Lattice QCD and rare decays

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Outline

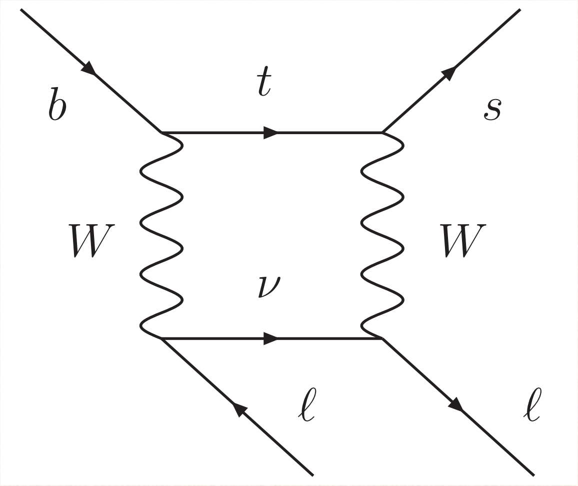
- Rare *b* decays
- Rare *s* decays

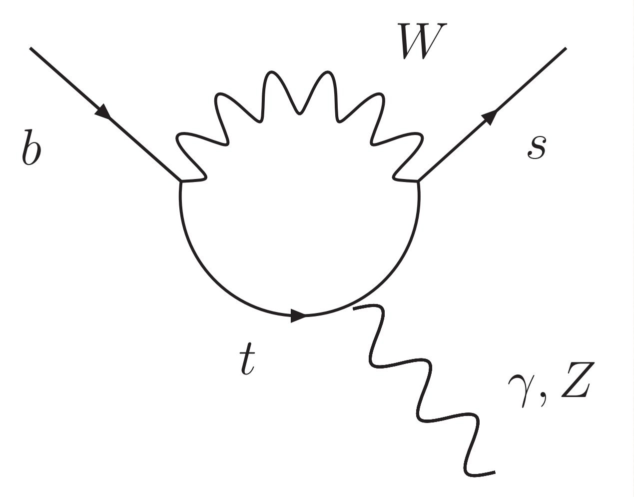
Other topics

- Hadronic matrix elements for $B_s \to \mu\mu$
  - E. Gamiz, Wednesday morning WG 4
- Ratio of $B_s \to D_s\, l\, \nu / B \to D\, l\, \nu$ for fragmentation ratio f_s/f_d
  - MW, Tuesday afternoon WG 2

Rare b decays

Rare b decays



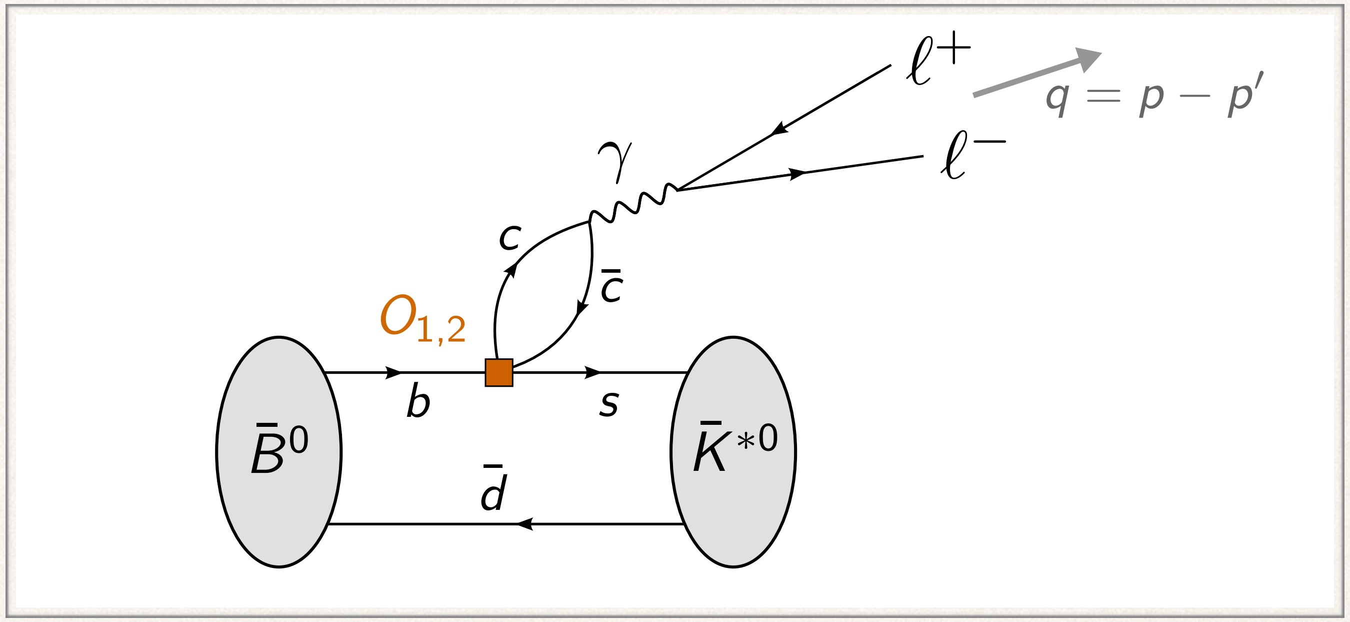


$$\mathcal{H}_{\text{eff}}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i (C_i \mathcal{O}_i + C_i' \mathcal{O}_i')$$

$$\mathcal{O}_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b\, \bar{\ell}\gamma_\mu \ell \qquad \mathcal{O}_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b\, \bar{\ell}\gamma_\mu \gamma_5 \ell$$

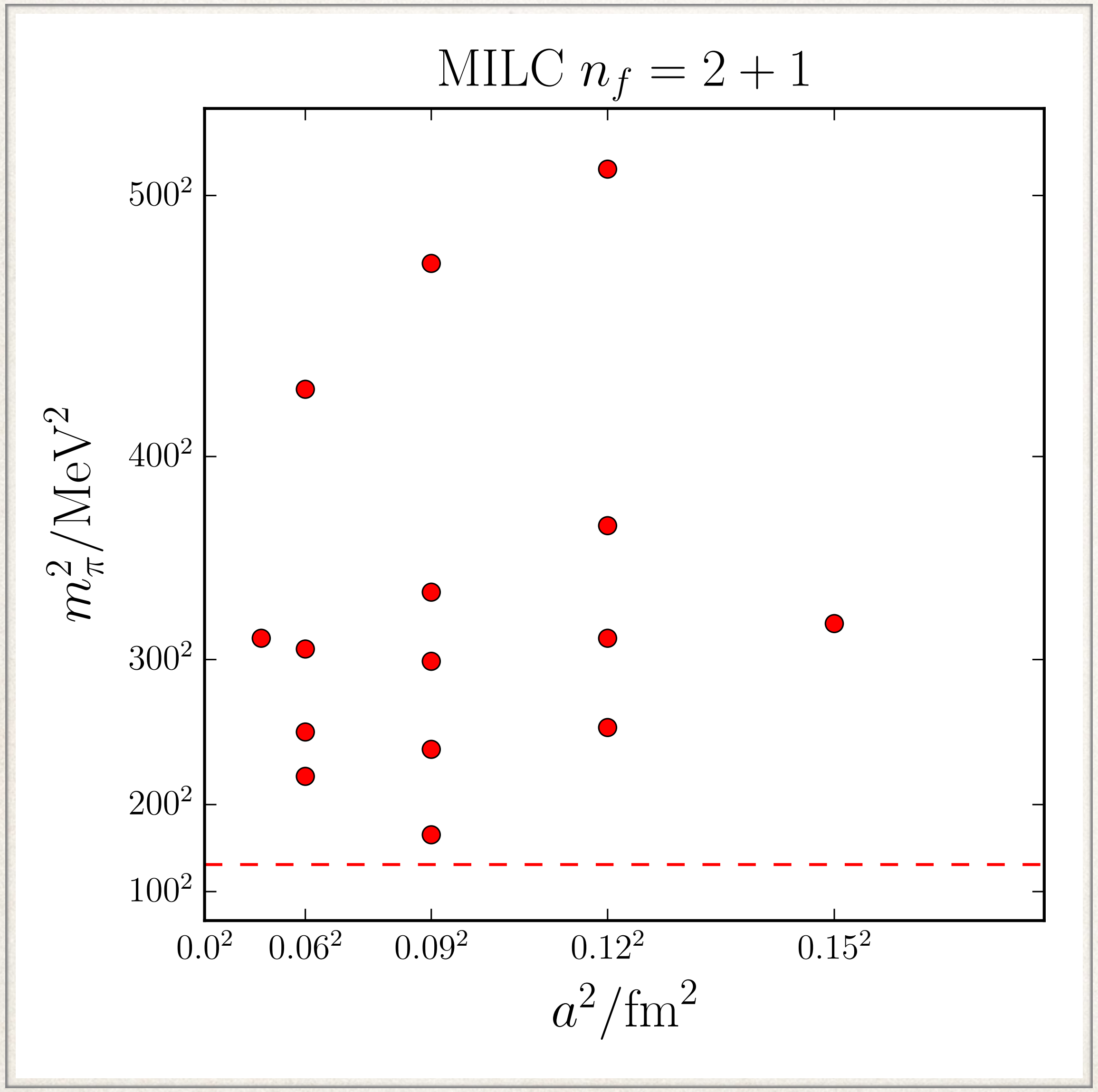
$$\mathcal{O}_7^{(\prime)} = \frac{m_b e}{16\pi^2} \bar{s}\sigma^{\mu\nu} P_{R(L)} b\, F_{\mu\nu}$$

