Theory of rare D decays

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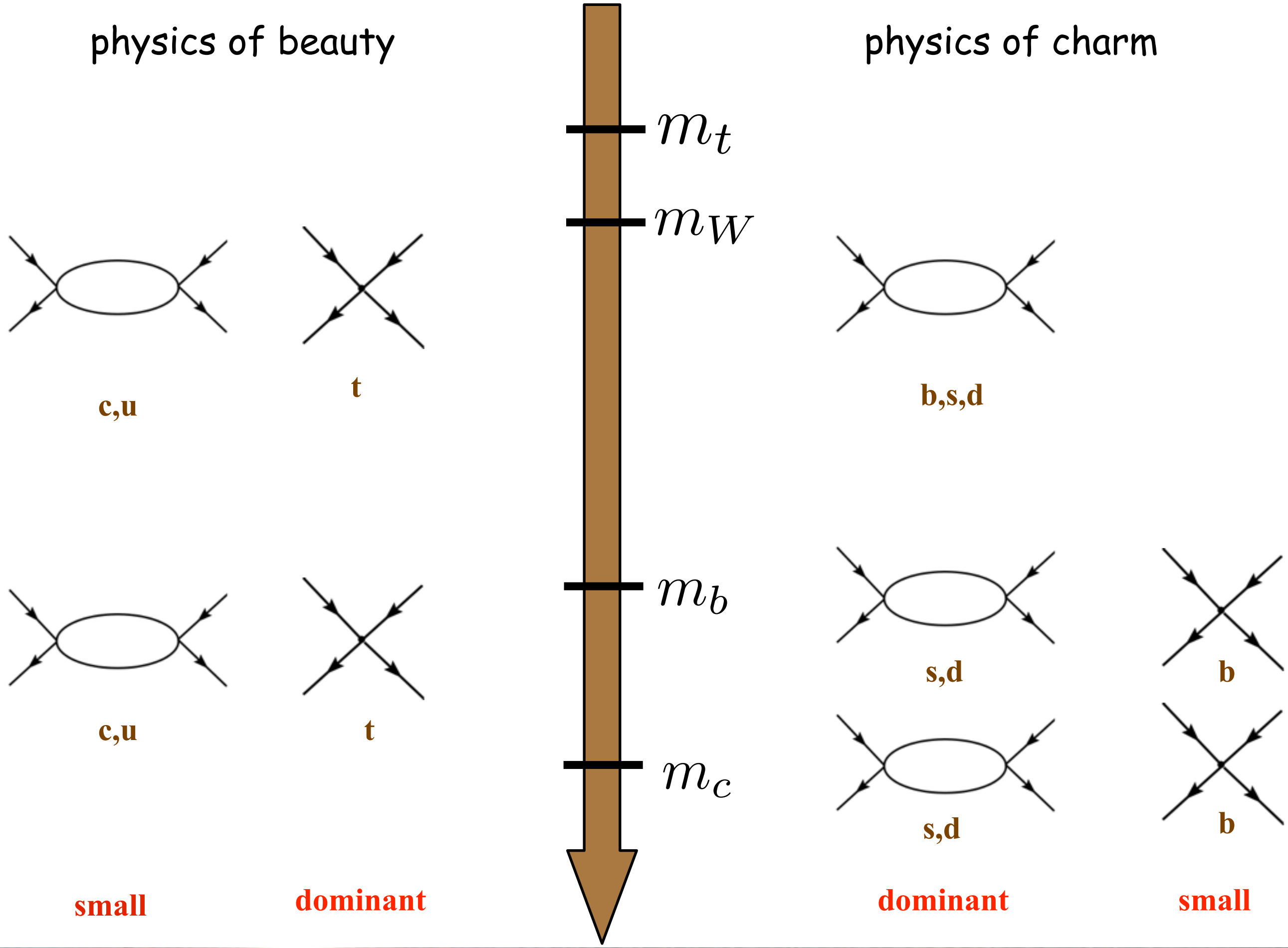
1. Introduction: energy scales

★ Main goal of the exercise: understand physics at the most fundamental scale

★ It is important to understand relevant energy scales for the problem at hand



2a. Rare decays: short distance

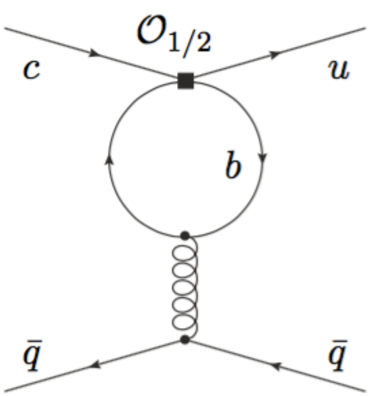
★ Effective Lagrangian can be obtained by integrating out heavy modes

- Contrary to b-physics, two matching scales (M_W and m_b)
- GIM mechanism is effective for light quarks

- Only two operators at M_W

$$H_{\rm eff}(M_W > \mu > m_b) = \frac{4G_F}{\sqrt{2}} \sum_{q=d,s,b} V_{cq}^* V_{uq} [C_1(\mu) \mathcal{O}_1^q + C_2(\mu) \mathcal{O}_2^q]$$

$$\mathcal{O}_1^q = (\bar{u}_L \gamma_\mu T^a q_L)(\bar{q}_L \gamma^\mu T^a c_L)\,,$$
$$\mathcal{O}_2^q = (\bar{u}_L \gamma_\mu q_L)(\bar{q}_L \gamma^\mu c_L)\,,$$



- Ten operators at m_b
- Those that correspond to rare decays

$$\mathcal{O}_9 = \frac{\alpha_{\rm em}}{4\pi} (\bar{u}_L \gamma_\mu c_L)(\bar{\ell} \gamma^\mu \ell)\,,$$
$$\mathcal{O}_{10} = \frac{\alpha_{\rm em}}{4\pi} (\bar{u}_L \gamma_\mu c_L)(\bar{\ell} \gamma^\mu \gamma_5 \ell)\,.$$

Short distance (cont.)

★ Effective Lagrangian can be obtained by integrating out heavy modes

- Most recent results: NNLL

De Boer, Muller, Seidel, 2016

	$\bar{C}_1$	$\bar{C}_2$	$\bar{C}_3$	$\bar{C}_4$	$\bar{C}_5$	$\bar{C}_6$
LL	-0.517	1.266	0.010	-0.025	0.007	-0.029
NLL	-0.356	1.157	0.014	-0.042	0.010	-0.045
NNLL	-0.317	1.140	0.013	-0.040	0.009	-0.045

- Short distance contribution for $\mu < m_b$ for $C_9(\mu)$ Wilson coefficient

$$C_9(\mu) = C_9(m_b) + W^{(n_f=4)}(\mu, m_b)\, R\, U^{(n_f=5)}(m_b, M_W)\, C(M_W)\,,$$

$$W^{(n_f=4)}(\mu, m_b) = -\frac{1}{2}\int_{a_s(m_b)}^{a_s(\mu)} da_s\, \frac{\kappa(a_s)}{\beta(a_s)}\, U^{(n_f=4)}(\mu, m_b)\,,$$

$$\frac{d}{d\ln\mu_1} U^{(n_f)}(\mu_1, \mu_2) = \gamma^T(n_f, \mu_1)\, U^{(n_f)}(\mu_1, \mu_2)$$

- The results are

	$C_7^{\rm eff}$	$C_8^{\rm eff}$	C_9	C_{10}	$C_9^{\rm NNLL}$	$C_{10}^{\rm NNLL}$
LL	0.078	-0.055	-0.098	0	-0.488	0
NLL	0.051	-0.062	-0.309	0		

An order of magnitude difference between leading log and NNLL results for C_9!

Short distance/long distance

★ Effective Lagrangian can be obtained by integrating out heavy modes

- Take into account light quarks: "effective Wilson coefficient" $C_9^{eff}(\mu)$

De Boer, Muller, Seidel, 2016

$$C_9^{\rm eff}(\mu,s) = (V_{cd}^* V_{ud} + V_{cs}^* V_{us}) \left(C_9(\mu) + Y^{(ds)}(\mu,s) \right) + V_{cd}^* V_{ud}\, Y^{(d)}(\mu,s) + V_{cs}^* V_{us}\, Y^{(s)}(\mu,s)\,,$$

- … where $Y^{(s)}(\mu,s)$, $Y^{(d)}(\mu,s)$ and $Y^{(ds)}(\mu,s)$ are functions of C and $\log(m_q^2/\mu^2)$

Burdman et al (02),
Fajfer et al (03), Paul et al (11)

★ Long distance effects: hadron resonances and others

- Since-particle resonances modify $C_9^{eff}(\mu)$ as $C_9^{\rm eff} \to C_9^{\rm eff} + \frac{3\pi}{\alpha^2} \sum_i \kappa_i \frac{m_{V_i} \Gamma_{V_i \to l^+ l^-}}{m_{V_i}^2 - s - i m_{V_i} \Gamma_{V_i}}$,

$$C_9^R = a_\rho e^{i\delta_\rho} \left(\frac{1}{q^2 - m_\rho^2 + i m_\rho \Gamma_\rho} - \frac{1}{3} \frac{1}{q^2 - m_\omega^2 + i m_\omega \Gamma_\omega} \right) + \frac{a_\phi e^{i\delta_\phi}}{q^2 - m_\phi^2 + i m_\phi \Gamma_\phi},$$

- Similar modifications are present for other "effective Wilson coefficients"
- In principle, should also contain contributions from two-particle states, etc.

