Recent progress and future prospects on lattice QCD studies of $D_{(s)}$ leptonic decays

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RBC-UKQCD Collaborations

CKM2016
TIFR, Mumbai

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THE UNIVERSITY *of* EDINBURGH

Outline

Quark Flavour Physics - Cabibbo-Kobayashi-Maskawa

The Cabibbo-Kobayashi-Maskawa matrix or *quark mixing matrix* determines the strength of flavour changing weak decays.

CKM Matrix

The CKM matrix relates flavour eigenstates (d', s', b') to mass eigenstates (d, s, b)

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Properties in SM

- complex
- unitarity
- 3 generations

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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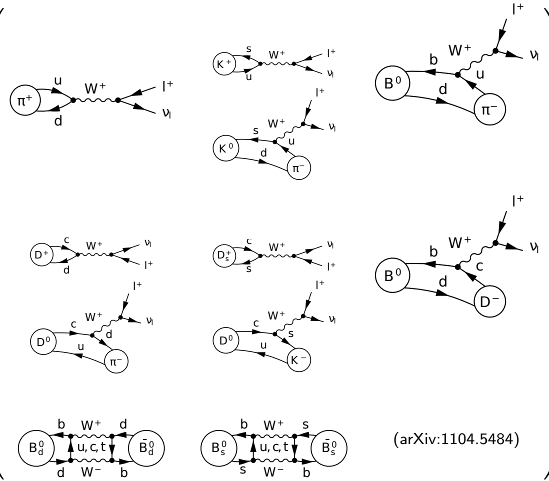
(some) CKM Processes on the Lattice

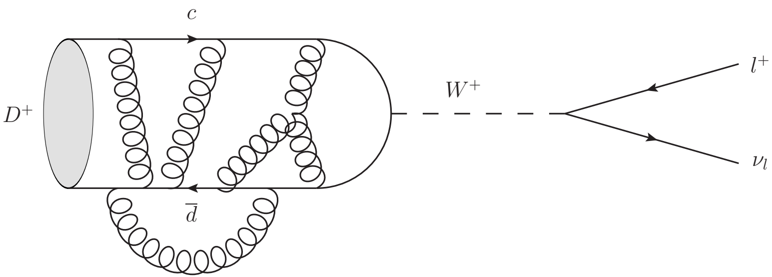
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim$$



(arXiv:1104.5484)

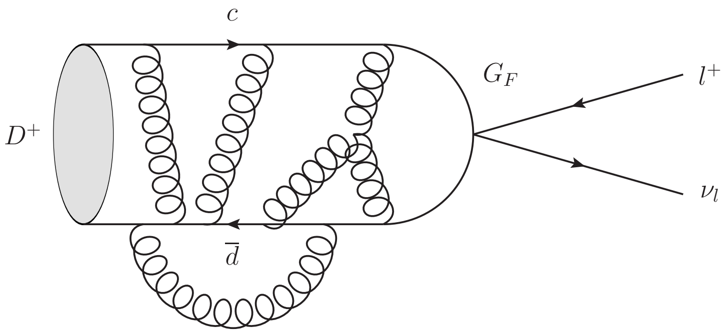
⇒ Focus of this talk: **Leptonic $\mathbf{D_{(s)}}$ decays**

Motivation: Leptonic $D_{(s)}$ decays



$$\Gamma_{\text{exp}} = V_{\text{CKM}}(\text{WEAK})(\text{EM})(\text{STRONG})$$

Motivation: Leptonic $D_{(s)}$ decays



$$\Gamma_{\text{exp}} = V_{\text{CKM}}(\text{WEAK})(\text{EM})(\text{STRONG})$$

$$\Gamma(D^+_{(s)} \to l^+ \nu_l) = \frac{G_F^2 \tau_{D_{(s)}}}{8\pi} |V_{cq}|^2 f^2_{D_{(s)}} m_l^2 m_{D_{(s)}} \left(1 - \frac{m_l^2}{m^2_{D_{(s)}}}\right)^2 + \mathcal{O}(\alpha_{\text{EM}})$$

Decay constants: $f_{D_{(s)}}$

Decay constants f_P are defined from **non-perturbative** matrix elements:

$$\langle 0|\, \mathcal{A}_0 \,|P(\mathbf{p})\rangle = iE_P(\mathbf{p}) f_P \quad \text{for } P = D, D_s$$

$\Rightarrow$ Lattice QCD is a systematically improvable tool to make predictions for these.

