Right-handed currents in $B \to K^* \ell^+ \ell^-$

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with Rahul Sinha, Thomas E. Browder,
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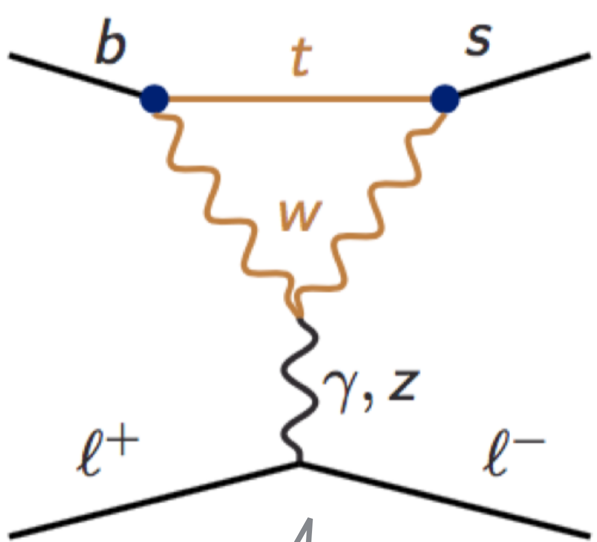
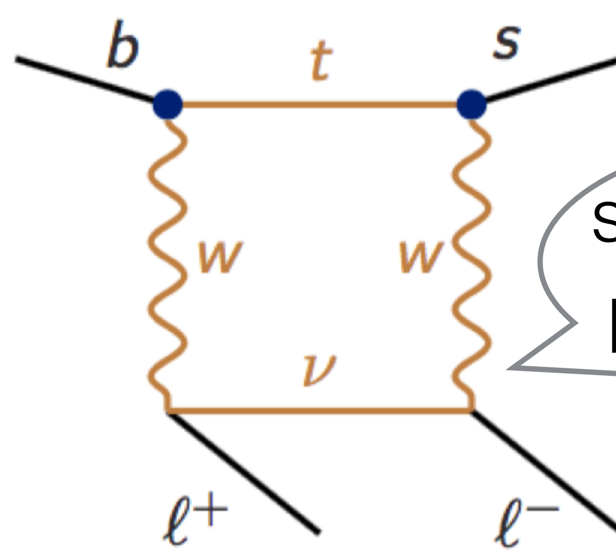
Outline

- Introduction
- Model Independent Framework
- Evidence of New Physics
- Summary

Introduction





loop and CKM suppressed SM amplitude

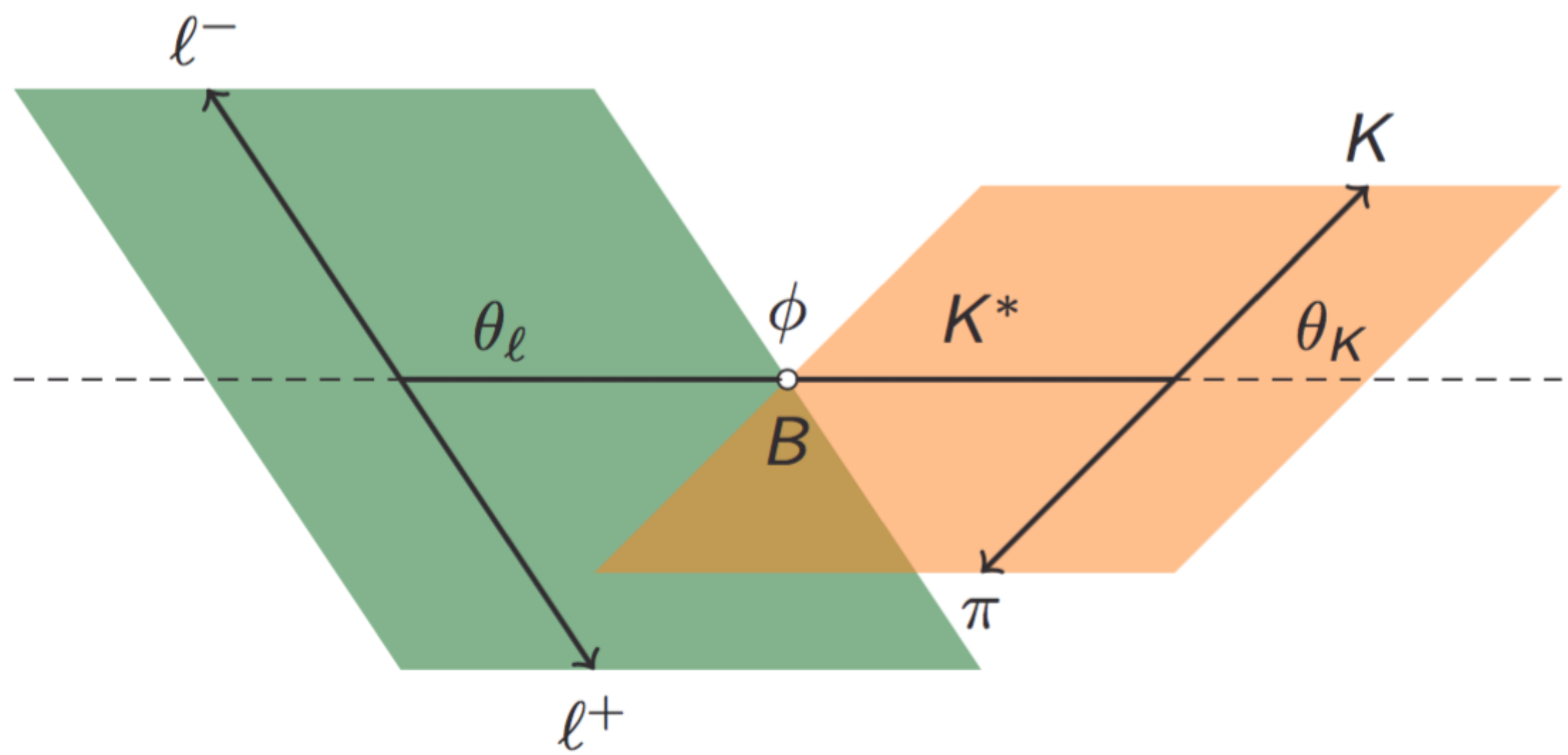
sensitive to new particle in loop

large no. of experimentally accessible observables

valuable probe for indirect search of NP

Introduction

Angular analysis in well known helicity frame [Kruger, Sehgal, Sinha, Sinha ‘99]



The differential distribution $\dfrac{d^4\Gamma(B \to K^*\ell^+\ell^-)}{dq^2\, d\cos\theta_l\, d\cos\theta_k\, d\phi}$

$$= \frac{9}{32\pi}\Big[I_1^s \sin^2\theta_K + I_1^c \cos^2\theta_K + (I_2^s \sin^2\theta_K + I_2^c \cos^2\theta_K)\cos 2\theta_l + I_3 \sin^2\theta_K \sin^2\theta_l \cos 2\phi$$
$$+ I_4 \sin 2\theta_K \sin 2\theta_l \cos\phi + I_5 \sin 2\theta_K \sin\theta_l \cos\phi + I_6^s \sin^2\theta_K \cos\theta_l$$
$$+ I_7 \sin 2\theta_K \sin\theta_l \sin\phi + I_8 \sin 2\theta_K \sin 2\theta_l \sin\phi + I_9 \sin^2\theta_K \sin^2\theta_l \sin 2\phi\Big]$$

Motivation

- $I_i =$ short distance + long distance

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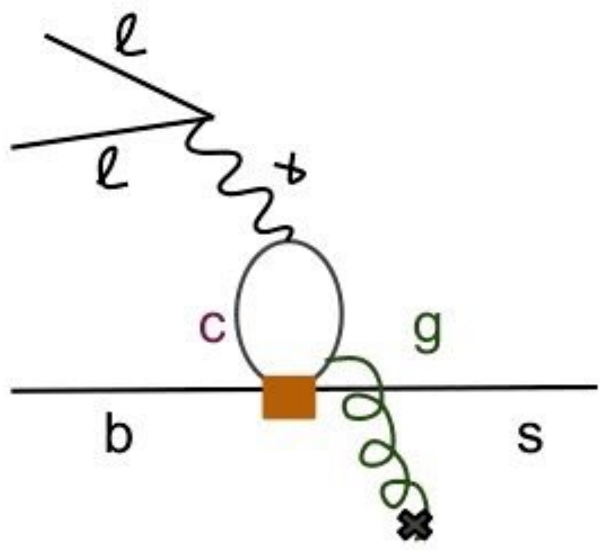
Non-factorizable
contributions:



no quantitative computation

- Challenge: either estimate accurately or *eliminate*

Model Independent Framework

- The amplitude $\mathcal{A}\left(B(p) \to K^*(k)\ell^+\ell^-\right)$ [RM, Sinha, Das '14]

$$= \frac{G_F\alpha}{\sqrt{2}\pi} V_{tb}V_{ts}^* \Bigg[\Bigg\{ C_9 \langle K^*|\bar{s}\gamma^\mu P_L b|\bar{B}\rangle - \frac{2C_7}{q^2} \langle K^*|\bar{s}i\sigma^{\mu\nu}q_\nu(m_b P_R + m_s P_L)b|\bar{B}\rangle$$

$$- \frac{16\pi^2}{q^2} \sum_{i=\{1-6,8\}} C_i \mathcal{H}_i^\mu \Bigg\} \bar{\ell}\gamma_\mu\ell + C_{10}\langle K^*|\bar{s}\gamma^\mu P_L b|\bar{B}\rangle \bar{\ell}\gamma_\mu\gamma_5\ell \Bigg]$$

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Wilson coefficients

non-local operator
for non factorization contributions

lorentz & gauge invariance allow general parametrization with form-factors $\mathcal{X}_j$, $\mathcal{Y}_j$

$$\mathcal{H}_i^\mu \sim \left\langle K^*|i\int d^4x\, e^{iq\cdot x} T\{j^\mu_{em}(x), \mathcal{O}_i(0)\}|\bar{B}\right\rangle \Rightarrow$$ parametrize with 'new' form-factors $\mathcal{Z}_j^i$

[Khodjamirian *et. al* '10]

Model Independent Framework

- Absorbing factorizable & non-factorizable contributions into

$$C_9 \longrightarrow \widetilde{C}_9^{(j)} = C_9 + \Delta C_9^{(\text{fac})}(q^2) + \underbrace{\Delta C_9^{(j),(\text{non-fac})}(q^2)}_{\sim \sum_i C_i \, \mathcal{Z}_j^i / \mathcal{X}_j}$$

$$\frac{2(m_b+m_s)}{q^2} C_7 \, \mathcal{Y}_j \longrightarrow \widetilde{\mathcal{Y}}_j = \frac{2(m_b+m_s)}{q^2} C_7 \, \mathcal{Y}_j + \cdots$$

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- Most general parametric form of amplitude in SM

$$\mathcal{A}_\lambda^{L,R} = (\widetilde{C}_9^\lambda \mp C_{10})\mathcal{F}_\lambda - \widetilde{\mathcal{G}}_\lambda \qquad \mathcal{A}_t\Big|_{m_\ell=0} = 0$$

