# Inclusive Semi-leptonic Penguin Decays

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# Motivation

- Radiative and semileptonic rare B decayse are highly sensitive probes for new physics

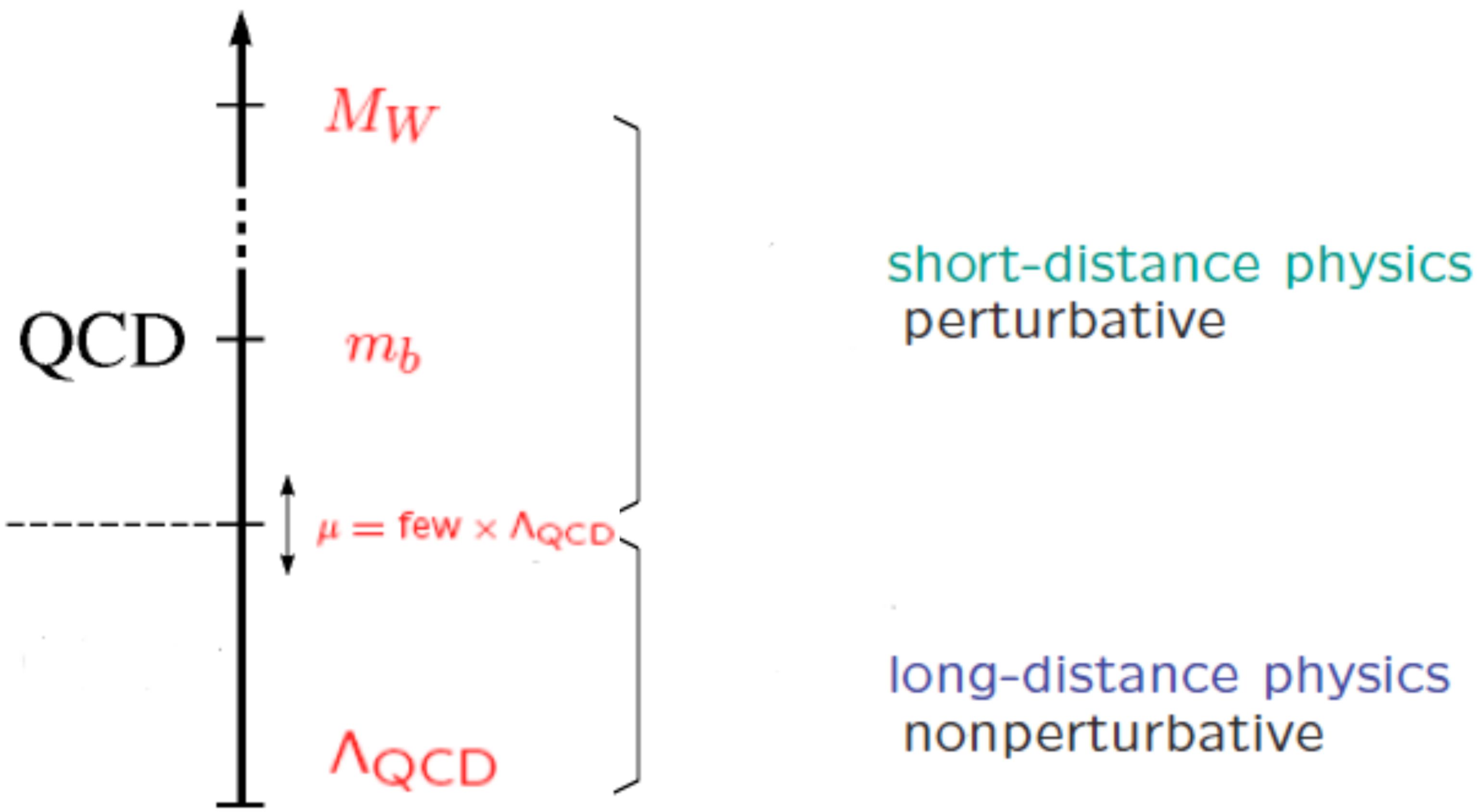
- Exclusive modes are experimentally easier (LHCb), but have larger theoretical uncertainties (issue of unknown power corrections !)

- Inclusive modes require Belle-II for full exploitation (complete angular analysis) but are theoretically very clean

- Inclusive modes allow for crosschecks of recent LHCb anomalies

# Theoretical Tools

# Theoretical tools for flavour precision observables




**Factorization theorems: separating long- and short-distance physics**

- Electroweak effective Hamiltonian: $H_{eff} = -\frac{4G_F}{\sqrt{2}} \sum C_i(\mu, M_{heavy})\ \mathcal{O}_i(\mu)$

- $\mu^2 \approx M_{New}^2 >> M_W^2$ : 'new physics' effects: $C_i^{SM}(M_W) + C_i^{New}(M_W)$

**How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$ ?**

# Inclusive modes $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

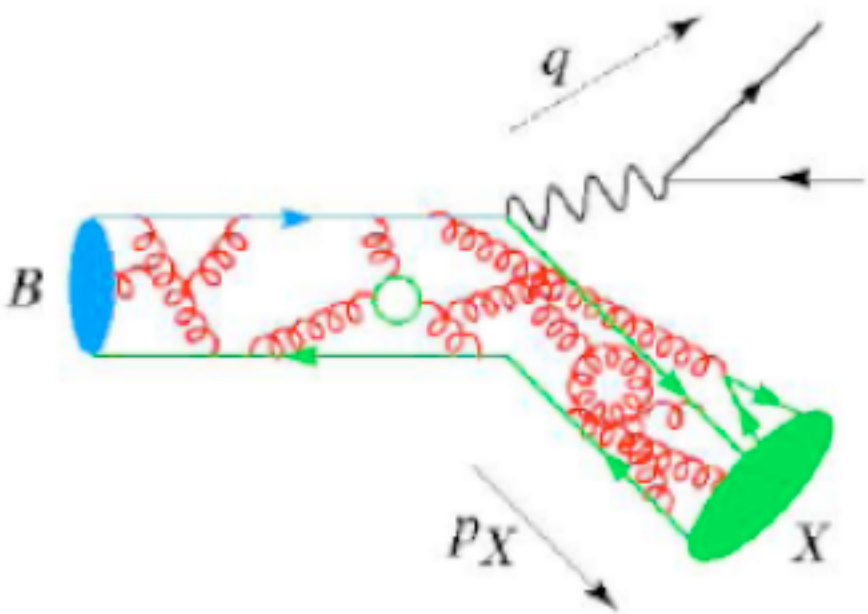
## How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$ ?

Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \to X_s\gamma) \stackrel{m_b\to\infty}{\longrightarrow} \Gamma(b \to X_s^{parton}\gamma)\,, \quad \Delta^{nonpert.} \sim \Lambda_{QCD}^2/m_b^2$$

No linear term $\Lambda_{QCD}/m_b$ (perturbative contributions dominant)

Chay,Georgi,Grinstein 1990

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No linear term $\Lambda_{QCD}/m_b$ (perturbative contributions dominant)

**An old story:**

– If one goes beyond the leading operator ($\mathcal{O}_7$, $\mathcal{O}_9$):
  breakdown of local expansion

**A new dedicated analysis:**

naive estimate of non-local matrix elements leads to 5% uncertainty.

Benzke,Lee,Neubert,Paz,arXiv:1003.5012

Analysis in $B \to X_s\ell\ell$ in this talk; Benzke,Fickinger,Hurth,Turczyk

# Exclusive modes $B \to K^{(*)}\ell\ell$

QCD-improved factorization: BBNS 1999

$$\mathcal{T}_a^{(i)} = C_a^{(i)}\xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K^*} + O(\Lambda/m_b)$$

(Soft-collinear effective theory)

– Separation of perturbative hard kernels from process-independent nonperturbative functions like form factors

– Relations between formfactors in large-energy limit

– Limitation: insufficient information on power-suppressed $\Lambda/m_b$ terms (breakdown of factorization: 'endpoint divergences')

# Difference between exclusive and inclusive $b \to s\gamma, \ell\ell$ modes:

## Inclusive

$\Lambda^2/m_b^2$ corrections can be calculated for the leading operators in the local OPE .

$\Lambda/m_b$ corrections to the subleading operators correspond to nonlocal matrix elements and can be estimated !

