Inclusive Semi-leptonic Penguin Decays

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Motivation

- Radiative and semileptonic rare B decayse are highly sensitive probes for new physics

- Exclusive modes are experimentally easier (LHCb), but have larger theoretical uncertainties (issue of unknown power corrections !)

- Inclusive modes require Belle-II for full exploitation (complete angular analysis) but are theoretically very clean

- Inclusive modes allow for crosschecks of recent LHCb anomalies
Theoretical Tools
Theoretical tools for flavour precision observables

Factorization theorems: separating long- and short-distance physics

- Electroweak effective Hamiltonian: \( H_{\text{eff}} = \frac{-4G_F}{\sqrt{2}} \sum C_i(\mu, M_{\text{heavy}}) \mathcal{O}_i(\mu) \)

- \( \mu^2 \approx M_{\text{New}}^2 >> M_W^2 \) : 'new physics' effects: \( C_i^{SM}(M_W) + C_i^{\text{New}}(M_W) \)

How to compute the hadronic matrix elements \( \mathcal{O}_i(\mu = m_b) \)?
Inclusive modes $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$

How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

Heavy mass expansion for inclusive modes:

\[ \Gamma(\bar{B} \to X_s \gamma) \xrightarrow{m_b \to \infty} \Gamma(b \to X_s^{\text{parton}} \gamma), \quad \Delta^{\text{nonpert.}} \sim \frac{\Lambda_{QCD}^2}{m_b^2} \]

No linear term $\Lambda_{QCD}/m_b$ (perturbative contributions dominant)

Chay, Georgi, Grinstein 1990
Inclusive modes $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$

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An old story:

- If one goes beyond the leading operator ($\mathcal{O}_7$, $\mathcal{O}_9$):
  breakdown of local expansion

A new dedicated analysis:

naive estimate of non-local matrix elements leads to 5% uncertainty.


Analysis in $B \rightarrow X_s \ell \ell$ in this talk; Benzke, Fickinger, Hurth, Turczyk
Exclusive modes $B \to K^{(*)} \ell \ell$

QCD-improved factorization: BBNS 1999

$$T_a^{(i)} = C_a^{(i)} \xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a, K^*} + O(\Lambda/m_b)$$

(Soft-collinear effective theory)

- Separation of perturbative hard kernels from process-independent nonperturbative functions like form factors

- Relations between formfactors in large-energy limit

- Limitation: insufficient information on power-suppressed $\Lambda/m_b$ terms (breakdown of factorization: 'endpoint divergences')
Difference between exclusive and inclusive $b \to s\gamma, \ell\ell$ modes:

**Inclusive**

$\Lambda^2/m_b^2$ corrections can be calculated for the leading operators in the local OPE.

$\Lambda/m_b$ corrections to the subleading operators correspond to nonlocal matrix elements and can be estimated!

**Exclusive**

No theory of $\Lambda/m_b$ corrections at all within QCD factorization formula (in the low-$q^2$ region); these corrections can only be "guesstimated"!
Crosscheck of LHCb anomalies with inclusive modes

Hurth, Mahmoudi, Neshatpour, arXiv:1410.4545

If SM deviations in $R_K$ and $P_5'$ persist until Belle-II

If NP then the effect of $C_9$ and $C'_9$ are large enough to be checked at Belle-II with theoretically clean modes.

Hurth, Mahmoudi, arXiv:1312.5267  Experimental extrapolation by Kevin Flood
Inclusive modes
"Latest" Belle measurement of branching ratio is based on less than 30% of the total luminosity

Belle hep-ex/0503044 (!!!) (based \(152 \times 10^6 B \bar{B}\) events)

**Integrated luminosity of B factories**

- **KEKB**
- **PEP-II**

\[ \text{fb}^{-1} \]

- \(> 1 \text{ ab}^{-1}\)
  - On resonance:
    - \(Y(5S): 121 \text{ fb}^{-1}\)
    - \(Y(4S): 711 \text{ fb}^{-1}\)
    - \(Y(3S): 3 \text{ fb}^{-1}\)
    - \(Y(2S): 25 \text{ fb}^{-1}\)
    - \(Y(1S): 6 \text{ fb}^{-1}\)