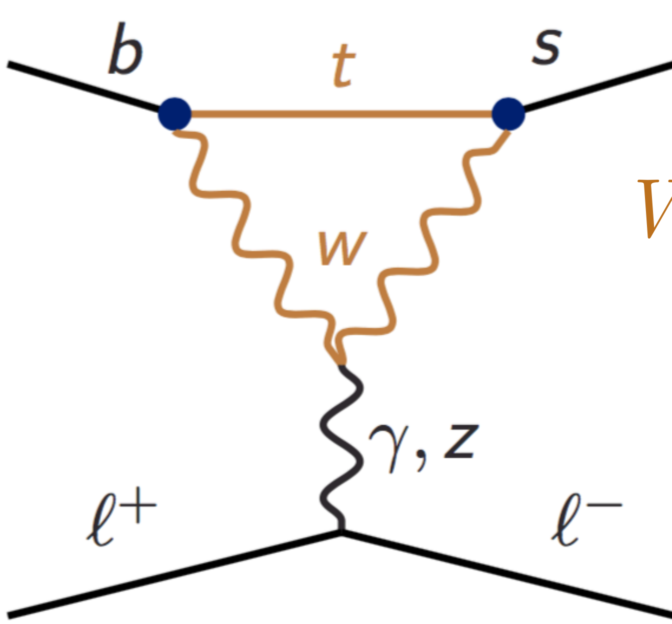
Form-factors: $\mathcal{F}_\lambda \equiv \mathcal{F}_\lambda(\mathcal{X}_j)$ and $\widetilde{\mathcal{G}}_\lambda \equiv \widetilde{\mathcal{G}}_\lambda(\widetilde{\mathcal{Y}}_j)$

Right-Handed Current

- Chirality flipped operators $\mathcal{O} \Leftrightarrow \mathcal{O}'$

$$\bar{s}\gamma_\mu P_L b \quad \Longleftrightarrow \quad \bar{s}\gamma_\mu P_R b$$

$$\bar{s}i\sigma_{\mu\nu} P_R b \quad \Longleftrightarrow \quad \bar{s}i\sigma_{\mu\nu} P_L b$$



$W_L \Leftrightarrow W_R$

- In presence of right-handed gauge boson or other kind of new particles like leptoquarks etc..

RH Current

- Amplitudes $\mathcal{A}_\perp^{L,R} = ((\widetilde{C}_9^\perp + C_9') \mp (C_{10} + C_{10}'))\mathcal{F}_\perp - \widetilde{\mathcal{G}}_\perp$

$$\mathcal{A}_{\parallel,0}^{L,R} = ((\widetilde{C}_9^{\parallel,0} - C_9') \mp (C_{10} - C_{10}'))\mathcal{F}_{\parallel,0} - \widetilde{\mathcal{G}}_{\parallel,0}$$

- Notation $r_\lambda = \dfrac{\mathrm{Re}(\widetilde{\mathcal{G}}_\lambda)}{\mathcal{F}_\lambda} - \mathrm{Re}(\widetilde{C}_9^\lambda) \qquad \xi = \dfrac{C_{10}'}{C_{10}} \qquad \xi' = \dfrac{C_9'}{C_{10}}$

- Variables $R_\perp = \dfrac{\frac{r_\perp}{C_{10}} - \xi'}{1+\xi}, \; R_\parallel = \dfrac{\frac{r_\parallel}{C_{10}} + \xi'}{1-\xi}, \; R_0 = \dfrac{\frac{r_0}{C_{10}} + \xi'}{1-\xi}.$

- HQET limit $\dfrac{\widetilde{\mathcal{G}}_\parallel}{\mathcal{F}_\parallel} = \dfrac{\widetilde{\mathcal{G}}_\perp}{\mathcal{F}_\perp} = \dfrac{\widetilde{\mathcal{G}}_0}{\mathcal{F}_0} = -\kappa \dfrac{2 m_b m_B C_7}{q^2},$ [Grinstein, Prijol ‘04] [Bobeth *et. al* ’10]

⟹ $r_0 = r_\parallel = r_\perp \equiv r$ ignoring non-factorisable corrections

⟹ $R_0 = R_\parallel \neq R_\perp$ *in presence of RH currents*

RH Current

At kinematic endpoint → ☑ exact HQET limit
☑ polarization independent non-factorisable correction

- Observables $F_L(q^2_{\rm max}) = \frac{1}{3}, F_{\|}(q^2_{\rm max}) = \frac{2}{3}, A_4(q^2_{\rm max}) = \frac{2}{3\pi},$

$$F_\perp(q^2_{\rm max}) = 0,\ \ A_{\rm FB}(q^2_{\rm max}) = 0,\ \ A_{5,7,8,9}(q^2_{\rm max}) = 0.$$

[Hiller, Zwicky '14]

- Taylor series expansion around $\delta \equiv q^2_{\rm max} - q^2$

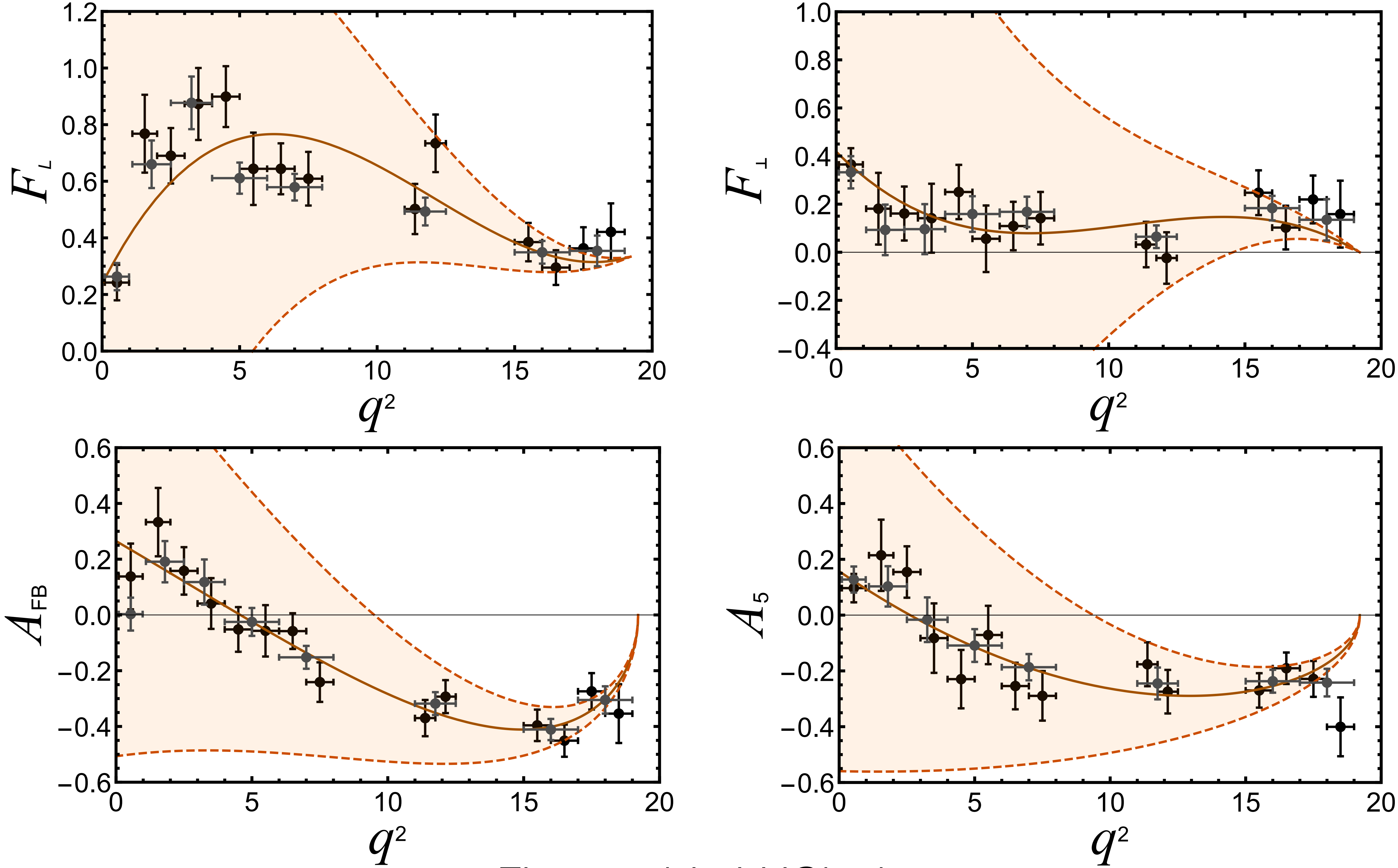
$$F_L = \frac{1}{3} + F_L^{(1)}\delta + F_L^{(2)}\delta^2 + F_L^{(3)}\delta^3$$

$$F_\perp = F_\perp^{(1)}\delta + F_\perp^{(2)}\delta^2 + F_\perp^{(3)}\delta^3$$

$$A_{\rm FB} = A_{\rm FB}^{(1)}\delta^{\frac{1}{2}} + A_{\rm FB}^{(2)}\delta^{\frac{3}{2}} + A_{\rm FB}^{(3)}\delta^{\frac{5}{2}}$$