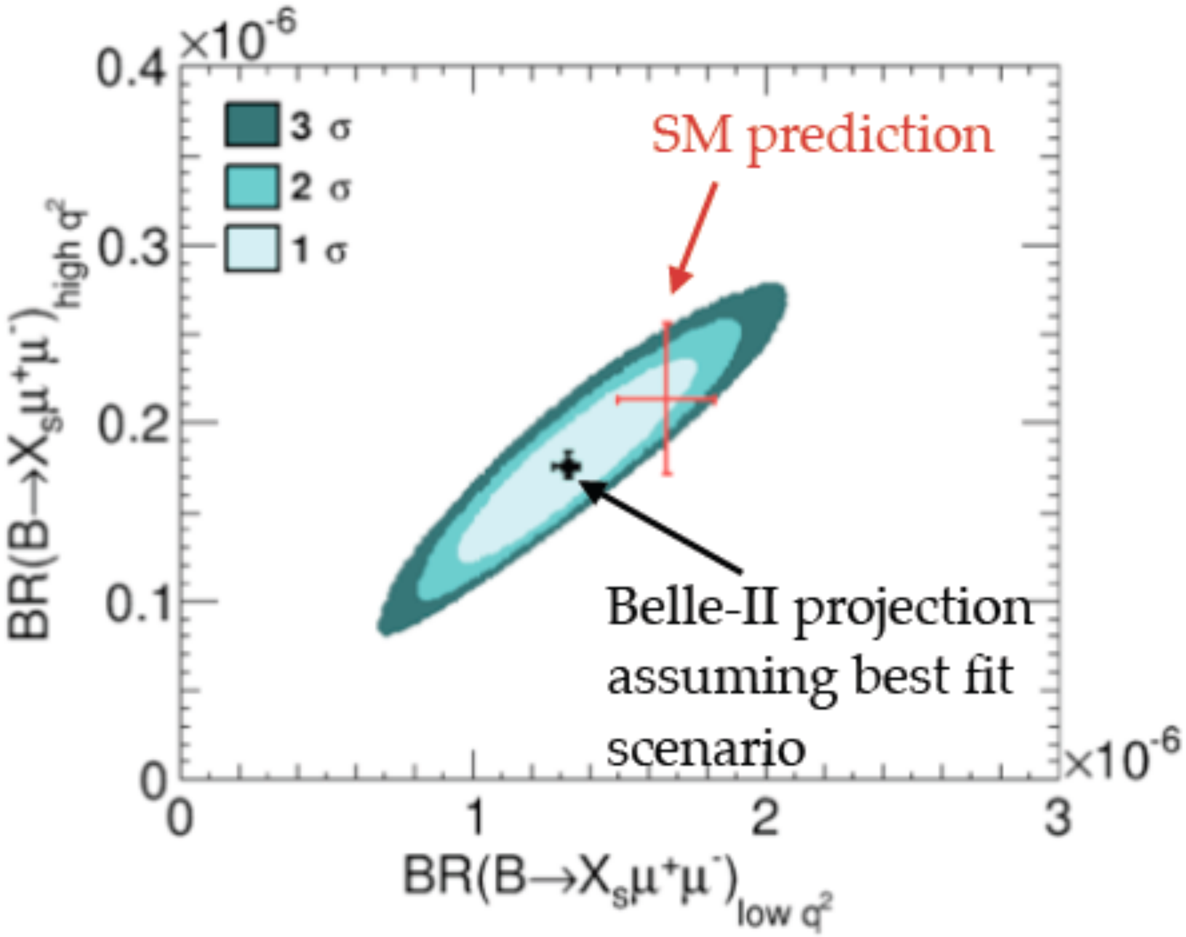
## Exclusive

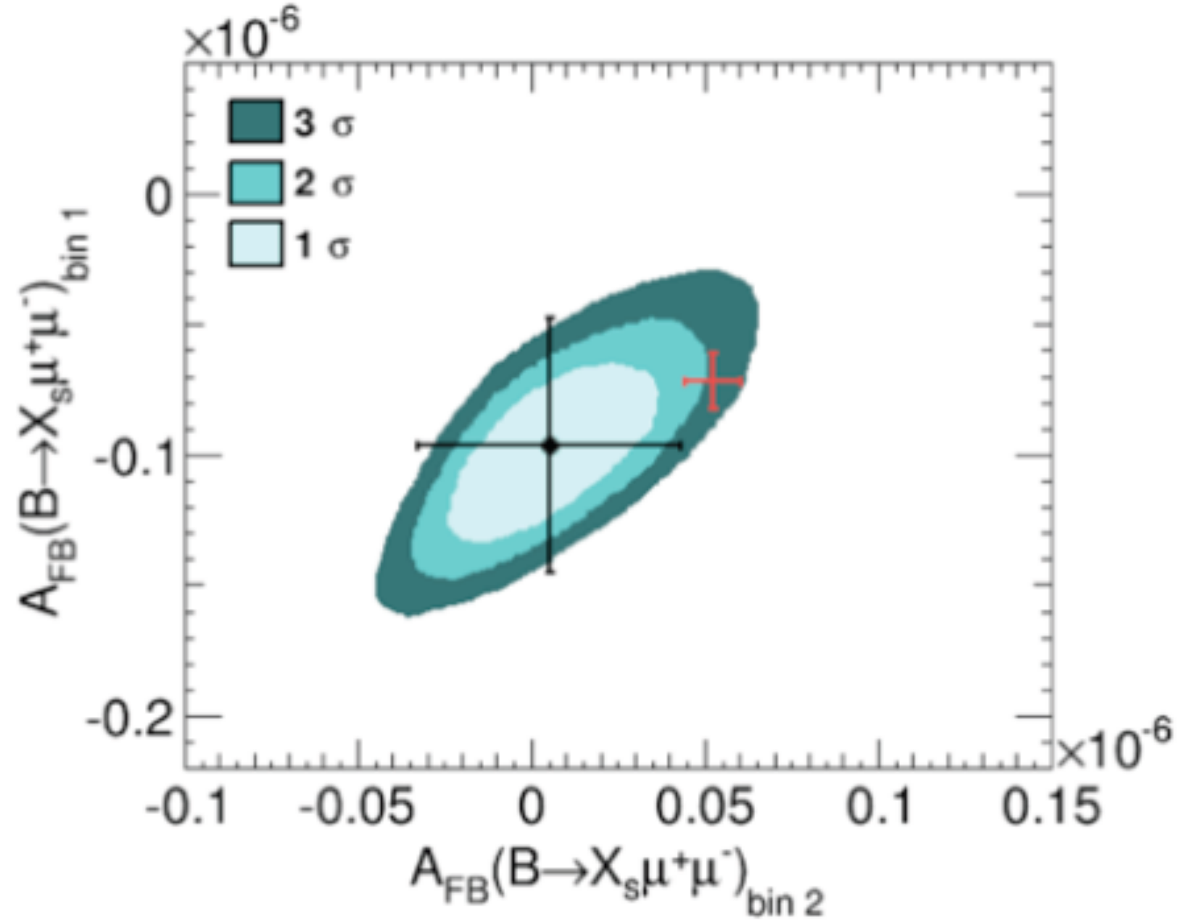
No theory of $\Lambda/mb$ corrections at all within QCD factorization formula (in the low-$q^2$ region); these corrections can only be "guesstimated" !

# Crosscheck of LHCb anomalies with inclusive modes

Hurth,Mahmoudi,Neshatpour,arXiv:1410.4545

if SM deviations in $R_K$ and $P_5'$ persist until Belle-II




**If NP then the effect of $C_9$ and $C_9'$ are large enough to be checked at Belle-II with theoretically clean modes.**

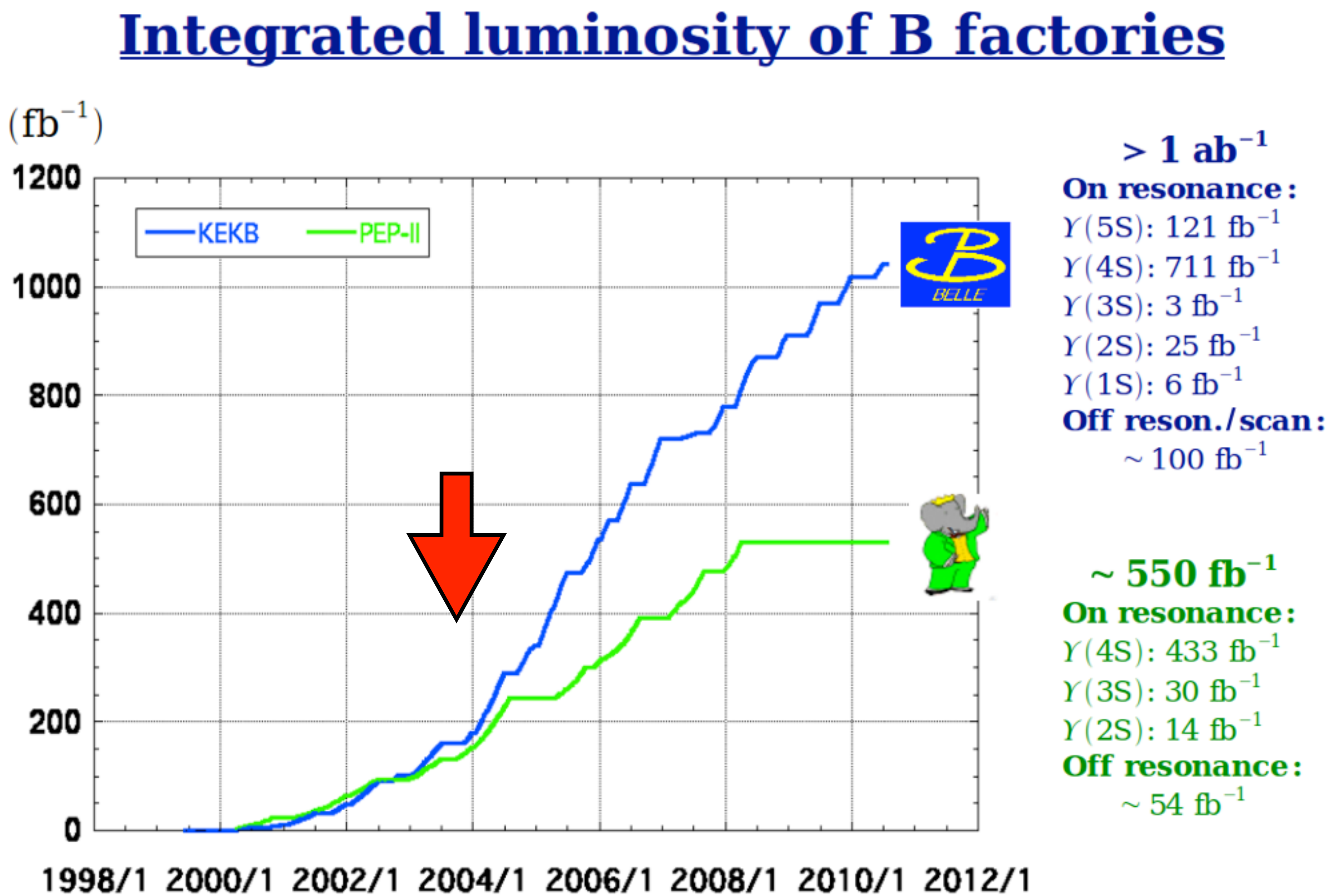
Hurth, Mahmoudi, arXiv:1312.5267 Experimental extrapolation by Kevin Flood

# Inclusive modes

# Experiment

- "Latest" Belle measurement of branching ratio is based on less than 30% of the total luminosity

Belle hep-ex/0503044 (!!!) (based $152 \times 10^6 B\bar{B}$ events)




New Babar analysis on dilepton spectrum arXiv:1312.3664

New Belle analysis on AFB arXiv:1402.7134

# Complete angular analysis of inclusive $B \to X_s \ell\ell$

Huber,Hurth,Lunghi, arXiv:1503.04849

- "Latest" Belle measurement of branching ratio is based on less than 30% of the total luminosity

$$\frac{d^2\Gamma}{dq^2\, dz} = \frac{3}{8}\left[(1+z^2)\, H_T(q^2) + 2\, z\, H_A(q^2) + 2\,(1-z^2)\, H_L(q^2)\right] \qquad (z = \cos\theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \qquad\qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\, H_A(q^2)$$

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events

Large logs $log(mb/m_\ell)$ different for muon and electron !

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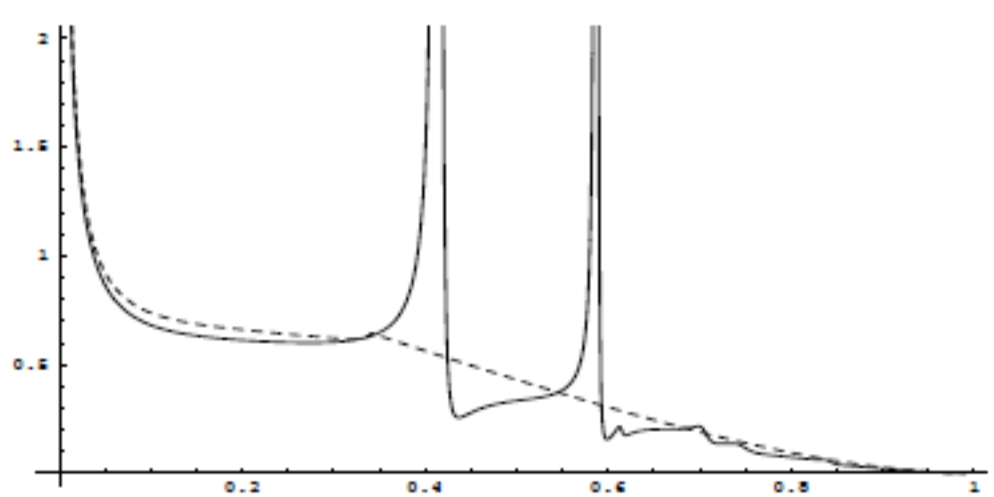
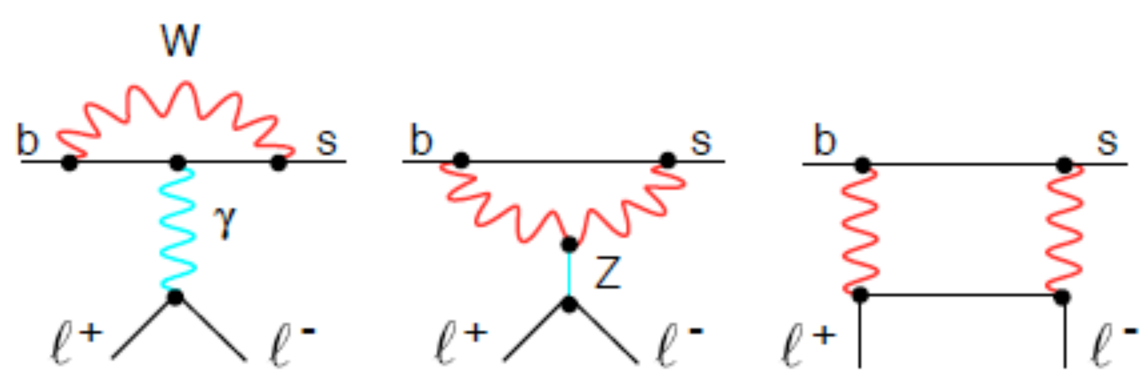
- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

