Theory of $\gamma/\phi_3$ from tree decays

Joachim Brod

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Partially based on
Brod, Zupan, JHEP 1401 (2014) 051
“Introduzione”
Definition of $\gamma$

- The fundamental equation: $\gamma \equiv \phi_3$

- The fundamental equation (extended version):
  \[ \gamma \equiv \phi_3 \equiv \arg(-V_{ud} V_{ub}^* / V_{cd} V_{cb}^*) \]

- Determination from $B \rightarrow DK$ theoretically extremely clean

- Fit hadronic parameters from data
Motivation: Tree vs. Loop

- From $B \to \pi\pi$ determine $\alpha = \pi - \beta - \gamma$ [Gronau, London 1990]
- Use $B \to \pi\pi$ and $B_s \to KK$ to extract $\gamma$ [Fleischer 1999]

- Compare to $\gamma$ from tree-level $B \to DK$ [Bigi, Sanda 1981]
Outline

- $\gamma$ from tree decays
- Theory uncertainty
- New-physics contributions?
\[ \gamma \text{ from tree decays – general idea} \]

- \( b \rightarrow c\bar{u}s(d), b \rightarrow u\bar{c}s(d) \)
- no penguin contribution
- interference from common \( D^0, \bar{D}^0 \) final states

\[
\begin{align*}
\propto & \quad V_{cb} V_{us}^*(d) \\
\propto & \quad V_{ub} V_{cs}^*(d)
\end{align*}
\]

\[
r_B^{D^0 h} e^{i(\delta_B - \gamma)} = \frac{A(B^{-} \rightarrow \bar{D}^0 h^-)}{A(B^{-} \rightarrow D^0 h^-)}
\]
Several choices for final state $f$ in $D$ decay

- **CP eigenstates** (e.g. $D \rightarrow K^+K^-, \pi^+\pi^-; K_S\pi^0$)
  

- **Flavor states** (e.g. $D \rightarrow \pi^-K^+, \pi^+K^-$) [Atwood, Dunietz, Soni 1997]

- **Many-body final states** (e.g. $D \rightarrow K_SK^+K^-, K_S\pi^+\pi^-$)
  
  [Giri, Grossman, Soffer, Zupan 2003; Poluektov 2004]

- **Many variants:**
  
  - Use $D^* \rightarrow D\pi^0, D\gamma$ [Bondar, Gershon 2004]
  
  - Many-body $B$ final states
    
    [Aleksan, Petersen, Soffer 2002; Gershon 2008; Gershon, Poluektov 2009]

  - **Neutral $B_d, B_s$** [Aleksan, Dunietz, Kayser 1992; Kayser, London 2000; Atwood, Soni 2003; Fleischer 2003; Gronau et al. 2004]

- ... 

- **Have** $\sim n_D n_B$ measurements, $\sim n_D + n_B$ unknowns
Why measure $\gamma$?

- Important SM input parameter
- Theoretically very clean
  - Hadronic parameters can be fitted from data
- In the (far) future can be used to search for NP at high scales
- $\Rightarrow$ what is the uncertainty?
The devil is in the detail

“Fuga del diavolo”
Precision in $\gamma$

- **Direct CP violation in $D$ decays**
  
  [E.g. Martone, Zupan 2012; Wang 2012; Bhattacharya et al. 2013; Bondar et al. 2013]

  - $\delta \gamma = O(r_f/r_B)$
    - $r_B(DK) = O(10\%)$, $\delta \gamma = O(\text{few }\%)$
    - $r_B(D\pi) = O(0.5\%)$, $\delta \gamma = O(1)$

  - Can still “solve the system”

  - (Shift symmetry: Need one CPC $D$ decay mode)

- $D - \bar{D}$, $K - \bar{K}$, $B_{(s)} - \bar{B}_{(s)}$ mixing


  - Shift of $\lesssim 3^\circ$

  - Can be included exactly if mixing is precisely measured!
So, why a theory talk?

“Silentium!”
What are we actually measuring?

- When is the phase extracted in $B \rightarrow Dh$ not $\gamma$?
  - Subleading terms in Wolfenstein expansion
    [Grossman, Savastio 2013, Brod 2014]
  - Electroweak corrections
    [Brod, Zupan 2013; Brod 2014]
  - Tree-level new physics
    [Brod, Lenz, Tetlalmatzi-Xolocotzi, Wiebusch 2014]
Subleading weak phases

- Extracting $\gamma$ from $B \to DK$ involves the ratio
  
  \[ V_{cd}V_{cs}/(V_{ud}V_{us}) = -1 + \lambda^4 A^2 (1 - (\rho + i\eta)) + O(\lambda^6) \]

- Extracting $\gamma$ from $B \to D\pi$ involves the ratio
  
  \[ V_{cd}^2/V_{ud}^2 = \lambda^2 [1 - \lambda^4 A^2 (1 - 2(\rho + i\eta)) + O(\lambda^6)] \]

- The $O(\lambda^4 \approx 2.6 \times 10^{-3})$ uncertainty can be removed by independent measurements of arg$[V_{cd}V_{cs}/(V_{ud}V_{us})]$, arg$[V_{cd}/V_{ud}]$
Irreducible theory uncertainty

- QED radiative corrections $\rightarrow$ CP conserving
- Electroweak corrections
  - No effect from $Z$ exchange
  - No effect from vertex corrections
- Box diagrams can change CKM structure
$B \rightarrow DK$

“Scherzo”
Box diagrams

- **$b \to u\bar{c}s$:**
  - tree level $\sim V_{ub} V_{cs}^*$
  - box diagram $\sim (V_{tb} V_{ts}^*) (V_{ub} V_{cb}^*)$
  - same weak phase, no shift in $\gamma$

