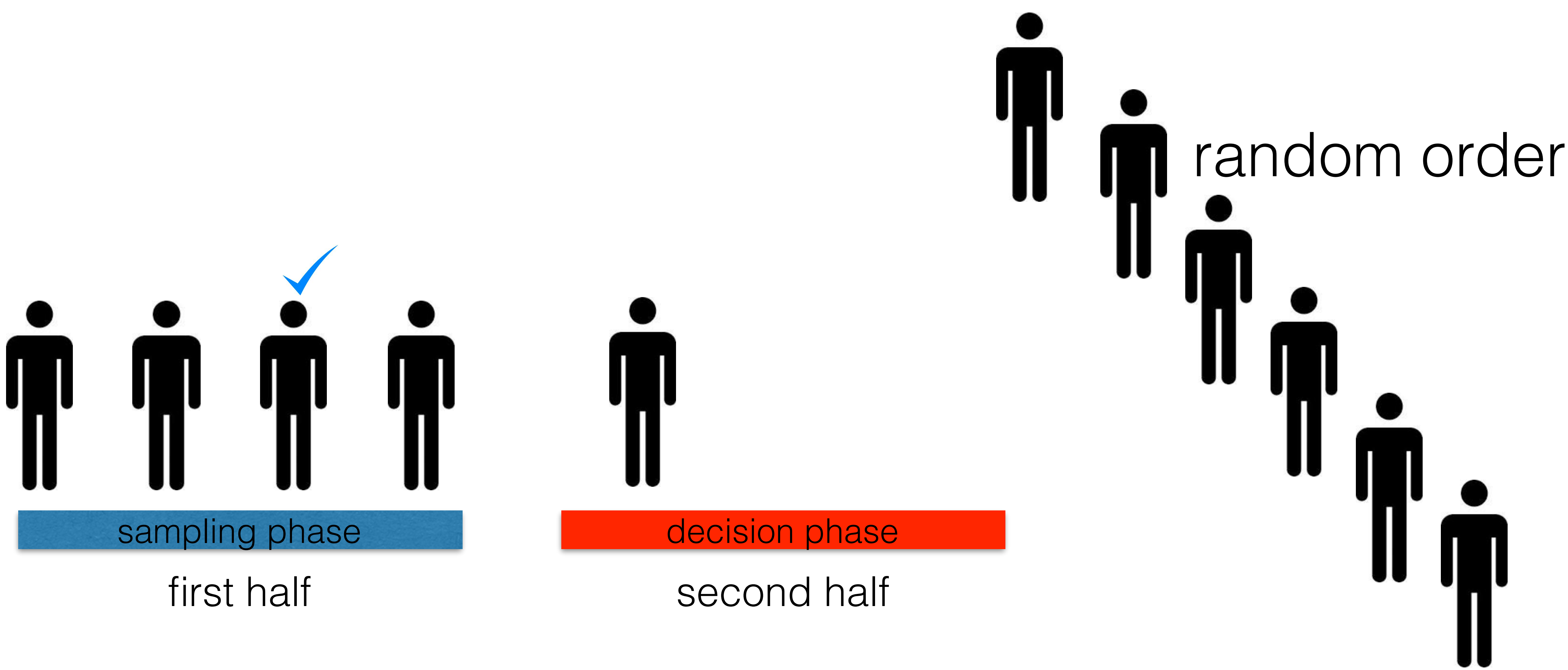
sampling phase

first half

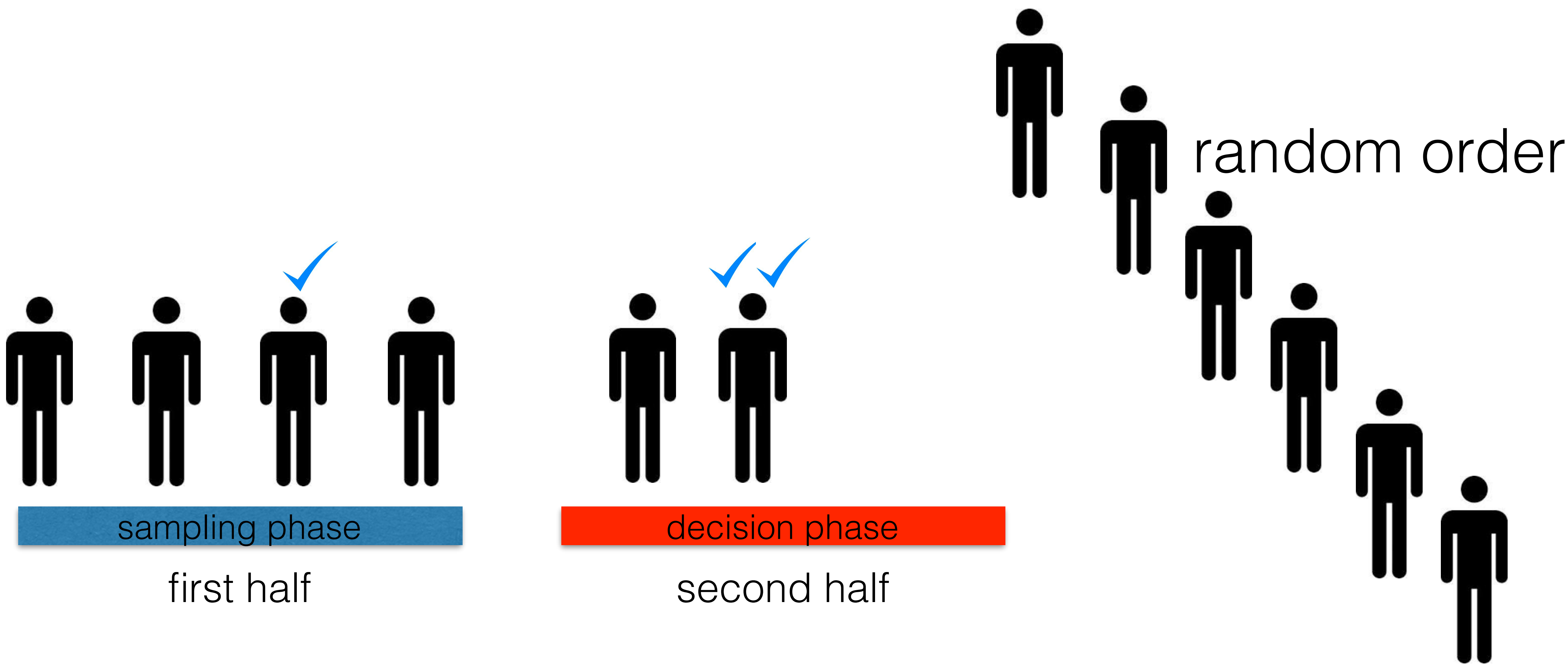
Hiring staff - not adversarial



Hiring staff - not adversarial

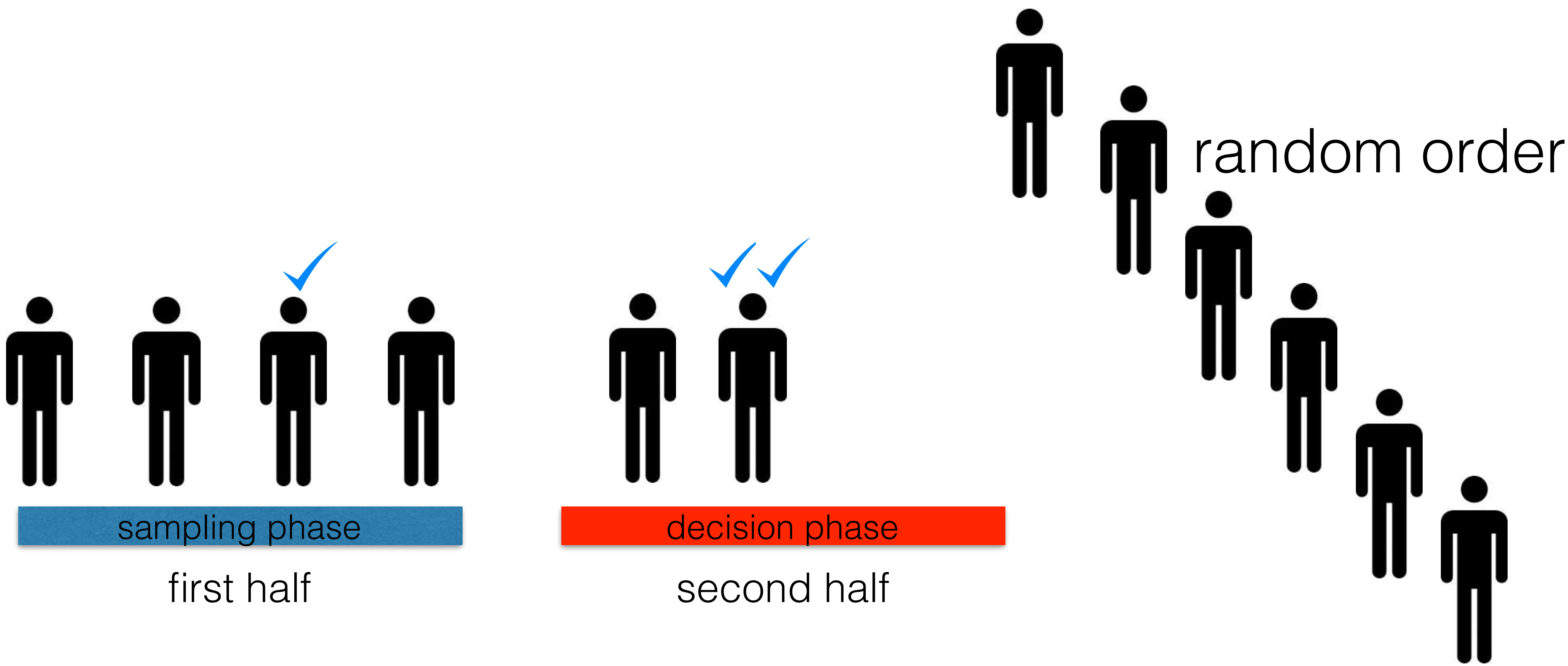


Hiring staff - not adversarial



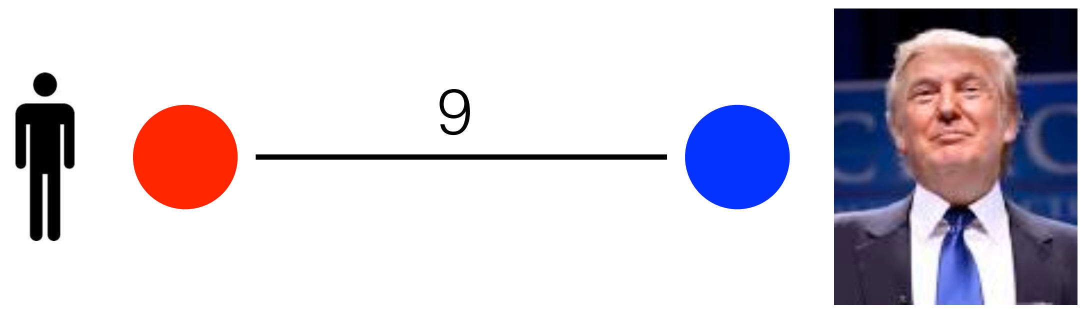
Hiring staff - not adversarial

Success with prob > 1/4

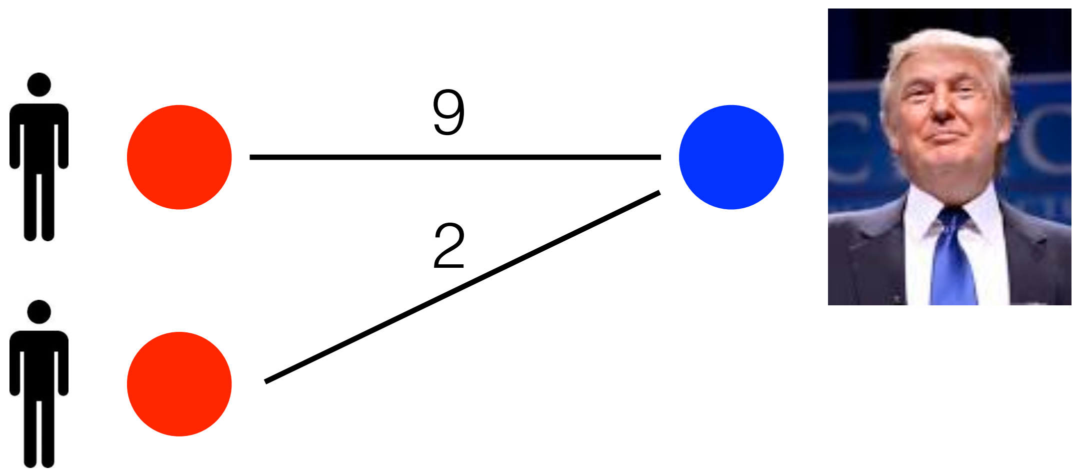


Actually Matching

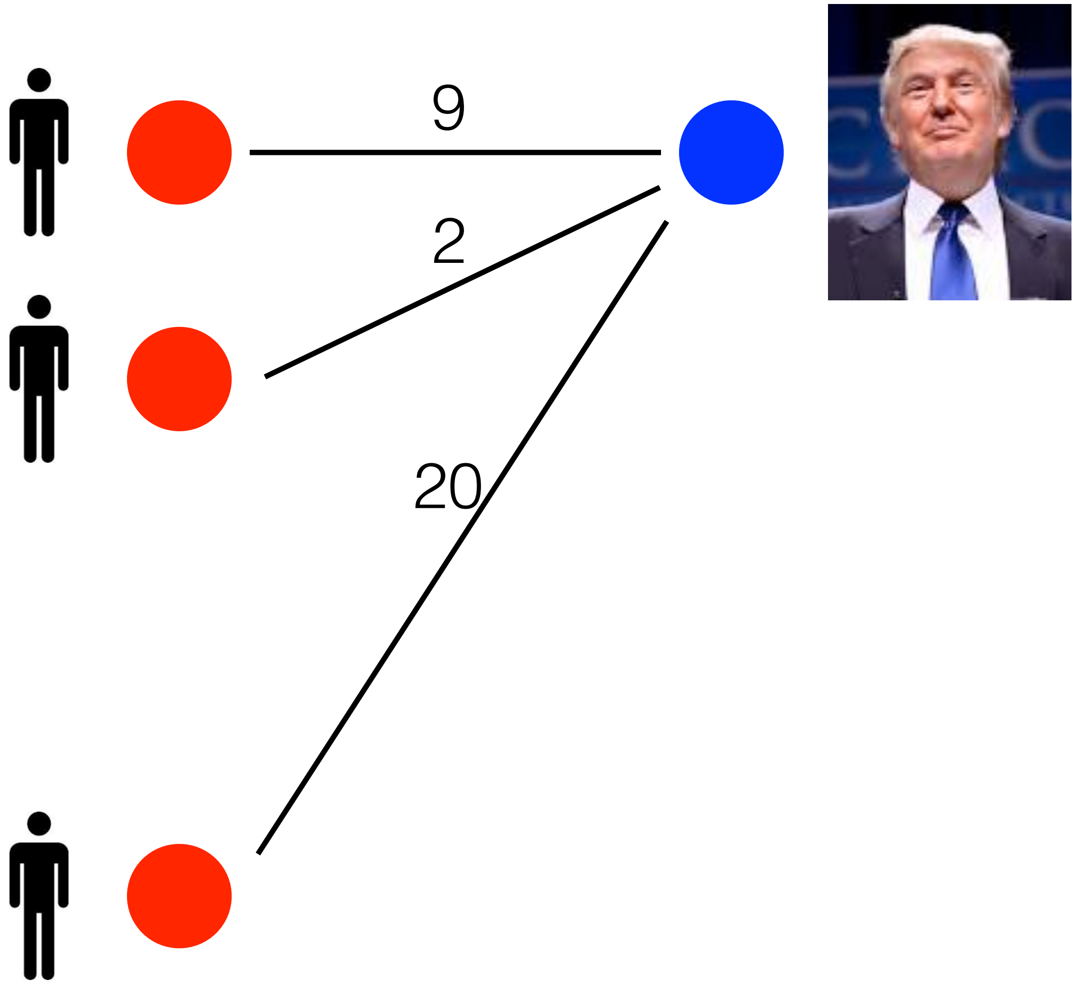
Actually Matching



Actually Matching

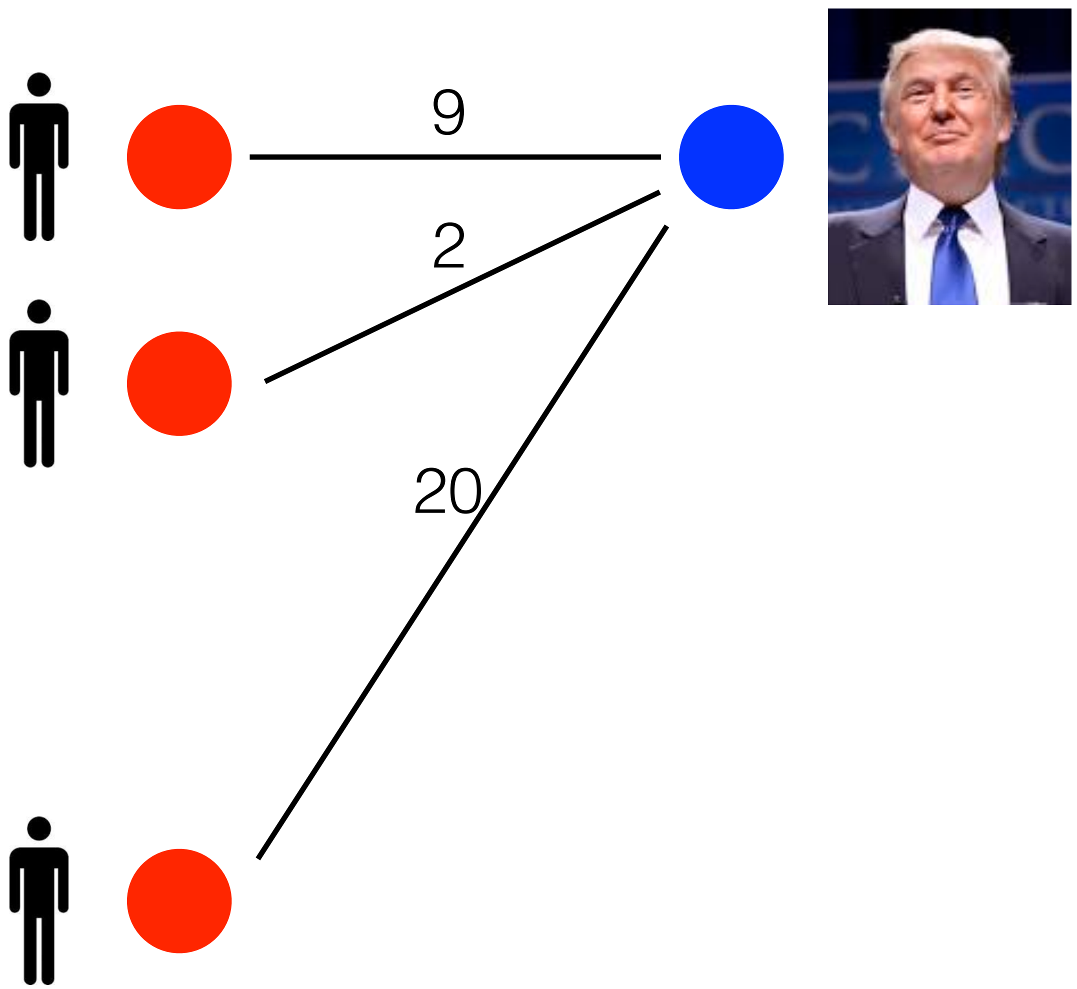


Actually Matching



Actually Matching

accept the edge with the largest weight instantaneously

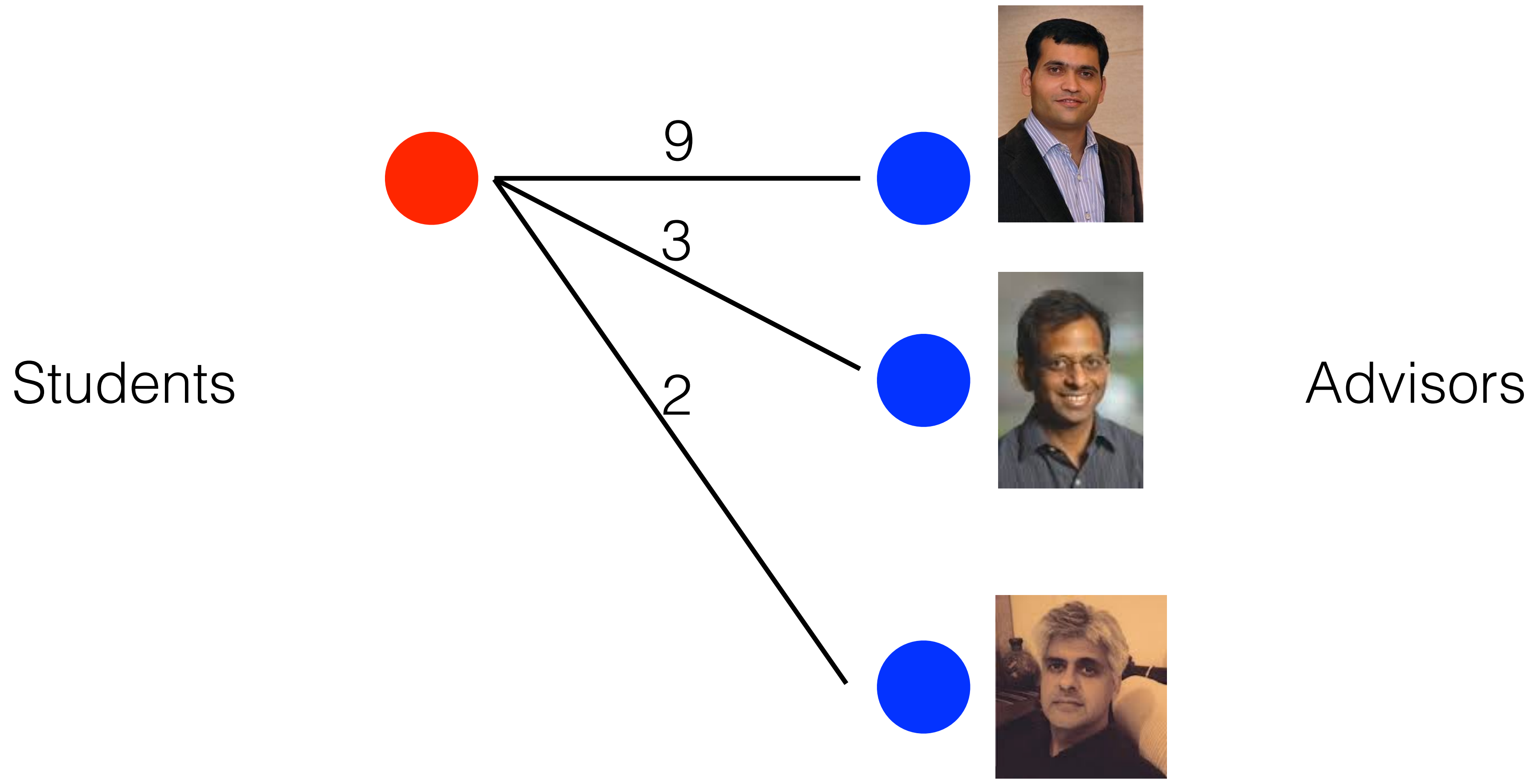


Natural Generalization

Students

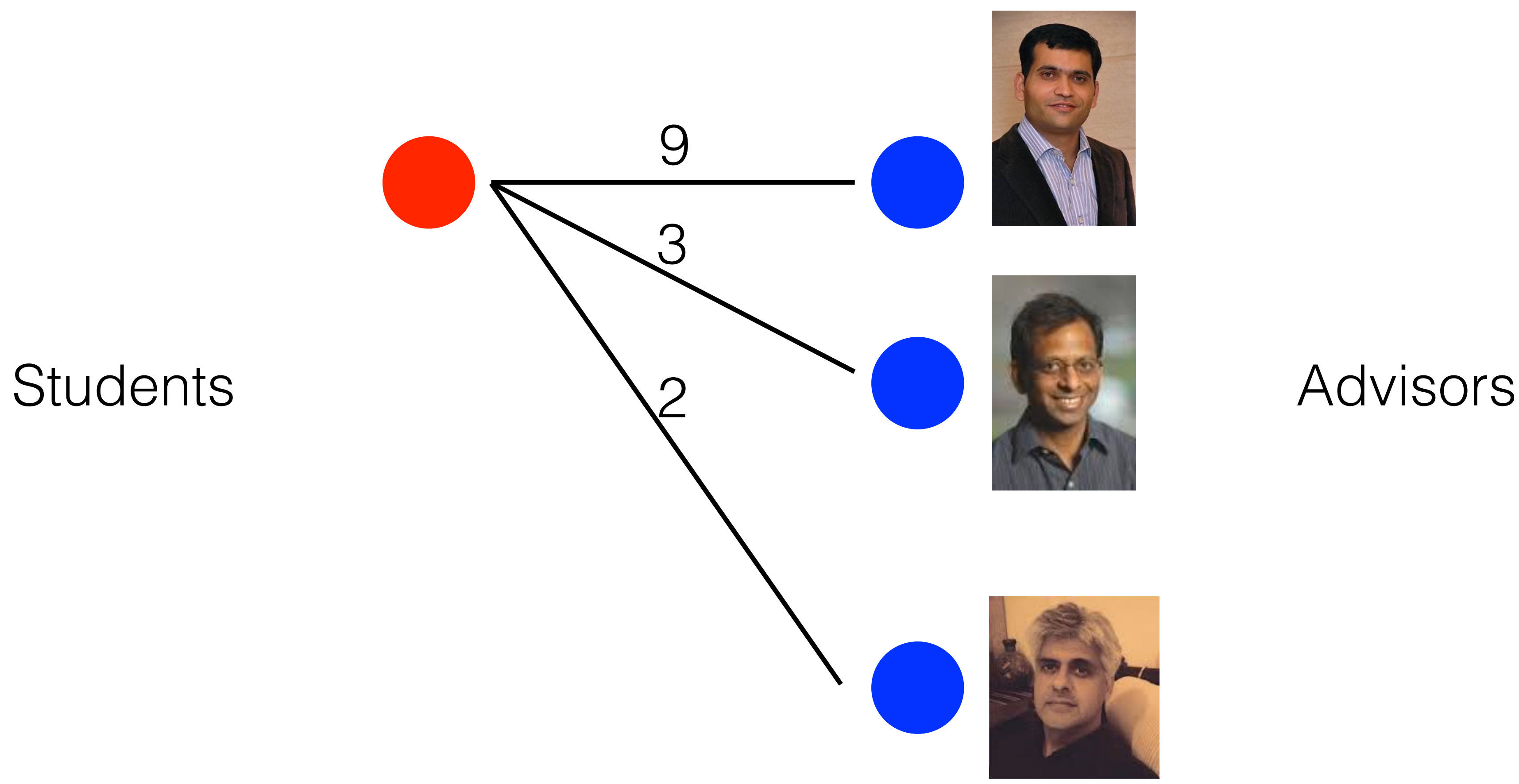
Advisors

Natural Generalization



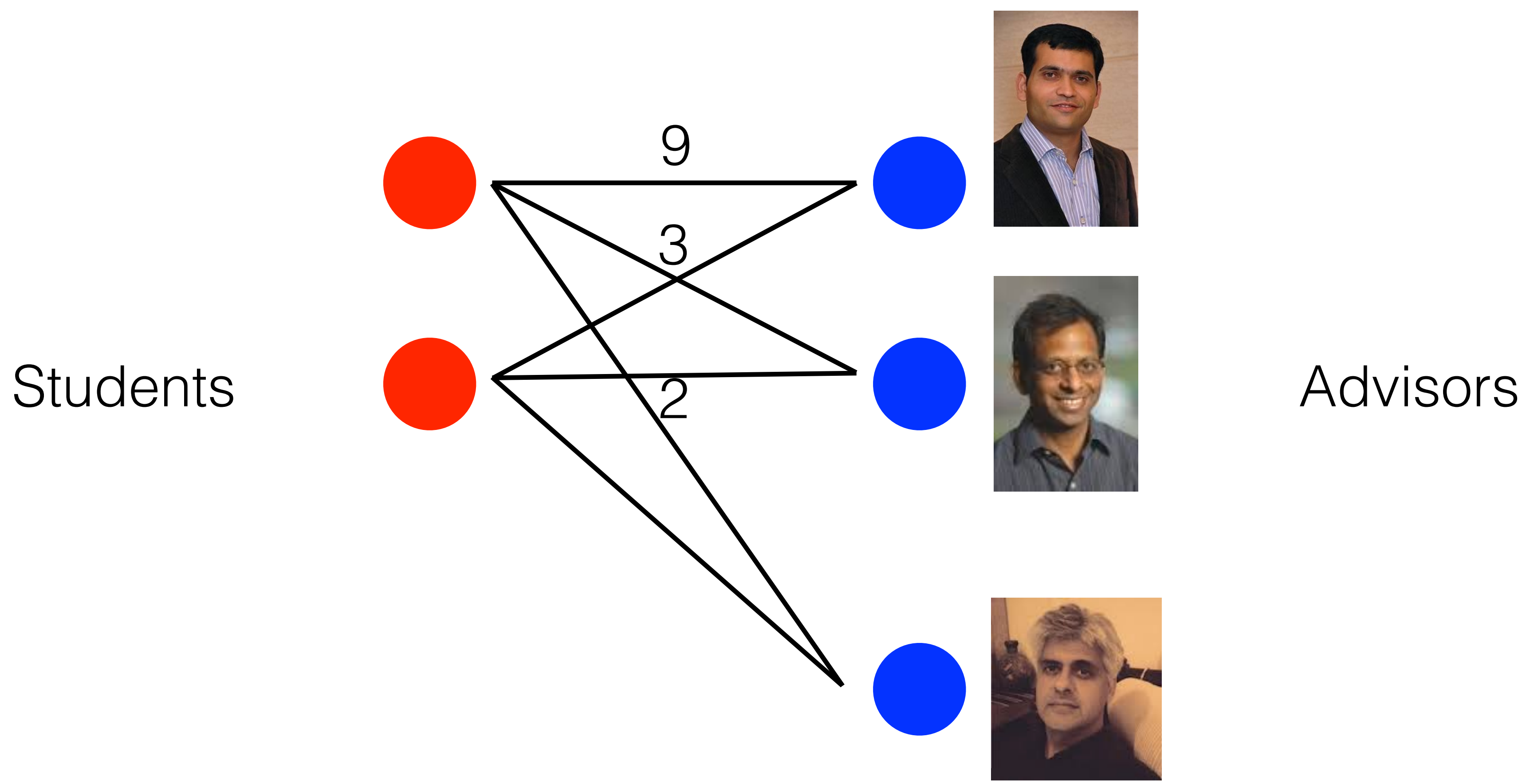
Natural Generalization

each advisor gets at most one student



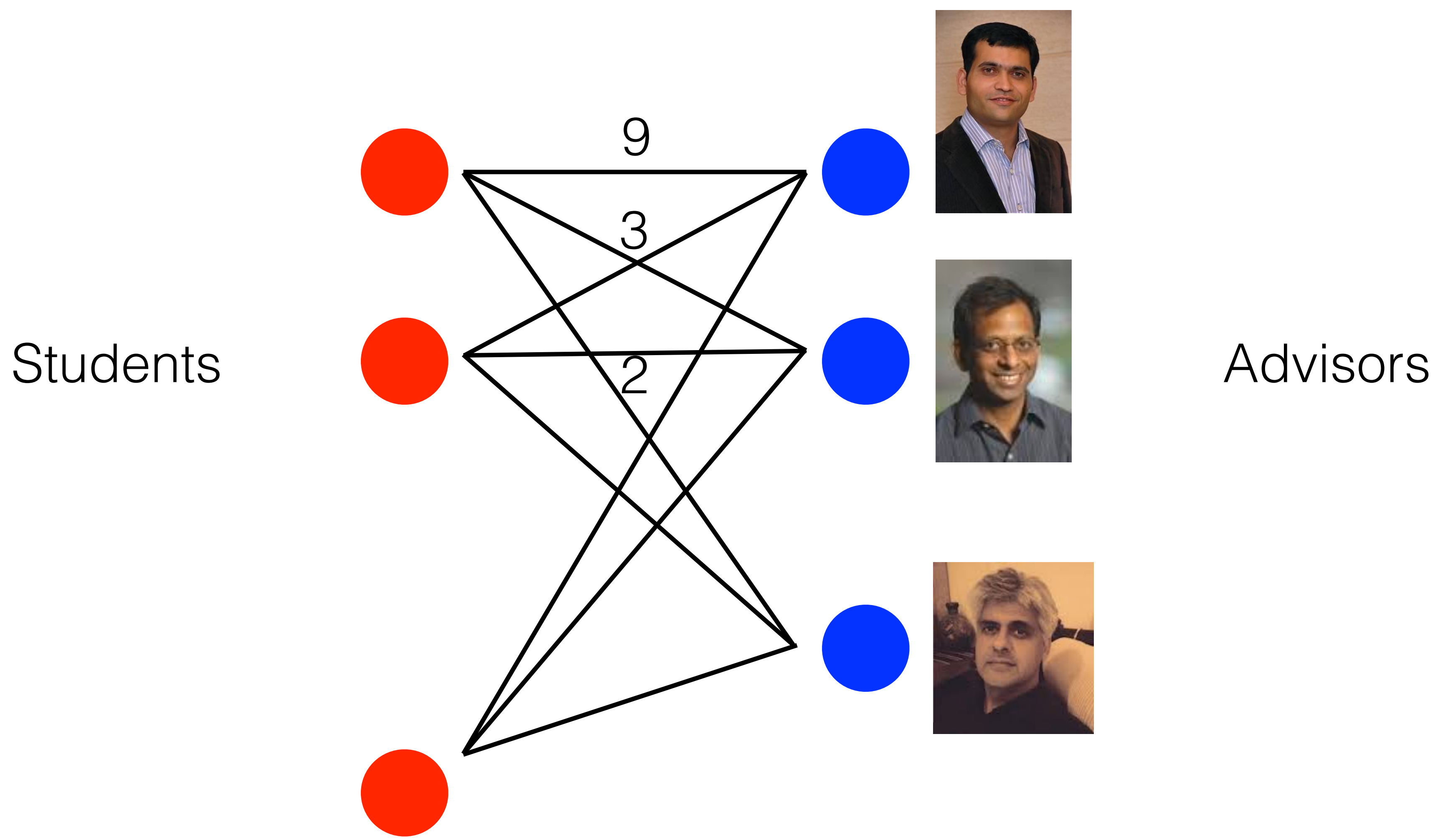
Natural Generalization

each advisor gets at most one student



Natural Generalization

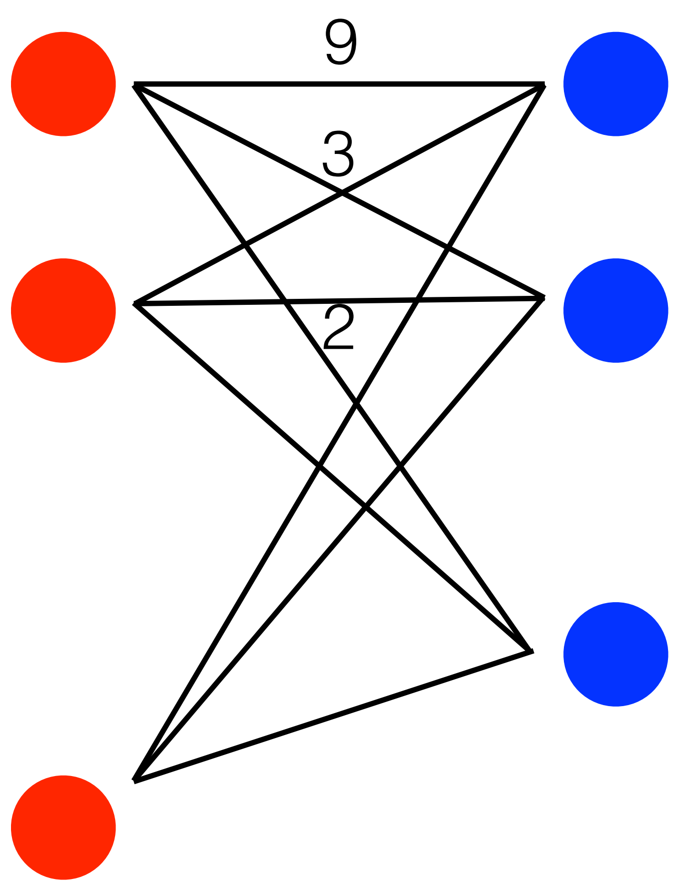
each advisor gets at most one student



Natural Generalization

each advisor gets at most one student - allocation made by 

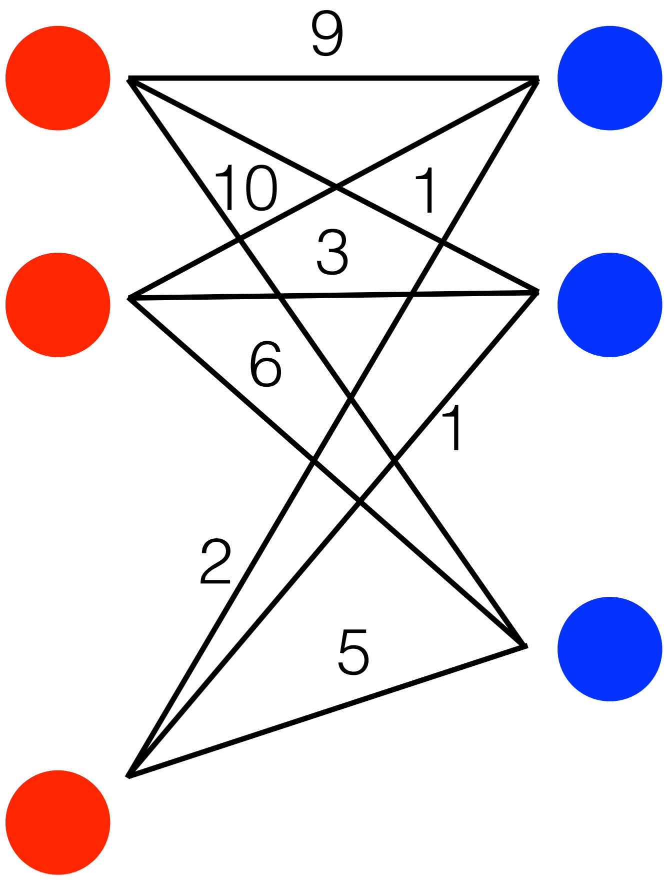
Students



Advisors

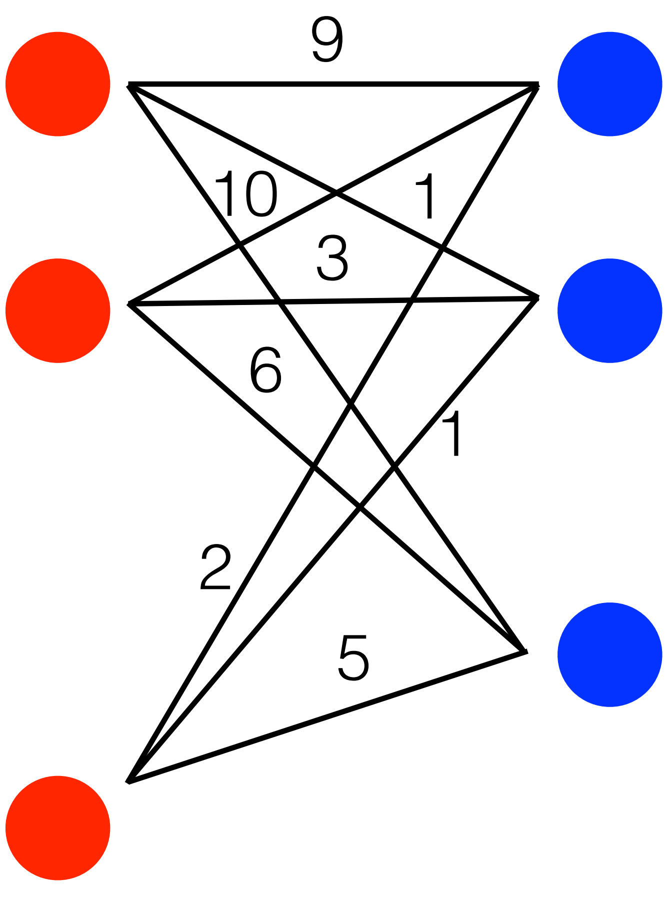
Objective: Matching with largest sum weight