Charmonium contributions



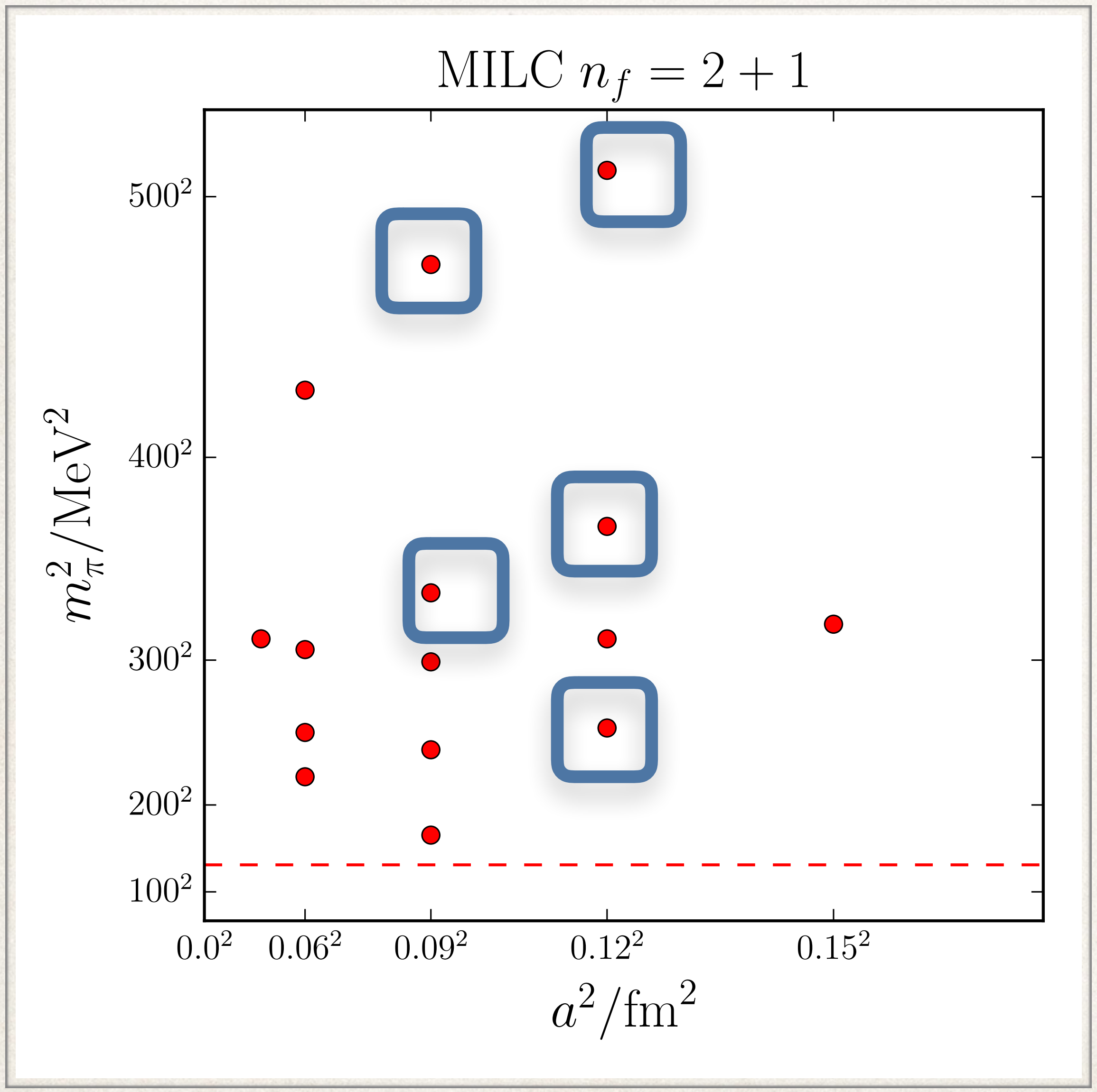
- Affects all b ➞ s decays, regardless of final state
- At high q^2, an OPE can be developed to include these effects perturbatively (Grinstein & Pirjol; Beylich, Buchalla, Feldmann)
- First correction in expansion simply augments C_7^{eff} and C_9^{eff} : Buras, Misiak, Münz, Pokorski (BMMP) ➞ Grinstein, Pirjol (GP)

Form factors: $b \to s$



- HPQCD subset for B → K
- FNAL/MILC used most of the set for B → π/K
- Cambridge subset for B → K*

Form factors: $b \to s$



- ▢ HPQCD subset for B → K
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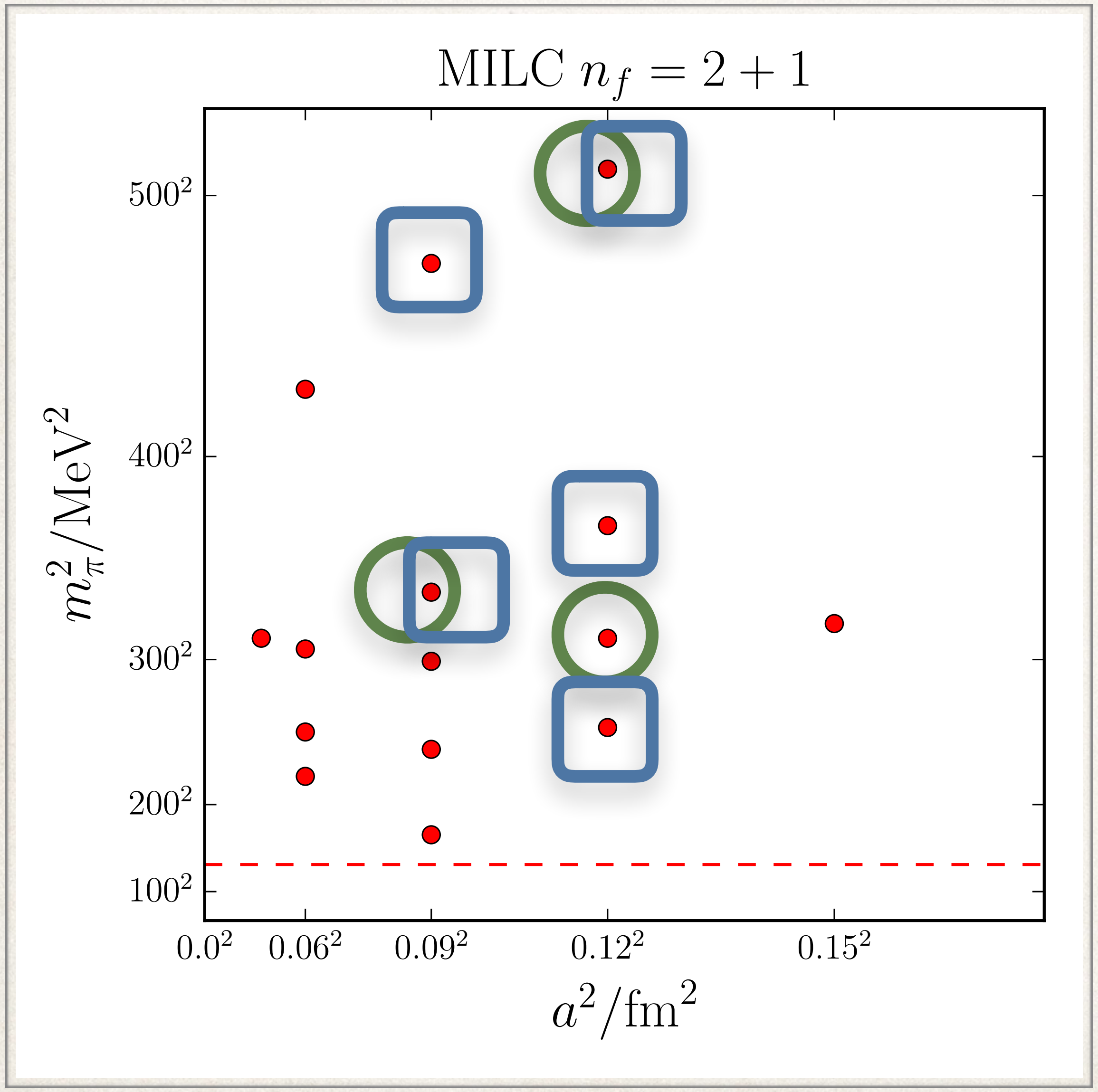
Form factors: $b \to s$