2b. Rare leptonic decays: phenomenology

★ Standard Model contribution to D → $\mu^+\mu^-$.

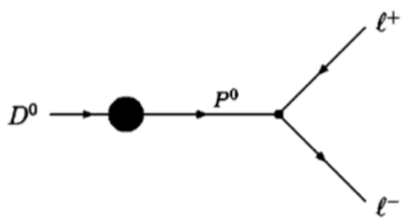
★ Short distance analysis

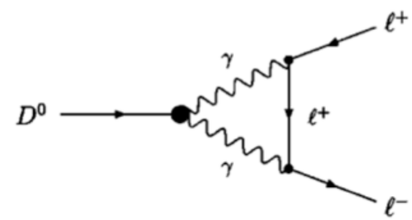
$$B^{(\text{s.d.})}_{D^0\ell^+\ell^-} \simeq \frac{G_F^2 M_W^2 f_D m_\ell}{\pi^2} F \,, \quad F = \sum_{i=d,s,b} V_{ui} V^*_{ci} \left[\frac{x_i}{2} + \frac{\alpha_s}{4\pi} x_i \cdot \left(\ln^2 x_i + \frac{4+\pi^2}{3} \right) \right]$$

- SD effects amount to Br ~ 10^{-18}
- single non-perturbative parameter (decay constant)

UKQCD, HPQCD; Jamin, Lange; Penin, Steinhauser; Khodjamirian

★ Long distance analysis





Burdman, Golowich, Hewett, Pakvasa; Fajfer, Prelovsek, Singer

Update soon: Healey, AAP

$$B^{(\text{mix})}_{D^0\ell^+\ell^-} = \sum_{P_n} \langle P_n | \mathcal{H}^{(\text{p.c.})}_{wk} | D^0 \rangle \frac{1}{M_D^2 - M_{P_n^2}} B_{P_n\ell^+\ell^-}$$

$$\mathcal{I}m\, \mathcal{M}_{D^0\to\ell^+\ell^-} = \frac{1}{2!} \sum_{\lambda_1,\lambda_2} \int \frac{d^3q_1}{2\omega_1(2\pi)^3} \frac{d^3q_2}{2\omega_2(2\pi)^3} \times \mathcal{M}_{D\to\gamma\gamma} \mathcal{M}^*_{\gamma\gamma\to\ell^+\ell^-} (2\pi)^4 \delta^{(4)}(p - q_1 - q_2)$$

- LD effects amount to Br ~ 10^{-13}
- BGHP (2002) paper probably overestimates LD contributions to D → μ+μ- .

Rare semileptonic decays: phenomenology

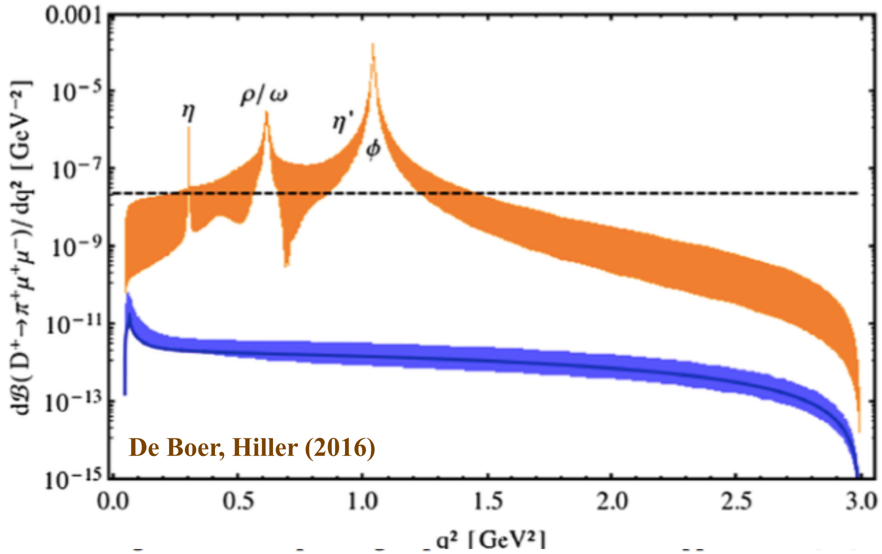
- These decays also proceed at one loop in the SM; GIM is very effective
  - SM rates are expected to be small

De Boer, Hiller (2016)

★ Rare decays $D \to M\, e^+e^-/\mu^+\mu^-$ just like $D \to e^+e^-/\mu^+\mu^-$ are mediated by $c \to u\, ll$

- SM contribution is dominated by LD effects, e.g.

Burdman, Golowich, Hewett, Pakvasa; Fajfer, Prelovsek, Singer



Carve out "resonance window" by comparing $f_{Resonance}/f_{Lead\ Resonance}$ to experimental sensitivity?

q^2 bin	$\mathcal{B}(D^+ \to \pi^+\mu^+\mu^-)^{SM}_{nr}$	90% C.L. [27]
Full q^2: $(2m_\mu)^2 \le q^2 \le (m_{D^+} - m_{\pi^+})^2$	$3.7 \times 10^{-12}(\pm1, \pm3, ^{+16}_{-15}, \pm1, ^{+3}_{-1}, ^{+158}_{-1}, ^{+16}_{-12})$	7.3×10^{-8}
Low q^2: $0.250^2\ \text{GeV}^2 \le q^2 \le 0.525^2\ \text{GeV}^2$	$7.4 \times 10^{-13}(\pm1, \pm4, ^{+23}_{-21}, ^{+10}_{-11}, ^{+10}_{-1}, ^{+238}_{-23}, ^{+6}_{-5})$	2.0×10^{-8}
High q^2: $q^2 \ge 1.25^2\ \text{GeV}^2$	$7.4 \times 10^{-13}(\pm1, \pm6, ^{+15}_{-14}, \pm6, ^{+0}_{-1}, ^{+136}_{-45}, ^{+27}_{-20})$	2.6×10^{-8}

Rare decays: new physics

★ Can New Physics be "hiding" in the up-type quark transitions?

- explicit models can be constructed where it can be done
- long-distance effects complicate interpretation
- must use exp and theo tricks to sort out

$$\begin{aligned}
\mathcal{O}_7 &= \frac{e}{8\pi^2} m_c F_{\mu\nu} \bar{u} \sigma^{\mu\nu} (1+\gamma_5) c, \\
\mathcal{O}_9 &= \frac{e^2}{16\pi^2} \bar{u}_L \gamma_\mu c_L \bar{\ell} \gamma^\mu \ell, \\
\mathcal{O}_{10} &= \frac{e^2}{16\pi^2} \bar{u}_L \gamma_\mu c_L \bar{\ell} \gamma^\mu \gamma_5 \ell, \\
\mathcal{O}_7' &= \frac{e}{8\pi^2} m_c F_{\mu\nu} \bar{u} \sigma^{\mu\nu} (1-\gamma_5) c, \\
\mathcal{O}_9' &= \frac{e^2}{16\pi^2} \bar{u}_R \gamma_\mu c_R \bar{\ell} \gamma^\mu \ell, \\
\mathcal{O}_{10}' &= \frac{e^2}{16\pi^2} \bar{u}_R \gamma_\mu c_R \bar{\ell} \gamma^\mu \gamma_5 \ell,
\end{aligned}$$

Maybe correlations between different measurements can help sorting out NP in charm?