Example



Example

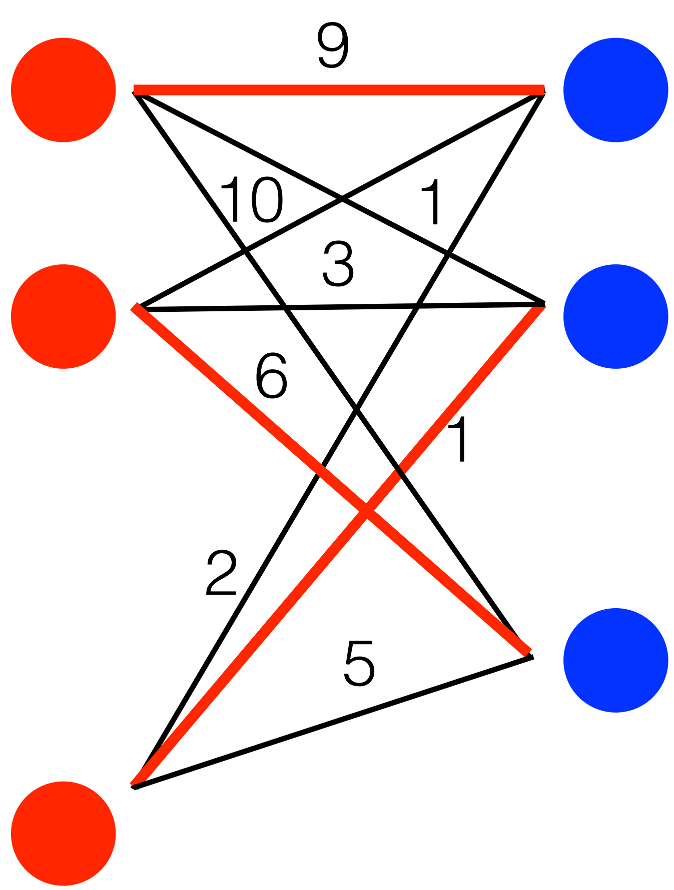
at most one accepted edge



Objective: Matching with largest sum weight

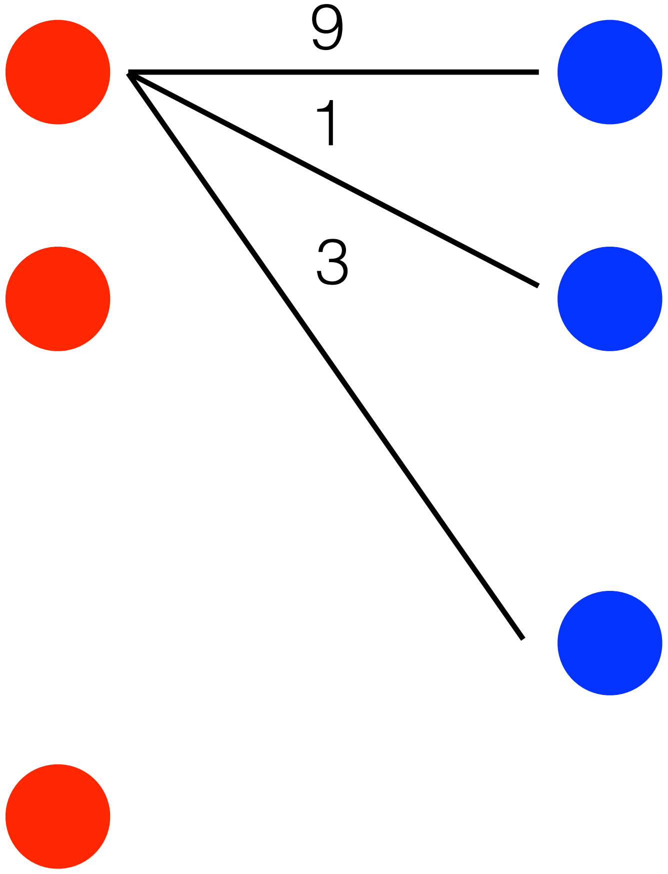
Example

at most one accepted edge

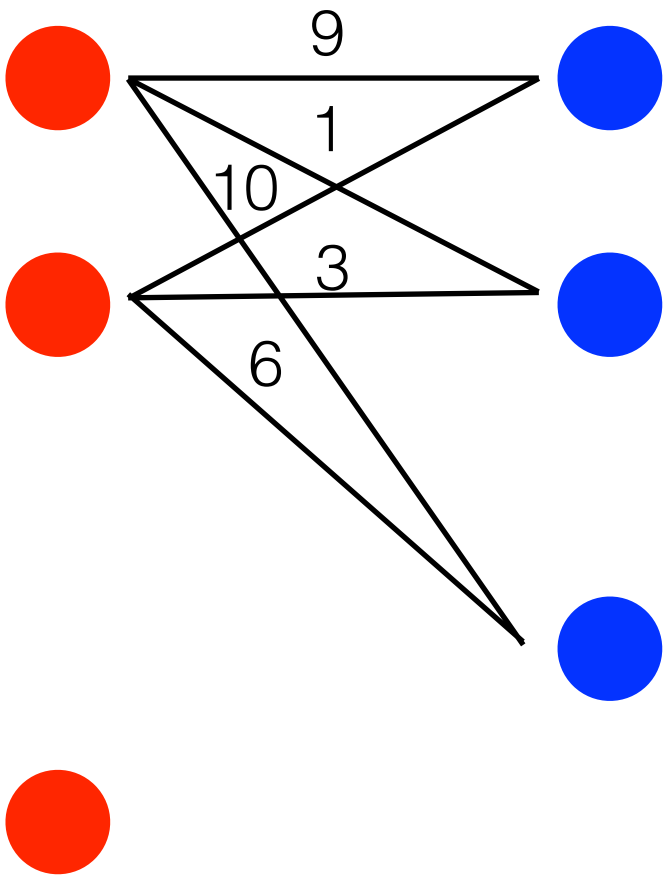


Objective: Matching with largest sum weight

How to solve this ONLINE

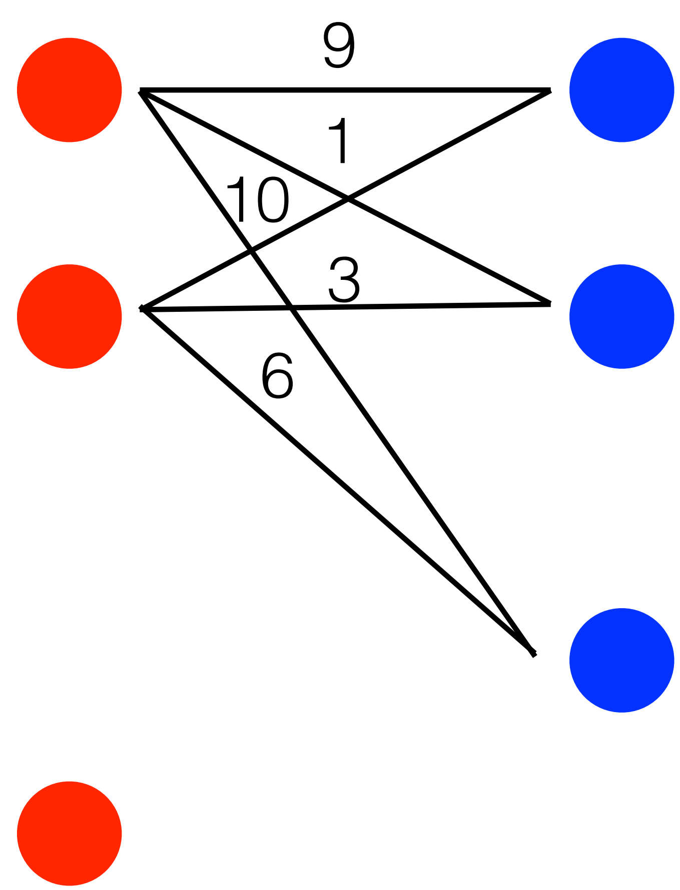


How to solve this ONLINE



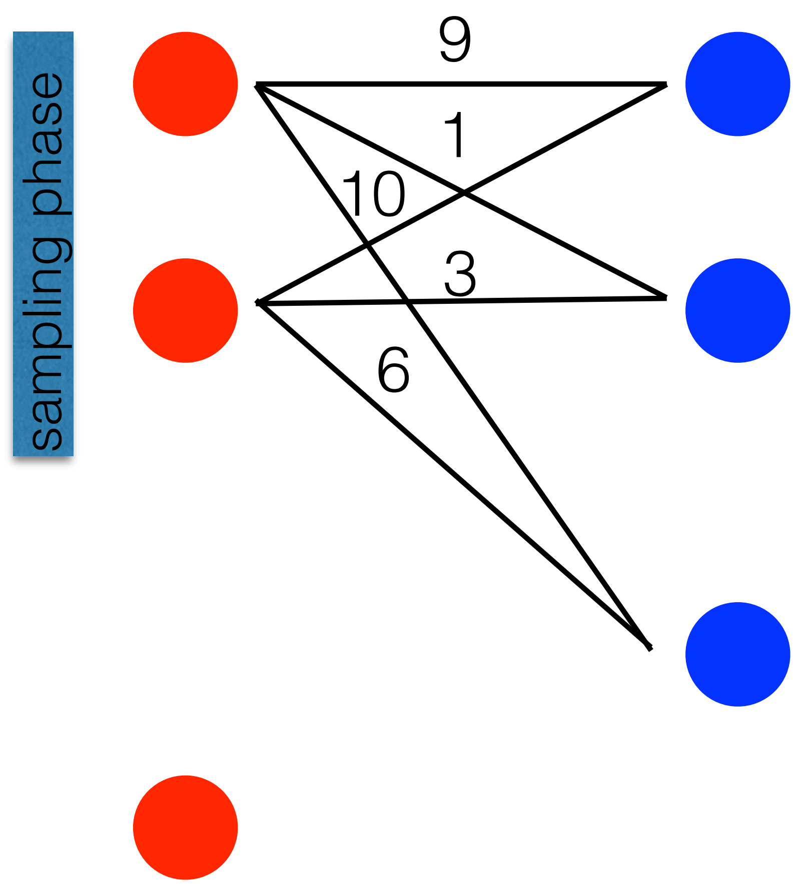
How to solve this ONLINE

Sampling idea as before



How to solve this ONLINE

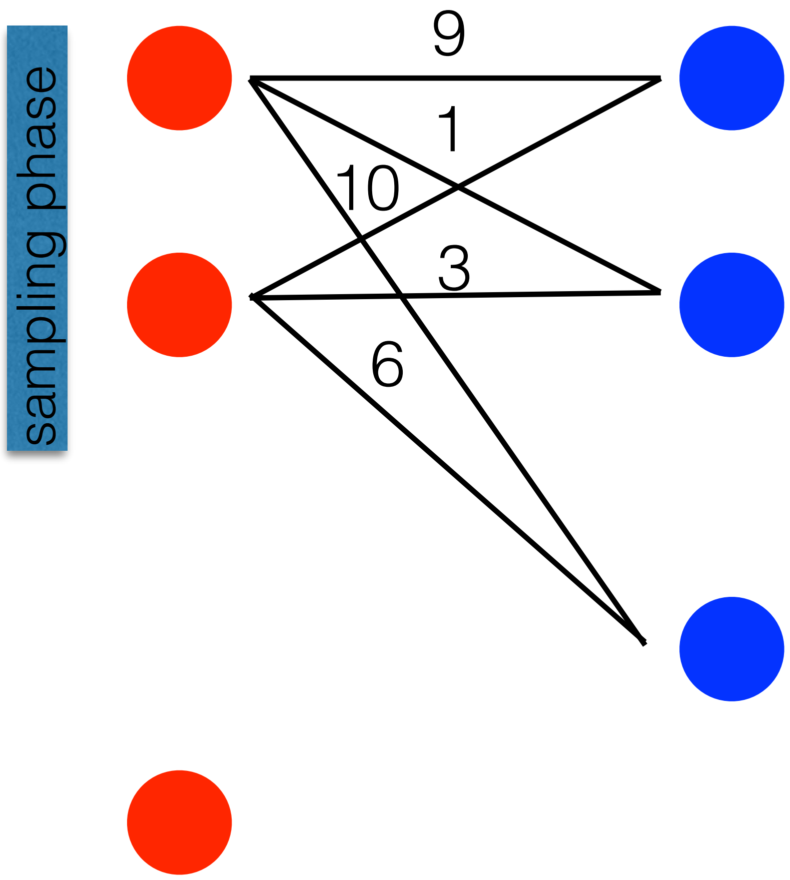
Sampling idea as before



How to solve this ONLINE

Sampling idea as before

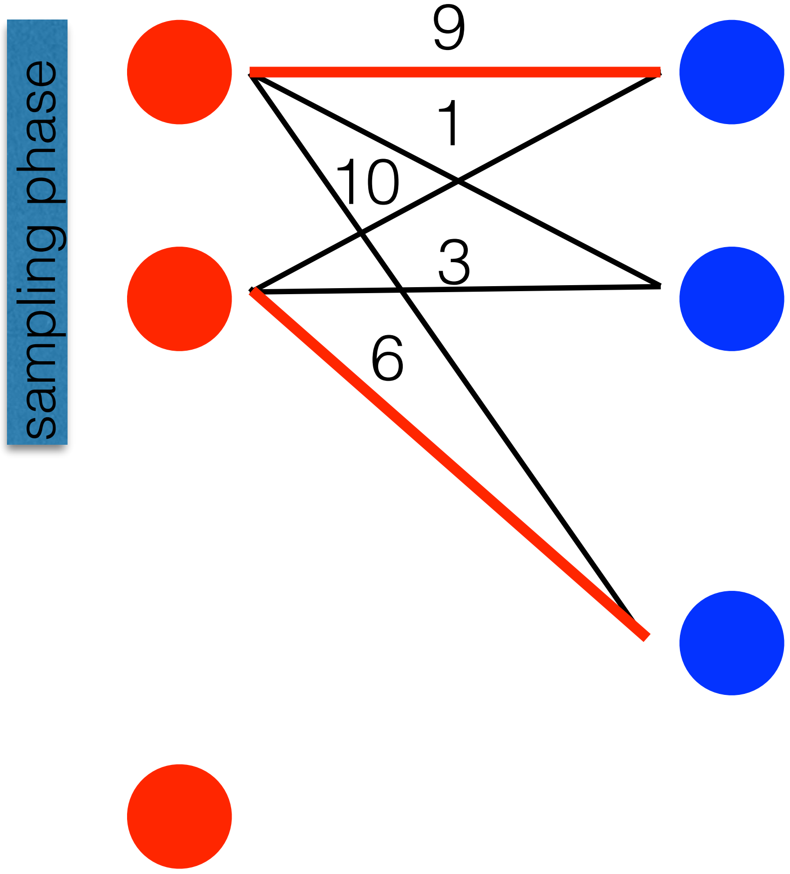
Find best matching



How to solve this ONLINE

Sampling idea as before

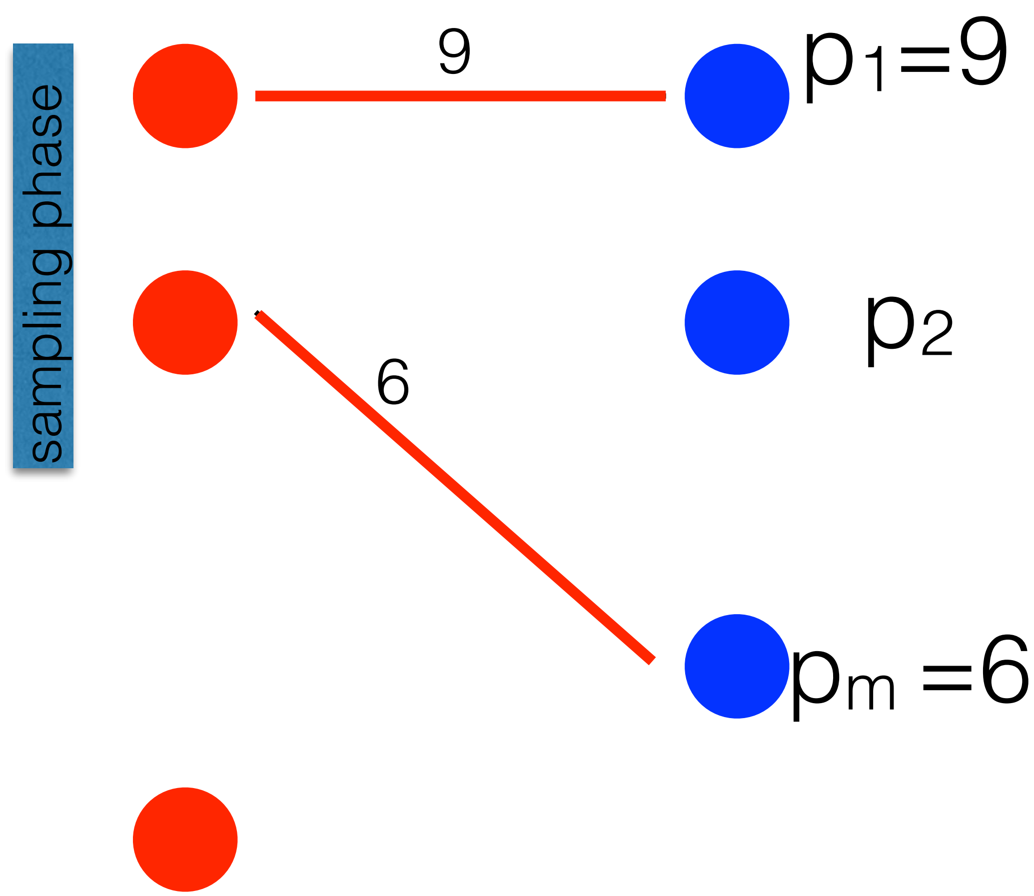
Find best matching



How to solve this ONLINE

Sampling idea as before

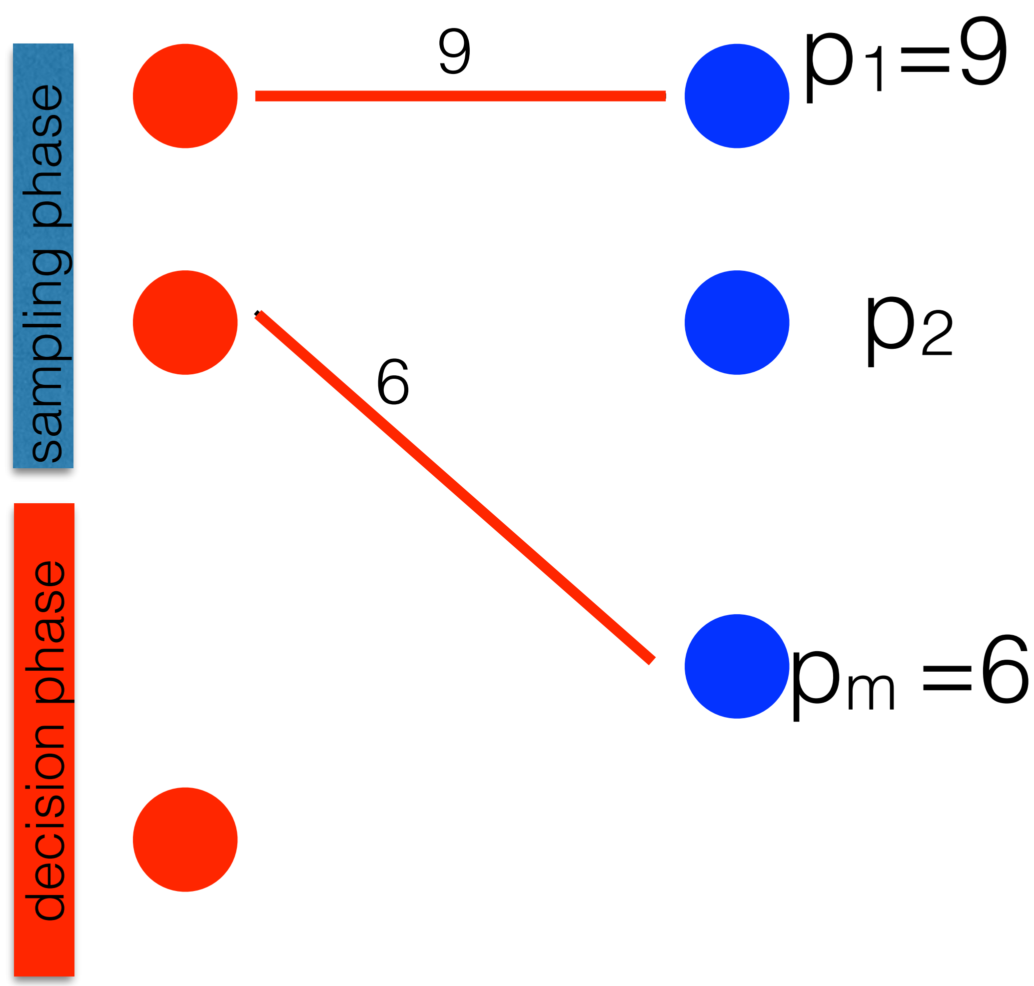
Find best matching



How to solve this ONLINE

Sampling idea as before

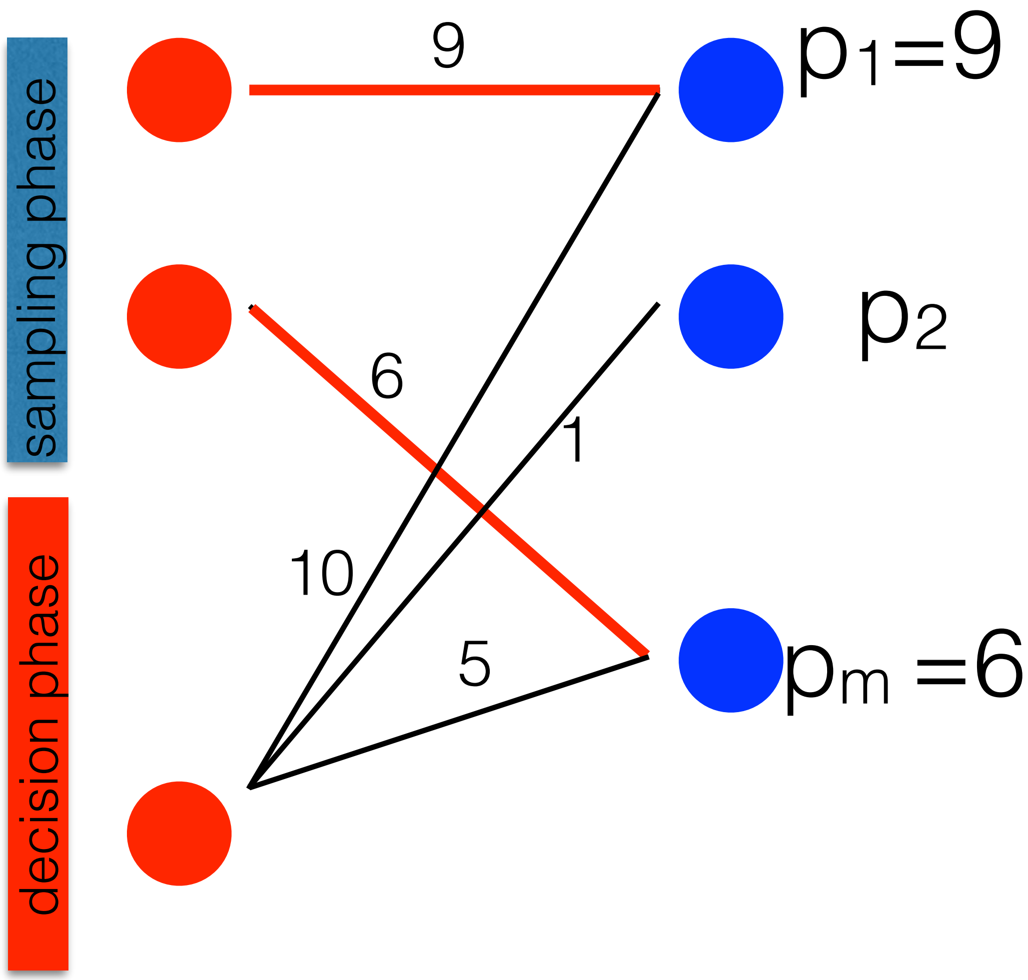
Find best matching



How to solve this ONLINE

Sampling idea as before

Find best matching

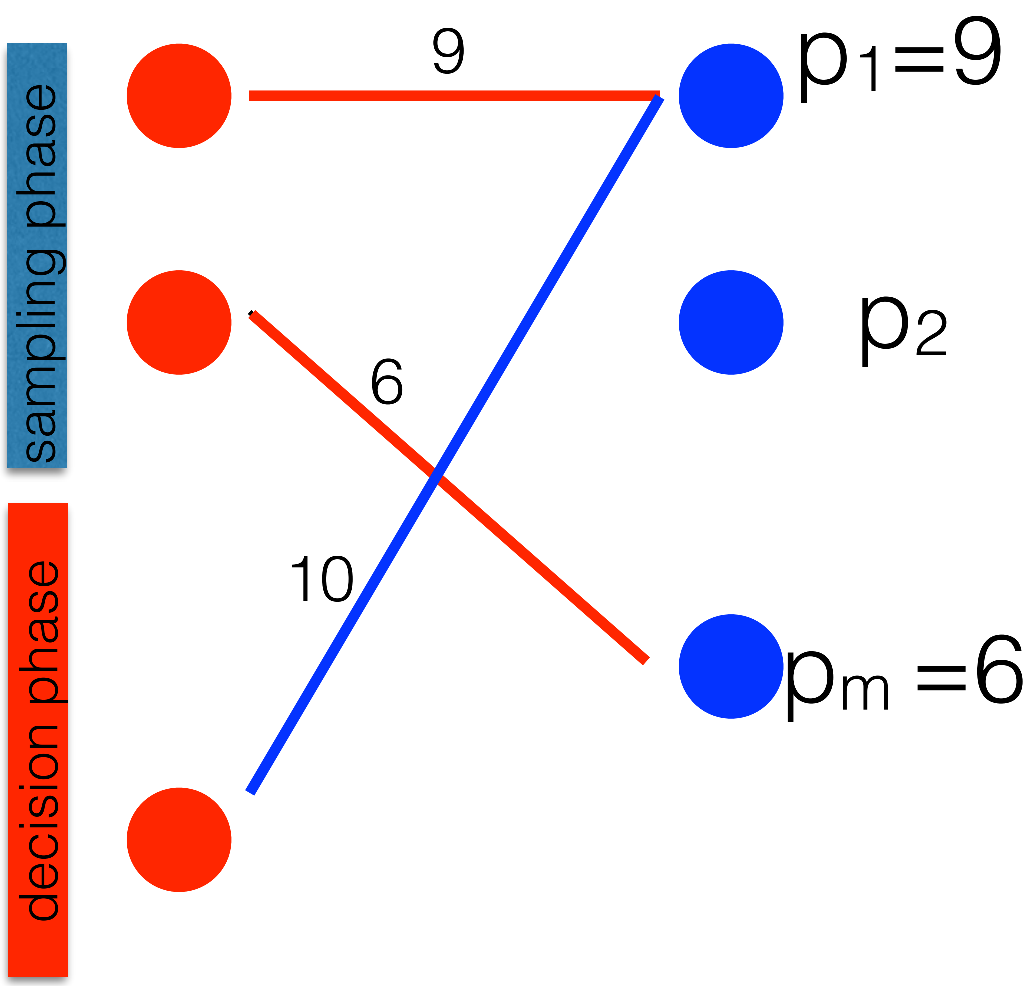


How to solve this ONLINE

Sampling idea as before

Find best matching

match the largest weight more than price