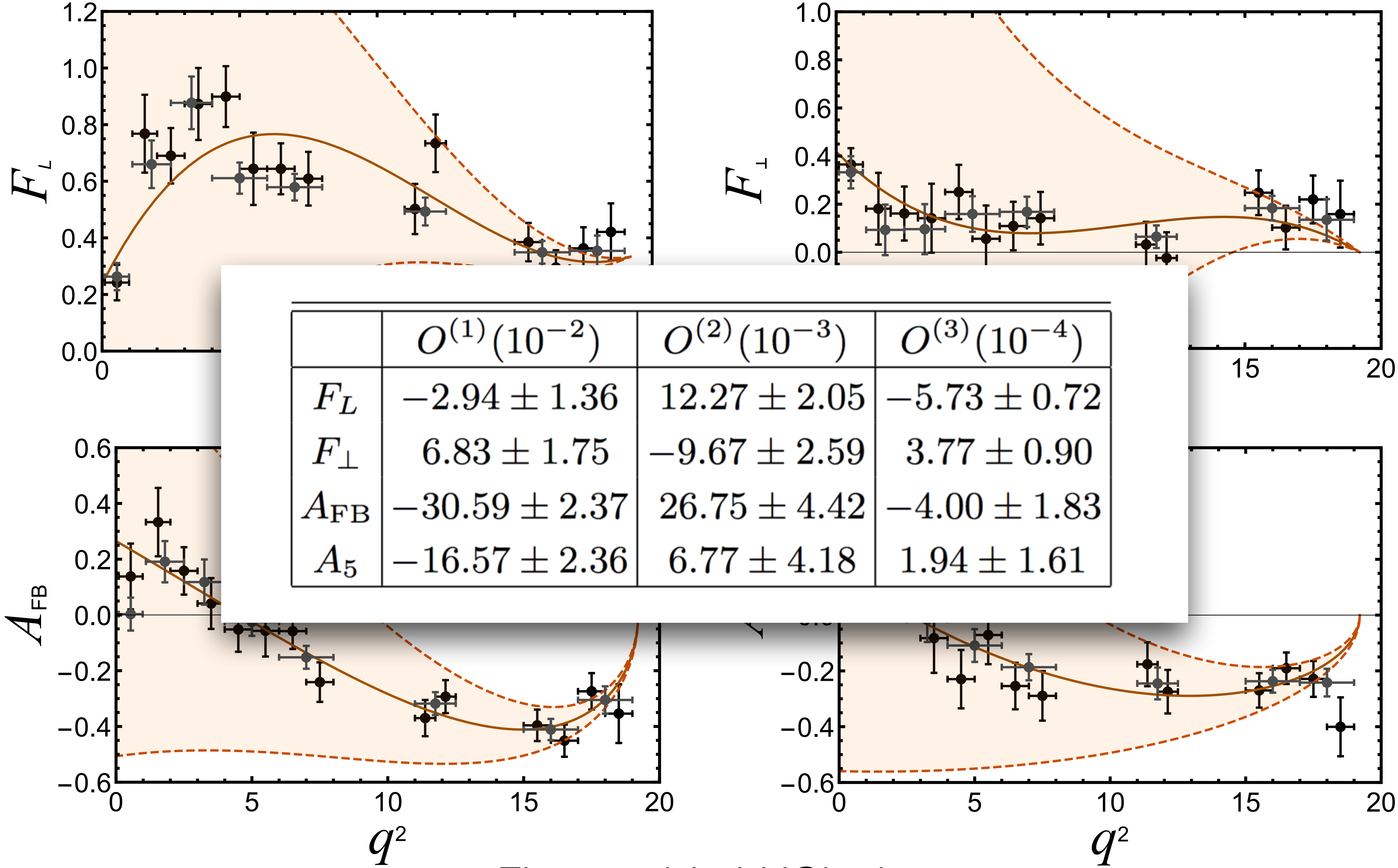
$$A_5 = A_5^{(1)}\delta^{\frac{1}{2}} + A_5^{(2)}\delta^{\frac{3}{2}} + A_5^{(3)}\delta^{\frac{5}{2}},$$

RH Current



Fit to 14 bin LHCb data

RH Current



	$O^{(1)}(10^{-2})$	$O^{(2)}(10^{-3})$	$O^{(3)}(10^{-4})$
F_L	-2.94 ± 1.36	12.27 ± 2.05	-5.73 ± 0.72
$F_\perp$	6.83 ± 1.75	-9.67 ± 2.59	3.77 ± 0.90
$A_{\rm FB}$	-30.59 ± 2.37	26.75 ± 4.42	-4.00 ± 1.83
A_5	-16.57 ± 2.36	6.77 ± 4.18	1.94 ± 1.61

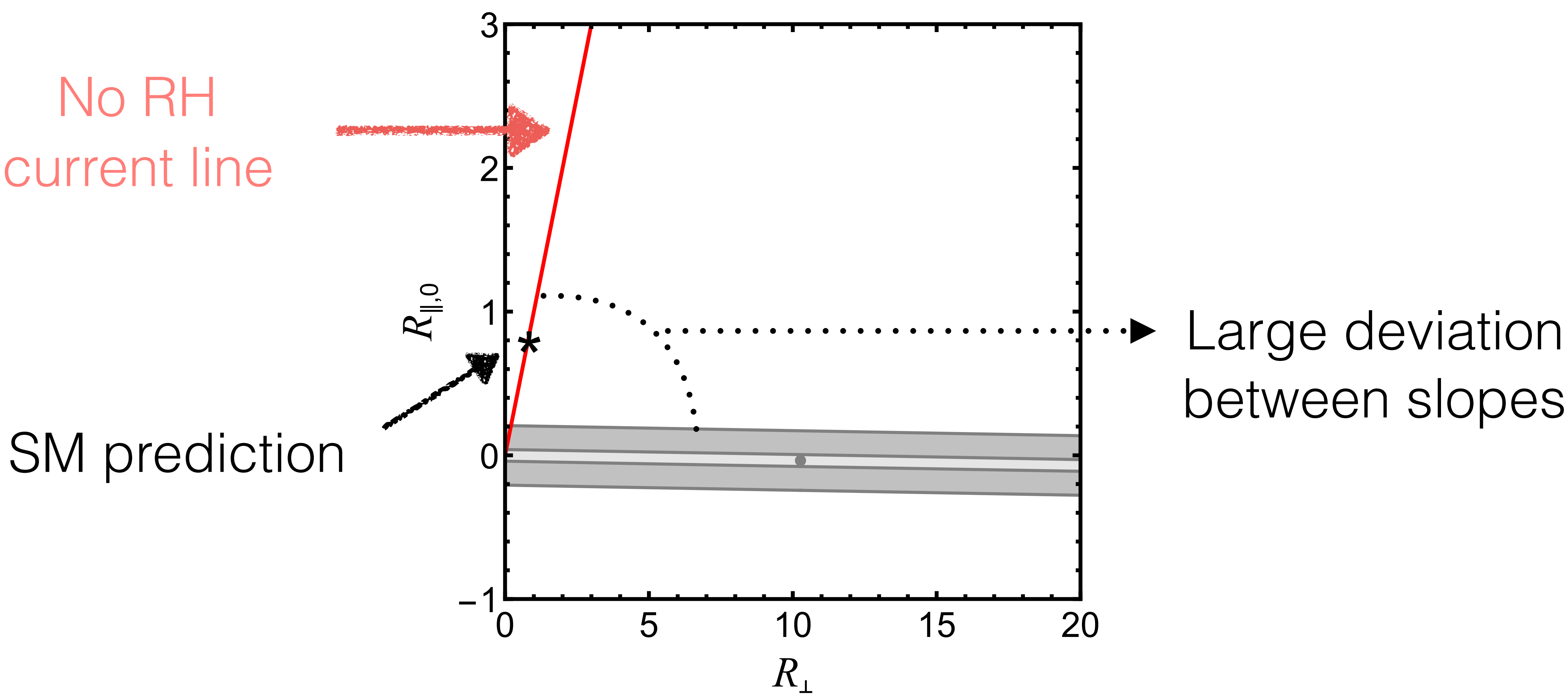
Fit to 14 bin LHCb data

RH Current

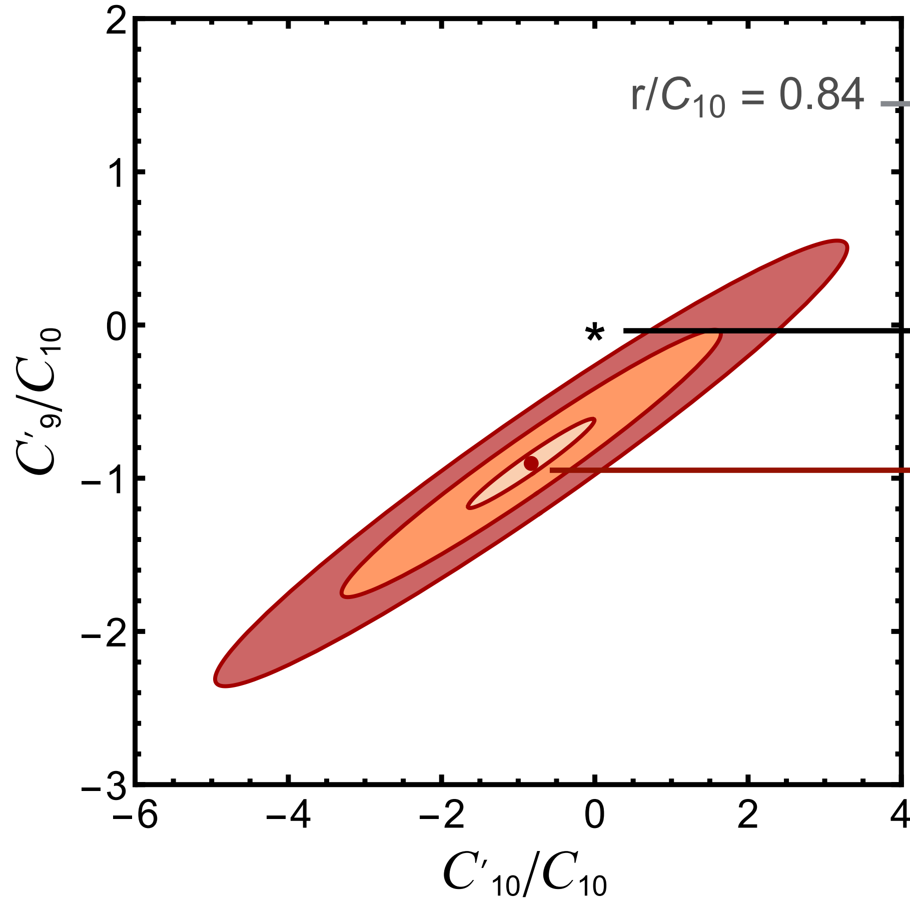
- Limiting analytic expressions

$$R_\perp(q^2_{\rm max}) = \frac{\omega_2 - \omega_1}{\omega_2\sqrt{\omega_1 - 1}}, \quad R_{||}(q^2_{\rm max}) = \frac{\sqrt{\omega_1 - 1}}{\omega_2 - 1} = R_0(q^2_{\rm max})$$

$$\omega_1 = \frac{3}{2}\frac{F_\perp^{(1)}}{A_{\rm FB}^{(1)\,2}} \quad \text{or} \quad \frac{3}{8}\frac{F_\perp^{(1)}}{A_5^{(1)\,2}} \quad \text{and} \quad \omega_2 = \frac{4\left(2A_5^{(2)} - A_{\rm FB}^{(2)}\right)}{3\,A_{\rm FB}^{(1)}\left(3F_L^{(1)} + F_\perp^{(1)}\right)} \quad \text{or} \quad \frac{4\left(2A_5^{(2)} - A_{\rm FB}^{(2)}\right)}{6\,A_5^{(1)}\left(3F_L^{(1)} + F_\perp^{(1)}\right)}$$