- On-shell-$c\bar{c}$-resonances $\Rightarrow$ cuts in dlepton mass spectrum necessary : $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ and $14.4\text{GeV}^2 < q^2$ $\Rightarrow$ perturbative contributions dominant

$\frac{d}{d\hat{s}} BR(\bar{B} \to X_s l^+ l^-) \times 10^{-5}$




$\hat{s} = q^2/m_b^2$



Huber,Hurth,Lunghi, arXiv:1503.04849

## Results

Low-$q^2$ $(1GeV^2 < q^2 < 6GeV^2)$

$BR(B \to X_s ee) = (1.67 \pm 0.10)\, 10^{-6}$

$BR(B \to X_s \mu\mu) = (1.62 \pm 0.09)\, 10^{-6}$

Babar: $BR(B \to X_s \ell\ell) =$

$= (1.60\ (+0.41 - 0.39)_{stat} (+0.17 - 0.13)_{syst} (\pm 0.18)_{mod})\, 10^{-6}$

good agreement with SM

Huber,Hurth,Lunghi, arXiv:1503.04849

## Results

High-$q^2$, Theory: $q^2 > 14.4 GeV^2$, Babar: $q^2 > 14.2 GeV^2$

$$BR(B \to X_s ee) = (0.220 \pm 0.070)\, 10^{-6}$$

$$BR(B \to X_s \mu\mu) = (0.253 \pm 0.070)\, 10^{-6}$$

Babar: $BR(B \to X_s \ell\ell) =$

$$(0.57\ (+0.16 - 0.15)_{stat}(+0.03 - 0.02)_{syst})\, 10^{-6}$$

$2\sigma$ higher than SM

Significant higher values predicted in Greub et al. due to missing power and QED corrections and different cut Greub,Pilipp,Schupbach,arXiv:0810.4077

(but perfect agreement if we use their prescriptions)

## Further refinement

Huber,Hurth,Lunghi, arXiv:1503.04849

Normalization to semileptonic $B \to X_u \ell \nu$ decay rate with the same cut reduces the impact of $1/m_b$ corrections in the high-$q^2$ region significantly.

Ligeti,Tackmann arXiv:0707.1694

Theory prediction for ratio

$$R(s_0)_{ee} = (2.25 \pm 0.31)\, 10^{-3}$$

$$R(s_0)_{\mu\mu} = (2.62 \pm 0.30)\, 10^{-3}$$

Largest source of error are CKM elements ($V_{ub}$)

Note: Additional $O(5\%)$ uncertainty due to nonlocal power corrections $O(\alpha_s \Lambda / m_b)$

# New physics sensitivity

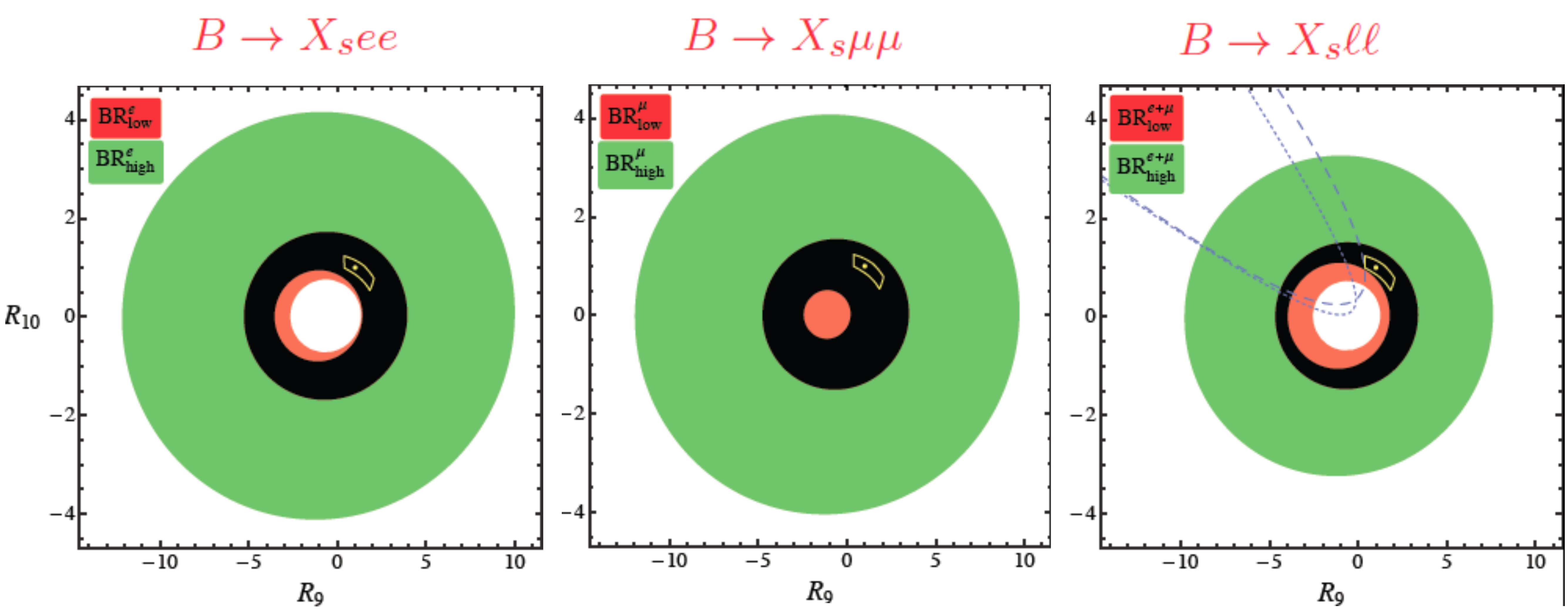
Huber,Hurth,Lunghi, arXiv:1503.04849

**Constraints on Wilson coefficients $C_9/C_9^{\text{SM}}$ and $C_{10}/C_{10}^{\text{SM}}$**

$$R_i = \frac{C_i(\mu_0)}{C_i^{\text{SM}}(\mu_0)}$$

that we obtain at 95% C.L. from present experimental data
(red low $q^2$, green high $q^2$)

that we will obtain at 95% C.L. from $50ab^{-1}$ data at Belle-II
(yellow)



# Subleading contributions in $B \to X_s \ell^+ \ell^-$

- On-shell-$c\bar{c}$-resonances $\Rightarrow$ cuts in dlepton mass spectrum necessary :
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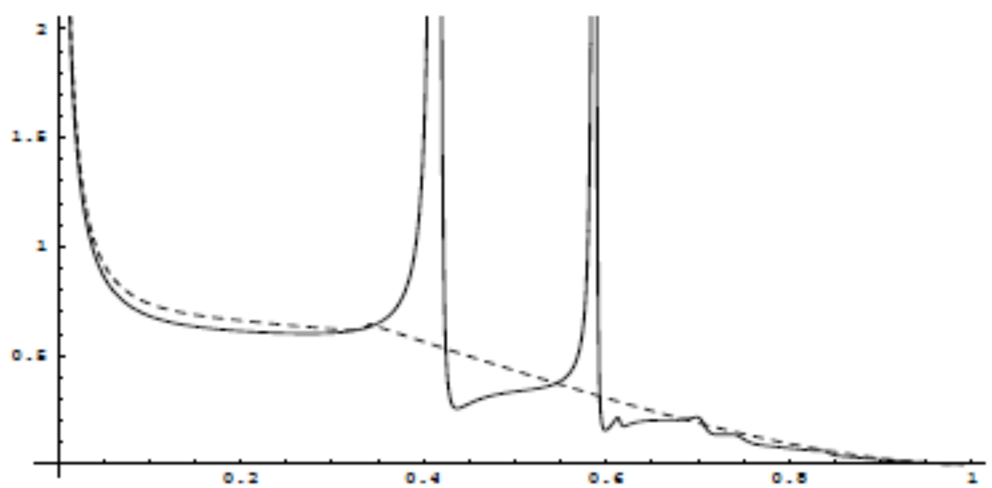
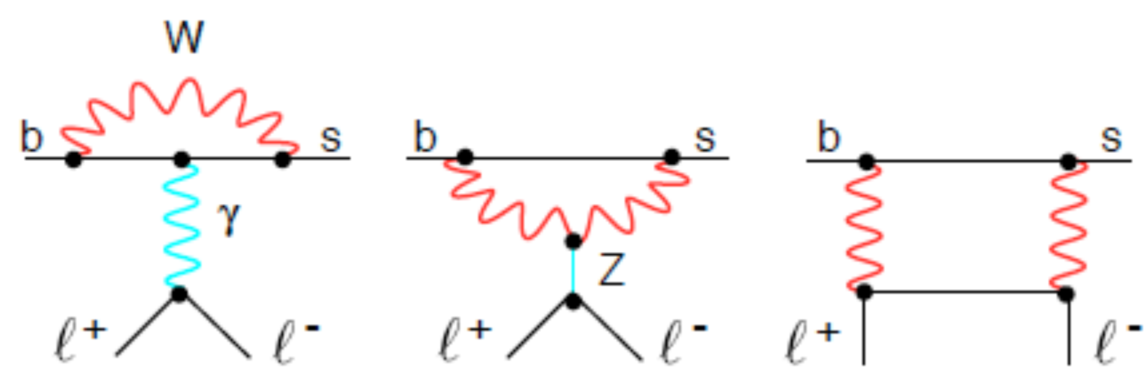





- Hadronic invariant-mass cut is imposed in order to eliminate the background like $b \to c\,(\to s e^+ \nu) e^- \bar{\nu} = b \to s e^+ e^- +$ missing energy
  - $*$ Babar,Belle: $m_X < 1.8\,\text{or}\,2.0 GeV$
  - $*$ high-$q^2$ region not affected by this cut
  - $*$ kinematics: $X_s$ is jetlike and $m_X^2 \leq m_b \Lambda_{QCD}$ $\Rightarrow$ shape function region
  - $*$ SCET analysis: universality of jet and shape functions found:
    the 10-30% reduction of the dilepton mass spectrum can be accurately computed using the $\bar{B} \to X_s \gamma$ shape function
    5% additional uncertainty for $2.0 GeV$ cut due to subleading shape functions

Lee,Stewart hep-ph/0511334
Lee,Ligeti,Stewart,Tackmann hep-ph/0512191
Lee,Tackmann arXiv:0812.0001 (effect of subleading shape functions)
Bell,Beneke,Huber,Li arXiv:1007.3758 (NNLO matching QCD $\to$ SCET)

# Subleading power factorization in $B \to X_s \ell^+ \ell^-$

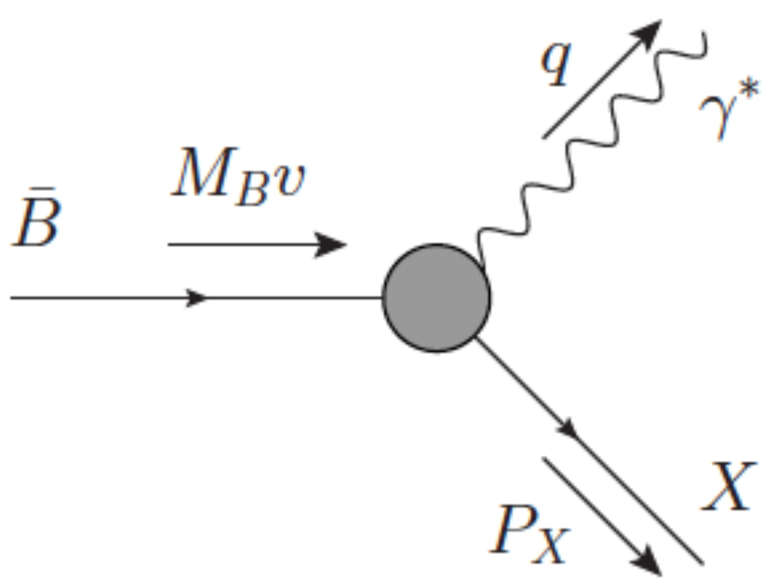
Benzke,Fickinger,Hurth,Turczyk, to appear

## Hadronic cut

Additional cut in $X_s$ necessary to reduce background
affects only low-$q^2$ region.