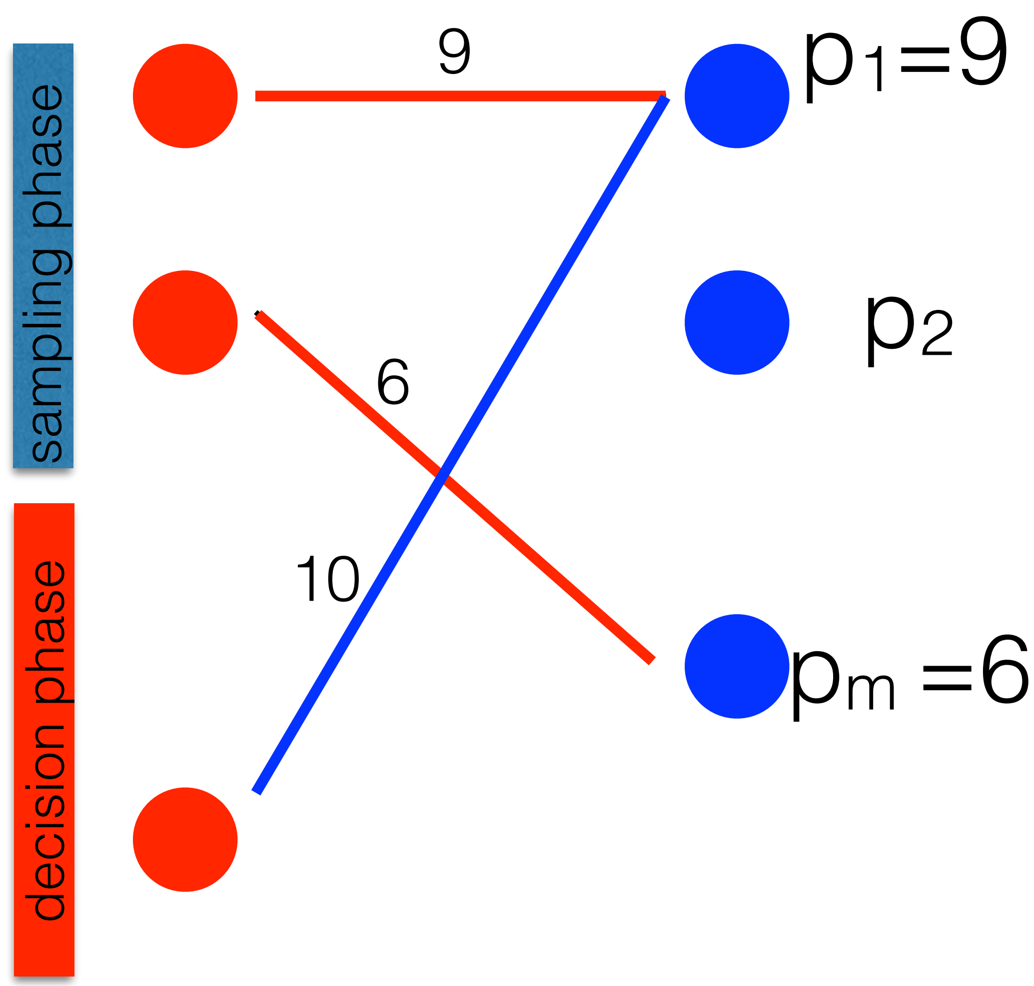


How to solve this ONLINE

Sampling idea as before

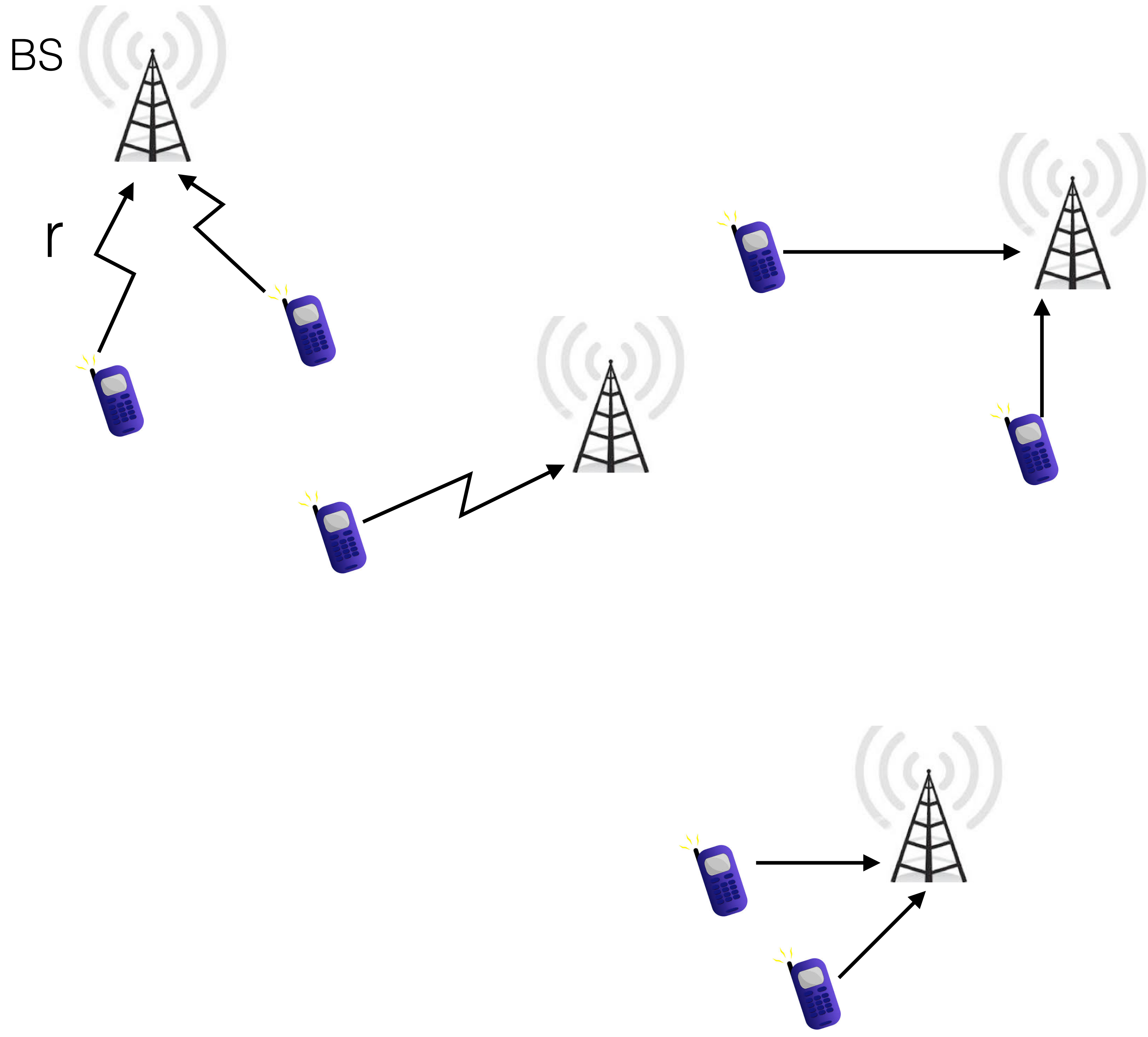
Find best matching

match the largest weight
more than price

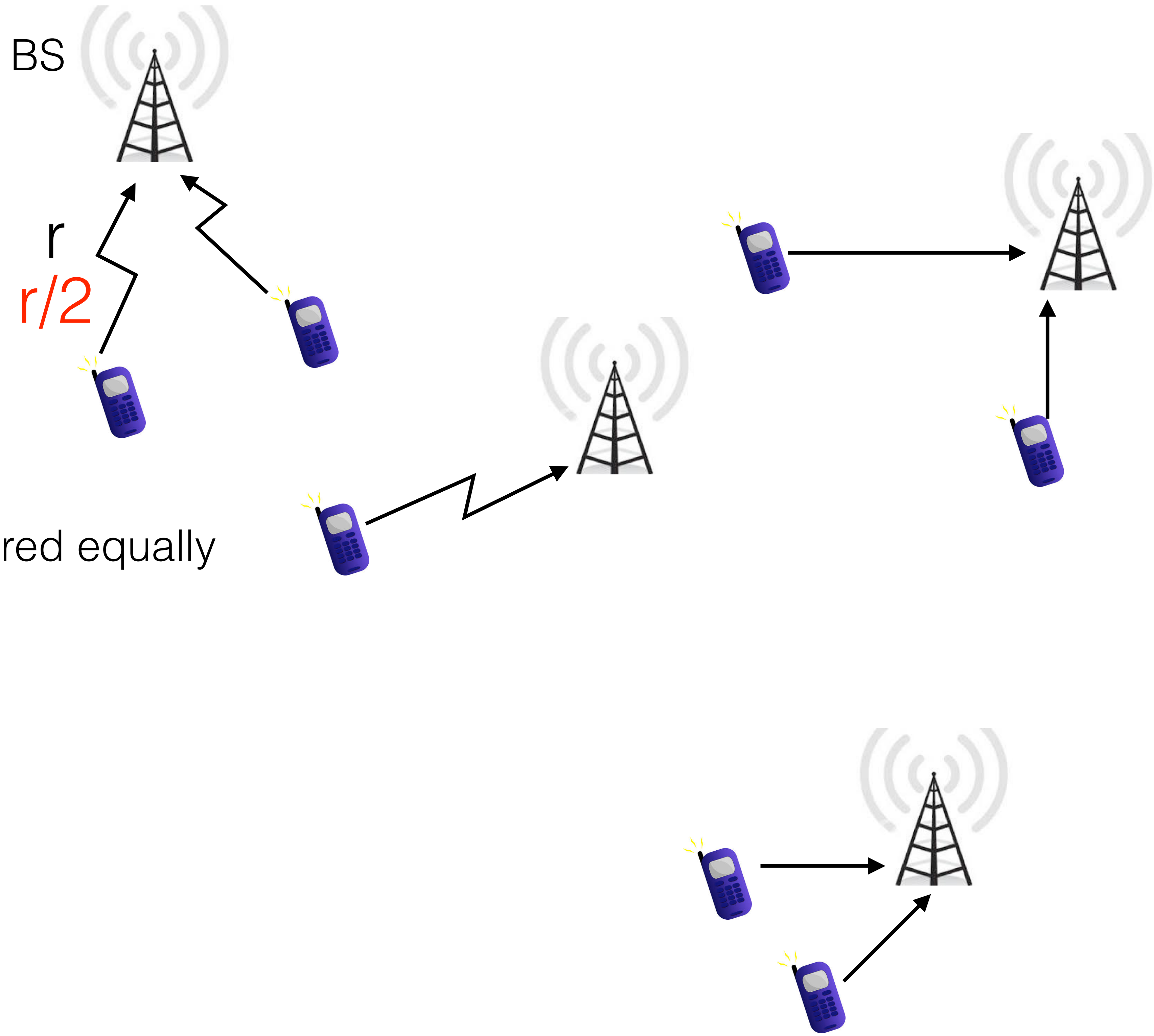


Result: 8-competitive/optimal [*Korula, Pal' 08***]**

Legacy Problem -Wireless Communication

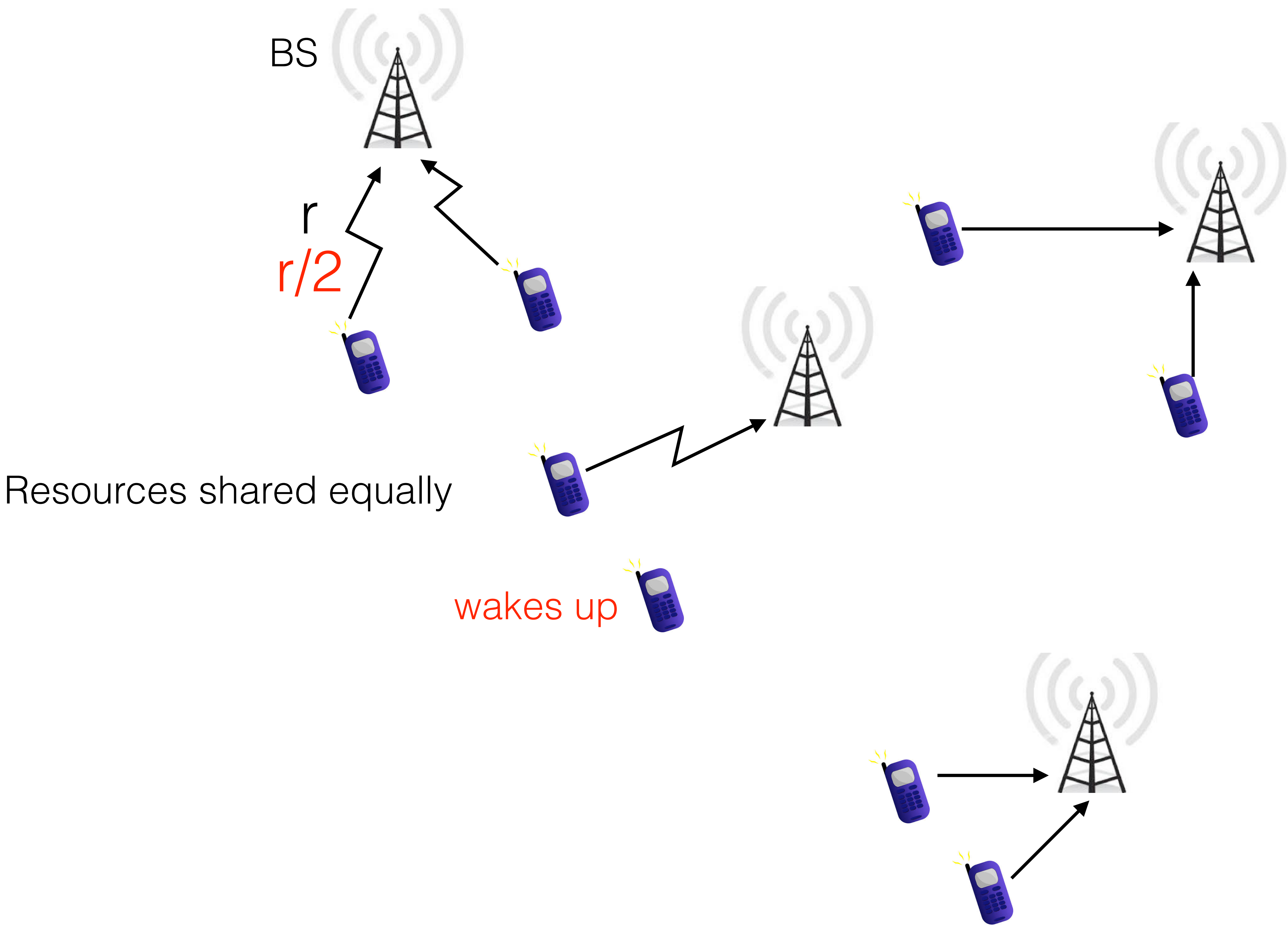


Legacy Problem -Wireless Communication

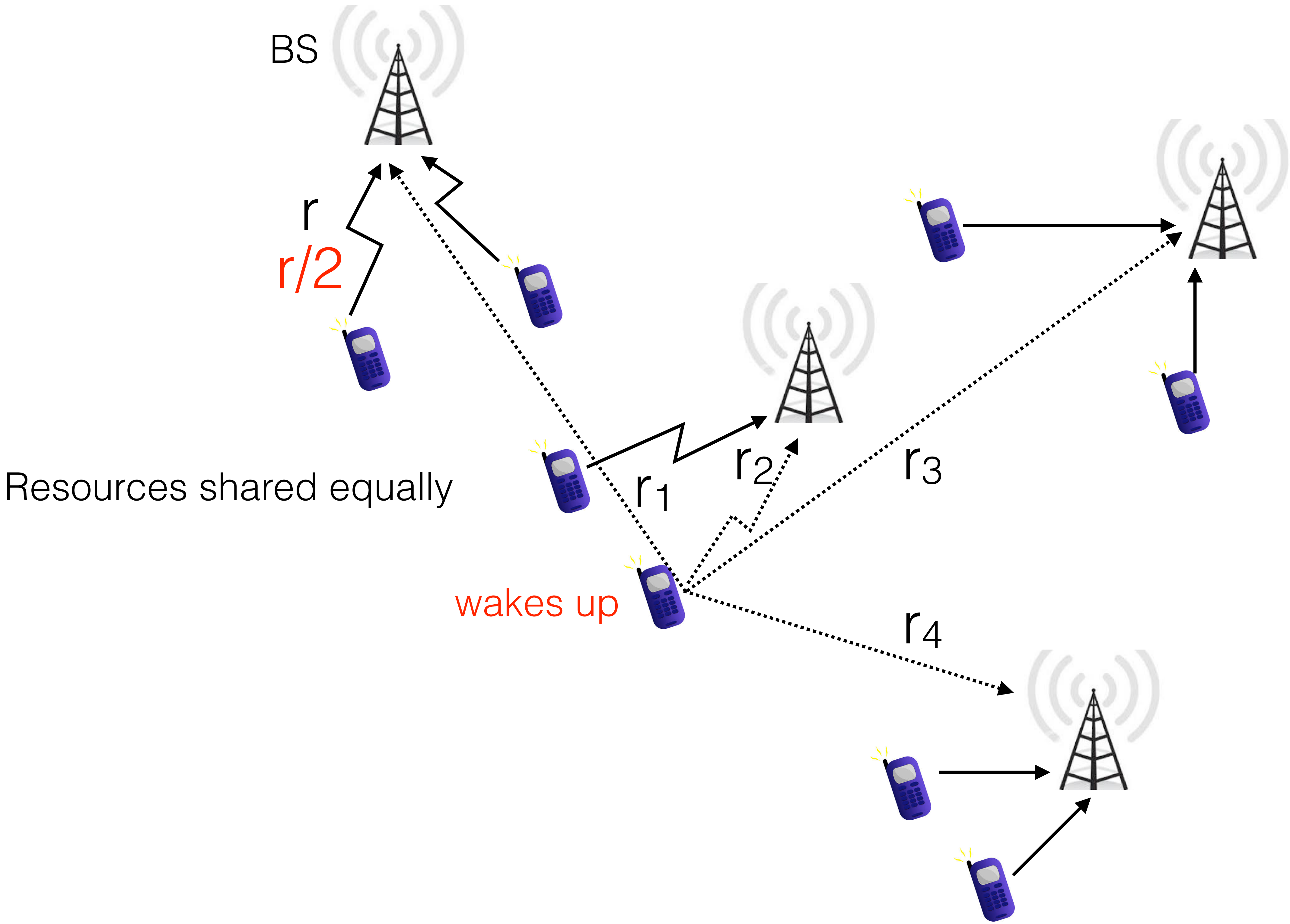


Resources shared equally

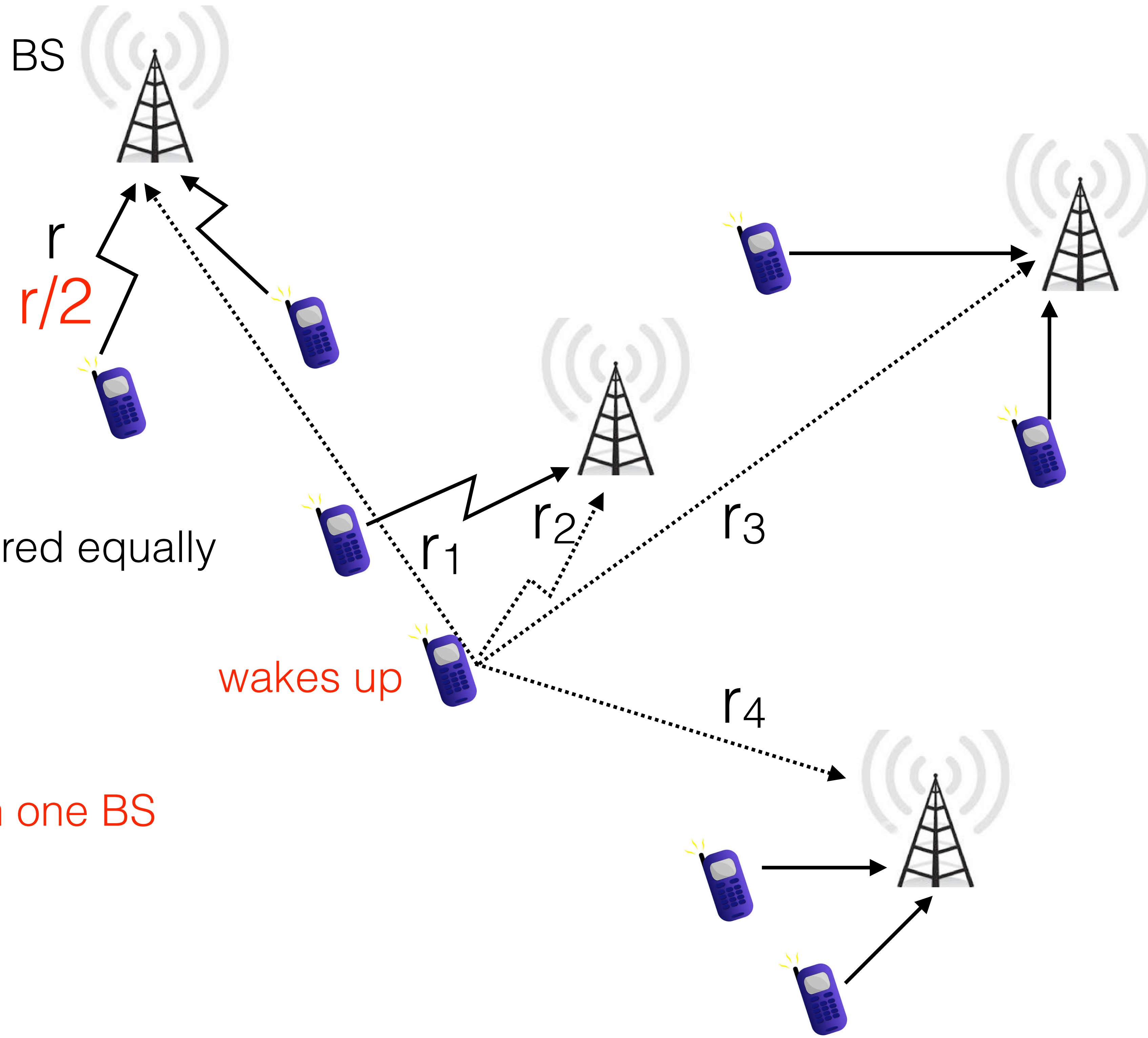
Legacy Problem -Wireless Communication



Legacy Problem -Wireless Communication



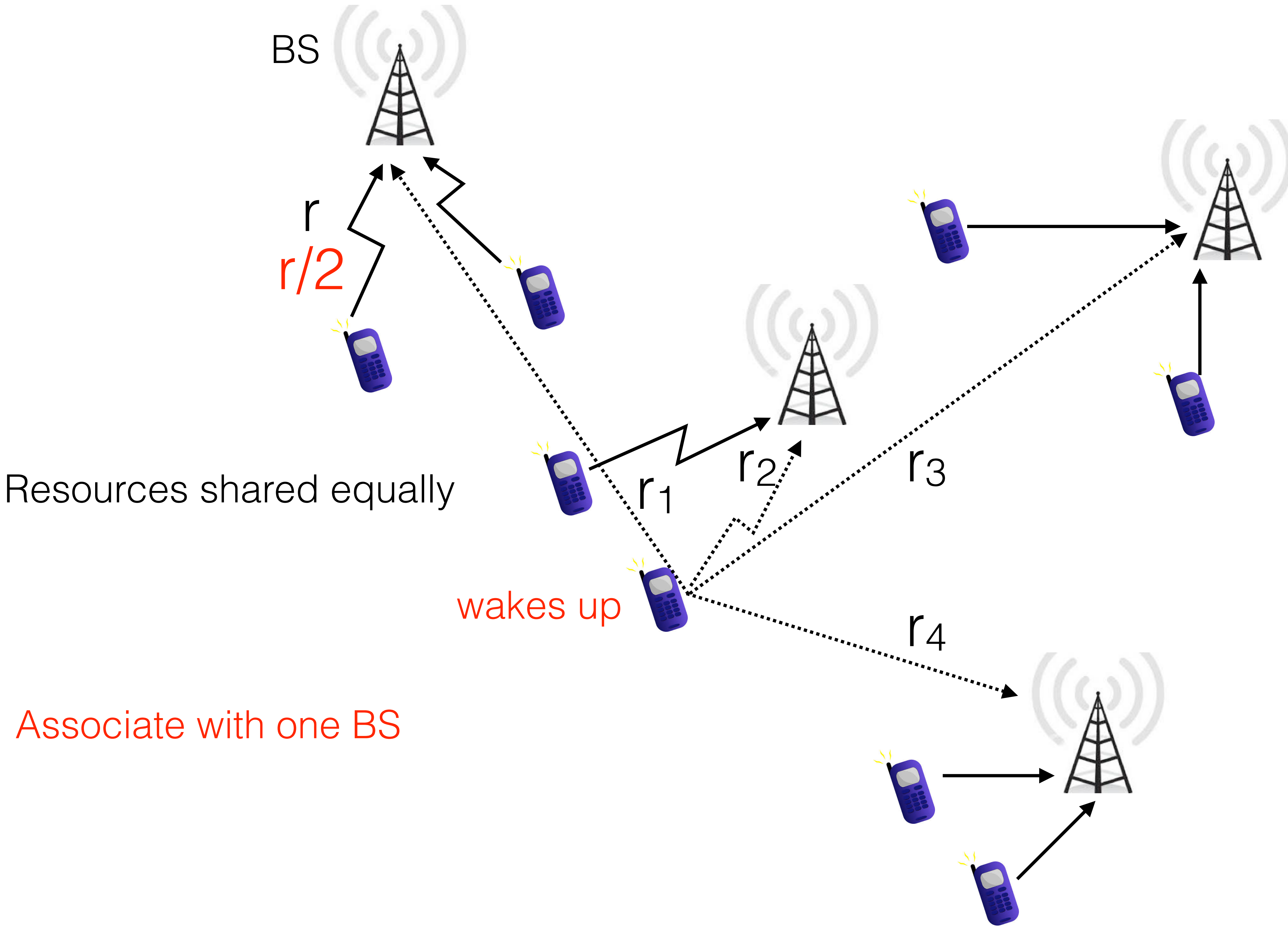
Legacy Problem -Wireless Communication



Resources shared equally

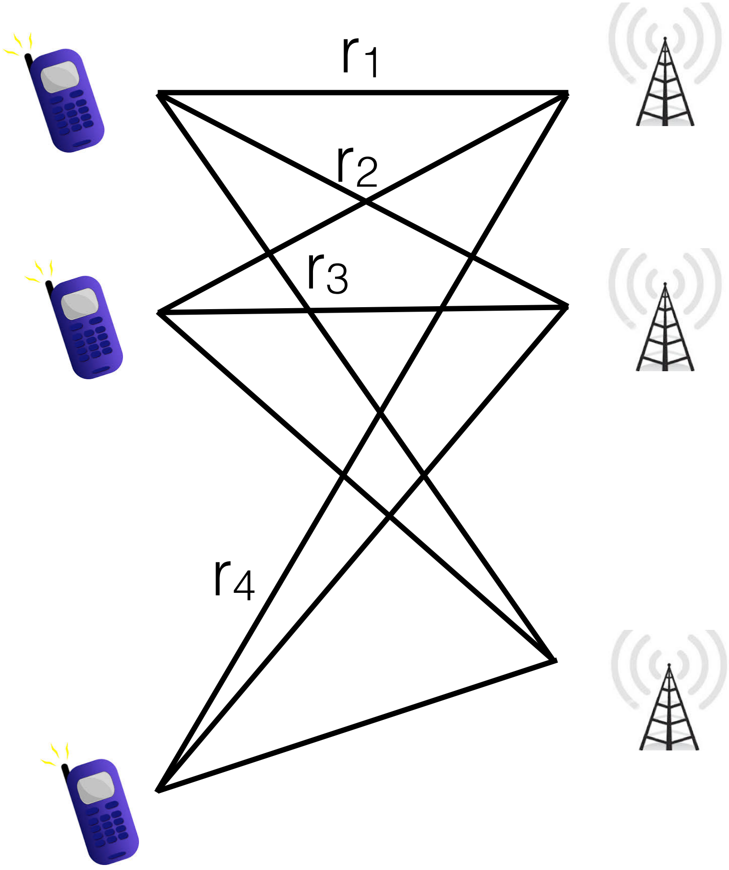
Associate with one BS

Legacy Problem -Wireless Communication



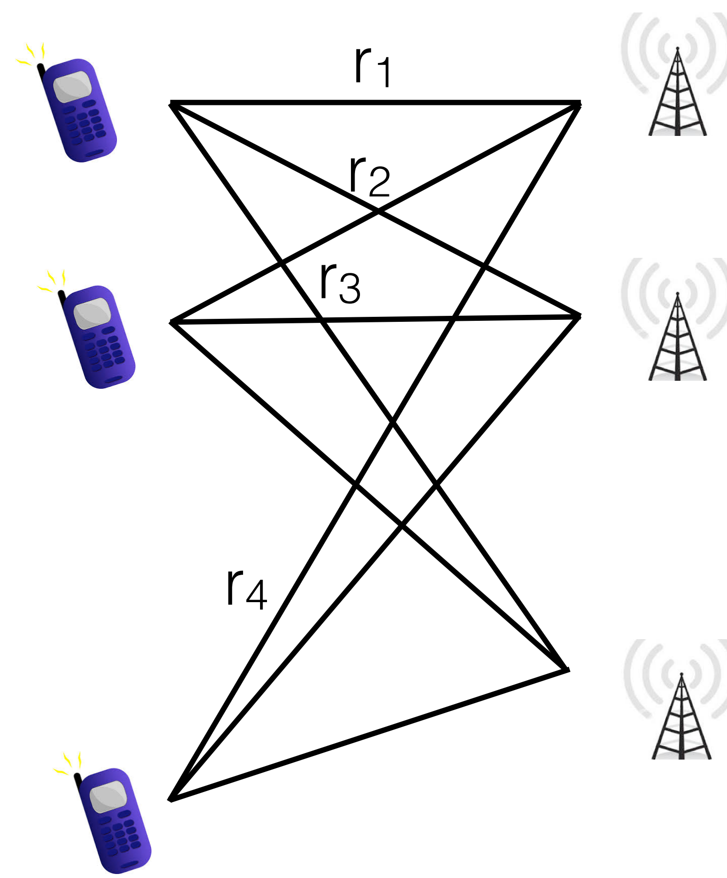
Find optimal BS allocation to maximize sum-rate

Example



Example

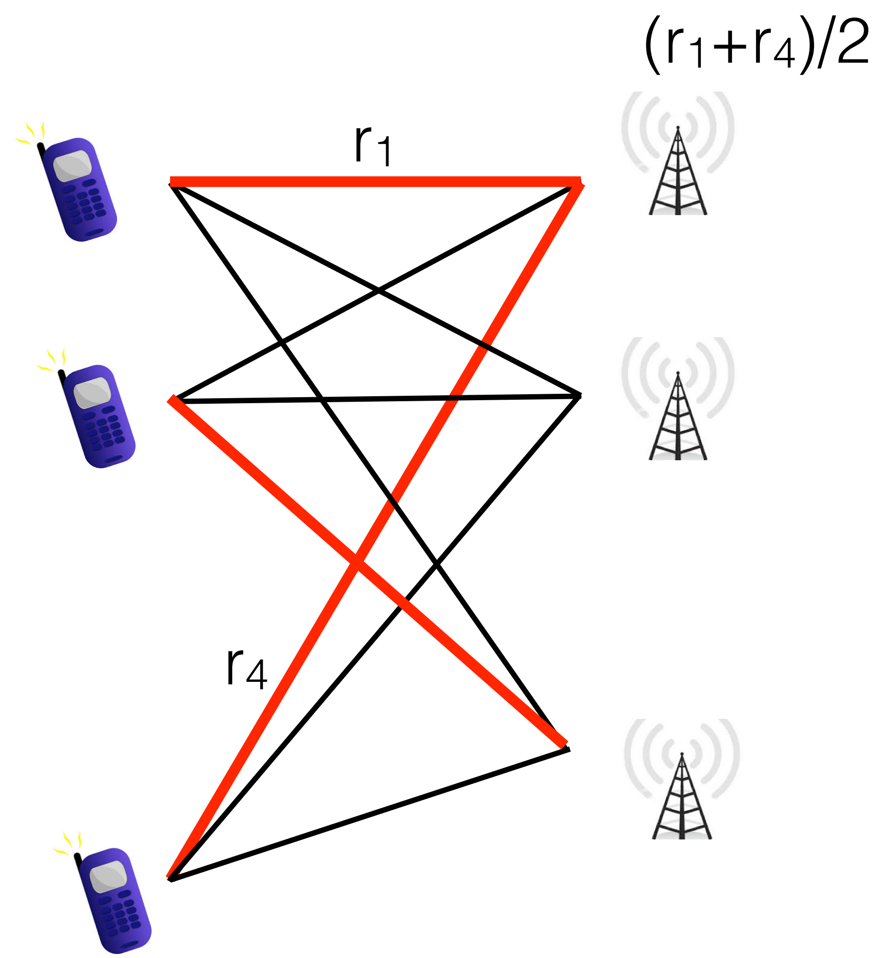
Still Interested in largest sum-weight but **No** longer MATCHING



Objective: Association with largest sum weight

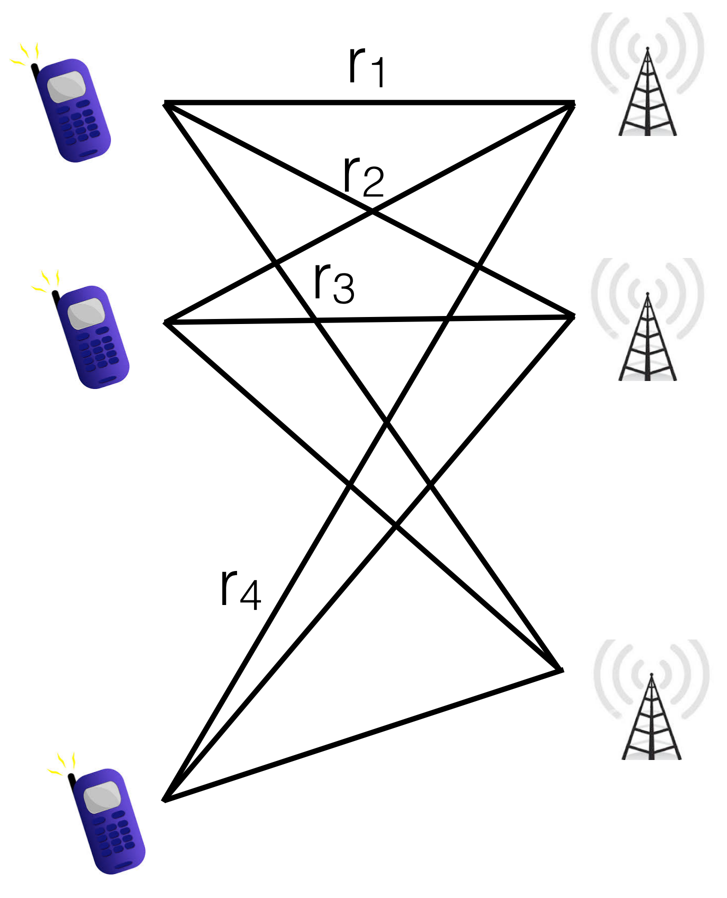
Example

Still Interested in largest sum-weight but **No** longer MATCHING



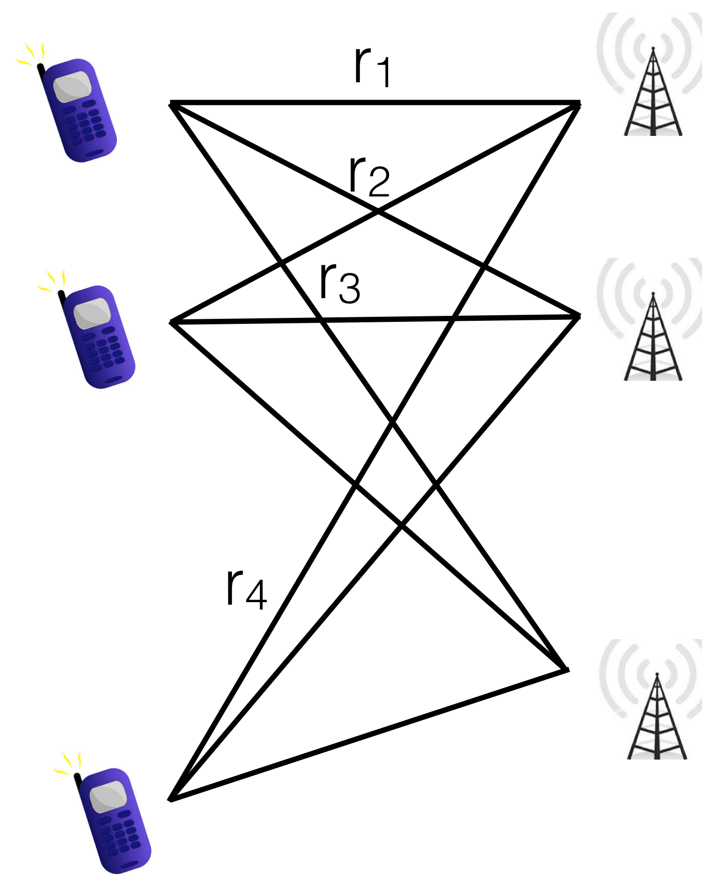
Objective: Association with largest sum weight