Results in $C'_{10}/C_{10} - C'_9/C_{10}$



SM input

More than 5σ deviation

$$C'_{10}/C_{10} = -0.83 \pm 0.82$$

$$C'_9/C_{10} = -0.90 \pm 0.28$$

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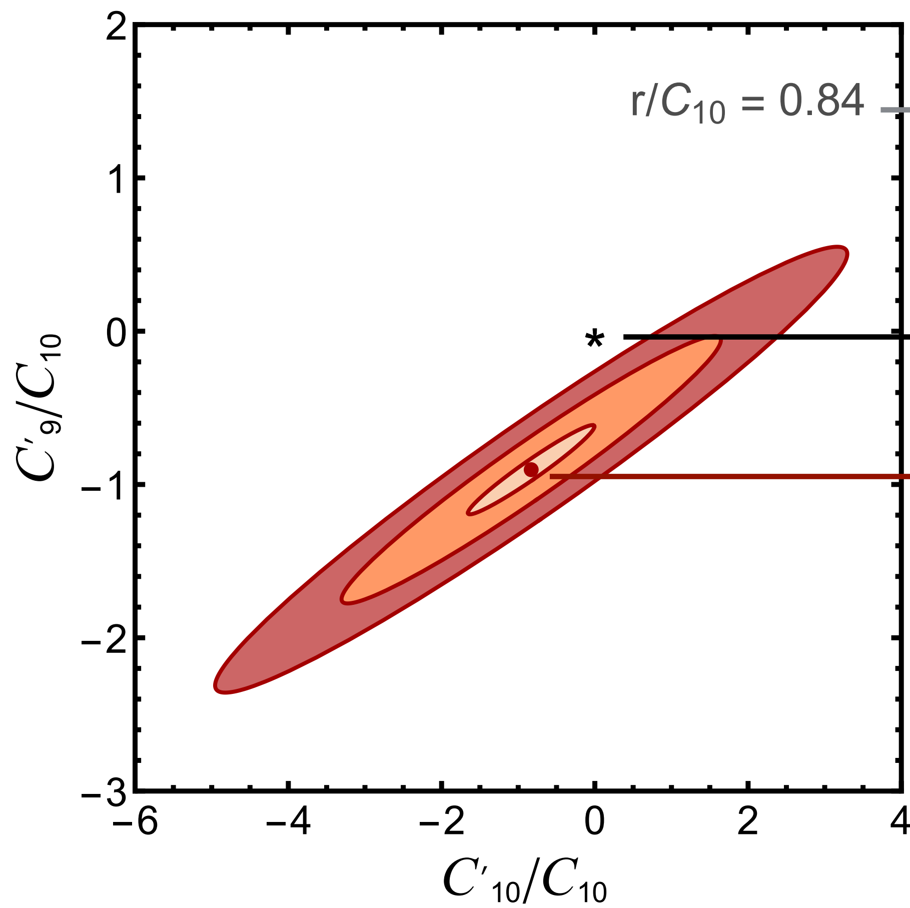

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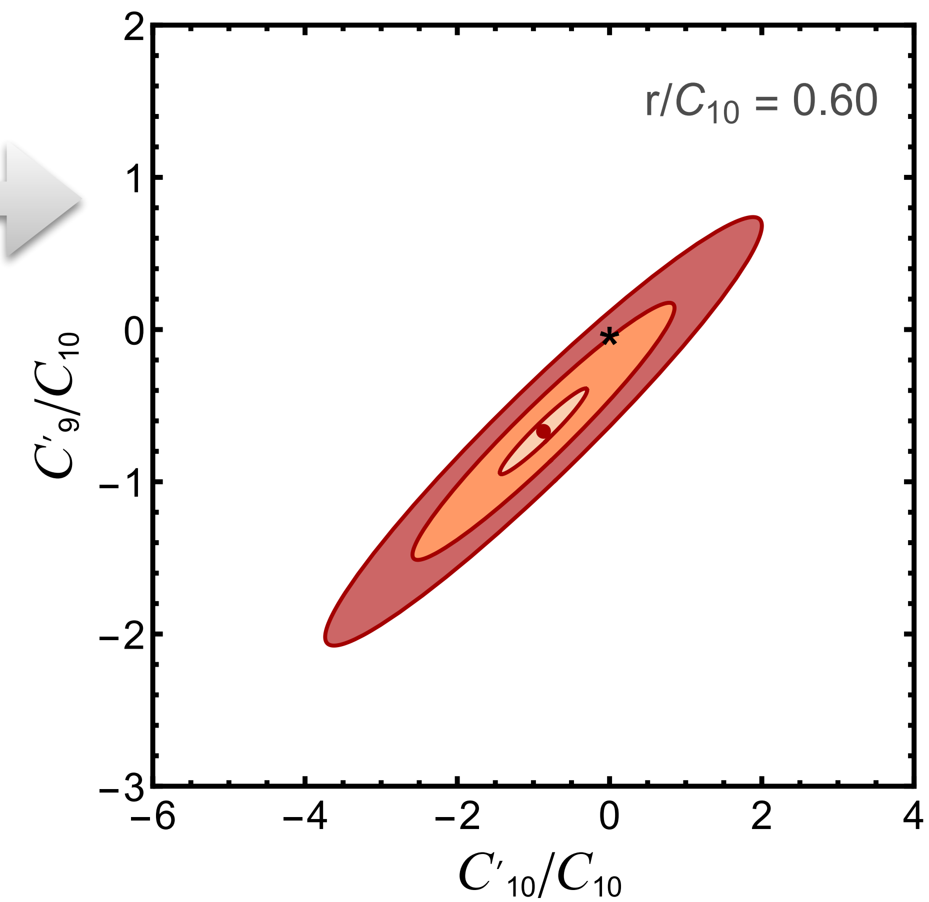
Significance of deviation is smaller for lower r/C_{10} values

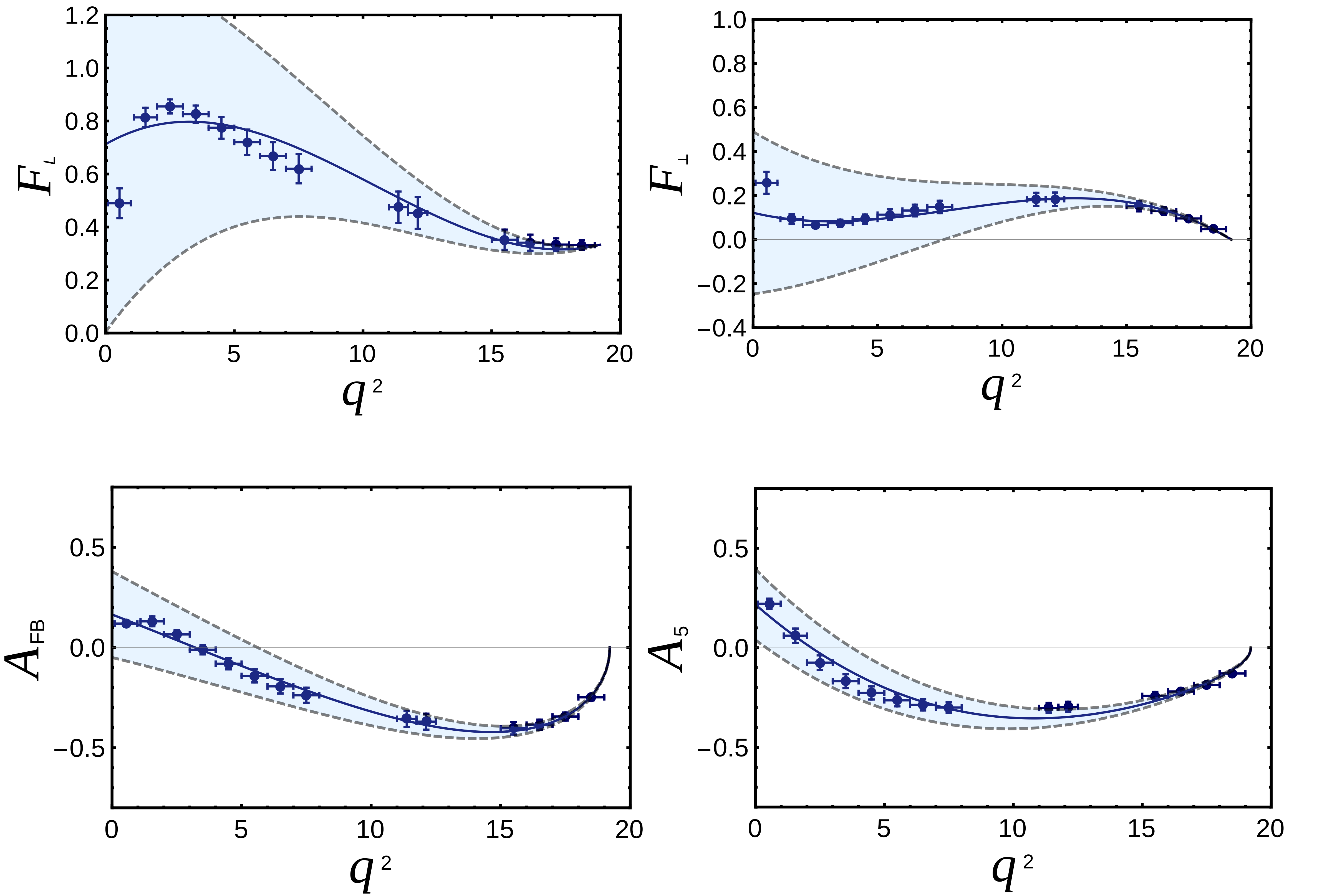


Other kind of NP like Z' as hinted in global fits

[Altmannshofer, Straub '14

& other groups also]