Generic NP contribution to D → $\mu^+\mu^-$

★ Most general effective Hamiltonian:

$$\langle f|\mathcal{H}_{NP}|i\rangle = G\sum_{i=1} \widetilde{C}_i(\mu)\ \langle f|Q_i|i\rangle(\mu)$$

$$\widetilde{Q}_1 = (\bar{\ell}_L\gamma_\mu\ell_L)\ (\bar{u}_L\gamma^\mu c_L)\ , \qquad \widetilde{Q}_4 = (\bar{\ell}_R\ell_L)\ (\bar{u}_R c_L)\ ,$$

$$\widetilde{Q}_2 = (\bar{\ell}_L\gamma_\mu\ell_L)\ (\bar{u}_R\gamma^\mu c_R)\ , \qquad \widetilde{Q}_5 = (\bar{\ell}_R\sigma_{\mu\nu}\ell_L)\ (\bar{u}_R\sigma^{\mu\nu} c_L)\ ,$$

$$\widetilde{Q}_3 = (\bar{\ell}_L\ell_R)\ (\bar{u}_R c_L)\ ,$$

plus L ↔ R

★ ... thus, the amplitude for D → $e^+e^-/\mu^+\mu^-$ decay is

$$\mathcal{B}_{D^0\to\ell^+\ell^-} = \frac{M_D}{8\pi\Gamma_D}\sqrt{1-\frac{4m_\ell^2}{M_D^2}}\left[\left(1-\frac{4m_\ell^2}{M_D^2}\right)|A|^2+|B|^2\right]$$

$$|A| = G\frac{f_D M_D^2}{4m_c}\left[\widetilde{C}_{3-8}+\widetilde{C}_{4-9}\right]\ ,$$

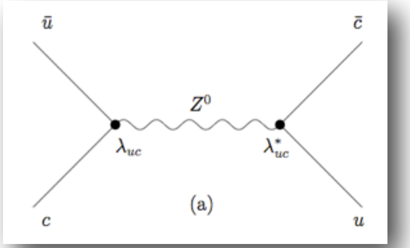
$$|B| = G\frac{f_D}{4}\left[2m_\ell\left(\widetilde{C}_{1-2}+\widetilde{C}_{6-7}\right)+\frac{M_D^2}{m_c}\left(\widetilde{C}_{4-3}+\widetilde{C}_{9-8}\right)\right], \quad \widetilde{C}_{i-k}\equiv\widetilde{C}_i-\widetilde{C}_k$$

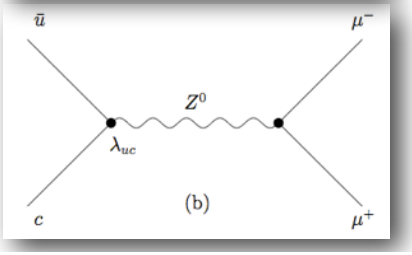
Many NP models give contributions to both D-mixing and D → $e^+e^-/\mu^+\mu^-$ decay: correlate!!!

Mixing vs rare decays: ruling out models

★ Relating mixing and rare decay
- consider an example: heavy vector-like quark (Q=+2/3)
  - appears in little Higgs models, etc.

E.Golowich, J. Hewett, S. Pakvasa and A.A.P. PRD79, 114030 (2009)





$$\lambda_{uc} \equiv -\left(V^*_{ud}V_{cd} + V^*_{us}V_{cs} + V^*_{ub}V_{cb}\right)$$

Mixing:

$$\mathcal{H}_{2/3} = \frac{g^2}{8\cos^2\theta_w M_Z^2}\lambda_{uc}^2\, Q_1 \;=\; \frac{G_F\lambda_{uc}^2}{\sqrt{2}}Q_1$$

$$x_{\rm D}^{(+2/3)} = \frac{2G_F\lambda_{uc}^2 f_D^2 M_D B_D r(m_c, M_Z)}{3\sqrt{2}\Gamma_D}$$

Rare decay:

$$A_{D^0\to\ell^+\ell^-} = 0 \qquad B_{D^0\to\ell^+\ell^-} = \lambda_{uc}\frac{G_F f_{\rm D} m_\mu}{2}$$

$$\mathcal{B}_{D^0\to\mu^+\mu^-} = \frac{3\sqrt{2}}{64\pi}\,\frac{G_F m_\mu^2 x_{\rm D}}{B_{\rm D} r(m_c, M_Z)}\left[1-\frac{4m_\mu^2}{M_{\rm D}}\right]^{1/2} \simeq 4.3\times 10^{-9} x_{\rm D} \;\le\; 4.3\times 10^{-11} \;.$$

Note: a NP parameter-free relation!

Rare semileptonic decays: New Physics

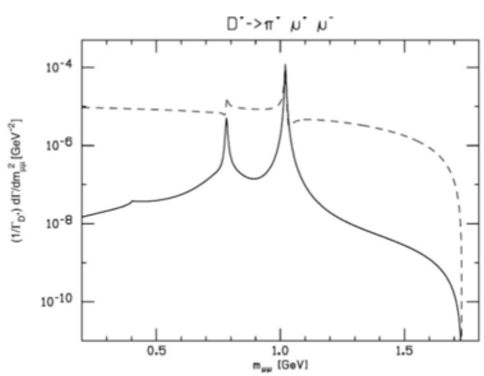
- Rare semileptonic decays can be used to study New Physics

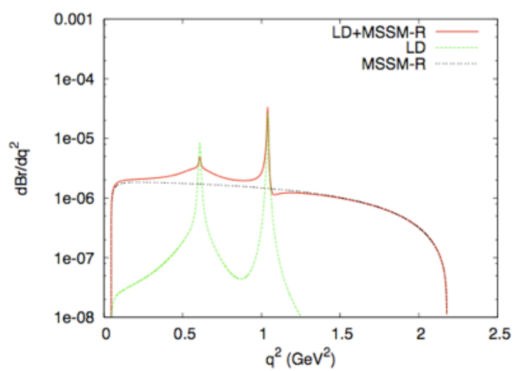
★ Example: R-partity-violating SUSY/leptoquarks
  - operators with the same parameters contribute to D-mixing
  - feed results into rare decays

Burdman, Golowich, Hewett, Pakvasa; Fajfer, Prelovsek, Singer

Mode	LD	Extra heavy q	LD + extra heavy q
$D^+ \to \pi^+ e^+ e^-$	2.0×10^{-6}	1.3×10^{-9}	2.0×10^{-6}
$D^+ \to \pi^+ \mu^+ \mu^-$	2.0×10^{-6}	1.6×10^{-9}	2.0×10^{-6}
Mode	MSSM$\not R$	LD + MSSM$\not R$	
$D^+ \to \pi^+ e^+ e^-$	2.1×10^{-7}	2.3×10^{-6}	
$D^+ \to \pi^+ \mu^+ \mu^-$	6.5×10^{-6}	8.8×10^{-6}	