Important Observation



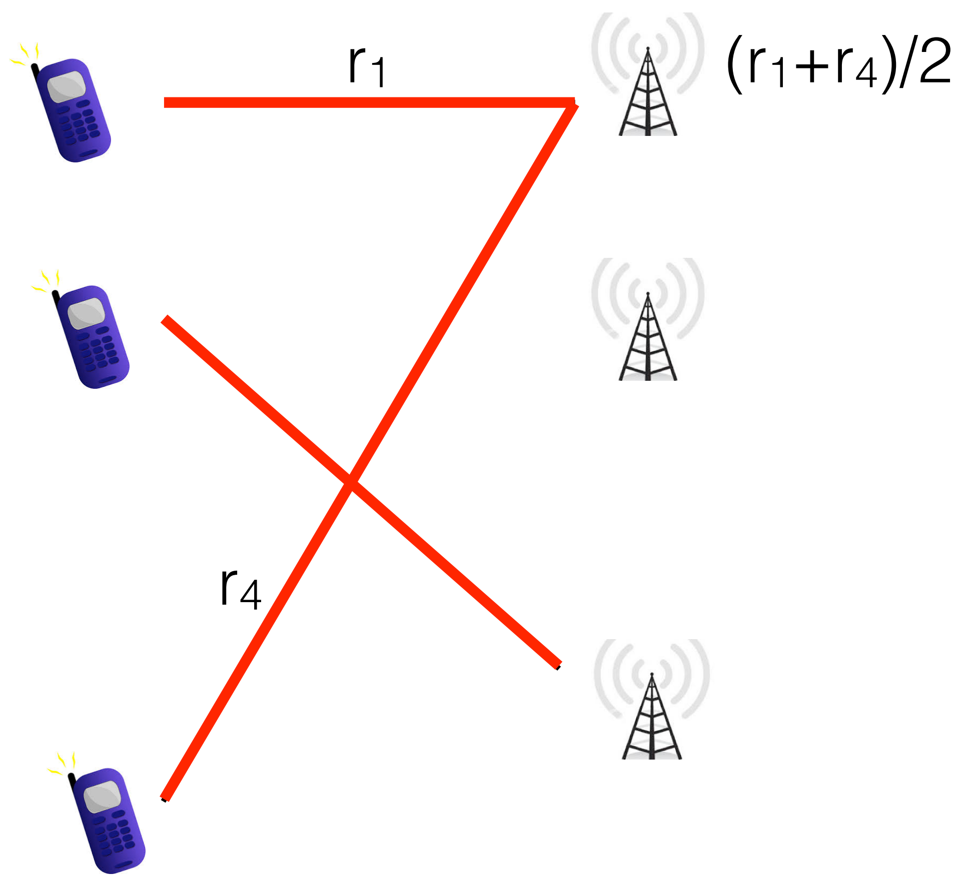
Important Observation

Note that sum-weight is still dominated by Max-Weight with MATCHING



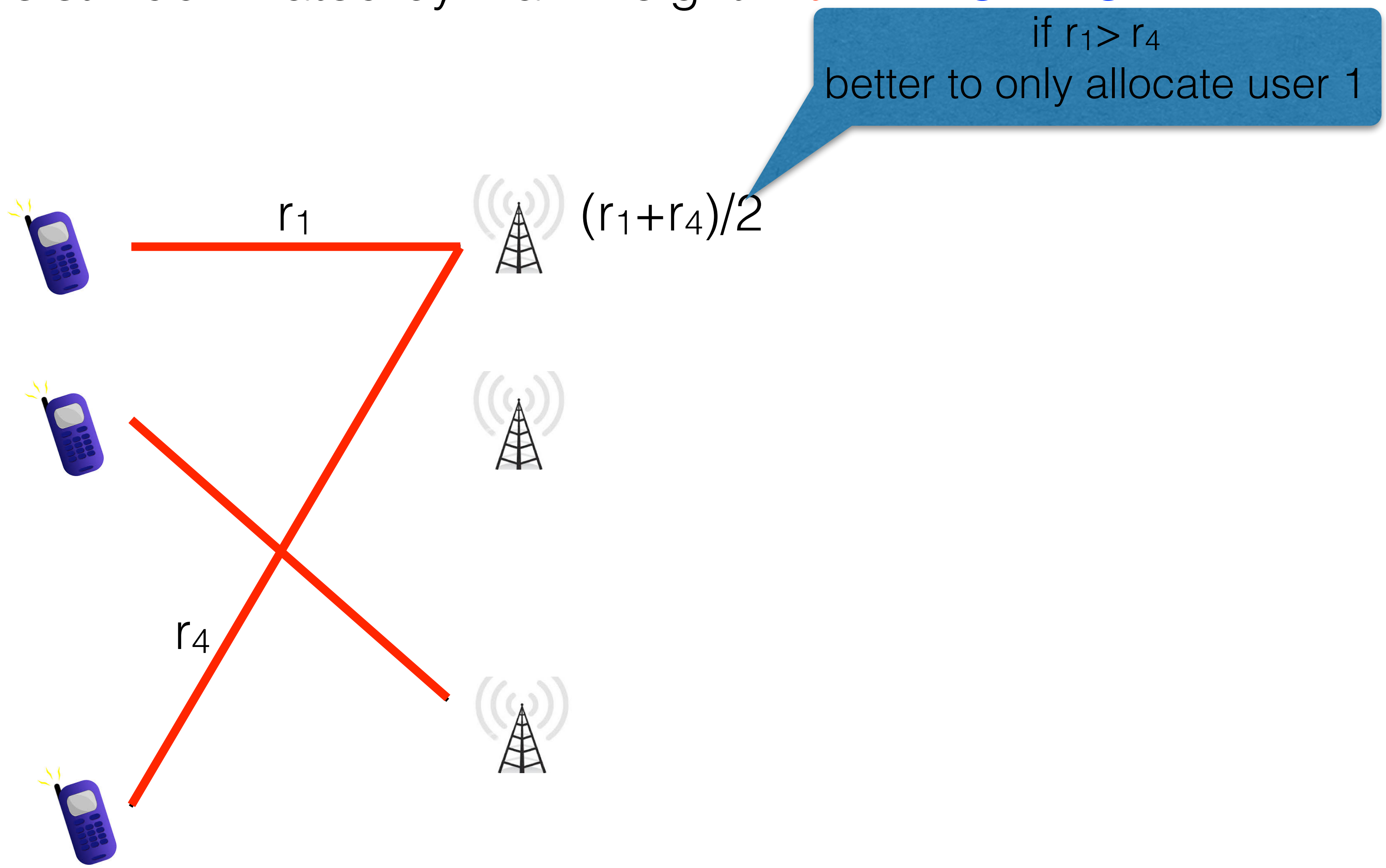
Important Observation

Note that sum-weight is still dominated by Max-Weight with MATCHING



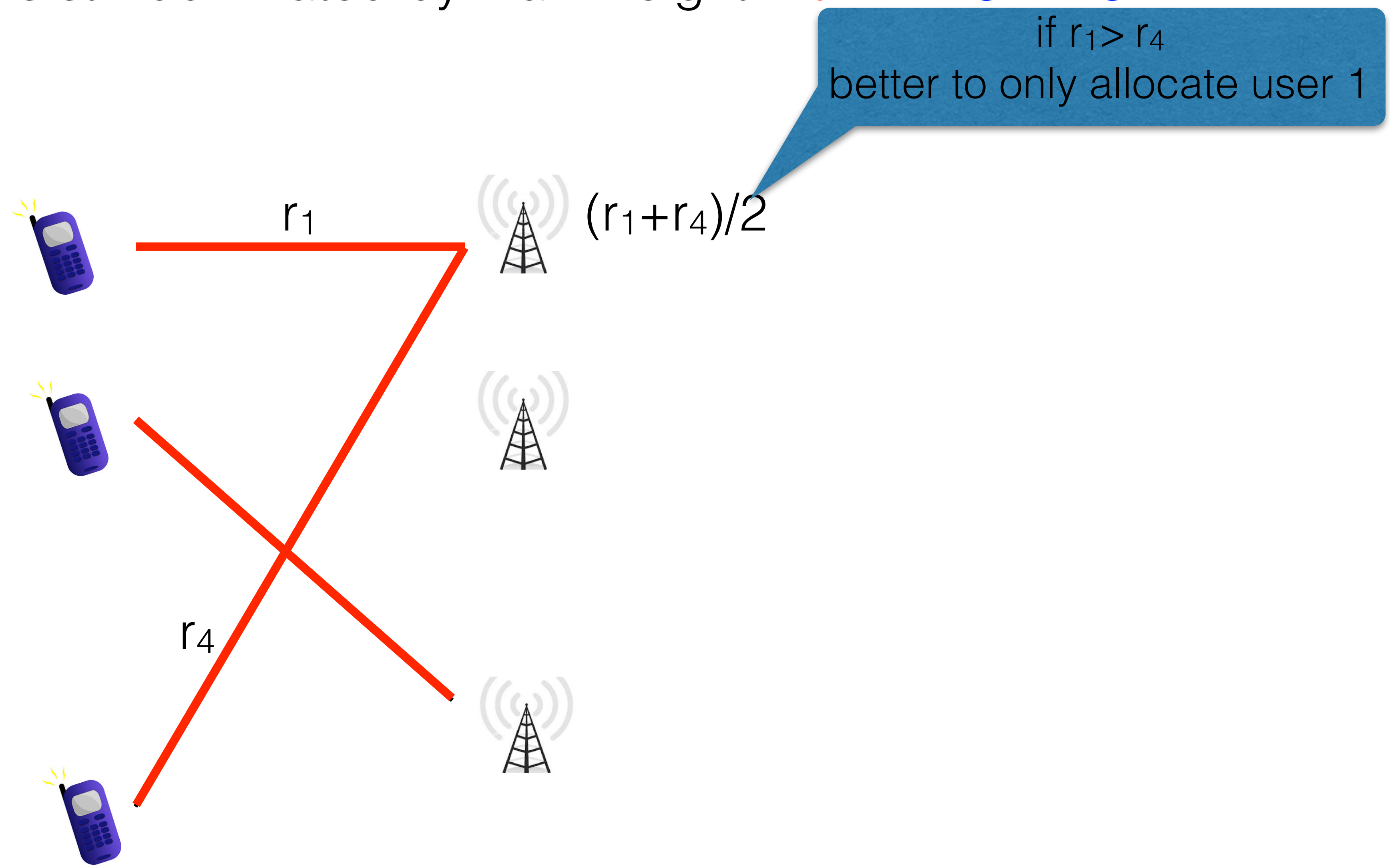
Important Observation

Note that sum-weight is still dominated by Max-Weight with MATCHING



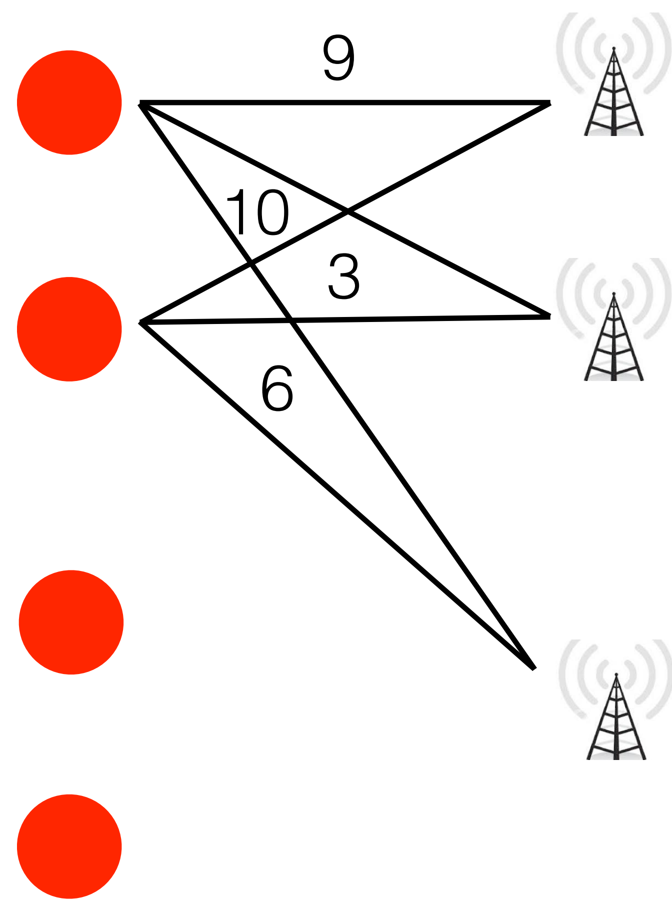
Important Observation

Note that sum-weight is still dominated by Max-Weight with MATCHING



Upper Bound : Max-Weight with **Matching**

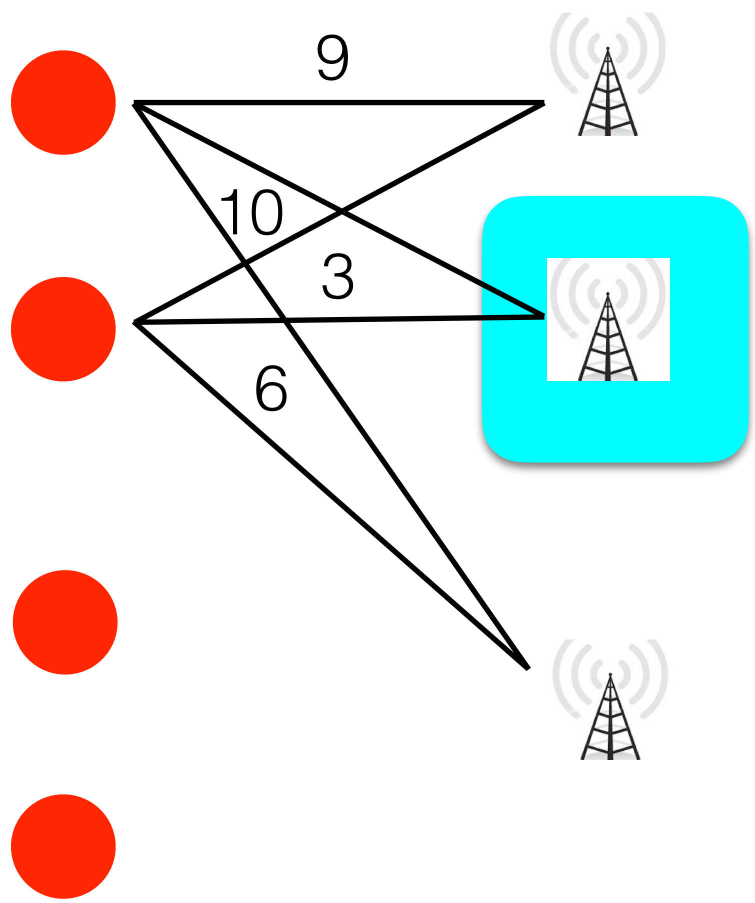
How to solve this ONLINE



How to solve this ONLINE

Choose one BS randomly and associate all users rejected by Matching

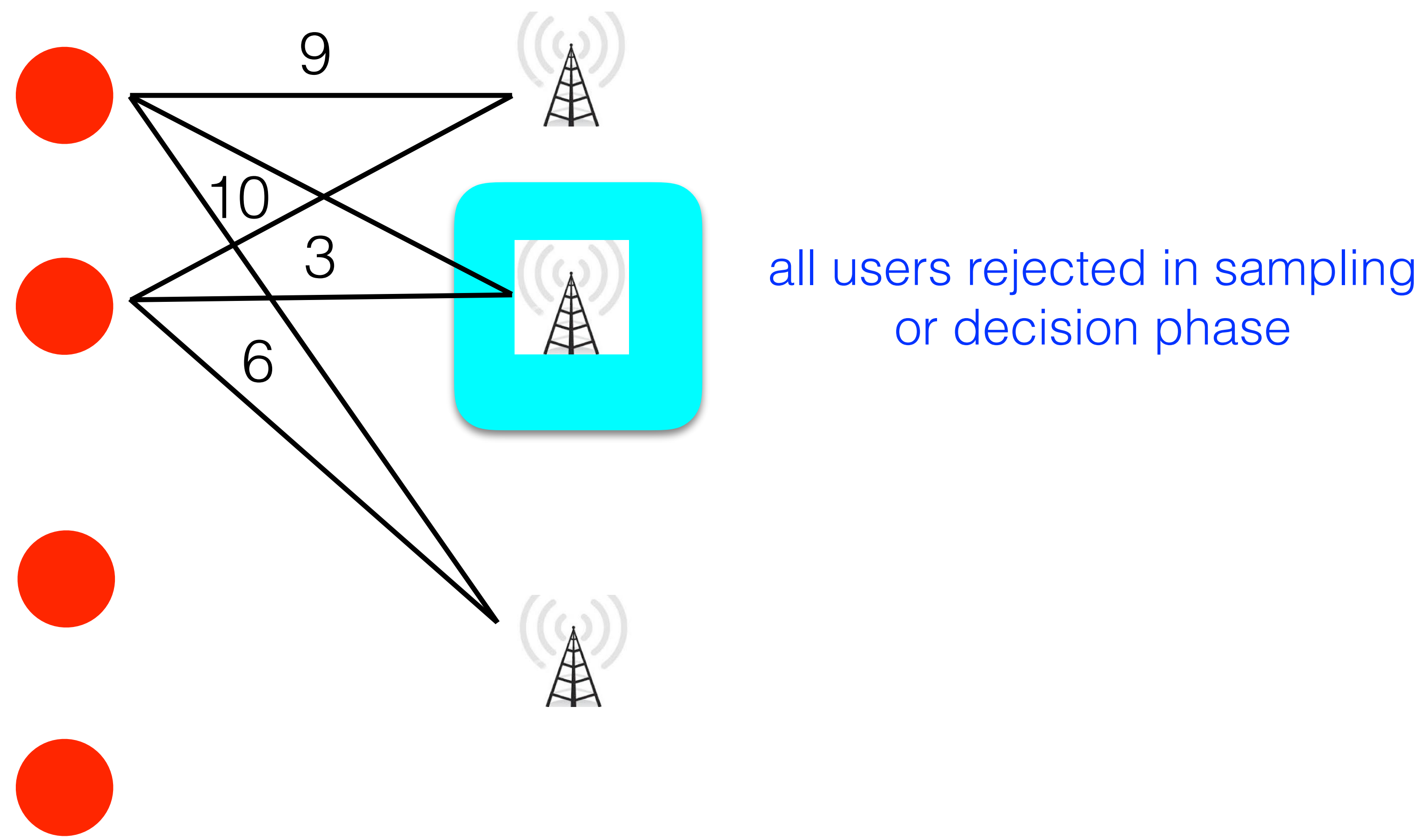
Sampling idea as before



How to solve this ONLINE

Choose one BS randomly and associate all users rejected by Matching

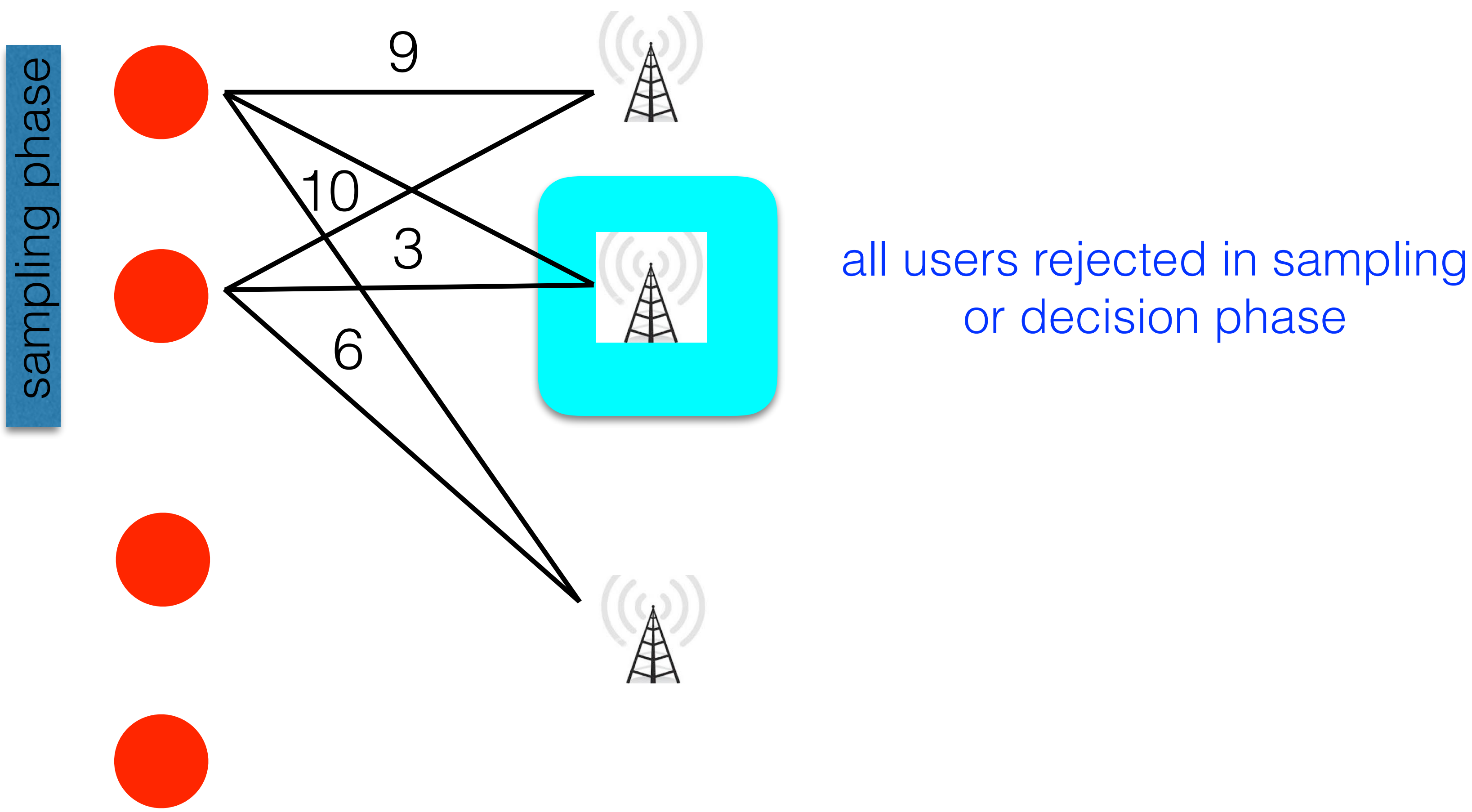
Sampling idea as before



How to solve this ONLINE

Choose one BS randomly and associate all users rejected by Matching

Sampling idea as before

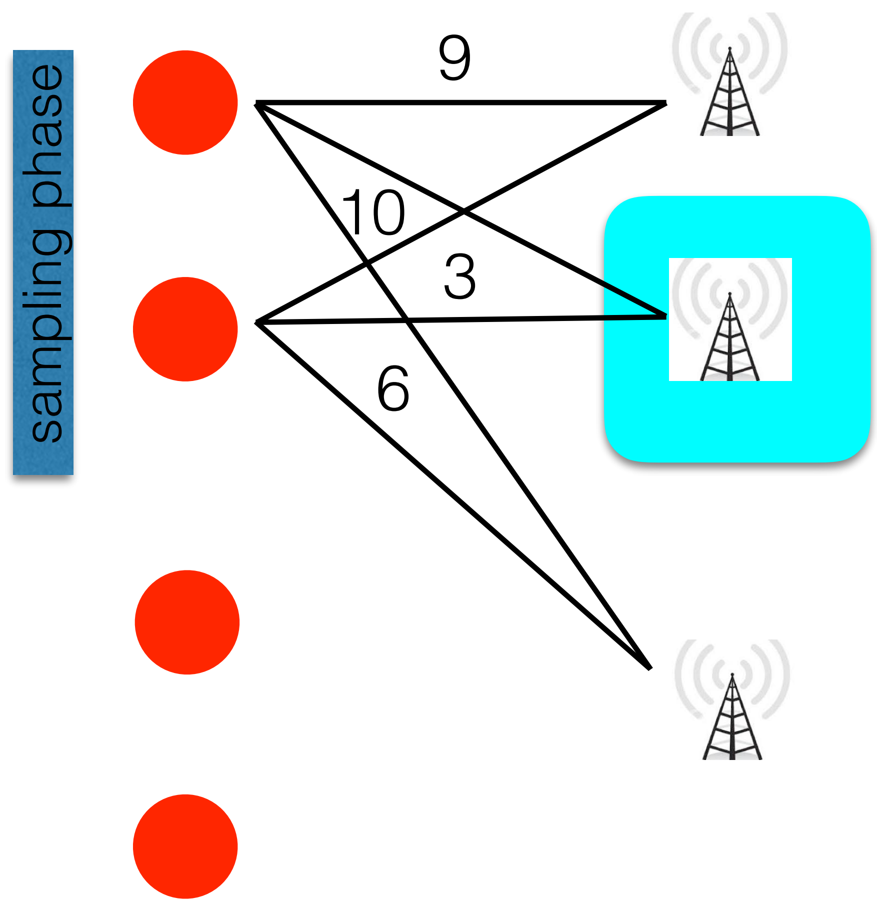


How to solve this ONLINE

Choose one BS randomly and associate all users rejected by Matching

Sampling idea as before

Find best matching



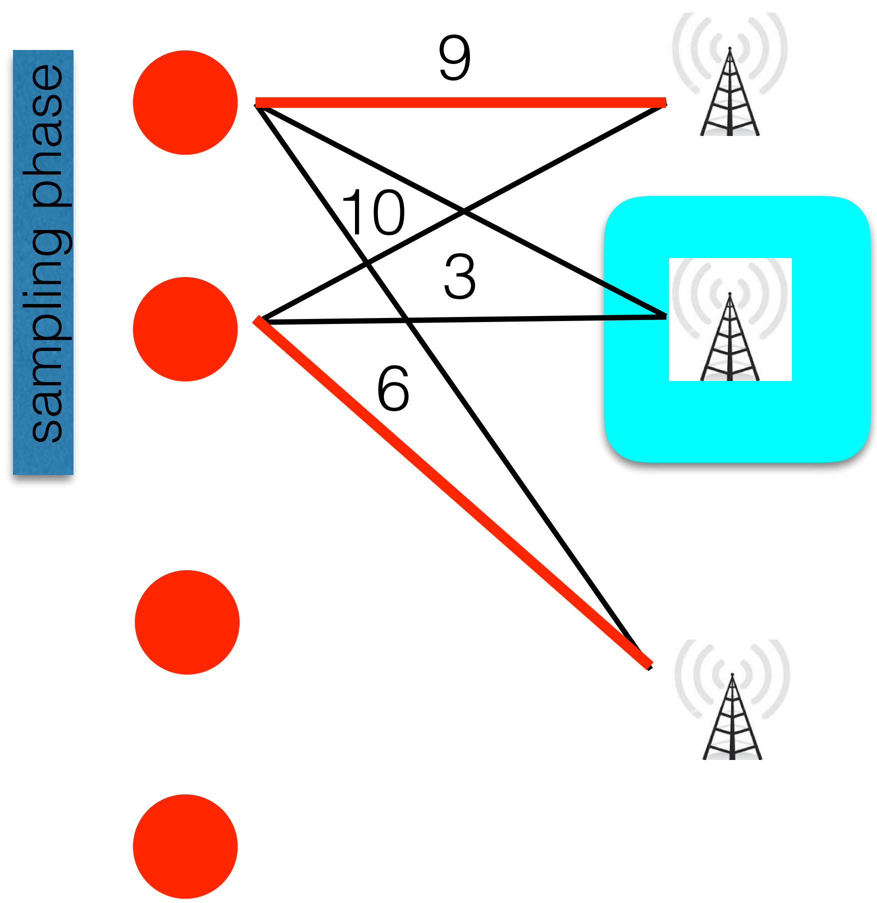
all users rejected in sampling
or decision phase

How to solve this ONLINE

Choose one BS randomly and associate all users rejected by Matching

Sampling idea as before

Find best matching



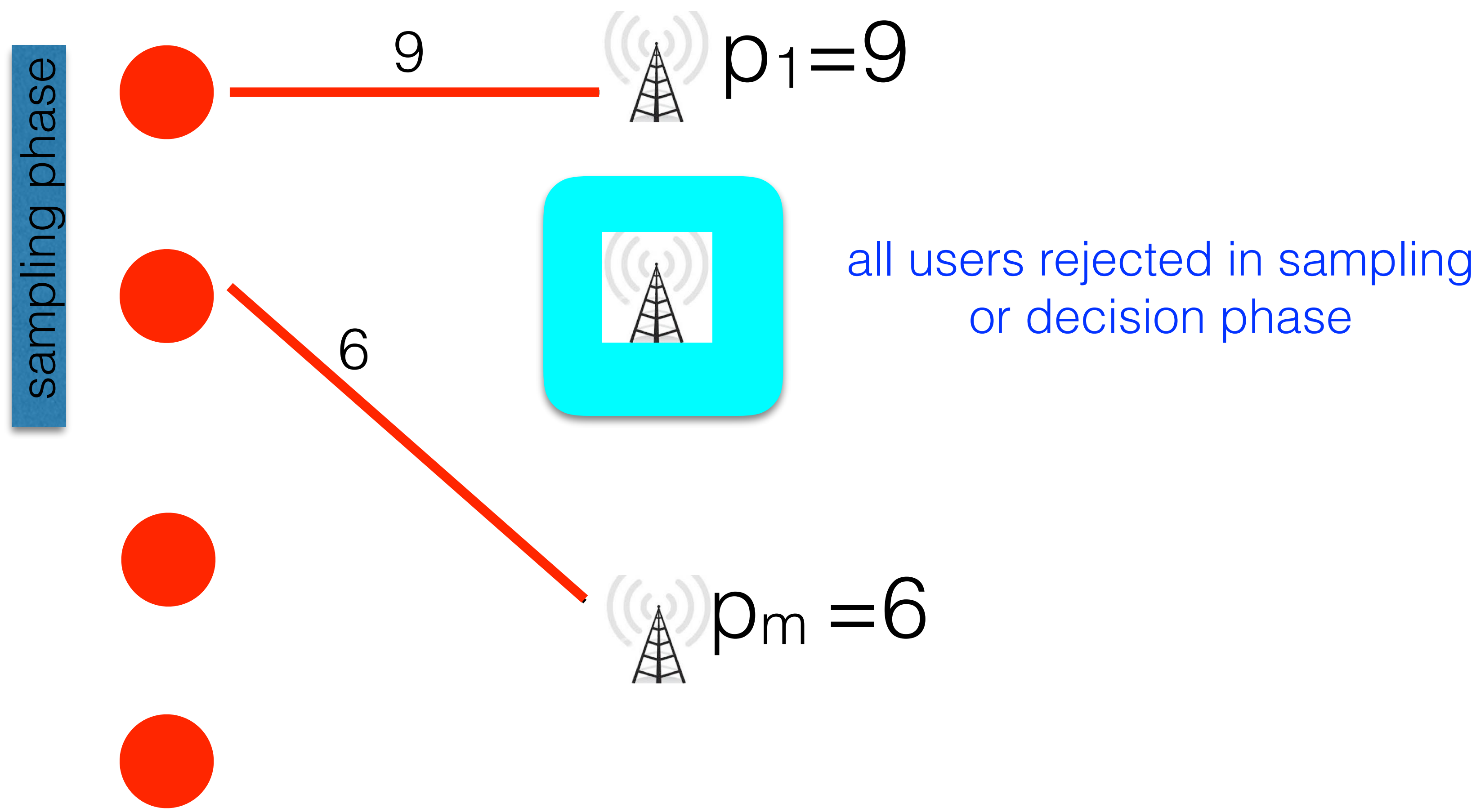
all users rejected in sampling
or decision phase

How to solve this ONLINE

Choose one BS randomly and associate all users rejected by Matching

Sampling idea as before

Find best matching

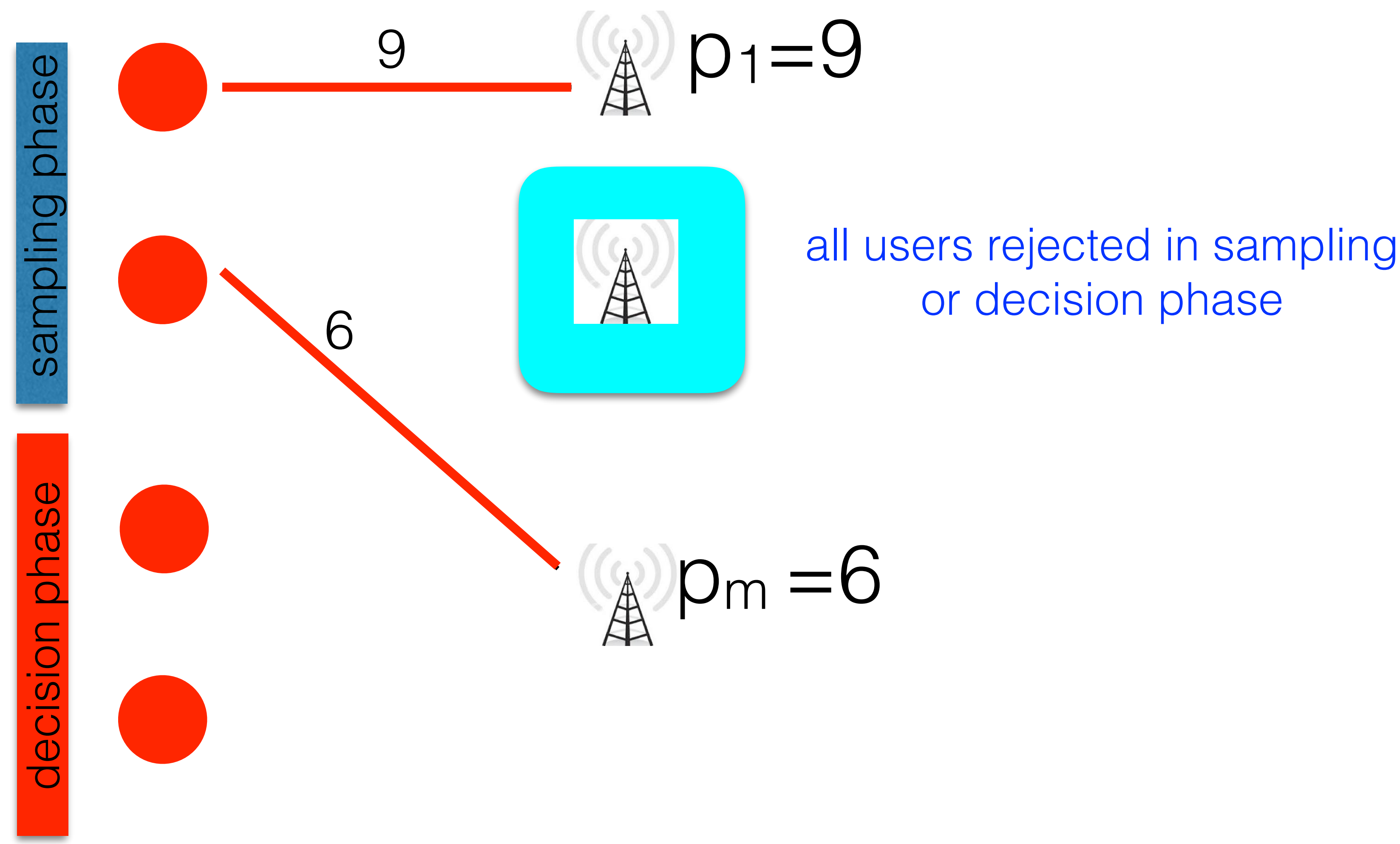


How to solve this ONLINE

Choose one BS randomly and associate all users rejected by Matching

Sampling idea as before

Find best matching



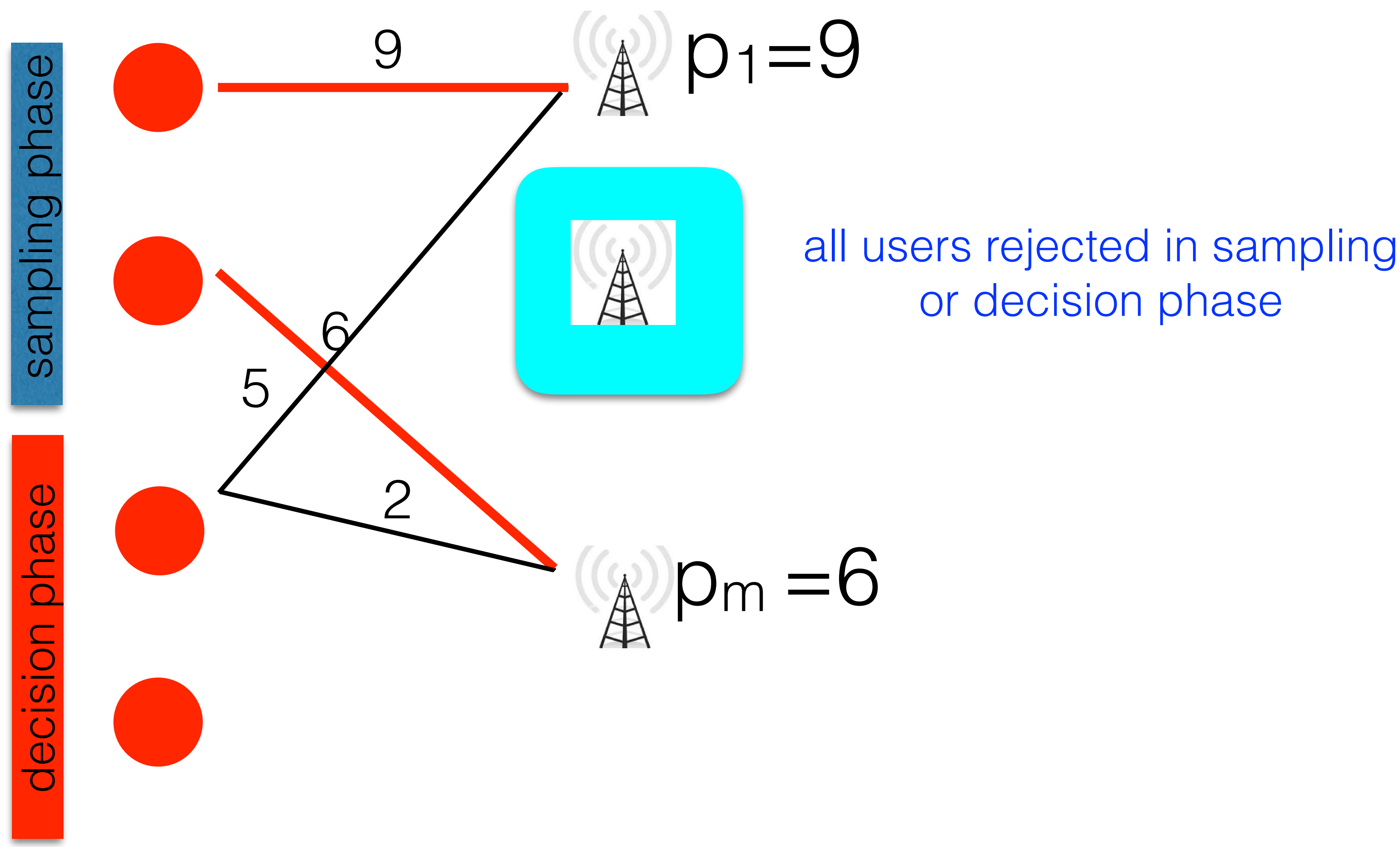
How to solve this ONLINE

Choose one BS randomly and associate all users rejected by Matching

Sampling idea as before

Find best matching

match the largest weight
more than price



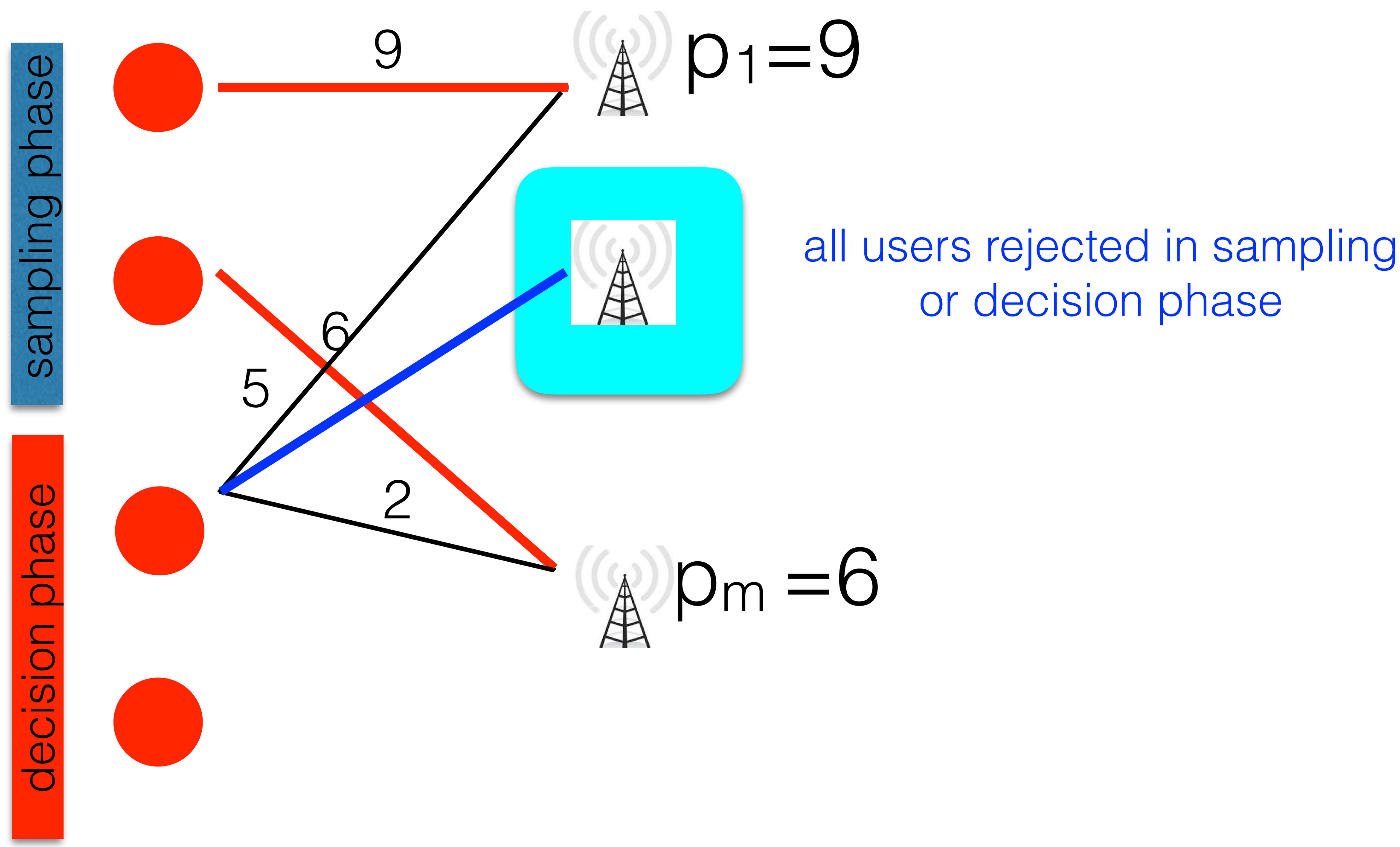
How to solve this ONLINE

Choose one BS randomly and associate all users rejected by Matching

Sampling idea as before

Find best matching

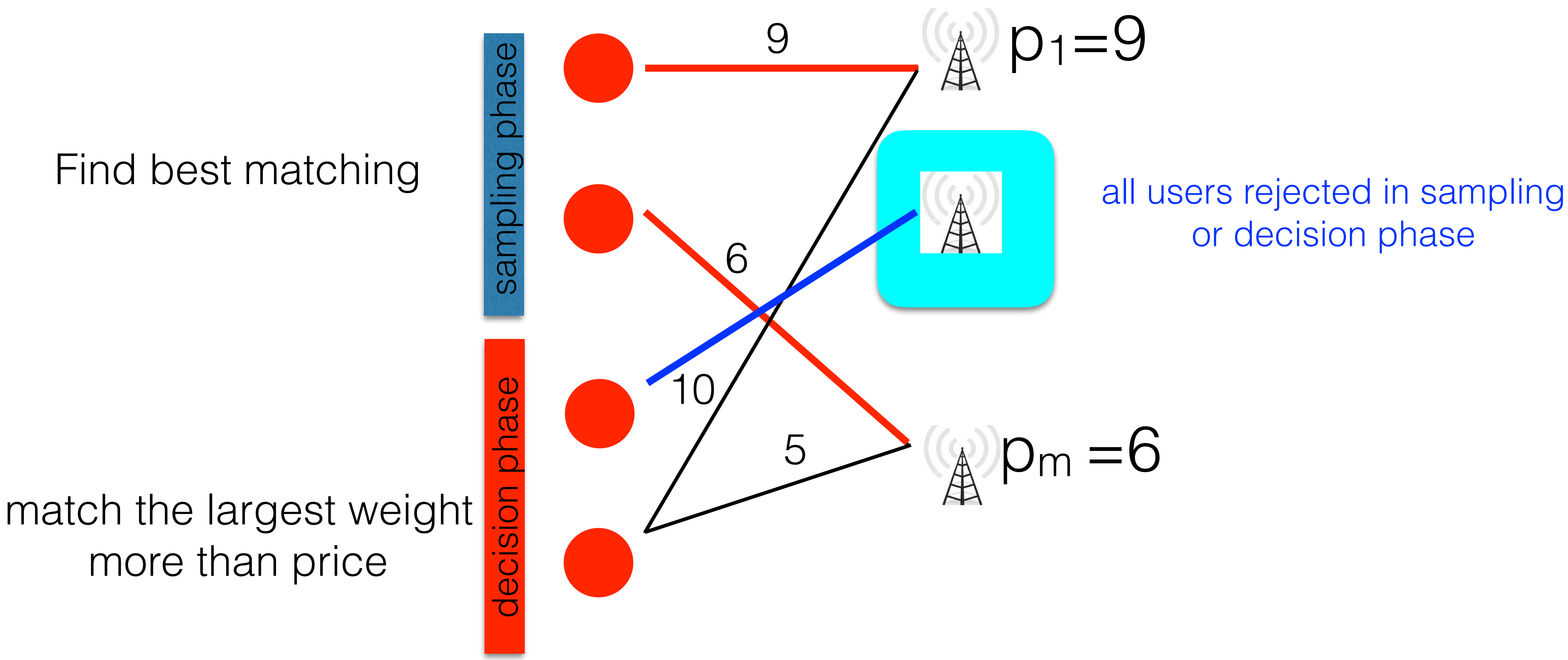
match the largest weight more than price