Fit to form factor observables



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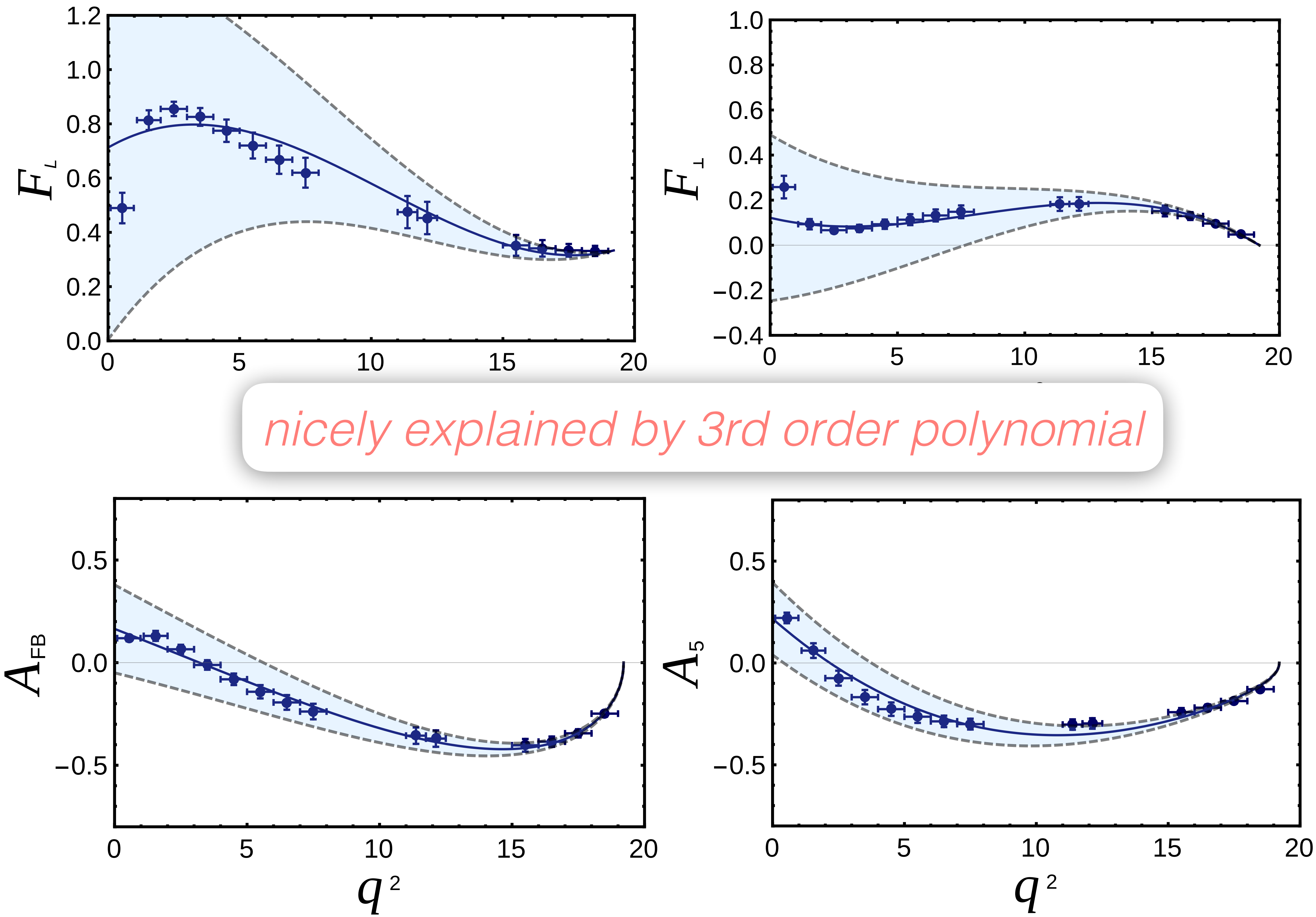
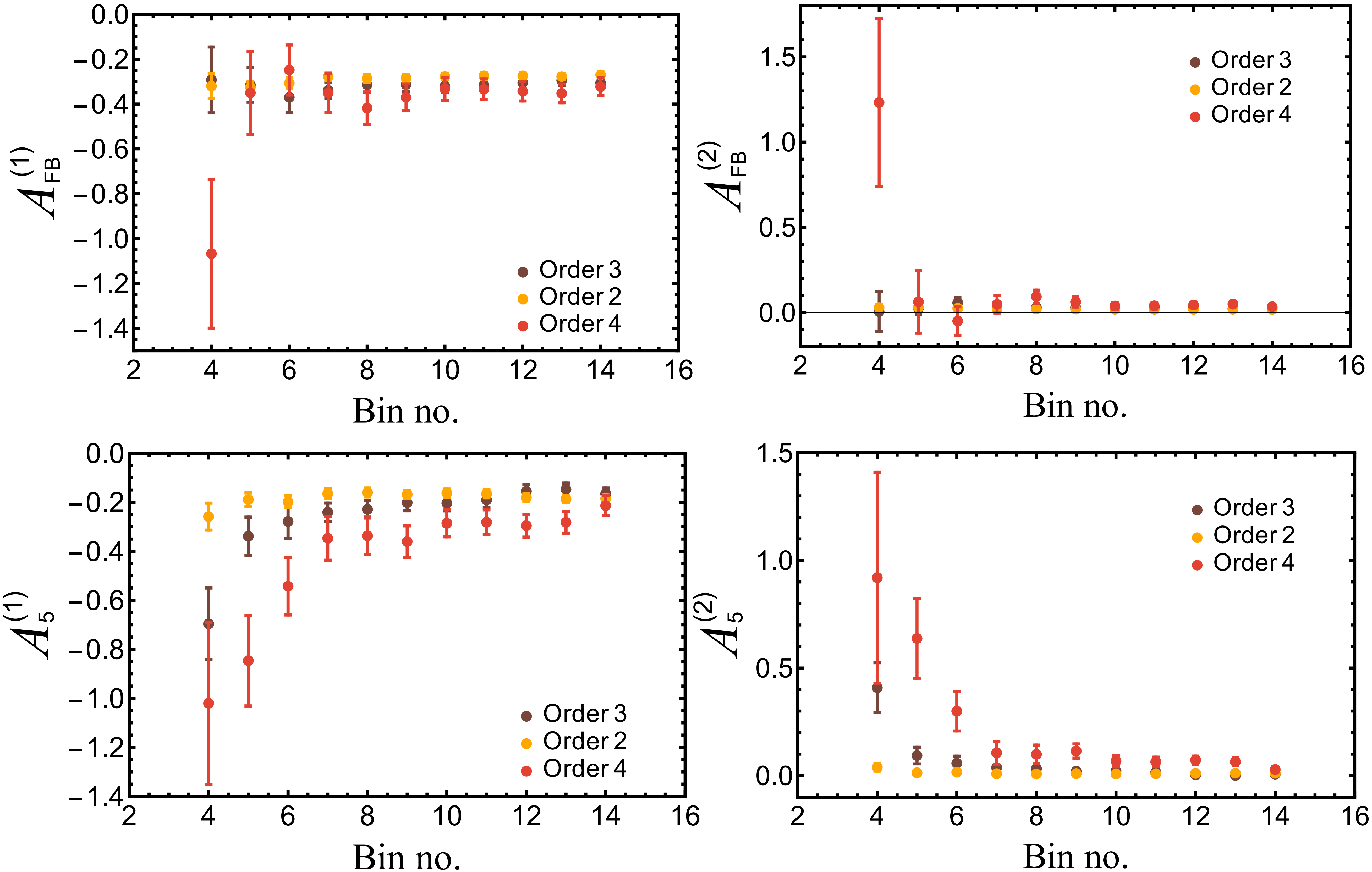


nicely explained by 3rd order polynomial

Convergence of coefficients



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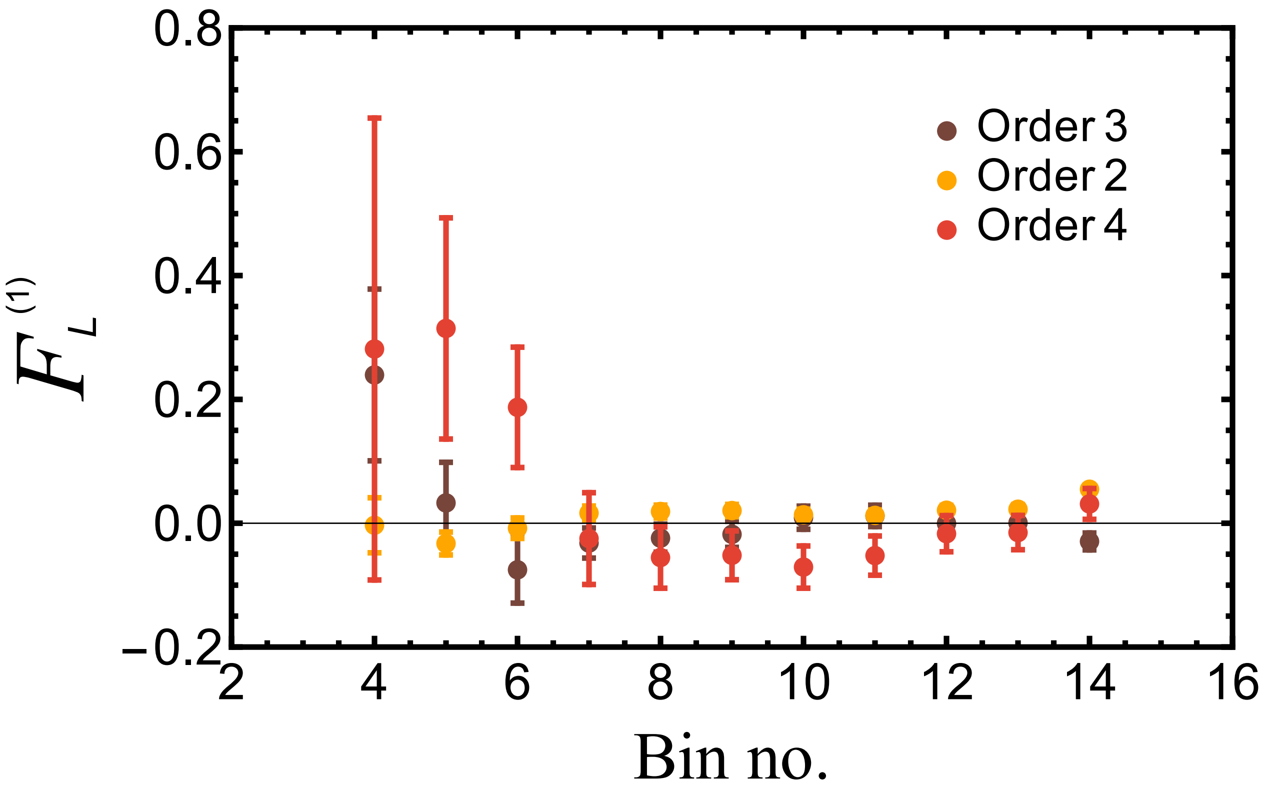
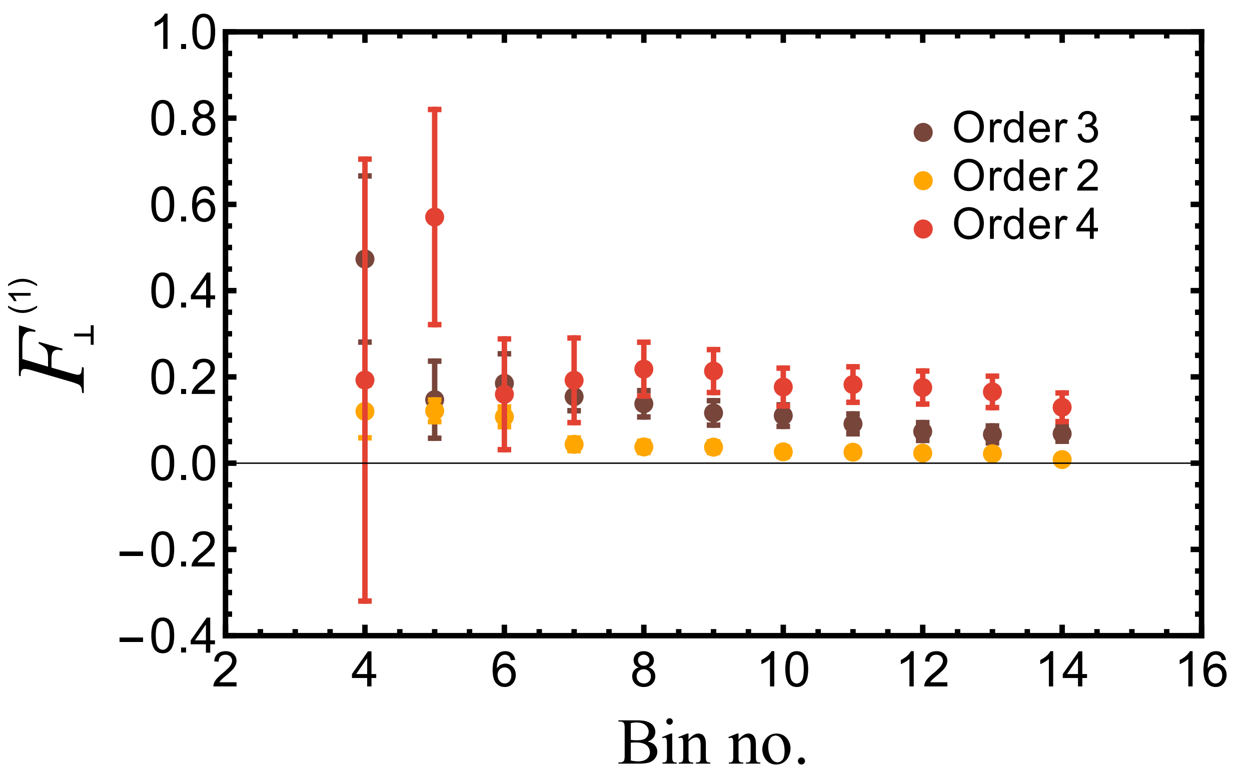