- Off resonance/scan:
  - \(\sim 100 \text{ fb}^{-1}\)

\(\sim 550 \text{ fb}^{-1}\)

- On resonance:
  - \(Y(4S): 433 \text{ fb}^{-1}\)
  - \(Y(3S): 30 \text{ fb}^{-1}\)
  - \(Y(2S): 14 \text{ fb}^{-1}\)

- Off resonance:
  - \(\sim 54 \text{ fb}^{-1}\)

New Babar analysis on dilepton spectrum arXiv:1312.3664
New Belle analysis on AFB arXiv:1402.7134
Complete angular analysis of inclusive $B \rightarrow X_s \ell \ell$

Huber, Hurth, Lunghi, arXiv:1503.04849

- "Latest" Belle measurement of branching ratio is based on less than 30% of the total luminosity

$$\frac{d^2\Gamma}{dq^2 \, dz} = \frac{3}{8} \left[ (1 + z^2) \, H_T(q^2) + 2 \, z \, H_A(q^2) + 2 \, (1 - z^2) \, H_L(q^2) \right] \quad (z = \cos \theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \quad \frac{dA_{FB}}{dq^2} = \frac{3}{4} \, H_A(q^2)$$

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events

Large logs $\log(mb/m_\ell)$ different for muon and electron!
Complete angular analysis of inclusive $B \rightarrow X_s \ell \ell$

Huber, Hurth, Lunghi, arXiv:1503.04849

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- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

- On-shell-$c\bar{c}$-resonances $\Rightarrow$ cuts in dilepton mass spectrum necessary:
  $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ and $14.4\text{GeV}^2 < q^2$ $\Rightarrow$ perturbative contributions dominant

$$\frac{d}{ds} BR(\bar{B} \rightarrow X_s \ell^+\ell^-) \times 10^{-5}$$
Results

Low-$q^2$ \((1\, GeV^2 < q^2 < 6\, GeV^2)\)

\[ BR(B \rightarrow X_{see}) = (1.67 \pm 0.10) \times 10^{-6} \]

\[ BR(B \rightarrow X_{s\mu\mu}) = (1.62 \pm 0.09) \times 10^{-6} \]

Babar: \[ BR(B \rightarrow X_{s\ell\ell}) = \]

\[ = (1.60^{+0.41}_{-0.39})_{stat}(^{+0.17}_{-0.13})_{syst}(^{\pm 0.18}_{mod}) \times 10^{-6} \]

good agreement with SM
Results

High-$q^2$, Theory: $q^2 > 14.4 GeV^2$, Babar: $q^2 > 14.2 GeV^2$

$$BR(\bar{B} \to X_{seee}) = (0.220 \pm 0.070) \times 10^{-6}$$

$$BR(\bar{B} \to X_{s\mu\mu}) = (0.253 \pm 0.070) \times 10^{-6}$$

Babar: $$BR(\bar{B} \to X_{s\ell\ell}) =$$

$$(0.57 \pm 0.16 - 0.15)_{\text{stat}} (\pm 0.03 - 0.02)_{\text{syst}} \times 10^{-6}$$

2$\sigma$ higher than SM

Significant higher values predicted in Greub et al. due to missing power and QED corrections and different cut \cite{Greub, Pilipp, Schupbach, arXiv:0810.4077}

(but perfect agreement if we use their prescriptions)
Further refinement

Normalization to semileptonic $B \to X_u \ell \nu$ decay rate with the same cut reduces the impact of $1/m_b$ corrections in the high-$q^2$ region significantly.