How to solve this ONLINE

Choose one BS randomly and associate all users rejected by Matching

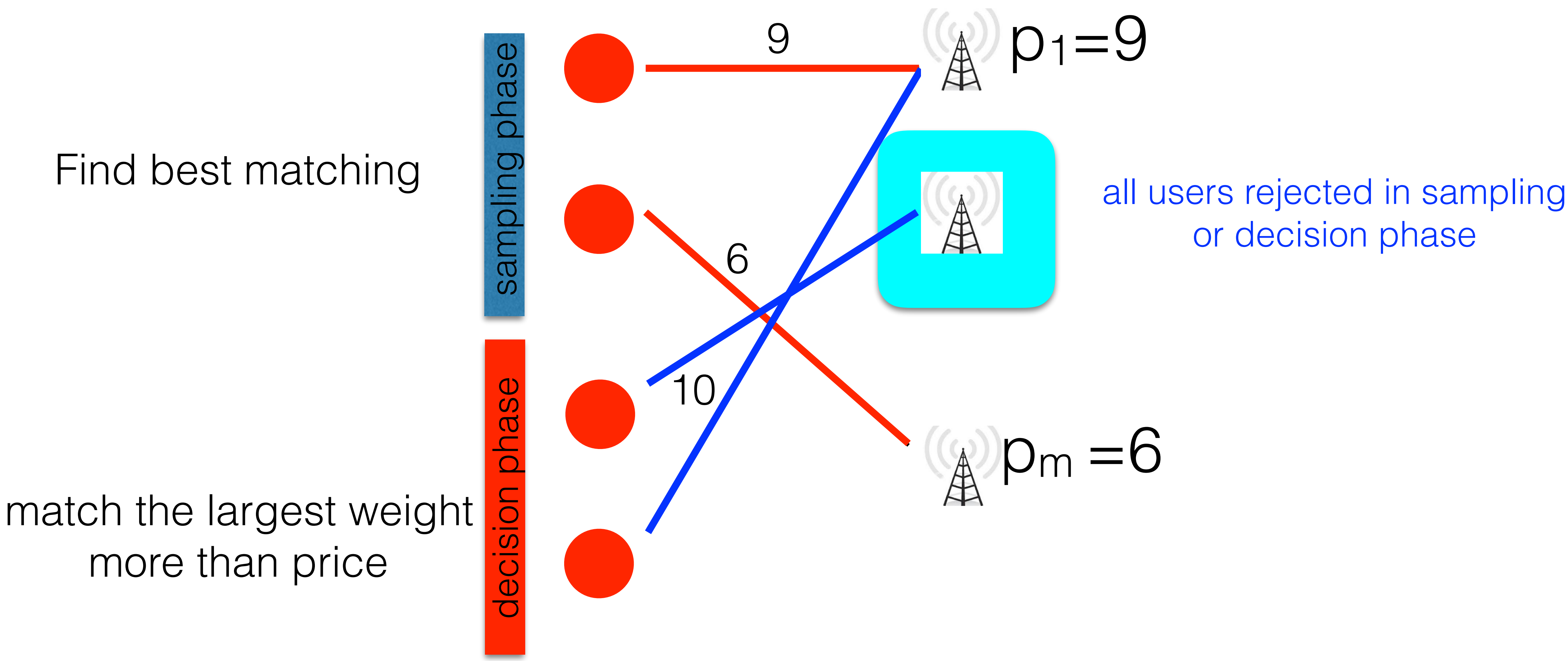
Sampling idea as before



How to solve this ONLINE

Choose one BS randomly and associate all users rejected by Matching

Sampling idea as before



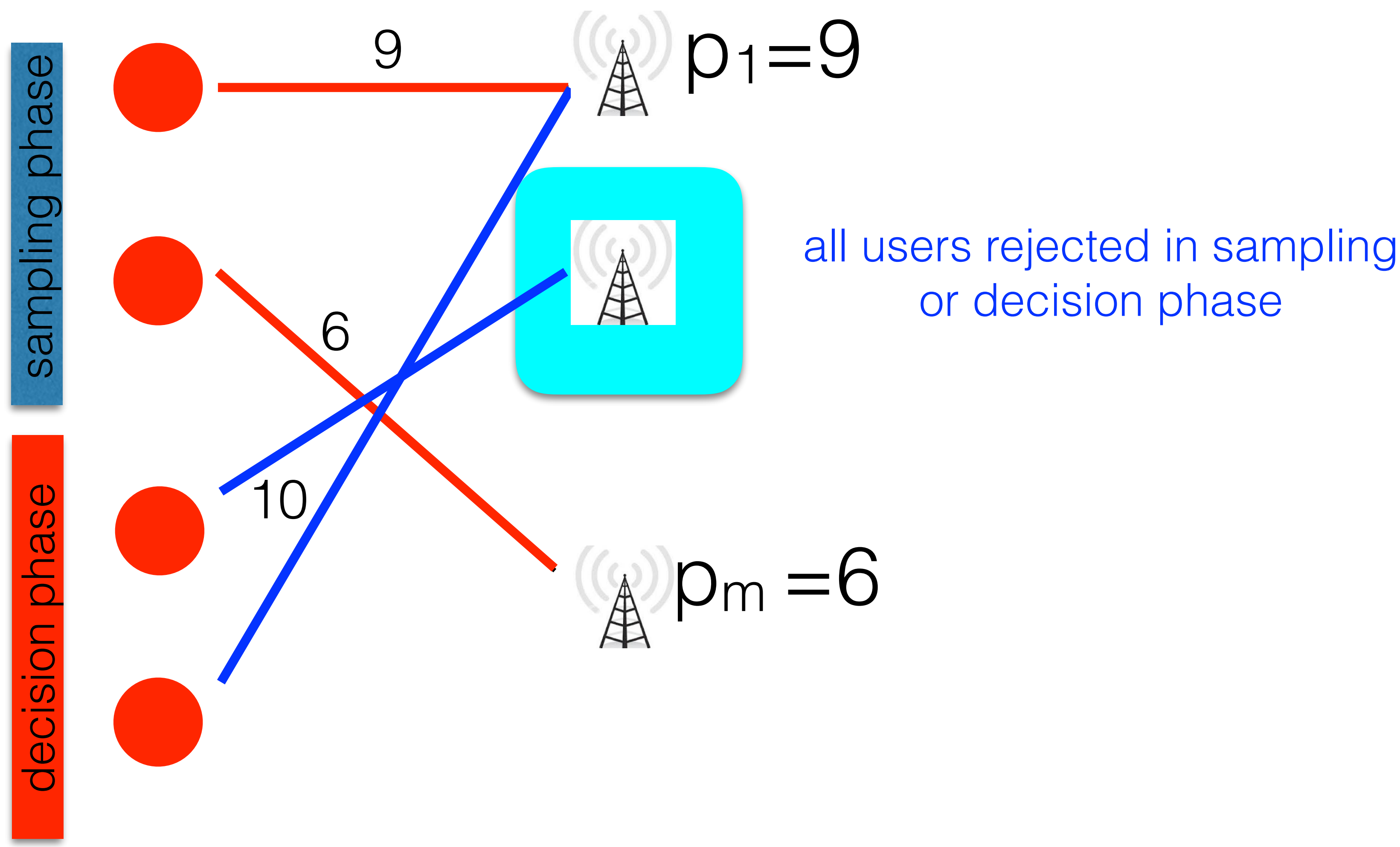
How to solve this ONLINE

Choose one BS randomly and associate all users rejected by Matching

Sampling idea as before

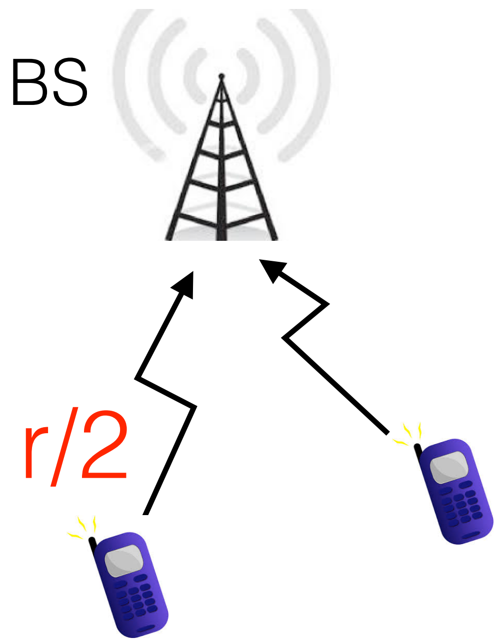
Find best matching

match the largest weight more than price



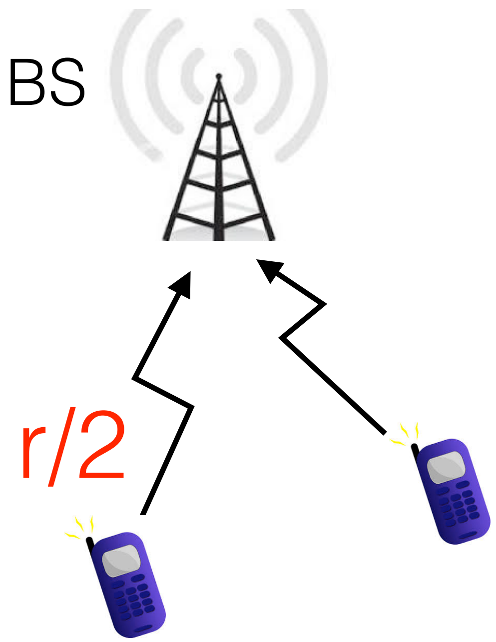
Result: 8m/(m-1)—competitive/optimal [*V, Thangaraj' 13***]**

Implication



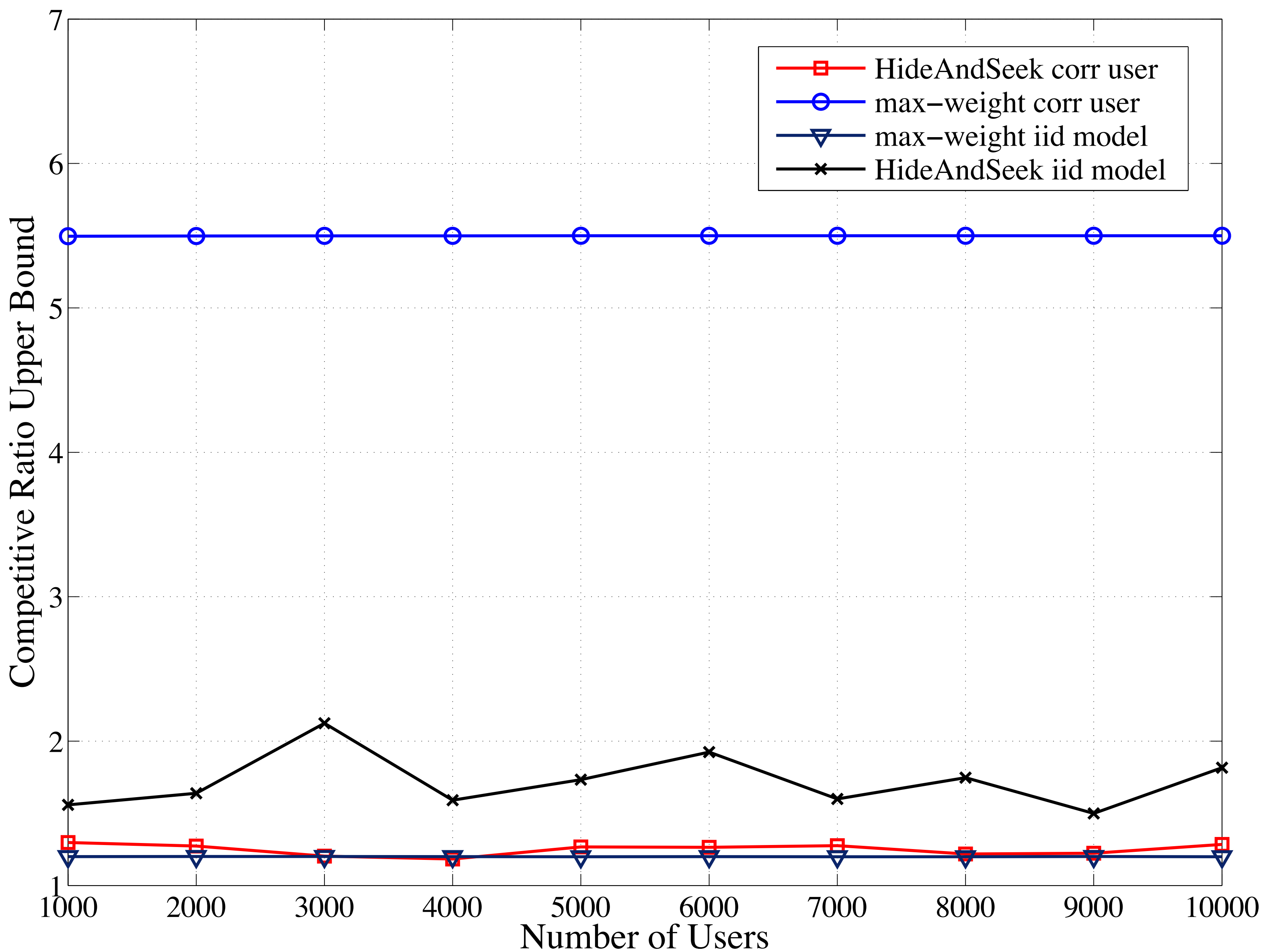
Lot of users get associated to just one BS

Implication



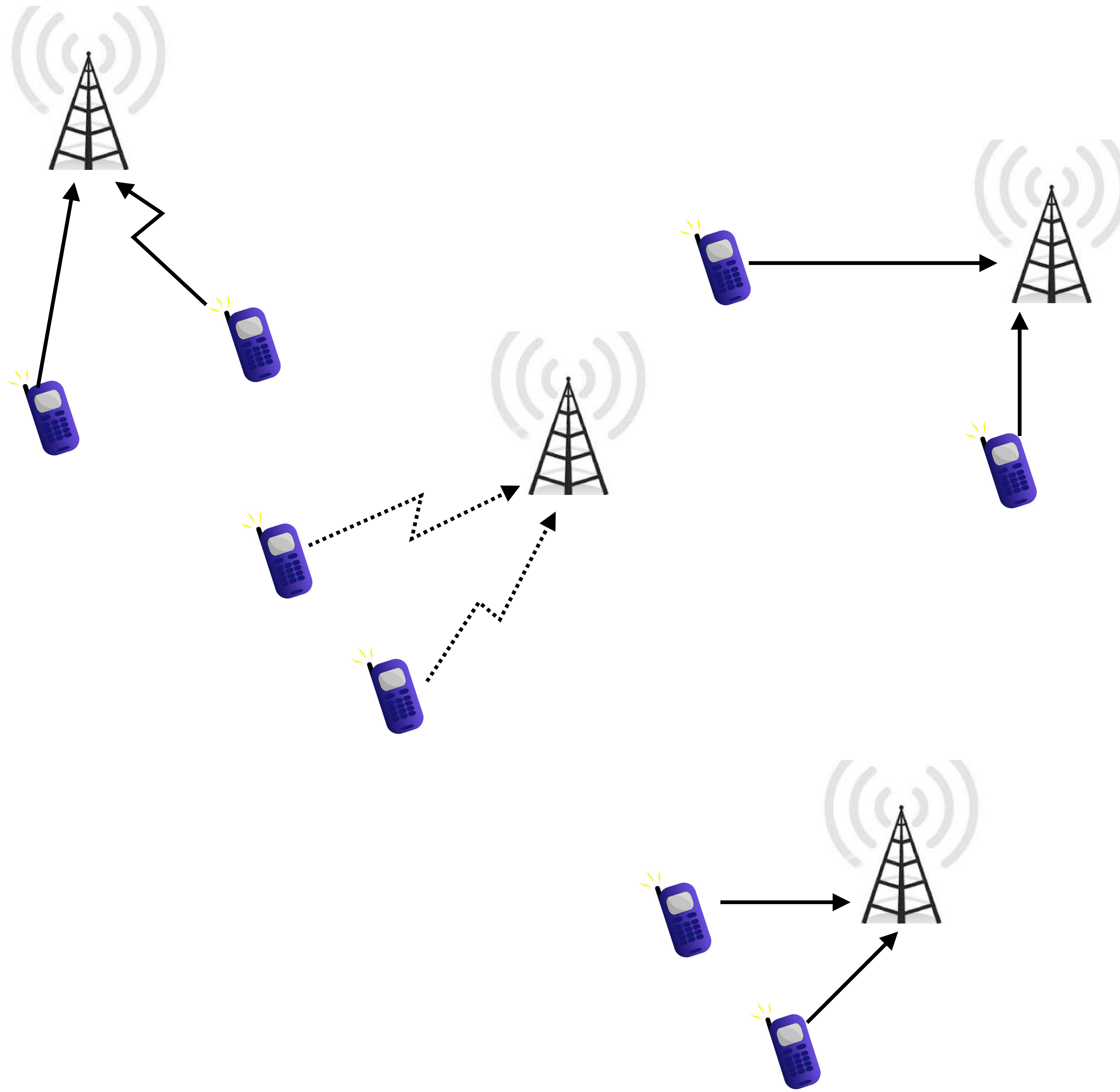
Lot of users get associated to just one BS

Still better than natural algorithm of connecting to the strongest BS

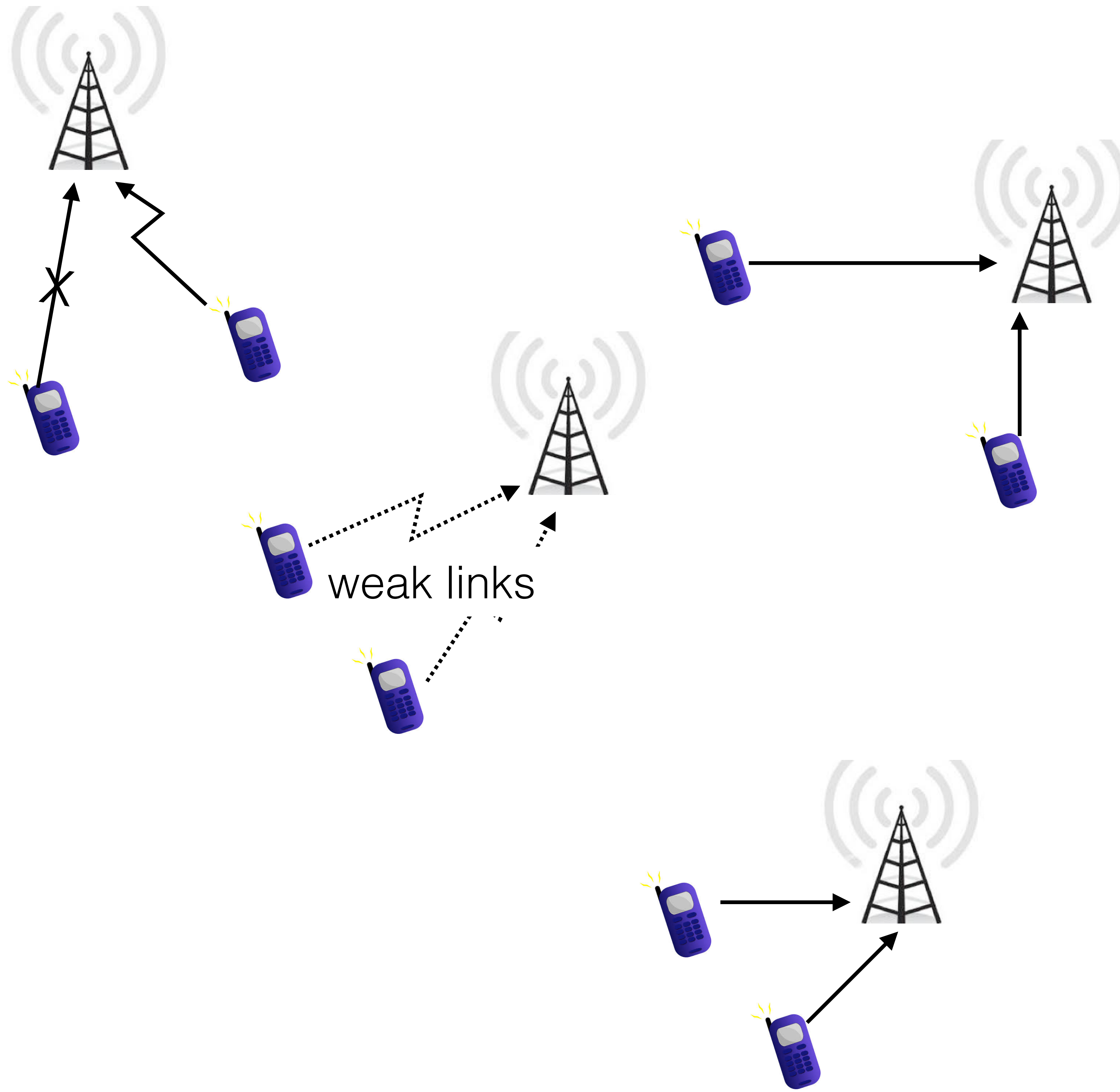


5G

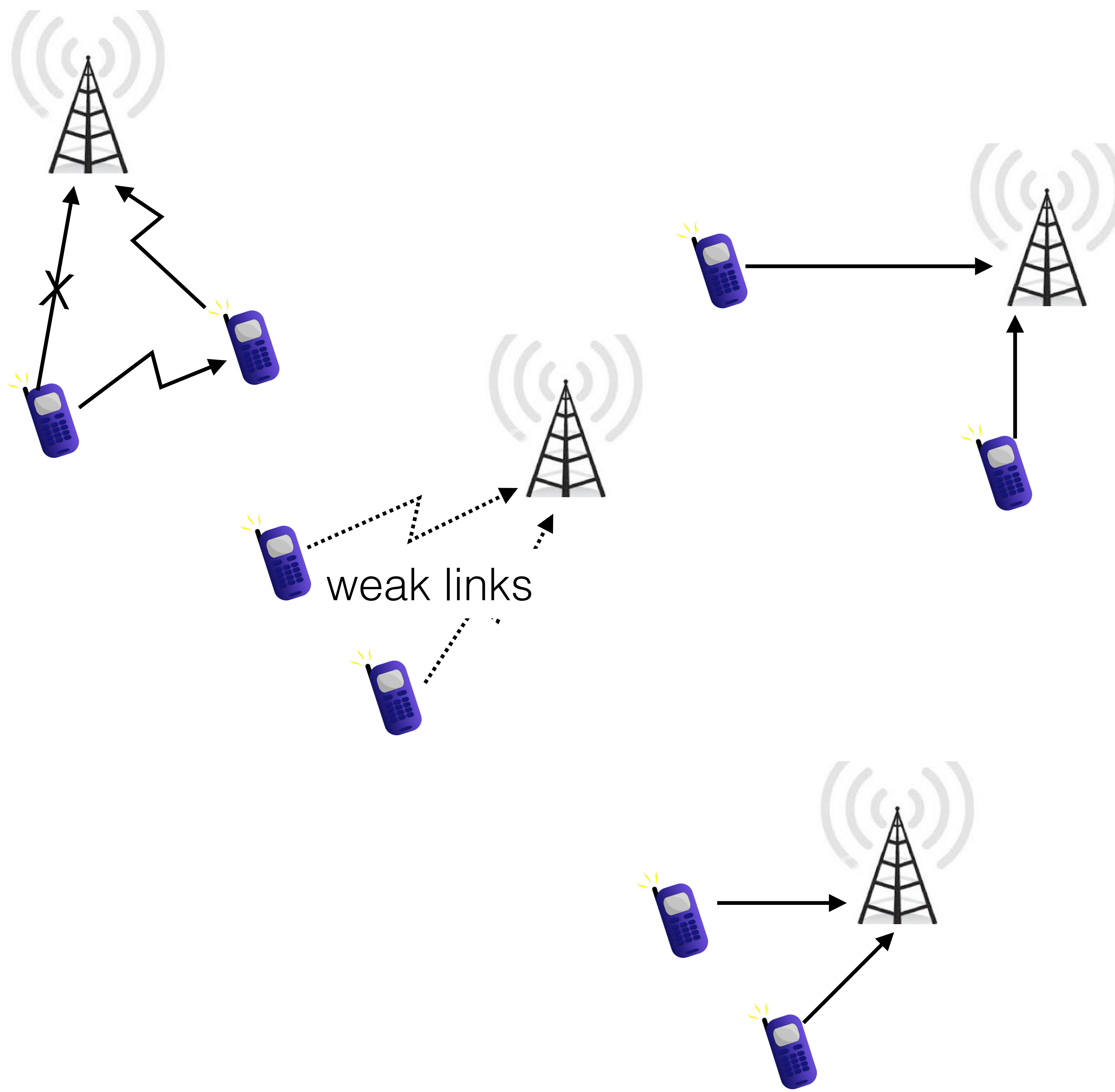
Modern Problem Device-2-Device Communication



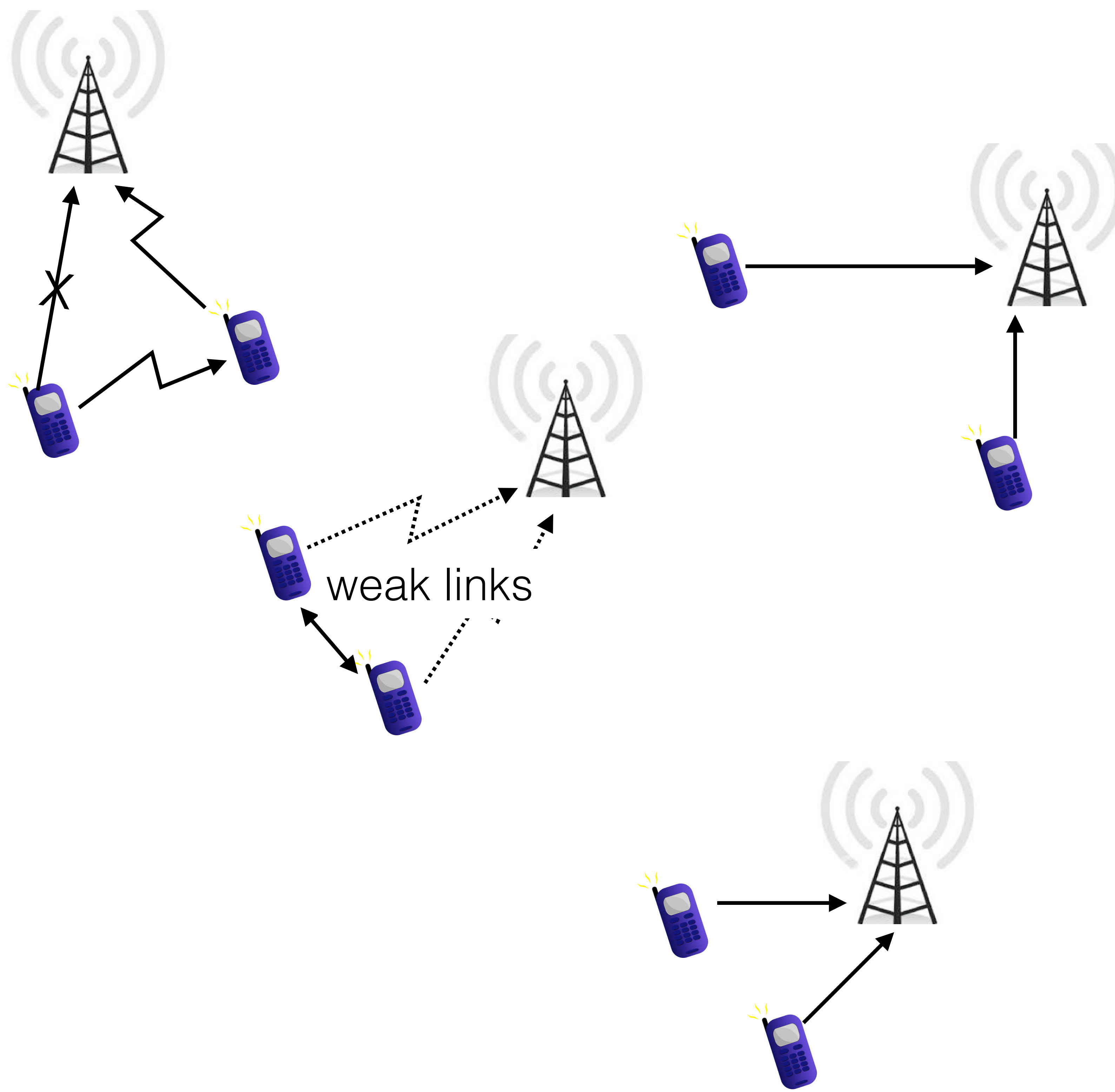
Modern Problem Device-2-Device Communication



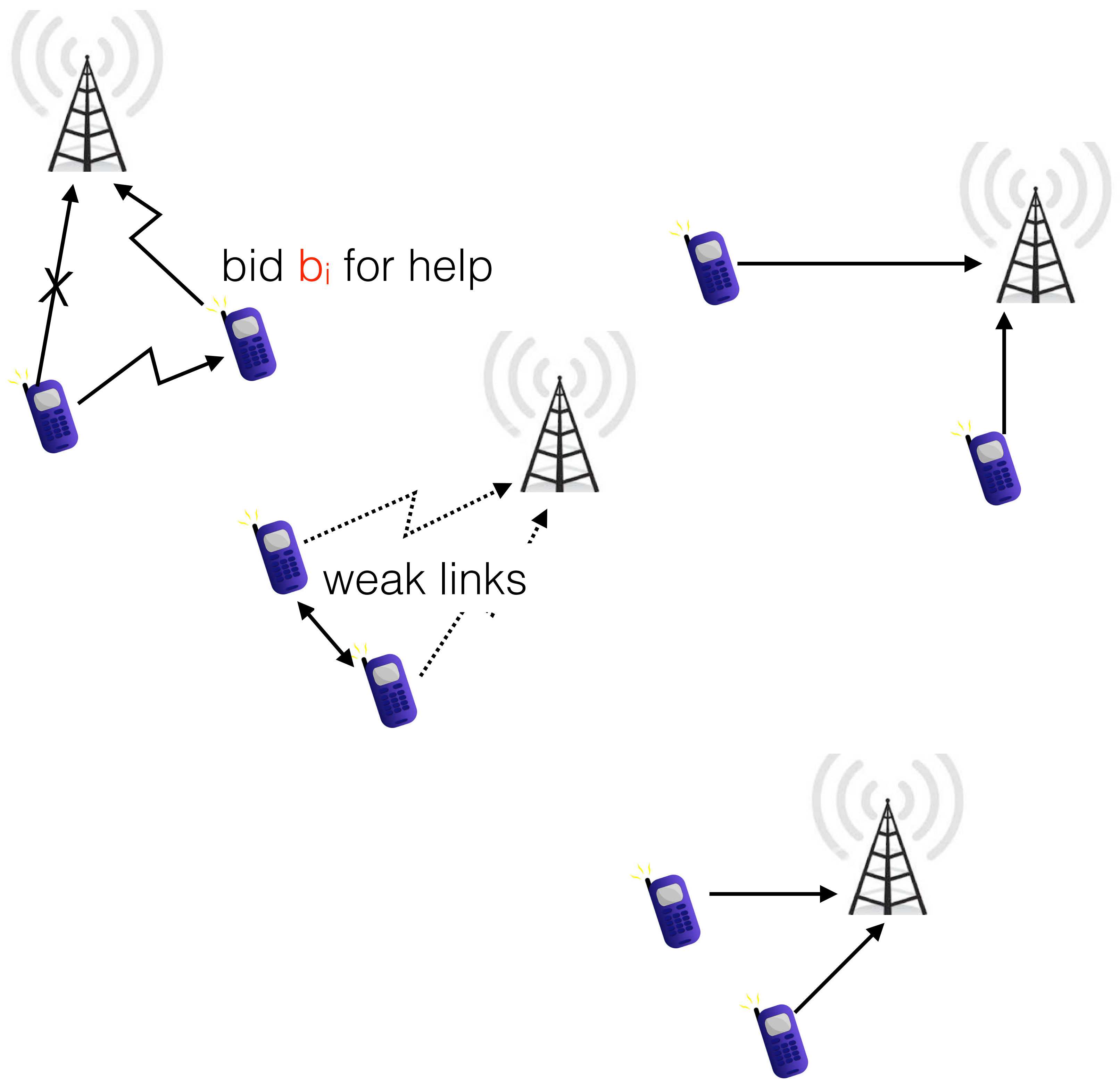
Modern Problem Device-2-Device Communication



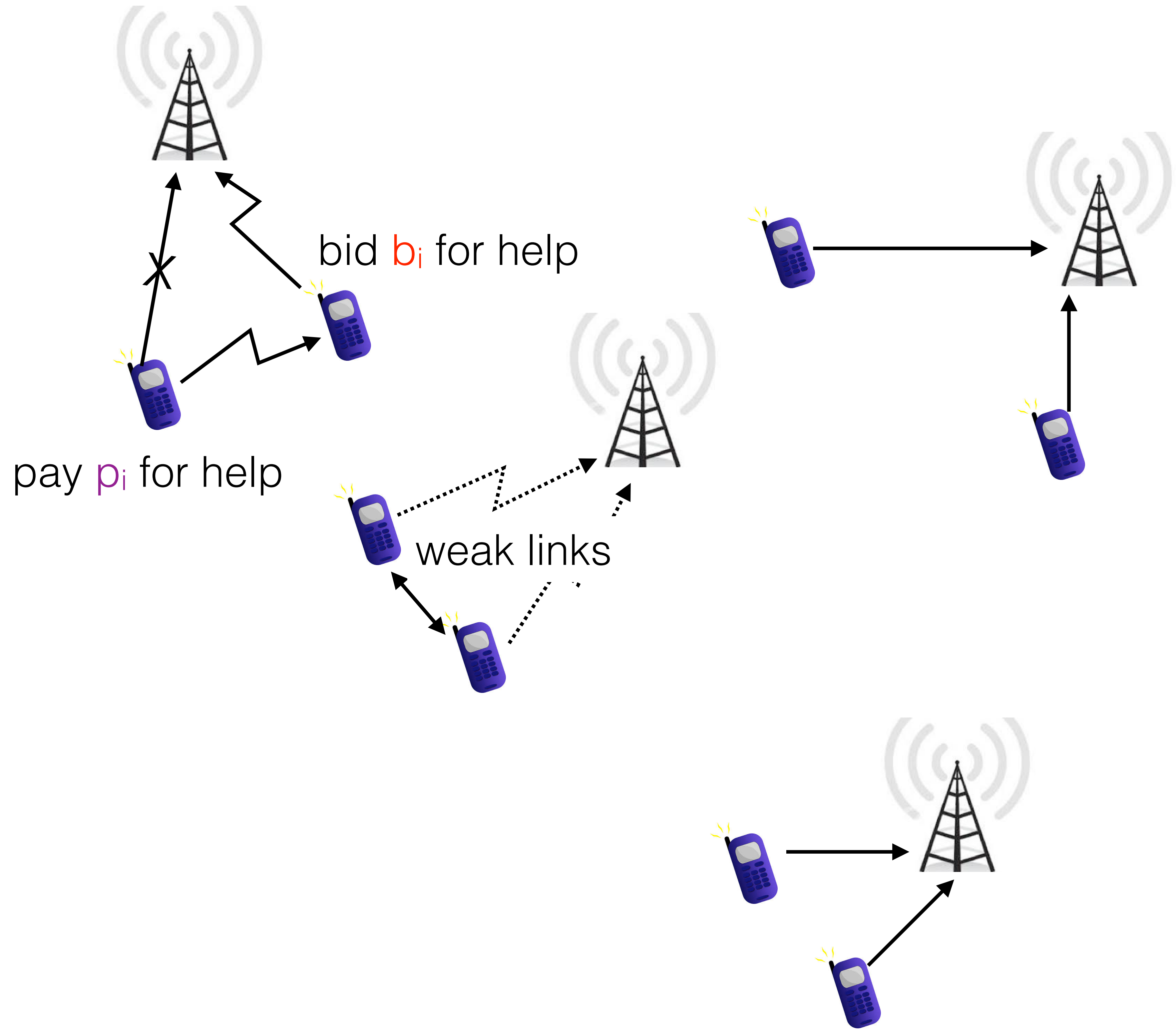
Modern Problem Device-2-Device Communication



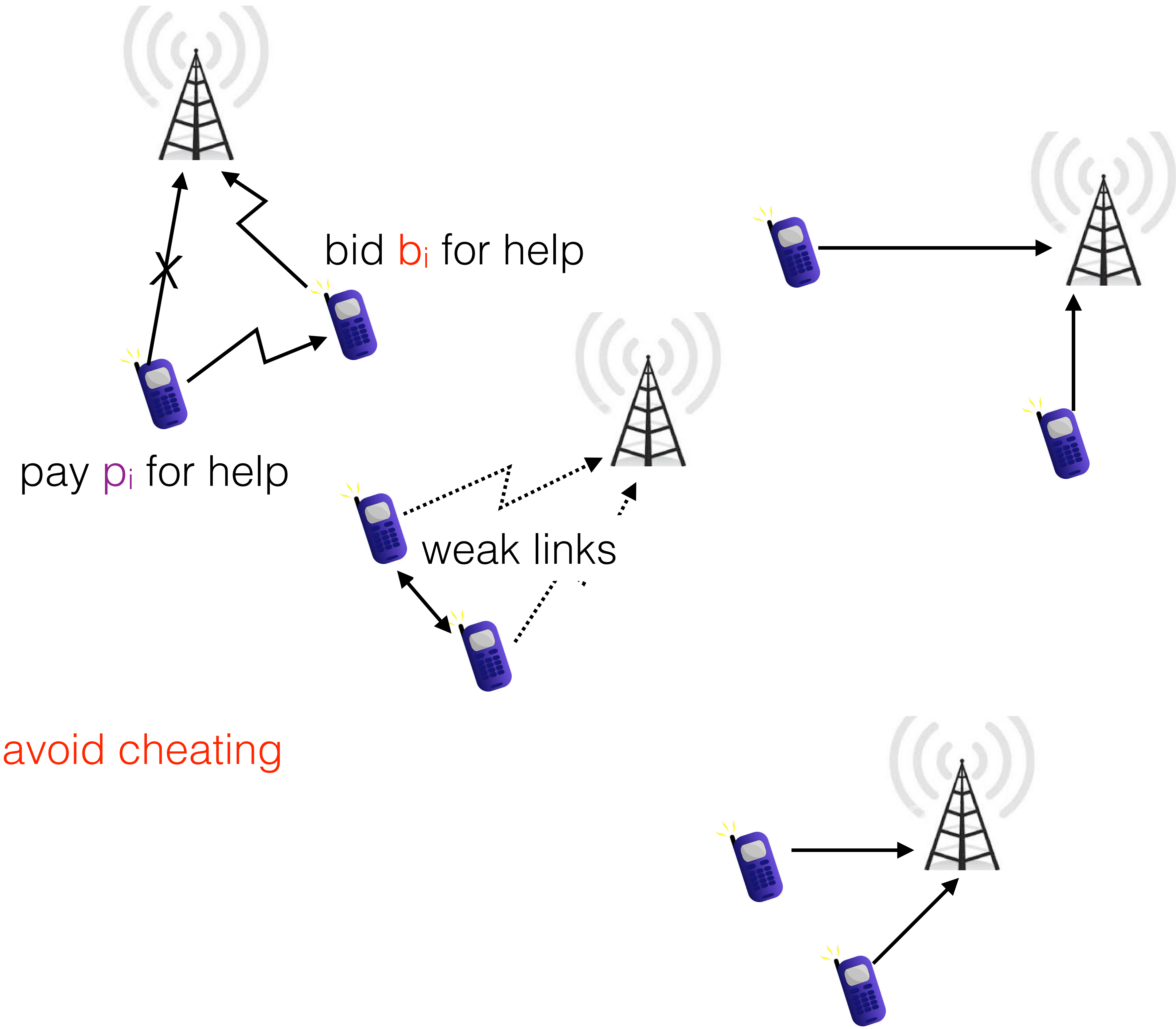
Modern Problem Device-2-Device Communication



