A new generation of ULFV observables from $B \to K^* \mu^+ \mu^-$

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CKM conference-Mumbai

In collaboration with: B. Capdevila, S. Descotes-Genon, L. Hofer and J. Virto

Based on: CDVM’16 (JHEP 1610 (2016) 075) and CDHM’16 (to appear).
Present situation concerning evidences of NP in $b \rightarrow s\ell\ell$
**$P'_5$ anomaly (but also $P_2$ or $A_{FB}$ is relevant)**

$P'_5$ was proposed in **DMRV, JHEP 1301(2013)048**

$$P'_5 = \sqrt{2} \frac{\text{Re}(A^L_0 A^L_0^* - A^R_0 A^R_0^*)}{\sqrt{|A_0|^2(|A_\perp|^2 + |A_\parallel|^2)}} = \sqrt{2} \frac{\text{Re}[n_0 n^\dagger_\perp]}{\sqrt{|n_0|^2(|n_\perp|^2 + |n_\parallel|^2)}} .$$

- 2013: 1 fb$^{-1}$ dataset LHCb found 3.7σ
- 2015: 3 fb$^{-1}$ dataset LHCb found 3σ in 2 bins.
- Belle confirmed it in a bin [4,8] few months ago.

1. Computed in i-QCDF + KMPW+ 4-types of correct. $F^{\text{full}}(q^2) = F^{\text{soft}}(\xi_\perp, \xi_\parallel) + \Delta F^{\alpha_s}(q^2) + \Delta F^{p.c.}(q^2)$

<table>
<thead>
<tr>
<th>type of correction</th>
<th>Factorizable</th>
<th>Non-Factorizable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_s$-QCDF</td>
<td>$\Delta F^{\alpha_s}(q^2)$</td>
<td></td>
</tr>
<tr>
<td>power-corrections</td>
<td>$\Delta F^{p.c.}(q^2)$</td>
<td>LCSR with single soft gluon contribution</td>
</tr>
</tbody>
</table>

2. Another group [BSZ] found using full-FF approach and BSZ-FF very similar result ($\lesssim$ errors).

**Joaquim Matias**  **Universitat Autònoma de Barcelona**  
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The agreement between 1 and 2 based on different methods + identification of the origin of the inflated errors in $P_5'$ by JC'14 (scheme choice+param. errors) deconstructed the attempt to explain this anomaly by means of factorizable p.c. (see more details in backup).

<table>
<thead>
<tr>
<th>ONLY power correction error of $\langle P'<em>5 \rangle</em>{[4,6]}$</th>
<th>error of f.f.+p.c. scheme-1 in transversity basis</th>
<th>error of f.f.+p.c. scheme-2 in helicity basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO correlations among errors of p.c. (hyp. 10%)</td>
<td>$\pm 0.05$</td>
<td>$\pm 0.15$</td>
</tr>
<tr>
<td>WITH correlations among errors of p.c.</td>
<td>$\pm 0.03$</td>
<td>$\pm 0.03$</td>
</tr>
</tbody>
</table>

Their scheme’s choice inflates error artificially.

... interesting evolution of error $\text{err}(\langle P'_5 \rangle_{[4,6]} = +0.48 \text{ (JC'14)}$ and $\text{err}(\langle P'_5 \rangle_{[4,8]} = +0.17 \text{ (hep-ex/1604.04042)}$ from C.
The agreement between 1 and 2 based on different methods + the understanding of the origin of the inflated errors in $P'_5$ by JC’14 (scheme choice+param. errors) deconstructed the attempt to explain this anomaly by means of factorizable p.c. (see more details in backup).

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and $\text{err}(\langle P'_5 \rangle_{[4,8]} = +0.17 \text{ (hep-ex/1604.04042)}$ from C.

Bin-by-bin global fit analysis of $C_9$ tells you if a residual $q^2$ dependence is present.

⇒ if the values obtained are flat, charm is well estimated.

We use KMPW. Notice the excellent agreement of bins [2,5], [4,6], [5,8].

$C_9^{NP[2,5]} = -1.6 \pm 0.7, \ C_9^{NP[4,6]} = -1.3 \pm 0.4, \ C_9^{NP[5,8]} = -1.3 \pm 0.3$

(see also F. Polci)

⇒ The lack of any indication for a $q^2$-dependence in $C_9$ in this plot disfavours the arguments based on a huge charm-loop $q^2$- dependent explanation.

More in CHDM’16 (to appear) to close this discussion
Other tensions beyond $P'_5$...

Systematic low-recoil small tensions:

<table>
<thead>
<tr>
<th>$b \to s\mu^+\mu^-$ (low-recoil)</th>
<th>bin</th>
<th>SM</th>
<th>EXP</th>
<th>Pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^7 \times \text{BR}(B^0 \to K^0\mu^+\mu^-)$</td>
<td>[15,19]</td>
<td>$0.91 \pm 0.12$</td>
<td>$0.67 \pm 0.12$</td>
<td>$+1.4$</td>
</tr>
<tr>
<td>$10^7 \times \text{BR}(B^0 \to K^{*0}\mu^+\mu^-)$</td>
<td>[16,19]</td>
<td>$1.66 \pm 0.15$</td>
<td>$1.23 \pm 0.20$</td>
<td>$+1.7$</td>
</tr>
<tr>
<td>$10^7 \times \text{BR}(B^+ \to K^{*+}\mu^+\mu^-)$</td>
<td>[15,19]</td>
<td>$2.59 \pm 0.25$</td>
<td>$1.60 \pm 0.32$</td>
<td>$+2.5$</td>
</tr>
<tr>
<td>$10^7 \times \text{BR}(B_s \to \phi\mu^+\mu^-)$</td>
<td>[15,18.8]</td>
<td>$2.20 \pm 0.17$</td>
<td>$1.62 \pm 0.20$</td>
<td>$+2.2$</td>
</tr>
</tbody>
</table>