Fajfer, Kosnik, Prelovsek

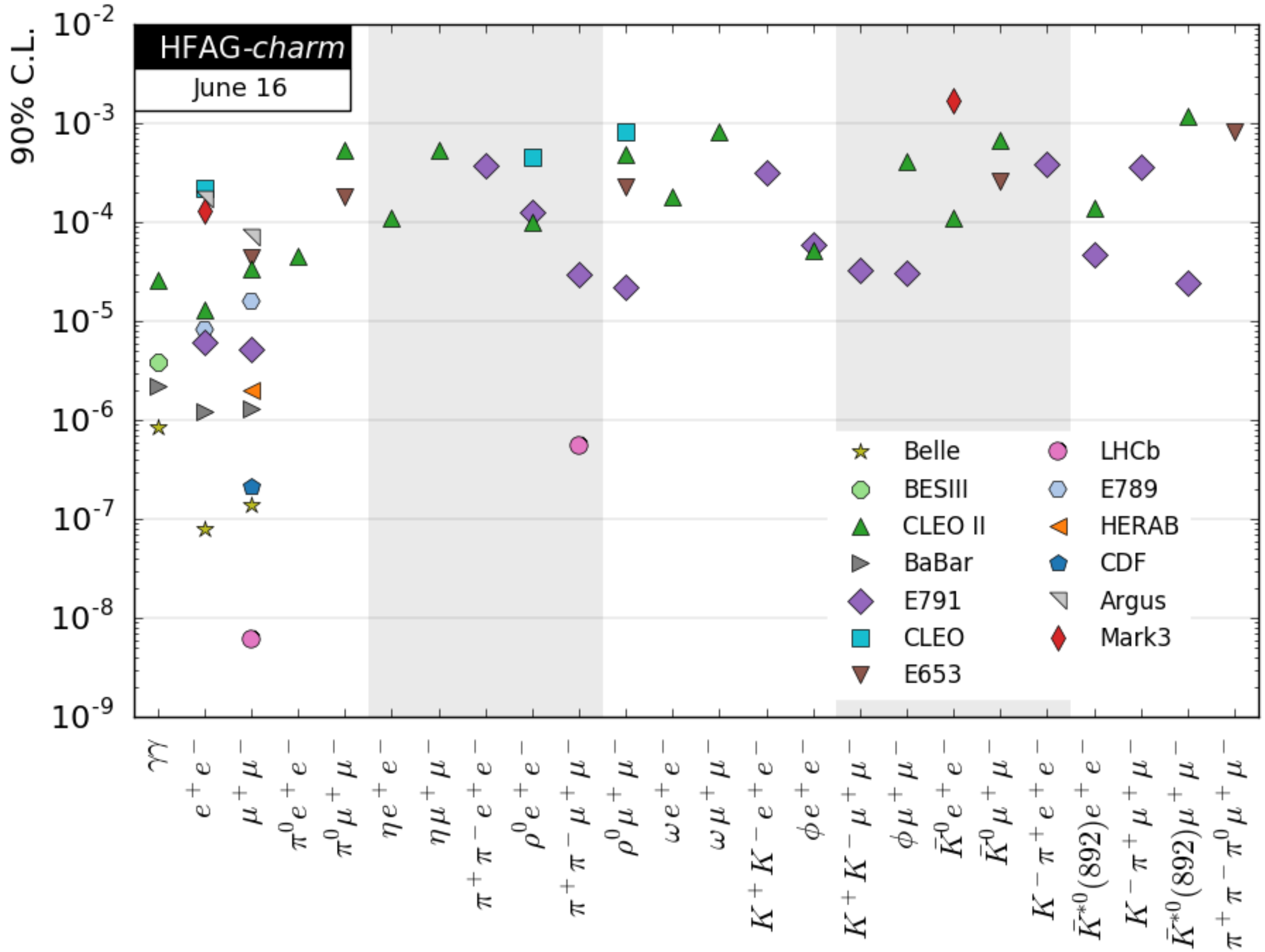




★ Direct CP-violating asymmetries in D -> $\pi\mu+\mu-$?

De Boer, Hiller (2016)

Rare charm decays: experiment



Any other ideas?

★ Two-body decays of B and D

- only one hadron to deal with: decay constant?
- but: probes limited number of operators, helicity suppression
  - e.g. not sensitive to vector-like New Physics (such as vector Z′)
- soft photon effects preclude studies of electron decay modes:

$$\frac{\mathcal{B}(D^0 \to \gamma \ell^+ \ell^-)}{\mathcal{B}(D^0 \to \ell^+ \ell^-)} \propto \alpha \frac{m_D^2}{m_\ell^2}$$

★ Three-body decays of B and D

- probes several operators, many different observables
- but: two hadrons: four form-factors, hard to calculate non-perturbatively
- recent "issues" with lepton universality in B-decays $R_K = \frac{\mathrm{BR}(B \to K\mu^+\mu^-)}{\mathrm{BR}(B \to Ke^+e^-)}$ $= 0.745^{+0.090}_{-0.074}\ (\mathrm{stat})\ \pm 0.036\ (\mathrm{syst})$

LHCb (2014)

Can one remove helicity suppression AND enlarge the set of probed operators by studying electroweak decays of excited states of D or B (like D* or B*)?

Can $D^*(B^*) \to e^+e^-$ or $\mu^+\mu^-$ decay be measured?

Can apparent superluminal neutrino speeds be explained as a quantum weak measurement?

M V Berry[1], N Brunner[1], S Popescu[1] and P Shukla[2]

[1] H H Wills Physics Laboratory, Tyndall Avenue, Bristol BS8 1TL, UK
[2] Department of Physics, Indian Institute of Technology, Kharagpur, India

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Abstract
Probably not.

PACS numbers: 03.65.Ta, 03.65.Xp, 14.60.Pq

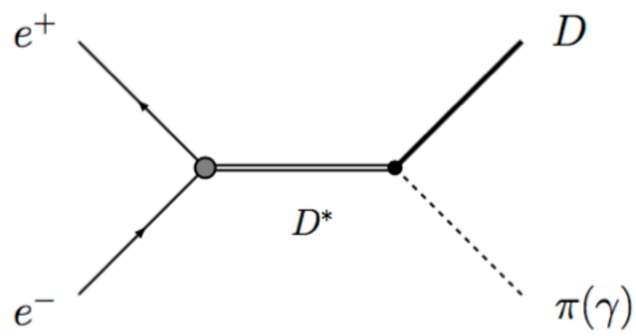
http://arxiv.org/abs/1110.2832

Studies of $D^*(B^*) \to e^+e^-$ in resonance production

★ Instead of searching for a decay of D*/B*, let's produce it!

- resonant enhancement possible if e+e- energy is tuned to $m_{D^*}(m_{B^*})$
- single heavy flavor + photon in the final state is a nice tag

Khodjamirian, Mannel, AAP
JHEP 11 (2015) 142



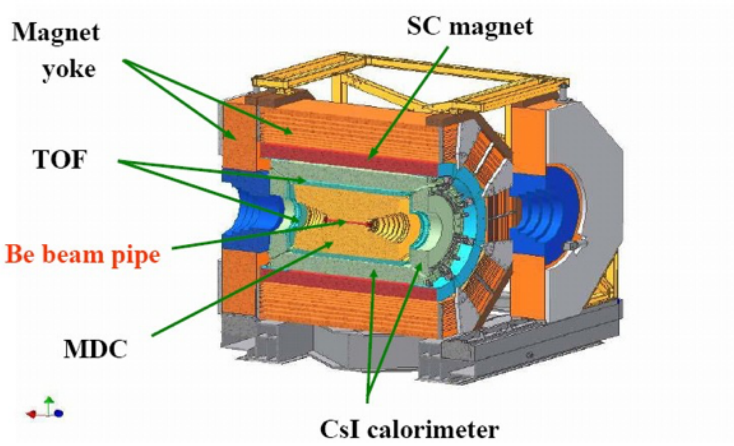
- contrary to a usual way of studying FCNC, production cross section is small

★ This way, the FCNC branching ratio for D*(2007) → e+e- is probed

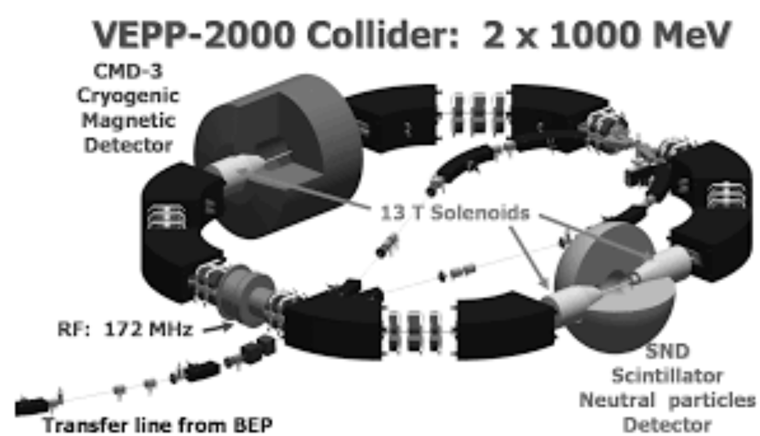
$$\sigma(e^+e^- \to D\pi)_{\sqrt{s}\simeq m_{D^*}} \equiv \sigma_{D^*}(s) = \frac{12\pi}{m_{D^*}^2} \mathcal{B}_{D^*\to e^+e^-} \mathcal{B}_{D^*\to D\pi} \frac{m_{D^*}^2 \Gamma_0^2}{(s - m_{D^*}^2)^2 + m_{D^*}^2 \Gamma_0^2},$$

Studies of $D^*(B^*) \to e^+e^-$ in resonance production

The BESIII Detector



★ BEPCII machine with BESIII detector (China)
- optimized for Psi(3770)
- already made scans $\sqrt{s} = 2.0 - 4.2$ GeV.
- luminosity is about $5\text{x}10^{31}$ $cm^{-2}s^{-1}$



★ VEPP-2000 machine (Novosibirsk, Russia)
- optimized for E_{CM} < 2000 MeV
- possible upgrade to E_{CM} > 2000 MeV
- luminosity is about $1\text{x}10^{32}$ $cm^{-2}s^{-1}$

★ HIEPA: new tau-charm factory in Hefei (if approved)
-luminosity is about $5\text{x}10^{34}$ $cm^{-2}s^{-1}$

Studies of $D^*(B^*) \to e^+e^-$ in the Standard Model

★ Standard Model, short distance:

- local O_9 and O_{10} operators $$O_9 = \frac{e^2}{16\pi^2}\left(\widetilde{Q}_1 + \widetilde{Q}_7\right), \quad O_{10} = \frac{e^2}{16\pi^2}\left(\widetilde{Q}_7 - \widetilde{Q}_1\right)$$
- additional dipole contribution $$H_{\text{eff}}^{(7\gamma)} = \frac{4G_F}{\sqrt{2}} C_7^{\text{c,eff}} \left(\frac{e}{16\pi^2} m_c\, \overline{u}_L \sigma^{\mu\nu} c_R F_{\mu\nu}\right)$$

★ Decay amplitude depends on additional non-perturbative parameter

$$\langle 0 | \overline{u} \sigma^{\mu\nu} c | D^*(p) \rangle = i f_{D^*}^T \left(\epsilon^\mu p^\nu - p^\mu \epsilon^\nu\right)$$

Khodjamirian, Mannel, AAP
JHEP 11 (2015) 142

★ Short-distance result is well-defined

$$\mathcal{B}_{D^* \to e^+e^-} = \frac{\alpha^2 G_F^2}{96\pi^3 \Gamma_0} m_{D^*}^3 f_{D^*}^2 \left(\left| C_9^{\text{c,eff}} + 2\frac{m_c}{m_{D^*}} \frac{f_{D^*}^T}{f_{D^*}} C_7^{\text{c,eff}} \right|^2 + |C_{10}^c|^2 \right)$$

★ … but the Br is small (the width is not though): $\mathcal{B}_{D^* \to e^+e^-}^{SD} = \frac{\Gamma(D^* \to e^+e^-)}{\Gamma_0} \approx 2.0 \times 10^{-19}$

★ LD contributions are of the same order of magnitude or less!!!