Ligeti, Tackmann arXiv:0707.1694

Theory prediction for ratio

$$R(s_0)_{ee} = (2.25 \pm 0.31) \times 10^{-3}$$

$$R(s_0)_{\mu\mu} = (2.62 \pm 0.30) \times 10^{-3}$$

Largest source of error are CKM elements ($V_{ub}$)

Note: Additional $O(5\%)$ uncertainty due to nonlocal power corrections $O(\alpha_s \Lambda/m_b)$
New physics sensitivity

Huber, Hurth, Lunghi, arXiv:1503.04849

Constraints on Wilson coefficients $C_9/C_9^{SM}$ and $C_{10}/C_{10}^{SM}$

that we obtain at 95% C.L. from present experimental data
(red low $q^2$, green high $q^2$)

that we will obtain at 95% C.L. from 50 $ab^{-1}$ data at Belle-II
(yellow)
Subleading contributions in $B \to X_s \ell^+ \ell^-$

- On-shell-$c\bar{c}$-resonances $\Rightarrow$ cuts in dilepton mass spectrum necessary: $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ and $14.4\text{GeV}^2 < q^2$ $\Rightarrow$ perturbative contributions dominant

$$\frac{d}{ds} BR(B \to X_s \ell^+ \ell^-) \times 10^{-5}$$

- Hadronic invariant-mass cut is imposed in order to eliminate the background like $b \to c (\to se^+\nu)e^-\bar{\nu} = b \to se^+e^- + \text{missing energy}$
  * Babar, Belle: $m_X < 1.8$ or $2.0 \text{GeV}$
  * high-$q^2$ region not affected by this cut
  * kinematics: $X_s$ is jetlike and $m_X^2 \leq m_b \Lambda_{QCD}$ $\Rightarrow$ shape function region
  * SCET analysis: universality of jet and shape functions found:
    the 10-30% reduction of the dilepton mass spectrum can be accurately computed using the $B \to X_s \gamma$ shape function
    5% additional uncertainty for $2.0 \text{GeV}$ cut due to subleading shape functions

Lee, Stewart hep-ph/0511334
Lee, Ligeti, Stewart, Tackmann hep-ph/0512191
Lee, Tackmann arXiv:0812.0001 (effect of subleading shape functions)
Bell, Beneke, Huber, Li arXiv:1007.3758 (NNLO matching QCD $\to$ SCET)
Subleading power factorization in $B \rightarrow X_s \ell^+ \ell^-$

Benzke, Fickinger, Hurth, Turczyk, to appear

**Hadronic cut**

Additional cut in $X_s$ necessary to reduce background affects only low-$q^2$ region.

Hadronic invariant $m_X^2 < 1.8(2.0) \text{GeV}^2$, jet-like $X_s\ E_X \sim O(m_b)$

Multiscale problem $\rightarrow$ SCET

![Diagram](image)

$M_B^2 \sim m_b^2 \gg m_X^2 \sim \Lambda_{\text{QCD}} m_b \gg \Lambda_{\text{QCD}}^2$

$m_X^2 = P_X^2 = (M_B - n \cdot q)(M_B - \bar{n} \cdot q)$

**Scaling**

$\lambda = \Lambda_{\text{QCD}} / m_b$
Kinematics

$B$ meson rest frame

$q = p_B - p_X \quad 2m_B E_X = m_B^2 + M_X^2 - q^2$

$X_s$ system is jet-like with $E_X \sim m_B$ and $m_X^2 \ll E_X^2$

two light-cone components $p^{-}_X p^{+}_X = m_X^2$

$\bar{n}p_X = p^-_X = E_X + |\vec{p}_X| \sim \mathcal{O}(m_B)$

$n p_X = p^+_X = E_X - |\vec{p}_X| \sim \mathcal{O}(\Lambda_{QCD})$

$q^+ = nq = m_B - p^+_X \quad q^- = \bar{n}q = m_B - p^-_X$
Scaling \[ \lambda = \frac{\Lambda_{\text{QCD}}}{m_b} \]

\[ m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda \]

For \( q^2 < 6 \text{GeV}^2 \) the scaling of \( np_X \) and \( \bar{n}p_X \) implies \( \bar{n}q \) is of order \( \lambda \), means \( q \) anti-hard-collinear (just kinematics).