After including the BSZ DA correction that affected the error of twist-4:

<table>
<thead>
<tr>
<th>$10^7 \times \text{BR}(B_s \to \phi\mu^+\mu^-)$</th>
<th>SM</th>
<th>EXP</th>
<th>Pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.1,2]</td>
<td>$1.56 \pm 0.35$</td>
<td>$1.11 \pm 0.16$</td>
<td>$+1.1$</td>
</tr>
<tr>
<td>[2,5]</td>
<td>$1.55 \pm 0.33$</td>
<td>$0.77 \pm 0.14$</td>
<td>$+2.2$</td>
</tr>
<tr>
<td>[5,8]</td>
<td>$1.89 \pm 0.40$</td>
<td>$0.96 \pm 0.15$</td>
<td>$+2.2$</td>
</tr>
</tbody>
</table>

Global fit to $\sim 90$ obs. (radiative+$b \to s\mu^+\mu^-$)

All deviations add up constructively

- A new physics contribution to $C_{9,\mu}=-1.1$ with a pull-SM of $4.5\sigma$ alleviates all anomalies and tensions.

- NP contributions to the rest of Wilson coefficient are not (for the moment) yet significantly different from zero.

No $b \to se^+e^-$ data included at this slide.

(see also J. Virto)
The strongest signal of NP in $C_9$

- This coefficient is affected by long-distance charm contributions.

$$C_{9}^{\text{eff},i} = C_{9}^{\text{eff,SM pert}}(q^2) + C_{9}^{\text{NP}} + C_{9}^{l.d. c\bar{c}(i)}(q^2)$$

Hints of lepton-flavour non-universal NP

- Observables probing ULFV are free from long-distance charm pollution in the SM, i.e., free from $C_{9}^{l.d. c\bar{c}(i)}(q^2)$.

- Only NP can explain tensions w.r.t SM in these observables and they appear to be consistent with $P_5^{\mu\mu}$.
Universal Lepton-Flavour Violating Observables

offer a second/complementary option to close the discussion about ‘SM alternatives’
Universal LFV observables: $R_K$’s hints

\[
R_K = \frac{\text{Br}(B^+ \rightarrow K^+\mu^+\mu^-)}{\text{Br}(B^+ \rightarrow K^+e^+e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036
\]

$\Rightarrow$ $R_K$ shows a $2.6\sigma$ tension with its SM prediction.

$\Rightarrow$ $R_K$ (but also future measurements of $R_{K^*}$, $R_\phi$, ...) represents the next step:

- This tension cannot be resolved within the SM, in particular long-distance charm cannot explain it.
- New ingredient of the puzzle: Is Nature Universal LFV?

If answer is YES:

**NP or Charm?** $\Rightarrow$ NP × Charm

(obsolete question) (disentangling type of NP)

New Physics only possible explanation and charm only enters into game when discussing type of New Physics

The gray box is the SM prediction and blue cross is data.
Universal LFV observables: $R_K$’s hints

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- This tension cannot be resolved within the SM, in particular long-distance charm cannot explain it.
- New ingredient of the puzzle: Is Nature Universal LFV?

A separated fit to $C_{9,\mu}^{NP}$ and $C_{9,e}^{NP}$ shows a preference for $C_{9,\mu}^{NP} \sim -1$ and $C_{9,e}^{NP}$ compatible with zero.
Universal LFV observables: $R_K$’s hints

$$R_K = \frac{\text{Br}(B^+ \to K^+ \mu^+\mu^-)}{\text{Br}(B^+ \to K^+ e^+e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

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- New ingredient of the puzzle: Is Nature Universal LFV?
- This tension cannot be resolved within the SM, in particular long-distance charm cannot explain it.

2 $R_K$ tension is coherent with the pattern of tensions observed in the $B \to K^*$ angular analysis.

3 Same $C_{9,\mu}^{\text{NP}} = -1.1$ alleviates both $R_K$ and $P'_5$ anomalies (with $C_{9,e}$ SM-like). $R_K$ adds coherently in the global fit $+0.4\sigma$ to this NP solution.
Universal LFV observables: $R_K$'s hints

\[ R_K = \frac{\text{Br}(B^+ \to K^+ \mu^+ \mu^-)}{\text{Br}(B^+ \to K^+ e^+ e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036 \]

⇒ $R_K$ shows a $2.6\sigma$ tension with its SM prediction.
⇒ $R_K$ (but also future measurements of $R_{K^*}$, $R_\phi$, ...) represents the next step:
   - New ingredient of the puzzle: Is Nature Universal LFV?
   - This tension cannot be resolved within the SM, in particular long-distance charm cannot explain it.

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BUT ALSO low-recoil tensions and $B_s \to \phi\mu\mu$. 