Hadronic invariant $m_X^2 < 1.8(2.0) GeV^2$, jet-like $X_s$ $E_X \sim \mathcal{O}(m_b)$

Multiscale problem $\to$ SCET




$$M_B^2 \sim m_b^2 \gg m_X^2 \sim \Lambda_{\rm QCD}\, m_b \gg \Lambda_{\rm QCD}^2$$

$$m_X^2 = P_X^2 = (M_B - n \cdot q)(M_B - \bar{n} \cdot q)$$

**Scaling** $\lambda = \Lambda_{\rm QCD}/m_b$

## Kinematics

$B$ meson rest frame

$$q = p_B - p_X \qquad 2\, m_B\, E_X = m_B^2 + M_X^2 - q^2$$

$X_s$ system is jet-like with $E_X \sim m_B$ and $m_X^2 \ll E_X^2$

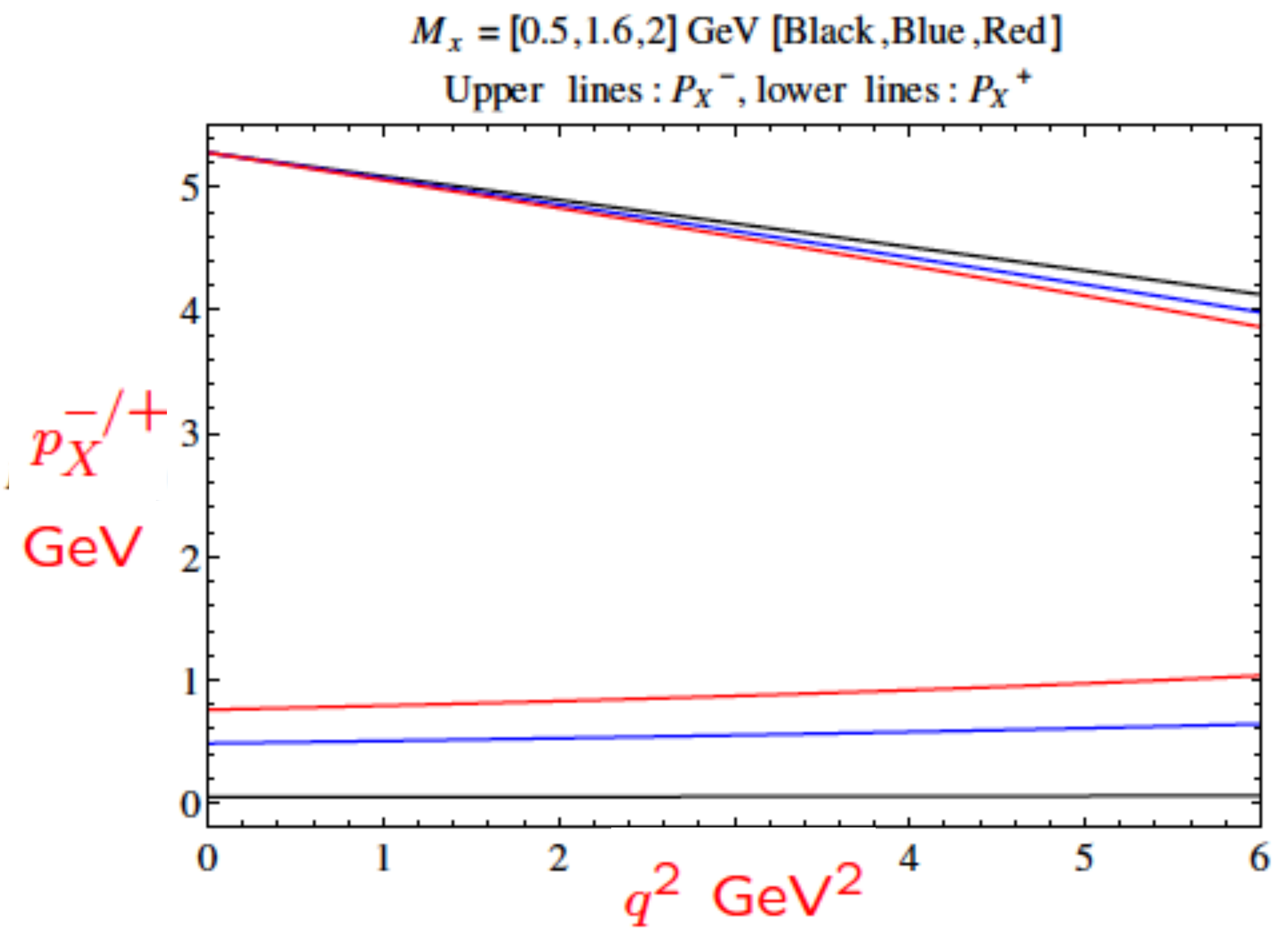
two light-cone components $\qquad p_X^- p_X^+ = m_X^2$

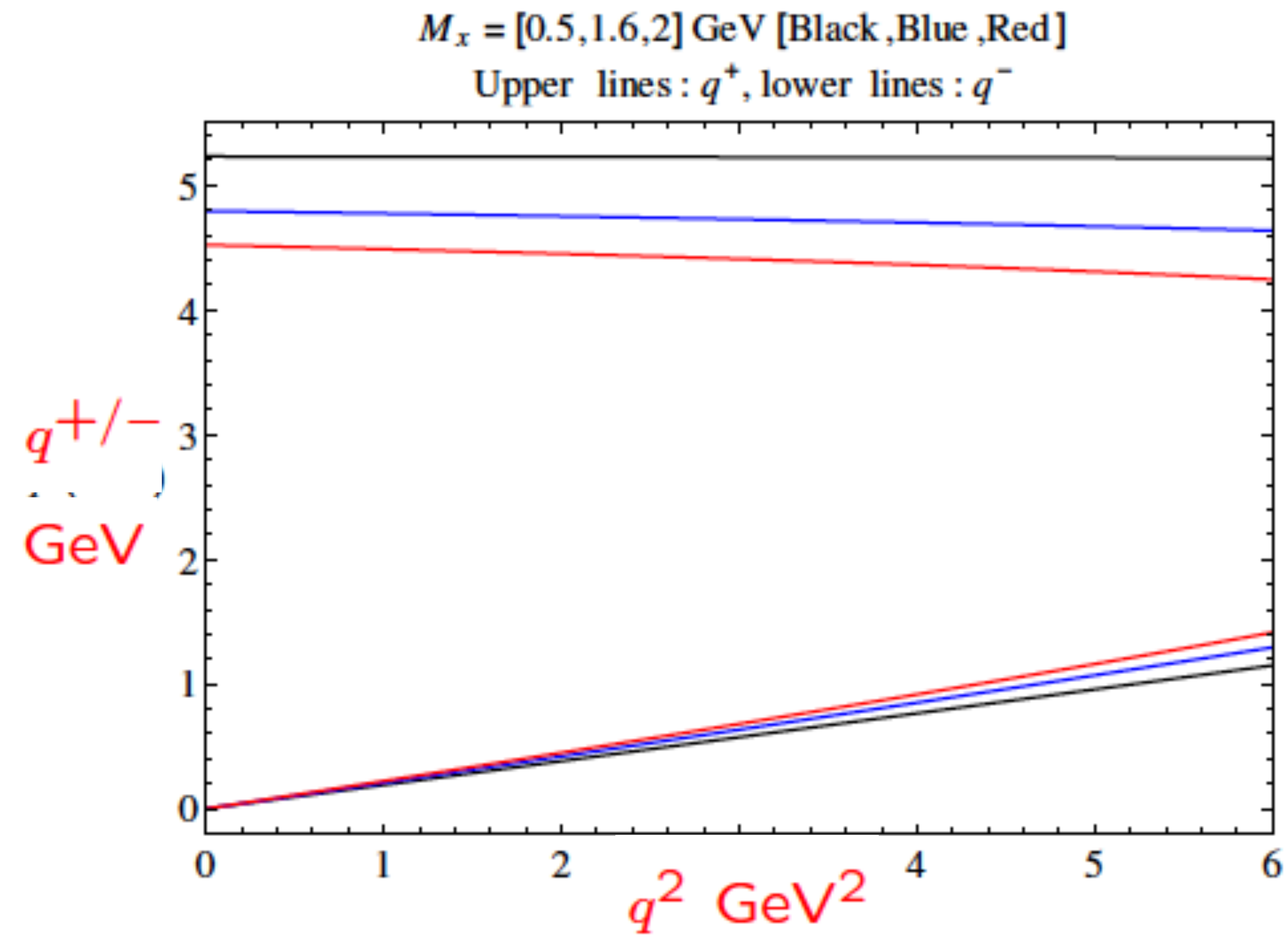
$$\bar{n} p_X = p_X^- = E_X + |\vec{p}_X| \sim \mathcal{O}(m_B)$$

$$n p_X = p_X^+ = E_X - |\vec{p}_X| \sim \mathcal{O}(\Lambda_{\mathrm{QCD}})$$

$$q^+ = nq = m_B - p_X^+ \qquad q^- = \bar{n} q = m_B - p_X^-$$

**Scaling** $\lambda = \Lambda_{\rm QCD}/m_b \qquad m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$



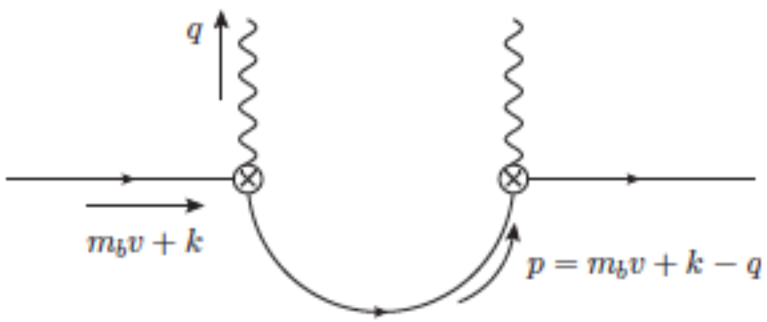

For $q^2 < 6GeV^2$ the scaling of $np_X$ and $\bar{n}p_X$ implies $\bar{n}q$ is of order $\lambda$, means $q$ anti-hard-collinear (just kinematics).

