Coherence of $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$ and consequences for the determination of $\phi_3$

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Outline

- Introduction
- CLEO-c and quantum correlation
- Calculation of CP content $F_+$
- Extraction of $c_i$ and $s_i$
- CPV sensitivity
- Summary
Introduction
Current best results for CKM angles

- $\phi_1 = 21.5^{+0.8}_{-0.7}$ deg.
- $\phi_2 = 85.4^{+4.0}_{-3.8}$ deg.
- $\phi_3 = 73.2^{+6.3}_{-7.0}$ deg.

Recent results from LHCb

- $\phi_3 = 72.2^{+6.8}_{-7.3}$ deg. [2]

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2. [arXiv:1611.03076v1 [hep-ex]]
- Determine $\phi_3$ via interference between $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$.

The above two amplitudes are related by

$$\frac{A(B^- \to \bar{D}^0 K^-)}{A(B^- \to D^0 K^-)} = r_B e^{i(\delta_B - \phi_3)}$$

$$r_B = \left| \frac{A(B^- \to \bar{D}^0 K^-)}{A(B^- \to D^0 K^-)} \right|, \delta_B = \delta(B^- \to \bar{D}^0 K^-) - \delta(B^- \to D^0 K^-).$$

No loop contribution ⇒ clean way to measure $\phi_3$. 
\( \phi_3 \) measurements - different methods

- **Gronau - London - Wyler (GLW) method** [3]
  - Modes with known CP content \((F_+ )\) [4] can be used along with CP eigenstates.

- **Giri - Grossman - Soffer - Zupan (GGSZ) method** [5]
  - Binned Dalitz plot analysis of multibody \( D \) final states like \( K_S^0 \pi^+ \pi^- \), \( K_S^0 K^+ K^- \), \( K_S^0 \pi^+ \pi^- \pi^0 \).
  - For the decay \( B^- \rightarrow D(K_S^0 h^+ h^-) K^- \)
    \[
    \Gamma_i^- = K_i + r_B^2 \bar{K}_i + 2\sqrt{K_i \bar{K}_i} (c_i x_- + s_i y_-),
    \]
    and for \( B^+ \rightarrow D(K_S^0 h^+ h^-) K^+ \),
    \[
    \Gamma_i^+ = \bar{K}_i + r_B^2 K_i + 2\sqrt{K_i \bar{K}_i} (c_i x_+ - s_i y_+).
    \]
  - \( x_\pm = r_B \cos(\delta_B \pm \phi_3) \); \( y_\pm = r_B \sin(\delta_B \pm \phi_3) \).
  - \( c_i, s_i \) - cos and sin of the strong phase difference between \( D^0 \) and \( \bar{D}^0 \) averaged over the region of phase space.

**Figure**: A typical Dalitz plot binning for a three body \( D \) decay.

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Motivation

- Information on the $D$ decay is required to determine $x, y$.
- Quantum correlated $D\bar{D}$ mesons produced in $e^+e^-$ collisions at an energy corresponding to $\Psi(3770)$ at CLEO-c can be used.
- A $D$ decay mode not yet used is $K_S^0\pi^+\pi^-\pi^0$.
- The decay $D^0 \rightarrow K_S^0\pi^+\pi^-\pi^0$ has a relatively large branching fraction of 5.2% which is almost twice that of $K_S^0\pi^+\pi^-$ [6].
- Interesting resonance substructure.
  - $K_S^0\omega$ - CP eigenstate - GLW like.
  - $K^-\pi^+\pi^0$ - Cabibbo-favored state (CF) - ADS like.
- As powerful as $K_S^0\pi^+\pi^-$ in the determination of $\phi_3$?

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CLEO-c and quantum correlation
Quantum correlated \( D \) mesons at CLEO-c

- \( \Psi \rightarrow D\bar{D} \) are produced coherently in the \( C = -1 \) state.

\[
\frac{\left( |D\rangle |\bar{D}\rangle - |\bar{D}\rangle |D\rangle \right)}{\sqrt{2}}
\]

- If \( \Psi(3770) \) decays into two states \( F \) and \( G \), then decay rate (\( \Gamma \)) depends on their CP eigenvalue.