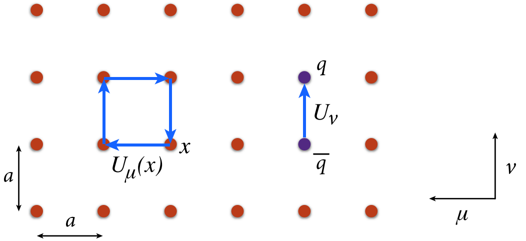
Lattice

- Wick rotate and stochastically estimate the PI

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[\Phi]\, \mathcal{O}(\Phi) \exp\left(-S_E[\Phi]\right)$$

- Discrete space, finite box
- Naive discretisation $\Rightarrow$ doublers.
- Different fermion actions (Wilson, TM, DWF, HISQ, ...)

From PDG



Lattice systematic errors

Lattice vs Continuum

Simulation:	Real World (SM)
2+1+1, 2+1, 2	6 distinct quark flavours
finite lattice spacing a	$a = 0$
finite volume L^3	$L = \infty$
lattice regularised	some continuum scheme
Some bare input quark masses am_l, am_s, am_h In general: $m_\pi \neq m_\pi^{\mathrm{phys}}$	$m_l = m_l^{\mathrm{phys}}$ $m_s = m_s^{\mathrm{phys}}$ $m_h = m_c^{\mathrm{phys}}$
Isospin symmetric	$m_u \neq m_d$, EW, EM

$\Rightarrow$ **We need to retrieve the continuum theory from the lattice simulations.**

Motivation: Experiment and Theory

Experimental Measurement of $|V_{cq}|^2 f_{D_{(s)}}^2$

- $\Rightarrow$ Belle, CLEO, BaBar
- $\Rightarrow$ BES III
- $\Rightarrow$ LHC
- $\Rightarrow$ Belle II

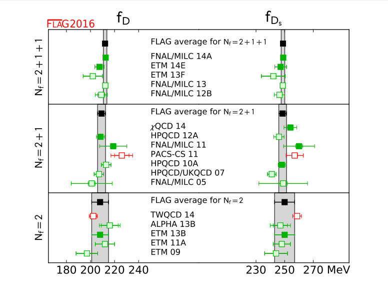
PDG average (March 2016):
$|V_{cd}| f_{D^+} = 45.91(1.05)\,\mathrm{MeV}$
$|V_{cs}| f_{D_s^+} = 250.9(4.0)\,\mathrm{MeV}$
(2.3% and 1.6% respectively)

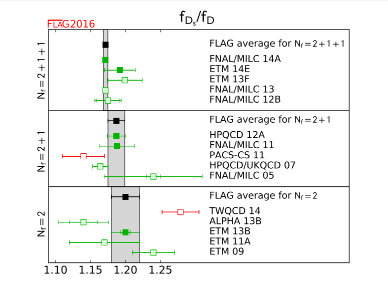
Lattice calculation of $f_{D_{(s)}}$

- $\Rightarrow$ Fermion formulations
- $\Rightarrow$ Number of sea quarks
- $\Rightarrow$ Scale setting
- $\Rightarrow$ Heavy quark treatment
- $\Rightarrow$ Renormalisation
- $\Rightarrow$ Systematic Errors

Summary of works: FLAG

FLAG Review (*arXiv* : 1607.00299)