Stewart and Lee assume $\bar{n}q$ to be order 1, means $q$ is hard.

This problematic assumption implies a different matching of SCET/QCD.

## Shapefunction region

Local OPE breaks down for $m_X^2 \sim \lambda$:




$$\frac{1}{(m_b v+k-q)^2} = \frac{1}{m_b - n\cdot q}\left(1 - \frac{n\cdot k}{m_b - n\cdot q} + \dots\right)\frac{1}{m_b - \bar{n}\cdot q}$$

Resummation of leading contributions into a shape function.

(scaling of $\bar{n}q$ does not matter here; zero in case of $B \to X_s\gamma$)

## Factorization theorem $d\Gamma \sim H \cdot J \otimes S$

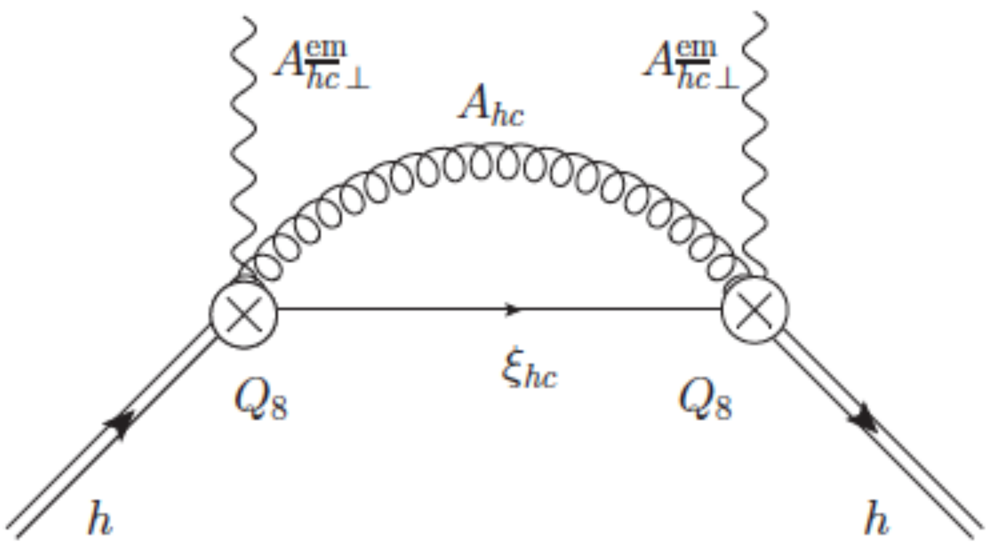
The hard function $H$ and the jet function $J$ are perturbative quantities.

The shape function $S$ is a non-perturbative non-local HQET matrix element.

(universality of the shape function, uncertainties due to subleading shape functions)

## Calculation at subleading power

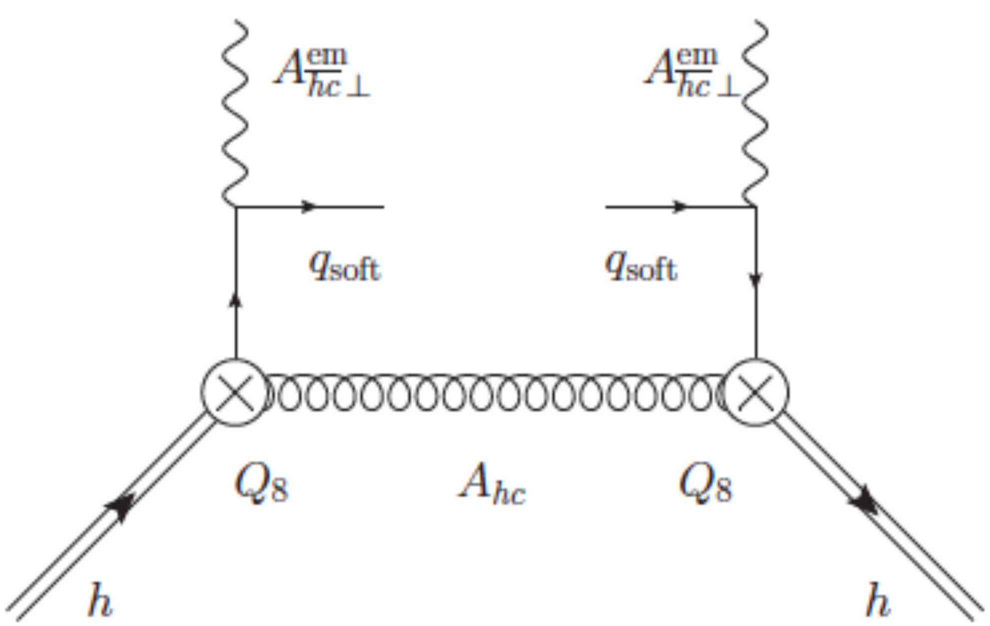
Example of **direct** photon contribution which factorizes $d\Gamma \sim H \cdot j \otimes S$




$\rightarrow \frac{\alpha_s}{m_b}$ in low $m_X^2$ region

Example of **resolved** photon contribution (double-resolved) which factorizes

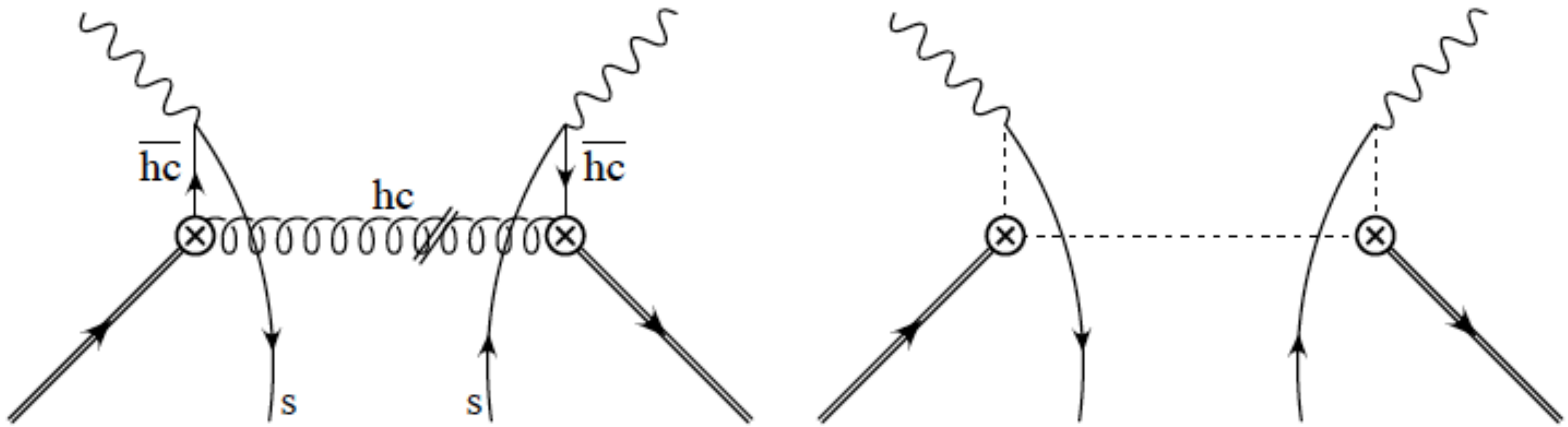
$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$




$\rightarrow \frac{\Lambda}{m_b}$

Shape function is non-local in two light-cone directions.
It survives $M_X \rightarrow 1$ limit (irreducible uncertainty).

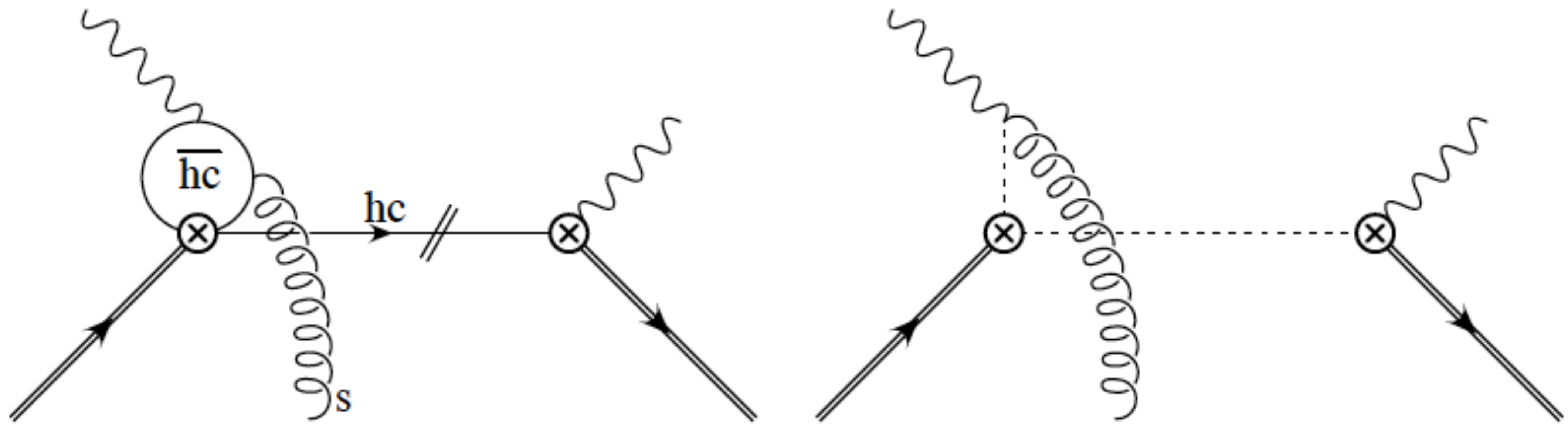
# Interference of $Q_8$ and $Q_8$




$$\frac{d\Gamma^{\rm res}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{e_s^2\alpha_s}{m_b}\int d\omega\, \delta(\omega+p_+)\int \frac{d\omega_1}{\omega_1+\bar{n}\cdot q+i\varepsilon}\int \frac{d\omega_2}{\omega_2+\bar{n}\cdot q-i\varepsilon} g_{88}(\omega,\omega_1,\omega_2)$$

$$g_{88}(\omega,\omega_1,\omega_2) = \frac{1}{M_B}\langle\bar{B}|\bar{h}(\mathbf{tn})\ldots s(\mathbf{tn}+\mathbf{u\bar{n}})\bar{s}(\mathbf{r\bar{n}})\ldots h(\mathbf{0})|\bar{B}\rangle_{\rm F.T.}$$

# Interference of $Q_1$ and $Q_7$




$$\frac{d\Gamma^{\rm res}}{dn\cdot q\,d\bar{n}\cdot q} \sim \frac{1}{m_b}\int d\omega\,\delta(\omega+p_+)\int\frac{d\omega_1}{\omega_1+i\varepsilon}$$

$$\frac{1}{\omega_1}\left[\bar{n}\cdot q\left(F\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right)-1\right)-(\bar{n}\cdot q+\omega_1)\left(F\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q+\omega_1)}\right)-1\right)\right.$$

$$\left.+\bar{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right)-G\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q+\omega_1)}\right)\right)\right]g_{17}(\omega,\omega_1)$$