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B → K form factors

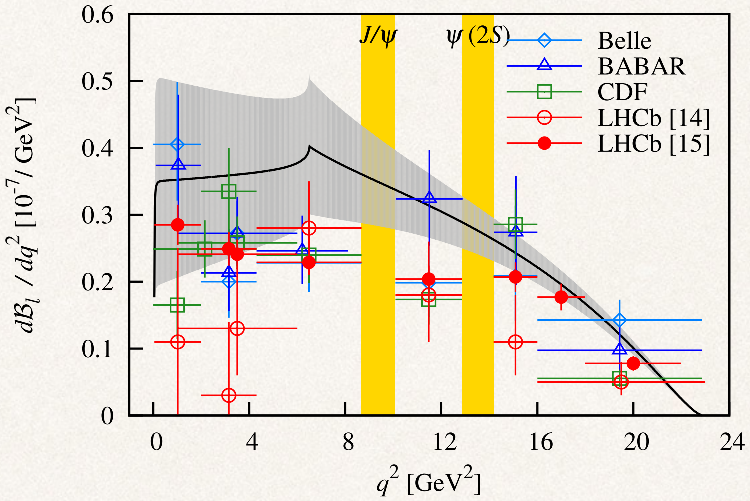
$$\langle K(k)|\bar{s}\gamma^\mu b|B(p)\rangle = \left[(p+k)^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu\right] f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2} q^\mu f_0(q^2)$$

$$\langle K(k)|\bar{s}\sigma^{\mu\nu} q_\nu b|B(p)\rangle = \frac{i f_T(q^2)}{m_B + m_K} \left[q^2 (p+k)^\mu - (m_B^2 - m_K^2) q^\mu\right]$$

- Gold-plated" matrix elements: QCD-stable | i ⟩ and | f ⟩ states
- Observables: differential branching fraction $d\Gamma/dq^2$, forward/backward asymmetry A_{FB} (zero in SM), and "flat term" F_H

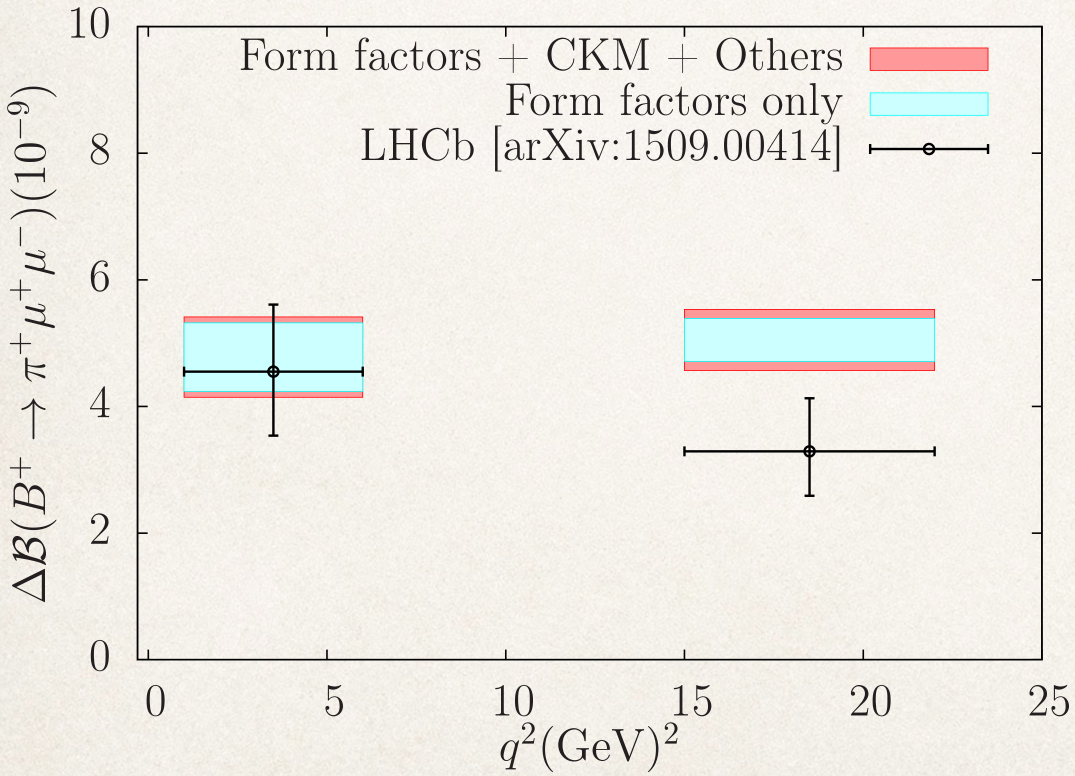
B → π μ⁺μ⁻ & B → K μ⁺μ⁻

Bouchard *et al.*, (HPQCD)
arXiv:1306.0434

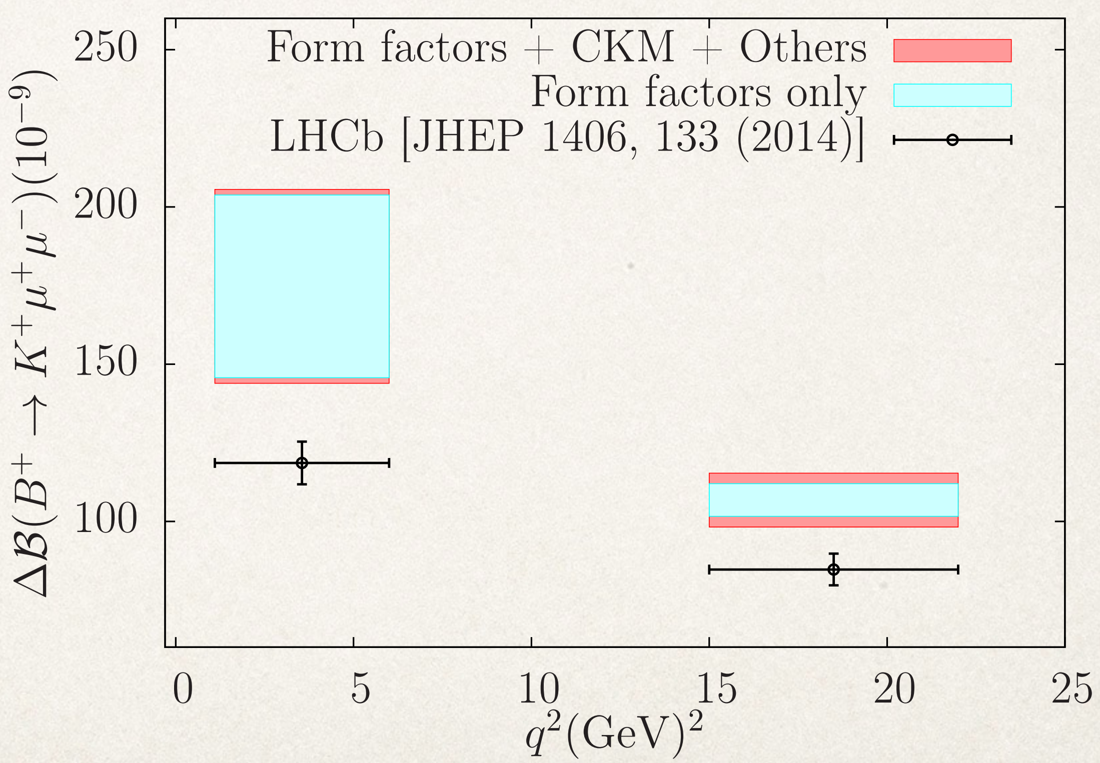


$B \to K\mu^+\mu^-$

$B^+ \to \pi^+\mu^+\mu^-$



$B^+ \to K^+\mu^+\mu^-$



Du *et al.*, (FNAL/MILC) arXiv:1510.02349

B → V form factors

$$\langle V(k,\varepsilon)|\bar{q}\hat{\gamma}^\mu b|B(p)\rangle = \frac{2iV(q^2)}{m_B+m_V}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma$$