- \( F = \text{CP even (odd)} \), \( G = \text{CP odd (even)} \) \( \Rightarrow \) two-fold enhancement.
- \( F = \text{CP even (odd)} \), \( G = \text{CP even (odd)} \) \( \Rightarrow \) zero.
- \( \Gamma \) changes with \( F \) or \( G \) being quasi CP states \( (\pi^+ \pi^- \pi^0) \) or self conjugate states \( (K^0_S \pi^+ \pi^-) \).

Figure: CLEO-c detector.
A total of 818 pb\(^{-1}\) data collected at the CLEO-c - \(D\bar{D}\) pairs from the \(\Psi(3770)\).

One of the \(D\) mesons reconstructed to \(K_S^0\pi^+\pi^-\pi^0\) (signal) and the other one to any other channel (tag).

Fully reconstructed modes - 
\(M_{bc}\) and \(\Delta E\).

Partially reconstructed modes - missing mass technique.

<table>
<thead>
<tr>
<th>Type</th>
<th>mode</th>
<th>yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP even tags</td>
<td>(K^+K^-)</td>
<td>200.7 ± 14.2</td>
</tr>
<tr>
<td></td>
<td>(\pi^+\pi^-)</td>
<td>91.45 ± 9.59</td>
</tr>
<tr>
<td></td>
<td>(K_S^0\pi^0\pi^0)</td>
<td>106.3 ± 10.9</td>
</tr>
<tr>
<td></td>
<td>(K_L^0\pi^0)</td>
<td>357.3 ± 20.2</td>
</tr>
<tr>
<td></td>
<td>(K_L^0\omega)</td>
<td>162.1 ± 13.7</td>
</tr>
<tr>
<td>CP odd tags</td>
<td>(K_S^0\pi^0)</td>
<td>93.97 ± 9.84</td>
</tr>
<tr>
<td></td>
<td>(K_S^0\eta)</td>
<td>11.64 ± 3.68</td>
</tr>
<tr>
<td></td>
<td>(K_S^0\eta')</td>
<td>7 ± 3</td>
</tr>
<tr>
<td>Quasi CP tags</td>
<td>(\pi^+\pi^-\pi^0)</td>
<td>428.8 ± 21.7</td>
</tr>
<tr>
<td>Self conjugate tags</td>
<td>(K_S^0\pi^+\pi^-)</td>
<td>504.8 ± 23.3</td>
</tr>
<tr>
<td></td>
<td>(K_L^0\pi^+\pi^-)</td>
<td>864.1 ± 46.1</td>
</tr>
<tr>
<td></td>
<td>(K_S^0\pi^+\pi^-\pi^0)</td>
<td>176.4 ± 14.8</td>
</tr>
<tr>
<td>Flavour tag</td>
<td>(K^{\pm}\ell^+\nu)</td>
<td>1010 ± 32</td>
</tr>
</tbody>
</table>

Figure: \(M^2_{\text{miss}}\) plot for \(K_L^0\pi^0\) tag for the data sample.
Calculation of $F_+$
The double tagged yield for the signal and tag

\[ M(S|T) = 2N_{D \bar{D}} \times BF(S) \times BF(T) \times \epsilon(S|T) \times [1 - \lambda_{CP}(2F_+ - 1)]. \]

The single tag yield

\[ S(T) = 2N_{D \bar{D}} \times BF(T) \times \epsilon(T). \]

If we assume \( \epsilon(S|T) = \epsilon(S)\epsilon(T) \), then we get \( N^+ \) for CP odd tag and \( N^- \) for CP even tag as follows:

\[ N^\pm = \frac{M(S|T)}{S(T)} = BF(S) \times \epsilon(S) \times [1 - \lambda_{CP}(2F_+ - 1)]. \]

From these, we can calculate \( F_+ \) as

\[ F_+ = \frac{N^+}{N^+ + N^-}; \quad F_+ = 1 \Rightarrow \text{CP even, } F_+ = 0 \Rightarrow \text{CP odd.} \]
Calculation of $F_+ - \text{CP tags}$

- The CP odd and CP even tags are used to evaluate $N^+$ and $N^-$ respectively.