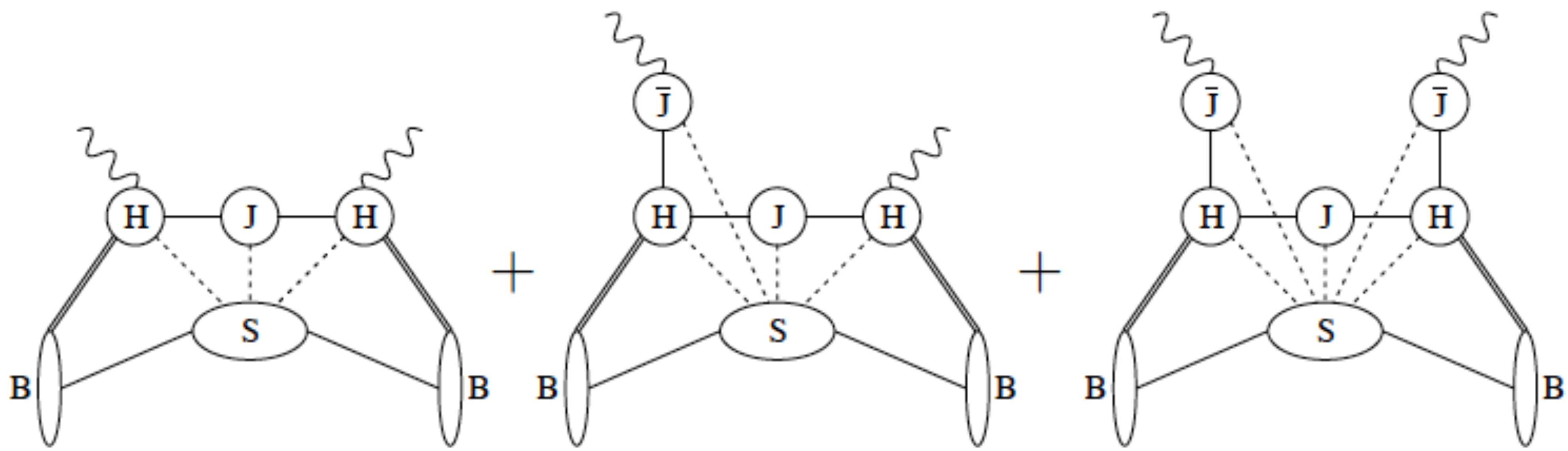
$$g_{17}(\omega,\omega_1)=\int\frac{dr}{2\pi}e^{-i\omega_1 r}\int\frac{dt}{2\pi}e^{-i\omega t}\frac{1}{M_B}\langle\bar{B}|\bar{h}(\mathbf{tn})\ldots G_s^{\alpha\beta}(\mathbf{r\bar{n}})\ldots h(\mathbf{0})|\bar{B}\rangle$$

Expansion for $m_c \sim m_b$ leads to Voloshin term in the total rate $(-\lambda_2/m_c^2)$, the terms stays non-local for $m_c < m_b$.

## Factorization formula

In the $m_X^2 \sim \lambda$ and $q^2 \sim \lambda$ region we have the following factorization formula

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum H \cdot j_i \otimes S + \frac{1}{m_b} \sum H \cdot J \otimes s_i$$
$$+ \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$




## Numerical evaluation (work in progress)

Similar subleading shape functions as in $B \to X_s \gamma$

Use vacuum insertion approximation, PT invariance,....

## Power corrections in the inclusive mode

- For q anti-hard-collinear we have identified a new type of subleading power corrections.

- In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

- They constitute an irreducible uncertainty because they survive the $M_X \to 1$ limit.

- If q was hard then these resolved contributions would not exist

**$M_X$ cut effects in the low-$q^2$ region with $q^2$ anti-hard-collinear**

(work in progress)

# Extra

# Semileptonic Penguin Decays

Based on

Huber,Hurth,Lunghi arXiv:1503.0449

Inclusive $B \to X_s \ell^+ \ell^-$: Complete angular analysis and a thorough study of collinear photons

Benzke,Fickinger,Hurth,Turczyk to appear

Subleading power factorization in $B \to X_s \ell^+ \ell^-$

Hurth,Mahmoudi,Neshatpour arXiv:1603.00865

On the anomalies in the latest LHCb data

# Alllowed regions

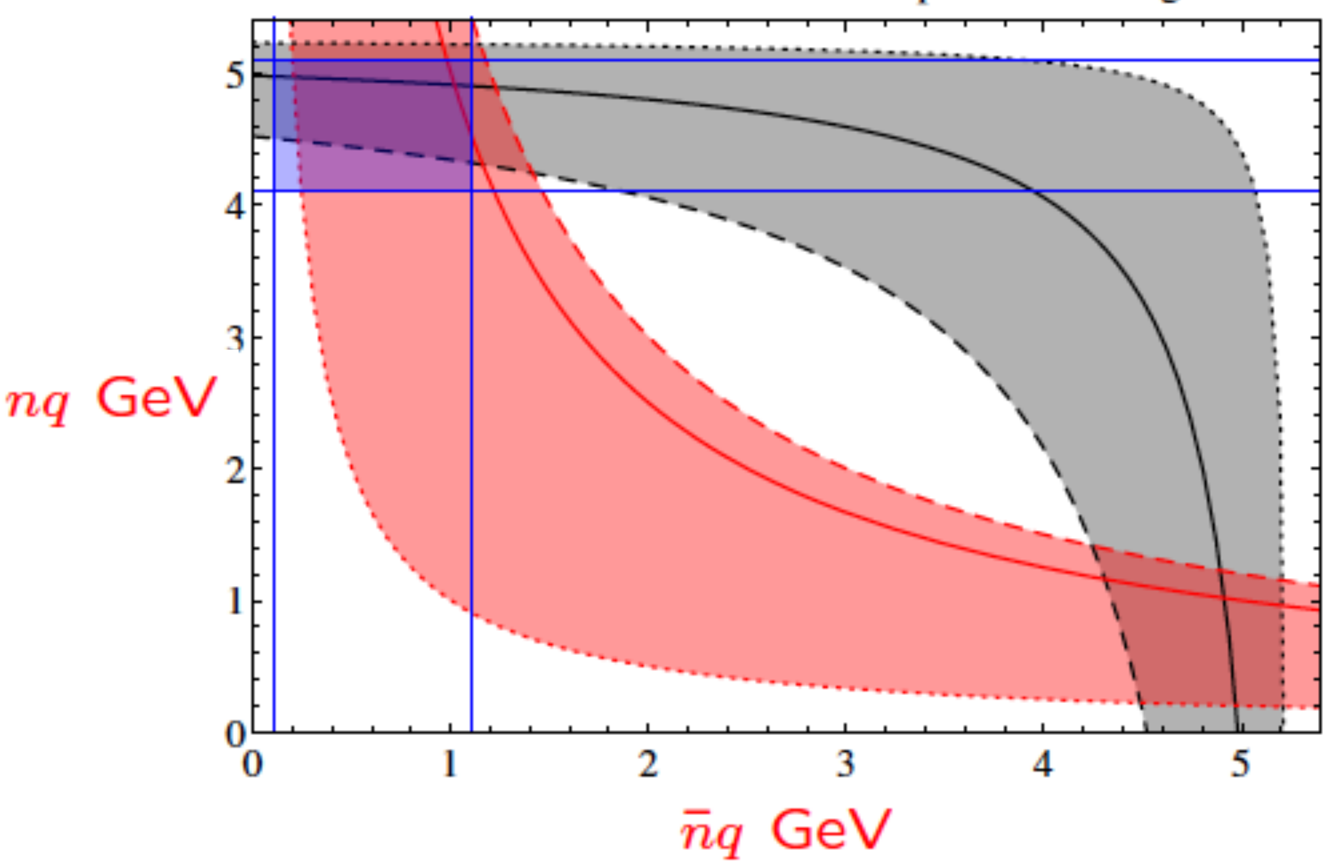
Benzke,Fickinger,Hurth,Turczyk, to appear

## low-$q^2$

Red: $q^2$= [1,5,6] GeV$^2$ [Dotted , Solid ,Dashed ]

Black: $M_x$ = [0.495 ,1.25 ,2] GeV [Dotted , Solid , Dashed ]

Blue : anti −hard −collinear component scaling

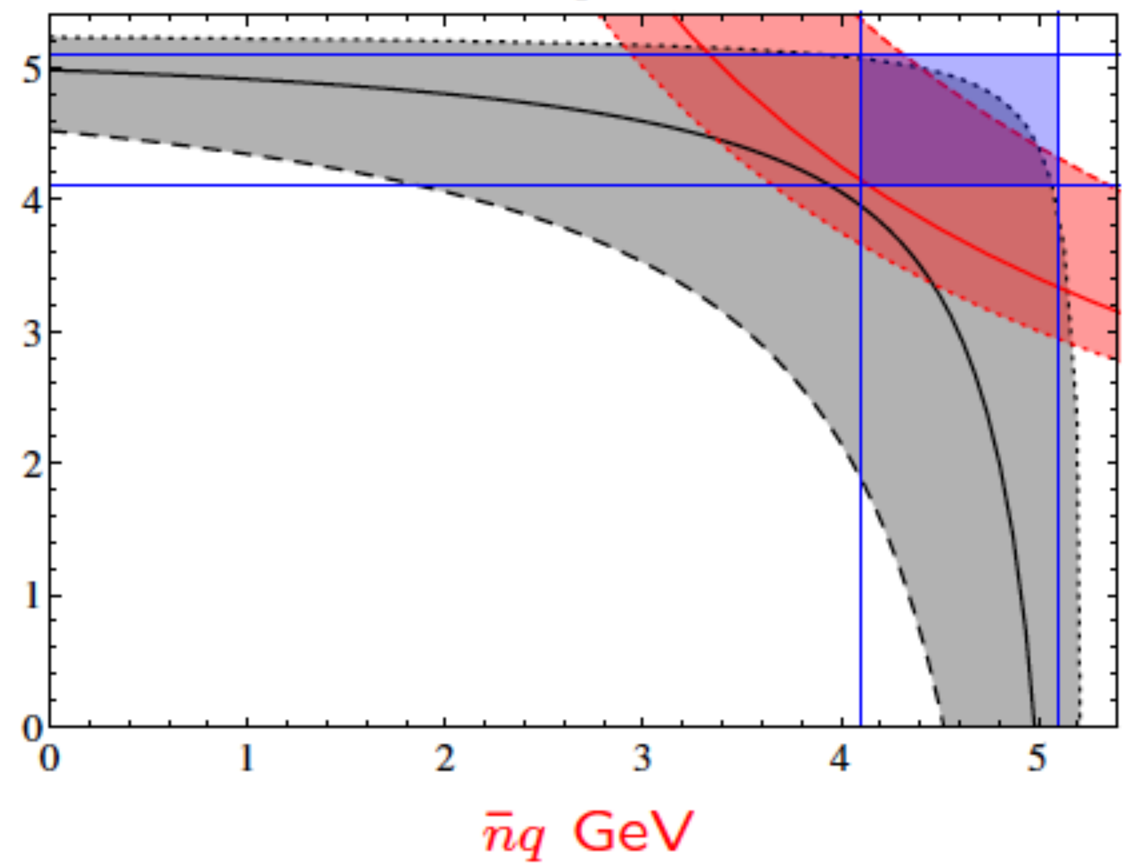



## high-$q^2$

Red: $q^2$= [15,17,22] GeV$^2$ [Dotted , Solid , Dashed ]