- **$b \to c\bar{u}s$:**
  - tree level $\sim V_{cb} V_{us}^*$
  - box diagram $\sim (V_{tb} V_{ts}^*) (V_{cb} V_{ub}^*)$
  - different weak phase, induces $\delta \gamma$
The resulting shift $\delta \gamma^{DK}$

$$\delta \gamma^{DK} = \frac{\text{Im}\Delta C_1}{C_1 + C_2 r_A} + \frac{\text{Im}\Delta C_2}{C_1/r_A + C_2}$$

- Here,

$$r_A = \frac{\langle K^- D^0 | Q_2 \bar{c}u | B^- \rangle}{\langle K^- D^0 | Q_1 \bar{c}u | B^- \rangle} \approx \frac{f_D F_{B \rightarrow K}^0(0)}{f_K F_{B \rightarrow D}^0(0)} \approx 0.4$$

- and $Q_1 \bar{c}u = (\bar{c}b)_{V-A}(\bar{s}u)_{V-A}$, $Q_2 \bar{c}u = (\bar{s}b)_{V-A}(\bar{c}u)_{V-A}$.

- Full RG analysis (resumming leading QCD logs) yields

$$\delta \gamma^{DK} / \gamma \lesssim \mathcal{O}(10^{-7})$$
$B \rightarrow D\pi$

“Forte vivace”


**Box diagrams**

- **$b \to u\bar{c}d$:**
  - Tree level: $V_{ub} V_{cd}^*$
  - Box diagram: $(V_{tb} V_{td}^*)(V_{ub} V_{cd}^*)$
  - Different weak phase, induces $\delta\gamma$

- **$b \to c\bar{u}d$:**
  - Tree level: $V_{cb} V_{ud}^*$
  - Box diagram: $(V_{tb} V_{td}^*)(V_{cb} V_{ub}^*)$
  - Different weak phase, induces $\delta\gamma$
  - CKM-suppressed by $10^{-2}$ – neglect!
The resulting shift \( \delta \gamma^{D\pi} \)

\[
\delta \gamma^{D\pi} = -\frac{\text{Im} \Delta C_1}{C_1 + C_2 r_{A'}} - \frac{\text{Im} \Delta C_2}{C_1/r_{A'} + C_2}
\]

- Here,

\[
r_{A'} = \frac{\langle \pi^- D^0 | Q_2 \bar{u}c | B^- \rangle}{\langle \pi^- D^0 | Q_1 \bar{u}c | B^- \rangle} = ?? N_c = 3 ??
\]

- problematic – note \( C_1(m_b) = 1.10, \ C_2(m_b) = -0.24 \)

- For \( r_{A'} \approx 4.6 \) the denominator in \( \delta \gamma^{D\pi} \) vanishes!

\[
\delta \gamma^{D\pi} / \gamma \lesssim \mathcal{O}(10^{-4})
\]
Tree-level New Physics?

“Fortissimo vivacissimo”
Tree-level New Physics?

\[ \delta \gamma \approx \frac{\text{Im} \Delta C_2 + r_{A'} \text{Im} \Delta C_1}{C_2 + r_{A'} C_1} - \frac{\text{Im} \Delta C_2 + r_A \text{Im} \Delta C_1}{C_2 + r_A C_1} \approx (r_{A'} - r_A) \frac{\text{Im} \Delta C_1}{C_2} \]

- Sizeable contributions to \( \text{Im} \Delta C_1 \), \( \text{Im} \Delta C_2 \) are not excluded from data
- \( \delta \gamma^{NP} = \mathcal{O}(5^\circ) \) for generic \( r_{A'} \)
- However, unknown \( r_{A'} \) makes contribution to \( \gamma \) hard to quantify
Tree-level New Physics?

- $\gamma$ from $B \to DK$ has built-in test for NP in decay amplitude
  
  [J. Zupan, talk at LHCb Implications 2012]

- Different $r_B$ for $B^+$ and $B^-$:

$$r_{B^+} \to |r_B e^{i(\delta_B+\gamma)} + r'_B e^{i(\delta'_B+\gamma)}|, \quad r_{B^-} \to |r_B e^{i(\delta_B-\gamma)} + r'_B e^{i(\delta'_B-\gamma)}|$$

- NP in $B \to DK$ amplitudes if $|r_{B^+}| \neq |r_{B^-}|$

- $x_{\pm} = r_B \cos(\gamma \pm \delta_B)$,
  $y_{\pm} = \pm r_B \sin(\gamma \pm \delta_B)$

- E.g. measured by Belle [Belle 2008]
γ from $B \rightarrow DK$ is **theoretically extremely clean**,

- $\delta\gamma^{DK}/\gamma \lesssim O(10^{-7})$

γ from $B \rightarrow D\pi$ is **most likely theoretically extremely clean**,

- $\delta\gamma^{D\pi}/\gamma \lesssim O(10^{-4})$

Sensitive test of NP
Bravo Bravissimo

[Image credit: Wilhelm Busch]
Backup
Tree vs. loop

- From $B \rightarrow \pi \pi$  [Gronau, London 1990]

\[
\text{Im} \xi_{+-} = \text{Im} \left[ e^{-2i(\beta+\gamma)} \frac{1 - \bar{A}_0/A_2}{1 - A_0/A_2} \right]
\]

\[
\text{Im} \xi_{00} = \text{Im} \left[ e^{-2i(\beta+\gamma)} \frac{1 + \bar{A}_0/2A_2}{1 + A_0/2A_2} \right]
\]

- Determines $\sin 2\alpha$, $\alpha = \pi - \beta - \gamma$

- From $B_d \rightarrow \pi^+\pi^-$ and $B_s \rightarrow K^+K^-$, using U spin, extract $\beta$ and $\gamma$ simultaneously  [Fleischer 1999]