<table>
<thead>
<tr>
<th>$b \to s\mu^+\mu^-$</th>
<th>bin</th>
<th>$\text{SM} \to \text{NP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{BR}(B^0 \to K^0 \mu^+ \mu^-)$</td>
<td>[15,19]</td>
<td>$+1.4\sigma \to +0.3\sigma$</td>
</tr>
<tr>
<td>$\text{BR}(B^0 \to K^{*0} \mu^+ \mu^-)$</td>
<td>[16,19]</td>
<td>$+1.7\sigma \to +0.4\sigma$</td>
</tr>
<tr>
<td>$\text{BR}(B^+ \to K^{*+} \mu^+ \mu^-)$</td>
<td>[15,19]</td>
<td>$+2.5\sigma \to +1.2\sigma$</td>
</tr>
<tr>
<td>$\text{BR}(B_s \to \phi\mu^+\mu^-)$</td>
<td>[15,18.8]</td>
<td>$+2.2\sigma \to +0.5\sigma$</td>
</tr>
</tbody>
</table>
Is it enough $R_K$ to disentangle different New Physics scenarios?

But, with current data, more information than $R_K$ alone is needed to distinguish between NP scenarios. E.g. $C_{9,\mu}^{\text{NP}} = -1.1$ (scenario 1) vs $C_{9,\mu}^{\text{NP}} = -C_{10,\mu}^{\text{NP}} = -0.65$ (scenario 2).

Blue cross is data and gray band is SM prediction
THE (near) FUTURE:

A new generation of ULFV charm-insensitive observables (in SM).
⇒ Assume Nature violates universal lepton flavour (muons vs electrons).

**Goal:** To probe the different NP scenarios suggested by global fits with the highest possible precision.

**How?** New observables matching the following criteria:

- Sensitivity only to the short distance part of $C_9$ (*charm free* in the SM).
- Capacity to test for lepton flavour universality violation between the electronic and muonic modes.
- Sensitivity to Wilson coefficients other than $C_9$.
- In presence of New Physics reduced hadronic uncertainties.

Exploiting the angular analyses of both $B \to K^* \mu\mu$ and $B \to K^* ee$ decays, certain combinations of the angular observables fulfill the requirements

$$
\langle Q_i \rangle = \langle P_i^\mu \rangle - \langle P_i^e \rangle \quad \hat{\langle Q_i \rangle} = \langle \hat{P}_i^\mu \rangle - \langle \hat{P}_i^e \rangle \\
\langle B_k \rangle = \frac{\langle J_k^\mu \rangle}{\langle J_k^e \rangle} - 1 \quad \hat{\langle B_k \rangle} = \frac{\langle J_k^\mu / \beta^2_{\mu} \rangle}{\langle J_k^e / \beta^2_{e} \rangle} - 1
$$

$i = 1, \ldots, 9$ & $k = 5, 6s$

where $\hat{\cdot}$ means correcting for lepton-mass effects in the first bin (backup slides).
How LFUV NP enter in Wilson coefficients:

\[ C_{i,\mu} = \begin{cases} C_i + \delta C_i, & i = 10, 9', 10' \\
C_9 + \delta C_9 + \Delta C_9^{(j)} & \end{cases} \]

\[ C_{i,e} = \begin{cases} C_i, & i = 10, 9', 10' \\
C_9 + \Delta C_9^{(j)} & \end{cases} \]

\[ j = \perp, \parallel, 0 \]

Notice \( C_{7,7'} \) is obviously lepton-mass independent.

⇒ \( \delta C_i = C_{i,\mu} - C_{i,e} \equiv \) amount of LFU violation.

⇒ \( C_i \equiv \) SM + LFU NP.

⇒ \( \Delta C_9^{(j)} \equiv \) long-distance charm. Two types:

- **Transversity Dependent**: \( \Delta C_9^{\perp,\parallel,0} \) different.
- **Transversity Independent**: \( \Delta C_9^{\perp} = \Delta C_9^{\parallel} = \Delta C_9^0. \)
$Q_i$ observables. The example: $P'_5$ versus $Q_5 = P'_{5\mu} - P'_{5e}$

Gray-SM, Red-NP $C_{9,\mu}^{NP} = -1.11$, $C_{9,e}^{NP} = 0$ and data

- Soft FF independent at LO exactly in SM
- Soft FF independent at LO exactly in NP.
- Large sensitivity to $C_{9,\mu}$. SM (DHMV’15):
  \[
  \langle P'_5 \rangle_{[4,6]} = -0.82 \pm 0.08
  \]
  \[
  \langle P'_5 \rangle_{[6,8]} = -0.94 \pm 0.08
  \]

- FF indep. at all orders in SM (up to $\Delta m_{\ell}^2 / q^2$).
  Soft FF indep. at LO exactly in NP.

- Long-distance charm insensitive in the SM.
  Large sensitivity to $\delta C_9 = C_{9,\mu} - C_{9,e}$.
  (CDMV’16): ($< 10^{-3}$ without lepton mass)
  \[
  \langle \hat{Q}_5 \rangle_{[4,6]} = -0.002 \pm 0.017
  \]
  \[
  \langle \hat{Q}_5 \rangle_{[6,8]} = +0.002 \pm 0.010
  \]
$Q_i$ observables. The example: $P_5'$ versus $Q_5 = P_5'^\mu - P_5'^e$ for $C_{9,\mu}^{\text{NP}} = -1.1$

Remark: In presence of NP hadronic uncertainties reemerge in $\hat{Q}_5$ ...