Modern Problem Device-2-Device Communication

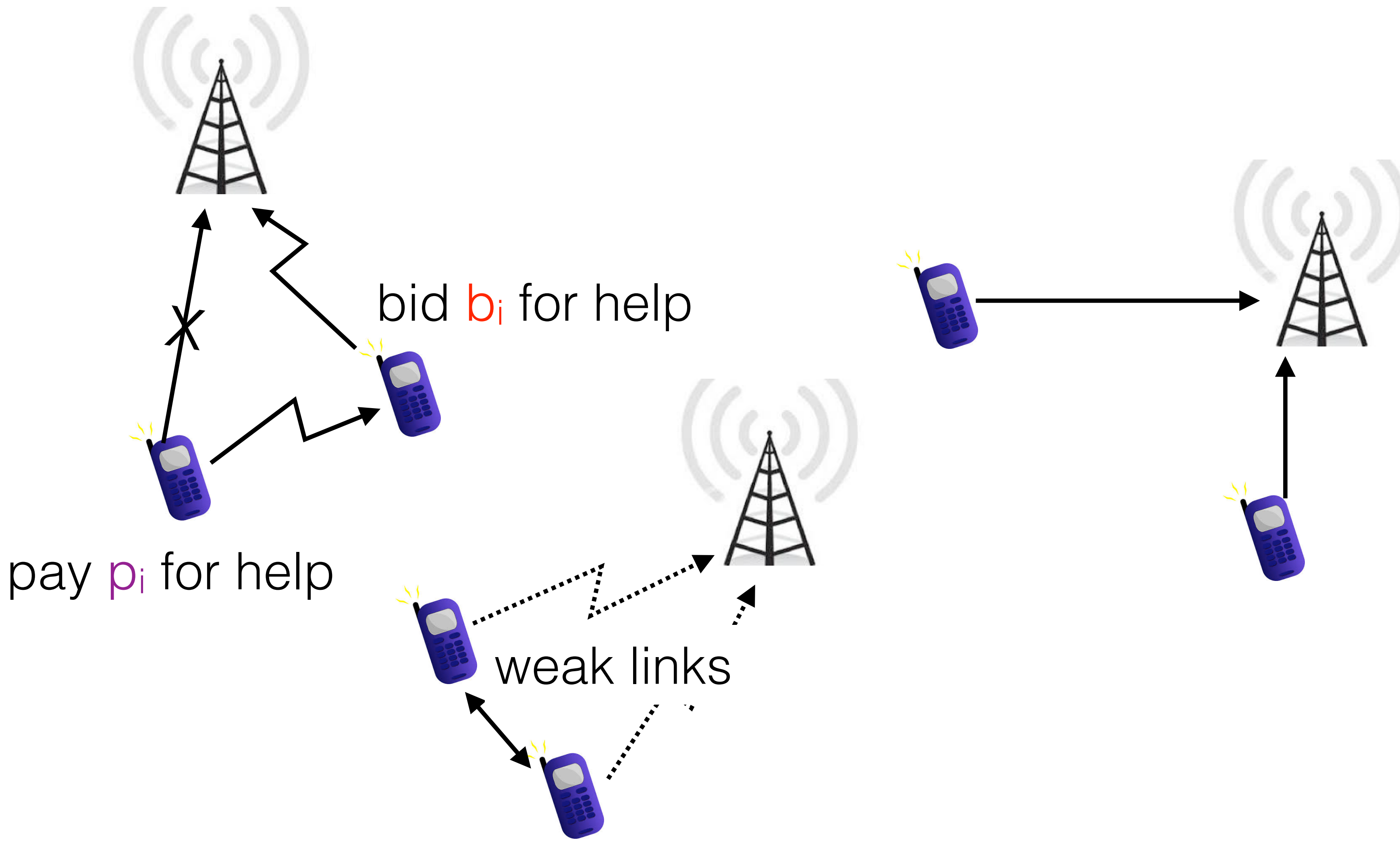


Modern Problem Device-2-Device Communication



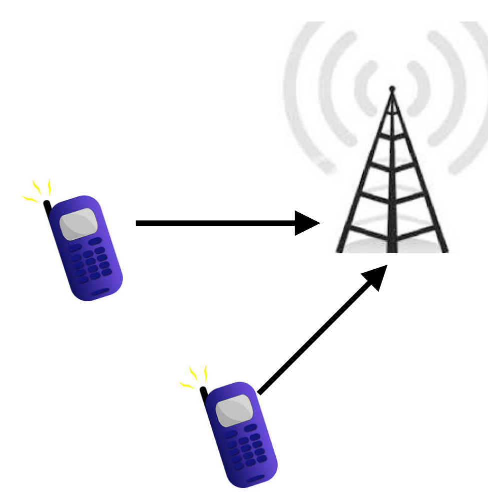
mechanism to avoid cheating

Modern Problem Device-2-Device Communication

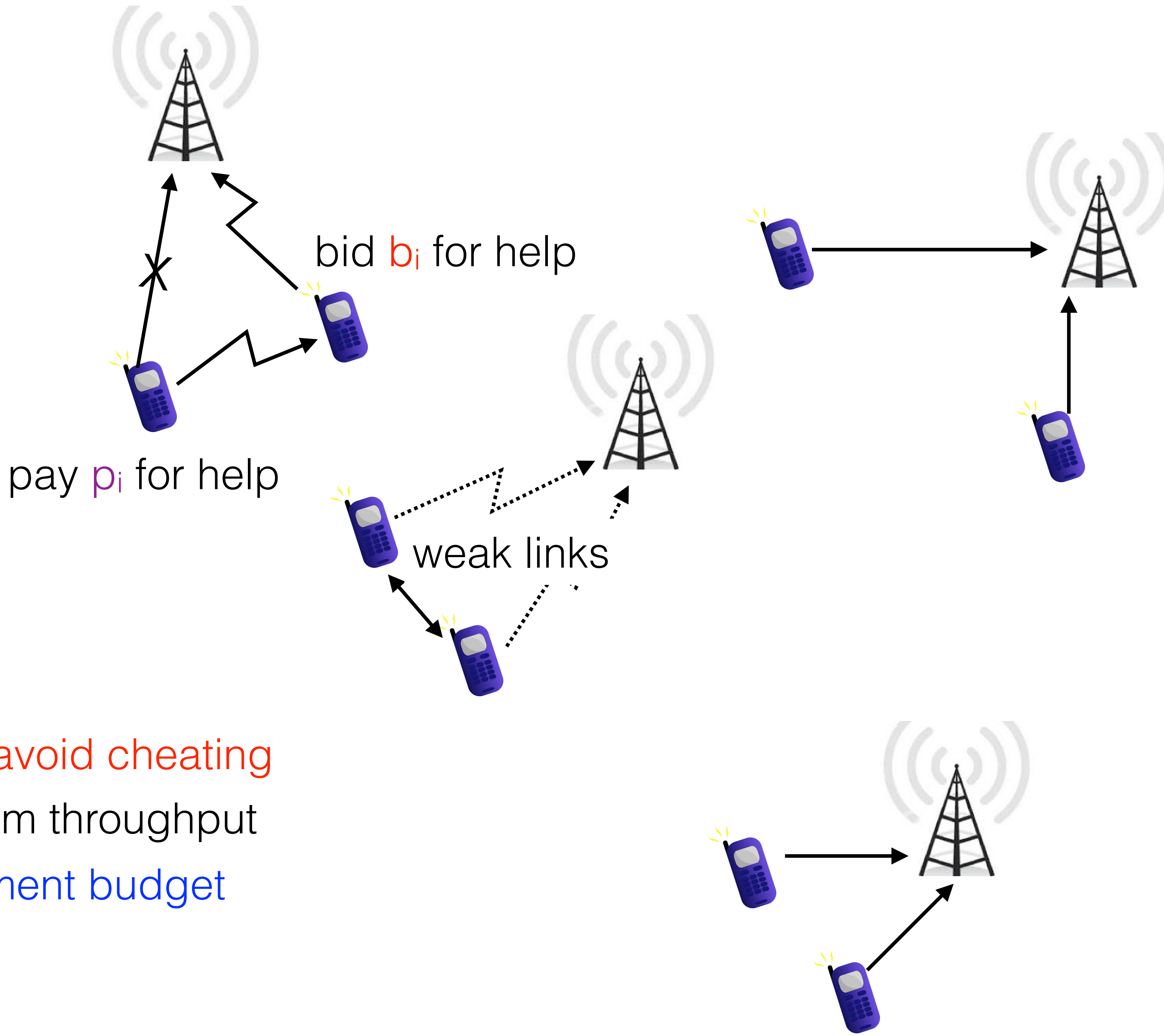


mechanism to avoid cheating

ensure maximum throughput



Modern Problem Device-2-Device Communication

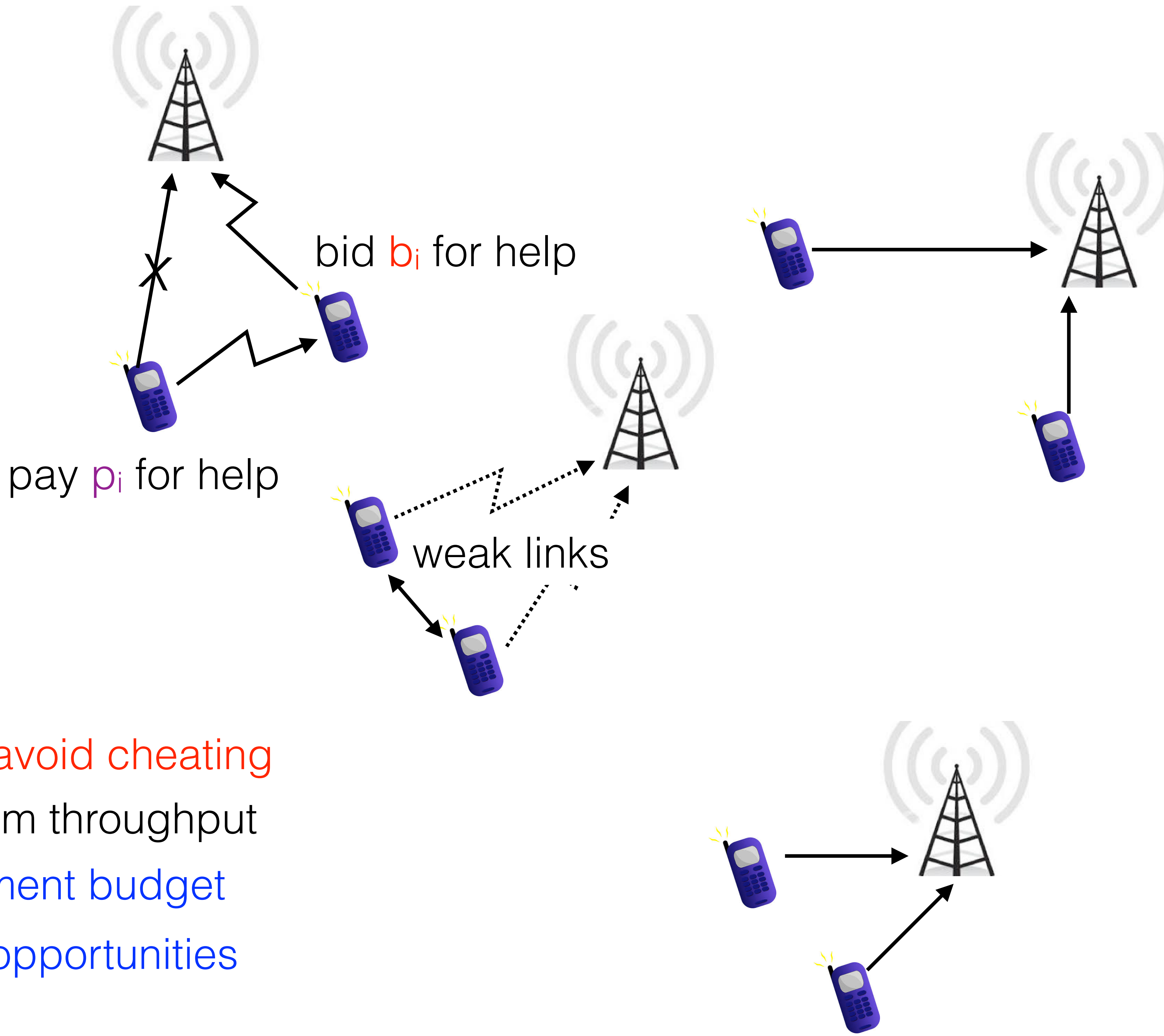


mechanism to avoid cheating

ensure maximum throughput

subject to payment budget

Modern Problem Device-2-Device Communication



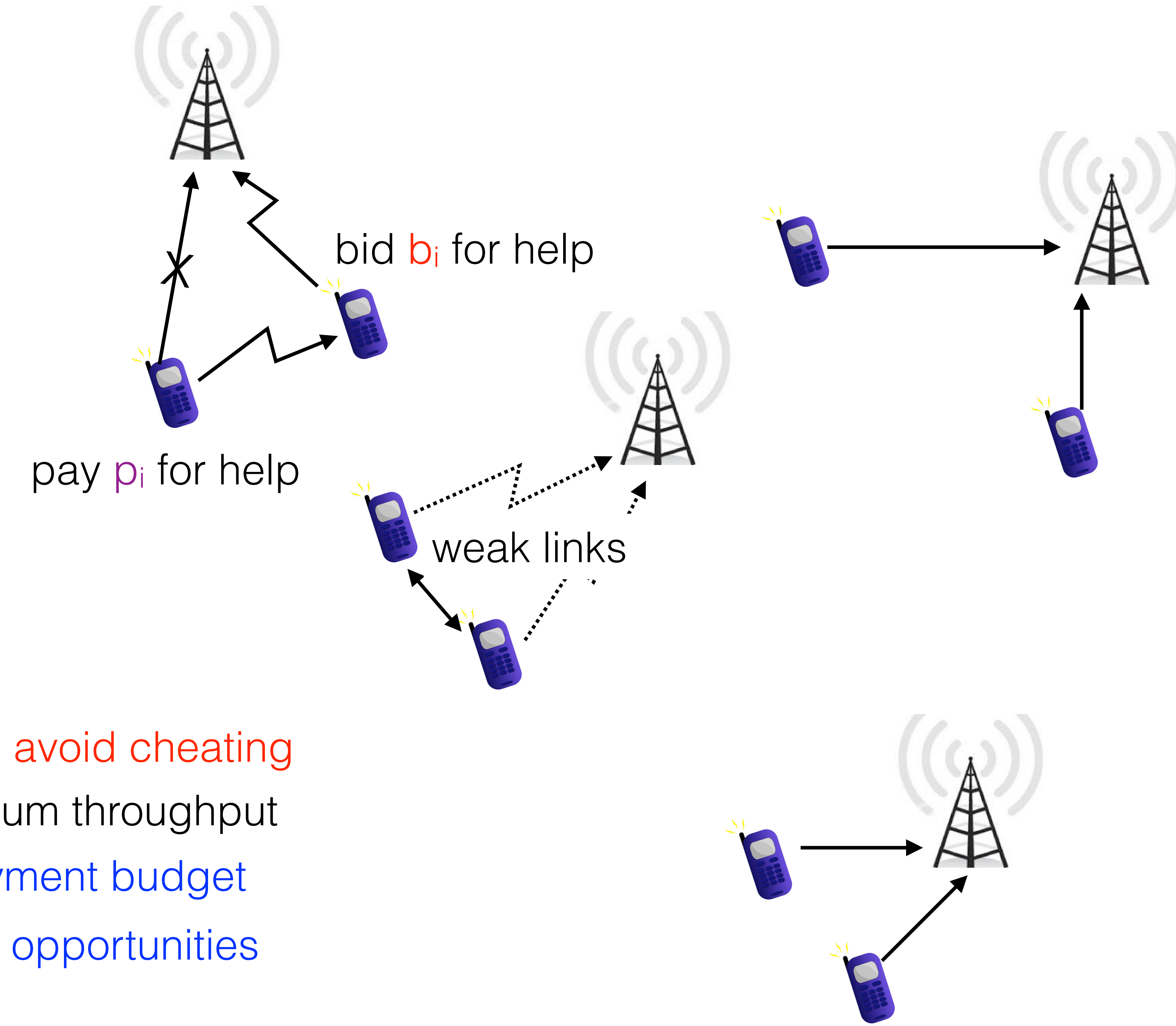
mechanism to avoid cheating

ensure maximum throughput

subject to payment budget

unknown help opportunities

Modern Problem Device-2-Device Communication



mechanism to avoid cheating

ensure maximum throughput

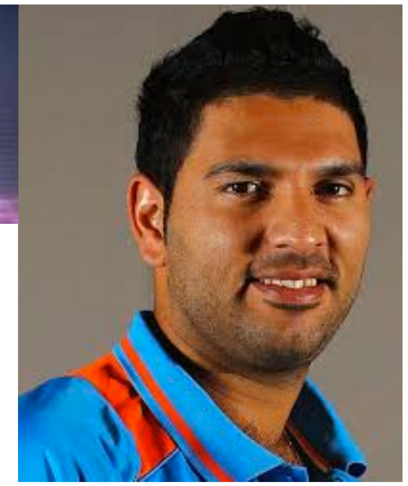
subject to payment budget

unknown help opportunities

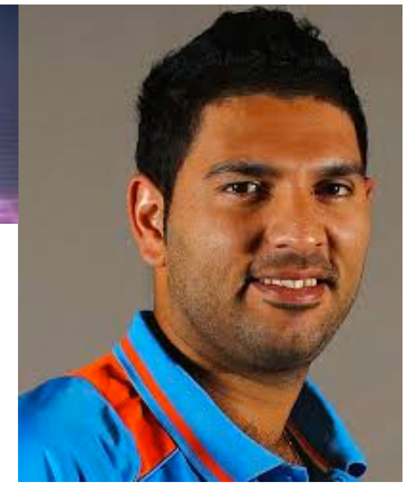
Find optimal helper association and incentive rule that is truthful

Why care about truthfulness ?

Why care about truthfulness ?

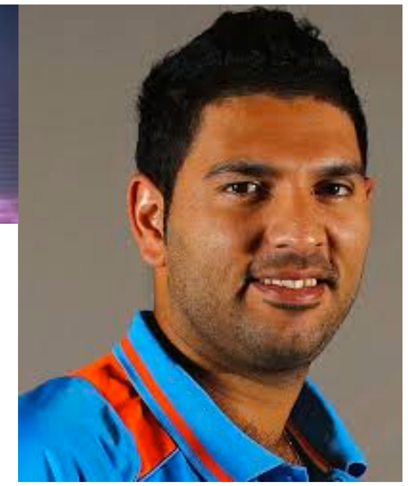


Why care about truthfulness ?



8.28 Crores

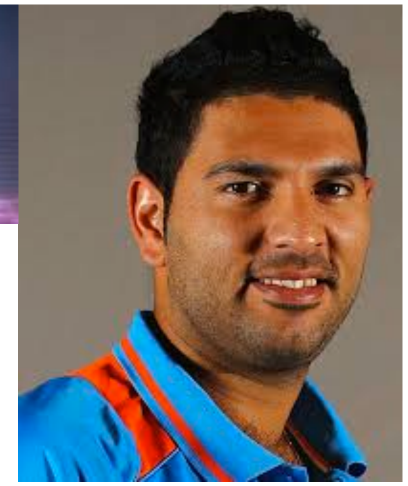
Why care about truthfulness ?



8.28 Crores

14 Crores

Why care about truthfulness ?

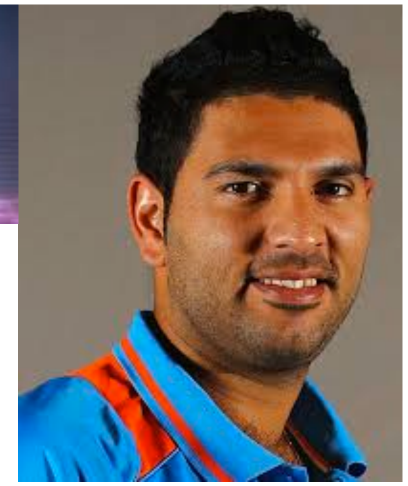


8.28 Crores

14 Crores

16 Crores

Why care about truthfulness ?



8.28 Crores

14 Crores

16 Crores

7 Crores

W. Vickerey

E. Clarke

T. Groves

Truthful Auction

Truthful Auction

Truthful Auction

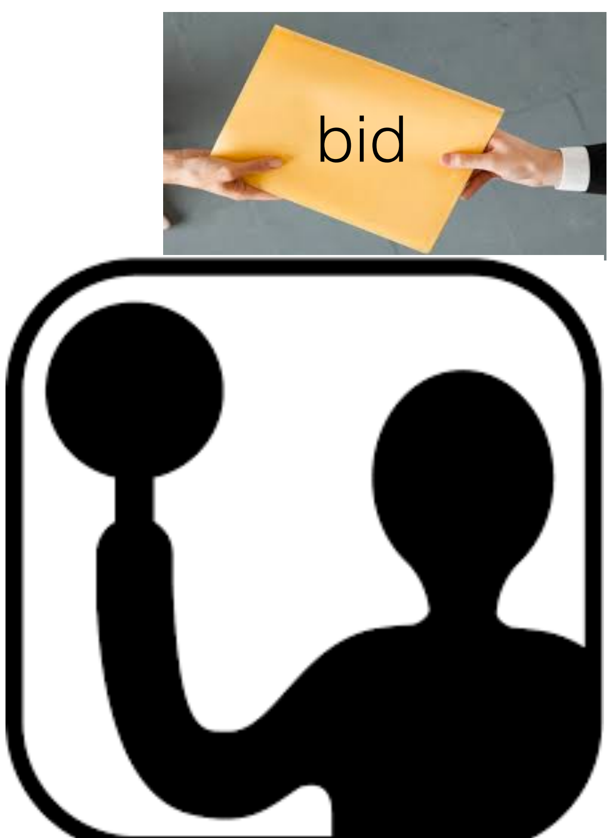
Winner: Largest bid

Price: Second-Largest bid

Truthful Auction

Winner: Largest bid

Price: Second-Largest bid



No incentive to bid more than private utility

Problem is equivalent to Crowdsourcing

Problem is equivalent to Crowdsourcing

multiple election tasks

Problem is equivalent to Crowdsourcing

multiple election tasks

bid

Problem is equivalent to Crowdsourcing

multiple election tasks

bid

payment

Problem is equivalent to Crowdsourcing

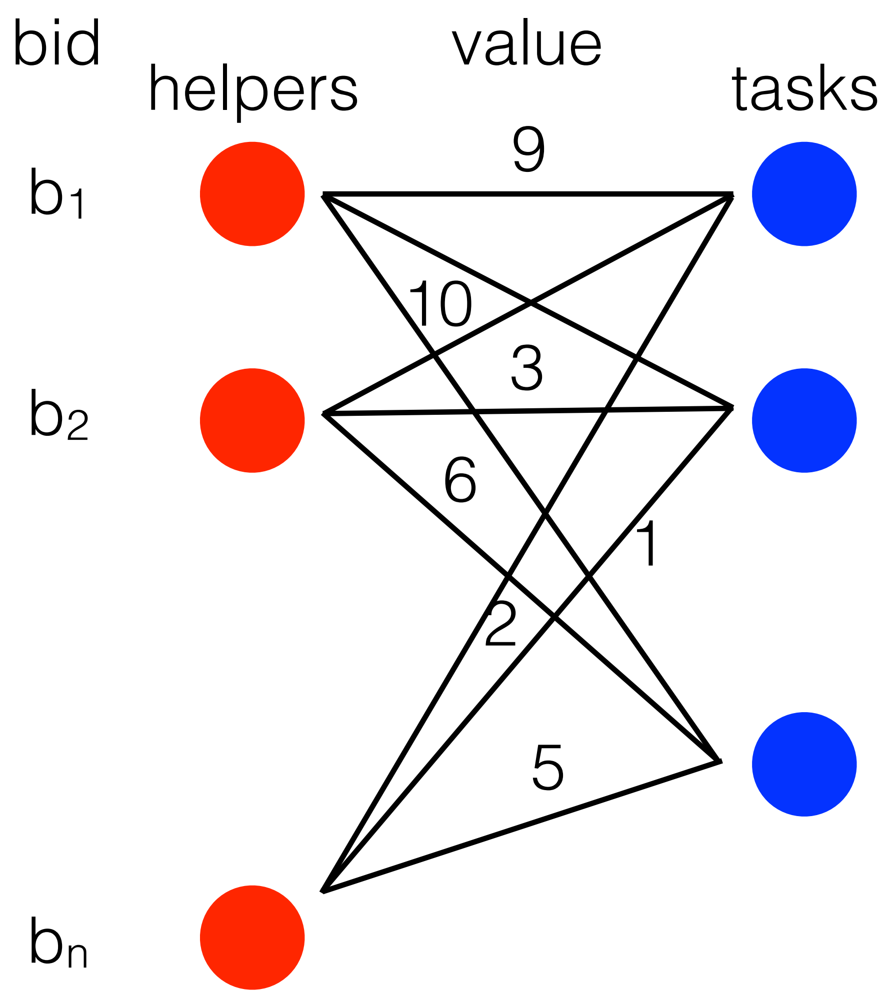
multiple election tasks

Total Budget

bid

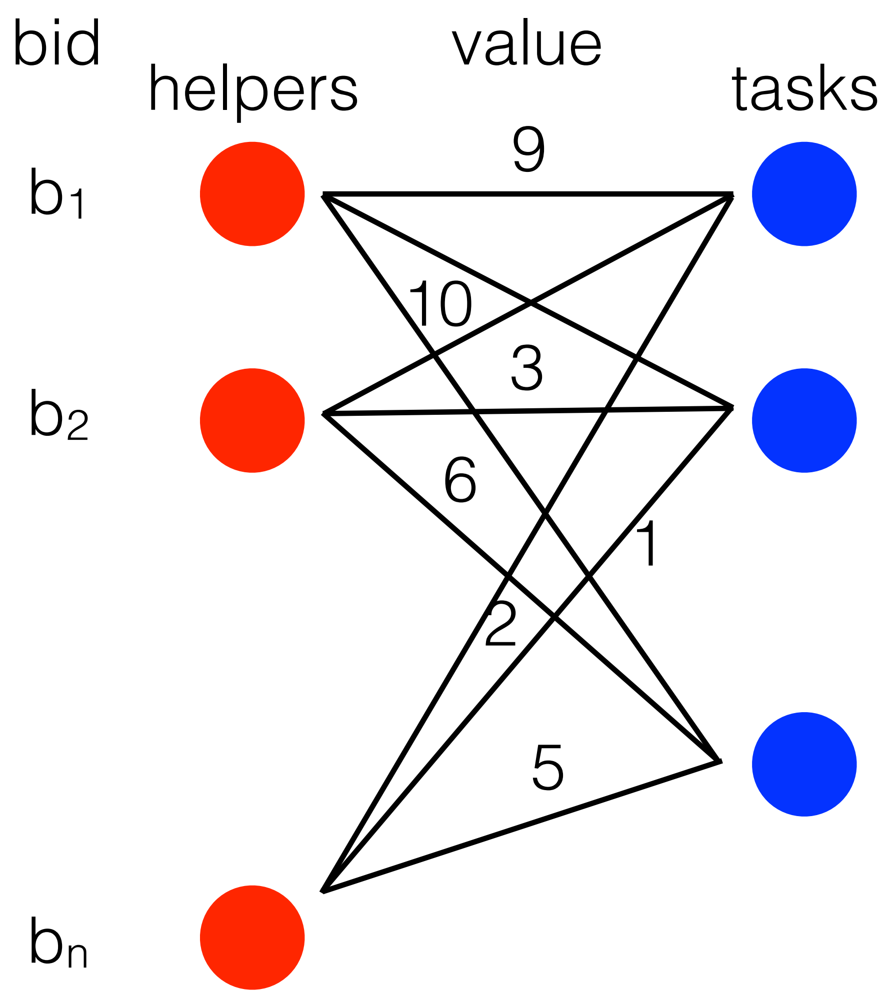
payment

Example



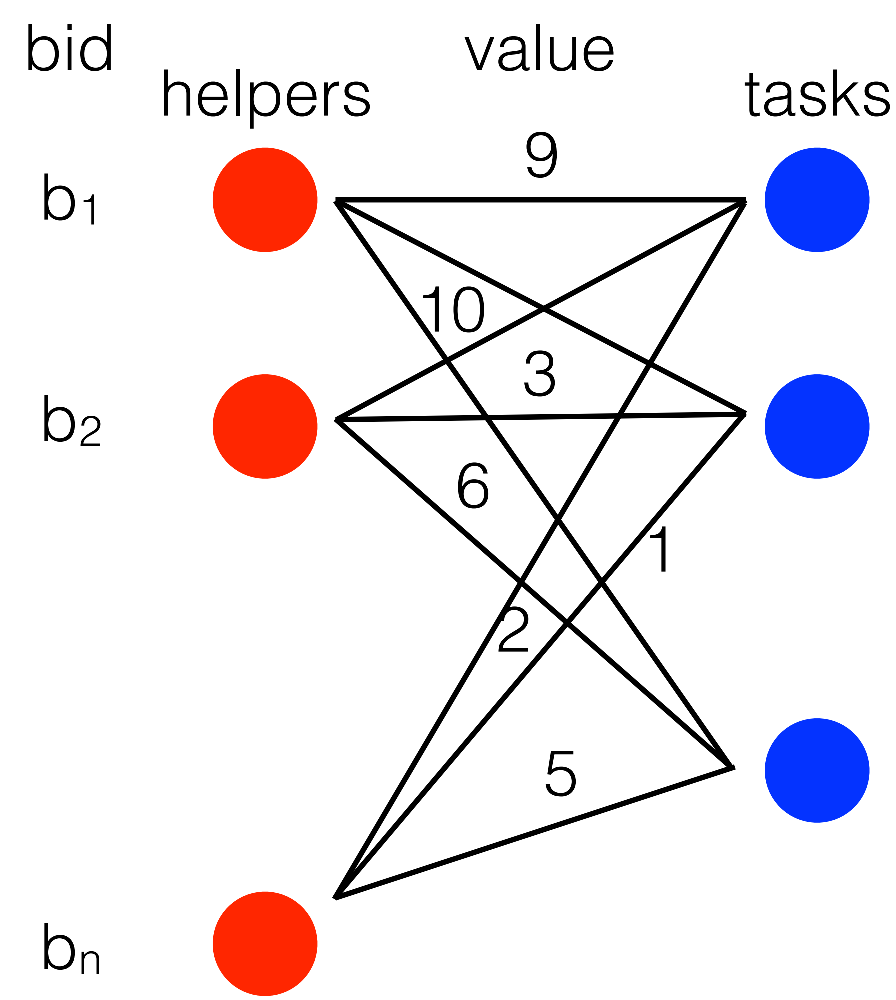
Example

For simplicity at most one task per helper and one helper per task



Example

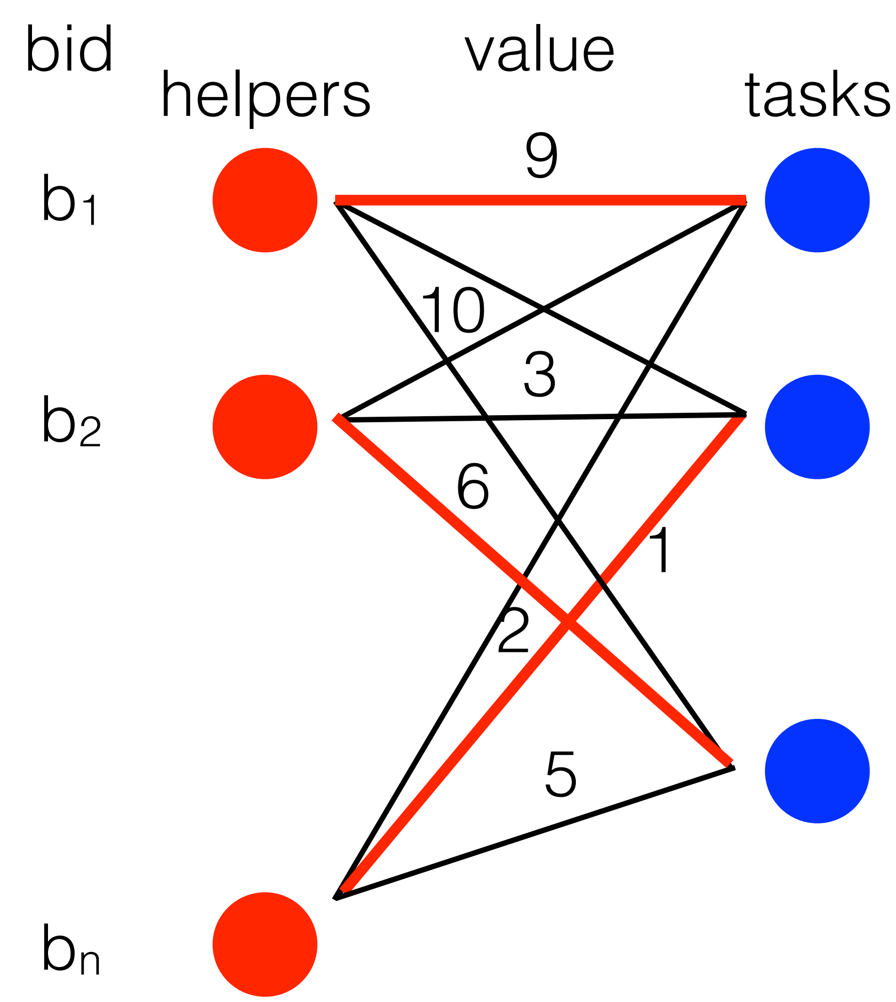
For simplicity at most one task per helper and one helper per task



Objective: Truthful Matching with largest sum weight under a budget constraint

Example

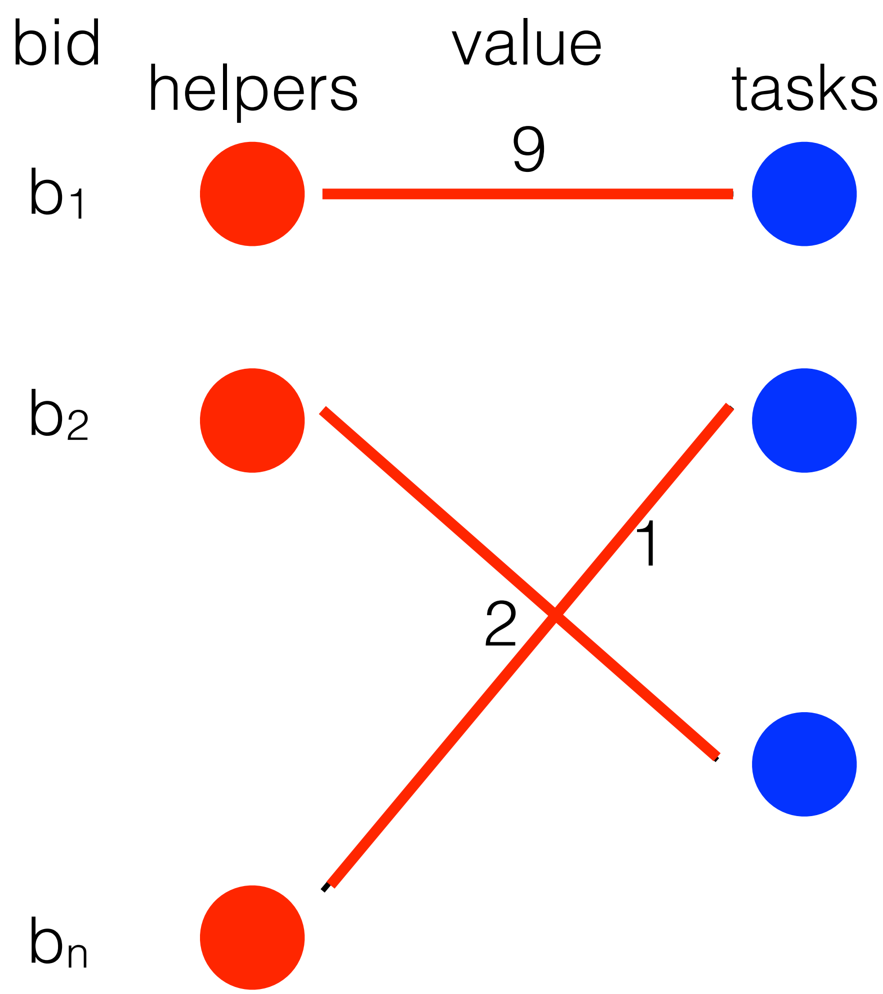
For simplicity at most one task per helper and one helper per task



Objective: Truthful Matching with largest sum weight under a budget constraint

Example

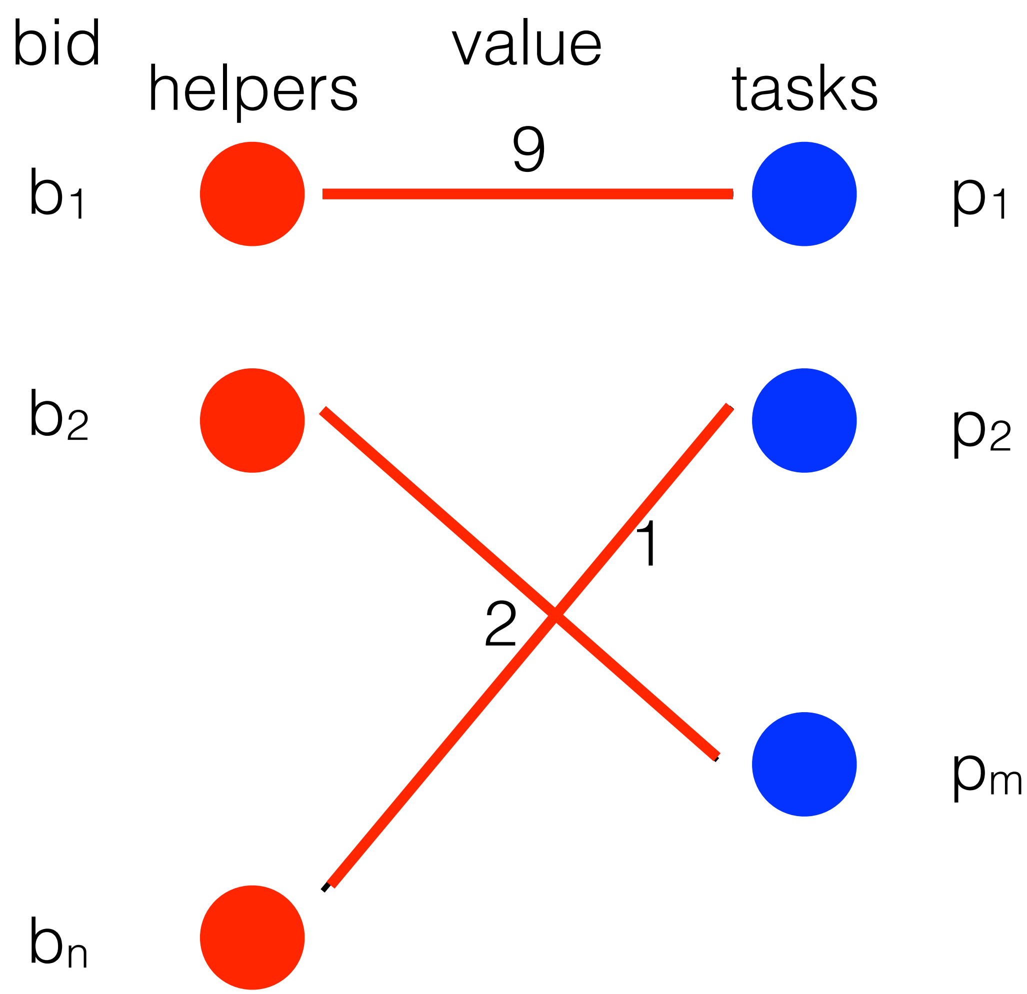
For simplicity at most one task per helper and one helper per task



Objective: Truthful Matching with largest sum weight under a budget constraint

Example

For simplicity at most one task per helper and one helper per task



$p_1 + p_2 + \ldots + p_m < B$

Objective: Truthful Matching with largest sum weight under a budget constraint

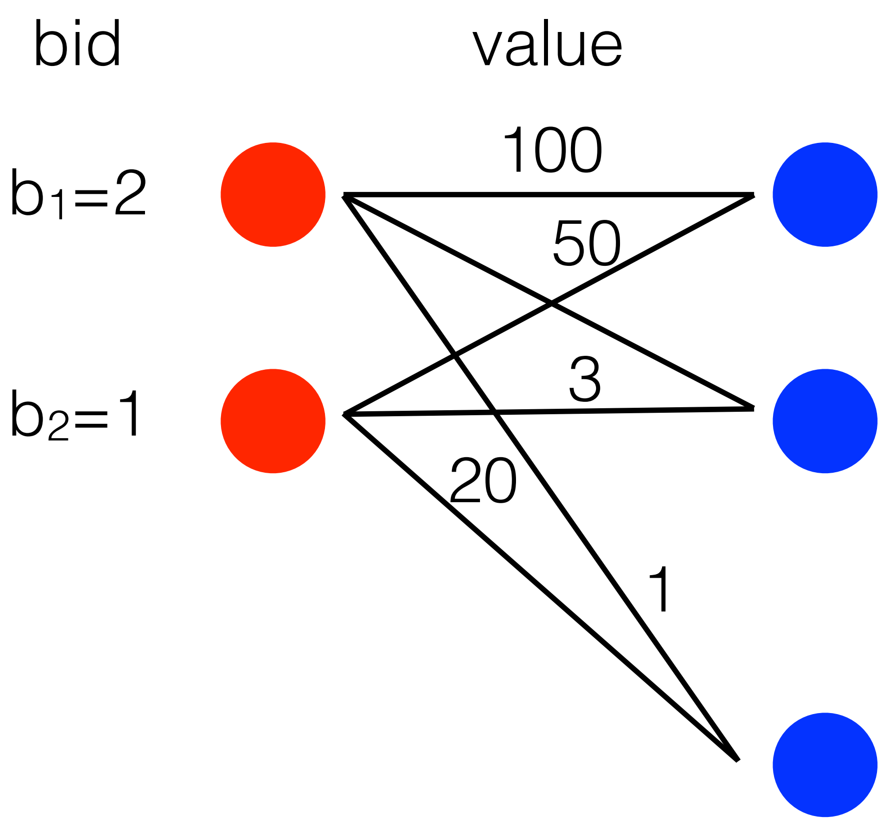
when is a reverse auction truthful ?