Black: $M_x$ = [0.495 ,1.25 ,2] GeV [Dotted , Solid , Dashed ]

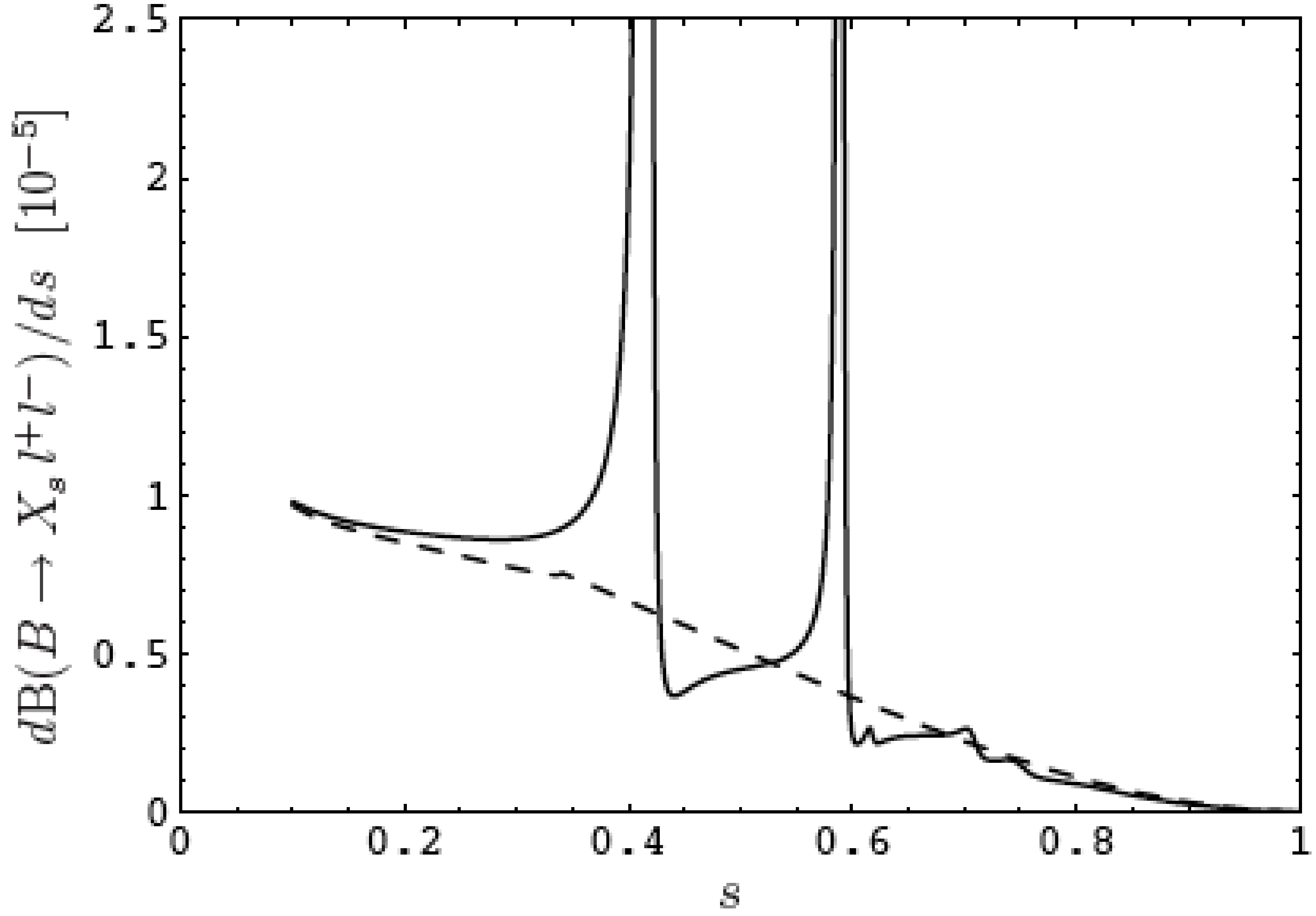
Blue : hard component scaling



## Quark-hadron duality violated in $\bar{B} \to X_s \ell^+ \ell^-$ ?

BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances $J/\psi$ and $\psi'$ exceed the perturbative contributions by two orders of magnitude.



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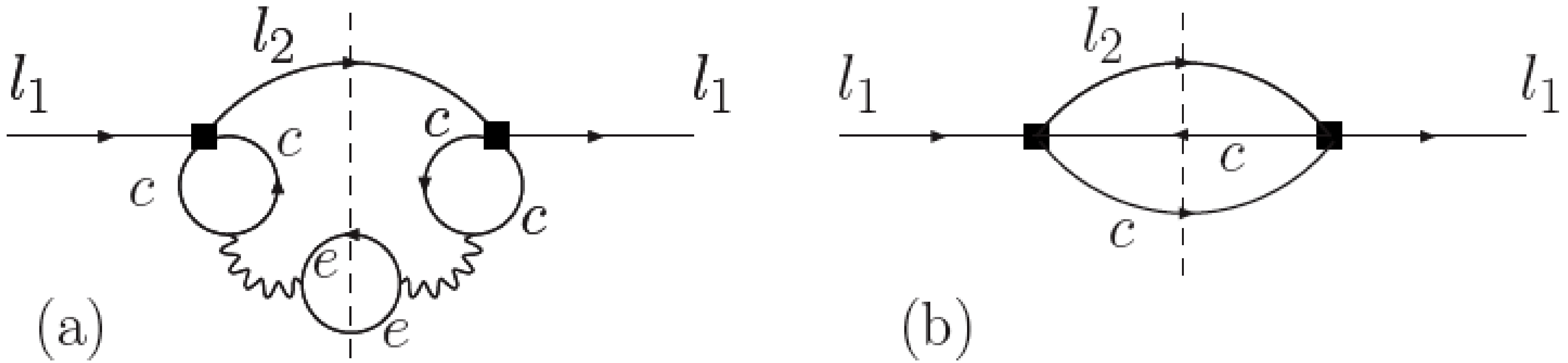



The rate $l_1 \to l_2 e^+ e^-$ (a) is connected to the integral over $|\Pi(q^2)|^2$ for which global duality is NOT expected to hold.

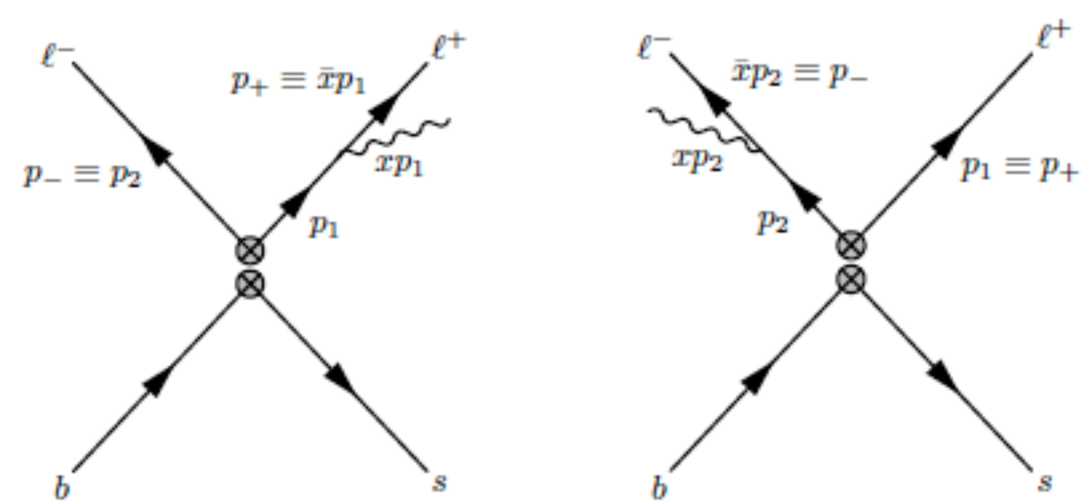
In contrast the inclusive hadronic rate $l_1 \to l_2 X$ (b) corresponds to the imaginary part of the correlator $\Pi(q^2)$.

Huber,Hurth,Lunghi, arXiv:1503.04849

- Collinear Photons give rise to log-enhanced QED corrections $\alpha_{\rm em} \log(m_b^2/m_\ell^2)$
- Higher powers of z in double differential decay width
  - Definition of $H_i$ ? Sensitivity for QED observables ?

We use Legendre poynomials for $H_T$ and $H_L$ and $Sign(z)$ for $H_A$

We can construct QED sensitive observables (vanish in absence of QED) by Legendre projectors $P_3(z)$ or $P_4(z)$: $10^{-8}$



Huber,Hurth,Lunghi, arXiv:1503.04849

- Collinear Photons give rise to log-enhanced QED corrections $\alpha_{\rm em} \log(m_b^2/m_\ell^2)$
- Higher powers of z in double differential decay width
  - Definition of $H_i$ ? Sensitivity for QED observables ?
- Size of logs depend on experimental set-up

$$q^2 = (p_{\ell^+} + p_{\ell^-})^2 \quad \text{vs.} \quad q^2 = (p_{\ell^+} + p_{\ell^-} + p_{\gamma,\rm coll})^2$$

  - We assume no photons are included in the definition of $q^2$ (di-muon channel at Babar/Belle, di-electron at Belle)
  - Babar's di-electron channel: Photons that are emitted in a cone of 35 mrad angular opening are included in $q^2$

  Monte Carlo techniques needed to estimate this effect

$$\frac{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 1.65\% \qquad \frac{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 6.8\%$$

Huber,Hurth,Lunghi, arXiv:1503.04849

- Dependence on Wilson coefficients

$$H_T(q^2) \propto 2s(1-s)^2\left[\left|C_9 + \frac{2}{s}\,C_7\right|^2 + |C_{10}|^2\right]$$

$$H_A(q^2) \propto -4s(1-s)^2\ \mathrm{Re}\left[C_{10}\left(C_9 + \frac{2}{s}\,C_7\right)\right]$$

$H_T$ suppressed in low-$q^2$ window

$$H_L(q^2) \propto (1-s)^2\left[\left|C_9 + 2\,C_7\right|^2 + |C_{10}|^2\right]$$

- Devide low-$q^2$ bin in two bins (zero of $H_A$ in low-$q^2$)

Lee,Ligeti,Stewart, Tackmann hep-ph/0612156

- Most important input parameters

$$m_b^{1S} = (4.691 \pm 0.037)\mathrm{GeV}\,, \qquad \overline{m}_c(\overline{m}_c) = (1.275 \pm 0.025)\mathrm{GeV}$$
$$|V_{ts}^* V_{tb}/V_{cb}|^2 = 0.9621 \pm 0.0027\,, \qquad BR^{exp.}_{b\to c\,e\,\nu} = (10.51 \pm 0.13)\,\%$$