<table>
<thead>
<tr>
<th>$P_5'$</th>
<th>Prediction $C_{9,\mu}^{\text{NP}} = -1.1$</th>
<th>$\hat{Q}_5$</th>
<th>SM-Prediction</th>
<th>$\hat{Q}_5$</th>
<th>Prediction $\delta C_9 = -1.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.1, 0.98]</td>
<td>$+0.80 \pm 0.14$</td>
<td>[0.1, 0.98]</td>
<td>$-0.097 \pm 0.023$</td>
<td>[0.1, 0.98]</td>
<td>$0.000 \pm 0.018$</td>
</tr>
<tr>
<td>[1.1, 2.5]</td>
<td>$+0.43 \pm 0.12$</td>
<td>[1.1, 2.5]</td>
<td>$-0.003 \pm 0.007$</td>
<td>[1.1, 2.5]</td>
<td>$0.227 \pm 0.023$</td>
</tr>
<tr>
<td>[2.5, 4]</td>
<td>$-0.12 \pm 0.13$</td>
<td>[2.5, 4]</td>
<td>$-0.005 \pm 0.017$</td>
<td>[2.5, 4]</td>
<td>$0.370 \pm 0.021$</td>
</tr>
<tr>
<td>[4, 6]</td>
<td>$-0.50 \pm 0.11$</td>
<td>[4, 6]</td>
<td>$-0.002 \pm 0.017$</td>
<td>[4, 6]</td>
<td>$0.314 \pm 0.046$</td>
</tr>
<tr>
<td>[6, 8]</td>
<td>$-0.73 \pm 0.12$</td>
<td>[6, 8]</td>
<td>$+0.002 \pm 0.010$</td>
<td>[6, 8]</td>
<td>$0.216 \pm 0.061$</td>
</tr>
</tbody>
</table>

**BUT**, it only matters when discussing the **type** of NP we can see.
In summary if $Q_5^{\text{exp}} \neq 0$ because $P_5^{\mu \text{exp}} \neq P_5^{\mu \text{SM}}$ while $P_5^{e \text{exp}} \simeq P_5^{e \text{SM}}$ we learn:

1. Nature is Universal Lepton Flavour Violation.

2. Any attempt to use:
   
   $\Rightarrow$ long distance $c\bar{c}$ loops that mimics New Physics

   as a possible explanation of $P_5^\prime$ within SM is ruled out!!

   .... all arguments in [CDHM’16] gets an independent confirmation.

Belle (according to T. Browder’s talk) can be the first in testing it (see S. Wehle’s talk).
Probing right-handed currents (RHC) with $Q_i$

SM predictions (grey boxes),
NP: $C_{9,\mu}^{\text{NP}} = -1.11$ & $C_{9,\mu}^{\text{NP}} = -C_{10,\mu}^{\text{NP}} = -0.65$ & $C_{9,\mu}^{\text{NP}} = -C_{9,\mu}^{\text{NP}} = -1.18$ & $C_{10,\mu}^{\text{NP}} = C_{10,\mu}^{\text{NP}} = 0.38$.

with $\delta C_i = C_{i,\mu} - C_{i,e}$ (and $C_{i,e}^{\text{SM}}$)

$\Rightarrow$ $Q_{1,4}$ provide excellent opportunities to probe RHC in $C_{9,\mu}^{'} & C_{10,\mu}^{'}$.

- $Q_1$ shows significant deviations in presence of RHC. If $C_{7}^{'} = 0$ at LO

$$s_0^{LO} = -2 \frac{C_7 \delta C_{9} m_b M_B}{C_{10,\mu}^{'} \delta C_{10}^{'} + \text{Re} C_{9,\mu}^{'} \delta C_{9}^{'}},$$

no zero (except $s = 0$) if $\delta C_{9}^{'} = 0$.
	no sensitivity to $C_i$ if $C_{i}^{'} = 0$.

- $Q_4$ at low-$q^2$ exhibits deviations for $C_{9,10,\mu}$ when accurate precision in measurements is achieved.
SM predictions (grey boxes),
NP: $C_{9,\mu}^{\text{NP}} = -1.11$ (scenario 1) & $C_{9,\mu}^{\text{NP}} = -C_{10,\mu}^{\text{NP}} = -0.65$ (scenario 2) with $\delta C_i = C_{i,\mu} - C_{i,e}$ (and $C_{i,e}^{\text{SM}}$)

$Q_2 = P_2^\mu - P_2^e$

$Q_5 = P_5'^\mu - P_5'^e$

$Q_4 = P_4'^\mu - P_4'^e$

⇒ $Q_2$, $Q_4$ & $Q_5$ show distinctive signatures for the two NP scenarios considered.