R. Myerson

Monotonicity - if an agent is selected with bid b, then he is always selected if he bids below b

Critical Price - there exists a threshold price such that if an agent bids above it, he is never selected

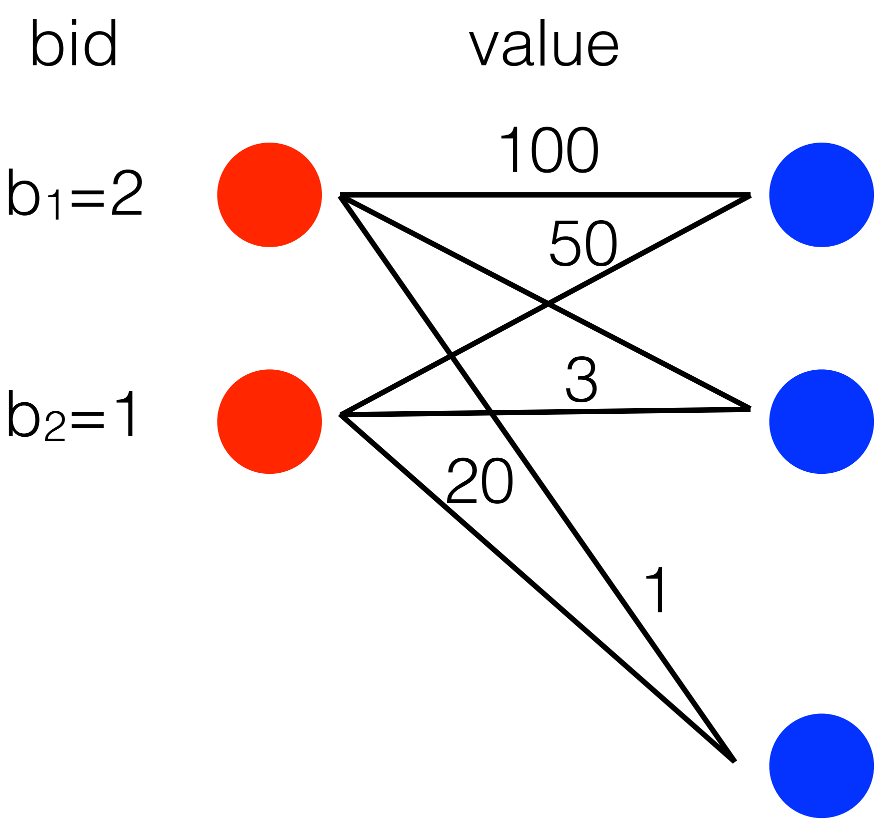
Algorithm



$p_1 + p_{2} + \ldots + p_m < B$

Algorithm

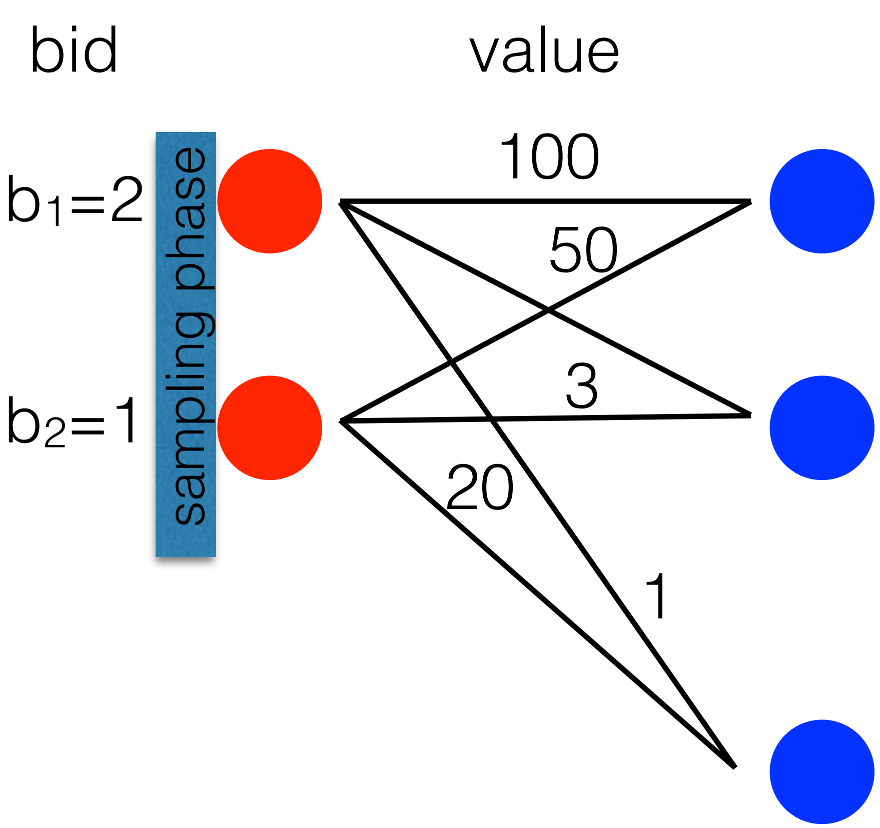
same idea as before, **sampling** followed by **decision**



$p_1 + p_2 + \ldots + p_m < B$

Algorithm

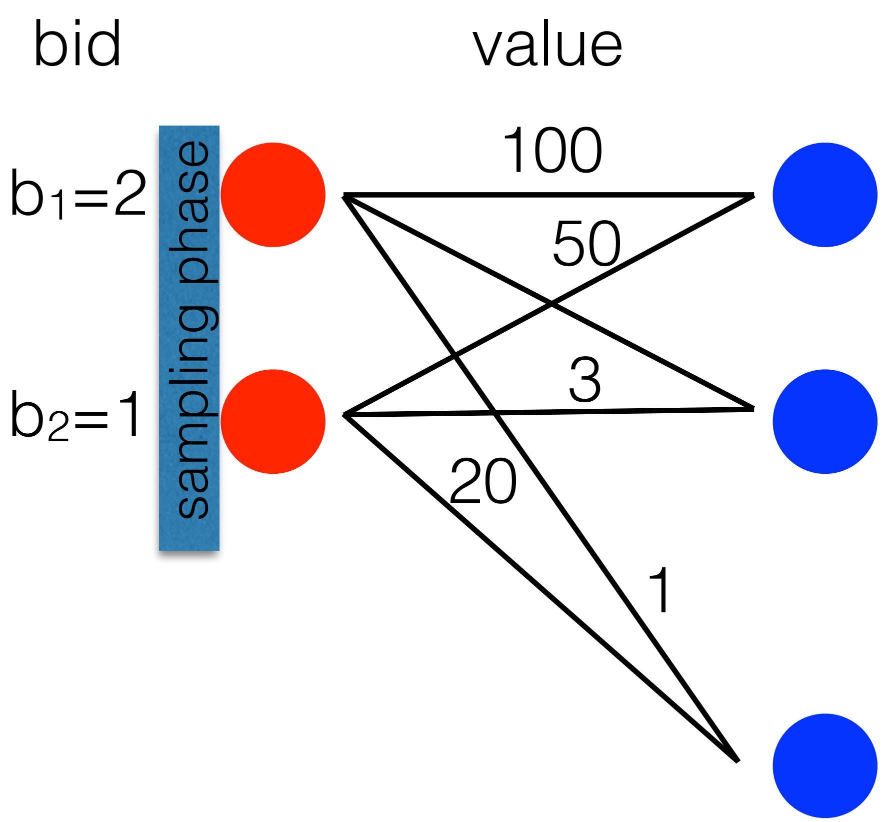
same idea as before, **sampling** followed by **decision**



$p_1 + p_2 + \ldots + p_m < B$

Algorithm

same idea as before, **sampling** followed by **decision**



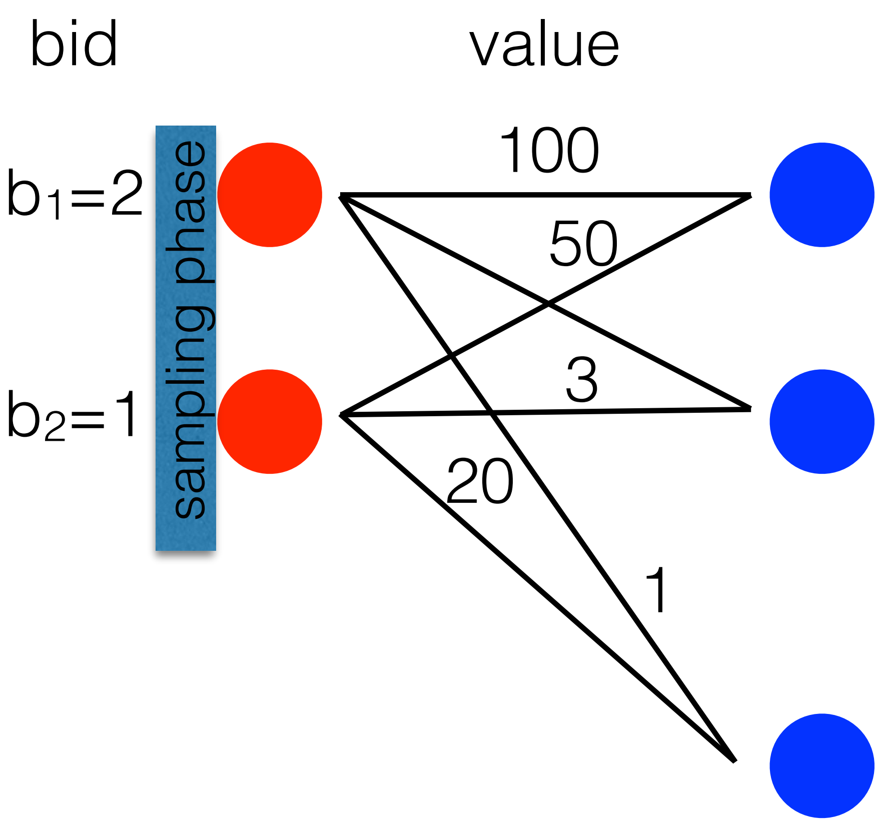
$p_1 + p_2 + \ldots + p_m < B$

In Sampling Phase

bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

Algorithm

same idea as before, **sampling** followed by **decision**



$p_1 + p_2 + \ldots + p_m < B$

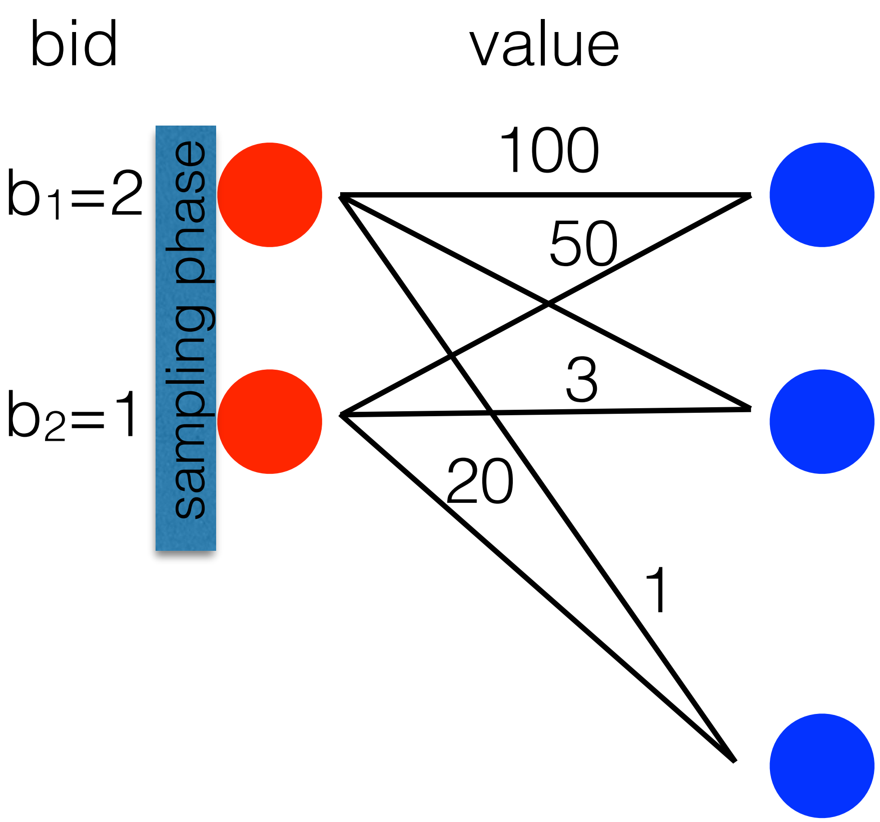
In Sampling Phase

bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

good Graph $\quad \mathrm{G}(\gamma) = \left\{ e \in G : \frac{b(e)}{v(e)} < \gamma \right\}$

Algorithm

same idea as before, **sampling** followed by **decision**



$$\gamma = 0.1$$

In Sampling Phase

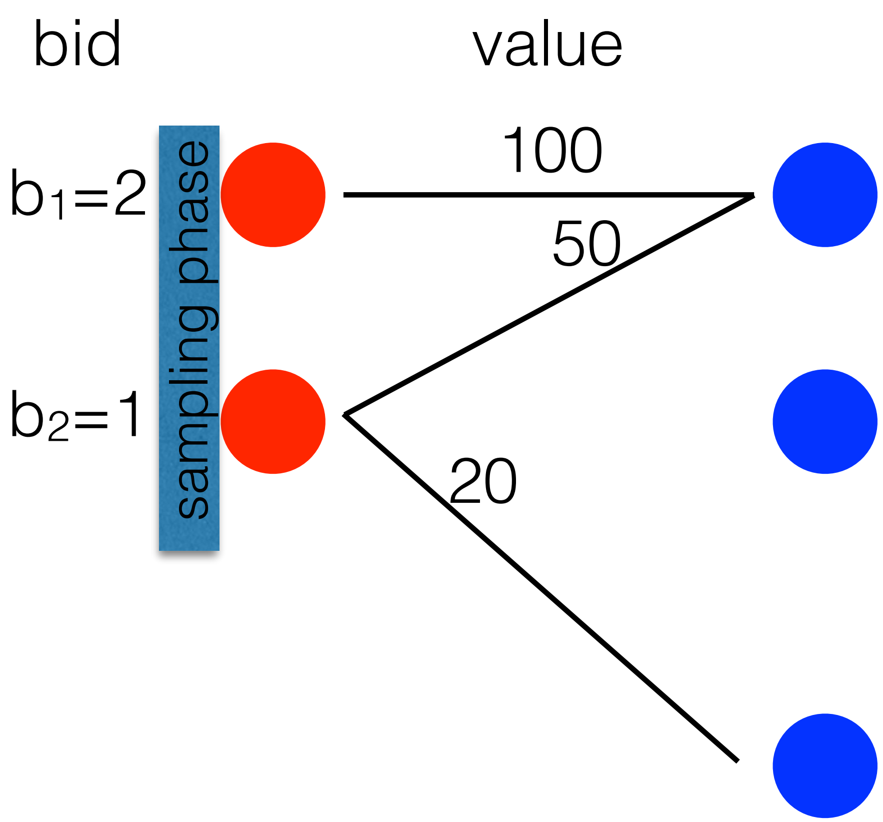
bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

good Graph $\mathrm{G}(\gamma) = \left\{ e \in G : \frac{b(e)}{v(e)} < \gamma \right\}$

$p_1 + p_2 + \ldots + p_m < B$

Algorithm

same idea as before, **sampling** followed by **decision**



$\gamma = 0.1$

In Sampling Phase

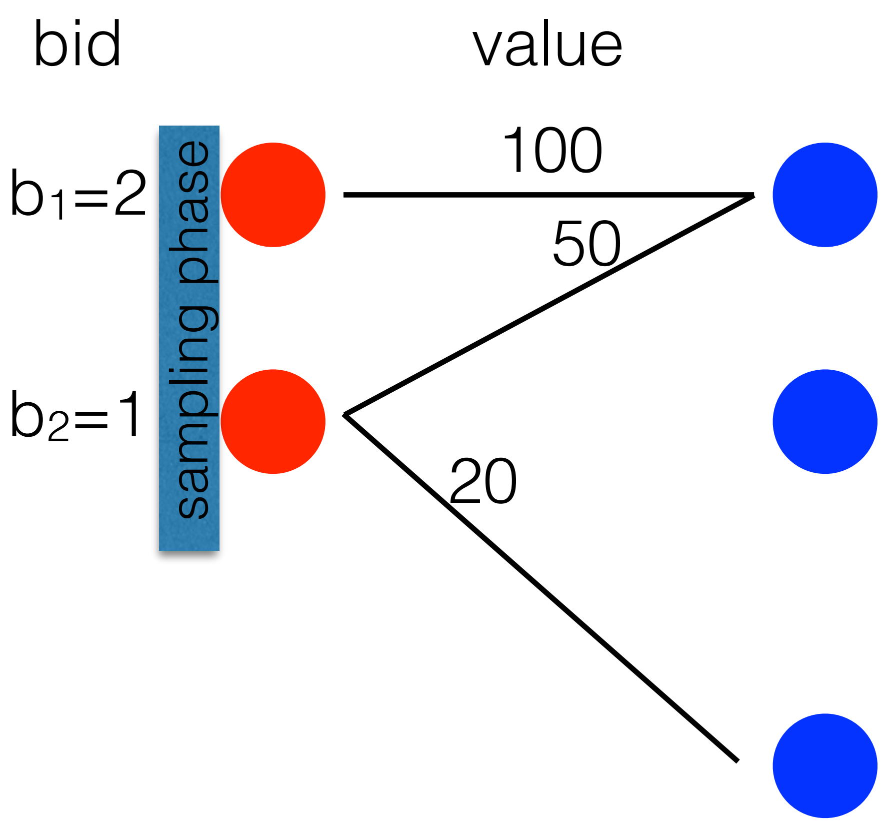
bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

good Graph $\mathrm{G}(\gamma) = \left\{ e \in G : \frac{b(e)}{v(e)} < \gamma \right\}$

$p_1 + p_2 + \ldots + p_m < B$

Algorithm

same idea as before, **sampling** followed by **decision**



In Sampling Phase

bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

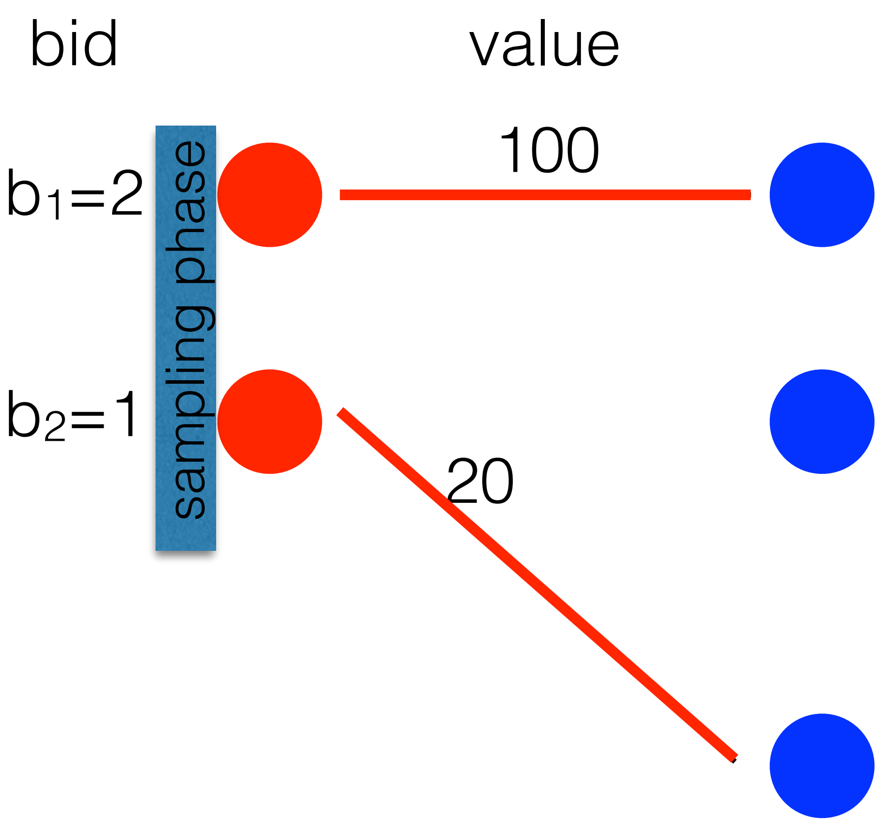
$\gamma = 0.1$ good Graph $\mathrm{G}(\gamma) = \left\{e \in G : \frac{b(e)}{v(e)} < \gamma\right\}$

$\mathrm{M}(\gamma)$ be greedy matching over $G(\gamma)$

$p_1 + p_2 + \ldots + p_m < B$

Algorithm

same idea as before, **sampling** followed by **decision**



In Sampling Phase

bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

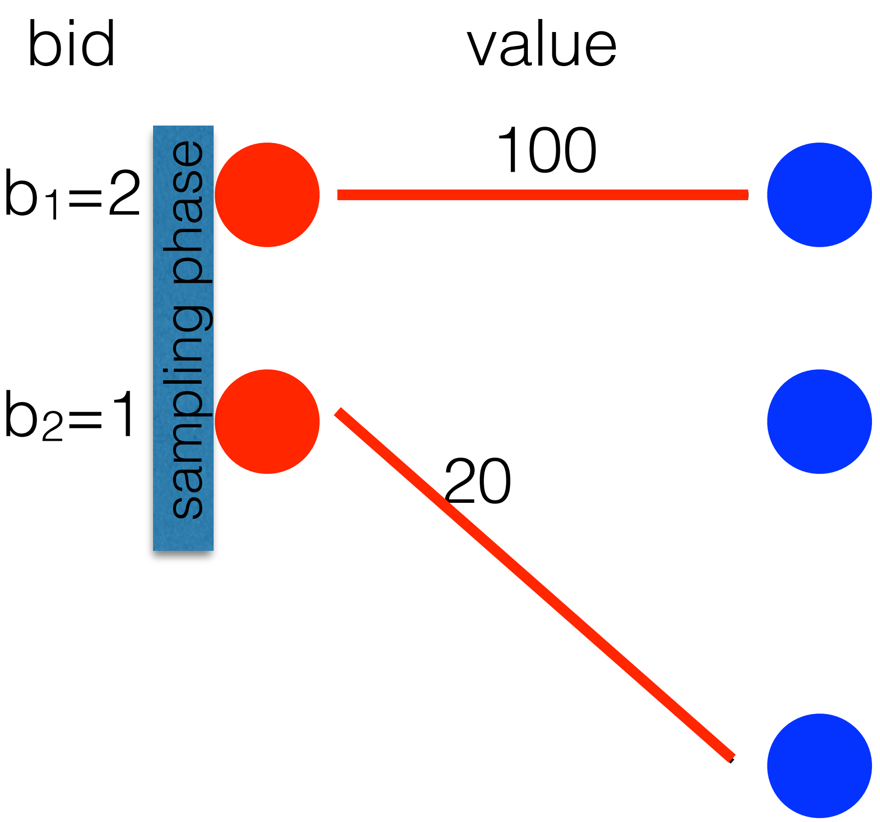
$\gamma = 0.1$ good Graph $\mathrm{G}(\gamma) = \left\{ e \in G : \frac{b(e)}{v(e)} < \gamma \right\}$

$\mathrm{M}(\gamma)$ be greedy matching over $G(\gamma)$

$p_1 + p_2 + \ldots + p_m < B$

Algorithm

same idea as before, **sampling** followed by **decision**



$p_1 + p_{2+\ldots+}p_m < B$

In Sampling Phase

bid to benefit ratio of an edge $\boxed{\frac{b(e)}{v(e)}}$

$\gamma = 0.1$ good Graph $\mathrm{G}(\gamma) = \left\{ e \in G : \frac{b(e)}{v(e)} < \gamma \right\}$

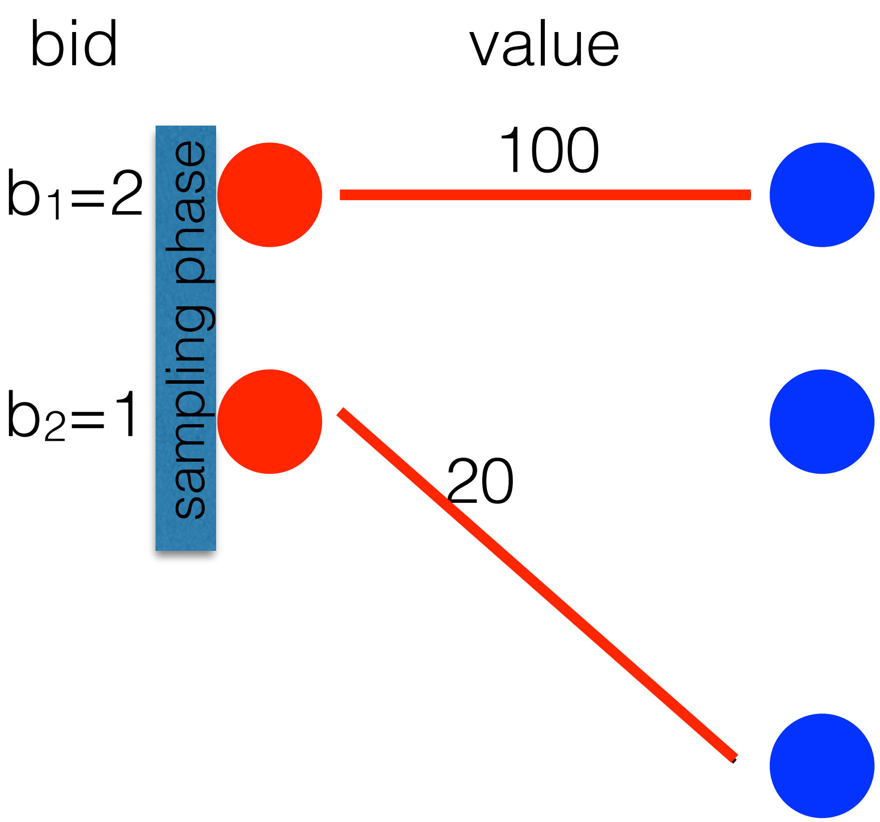
$\mathrm{M}(\gamma)$ be greedy matching over $G(\gamma)$

Find largest γ

$\gamma \sum_{e \in M(\gamma)} v(e) \leq B$

Algorithm

same idea as before, **sampling** followed by **decision**



In Sampling Phase

bid to benefit ratio of an edge $\frac{b(e)}{v(e)}$

good Graph $\mathrm{G}(\gamma) = \left\{ e \in G : \frac{b(e)}{v(e)} < \gamma \right\}$

$\mathrm{M}(\gamma)$ be greedy matching over $G(\gamma)$

Find largest γ

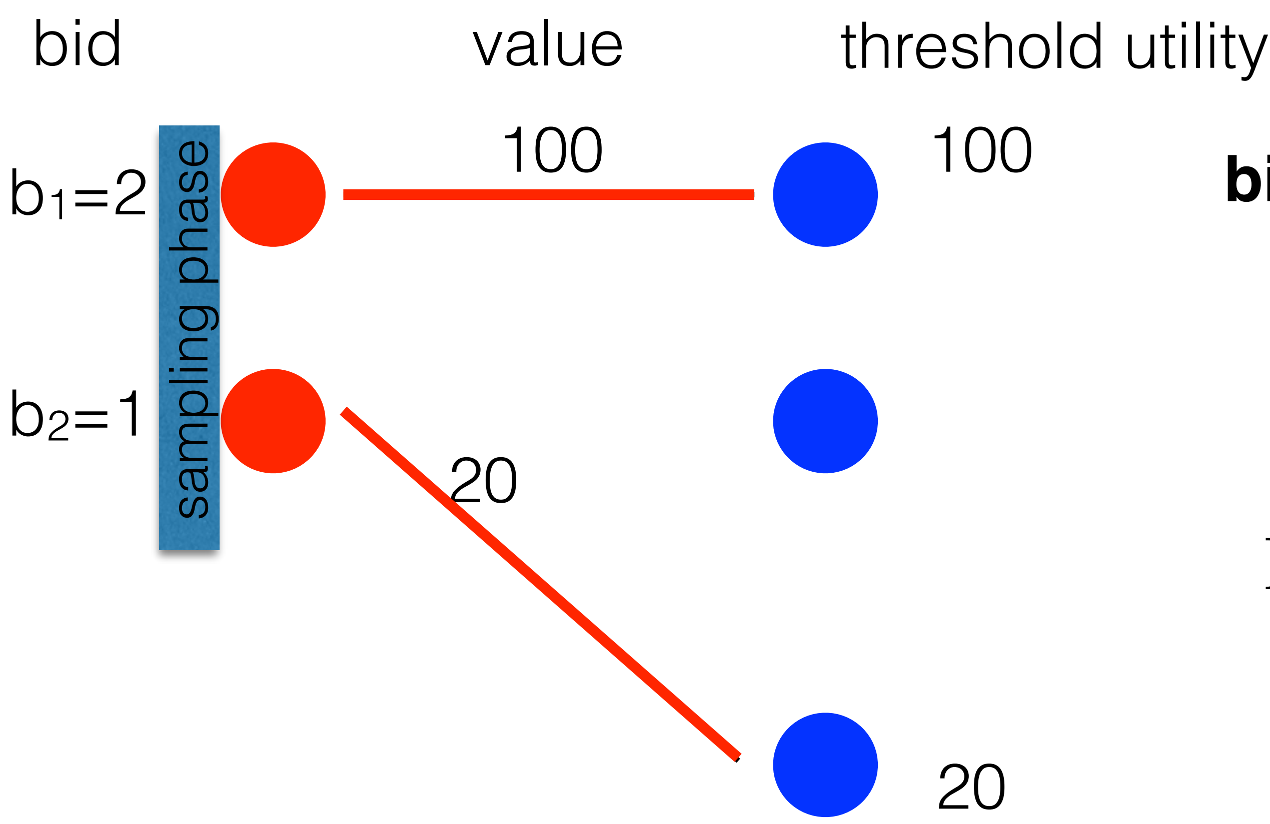
$\gamma \sum_{e \in M(\gamma)} v(e) \leq B$

$p_1 + p_2 + \ldots + p_m < B$

For decision phase utility threshold of each blue node to be value in Matching $\mathrm{M}(\gamma)$

Algorithm

same idea as before, **sampling** followed by **decision**



In Sampling Phase

bid to benefit ratio of an edge $\boxed{\frac{b(e)}{v(e)}}$

good Graph $\mathrm{G}(\gamma) = \left\{ e \in G : \frac{b(e)}{v(e)} < \gamma \right\}$

$\mathrm{M}(\gamma)$ be greedy matching over $G(\gamma)$

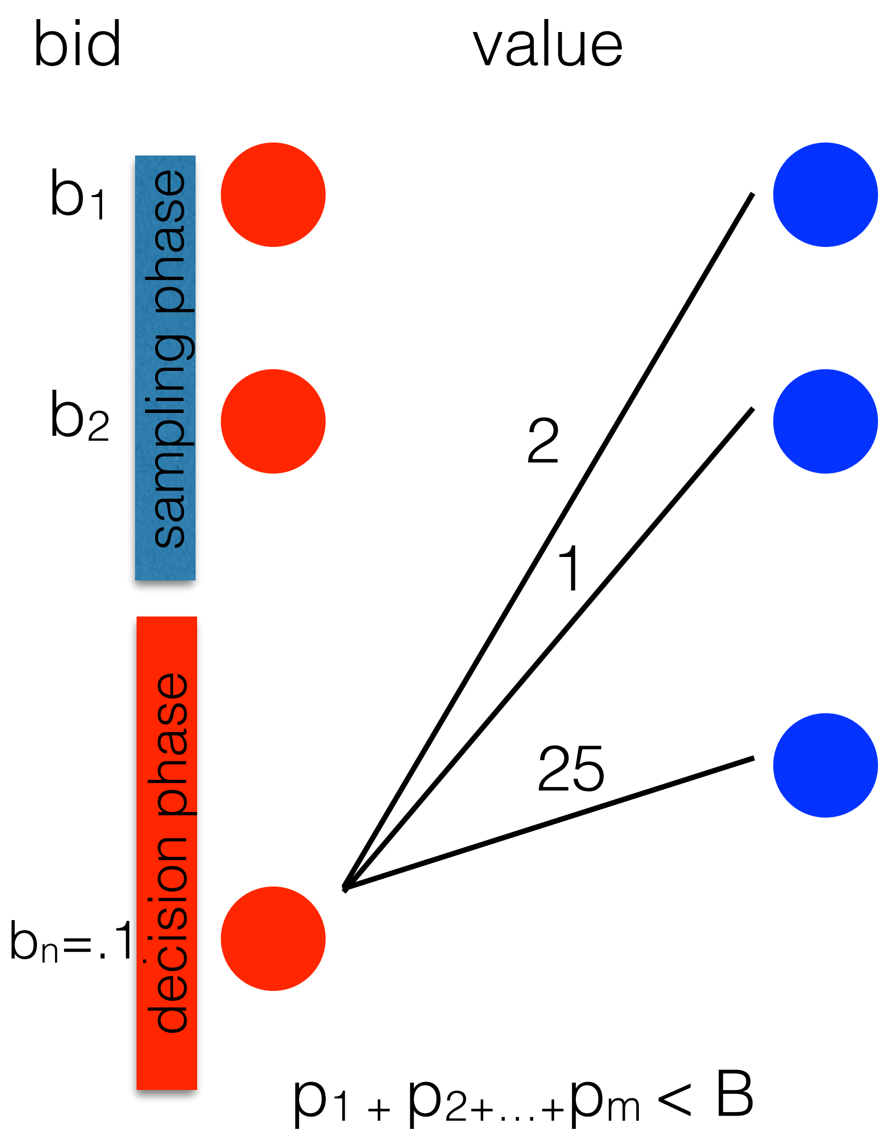
Find largest γ

$\gamma \sum_{e \in M(\gamma)} v(e) \leq B$

$p_1 + p_2 + \ldots + p_m < B$

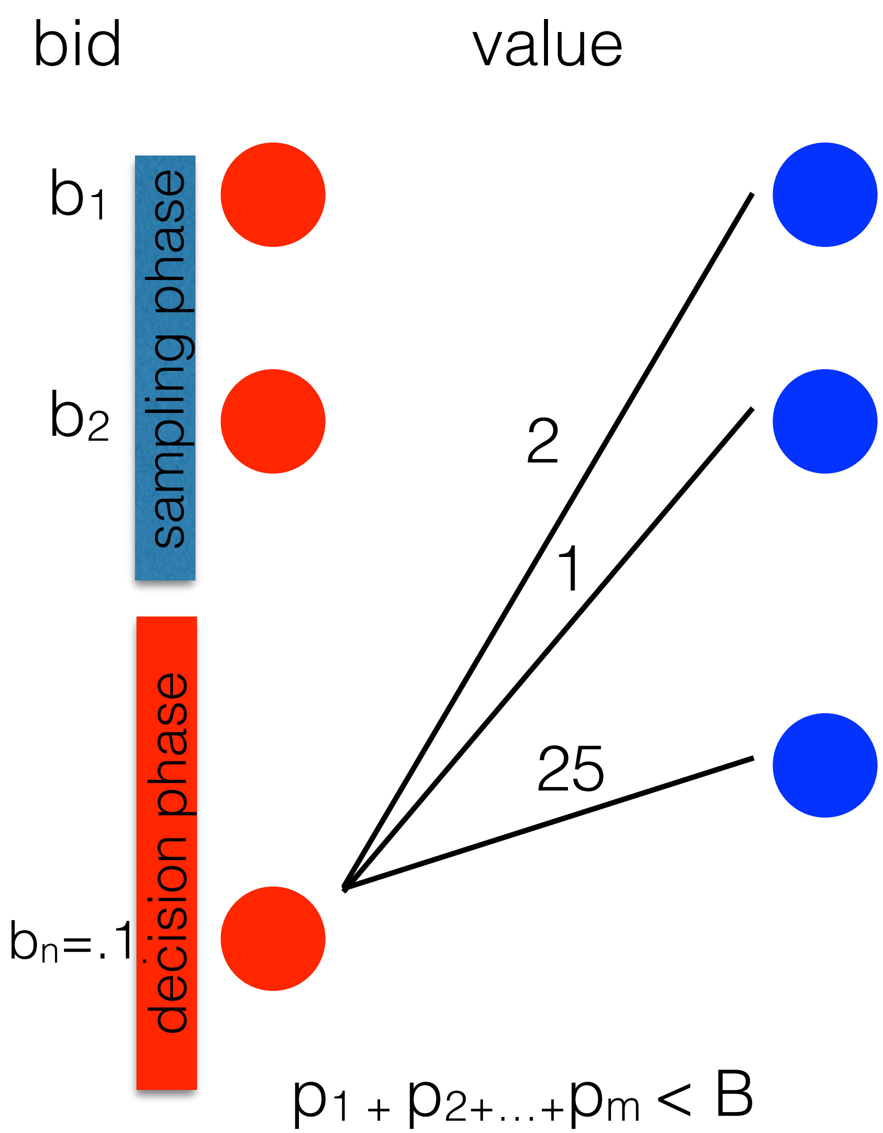
For decision phase utility threshold of each blue node to be value in Matching $\mathrm{M}(\gamma)$