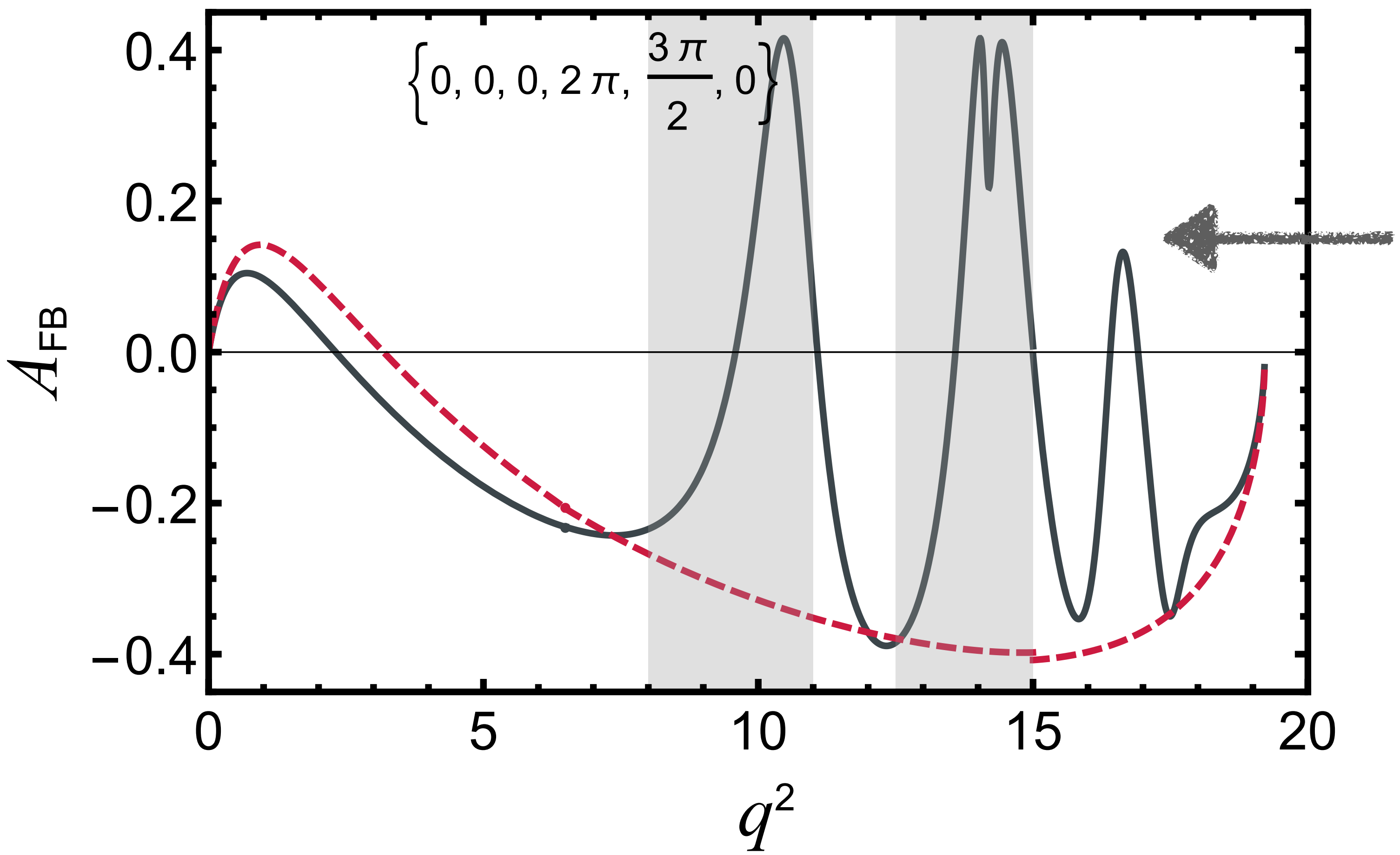


Shows a good convergence with variation in polynomial order & no. of bins used for the data fit

Resonances

$c\bar{c}$ bound states added: J/ψ, $\psi(2S)$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, $\psi(4415)$.

Observable $=$ Form-factors $+$ Kruger & Sehgal parametrization



Asymmetries decrease in high q^2 region

makes observable ω_1 unphysical

Random variation of each strong phases

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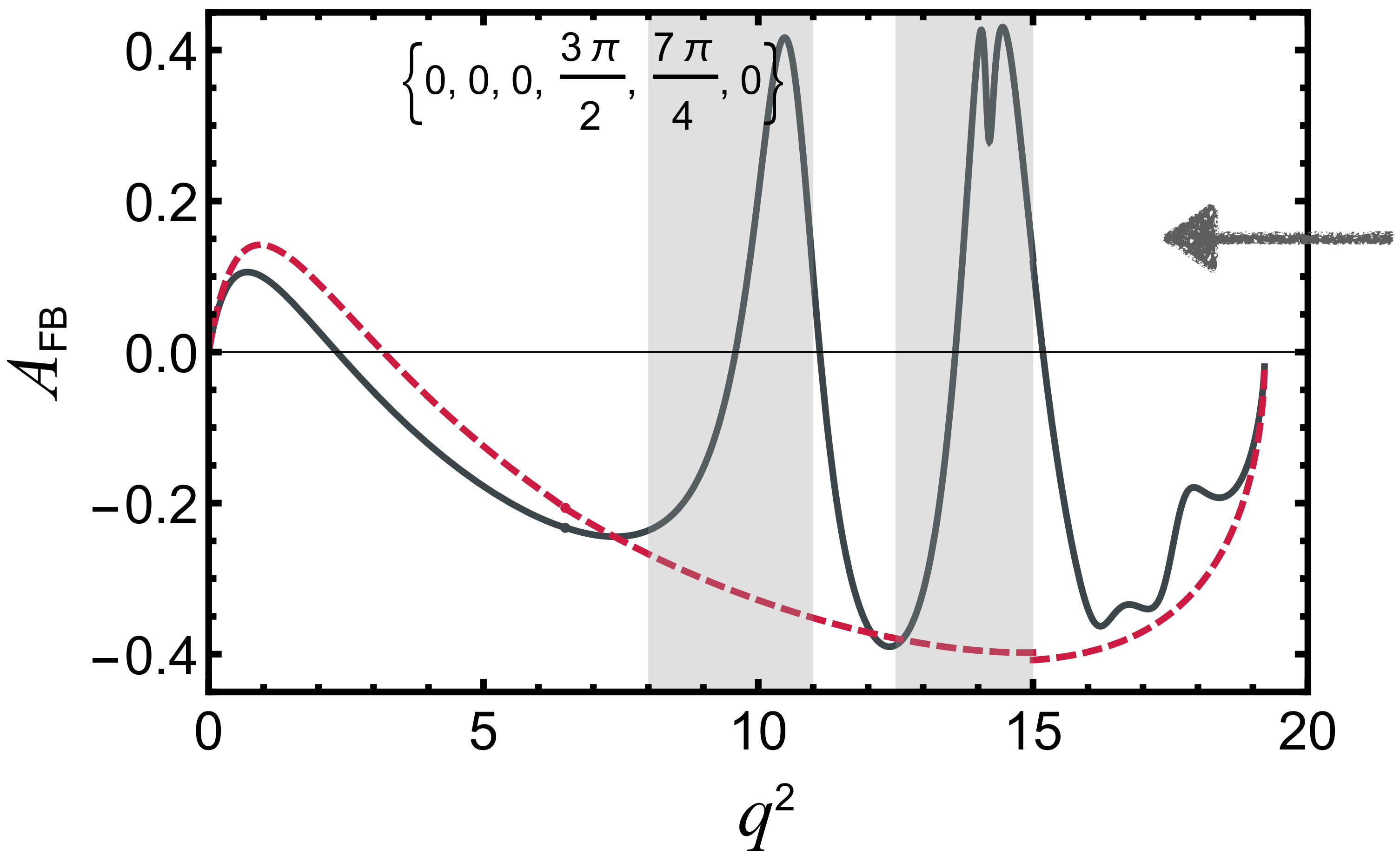

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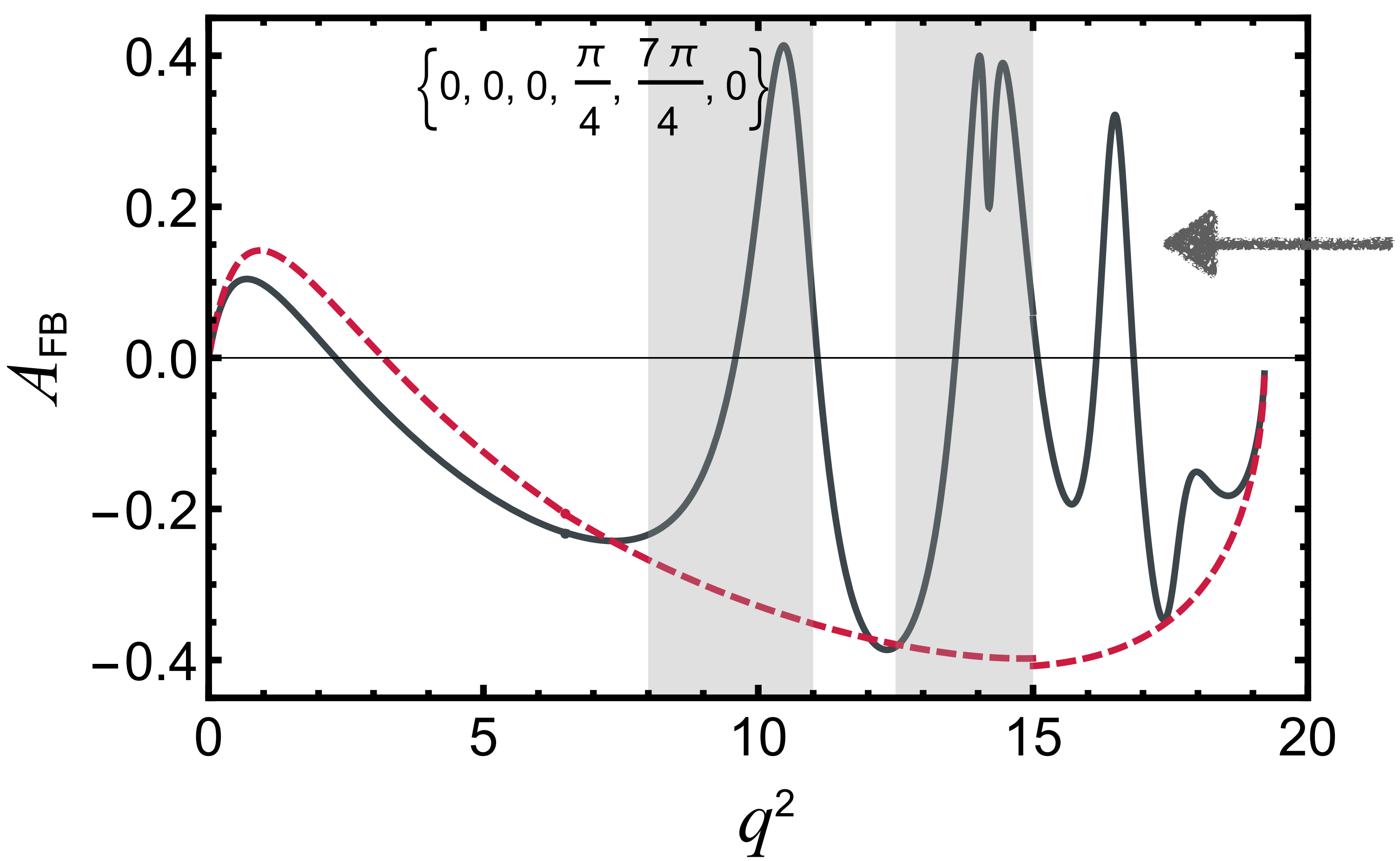