- Differences in the high-$q^2$ bins of the large recoil region of $Q_2$ & $Q_5$ are quite significant. Lack of difference between scenario 2 and SM same reason why $P_5'$ in scenario 2 is worst than scenario 1.
- $Q_4$ at very low-$q^2$ (second bin) is very promising to disentangle scenario 1 from 2.
**Observables (unique properties)**

**Idea:** Combine $J_i^\mu$ & $J_i^e$ to build combinations sensitive to some $C_i$, with controlled sensitivity to long-distance charm.

\[
\begin{align*}
\beta_\ell J_5 - 2iJ_8 &= 8\beta_\ell^2 N^2 m_B^2 (1 - \hat{s})^3 \frac{\hat{m}_{K^*}}{\hat{s}\sqrt{\hat{s}}} C_{10}^\ell \left[ C_7 \hat{m}_b (1 + \hat{s}) + \hat{s} C_9^\ell \right] \xi_\perp \xi_\parallel + \ldots \\
\beta_\ell J_{6s} - 2iJ_9 &= 16\beta_\ell^2 N^2 m_B^2 (1 - \hat{s})^2 \frac{1}{\hat{s}} C_{10}^\ell \left[ 2C_7 \hat{m}_b + \hat{s} C_9^\ell \right] \xi_\perp^2 + \ldots
\end{align*}
\]

where $\beta_\ell = \sqrt{1 - 4m_\ell^2/q^2}$.

Assuming real NP & maximal LFUV $\mu$ vs $e$, natural combinations are

\[
B_5 = \frac{J_5^\mu}{J_5^e} - 1 \quad B_{6s} = \frac{J_{6s}^\mu}{J_{6s}^e} - 1
\]

- Form factor independent at all orders (up to $\triangle$ lepton mass).
- Full charm insensitive in the SM.
- Linear sensitivity to $\delta C_9$ **kinematically suppressed.**
In the large-recoil limit and in absence of RHC currents [CDMV'16]:

\[
B_5 = \frac{J_5^\mu - J_5^e}{J_5^e} = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} \delta C_{10} + \frac{\beta_\mu^2}{\beta_e^2} \frac{2(C_{10} + \delta C_{10}) \delta C_9}{C_{10}} \hat{s} + \ldots
\]

\[
B_{6s} = \frac{J_{6s}^\mu - J_{6s}^e}{J_{6s}^e} = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} \delta C_{10} + \frac{\beta_\mu^2}{\beta_e^2} \frac{2(C_{10} + \delta C_{10}) \delta C_9}{C_{10}} \left(4C_7 \hat{m}_b + (2C_9 + \Delta C_9,\perp + \Delta C_9,\parallel) \hat{s}\right) + \ldots
\]

In the limit of \( s \to 0 \), \( \delta C_{10} \) is cleanly disentangled:

\[
B_5(s \to 0) = B_{6s}(s \to 0) = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} + \frac{\beta_\mu^2}{\beta_e^2} \frac{\delta C_{10}}{C_{10}} + \ldots
\]

This shows the IMPORTANCE of the normalization to the electronic mode. IF NOT normalized:

\[
J_5^\mu - J_5^e \propto C_7 \delta C_{10} \xi_\perp \xi_\parallel
\]

Several PROBLEMS in extracting \( \delta C_{10} \) if not normalized:

1) \( \xi_\perp \xi_\parallel \): SFF error? KMPW or BSZ
2) Charm contribution possible inside \( C_7 \).
$B_5$ & $B_{6s}$ are **not identically** 0 in the SM.

**Lepton mass differences** generates a non-zero contribution mainly in the first bin.

\[
\Rightarrow \text{If on an event-by-event basis experimentalist can measure } \langle J_\mu^i / \beta_\mu^2 \rangle:
\]

\[
\langle \tilde{B}_5 \rangle = \frac{\langle J_5^\mu / \beta_\mu^2 \rangle}{\langle J_5^e / \beta_e^2 \rangle} - 1 \quad \langle \tilde{B}_{6s} \rangle = \frac{\langle J_{6s}^\mu / \beta_\mu^2 \rangle}{\langle J_{6s}^e / \beta_e^2 \rangle} - 1
\]

- **SM Predictions:** $\langle \tilde{B}_i \rangle = 0.00 \pm 0.00$.
- **All good properties of $B_{5,6s}$ + simpler structure $\beta_i \rightarrow 1$.**

- **When $\hat{s} \rightarrow 0$, $\tilde{B}_5 = \tilde{B}_{6s} = \delta C_{10}/C_{10}$**

\[
\Rightarrow \text{Sensitivity to } \delta C_{10}!
\]

Exactly as $B_5$, $B_{6s}$ but simpler.

- **1st Bins:** Capacity to distinguish $C_{9,\mu}^{NP} = -1.11$ from $C_{9,\mu}^{NP} = -C_{10,\mu}^{NP} = -0.65$. 

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A new generation of ULFV observables from $B \rightarrow K^* \mu^+ \mu^-$
**Goals:** Can one construct a ULFV observable not only free from hadronic uncertainties in the SM but also free from long-distance charm in presence of New Physics? **Yes** BUT only under two conditions:

- Only if New Physics is dominated by $\delta C_9$.
- Only if long-distance charm is transversity independent $\triangle C_9^\perp = \triangle C_9^\parallel = \triangle C_9^0 = \triangle C_9$.

\[
\tilde{M} = \frac{B_5 \bar{B}_{6s}}{B_{6s} - B_5} = -\frac{\delta C_9 \hat{s}}{C_7 \hat{m}_b (1 - \hat{s})} + \delta C_{10} \text{ terms} + \delta C_{10} \triangle C_9 \text{ terms} + \ldots
\]

- If charm is transversity dependent (as expected) is impossible to remove it in presence of NP.

\[
\tilde{M} = \frac{B_5 \bar{B}_{6s}}{B_{6s} - B_5} = -\frac{\delta C_9 \hat{s}}{C_7 \hat{m}_b (1 - \hat{s}) - (\triangle C_9^0 - \triangle C_9^\parallel) \hat{s}/2} + \ldots
\]

(Leading order expression)

- Maximal sensitivity to NP at very low-$q^2$.
- **Even if for** $\delta C_{10} \neq 0$ **⇒** long-distance charm reemerges, this observable is particularly promising to measure $\delta C_{10}$.
- Singular in the region where $B_5 \simeq B_{6s}$.  