$$\langle V(k,\varepsilon)|\bar{q}\hat{\gamma}^\mu\hat{\gamma}^5 b|B(p)\rangle = 2m_V A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu + (m_B+m_V)A_1(q^2)\left(\varepsilon^{*\mu} - \frac{\varepsilon^*\cdot q}{q^2}q^\mu\right)$$
$$-A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_V}\left((p+k)^\mu - \frac{m_B^2-m_V^2}{q^2}q^\mu\right)$$

$$q^\nu\langle V(k,\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}b|B(p)\rangle = 2T_1(q^2)\epsilon_{\mu\rho\tau\sigma}\varepsilon^{*\rho}p^\tau k^\sigma$$

$$-q^\nu\langle V(k,\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}\hat{\gamma}^5 b|B(p)\rangle = iT_2(q^2)[\varepsilon^*_\mu(m_B^2-m_V^2) - (\varepsilon^*\cdot q)(p+k)_\mu]$$
$$+iT_3(q^2)(\varepsilon^*\cdot q)\left[q_\mu - \frac{q^2}{m_B^2-m_V^2}(p+k)_\mu\right]$$

$$A_{12}(q^2) = \frac{(m_B+m_V)^2(m_B^2-m_V^2-q^2)A_1(q^2) - \lambda A_2(q^2)}{16m_B m_V^2(m_B+m_V)}$$

$$T_{23}(q^2) = \frac{m_B+m_V}{8m_B m_V^2}\left[\left(m_B^2+3m_V^2-q^2\right)T_2(q^2) - \frac{\lambda T_3(q^2)}{m_B^2-m_V^2}\right]$$

with $\lambda = (t_+-t)(t_--t) \qquad t=q^2 \qquad t_\pm = (m_B\pm m_V)^2$

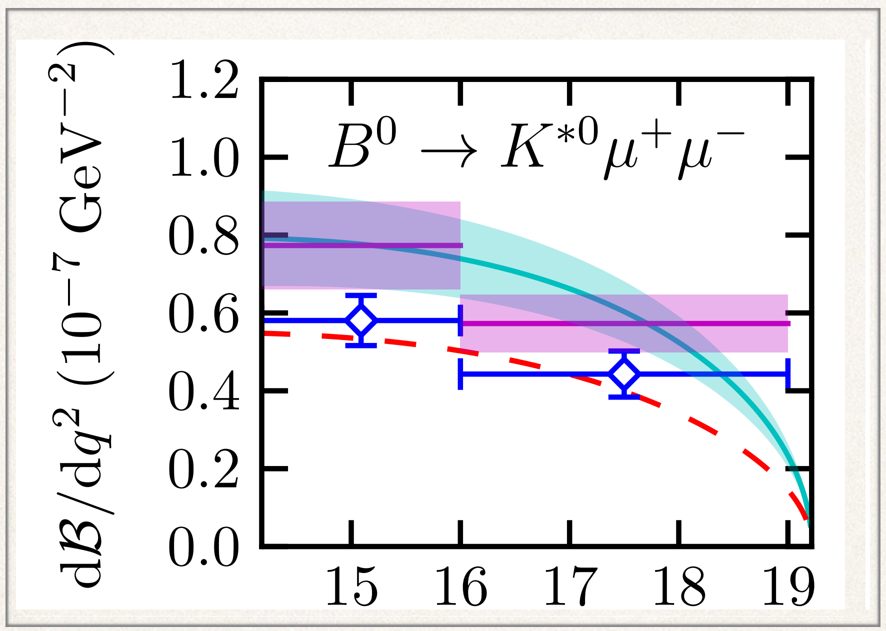
$B \to K^* \mu^+ \mu^-$

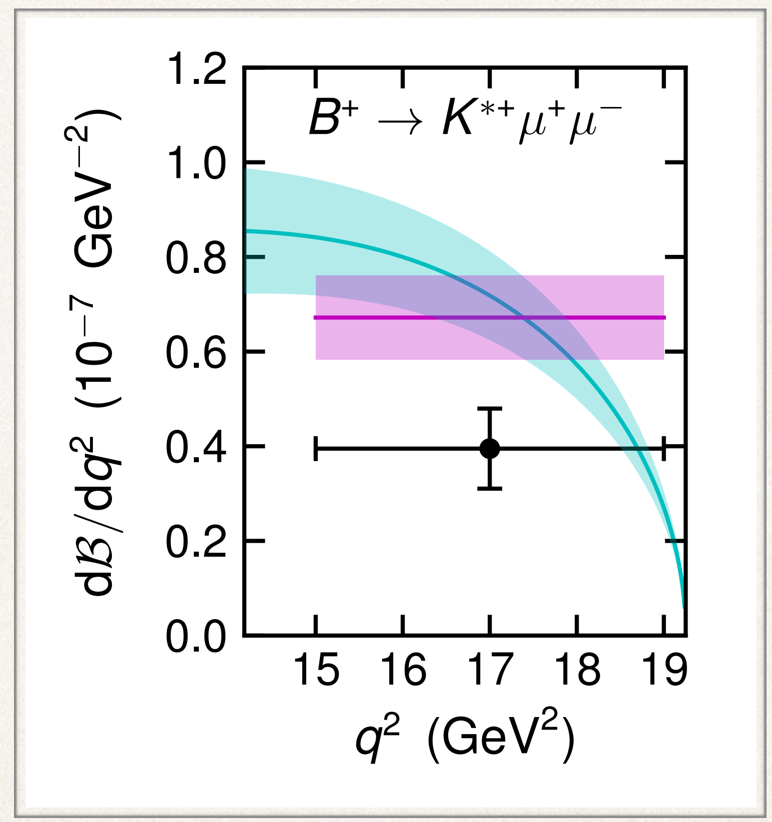
SM

SM (binned)

Expt: LHCb, CMS & CDF (K^*)
LHCb, CDF (ϕ)