![Graph showing $N^+$ values for the CP odd tags. The yellow region shows the average value.](image)

**Figure:** $N^+$ values for the CP odd tags. The yellow region shows the average value.

![Graph showing $N^-$ values for the CP even tags. The yellow region shows the average value.](image)

**Figure:** $N^-$ values for the CP even tags. The yellow region shows the average value.

Note: The x-axis scale for $N^+$ is much smaller than that of $N^-$. 

- The value of $F_+$ is obtained to be $0.240 \pm 0.021$, i.e. $K_S^{0} \pi^+ \pi^- \pi^0$ is significantly CP odd.
Calculation of $F_+ - \pi^+\pi^-\pi^0$ tag

- $F_+$ for $\pi^+\pi^-\pi^0 = 0.973 \pm 0.017$ [7].

- Define $N^{\pi^+\pi^-\pi^0}$ as the ratio of double tagged events and $\pi^+\pi^-\pi^0$ single tag events

\[
N^{\pi^+\pi^-\pi^0} = \frac{M(K_S^0 K^{+}\pi^+\pi^-\pi^0 | \pi^+\pi^-\pi^0)}{S(\pi^+\pi^-\pi^0)}.
\]

- Then with $N^+$ from CP tags, we can get

\[
F_{K_S^0 K^{+}\pi^+\pi^-\pi^0} = \frac{N^+ F_+^{\pi^+\pi^-\pi^0}}{N^{\pi^+\pi^-\pi^0} - N^+ + 2N^+ F_+^{\pi^+\pi^-\pi^0}}.
\]

- With CP and $\pi^+\pi^-\pi^0$ tags, $F_+$ is $0.244 \pm 0.021$.

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Calculation of $F_+ - K^0_S \pi^+\pi^-$ and $K^0_L \pi^+\pi^-$ tags

- The $K^0_S \pi^+\pi^-$ and $K^0_L \pi^+\pi^-$ Dalitz plots are binned according to Equal $\delta_D$ BABAR 2008 scheme [8].

Figure: $D^0 \rightarrow K^0_S \pi^+\pi^-$ Dalitz plot.

$$Y_{i}^{K^0_S \pi^+\pi^-} = h_{K^0_S \pi^+\pi^-} (K_i^{K^0_S \pi^+\pi^-} + K_{-i}^{K^0_S \pi^+\pi^-} - 2c_i \sqrt{K_i^{K^0_S \pi^+\pi^-} K_{-i}^{K^0_S \pi^+\pi^-}} (2F_+^{K^0_S \pi^+\pi^- \pi^0} - 1)).$$

$$Y_{i}^{K^0_L \pi^+\pi^-} = h_{K^0_L \pi^+\pi^-} (K_i^{K^0_L \pi^+\pi^-} + K_{-i}^{K^0_L \pi^+\pi^-} + 2c_i \sqrt{K_i^{K^0_L \pi^+\pi^-} K_{-i}^{K^0_L \pi^+\pi^-}} (2F_+^{K^0_S \pi^+\pi^- \pi^0} - 1)).$$

B. Aubert et al. (BaBar collaboration), Phys. Rev. D 78, 034023 (2008).
Calculation of $F_+ - K^0_S\pi^+\pi^-$ and $K^0_L\pi^+\pi^-$ tags

- Fit with 64 observables; $\frac{\chi^2}{\text{DoF}} = 1.3$.

**Figure** : The predicted and measured yields for $K^0_S\pi^+\pi^-$ (left) and $K^0_L\pi^+\pi^-$ (right).

- $F_+$ is found to be $0.265 \pm 0.029$.
- With all the three methods, the average $F_+$ is $0.246 \pm 0.018$. 
Extraction of $c_i$ and $s_i$
Binning $K_S^0 \pi^+ \pi^- \pi^0$ phase space

- $N_{\text{bins}} > 4 \Rightarrow \phi_3$ extraction in $B^\pm \rightarrow DK^\pm$ data in GGSZ framework - requires $c_i, s_i, K_i$ and $\bar{K}_i$.