No helicity suppression: no issues with testing lepton universality!

$D^*(B^*) \to e^+e^-$: example of NP contribution

★ A plethora of NP models that realize charm (beauty) FCNC interactions can be probed
- consider a model with a Z′ coupling to a left-handed FCNC quark currents

$$\mathcal{L}_{Z'} = -g'_{Z'1}\bar{\ell}_L\gamma_\mu\ell_L Z'^\mu - g'_{Z'2}\bar{\ell}_R\gamma_\mu\ell_R Z'^\mu$$
$$- g^{cu}_{Z'1}\bar{u}_L\gamma_\mu c_L Z'^\mu - g^{cu}_{Z'2}\bar{u}_R\gamma_\mu c_R Z'^\mu.$$

★ At low energies integrate out Z′:

$$\mathcal{L}^{Z'}_{\text{eff}} = -\frac{1}{M^2_{Z'}}\left[g'_{Z'1}g^{cu}_{Z'1}\widetilde{Q}_1 + g'_{Z'1}g^{cu}_{Z'2}\widetilde{Q}_2 + g'_{Z'2}g^{cu}_{Z'2}\widetilde{Q}_6 + g'_{Z'2}g^{cu}_{Z'1}\widetilde{Q}_7\right]$$

★ …which leads to a branching ratio (for $g'_{Z'1} = \frac{g}{\cos\theta_W}\left(-\frac{1}{2} + \sin^2\theta_W\right), \qquad g'_{Z'2} = \frac{g\sin^2\theta_W}{\cos\theta_W},$)

$$\mathcal{B}^{Z'}_{D^*\to e^+e^-} = \frac{\sqrt{2}G_F}{3\pi\Gamma_0} m^3_{D^*} f^2_{D^*} \frac{|g^{cu}_{Z'1}|^2}{M^2_{Z'}} \frac{M^2_Z}{M^2_{Z'}} \left(\frac{1}{4} - \sin^2\theta_W + 2\sin^4\theta_W\right)$$

★ … and current constraint of $\mathcal{B}^{Z'}_{D^*\to e^+e^-} < 2.5 \times 10^{-11}$

Plenty of room in the parameter space to constrain

2c. Studies of lepton flavor violation with charm

- Non-diagonal lepton flavor Lagrangians can be studies with charm
  - large amounts of charm data and clean experimental signature
  - (almost) no SM background: clear sign of New Physics

$$\widetilde{Q}_1 = (\overline{\ell}_L \gamma_\mu \ell'_L)\ (\overline{u}_L \gamma^\mu c_L)\ , \qquad \widetilde{Q}_4 = (\overline{\ell}_R \ell'_L)\ (\overline{u}_R c_L)\ ,$$
$$\widetilde{Q}_2 = (\overline{\ell}_L \gamma_\mu \ell'_L)\ (\overline{u}_R \gamma^\mu c_R)\ , \qquad \widetilde{Q}_5 = (\overline{\ell}_R \sigma_{\mu\nu} \ell'_L)\ (\overline{u}_R \sigma^{\mu\nu} c_L)\ ,$$
$$\widetilde{Q}_3 = (\overline{\ell}_L \ell'_R)\ (\overline{u}_R c_L)\ ,$$

plus L ↔ R

- These operators drive LFV D-decays, e.g.

$$\mathcal{B}_{D^0 \to \mu^+ e^-} = \frac{M_D}{8\pi \Gamma_{\rm D}} \left(1 - \frac{m_\mu^2}{M_D^2}\right)^2 \left[|A|^2 + |B|^2\right]$$

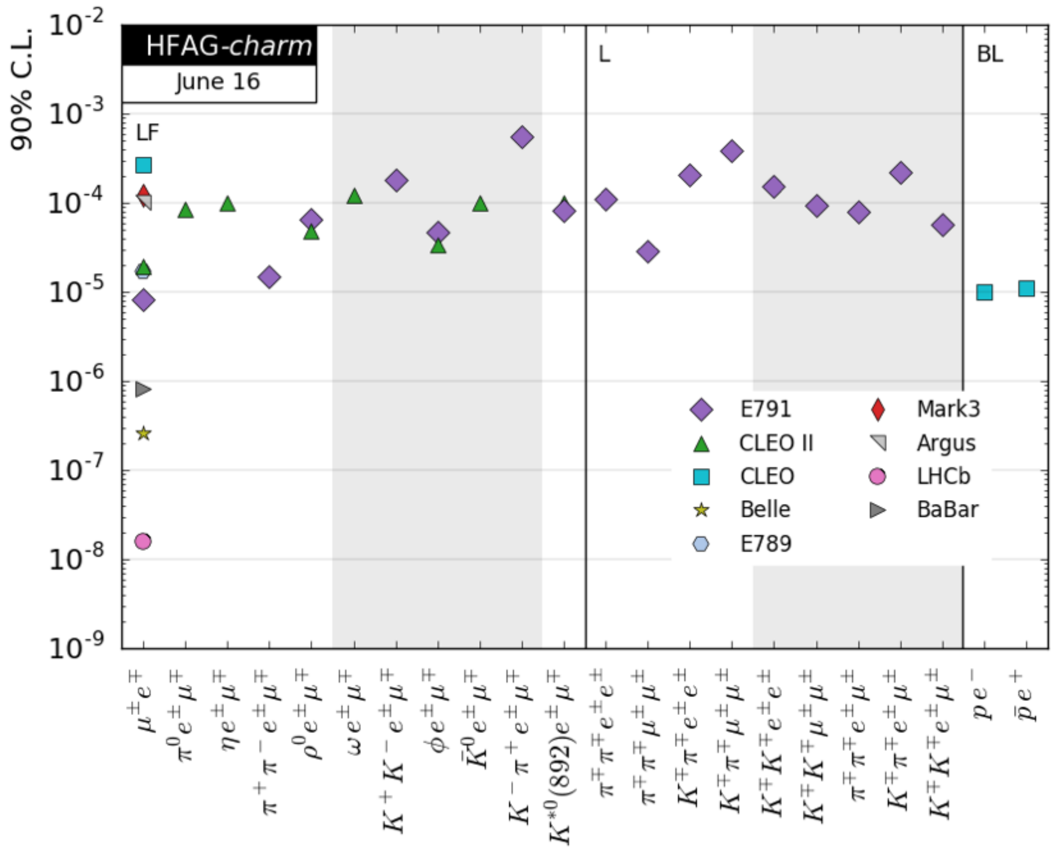
$$|A| = G \frac{f_D M_D^2}{4 m_c} \left[\widetilde{C}_{3-8} + \widetilde{C}_{4-9}\right]\ ,$$
$$|B| = G \frac{f_D}{4} \left[2 m_\ell \left(\widetilde{C}_{1-2} + \widetilde{C}_{6-7}\right) + \frac{M_D^2}{m_c}\left(\widetilde{C}_{4-3} + \widetilde{C}_{9-8}\right)\right], \quad \widetilde{C}_{i-k} \equiv \widetilde{C}_i - \widetilde{C}_k$$

Note: either still helicity suppressed or depends on the Wilson coefficient of the scalar/pseudoscalar operator which usually contains lepton mass!

Studies of lepton flavor violation with charm

- To lift helicity suppression, add more particles to the final state: γ, π, ρ, e.g.

$$D^0 \to \gamma \mu^\pm e^\mp, D^0 \to \pi \mu^\pm e^\mp, D^0 \to \rho \mu^\pm e^\mp, \text{etc.}$$



- … or consider other hadronic systems, e.g. LFV quarkonia decays $M \to \ell_1 \bar{\ell}_2$ or $M \to \gamma \ell_1 \bar{\ell}_2$

Unfortunately, very scarce experimental data!