N_f	f_D[MeV]	f_{D_s}[MeV]	f_{D_s}/f_D
2+1+1	212.15(1.45)	248.83(1.27)	1.1716(32)
2+1	209.2(3.3)	249.8(2.3)	1.187(12)
2	208(7)	250(7)	1.20(2)

Summary of works: FLAG

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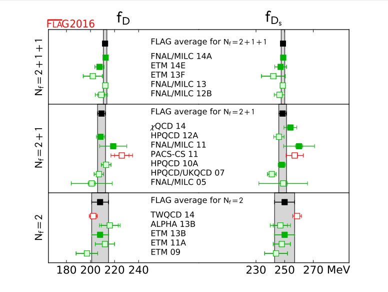
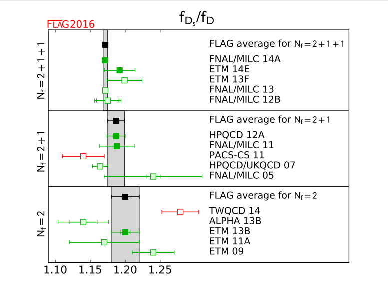




N_f	f_D[MeV]	f_{D_s}[MeV]	f_{D_s}/f_D
2+1+1	0.68%	0.51%	0.27%
2+1	1.58%	0.92%	1.01%
2	3.37%	2.80%	1.67%

Summary of works: What happened since CKM 2014?

Coll.	N_f	f_D[MeV]	f_{D_s}[MeV]	f_{D_s}/f_D
ETM 14E	2+1+1	207.4(3.8)	247.2(4.1)	1.192(22)
χQCD 14	2+1	-	254(5)	-
JLQCD 15	2+1	209.6(5.2)	244.4(4.1)	-
JLQCD 16	2+1	212.8(4.0)	244.4(4.2)	-
RBC/UKQCD 16	2+1	$208.7(^{+3.5}_{-3.3})$	$246.4(^{+2.3}_{-2.7})$	$1.1667(^{+98}_{-87})$
TWQCD 14	2	202.3(3.4)	258.7(3.1)	1.279(26)

- χQCD: no light props, uses RBC/UKQCD ensembles.
- JLQCD: stat and scale error only. No systematics yet.
- RBC/UKQCD: In preparation (preliminary)
- TWQCD: Single lattice spacing ($a = 0.061\mathrm{fm}$). $m_{\pi,\mathrm{min}} L \approx 1.9$

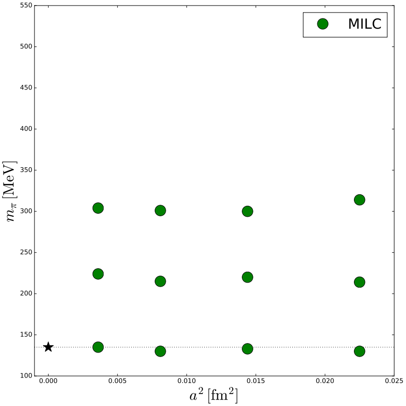
This talk

I will focus on the following works:

1. FNAL/MILC 2014A ($N_f = 2+1+1$, $m_\pi = m_\pi^{\mathrm{phys}}$)
2. ETM 14E ($N_f = 2+1+1$, **new**)
3. χQCD ($N_f = 2+1$, **new**)
4. RBC/UKQCD ($N_f = 2+1$, $m_\pi = m_\pi^{\mathrm{phys}}$, **new**)
5. JLQCD ($N_f = 2+1$, $a^{\mathrm{min}} = 0.044\mathrm{fm}$, **new**)

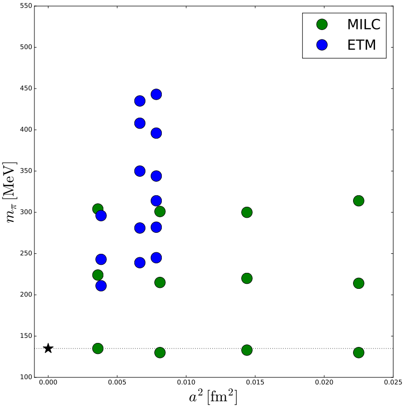
- Differences between different calculations
- Full systematic error analysis
  $\Rightarrow$ What are the leading systematic errors
  $\Rightarrow$ Any additional assumptions
- Restrictions of current calculations
  $\Rightarrow$ What can be improved?