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Transversity Independent Charm Free Observables at low $q^2$

Error size comes from TD charm suppressed at low-$q^2$

Error size comes from all type of charm TD and TI (due to $\delta C_{10} \neq 0$)

**Scenario 1:**

$\delta C^{NP}_{9\mu} = -1.11$

**Scenario 2:**

$\delta C^{NP}_{9\mu} = -\delta C^{NP}_{10\mu} = -0.65$

**Figure:** SM predictions (grey boxes) and NP predictions (red boxes) for $\langle \tilde{M} \rangle$ in the 2 scenarios.
Conclusions

■ Global point of view: We have shown that the same NP solution $C_{9,\mu}^{\text{NP}} = -1.1$, $C_{9,e}^{\text{NP}} = 0$ alleviates all tensions: $P_5', R_K$, low-recoil, $B_s \rightarrow \phi \mu^+ \mu^-$,...

   $\rightarrow$ SM ‘alternative explanations’ are in trouble from a global point of view.

■ Local point of view:
   - Factorizable p.c.: We have proven that an inappropriate scheme’s choice if correlations among p.c. are not considered inflates artificially the errors.
   - Long-distance charm: Explicit computation by KMPW do not explain the anomaly and a bin-by-bin analysis does not find any indication for a $q^2$-dependence.

■ We have proposed different sets of ULFV observables comparing $B \rightarrow K^* ee$ & $B \rightarrow K^* \mu \mu$
   (totally free from any long distance charm in the SM).
   ■ $Q_i$ Observables: $Q_i \leftrightarrow P_i$
   ■ $C_{9\ell}$ linear Observables: $B_{5,6s}$, $\bar{B}_{5,6s} \leftrightarrow J_{5,6s}$
   ■ TI charm free Observables: $M$ ($\bar{M}$)

■ $\langle Q_i \rangle$ observables allows us to distinguish different NP scenarios: RHC or $\delta C_9$ versus $\delta C_9 = -\delta C_{10}$.

■ $\langle B_5 \rangle$ & $\langle B_{6s} \rangle$ but also $\langle \bar{M} \rangle$ can be used to measure $\delta C_{10}$ at very low-$q^2$. 

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Backup Slide
LHCb currently determines $F_{L,T}$ using a simplified description of the angular kinematics:

$$\begin{align*}
J_{2s} & \\
J_{2c} & \end{align*} \quad \rightarrow \quad J_1c \quad \text{(equivalent in the massless limit)}$$

Then, to match this convention, the angular observables are redefined in the following way:

$$\begin{align*}
F_L &= \frac{-J_{2c}}{dG/dq^2} \quad \rightarrow \quad \hat{F}_L = \frac{J_1c}{dG/dq^2} \\
F_T &= \frac{4J_{2s}}{dG/dq^2} \quad \rightarrow \quad \hat{F}_T = 1 - \hat{F}_L \\
\hat{P}_1 &= \frac{J_3}{2J_{2s}} \quad \rightarrow \quad \hat{P}_1 = \frac{J_3}{2\hat{J}_{2s}} \\
\hat{P}_2 &= \frac{J_{6s}}{8J_{2s}} \quad \rightarrow \quad \hat{P}_2 = \frac{J_{6s}}{8\hat{J}_{2s}} \\
\hat{P}_3 &= -\frac{J_9}{4J_{2s}} \quad \rightarrow \quad \hat{P}_3 = -\frac{J_9}{4\hat{J}_{2s}} \\
\hat{P}_4 &= \frac{J_4}{\sqrt{-J_{2s}J_{2c}}} \quad \rightarrow \quad \hat{P}_4 = \frac{J_4}{\sqrt{\hat{J}_{2s}\hat{J}_{1c}}} \\
\hat{P}_5' &= \frac{J_5}{2\sqrt{-J_{2s}J_{2c}}} \quad \rightarrow \quad \hat{P}_5' = \frac{J_5}{2\sqrt{\hat{J}_{2s}\hat{J}_{1c}}} \\
\hat{P}_6' &= -\frac{J_7}{2\sqrt{-J_{2s}J_{2c}}} \quad \rightarrow \quad \hat{P}_6' = -\frac{J_7}{2\sqrt{\hat{J}_{2s}\hat{J}_{1c}}} \\
& \text{with } \hat{J}_{2s} = \frac{1}{16}(6J_{1s} - J_{1c} - 2J_{2s} - J_{2c})
\end{align*}$$
Why is there a need to compute the predictions from $\hat{F}_{L,T}$ instead of $F_{L,T}$? Let’s consider the decay distribution

$$\frac{1}{d(G + \tilde{G})/dq^2} \frac{d^3(G + \tilde{G})}{dO} = \frac{9}{32\pi} \left[ \frac{3}{4} \hat{F}_T \sin^2 \theta_K + \hat{F}_L \cos^2 \theta_K \\
+ \frac{1}{4} F_T \sin^2 \theta_K \cos 2\theta_I - F_L \cos^2 \theta_K \cos 2\theta_I + \ldots \right]$$

- With the current limited statistics, $\hat{F}_{L,T}$ and $F_{L,T}$ cannot be distinguished by LHCb.
- $\cos \theta_K^2$ is the dominant term, so it is the natural place to extract $F_L$. 