D. Hazard and A.A.P., PRD94 (2016), 074023

Studies of lepton flavor violation with charm

★ Multitude of NP operators: single operator dominance hypothesis (SODH)

- but it is not often that only a single operator contributes, e.g. for quarkonia

$$\mathcal{L}_{lq} = -\frac{1}{\Lambda^2}\sum_q \Big[\Big(C^q_{VR}\bar{\ell}_1\gamma^\mu P_R \ell_2 + C^q_{VL}\bar{\ell}_1\gamma^\mu P_L \ell_2\Big)\bar{q}\gamma_\mu q$$

$$+\Big(C^q_{AR}\bar{\ell}_1\gamma^\mu P_R \ell_2 + C^q_{AL}\bar{\ell}_1\gamma^\mu P_L \ell_2\Big)\bar{q}\gamma_\mu\gamma_5 q$$

$$+ m_2 m_q G_F\Big(C^q_{SR}\bar{\ell}_1 P_R \ell_2 + C^q_{SL}\bar{\ell}_1 P_L \ell_2\Big)\bar{q}q$$

$$+ m_2 m_q G_F\Big(C^q_{PR}\bar{\ell}_1 P_R \ell_2 + C^q_{PL}\bar{\ell}_1 P_L \ell_2\Big)\bar{q}\gamma_5 q + h.c.\Big]$$

- Can (partially) do away with SODH if designer initial states are used

Vector:

$$\mathcal{A}(V \to \ell_1\bar{\ell}_2) = \bar{u}(p_1, s_1)\left[A_V^{\ell_1\ell_2}\gamma_\mu + B_V^{\ell_1\ell_2}\gamma_\mu\gamma_5 + \frac{C_V^{\ell_1\ell_2}}{m_V}(p_2-p_1)_\mu + \frac{iD_V^{\ell_1\ell_2}}{m_V}(p_2-p_1)_\mu\gamma_5\right] v(p_2, s_2)\ \epsilon^\mu(p).$$

Scalar:

$$\mathcal{A}(S \to \ell_1\bar{\ell}_2) = \bar{u}(p_1, s_1)\left[E_S^{\ell_1\ell_2} + iF_S^{\ell_1\ell_2}\gamma_5\right] v(p_2, s_2)$$

- Scalar states can be accessed via radiative decays $\psi(2S) \to \gamma\chi_{c0}$ (O(10%) fractions)
- No data (other than J/psi) exist!!!

D. Hazard and A.A.P., PRD94 (2016), 074023

- To follow Prof. Browder's lead (see Monday's plenary talk), we need more data

Apologies to Q. Tarantino

3. Rare D(B)-decays with missing energy

➢ D-decays with missing energy can probe both heavy and light (DM) NP

★ SM process: D → $\nu\nu$ and D → $\nu\nu\gamma$:

- for B-decays $J^{\mu}_{Qq} = \bar{q}_L \gamma^{\mu} b_L$
- for D-decays $J^{\mu}_{Qq} = \bar{u}_L \gamma^{\mu} c_L$

★ For B(D) → $\nu\nu$ decays SM branching ratios are tiny

- SM decay is helicity suppressed, e.g.

$$\mathcal{B}(B_s \to \nu\bar{\nu}) = \frac{G_F^2 \alpha^2 f_B^2 M_B^3}{16\pi^3 \sin^4\theta_W \Gamma_{B_s}} |V_{tb} V_{ts}^*|^2 X(x_t)^2 x_\nu^2$$

Decay	Branching ratio
$B_s \to \nu\bar{\nu}$	3.07×10^{-24}
$B_d \to \nu\bar{\nu}$	1.24×10^{-25}
$D^0 \to \nu\bar{\nu}$	1.1×10^{-30}

- NP: other ways of flipping helicity?
- add a third particle to the final state?

What would happen if a photon is added to the final state?

Decay	Branching ratio
$B_s \to \nu\bar{\nu}\gamma$	3.68×10^{-8}
$B_d \to \nu\bar{\nu}\gamma$	1.96×10^{-9}
$D^0 \to \nu\bar{\nu}\gamma$	3.96×10^{-14}

★ For B(D) → $\nu\nu\gamma$ decays SM branching ratios are still tiny

- need form-factors to describe the transition
- helicity suppression is lifted

★ BUT: missing energy does not always mean neutrinos

- nice constraints on light Dark Matter properties!!!

Badin, AAP (2010)

Rare D(B)-decays: scalar DM

- Let us discuss B and D-decays simultaneously: physics is similar

Badin, AAP (2010)

★ Generic interaction Lagrangian: $\mathcal{H}_{eff} = \sum_i \frac{2C_i^{(s)}}{\Lambda^2} O_i$

- respective neutral currents for B-and D-decays

$$O_1 = m_Q \left(J_{Qq}\right)_{RL} \ (\chi_0^* \chi_0)$$
$$O_2 = m_Q \left(J_{Qq}\right)_{LR} \ (\chi_0^* \chi_0)$$
$$O_3 = \left(J_{Qq}^\mu\right)_{LL} \ \left(\chi_0^* \overleftrightarrow{\partial}_\mu \chi_0\right)$$
$$O_4 = \left(J_{Qq}^\mu\right)_{RR} \ \left(\chi_0^* \overleftrightarrow{\partial}_\mu \chi_0\right)$$

★ Scalar DM does not exhibit helicity suppression

- B(D) → E_{mis} is more powerful than B(D) → $E_{mis}\,\gamma$

$$\mathcal{B}(B_q \to \chi_0 \chi_0) = \frac{(C_1^{(s)} - C_2^{(s)})^2}{4\pi M_{B_q} \Gamma_{B_q}} \left(\frac{f_{B_q} M_{B_q}^2 m_b}{\Lambda^2 (m_b + m_q)}\right)^2 \times \sqrt{1 - 4x_\chi^2},$$

$$\mathcal{B}(B_q \to \chi_0^* \chi_0 \gamma) = \frac{f_{B_q}^2 \alpha C_3^{(s)} C_4^{(s)} M_{B_q}^5}{6\Lambda^4 \Gamma_{B_q}} \left(\frac{F_{B_q}}{4\pi}\right)^2 \times \left(\frac{1}{6}\sqrt{1 - 4x_\chi^2}(1 - 16x_\chi^2 - 12x_\chi^4) - 12x_\chi^4 \log \frac{2x_\chi}{1 + \sqrt{1 - 4x_\chi^2}}\right).$$

$$\left(\frac{C_1^{(s)} - C_2^{(s)}}{\Lambda^2}\right)^2 \leq 2.07 \times 10^{-16}\ \text{GeV}^{-4} \quad \text{for } m_\chi = 0.1 \times M_{B_d},$$

$$\frac{C_3^{(s)}}{\Lambda^2} \frac{C_4^{(s)}}{\Lambda^2} \leq 1.55 \times 10^{-12}\ \text{GeV}^{-4} \quad \text{for } m = 0,$$

$$\frac{C_3^{(s)}}{\Lambda^2} \frac{C_4^{(s)}}{\Lambda^2} \leq 7.44 \times 10^{-11}\ \text{GeV}^{-4} \quad \text{for } m = 0.4 \times M_{B_d}$$

These general bounds translate into constraints onto constraints for particular models

Example of a particular model of scalar DM

★ Several different models of light scalar DM
- simplest: singlet scalar DM
- more sophisticated - less restrictive

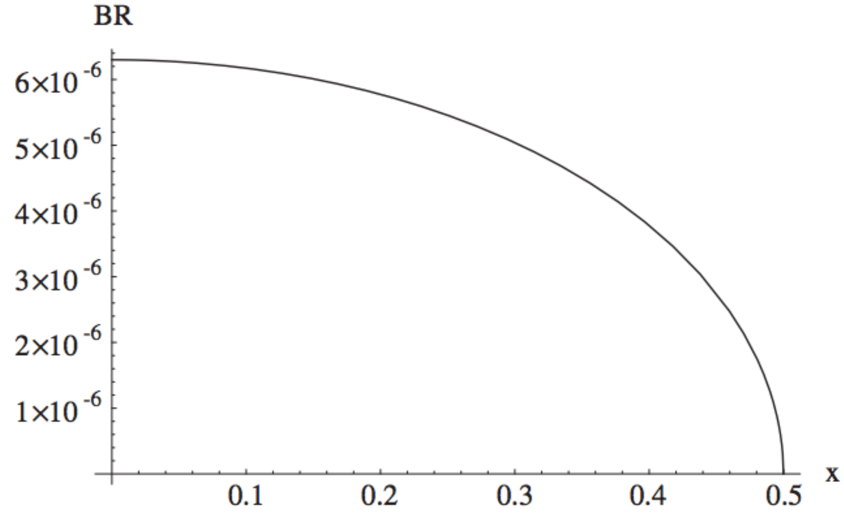
$$-\mathcal{L}_S = \frac{\lambda_S}{4}S^4 + \frac{m_0^2}{2}S^2 + \lambda S^2 H^\dagger H$$
$$= \frac{\lambda_S}{4}S^4 + \frac{1}{2}(m_0^2 + \lambda v_{\rm EW}^2)S^2 + \lambda v_{\rm EW} S^2 h$$
$$+ \frac{\lambda}{2}S^2 h^2,$$