Ensemble details



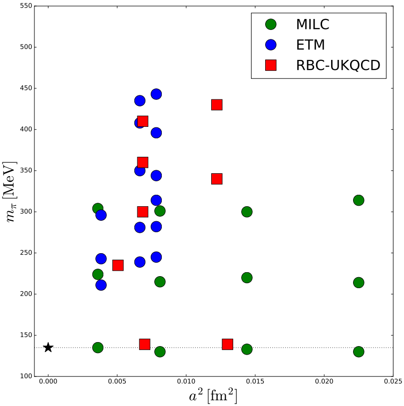
Collaboration	Fermion Action
MILC	HISQ

Ensemble details



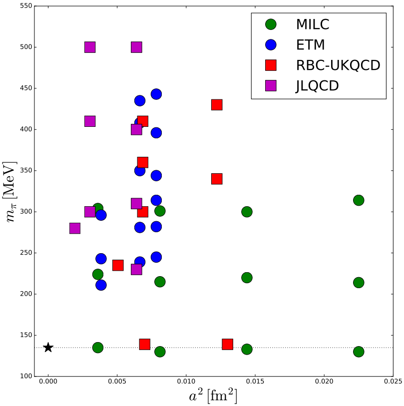
Collaboration	Fermion Action
MILC	HISQ
ETM	Twisted Mass

Ensemble details



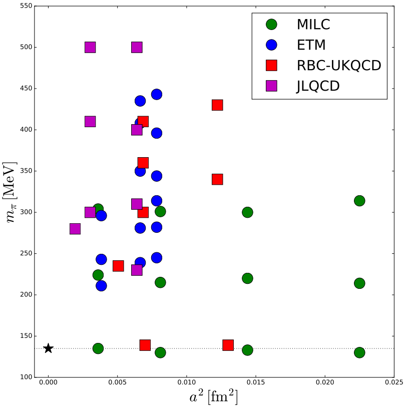
Collaboration	Fermion Action
MILC	HISQ
ETM	Twisted Mass
RBC/UKQCD	Domain Wall

Ensemble details



Collaboration	Fermion Action
MILC	HISQ
ETM	Twisted Mass
RBC/UKQCD	Domain Wall
JLQCD	Domain Wall

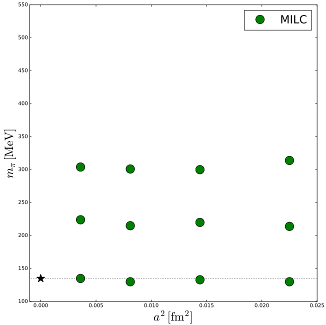
Ensemble details



- All automatically $\mathcal{O}(a)$-improved
- Only two results with $m_\pi = m_\pi^{\rm phys}$
- $a_{\rm min}$:
  $\sim 0.044\rm fm$ (JLQCD),
  $\sim 0.06\rm fm$ (MILC, ETM),
  $\sim 0.07\rm fm$ (RBC/UKQCD)

FNAL/MILC - decay constants arXiv:1407.3772

- Discussed at CKM 2014
- 4 lattice spacings
- 4 physical pion mass ensembles
- HISQ for light, strange and charm in *sea and valence*
- absolute scale set by f_π



	stat	FV	EM	chiral+CL
Φ_D	0.36	0.10	0.04	$^{+0.18}_{-0.49}$
Φ_{D_s}	0.09	0.08	0.04	$^{+0.47}_{-0.26}$
f_{D_s}/f_D	0.31	0.03	0.04	$^{+0.03}_{-0.13}$

All errors listed in %.