Stewart and Lee assume \( \bar{n}q \) to be order 1, means \( q \) is hard. This problematic assumption implies a different matching of SCET/QCD.
Shapefunction region

Local OPE breaks down for $m_X^2 \sim \lambda$:

$$\frac{1}{(m_b v + k - q)^2} = \frac{1}{m_b - n \cdot q} \left( 1 - \frac{n \cdot k}{m_b - n \cdot q} + \ldots \right) \frac{1}{m_b - \bar{n} \cdot q}$$

Resummation of leading contributions into a shape function.
(scaling of $\bar{n} q$ does not matter here; zero in case of $B \to X_s \gamma$)

Factorization theorem

$$d \Gamma \sim H \cdot J \otimes S$$

The hard function $H$ and the jet function $J$ are perturbative quantities.
The shape function $S$ is a non-perturbative non-local HQET matrix element.
(universality of the shape function, uncertainties due to subleading shape functions)
Calculation at subleading power

Example of **direct** photon contribution which factorizes

\[ d\Gamma \sim H \cdot j \otimes S \]

\[ \rightarrow \frac{\alpha_s}{m_b} \text{ in low } m_X^2 \text{ region} \]

Example of **resolved** photon contribution (double-resolved) which factorizes

\[ d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J} \]

\[ \rightarrow \frac{\Lambda}{m_b} \]

Shape function is non-local in two light-cone directions.
It survives \( M_X \to 1 \) limit (irreducible uncertainty).
Interference of $Q_8$ and $Q_8$

\[
\frac{d\Gamma^{\text{res}}}{dn \cdot q \, d\bar{n} \cdot q} \sim \frac{e_s^2 \alpha_s}{m_b} \int d\omega \, \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + \bar{n} \cdot q + i\varepsilon} \int \frac{d\omega_2}{\omega_2 + \bar{n} \cdot q - i\varepsilon} g_{88}(\omega, \omega_1, \omega_2)
\]

\[
g_{88}(\omega, \omega_1, \omega_2) = \frac{1}{M_B} \langle \bar{B} | \bar{h}(t_n) \ldots s(t_n + u\bar{n}) \bar{s}(r\bar{n}) \ldots h(0) | \bar{B} \rangle_{\text{F.T.}}
\]
Interference of $Q_1$ and $Q_7$

\[ \frac{d\Gamma^{\text{res}}}{dn \cdot q \; d\bar{n} \cdot q} \sim \frac{1}{m_b} \int d\omega \; \delta(\omega + p_{+}) \int \frac{d\omega_1}{\omega_1 + i\varepsilon} \]

\[ \frac{1}{\omega_1} \left[ \bar{n} \cdot q \left( F \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) - 1 \right) - (\bar{n} \cdot q + \omega_1) \left( F \left( \frac{m_c^2}{n \cdot q(\bar{n} \cdot q + \omega_1)} \right) - 1 \right) \right. \]

\[ \left. + \bar{n} \cdot q \left( G \left( \frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) - G \left( \frac{m_c^2}{n \cdot q(\bar{n} \cdot q + \omega_1)} \right) \right) \right] \]

\[ g_{17}(\omega, \omega_1) = \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{dt}{2\pi} e^{-i\omega t} \frac{1}{M_B} \langle \bar{B}|\bar{h}(tn) \ldots G_s^{\alpha\beta}(r\bar{n}) \ldots h(0)|B\rangle \]

Expansion for $m_c \sim m_b$ leads to Voloshin term in the total rate ($-\lambda_2/m_c^2$), the terms stays non-local for $m_c < m_b$. 
Factorization formula

In the $m_X^2 \sim \lambda$ and $q^2 \sim \lambda$ region we have the following factorization formula

\[ d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum H \cdot j_i \otimes S + \frac{1}{m_b} \sum H \cdot J \otimes s_i + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes J + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right) \]

Numerical evaluation (work in progress)

Similar subleading shape functions as in $B \to X_s \gamma$

Use vacuum insertion approximation, PT invariance,....
Power corrections in the inclusive mode

- For q anti-hard-collinear we have identified a new type of subleading power corrections.

- In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

- They constitute an irreducible uncertainty because they survive the $M_X \to 1$ limit.