★ B(D) decays rate in this model

$$\mathcal{B}(B_q \to SS) = \left[\frac{3g_w^2 V_{tb}V_{tq}^* x_t m_b}{128\pi^2}\right]^2 \frac{\sqrt{1-4x_S^2}}{16\pi M_B \Gamma_{B_q}}\left(\frac{\lambda^2}{M_H^4}\right)$$
$$\times \left(\frac{f_{B_q}M_{B_q}^2}{m_b + m_q}\right)^2,$$

- fix λ from relic density

$$\sigma_{\rm ann} v_{\rm rel} = \frac{8 v_{\rm EW}^2 \lambda^2}{M_H^2} \times \lim_{m_{h^*}\to 2m_S} \frac{\Gamma_{h^* X}}{m_h^*}$$



These results are complimentary to constraints from quarkonium decays with missing energy

Rare D(B)-decays: fermionic DM

★ Generic interaction Lagrangian: $\mathcal{H}_{eff} = \sum_i \frac{4C_i}{\Lambda^2} O_i$

- respective neutral currents for B-and D-decays

$$O_1 = \left(J^\mu_{Qq}\right)_{LL} \left(\bar{\chi}_{1/2L}\gamma_\mu \chi_{1/2L}\right)$$
$$O_2 = \left(J^\mu_{Qq}\right)_{LL} \left(\bar{\chi}_{1/2R}\gamma_\mu \chi_{1/2R}\right)$$
$$O_3 = O_{1(L\leftrightarrow R)}, \quad O_4 = O_{2(L\leftrightarrow R)}$$
$$O_5 = \left(J_{Qq}\right)_{LR} \left(\bar{\chi}_{1/2L}\, \chi_{1/2R}\right)$$
$$O_6 = \left(J_{Qq}\right)_{LR} \left(\bar{\chi}_{1/2R}\, \chi_{1/2L}\right)$$
$$O_7 = O_{5(L\leftrightarrow R)}, \quad O_8 = O_{6(L\leftrightarrow R)}$$
$$+ \ \text{tensor operators}$$

★ Scalar DM does exhibit helicity suppression

- B(D) → E_{mis} maybe less powerful than B(D) → $E_{mis}\,\gamma$
- … but it really depends on the DM mass!

Badin, AAP

$$\mathcal{B}(B_q \to \bar{\chi}_{1/2}\chi_{1/2}) = \frac{f^2_{B_q} M^3_{B_q}}{16\pi \Gamma_{B_q} \Lambda^2}\sqrt{1-4x^2_\chi} \times \left[C_{57}C_{68}\frac{4M^2_{B_q}x^2_\chi}{(m_b+m_q)^2} - (C^2_{57}+C^2_{68}) \times \frac{M^2_{B_q}(2x^2_\chi - 1)}{(m_b+m_q)^2} - 2\tilde{C}_{1-8}\frac{x_\chi M_{B_q}}{m_b+m_q} + 2(C_{13}+C_{24})^2 x^2_\chi \right],$$

Lots of operators — less so in particular models

Rare D(B)-decays: fermionic DM

★ Constraints from B decays are the best at the moment

TABLE I. Constraints (upper limits) on the Wilson coefficients of operators of Eq. (43) from the $B_q \to \chi_{1/2}\bar{\chi}_{1/2}$ transition. Note that operators Q_9–Q_{12} give no contribution to this decay.

x_χ	C_1/Λ^2, GeV^{-2}	C_2/Λ^2, GeV^{-2}	C_3/Λ^2, GeV^{-2}	C_4/Λ^2, GeV^{-2}	C_5/Λ^2, GeV^{-2}	C_6/Λ^2, GeV^{-2}	C_7/Λ^2, GeV^{-2}	C_8/Λ^2, GeV^{-2}
0	...	...	...	...	2.3×10^{-8}	2.3×10^{-8}	2.3×10^{-8}	2.3×10^{-8}
0.1	1.9×10^{-7}	1.9×10^{-7}	1.9×10^{-7}	1.9×10^{-7}	2.3×10^{-8}	2.3×10^{-8}	2.3×10^{-8}	2.3×10^{-8}
0.2	9.7×10^{-8}	9.7×10^{-8}	9.7×10^{-8}	9.7×10^{-8}	2.5×10^{-8}	2.5×10^{-8}	2.5×10^{-8}	2.5×10^{-8}
0.3	6.9×10^{-8}	6.9×10^{-8}	6.9×10^{-8}	6.9×10^{-8}	2.8×10^{-8}	2.8×10^{-8}	2.8×10^{-8}	2.8×10^{-8}
0.4	6.0×10^{-8}	6.0×10^{-8}	6.0×10^{-8}	6.0×10^{-8}	3.6×10^{-8}	3.6×10^{-8}	3.6×10^{-8}	3.6×10^{-8}

★ … the same is true for the radiative decays with missing energy

TABLE II. Constraints (upper limits) on the Wilson coefficients of operators of Eq. (43) from the $B_q \to \chi_{1/2}\bar{\chi}_{1/2}\gamma$ transition. Note that operators Q_5–Q_8 give no contribution to this decay.

x_χ	C_1/Λ^2, GeV^{-2}	C_2/Λ^2, GeV^{-2}	C_3/Λ^2, GeV^{-2}	C_4/Λ^2, GeV^{-2}
0	6.3×10^{-7}	6.3×10^{-7}	6.3×10^{-7}	6.3×10^{-7}
0.1	7.0×10^{-7}	7.0×10^{-7}	7.0×10^{-7}	7.0×10^{-7}
0.2	9.2×10^{-7}	9.2×10^{-7}	9.2×10^{-7}	9.2×10^{-7}
0.3	1.5×10^{-6}	1.5×10^{-6}	1.5×10^{-6}	1.5×10^{-6}
0.4	3.4×10^{-6}	3.4×10^{-6}	3.4×10^{-6}	3.4×10^{-6}

These general bounds translate into constraints onto constraints for particular models

4. Things to take home

- Indirect probes for new physics compete well with direct searches
  - for some observables sensitive to scales way above LHC
- Calculational techniques for heavy flavors are well-established
  - but don't always work well: "heavy-quark-expansion" techniques for charm often miss threshold effects
  - "hadronic" techniques that sum over large number of intermediate states can be used, BUT one cannot use current experimental data on D-decays
- Calculations of New Physics contributions to mixing are in better shape
- Can correlate mixing and rare decays with New Physics models
  - signals in B/D-mixing vs B/D rare decays help differentiate among models
- New reach: $D^*(B^*) \to e^+e^-$ can be studied with resonance production
  - plenty of parameter space for New Physics reach
  - probes models that $D(B) \to e^+e^-/\mu^+\mu^-$ are not sensitive to
- Rare decays with missing energy provide excellent opportunities to constrain parameters of models with light Dark Matter
  - both scalar and sermonic DM models can be constrained


EXTRA!

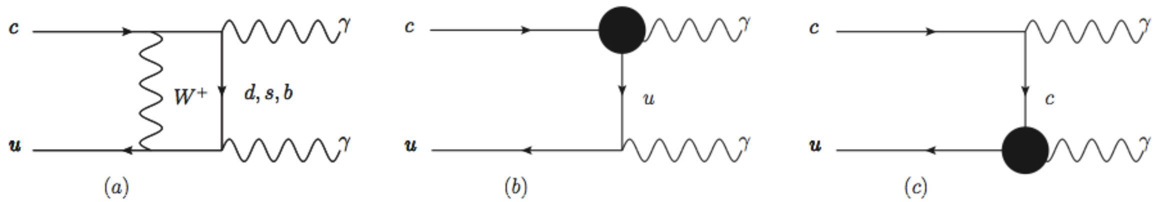
Rare radiative decays of charm I

★ Standard Model contribution to D → γγ

$$A(D \to \gamma\gamma) = \epsilon_{1\mu}\epsilon_{2\nu}\left[A_{PC}\epsilon^{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta} + iA_{PV}\left(g^{\mu\nu} - \frac{k_2^{\mu}k_1^{\nu}}{k1\cdot k_2}\right)\right]$$

$$\Gamma(D \to \gamma\gamma) = \frac{m_D^3}{64\pi}\left[\left|A_{PC}\right|^2 + \frac{4}{m_D^4}\left|A_{PV}\right|^2\right]$$