Expt: Aaij *et al.*, (LHCb) arXiv:1403.8044





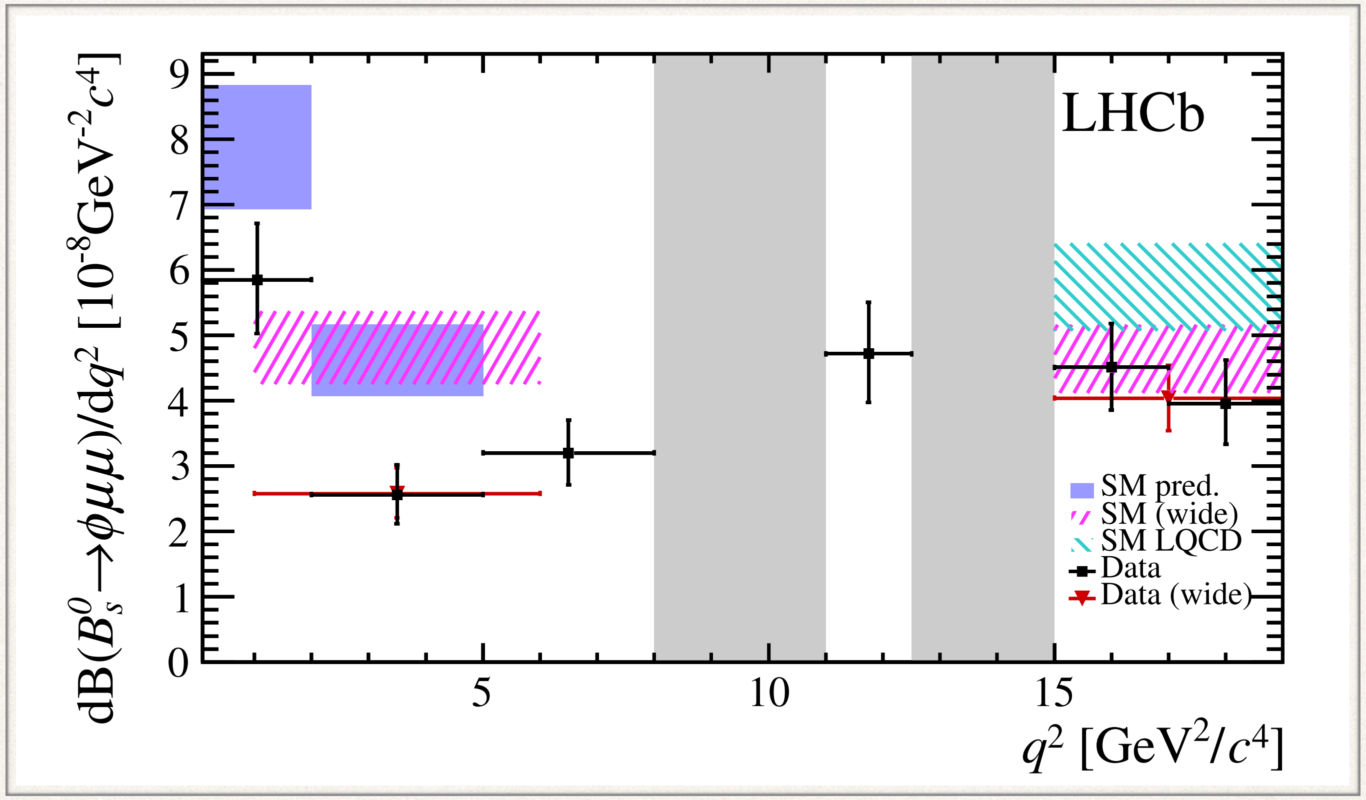
$q^2\ (\mathrm{GeV}^2)$

$C_9^{\mathrm{NP}} = -1.0, \quad C_9' = 1.2$

Horgan *et al.*, arXiv:1310.3887; S Meinel, Paris Workshop 2014

$B_s \to \phi\, \mu^+\mu^-$

Expt. measurement from Aaij *et al.*, (LHCb), arXiv:1506.08777

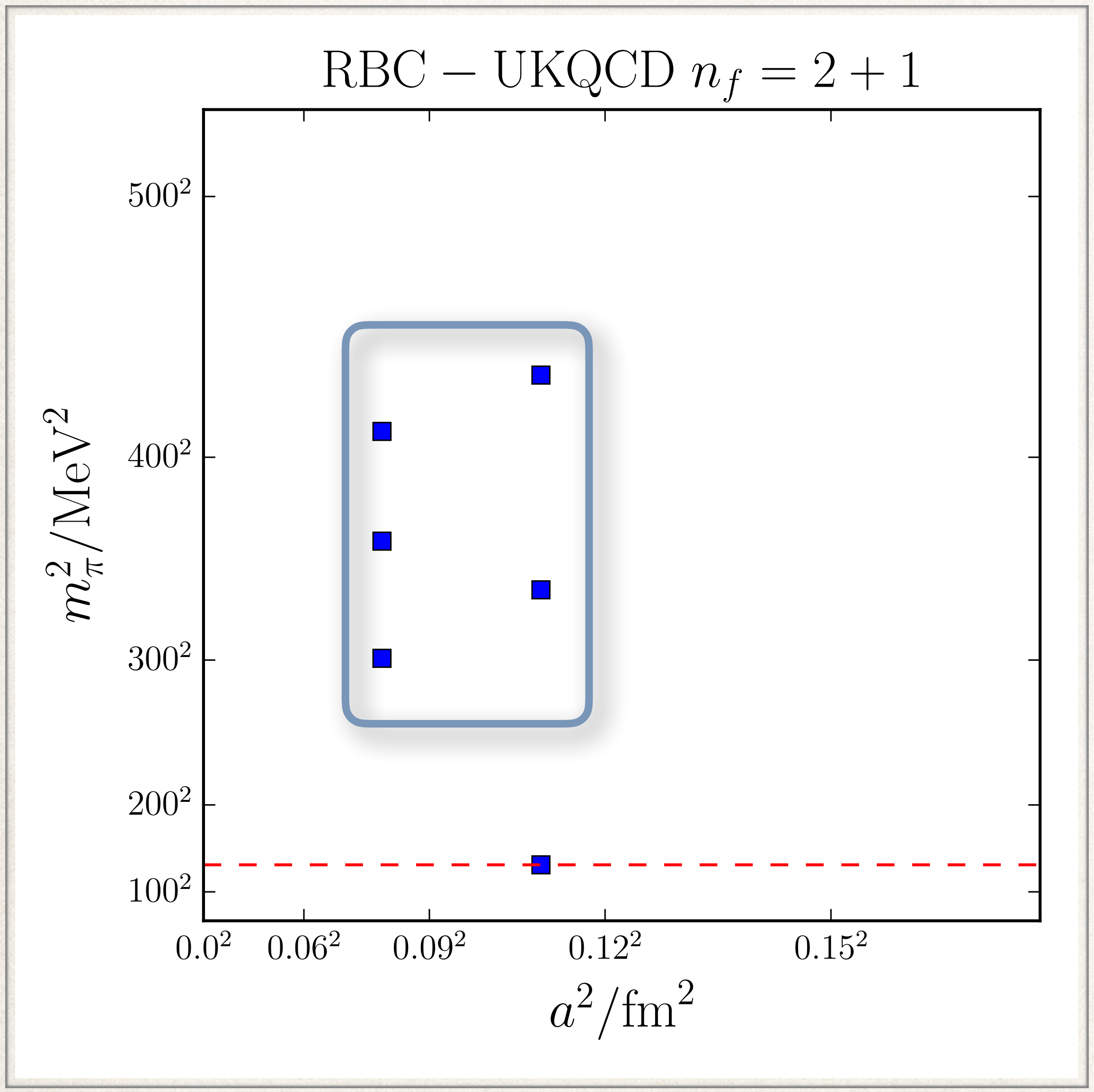


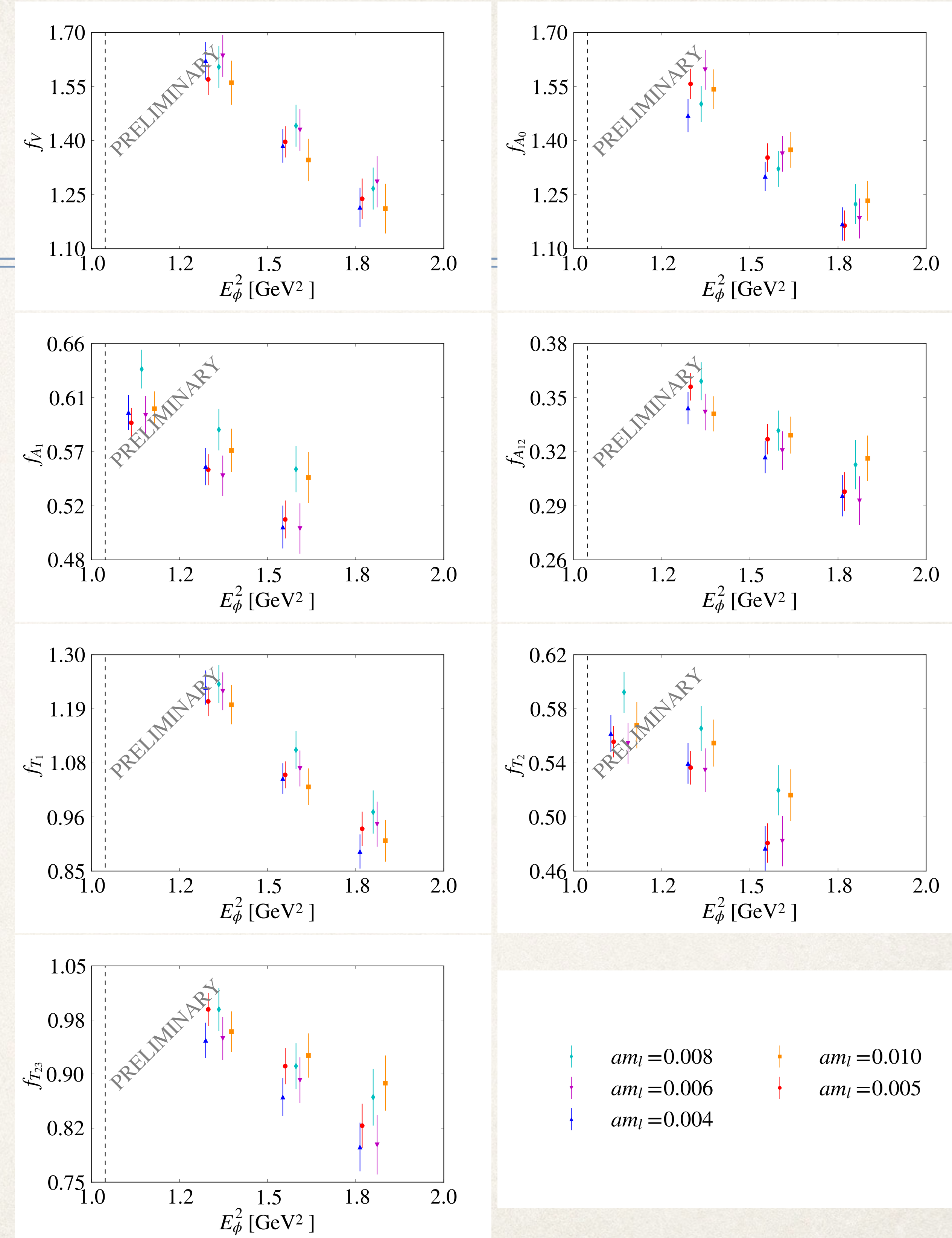
Bharucha, Straub, Zwicky, arXiv:1503.05534
Altmannshoher & Straub, arXiv:1411.3161