Algorithm - decision phase



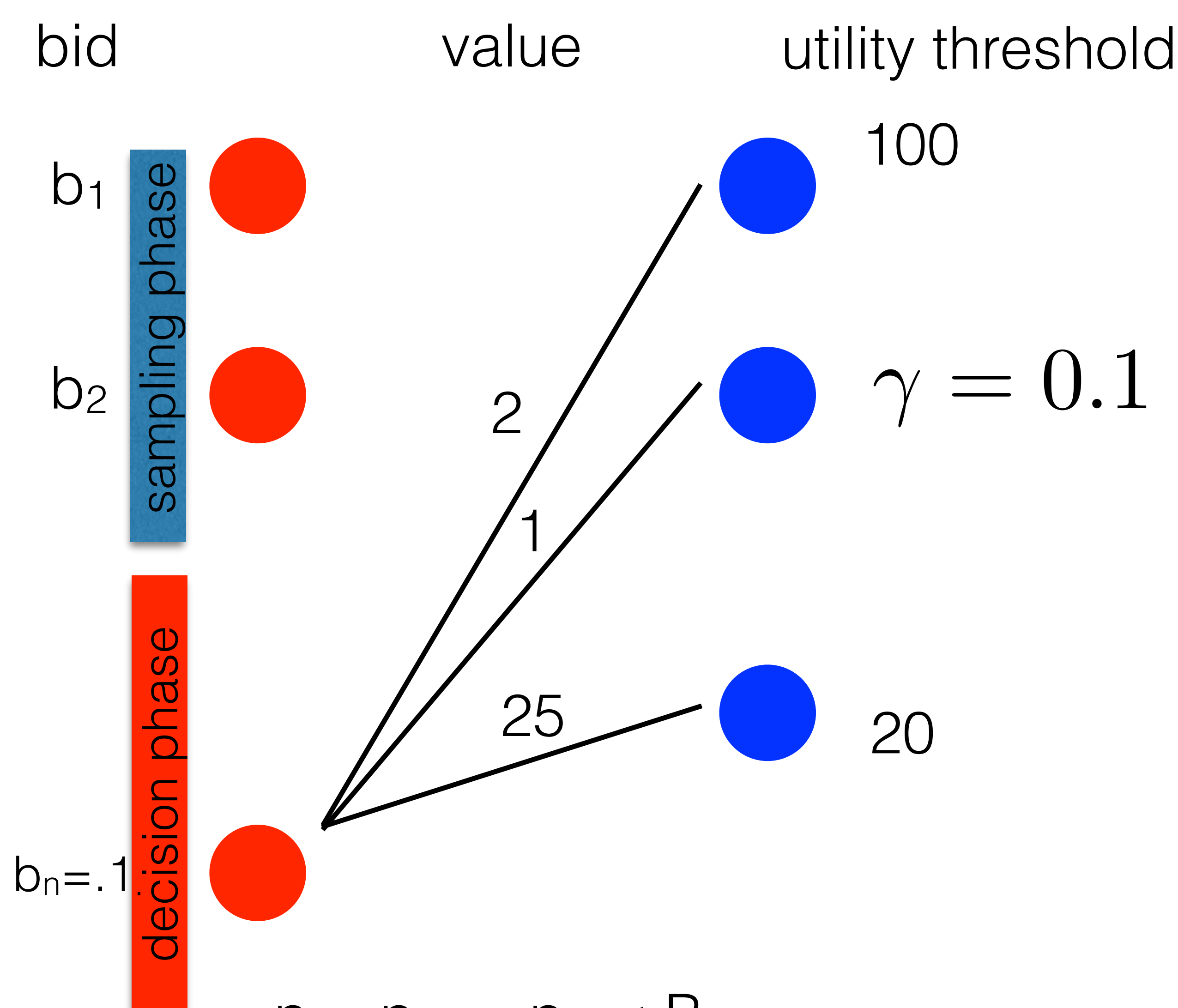
Algorithm - decision phase

γ and threshold utility obtained from sampling phase



Algorithm - decision phase

γ and threshold utility obtained from sampling phase

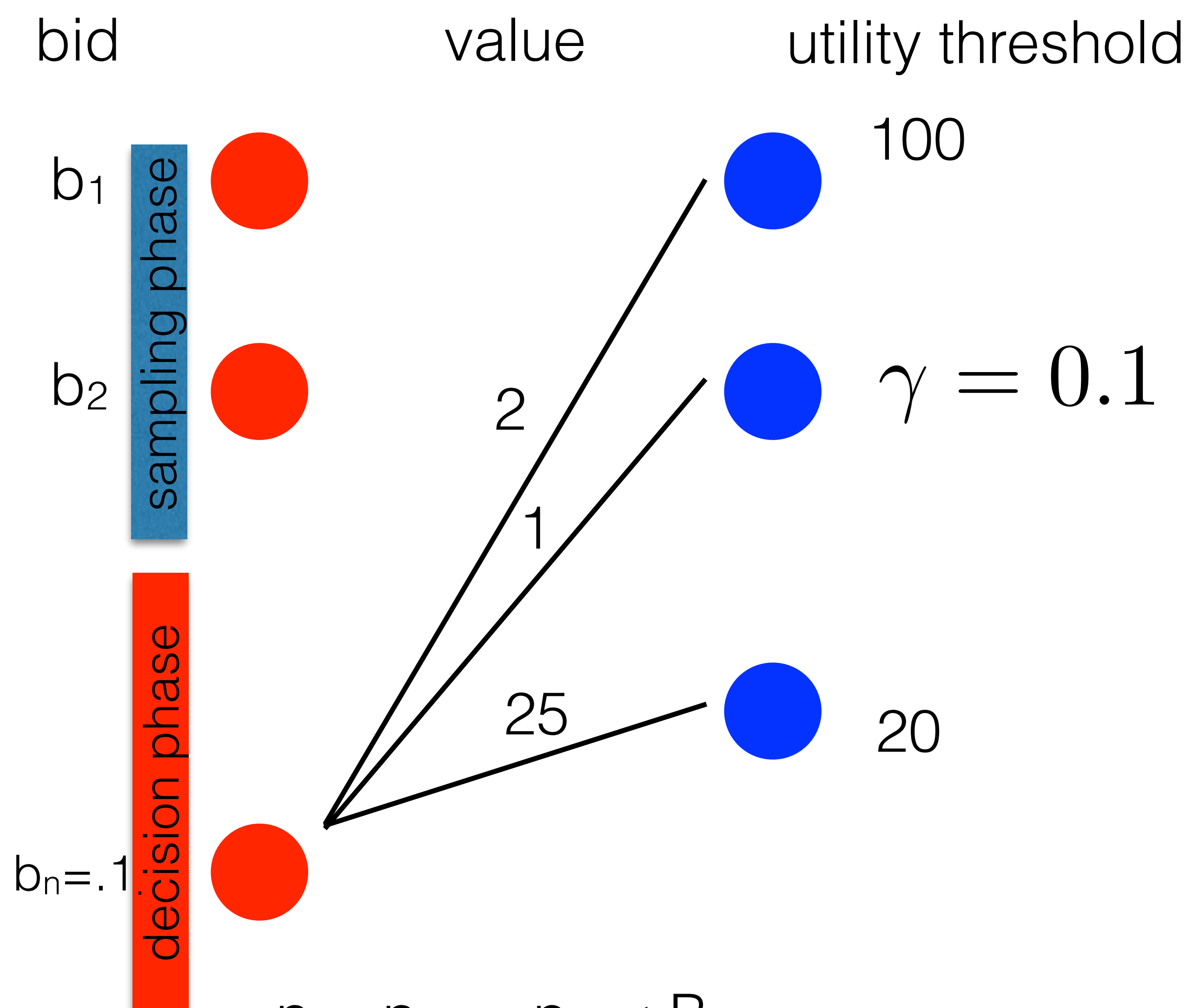


$p_1 + p_2 + \ldots + p_m < B$

Algorithm - decision phase

γ and threshold utility obtained from sampling phase

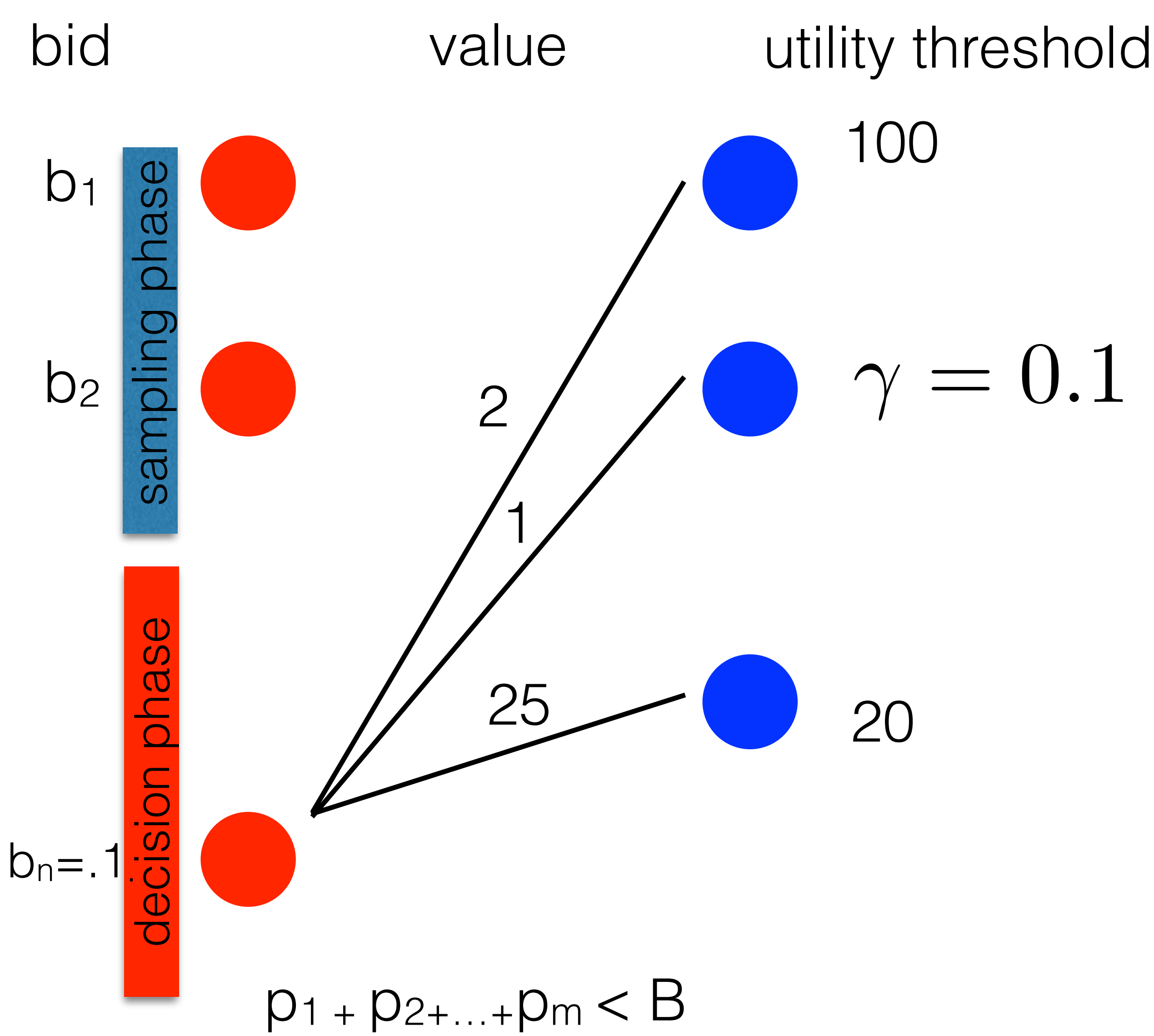
In Decision Phase



$p_1 + p_2 + \ldots + p_m < B$

Algorithm - decision phase

γ and threshold utility obtained from sampling phase

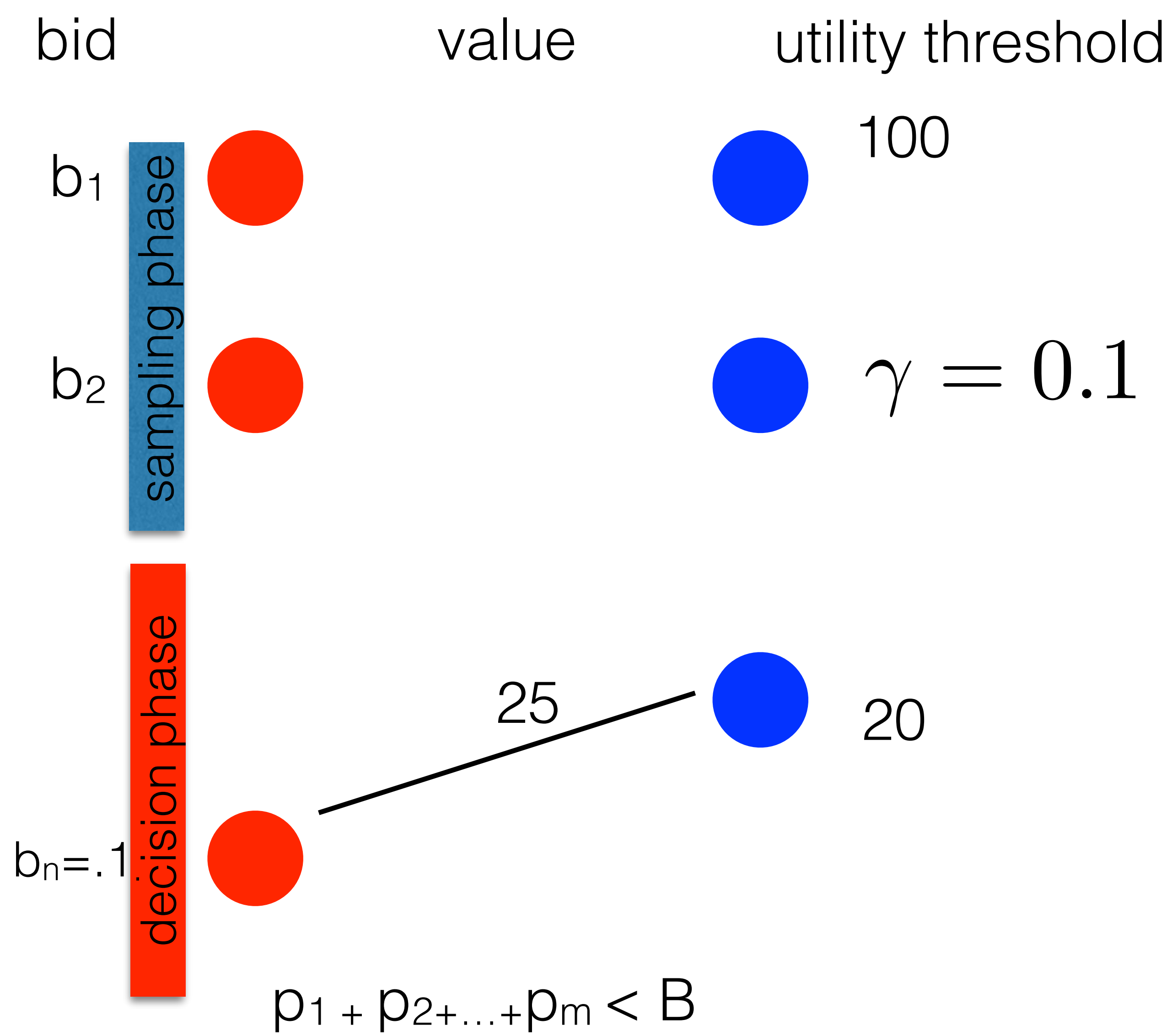


In Decision Phase

remove all edges with **bid to benefit** ratio $> \gamma$

Algorithm - decision phase

γ and threshold utility obtained from sampling phase

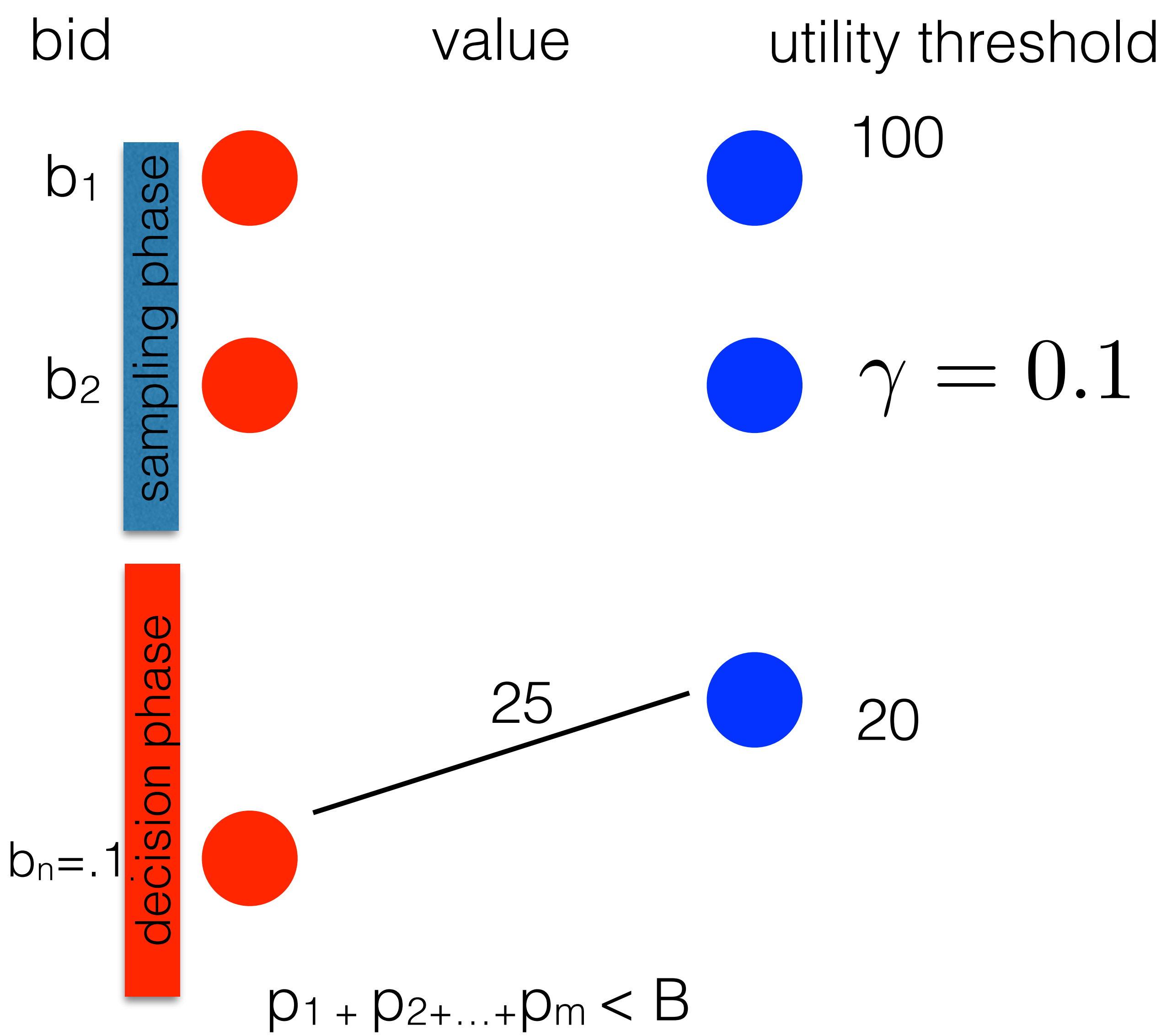


In Decision Phase

remove all edges with **bid to benefit** ratio $> \gamma$

Algorithm - decision phase

γ and threshold utility obtained from sampling phase



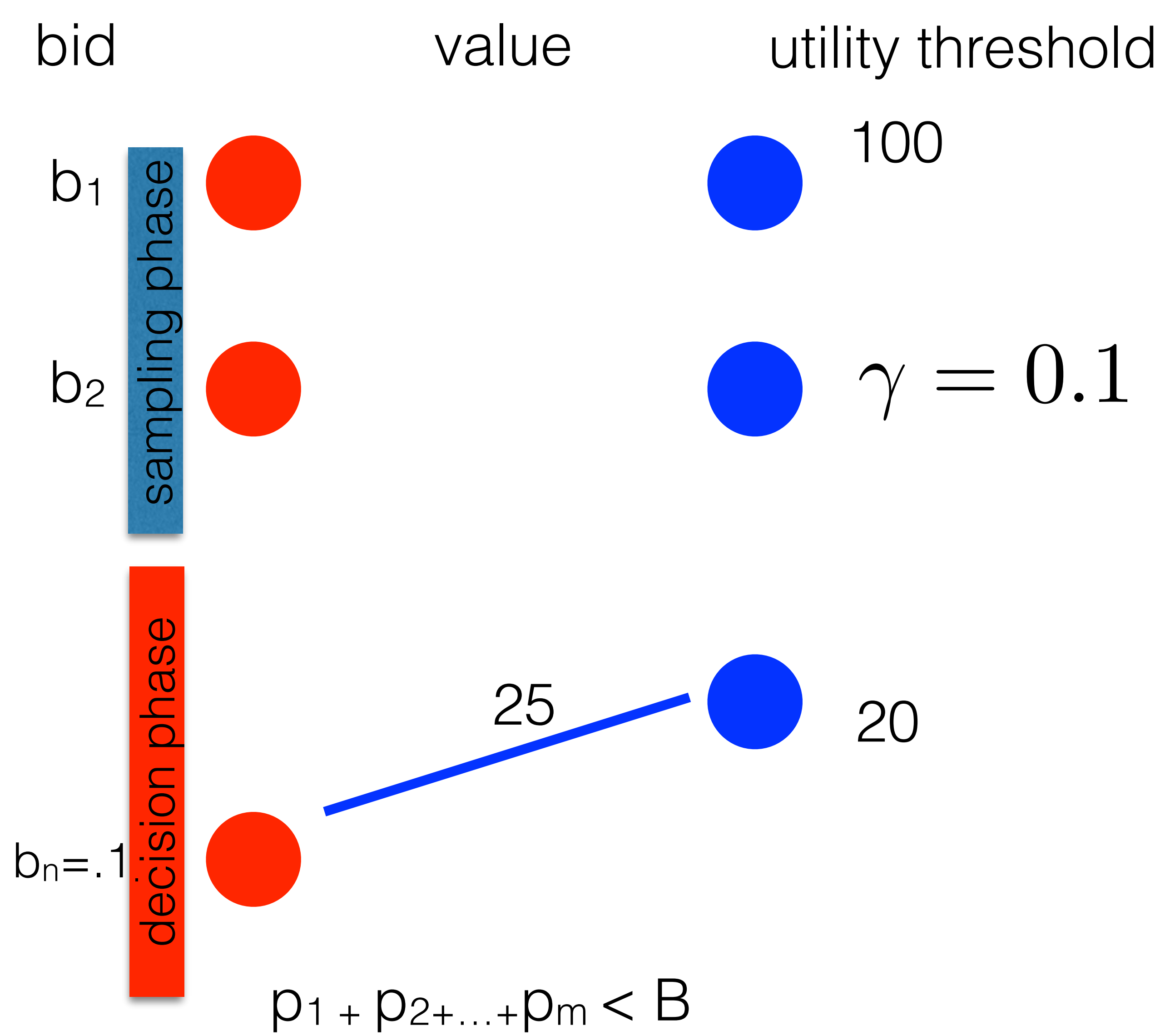
In Decision Phase

remove all edges with **bid to benefit** ratio $> \gamma$

match the largest weight more than utility threshold

Algorithm - decision phase

γ and threshold utility obtained from sampling phase



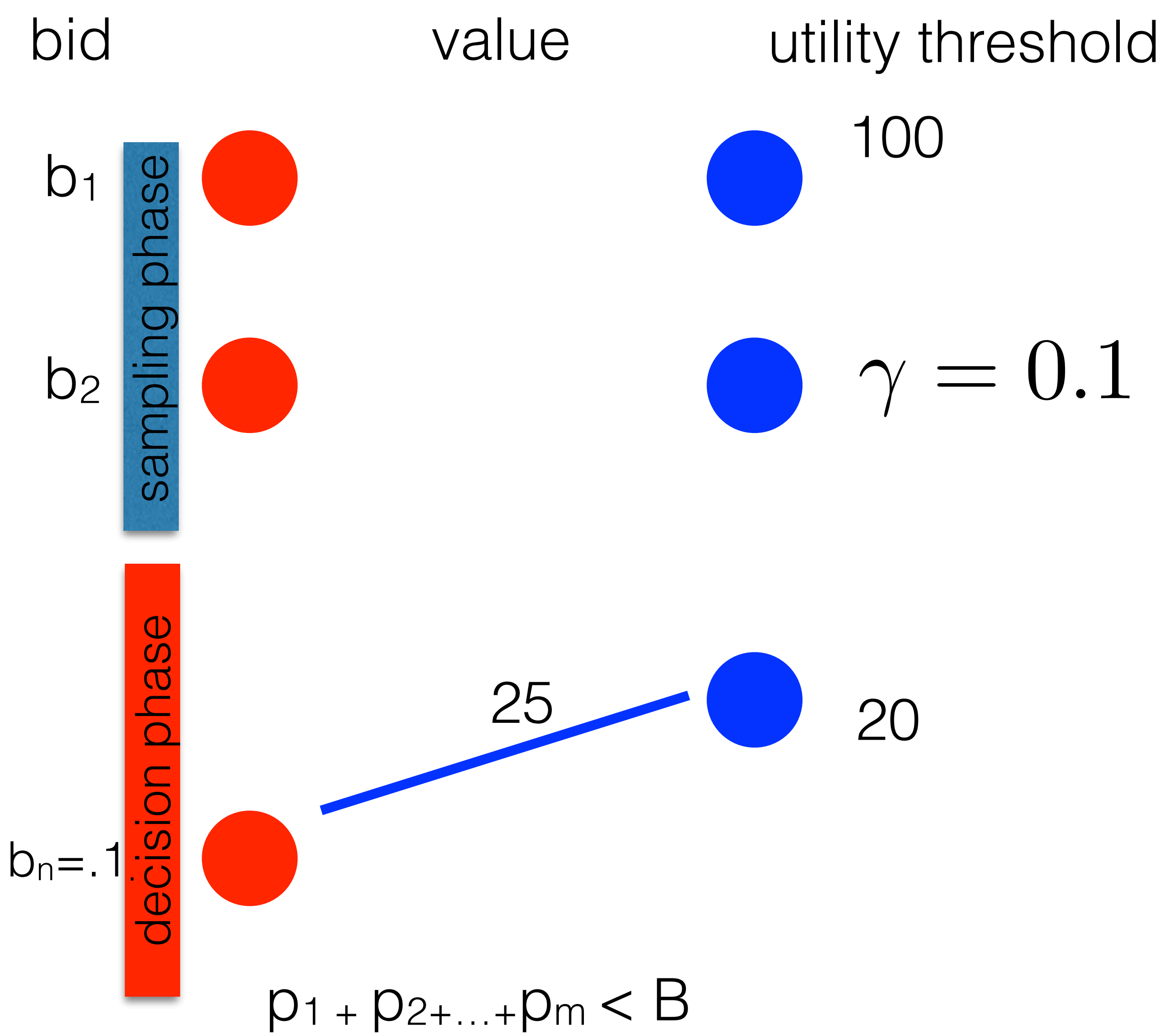
In Decision Phase

remove all edges with **bid to benefit** ratio $> \gamma$

match the largest weight more than utility threshold

Algorithm - decision phase

γ and threshold utility obtained from sampling phase



In Decision Phase

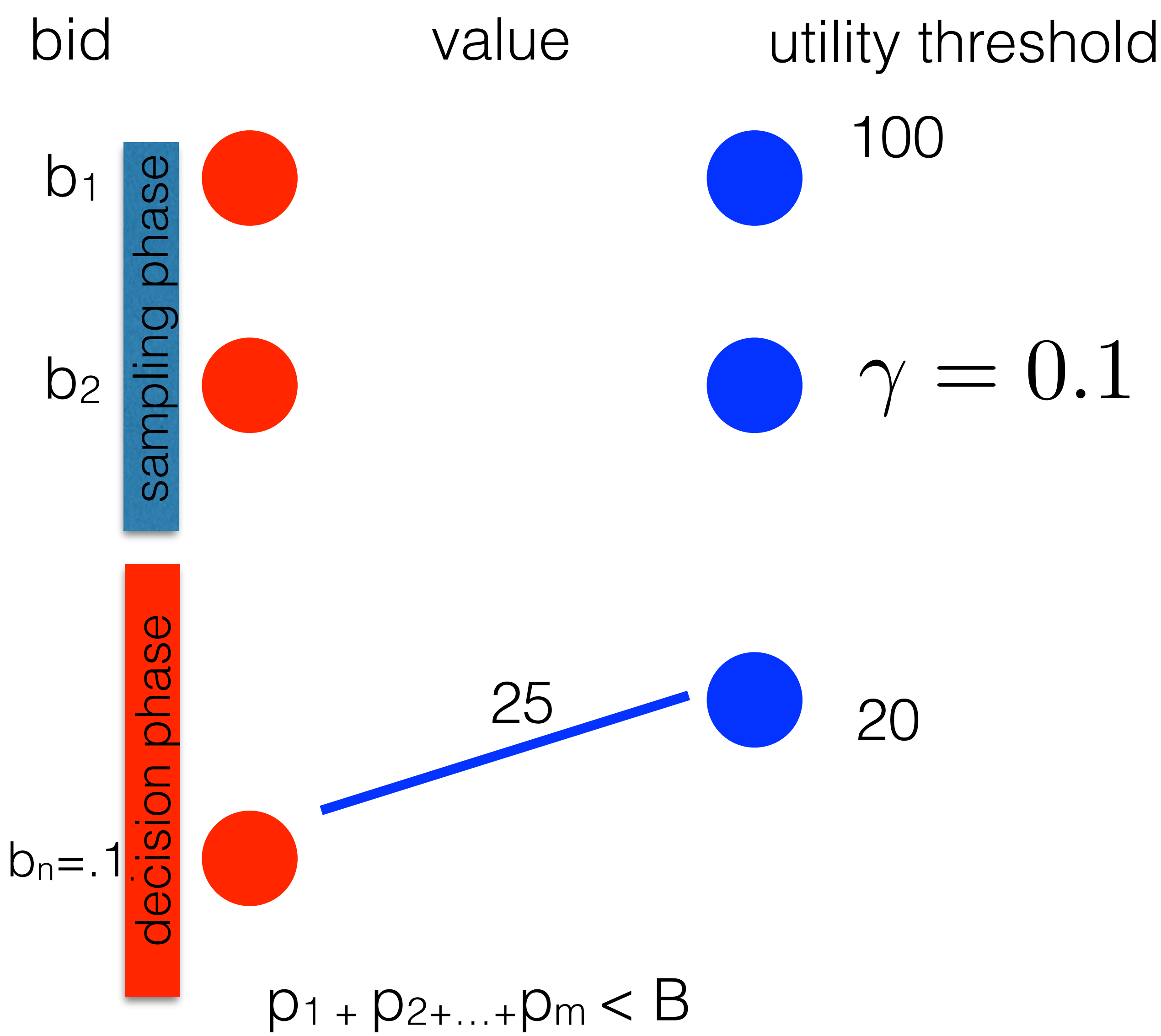
remove all edges with **bid to benefit** ratio $> \gamma$

match the largest weight more than utility threshold

Payment for each selected e, $p(e) = \gamma v(e)$

Algorithm - decision phase

γ and threshold utility obtained from sampling phase



In Decision Phase

remove all edges with **bid to benefit** ratio $> \gamma$

match the largest weight more than utility threshold

Payment for each selected e, $p(e) = \gamma v(e)$

Result: 144—competitive/optimal and truthful [*V, Coupechoux*]

Acknowledgments

Andrew Thangaraj
IIT-M

Marceau Coupechoux
Telecom ParisTech

Funding

CEFIPRA


SON, NOT EVERYTHING IS BETTER WIRELESS
1-25
offthemark.com


I don't always fail
But when I do, I make sure that you're in the middle of something important.

NATION, IN OTHER NEWS

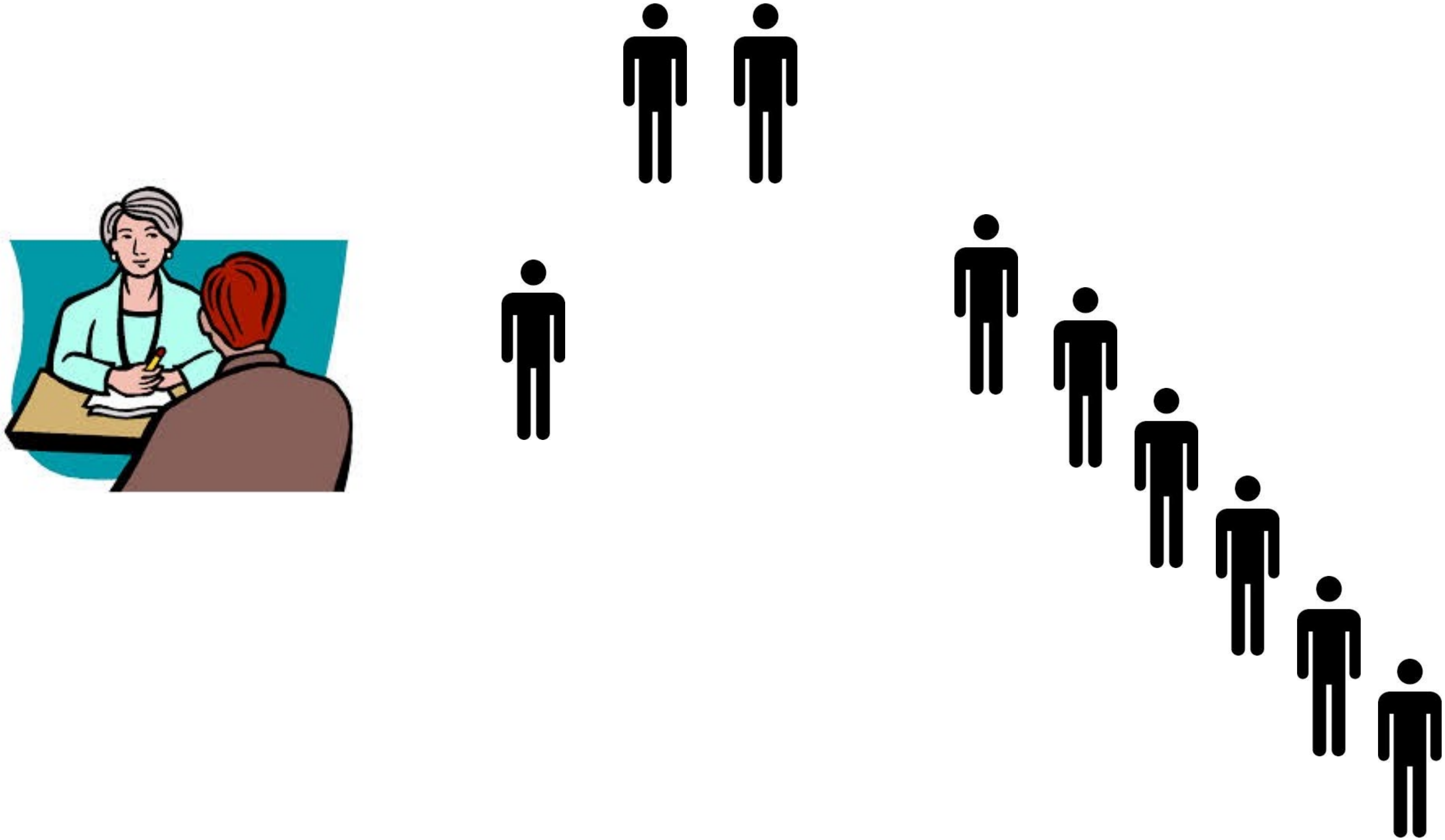
Kerala IAS officer lures public with biryani to clean lake

DECCAN CHRONICLE

Published Jan 27, 2016, 5:54 pm IST **Updated** Jan 27, 2016, 5:57 pm IST

Volunteers cleaned up the 14-acre lake and were rewarded with a plate of Malabar biryani.

IAS officer and collector Prasanth Nair (Photo Courtesy: Facebook.com/Prasanth Nair)



Secretary Problem

1. Why arbit doesn't work
2. Randomized Model
3. Simple Algo 1/2

Sec. Prob as Matching with only one left vertex
Bipartite matching problem
Greedy 1/2 algo
Philosophy from Sec problem Hide the first half Set the price and select above the threshold

Wireless Problem -BS assoc
Equal weight case- Offline is to keep one per good BS
Use the same philosophy show that OFF < Max Weight
Designate one BS as garbage and do Online Matching on the rest Guarantee (M-1)/8M