FNAL/MILC - CKM arXiv:1407.3772

adjust experimental decay rates for

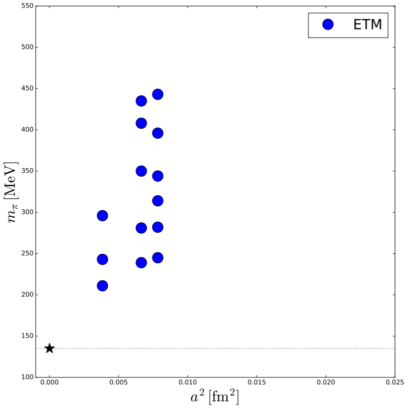
- long-distance EW
- short-distance EW
- structure dependent EM corrections

	central	lattice	experiment	EM
V_{cd}	0.217	1	**5**	1
V_{cs}	1.010	5	**18**	6

ETM - set up arXiv:1411.7908

- 3 lattice spacings
- $m_\pi^{\text{min}} = 210\text{MeV}$
- TM for light, strange and charm in *sea*
- TM for light in *valence*
- Osterwalder Schrader for strange and charm in *valence*

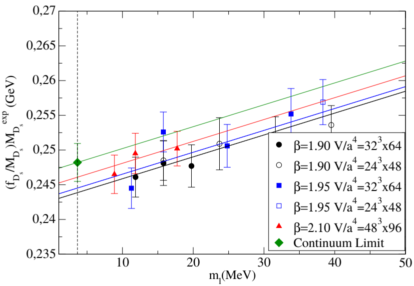
$\Rightarrow$ **MIXED ACTION**

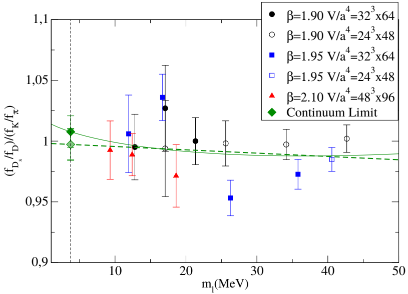


ETM - analysis arXiv:1411.7908

f_{D_s} from extrapolating $\frac{f_{D_s}}{m_{D_s}}$

$\frac{f_{D_s}}{f_D}$ from fitting $\frac{f_{D_s}/f_D}{f_K/f_\pi}$





$\Rightarrow f_D$ from combining this

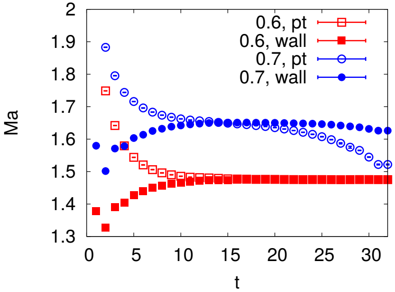
ETM - results arXiv:1411.7908

	stat+fit	chiral	scale+disc	renorm.
Φ_D	1.78	0.29	0.34	0.05
Φ_{D_s}	1.58	0.28	0.49	0.12
f_{D_s}/f_D	1.59	0.67	0.67	0.08

All errors listed in %.

χQCD

- 2 lattice spacings (RBC/UKQCD)
- $m_\pi^{\min} = 300\text{MeV}$
- 2+1f DW in sea
- Overlap in *valence*
- Wide range of charm quark masses



	stat	m_h	m_s	rest (scale, disc,χ,...)
Φ_{D_s}	0.88	0.99	0.83	0.60

All errors listed in %.