- Dividing the 5-D phase space of $K_S^0 \pi^+ \pi^- \pi^0$ - not as trivial as the 2-D phase space of $K_S^0 \pi^+ \pi^- \Rightarrow i$ and $-i$ symmetry non-trivial.

- Amplitude model not available $\Rightarrow$ a proper optimisation difficult.

- Split the phase-space into a series of bins around the resonances and work out partial rates in each.

- Exclusive binning.

**Figure**: Invariant mass distribution for $\pi^+ \pi^- \pi^0$ (left) and 2-D distribution between the invariant masses of $K_S^0 \pi^-$ and $\pi^+ \pi^0$ (right).
Extraction of $c_i$ and $s_i$

- For a CP tag, the double tagged yield is given by

\[ M_i^{\pm} = h_{CP} \left[ K_i + \bar{K}_i \pm 2 \sqrt{K_i \bar{K}_i c_i} \right]. \]

For $\pi^+ \pi^- \pi^0$ tag, the $c_i$ sensitive term is scaled by $(2F_+ - 1)$ rather than 1.

- For $K^0_S \pi^+ \pi^- \pi^0$ double tagged events, the yield is given by

\[ M_{ij} = h_{corr} \left[ K_i \bar{K}_j + \bar{K}_i K_j - 2 \sqrt{K_i \bar{K}_j \bar{K}_i K_j (c_i c_j + s_i s_j)} \right]. \]

- For $K^0_S \pi^+ \pi^-$ tag

\[ M_{i \pm j}^{K_S \pi\pi} = h_{K_S \pi\pi} \left[ K_i K_{\mp j}^{K_S \pi\pi} + \bar{K}_i K_{\pm j}^{K_S \pi\pi} - 2 \sqrt{K_i K_{\mp j}^{K_S \pi\pi} \bar{K}_i K_{\pm j}^{K_S \pi\pi} (c_i c_j + s_i s_j)} \right]. \]

- Similarly for $K^0_L \pi^+ \pi^-$ tag,

\[ M_{i \pm j}^{K_L \pi\pi} = h_{K_L \pi\pi} \left[ K_i K_{\mp j}^{K_L \pi\pi} + \bar{K}_i K_{\pm j}^{K_L \pi\pi} + 2 \sqrt{K_i K_{\mp j}^{K_L \pi\pi} \bar{K}_i K_{\pm j}^{K_L \pi\pi} (c_i c_j + s_i s_j)} \right]. \]
### Extraction of $c_i$ and $s_i$

<table>
<thead>
<tr>
<th>Bin number</th>
<th>Specification</th>
<th>$K_i$</th>
<th>$\bar{K}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$m(\pi^+\pi^-\pi^0) \approx m(\omega)$</td>
<td>0.222 ± 0.019</td>
<td>0.176 ± 0.017</td>
</tr>
<tr>
<td>2</td>
<td>$m(K^0_S\pi^-) \approx m(K^{*-})$ &amp; $m(\pi^+\pi^0) \approx m(\rho^+)$</td>
<td>0.394 ± 0.022</td>
<td>0.190 ± 0.017</td>
</tr>
<tr>
<td>3</td>
<td>$m(K^0_S\pi^+)$ &amp; $m(\pi^-\pi^0) \approx m(\rho^-)$</td>
<td>0.087 ± 0.013</td>
<td>0.316 ± 0.021</td>
</tr>
<tr>
<td>4</td>
<td>$m(K^0_S\pi^-) \approx m(K^{*-})$</td>
<td>0.076 ± 0.012</td>
<td>0.046 ± 0.009</td>
</tr>
<tr>
<td>5</td>
<td>$m(K^0_S\pi^+)$ &amp; $m(K^{*-})$</td>
<td>0.057 ± 0.010</td>
<td>0.065 ± 0.011</td>
</tr>
<tr>
<td>6</td>
<td>$m(K^0_S\pi^0)$ &amp; $m(K^{*-})$</td>
<td>0.059 ± 0.011</td>
<td>0.092 ± 0.013</td>
</tr>
<tr>
<td>7</td>
<td>$m(\pi^+\pi^0) \approx m(\rho^+)$</td>
<td>0.045 ± 0.009</td>
<td>0.045 ± 0.009</td>
</tr>
<tr>
<td>8</td>
<td>Remainder</td>
<td>0.061 ± 0.011</td>
<td>0.070 ± 0.011</td>
</tr>
</tbody>
</table>

- The semileptonic tag $K^\pm e^\mp \nu$ is used to calculate $K_i$ and $\bar{K}_i$, the fraction of decays in each bin.
- The double tagged yields are given to the fitter along with the $c_i$, $s_i$, $K_i$ and $\bar{K}_i$ values for $K^0_S\pi^+\pi^-$ and $K^0_L\pi^+\pi^-$ [9] as input.
- Corrected for bin-to-bin migration.