★ Short distance analysis $\mathcal{L} = -\frac{G_f}{\sqrt{2}}V_{us}V_{cs}^{*}C_{7\gamma}^{eff}\frac{e}{4\pi^2}F_{\mu\nu}m_c\left(\bar{u}\sigma^{\mu\nu}\frac{1}{2}(1+\gamma_5)c\right)$



Paul, Bigi, Recksiegel (2011)

- only one operator contributes
- including QCD corrections, SD effects amount to Br = $(3.6\text{-}8.1)\times10^{-12}$

★ Long distance analysis

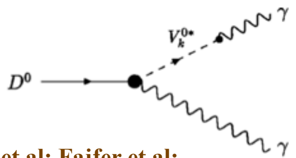
- long distance effects amount to Br = $(1\text{-}3)\times10^{-8}$

Burdman, Golowich, Hewett, Pakvasa (02);
Fajfer, Singer, Zupan (01)

Rare radiative decays of charm II

- Hope to isolate penguin-like contribution: BUT SM GIM is very effective
  - SM penguin contributions are expected to be small

★ Radiative decays $D \to \gamma X$, $\gamma\gamma$: FCNC transition $c \to u\,\gamma$



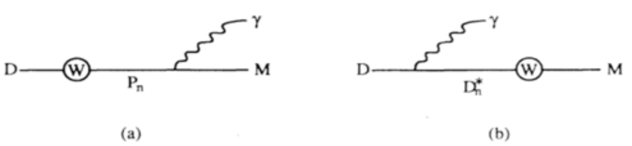
+ others

Burdman et al; Fajfer et al; Greub, Hurth, Misiak, Wyler

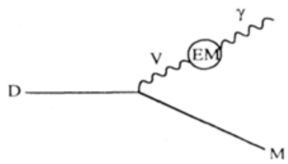
$$Q_7 = \frac{e}{8\pi^2} m_c F_{\mu\nu} \bar{u} \sigma^{\mu\nu} (1 + \gamma_5) c,$$

- SM contribution is dominated by LD effects
- dominated by SM anyway: useless for NP studies?

★ Examples of long-distance contributions



pole amplitudes (WA/WS)



VMD amplitudes

Rare radiative decays of charm II

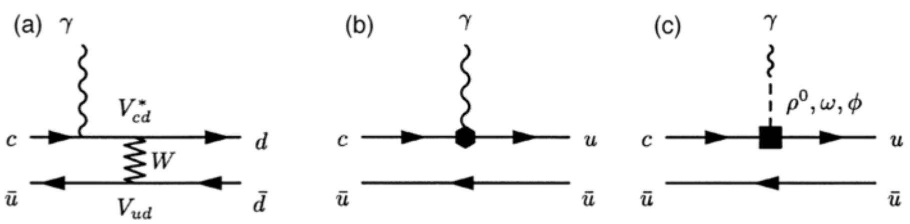
★ Theoretical predictions and experimental bounds (x 10^5)

$D \to V\gamma$	Burdman et al	Fajfer et al	Khodjamirian et al
$D_s^+ \to \rho^+\gamma$	6–38	20–80	4.4
$D^0 \to \bar{K}^{*0}\gamma$	7–12	6–36	0.18
$D^0 \to \rho^0\gamma$	0.1–0.5	0.1–1	0.38
$D^0 \to \omega^0\gamma$	$\simeq$0.2	0.1–0.9	—
$D^0 \to \phi^0\gamma$	0.1–3.4	0.4–0.9	—
$D^+ \to \rho^+\gamma$	2–6	0.4–6.3	0.43
$D_s^+ \to K^{*+}\gamma$	0.8–3	1.2–5.1	—

Rare radiative decays of charm II

★ Try to find combinations of decays where LD contributions cancel

★ Consider exclusive decays D → γϱ, γω: $\omega^{(I=0)} = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$, $\rho^{(I=1)} = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$



$V_{cd}^* V_{ud} \sim \lambda$ $V_{cb}^* V_{ub} \sim \lambda^5$

+ QCD penguins and diagrams with photon emission from spectators

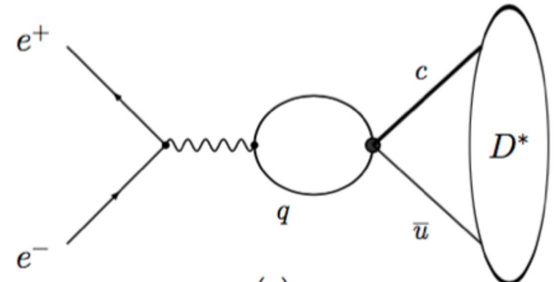
- Extract c → uu γ: LD contribution cancels in $R_{uu\gamma} = \frac{\Gamma(D^0 \to \omega\gamma) - \Gamma(D^0 \to \rho\gamma)}{\Gamma(D^0 \to \omega\gamma)}$

- Consider isospin asymmetries $R_I = \frac{2\Gamma(D^0 \to \rho^0\gamma) - \Gamma(D^+ \to \rho^+\gamma)}{2\Gamma(D^0 \to \rho^0\gamma) + \Gamma(D^+ \to \rho^+\gamma)}$ (same with omega)

- isospin asymmetries are sensitive to 4-fermion operators with photon emissions from "spectators"

Studies of $D^*(B^*) \to e^+e^-$ in the Standard Model

★ Standard Model, long distance:
- local O_1 and O_2 operators
- additional penguin-like contribution



★ Decay amplitude:

$$\langle e^+e^-|\mathcal{H}_w|D^*(p)\rangle = -e^2\bar{u}(p_-,s_-)\gamma^\mu v(p_+,s_+)\left(\frac{\Sigma_\mu(p^2)}{p^2}\right)\Bigg|_{p^2=m^2_{D^*}}$$

with $\Sigma_\mu(p^2) = i\int d^4x e^{ip\cdot x}\langle 0|T\left\{j^{em}_\mu(x)\mathcal{H}_w(0)\right\}|D^*(p)\rangle$

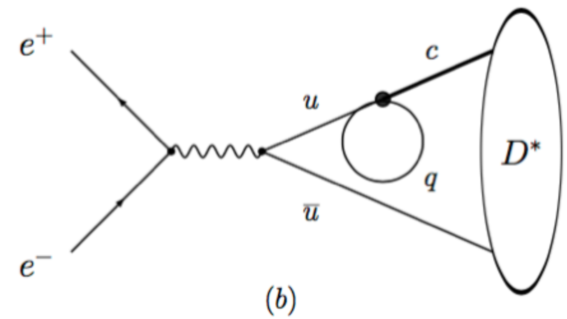
$$\Sigma^{(a)}_\mu(p^2) = \frac{G_F}{\sqrt{2}}\sum_{q=d,s}Q_q\left(C_1^{c(q)}+\frac{C_2^{c(q)}}{N_c}\right)\left\{i\int d^4x e^{ip\cdot x}\langle 0|T\left\{\bar{q}\gamma_\mu q(x)\ \bar{q}\gamma_\nu q(0)\right\}|0\rangle\right\} \times \langle 0|\bar{u}\gamma^\nu c|D^*(p)\rangle\,,$$

$$\Pi^{(q)}_{\mu\nu}(p) = \left(-g_{\mu\nu}p^2 + p_\mu p_\nu\right)\Pi^{(q)}(p^2)$$

$$\Pi^{(q)}(p^2) = \frac{p^2}{12\pi^2 Q_q^2}\int_0^\infty ds\frac{R^{(q)}(s)}{s(s-p^2-i\epsilon)}$$ with $$R(s) \equiv \frac{\sigma(e^+e^-\to hadrons)}{\sigma(e^+e^-\to\mu^+\mu^-)} = \sum_{q=u,d,s}R^{(q)}(s)$$

Studies of $D^*(B^*) \to e^+e^-$ in the Standard Model

★ Standard Model, long distance:
- local O_1 and O_2 operators
- additional penguin-like contribution



★ As a result:

$$\mathcal{B}^{LD,A}_{D^*\to e^+e^-} \simeq \begin{cases} 4.7\times 10^{-20} & \text{(NLO)} \\ 5.7\times 10^{-18} & \text{(LO)} \end{cases} \qquad \mathcal{B}^{(LD,b)}_{D^*\to e^+e^-} \geq (0.1-5.0)\times 10^{-19}$$

… and recall that the short distance $\mathcal{B}^{SD}_{D^*\to e^+e^-} \approx 2.0\times 10^{-19}$

★ Overall, the Standard Model contribution to $D^* \to e+e-$ is rather small, but
- it is four orders of magnitude higher than the $Br(D \to e+e-)$!
- the long-distance contribution is moderate
- there is a large window to probe New Physics, as e.g. with BES-III

$$\mathcal{B}_{D^*\to e^+e^-} > 4\times 10^{-13}$$

Khodjamirian, Mannel, AAP (2015)

Any interesting New Physics scenarios?