Update of Horgan *et al.*, arXiv:1310.3887

Difference in high q^2 SM prediction due in part to: inclusion of low q^2 LCSR form factors, formulation for virtual corrections from O_1, O_2; also inputs.

$B_s \to \phi\, \mu^+ \mu^-$

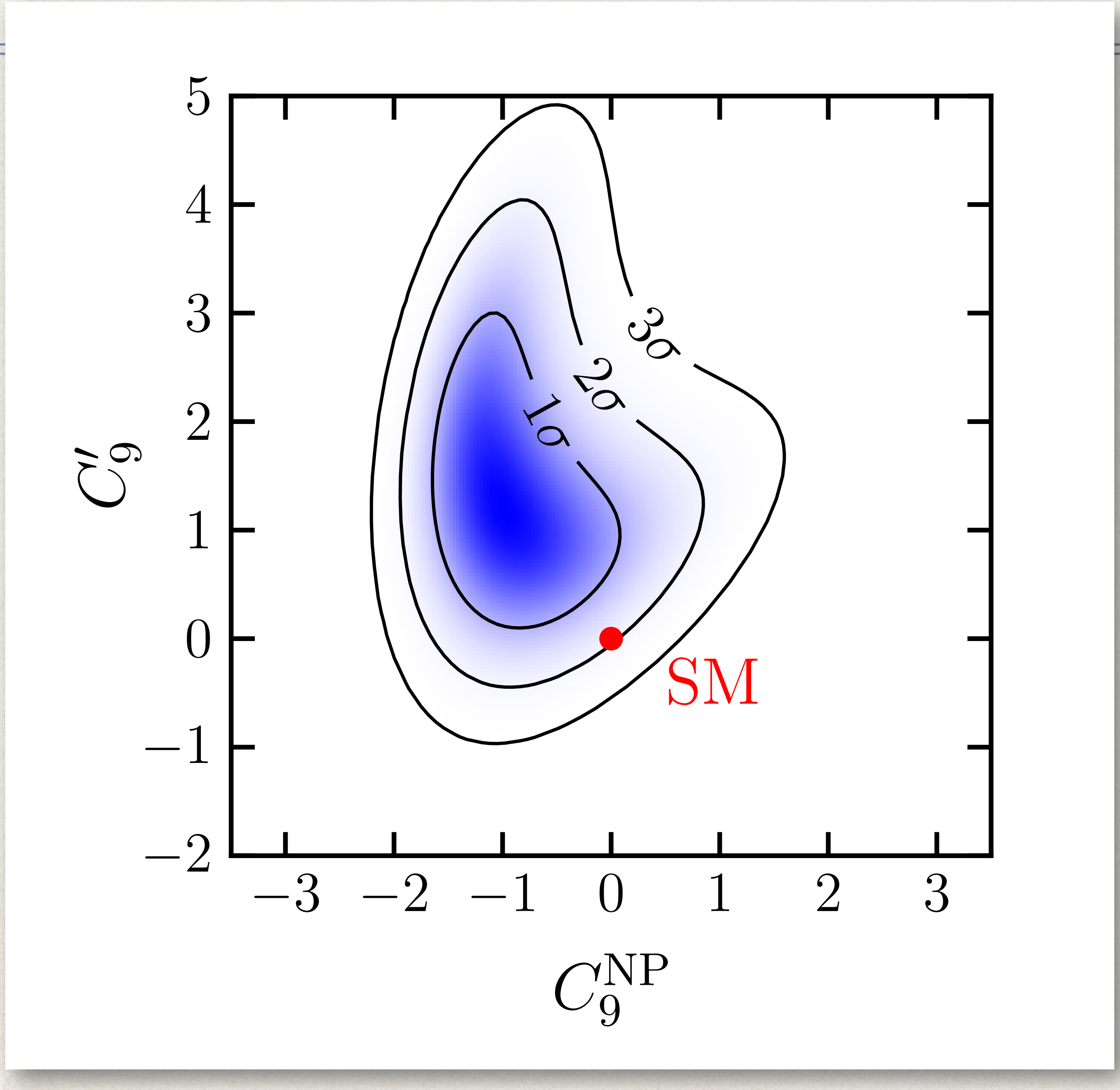




Witzel et al (RBC-UKQCD) Lattice 2016 proceedings

Fit to low recoil data (2013)

Best fit: $C_9^{\mathrm{NP}} = -1.0 \pm 0.6 \qquad C_9' = 1.2 \pm 1.0$



Likelihood function

- C_9, C_9' assumed to be real
- Data in 2 highest q^2 bins
  - $B \to K^*\mu\mu$ (neutral mode): dB/dq^2, F_L, S_3, S_4, S_5, A_{FB}
  - $B_s \to \varphi\mu\mu$: dB/dq^2, F_L, S_3
- Theory correlations between observables & bins taken into account