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9 J. Libby et al. (CLEO collaboration), Phys. Rev. D 82, 112006 (2010).
$c_i$ and $s_i$ results - preliminary

- The combined fit: 472 observables including different tag yields in each bin; $\frac{\chi^2}{\text{DoF}} = 1.04$.

<table>
<thead>
<tr>
<th>Bin</th>
<th>$c_i$</th>
<th>$s_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-1.12 \pm 0.12$</td>
<td>$0.12 \pm 0.17$</td>
</tr>
<tr>
<td>2</td>
<td>$-0.29 \pm 0.07$</td>
<td>$0.11 \pm 0.13$</td>
</tr>
<tr>
<td>3</td>
<td>$-0.41 \pm 0.09$</td>
<td>$-0.08 \pm 0.18$</td>
</tr>
<tr>
<td>4</td>
<td>$-0.84 \pm 0.12$</td>
<td>$-0.73 \pm 0.34$</td>
</tr>
<tr>
<td>5</td>
<td>$-0.54 \pm 0.13$</td>
<td>$0.65 \pm 0.13$</td>
</tr>
<tr>
<td>6</td>
<td>$-0.22 \pm 0.12$</td>
<td>$1.37 \pm 0.22$</td>
</tr>
<tr>
<td>7</td>
<td>$-0.90 \pm 0.16$</td>
<td>$-0.12 \pm 0.40$</td>
</tr>
<tr>
<td>8</td>
<td>$-0.70 \pm 0.14$</td>
<td>$-0.03 \pm 0.44$</td>
</tr>
</tbody>
</table>

The uncertainties shown are statistical only.

- $c_i < 0 \Rightarrow \textbf{CP oddness}$ of $K_S^0 \pi^+ \pi^- \pi^0$. 
CPV sensitivity and summary
Estimates of $\phi_3$ sensitivity with $B^{\pm} \rightarrow D(K_S^0\pi^+\pi^-\pi^0)K^{\pm}$

- Assumed increase in BF compensated by loss of efficiency due to $\pi^0$ in final state.
- With 1200 events (Belle sample of $B^{\pm} \rightarrow D(K_S^0\pi^+\pi^-)K^{\pm}$)
  $\sigma_{\phi_3} = 25^\circ - 1000$ pseudo experiments using $c_i$, $s_i$, $K_i$ and $\bar{K}_i$ measurements reported.
- Project to a 50 $ab^{-1}$ sample $\sigma_{\phi_3} = 3.5^\circ$.
- Compare to $B^{\pm} \rightarrow D(K_S^0\pi^+\pi^-)K^{\pm}$ $\sigma_{\phi_3} \sim 2^\circ$.

Improvements:
- Optimized binning once a $D^0 \rightarrow K_S^0\pi^+\pi^-\pi^0$ amplitude model developed.
- Finer binning possible with 10 $fb^{-1}$ of BESIII data.
- Caveat: background to be studied.

Figure: $\phi_3$ sensitivity with 50 $ab^{-1}$ Belle II sample.
Calculated the CP content $F_+$ for the decay $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$ from CLEO-c data to be $0.246 \pm 0.018$.

Addition of this mode to quasi-GLW methods to determine $\phi_3$.

Extracted the strong phase differences by introducing an eight bin scheme for the $K_S^0 \pi^+ \pi^- \pi^0$ phase space.

Addition to GGSZ formalism to determine $\phi_3$.

Sensitivity to $\phi_3$ from a 50 ab$^{-1}$ sample, $\sigma_{\phi_3} = 3.5^\circ$. 