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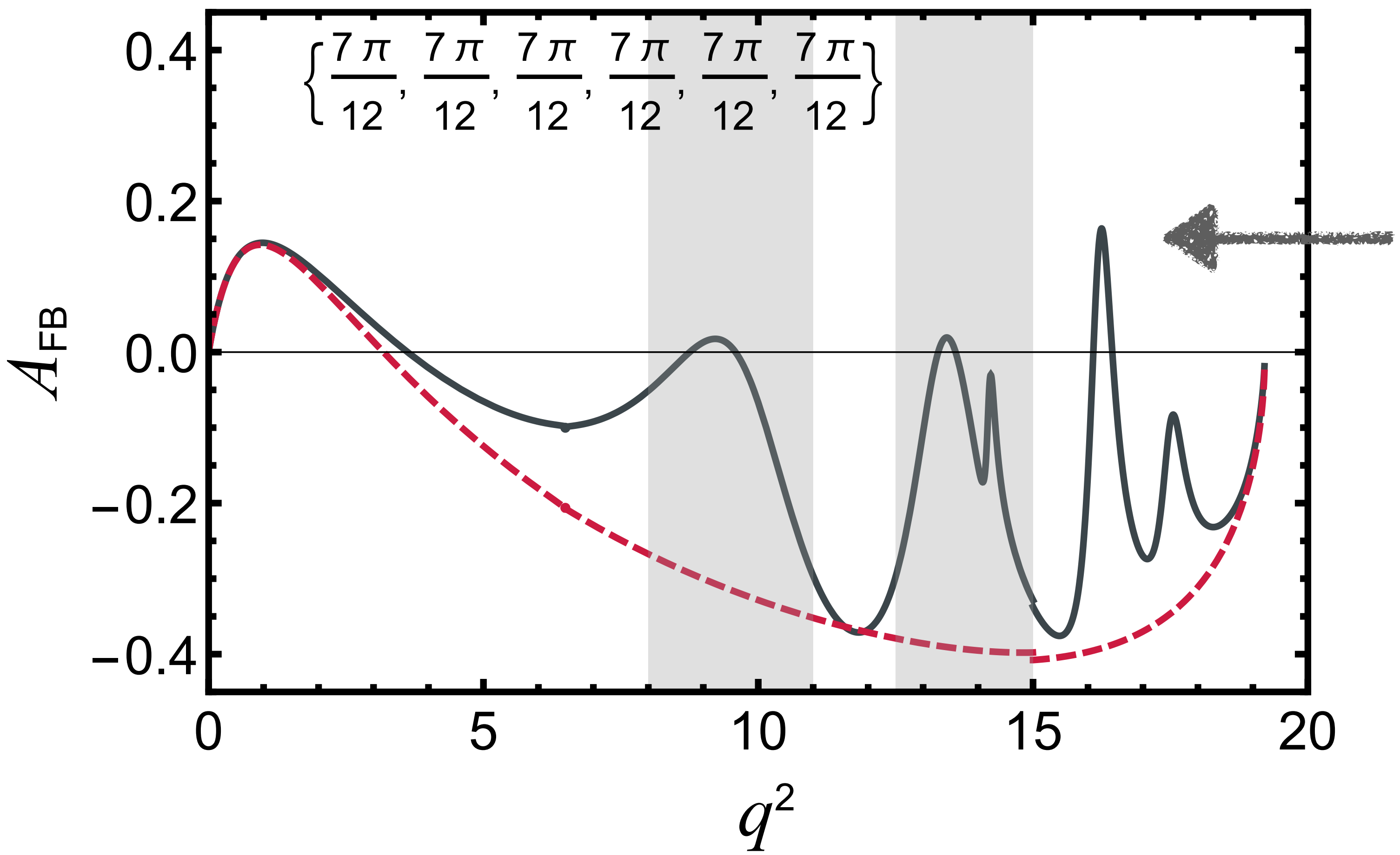

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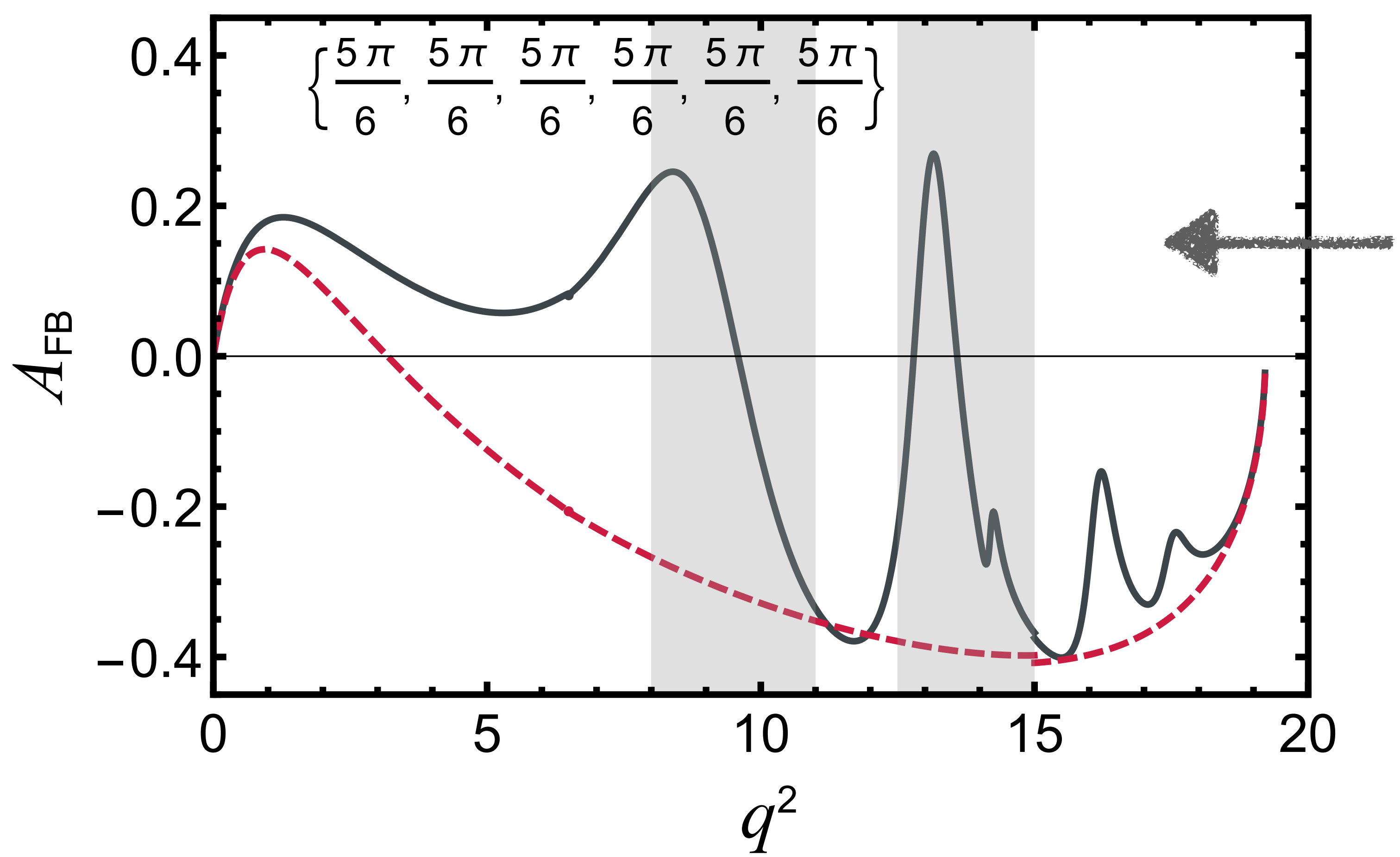