- Perturbative expansion (NNLO QCD + NLO QED) $\alpha_s$ $\kappa = \alpha_{\mathrm{em}}/\alpha_s$

$$\begin{aligned} A &= \kappa\left[A_{LO} + \alpha_s\,A_{NLO} + \alpha_s^2\,A_{NNLO} + \mathcal{O}(\alpha_s^3)\right] \\ &+ \kappa^2\left[A_{LO}^{em} + \alpha_s\,A_{NLO}^{em} + \alpha_s^2\,A_{NNLO}^{em} + \mathcal{O}(\alpha_s^3)\right] + \mathcal{O}(\kappa^3) \end{aligned}$$

LO = $\alpha_{\mathrm{em}}/\alpha_s$, NLO = $\alpha_{\mathrm{em}}$, NNLO = $\alpha_{\mathrm{em}}\,\alpha_s$

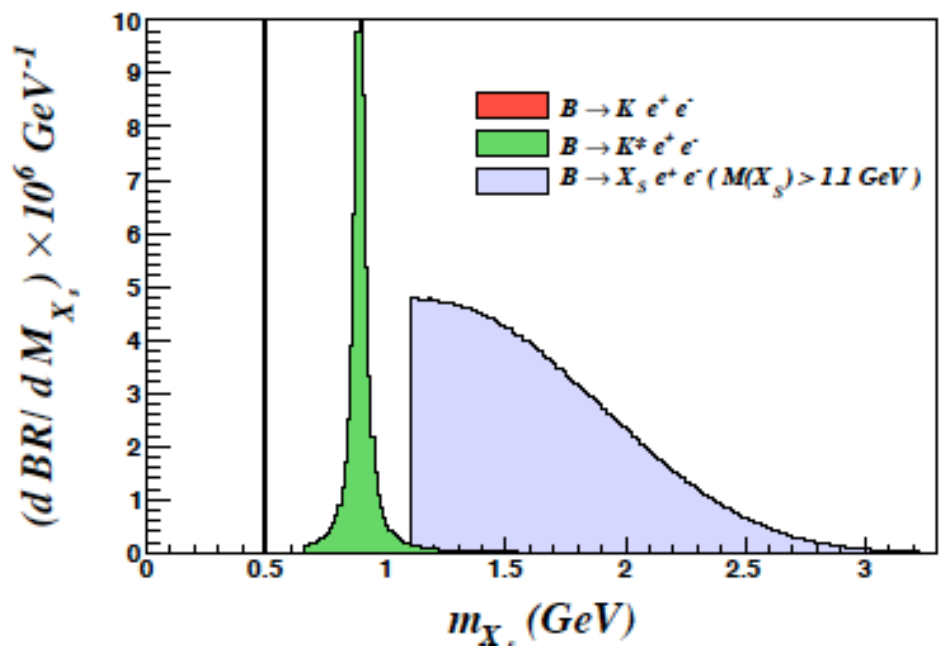
# Monte Carlo analysis

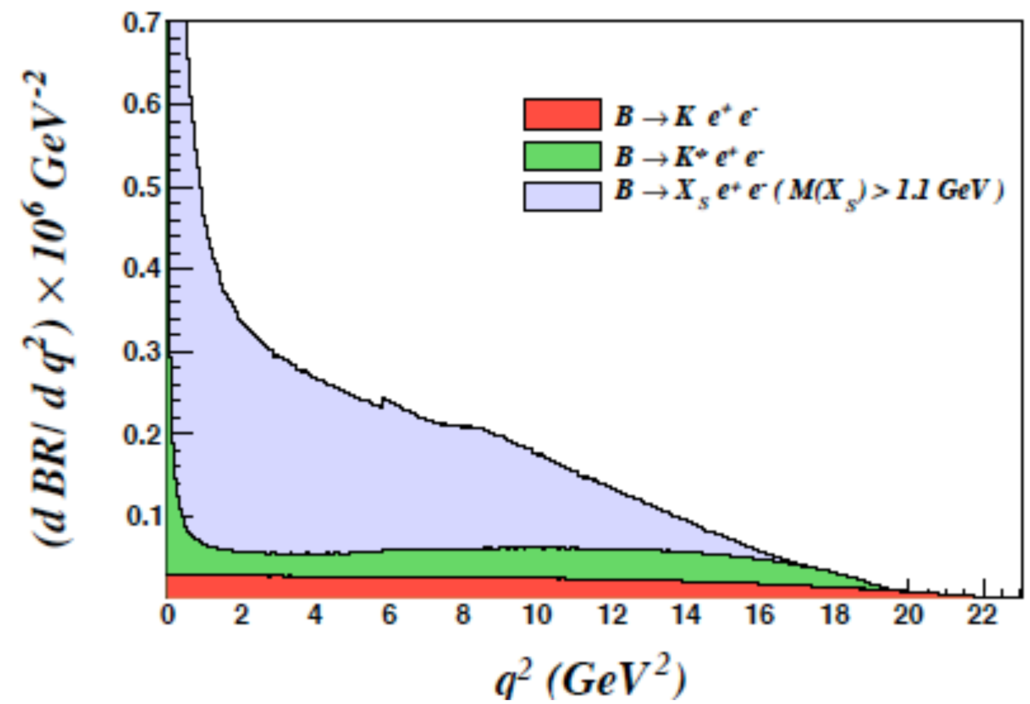
Huber,Hurth,Lunghi, arXiv:1503.04849

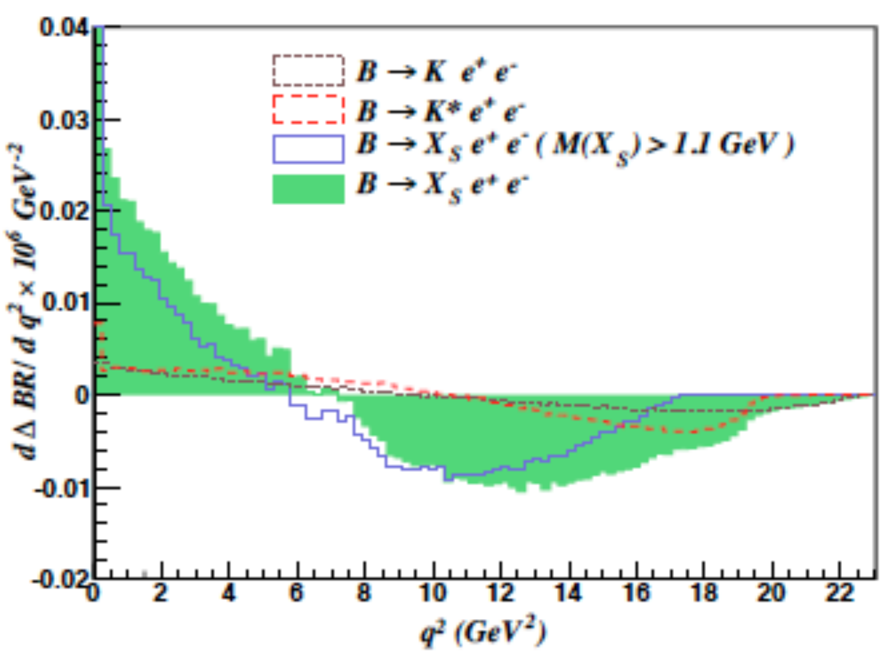
(event generator EVTGEN, hadronization JETSET, EM radiation PHOTOS)

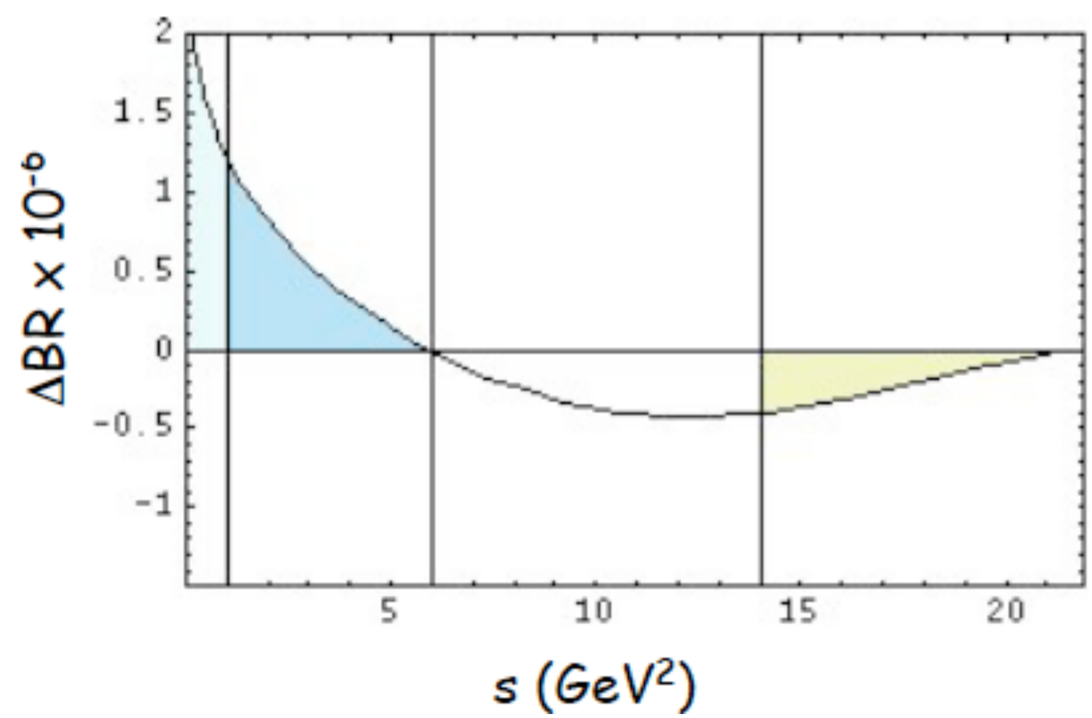
$$\frac{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 1.65\%$$

$$\frac{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 6.8\%$$



Huber,Hurth,Lunghi, arXiv:1503.04849

## Further results in units of $10^{-6}$

$$H_L[1,3.5]_{ee} = 0.64 \pm 0.03 \qquad H_L[1,3.5]_{\mu\mu} = 0.68 \pm 0.04$$

$$H_L[3.5,6]_{ee} = 0.50 \pm 0.03 \qquad H_L[3.5,6]_{\mu\mu} = 0.53 \pm 0.03$$

$$H_L[1,6]_{ee} = 1.13 \pm 0.06 \qquad H_L[1,6]_{\mu\mu} = 1.21 \pm 0.07$$

$$H_T[1,3.5]_{ee} = 0.29 \pm 0.02 \qquad H_T[1,3.5]_{\mu\mu} = 0.21 \pm 0.01$$

$$H_T[3.5,6]_{ee} = 0.24 \pm 0.02 \qquad H_T[3.5,6]_{\mu\mu} = 0.19 \pm 0.02$$

$$H_T[1,6]_{ee} = 0.53 \pm 0.04 \qquad H_T[1,6]_{\mu\mu} = 0.40 \pm 0.03$$

$$H_A[1,3.5]_{ee} = -0.103 \pm 0.005 \qquad H_A[1,3.5]_{\mu\mu} = -0.110 \pm 0.005$$

$$H_A[3.5,6]_{ee} = +0.073 \pm 0.012 \qquad H_A[3.5,6]_{\mu\mu} = +0.067 \pm 0.012$$

$$H_A[1,6]_{ee} = -0.029 \pm 0.016 \qquad H_A[1,6]_{\mu\mu} = -0.042 \pm 0.016$$

Total error $\mathcal{O}(5-8\%)$. Still dominated by scale uncertainty.