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The size of power corrections

The ratio $A_1/V$ is particularly relevant. Let’s illustrate that the size of the error associated to power corrections is much below 10%. We use BSZ for this example.

Ratio of FF computed in BSZ including correlations using full-FF.

![Graph 1](image1)

Ratio of FF computed in BSZ taking 5%, 10% and 20% for the error associated to p.c.

![Graph 2](image2)

Notice that already a 5% error of power correction is of the same size of the error of the full-FF.
A brief (or not so) parenthesis on hadronic uncertainties

There are two ways to discard attempts of explanation (factorizable p.c, charm) of the anomaly in $P'_5$ within the SM:

1. Direct deconstruction of arguments (→ the case of factorizable power corrections) or by comparison with data of explicit computations (not fits) of long-distance charm contributions (KMPW).

2. With the help of ULFV observables: if $P'_5$ and ULFV observables share the same new physics explanation, no space for long-distance charm or other unknown hadronic uncertainties is left in $P'_5$.

let’s play a bit first with 1....
Factorizable power corrections

\[ F_{\text{full}}(q^2) = F_{\text{soft}}(\xi_\perp, \xi_\parallel) + \triangle F^{\alpha_8}(q^2) + \triangle F^{p.c.}(q^2) \]

with \[ \triangle F^{p.c.}(q^2) = a_F + b_F\left(\frac{q^2}{m_B^2}\right) + \ldots \]

1) **Power correction error size**: In JC’14 (and DHMV’14) they take uncorrelated errors among p.c. missing that this choice introduces scheme (definition of SFF) dependence. **Numerically**:

<table>
<thead>
<tr>
<th>ONLY power correction error of ( P'_5 ) [4,6]</th>
<th>error of f.f.+p.c. scheme-1 in transversity basis</th>
<th>error of f.f.+p.c. scheme-2 in helicity basis</th>
</tr>
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<td>NO correlations among errors of p.c. (hyp. 10%)</td>
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Their scheme’s choice inflates error **artificially**.

\( \Rightarrow \) Analytically (see CHDM’16) we found that JC’14 missed the most relevant term in \( P'_5 \) that in transversity basis makes manifest scheme-dependence. Numerically it was proven in [DHMV’14]

The weights of power corrections \( aV \) & \( aT_1 \) are MANIFESTLY different:

\[ P'_5(q^2=6) = P'_5|_\infty(1 + [0.78 aV - 0.20 aT_1]/\xi_\perp(6) + \ldots \]

\[ \xi_\perp^{(1)}(q^2) \equiv \frac{m_B}{m_B + m_{K^*}} V(q^2) \Rightarrow aV = 0 \text{ (our) or} \quad \xi_\perp^{(2)}(q^2) \equiv T_1(q^2) \Rightarrow aT_1 = 0 \text{ (JC) > 3 times bigger} \]

- JC’14 enters in conflict not only with DHMV but also with BSZ that uses full-FF method.
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• JC’14 enters in conflict not only with DHMV but also with BSZ that uses full-FF method.
2) BUT also **Parametric errors** from \((m_q, f_{K^*}, \mu, a_i,...)\) and soft FF are abnormally large.

- DHMV’14 a random scan over all parameters and take max and min.
- JC’12 (same approach) error is factor 2 larger than: DHMV’14, BSZ’15 and also Bobeth et al.’13.

\[
\text{err}[\langle P_5'\rangle_{\text{DHMV'14}}^{1.6}] = \pm 0.066 \quad \text{err}[\langle P_5'\rangle_{\text{BSZ}}^{[1.1,6]}] = \pm 0.035 \quad \text{err}[\langle P_5'\rangle_{\text{BBD'14}}^{[1,6]}] = \leq \pm 0.08
\]

\[
\text{err}[\langle P_5'\rangle_{\text{JC'12}}^{[1,6]}] = \pm 0.12
\]

This is strange considering the undervalued error of JC’14: \(\xi_\perp = 0.31 \pm 0.04\)

compared to our DHMV: \(\xi_\perp = 0.31^{+0.20}_{-0.10}\)

1) and 2) explains the artificially large errors in FFI observables \(P_i\) in JC’12 and ’14.

... an appropriate choice of scheme is mandatory not to artificially inflate your errors.

*The scheme’s choice by JC’14 would be excellent if one is interested in analyzing lepton-mass observables \(M_{1,2}\) function of \(J_{1c,2c,1s,2s}\)!!!! or in a global analysis carefully including all correlations, otherwise it is inappropriate to analyze separated relevant observables like \(P_5'\).*
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\]

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Long-distance contributions from $c\bar{c}$ loops where the lepton pair is created by an electromagnetic current.