Horgan, Liu, Meinel, Wingate, PRL 112, arXiv:1310.3887

$\Lambda_b \to \Lambda$ form factors

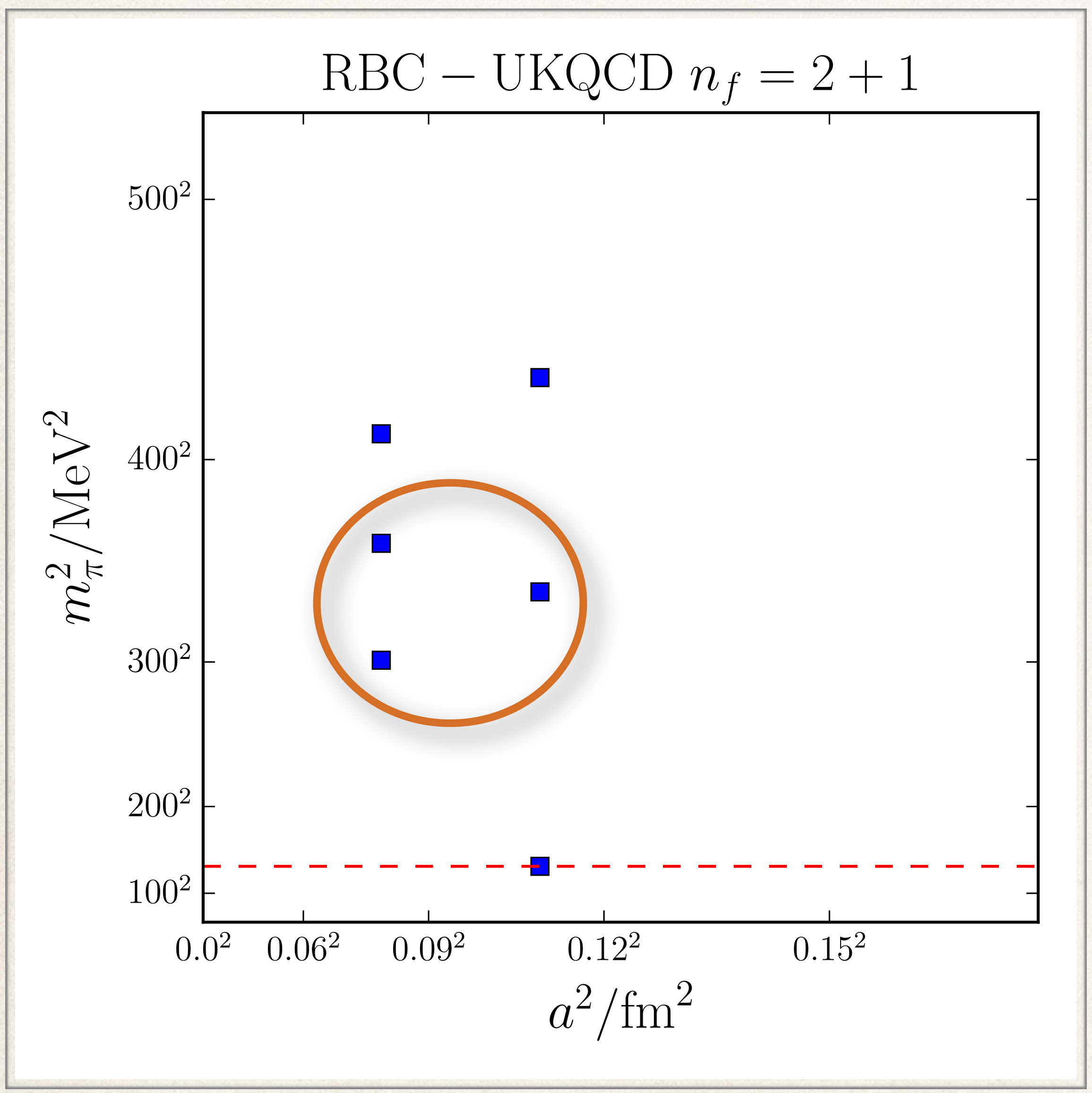
$$\langle \Lambda | \bar{s}\gamma^\mu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^V \gamma^\mu - f_2^V \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Lambda_b}} + f_3^V \frac{q^\mu}{m_{\Lambda_b}} \right] u_{\Lambda_b}$$

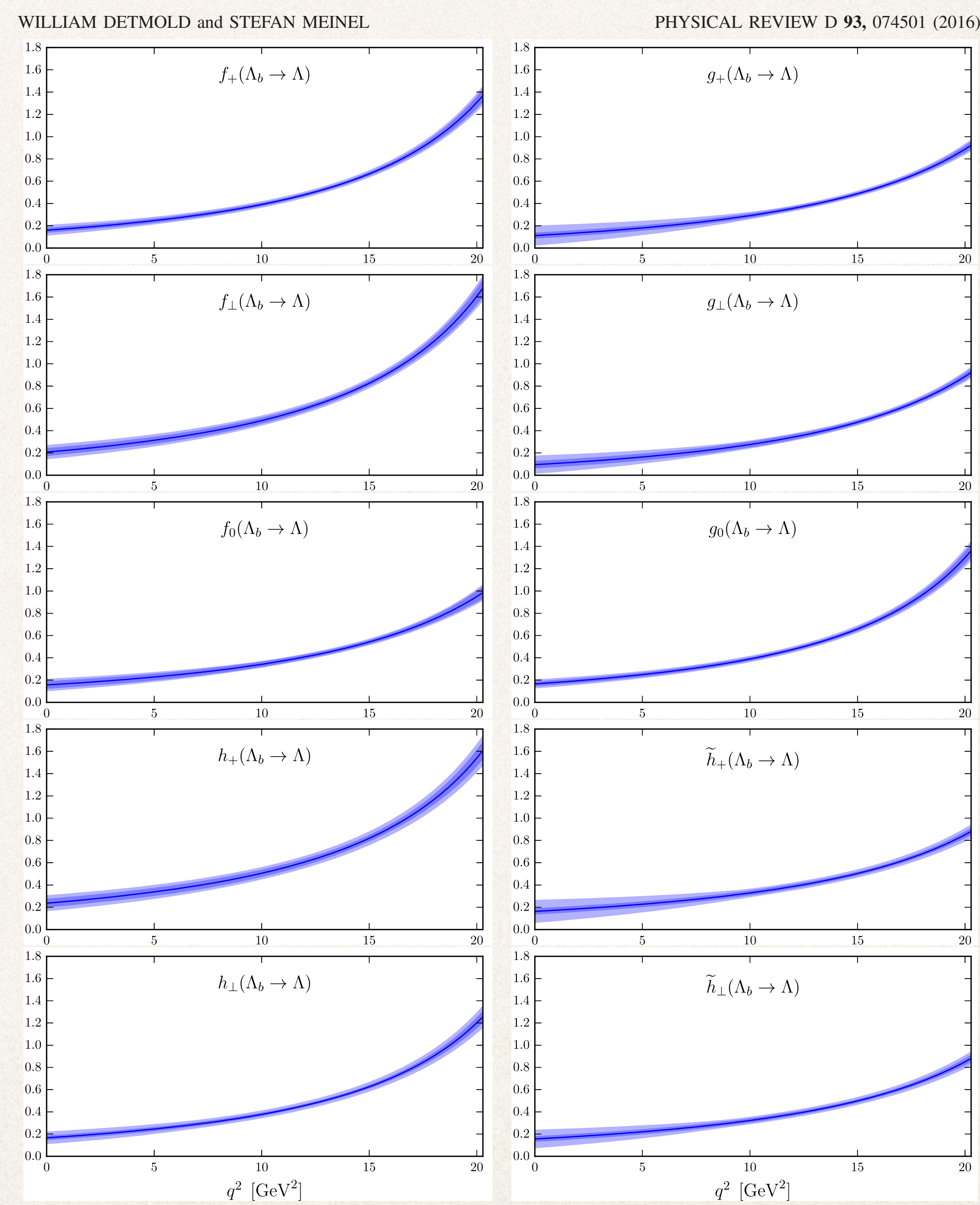
$$\langle \Lambda | \bar{s}\gamma^\mu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^A \gamma^\mu - f_2^A \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Lambda_b}} + f_3^A \frac{q^\mu}{m_{\Lambda_b}} \right] \gamma_5 u_{\Lambda_b}$$

$$\langle \Lambda | \bar{s} i\sigma^{\mu\nu} q_\nu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^{TV} \frac{\gamma^\mu q^2 - q^\mu \not{q}}{m_{\Lambda_b}} - f_2^{TV} \frac{q^\mu}{m_{\Lambda_b}} \right] u_{\Lambda_b}$$

$$\langle \Lambda | \bar{s} i\sigma^{\mu\nu} q_\nu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda \left[f_1^{TA} \frac{\gamma^\mu q^2 - q^\mu \not{q}}{m_{\Lambda_b}} - f_2^{TA} \frac{q^\mu}{m_{\Lambda_b}} \right] \gamma_5 u_{\Lambda_b}$$