- If q was hard then these resolved contributions would not exist

$M_X$ cut effects in the low-$q^2$ region with $q^2$ anti-hard-collinear

(work in progress)
Extra
Semileptonic Penguin Decays

Based on

Huber, Hurth, Lunghi arXiv:1503.0449

Inclusive $B \to X_s \ell^+\ell^-$: Complete angular analysis and a thorough study of collinear photons

Benzke, Fickinger, Hurth, Turczyk to appear

Subleading power factorization in $B \to X_s \ell^+\ell^-$

Hurth, Mahmoudi, Neshatpour arXiv:1603.00865

On the anomalies in the latest LHCb data
Allowed regions

**low-\(q^2\)**
- Red: \(q^2 = [1,5,6] \text{ GeV}^2\) [Dotted, Solid, Dashed]
- Black: \(M_x = [0.495,1.25,2] \text{ GeV}\) [Dotted, Solid, Dashed]
- Blue: anti-hard-collinear component scaling

**high-\(q^2\)**
- Red: \(q^2 = [15,17,22] \text{ GeV}^2\) [Dotted, Solid, Dashed]
- Black: \(M_x = [0.495,1.25,2] \text{ GeV}\) [Dotted, Solid, Dashed]
- Blue: hard component scaling
Quark-hadron duality violated in $\bar{B} \rightarrow X_s \ell^+\ell^-$? BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances $J/\psi$ and $\psi'$ exceed the perturbative contributions by two orders of magnitude.
Quark-hadron duality violated in $\bar{B} \to X_s \ell^+ \ell^-$?

Within integrated branching ratio the resonances $J/\psi$ and $\psi'$ exceed the perturbative contributions by two orders of magnitude.

The rate $l_1 \to l_2 e^+ e^-$ (a) is connected to the integral over $|\Pi(q^2)|^2$ for which global duality is NOT expected to hold.

In contrast the inclusive hadronic rate $l_1 \to l_2 X$ (b) corresponds to the imaginary part of the correlator $\Pi(q^2)$. 
Collinear Photons give rise to log-enhanced QED corrections \( \alpha_{\text{em}} \log(m_b^2/m_\ell^2) \)

Higher powers of \( z \) in double differential decay width

- Definition of \( H_i \)? Sensitivity for QED observables?

We use Legendre polynomials for \( H_T \) and \( H_L \) and \( \text{Sign}(z) \) for \( H_A \)

We can construct QED sensitive observables (vanish in absence of QED) by Legendre projectors \( P_3(z) \) or \( P_4(z) \): \( 10^{-8} \)

![Diagram of particle interactions](image)
- Collinear Photons give rise to log-enhanced QED corrections $\alpha_{\text{em}} \log(m_b^2/m_{\ell}^2)$
- Higher powers of $z$ in double differential decay width
  - Definition of $H_i$? Sensitivity for QED observables?
- Size of logs depend on experimental set-up
  $$q^2 = (p_{\ell^+} + p_{\ell^-})^2 \quad \text{vs.} \quad q^2 = (p_{\ell^+} + p_{\ell^-} + p_{\gamma,\text{coll}})^2$$
  - We assume no photons are included in the definition of $q^2$ (di-muon channel at Babar/Belle, di-electron at Belle)
  - Babar’s di-electron channel: Photons that are emitted in a cone of 35 mrad angular opening are included in $q^2$
  
  Monte Carlo techniques needed to estimate this effect

\[
\frac{[B_{ee}^{\text{low}}]_{q=p_{\ell^+}+p_{\ell^-}+p_{\gamma,\text{coll}}}}{[B_{ee}^{\text{low}}]_{q=p_{\ell^+}+p_{\ell^-}}} - 1 = 1.65\% \\
\frac{[B_{ee}^{\text{high}}]_{q=p_{\ell^+}+p_{\ell^-}+p_{\gamma,\text{coll}}}}{[B_{ee}^{\text{high}}]_{q=p_{\ell^+}+p_{\ell^-}}} - 1 = 6.8\% 
\]
- Dependence on Wilson coefficients

\[ H_T(q^2) \propto 2s(1-s)^2 \left[ |C_9 + \frac{2}{s}C_7|^2 + |C_{10}|^2 \right] \]

\[ H_A(q^2) \propto -4s(1-s)^2 \text{Re} \left[ C_{10} (C_9 + \frac{2}{s}C_7) \right] \]

\[ H_L(q^2) \propto (1-s)^2 \left[ |C_9 + 2C_7|^2 + |C_{10}|^2 \right] \]