1. The $\gamma$ couples universally to $\mu^\pm$ and $e^\pm$: $R_K$ nor any LFVU cannot be explained by charm-loops.

2. KMPW is the only real computation of long-distance charm.

$$C_{9}^{\text{eff}} = C_{9}^{\text{eff SM pert}}(q^2) + C_{9}^{\text{NP}} + s_i \delta C_{9}^{c\bar{c}(i)}_{\text{KMPW}}(q^2)$$

KMPW implies $s_i = 1$, but we vary $s_i = 0 \pm 1$, $i = 0, \perp, \parallel$. 

$$\delta C_{9}^{\text{LD,}(\perp,\parallel)}(q^2) = \frac{a^{(\perp,\parallel)} + b^{(\perp,\parallel)} q^2 [c^{(\perp,\parallel)} - q^2]}{b^{(\perp,\parallel)} q^2 [c^{(\perp,\parallel)} - q^2]}$$

$$\delta C_{9}^{\text{LD,0}}(q^2) = \frac{a^0 + b^0 [q^2 + s_0] [c^0 - q^2]}{b^0 [q^2 + s_0] [c^0 - q^2]}$$
Bin-by-bin global fit analysis of $C_9$ tells you if a residual $q^2$ dependence is present.

⇒ if the values obtained are flat, charm is well estimated.

![Graph showing $C_9^{NP}$ values against $q^2$](image)

- We use KMPW. Notice the excellent agreement of bins [2,5], [4,6], [5,8].
  
  $C_9^{NP[2,5]} = -1.6 \pm 0.7$, $C_9^{NP[4,6]} = -1.3 \pm 0.4$, $C_9^{NP[5,8]} = -1.3 \pm 0.3$

- We do not find any indication for a $q^2$-dependence in $C_9$ neither in the plots nor in a 6D fit adding $a^i + b^i$ to $C_9^{\text{eff}}$ for $i = K^*, K, \phi$. → disfavours again charm explanation.

Another group [Silvestrini et al.] argue that maybe there is an unknown and very hard to compute charm contribution (that they do not even try to compute or estimate) that explain only one anomaly.
There is certain confusion in the literature related to the correct interpretation of [Ciuchini et al.’15].

1) **Arbitrary** parametrization \( h_\lambda = h_\lambda^{(0)} + h_\lambda^{(1)} q^2 + h_\lambda^{(2)} q^4 \) and fit ONLY LHCb data @low-\(q^2\).

**THIS IS JUST A FIT TO DATA**: No dynamics is involved. If one adds 18 free parameters one can fit easily anything.

Can one get a solid conclusion out of this result?

In v1 of that work we found an internal inconsistency of more than 4\(\sigma\) between their predictions.

→ Reason error in \(S_4^{theory}\). Example in bin \([4,6]\): 

\[
S_4^{\nu 1} = -0.120 \pm 0.008 \text{ versus } S_4^{\nu 2} = -0.241 \pm 0.014 \text{ they differ by } 7.5\sigma \text{!!!!}
\]

Surprisingly in abstract v1: ”**good description** of current experimental data within SM” (also in v2...)

→ Difficult to get a robust conclusion. So many parameters can swallow anything (real or spurious).
The paper has basically two parts:

1) Part-I Unconstrained fit: They simply confirm our results of the global fit (we obviously agree).

Consider again:

\[ C_{9}^{\text{eff}} = C_{9}^{\text{eff \ SM \ pert}}(q^2) + C_{9}^{\text{NP}} + \delta C_{9}^{c\bar{c}(i)}^{\text{KMPW}}(q^2) \]

where

\[ \delta C_{9}^{c\bar{c}(i)}^{\text{KMPW}}(q^2) \rightarrow |2C_{1}\tilde{g}_{i}^{\text{CFMPSV}}| \rightarrow h_{\lambda} \]

Blue: Their fit to \( \delta C_{9}^{c\bar{c}(i)}^{\text{KMPW}}(q^2) \)
Green: The computation of Khodjamirian et al.

They show a constant shift everywhere. Two options:

...this universal shift is \( C_{9}^{NP} \) (same as \( R_{K} \)).

...or a universal charm \( q^2 \)-independent coming from?? unable to explain nor \( R_{K} \) neither any LFVU. (weird)
More on (Ciuchini et al.’15)… an anatomy

II) Part-II Constrained fit: This part of the paper is highly ‘controversial’.

They consider the result of KMPW at $q^2 \lesssim 1$ GeV$^2$ as an estimate of the charm loop effect.

- **Problem 1**: They tilt the fit at very-low $q^2$ inducing artificially a high-$q^2$ effect.
- **Problem 2**: Precisely below 1 GeV$^2$ there are well known lepton mass effects not considered here.
- **Problem 3**: KMPW computed the soft gluon effect with respect to LO factorizable (no imaginary part included) but CFFMPSV imposes

$$|g_i|^{LHCb} \simeq |g_i|^{KMPW} \text{ at } q^2 \lesssim 1 \text{ GeV}^2$$

This makes no sense since on the RHS the imaginary part is not computed.

KMPW (left): Dashed is $2C_1 \tilde{g}_1$ indistinguishable from $2C_1 \tilde{g}_2$. 
Independently of ULFV observables a closer look to these anomalies shows that they cannot be explained neither by factorizable power corrections nor by known long distance charm contributions.

let's now explore $2$...this is the goal of this seminar