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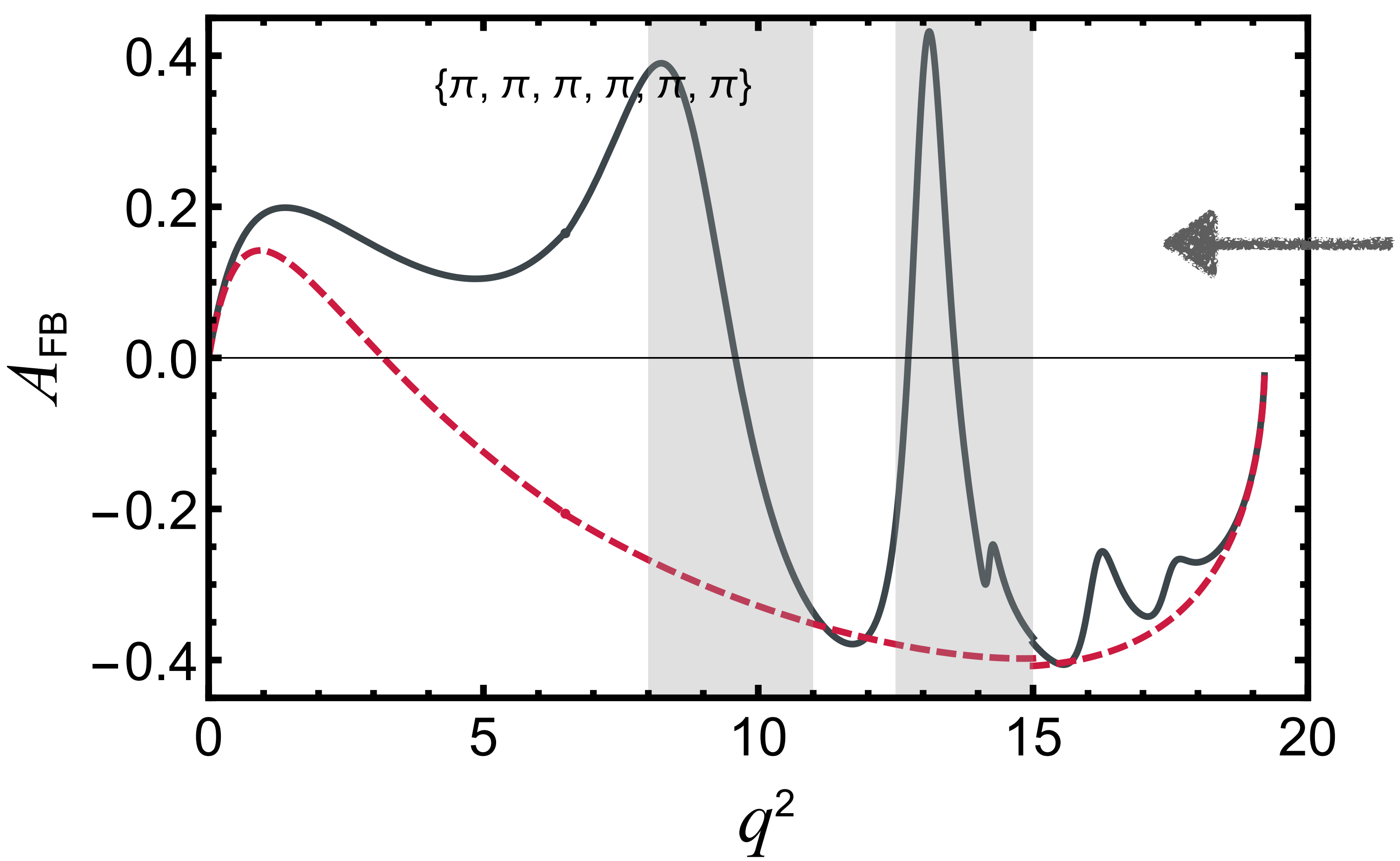

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Random variation of each strong phases

Summary

Popular approaches	Our approach
☑ Combine all $b \to s$ transitions	☑ Most general parametric form of SM amplitude
↓	$+$
many decay modes i.e observables	$B \to K^* \ell^+ \ell^-$ observables
$+$	$+$
more hadronic uncertainties	eliminate hadronic uncertainties
$+$	↓
conservative assumption of non-factorisable contributions	no/minimal dependency on form-factors & independent of non-factorisable contributions
☑ Focusing on low q^2 region	☑ Conclusion derived at endpoint

Summary

- Formalism developed to include all possible effects within SM
- Strong evidence of RH currents derived at endpoint limit —
  - systematics studied by varying polynomial order & bin no.
  - finite K^* width effect considered
  - resonance systematics & experimental correlation can reduce significance of deviation
- Fluctuation? Wait for more data to be accumulated!

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Back Up

Complex part of amplitudes

- SM amplitude $\mathcal{A}_\lambda^{L,R} = (\widetilde{C}_9^\lambda \mp C_{10})\mathcal{F}_\lambda - \widetilde{\mathcal{G}}_\lambda$
- Complex part $\varepsilon_\lambda \equiv \mathrm{Im}(\widetilde{C}_9^\lambda)\mathcal{F}_\lambda - \mathrm{Im}(\widetilde{\mathcal{G}}_\lambda)$
- Iterative solutions

$$\varepsilon_\perp = \frac{\sqrt{2}\pi\Gamma_f}{(r_0 - r_\|)\mathcal{F}_\perp}\left[\frac{A_9 \mathrm{P}_1}{3\sqrt{2}} + \frac{A_8 \mathrm{P}_2}{4} - \frac{A_7 \mathrm{P}_1 \mathrm{P}_2 r_\perp}{3\pi \widehat{C}_{10}}\right],$$

$$\varepsilon_\| = \frac{\sqrt{2}\pi\Gamma_f}{(r_0 - r_\|)\mathcal{F}_\perp}\left[\frac{A_9 r_0}{3\sqrt{2} r_\perp} + \frac{A_8 \mathrm{P}_2 r_\|}{4 \mathrm{P}_1 r_\perp} - \frac{A_7 \mathrm{P}_2 r_\|}{3\pi \widehat{C}_{10}}\right],$$

$$\varepsilon_0 = \frac{\sqrt{2}\pi\Gamma_f}{(r_0 - r_\|)\mathcal{F}_\perp}\left[\frac{A_9 \mathrm{P}_1 r_0}{3\sqrt{2}\mathrm{P}_2 r_\perp} + \frac{A_8 r_\|}{4 r_\perp} - \frac{A_7 \mathrm{P}_1 r_0}{3\pi \widehat{C}_{10}}\right].$$

Complex part of amplitudes

q^2 range in GeV^2	$\varepsilon_\perp/\sqrt{\Gamma_f}$	$\varepsilon_\parallel/\sqrt{\Gamma_f}$	$\varepsilon_0/\sqrt{\Gamma_f}$
$0.1 \leq q^2 \leq 0.98$	-0.048 ± 0.116	-0.047 ± 0.103	0.020 ± 0.111
$1.1 \leq q^2 \leq 2.5$	-0.010 ± 0.078	-0.010 ± 0.078	0.078 ± 0.172
$2.5 \leq q^2 \leq 4.0$	-0.009 ± 0.079	-0.008 ± 0.080	-0.025 ± 0.212
$4.0 \leq q^2 \leq 6.0$	-0.026 ± 0.097	0.014 ± 0.093	0.032 ± 0.234
$6.0 \leq q^2 \leq 8.0$	-0.011 ± 0.088	-0.046 ± 0.078	-0.132 ± 0.129
$11.0 \leq q^2 \leq 12.5$	-0.011 ± 0.050	0.038 ± 0.074	-0.078 ± 0.114
$15.0 \leq q^2 \leq 17.0$	-0.0003 ± 0.067	-0.027 ± 0.071	0.020 ± 0.072
$17.0 \leq q^2 \leq 19.0$	0.006 ± 0.076	-0.090 ± 0.090	-0.040 ± 0.088

$\frac{\varepsilon_\lambda}{\sqrt{\Gamma_f}}$ values with errors are consistent with zero

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RH Current

- Limiting analytic expressions

$$R_\perp(q^2_{\rm max}) = \frac{\omega_2 - \omega_1}{\omega_2\sqrt{\omega_1 - 1}}, \quad R_\parallel(q^2_{\rm max}) = \frac{\sqrt{\omega_1 - 1}}{\omega_2 - 1} = R_0(q^2_{\rm max})$$

$$\omega_1 = \frac{3}{2}\frac{F_\perp^{(1)}}{A_{\rm FB}^{(1)\,2}} \quad \text{or} \quad \frac{3}{8}\frac{F_\perp^{(1)}}{A_5^{(1)\,2}} \quad \text{and} \quad \omega_2 = \frac{4\left(2A_5^{(2)} - A_{\rm FB}^{(2)}\right)}{3\,A_{\rm FB}^{(1)}\left(3F_L^{(1)} + F_\perp^{(1)}\right)} \quad \text{or} \quad \frac{4\left(2A_5^{(2)} - A_{\rm FB}^{(2)}\right)}{6\,A_5^{(1)}\left(3F_L^{(1)} + F_\perp^{(1)}\right)}$$

	Real limit	Complex limit	Adding finite K^* width
ω_1	1.09 ± 0.33 0.93 ± 0.36	0.98 ± 0.33 0.85 ± 0.30	1.18 ± 0.35 1.02 ± 0.40
ω_2	-2.87 ± 6.69 -2.65 ± 6.18	-2.85 ± 12.54 -2.59 ± 6.22	-2.48 ± 5.95 -2.30 ± 5.51

Resonances

Parametrization in Wilson coefficient C_9 [Kruger, Sehgal '96]

$$g(m_c, q^2) = -\frac{8}{9}\ln\frac{m_c}{m_b} - \frac{4}{9} + \frac{q^2}{3}\, P\int_{4\hat{m}_D^2}^{m_b^2} \frac{R_{\rm had}^{c\bar{c}}(x)}{x(x-q^2)}dx + i\frac{\pi}{3}R_{\rm had}^{c\bar{c}}(q^2)$$

$$R_{\rm had}^{c\bar{c}}(q^2) = R_{\rm cont}^{c\bar{c}}(q^2) + \sum_{V=J/\psi,\psi'\ldots} \frac{9q^2}{\alpha}\frac{{\rm Br}(V\to l^+l^-)\Gamma_{\rm total}^V\Gamma_{\rm had}^V}{(q^2-m_V^2)^2+m_V^2{\Gamma_{\rm total}^V}^2}e^{i\delta_V}$$

Solutions

$$R_\perp = \pm\frac{3}{2}\frac{\left(\frac{1-\xi}{1+\xi}\right)F_\perp + \frac{1}{2}\mathsf{P}_1 Z_1}{\mathsf{P}_1 A_{\rm FB}}$$

$$R_\parallel = \pm\frac{3}{2}\frac{\left(\frac{1+\xi}{1-\xi}\right)\mathsf{P}_1 F_\parallel + \frac{1}{2}Z_1}{A_{\rm FB}}$$

$$R_0 = \pm\frac{3}{2\sqrt{2}}\frac{\left(\frac{1+\xi}{1-\xi}\right)\mathsf{P}_2 F_L + \frac{1}{2}Z_2}{A_5}$$

$$\mathsf{P}_2 = \frac{\left(\frac{1-\xi}{1+\xi}\right)2\mathsf{P}_1 A_{\rm FB} F_\perp}{\sqrt{2}A_5\left(\left(\frac{1-\xi}{1+\xi}\right)2F_\perp + Z_1\mathsf{P}_1\right) - Z_2\mathsf{P}_1 A_{\rm FB}}$$

$$F_\perp = 2\zeta\,(1+\xi)^2(1+R_\perp^2)$$

$$F_\parallel \mathsf{P}_1^2 = 2\zeta\,(1-\xi)^2(1+R_\parallel^2)$$

$$F_L \mathsf{P}_2^2 = 2\zeta\,(1-\xi)^2(1+R_0^2)$$

$$A_{\rm FB}\mathsf{P}_1 = 3\zeta\,(1-\xi^2)\left(R_\parallel + R_\perp\right)$$

$$\sqrt{2}A_5\mathsf{P}_2 = 3\zeta\,(1-\xi^2)\left(R_0 + R_\perp\right)$$

$$Z_1 = \sqrt{4F_\parallel F_\perp - \frac{16}{9}A_{\rm FB}^2} \quad Z_2 = \sqrt{4F_L F_\perp - \frac{32}{9}A_5^2}$$