- \( H_T \) suppressed in low-\( q^2 \) window

- Devide low-\( q^2 \) bin in two bins (zero of \( H_A \) in low-\( q^2 \))

Lee,Ligeti,Stewart, Tackmann hep-ph/0612156

- Most important input parameters

\[ m_b^{ls} = (4.691 \pm 0.037) \text{GeV} , \quad \overline{m}_c(\overline{m}_c) = (1.275 \pm 0.025) \text{GeV} \]

\[ |V_{ts}V_{tb}/V_{cb}|^2 = 0.9621 \pm 0.0027 , \quad \text{BR}_{b\rightarrow c e \nu}^{\text{exp.}} = (10.51 \pm 0.13) \% \]

- Perturbative expansion (NNLO QCD + NLO QED)

\[ \kappa = \alpha_{em}/\alpha_s \]

\[ A = \kappa \left[ A_{LO} + \alpha_s A_{NLO} + \alpha_s^2 A_{NNLO} + \mathcal{O}(\alpha_s^3) \right] + \kappa^2 \left[ A_{LO}^{em} + \alpha_s A_{NLO}^{em} + \alpha_s^2 A_{NNLO}^{em} + \mathcal{O}(\alpha_s^3) \right] + \mathcal{O}(\kappa^3) \]

\[ \text{LO} = \alpha_{em}/\alpha_s , \quad \text{NLO} = \alpha_{em} , \quad \text{NNLO} = \alpha_{em} \alpha_s \]
Monte Carlo analysis

(event generator EVTGEN, hadronization JETSET, EM radiation PHOTOS)

\[
\frac{[B_{ee}^{\text{low}}]}{[B_{ee}^{\text{high}}]} = \frac{q = p_{e^+} + p_{e^-}}{q = p_{e^+} + p_{e^-}} = 1.65% 
\]

\[
\frac{[B_{ee}^{\text{low}}]}{[B_{ee}^{\text{high}}]} = \frac{q = p_{e^+} + p_{e^-} + p_{\gamma\text{coll}}}{q = p_{e^+} + p_{e^-}} = 6.8% 
\]
Further results in units of $10^{-6}$

\[
\begin{align*}
H_L[1, 3.5]_{ee} &= 0.64 \pm 0.03 \\
H_L[3.5, 6]_{ee} &= 0.50 \pm 0.03 \\
H_L[1, 6]_{ee} &= 1.13 \pm 0.06 \\
H_T[1, 3.5]_{ee} &= 0.29 \pm 0.02 \\
H_T[3.5, 6]_{ee} &= 0.24 \pm 0.02 \\
H_T[1, 6]_{ee} &= 0.53 \pm 0.04 \\
H_A[1, 3.5]_{ee} &= -0.103 \pm 0.005 \\
H_A[3.5, 6]_{ee} &= +0.073 \pm 0.012 \\
H_A[1, 6]_{ee} &= -0.029 \pm 0.016 \\
H_L[1, 3.5]_{\mu\mu} &= 0.68 \pm 0.04 \\
H_L[3.5, 6]_{\mu\mu} &= 0.53 \pm 0.03 \\
H_L[1, 6]_{\mu\mu} &= 1.21 \pm 0.07 \\
H_T[1, 3.5]_{\mu\mu} &= 0.21 \pm 0.01 \\
H_T[3.5, 6]_{\mu\mu} &= 0.19 \pm 0.02 \\
H_T[1, 6]_{\mu\mu} &= 0.40 \pm 0.03 \\
H_A[1, 3.5]_{\mu\mu} &= -0.110 \pm 0.005 \\
H_A[3.5, 6]_{\mu\mu} &= +0.067 \pm 0.012 \\
H_A[1, 6]_{\mu\mu} &= -0.042 \pm 0.016
\end{align*}
\]

Total error $\mathcal{O}(5 - 8\%)$. Still dominated by scale uncertainty.