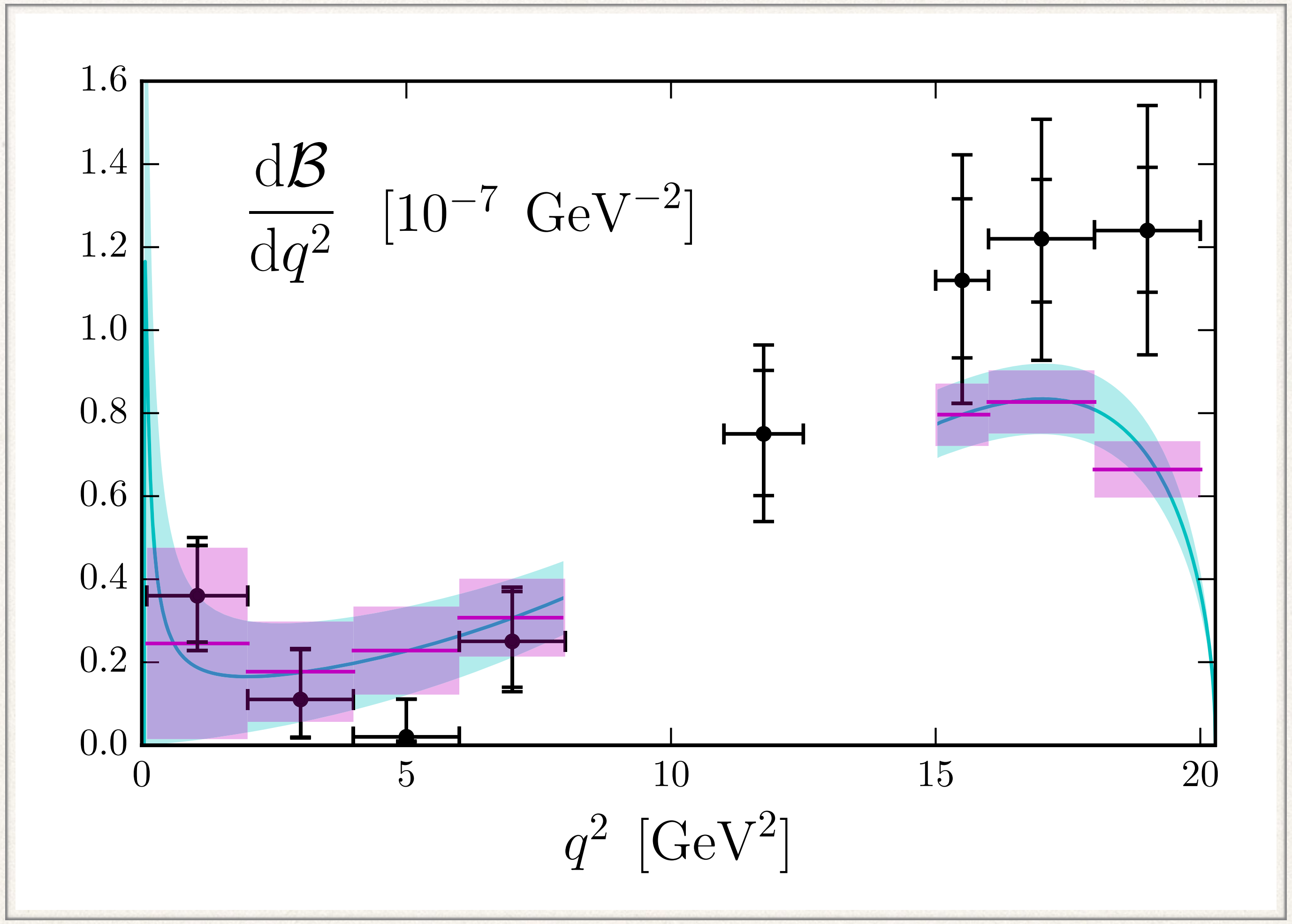
$\Lambda_b \to \Lambda\, \mu^+\mu^-$





Detmold & Meinel, PRD 93 (2016)

$\Lambda_b \to \Lambda\, \mu^+\mu^-$



- Contrary to rare B branching fractions, here the measured data at low recoil exceed the SM prediction.

Detmold & Meinel, PRD 93 (2016)

Rare *s* decays

Neutral current K → π decays

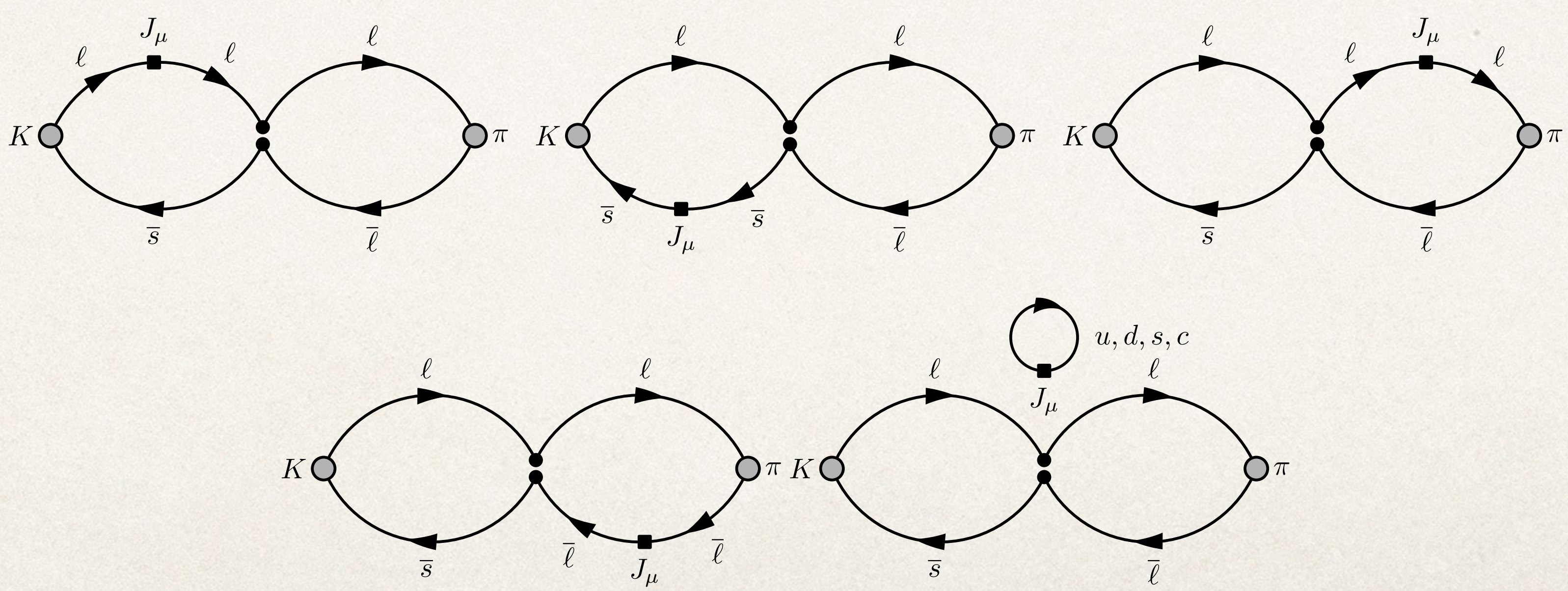
- Recent work by RBC-UKQCD: Xu Feng was due to speak here
- Long distance contributions important for the decays

$$K^{\pm} \to \pi^{\pm}\ell^{+}\ell^{-} \qquad K_S \to \pi^0\ell^+\ell^- \qquad K^{\pm} \to \pi^{\pm}\nu\bar{\nu}$$

- Amplitude

$$\mathcal{A}^i_\mu\left(q^2\right) = \int d^4x \left\langle \pi^i\left(\mathbf{p}\right)\left|T\left[J_\mu\left(0\right)\mathcal{H}_W\left(x\right)\right]\right|K^i\left(\mathbf{k}\right)\right\rangle$$

- Example contractions



Exploratory calculation

- Correlation functions include contributions from multi-pion states
- These lead to exponentially growing contributions (when intermediate energy < E_K) which must be removed

$$\Gamma_\mu^{(4)}\left(t_H, t_J, \mathbf{k}, \mathbf{p}\right) = \int d^3\mathbf{x} \int d^3\mathbf{y}\; e^{-i\mathbf{q}\cdot\mathbf{x}} \left\langle \phi_\pi\left(t_\pi, \mathbf{p}\right) T\left[J_\mu\left(t_J, \mathbf{x}\right) H_W\left(t_H, \mathbf{y}\right)\right] \phi_K^\dagger\left(t_K, \mathbf{k}\right)\right\rangle$$

$$I_\mu\left(T_a, T_b, \mathbf{k}, \mathbf{p}\right) = -\int_0^\infty dE\, \frac{\rho\left(E\right)}{2E} \frac{\langle \pi\left(\mathbf{p}\right)|J_\mu|E, \mathbf{k}\rangle \langle E, \mathbf{k}|H_W|K\left(\mathbf{k}\right)\rangle}{E_K\left(\mathbf{k}\right) - E}\left(1 - e^{\left(E_K(\mathbf{k}) - E\right)T_a}\right) + \int_0^\infty dE\, \frac{\rho_S\left(E\right)}{2E} \frac{\langle \pi\left(\mathbf{p}\right)|H_W|E, \mathbf{p}\rangle \langle E, \mathbf{p}|J_\mu|K\left(\mathbf{k}\right)\rangle}{E - E_\pi\left(\mathbf{p}\right)}\left(1 - e^{-\left(E - E_\pi(\mathbf{p})\right)T_b}\right)$$

- 2 subtraction methods tested
- Approach looks feasible

Christ et al., PRD 92 (2015)
Christ et al., arXiv:1608.07585

Conclusions

- Matrix elements of short-distance rare-decay operators
- Baryon decay rate currently spoiling picture of single new C_9
- Progress on matrix elements of nonlocal operators in K sector