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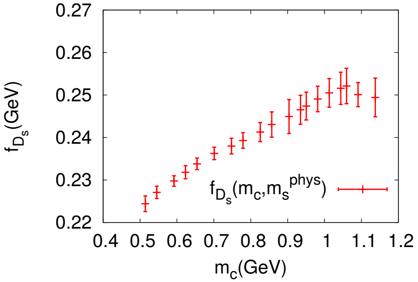

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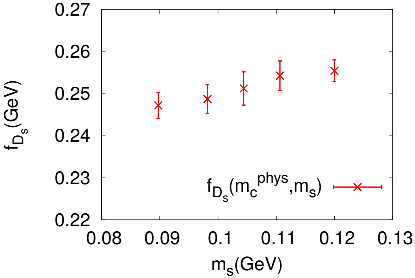


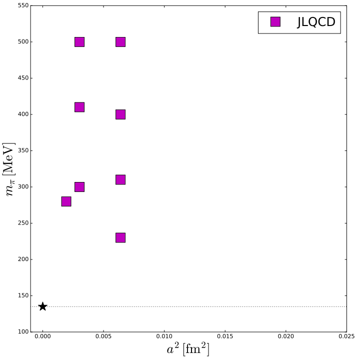
	stat	m_h	m_s	rest (scale, disc,χ,...)
Φ_{D_s}	0.88	0.99	0.83	0.60

All errors listed in %.

JLQCD

- 3 lattice spacings ($a_{\min} = 0.44\mathrm{fm}$)
- $m_\pi^{\min} = 230\,\mathrm{MeV}$
- $N_f = 2 + 1$
- DWF for sea and valence.
- Stout smeared with three hits ($\rho = 0.1$)
- Linear in m_π^2 and a^2
- No systematic error yet

$f_D = 212.8(1.7)_{\mathrm{stat}}(3.6)_{\mathrm{scale}}\,\mathrm{MeV}$
$f_{D_s} = 244.0(0.84)_{\mathrm{stat}}(4.1)_{\mathrm{scale}}\,\mathrm{MeV}$



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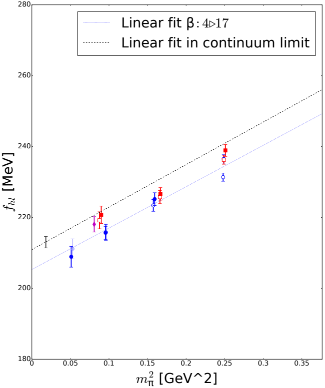
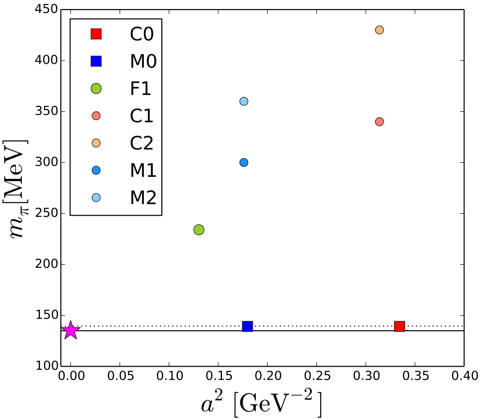
B. Fahy at Lattice 2016



RBC/UKQCD - set up

- 3 lattice spacings ($a_{\rm min} = 0.07\rm fm$)
- 2 physical pion mass ensembles
- $N_f = 2+1$
- DWF for sea and valence



RBC/UKQCD - set up

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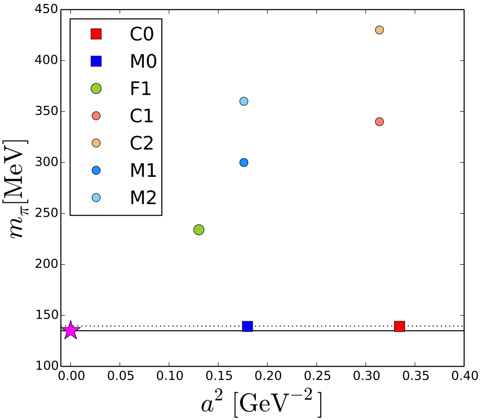
- 2 physical pion mass ensembles
- $N_f = 2+1$
- DWF for sea and valence
- Modification of DW parameters for charm: $M_5 = 1.6, am_h \lesssim 0.4$ $\Rightarrow$ Mixed Action



RBC/UKQCD - set up

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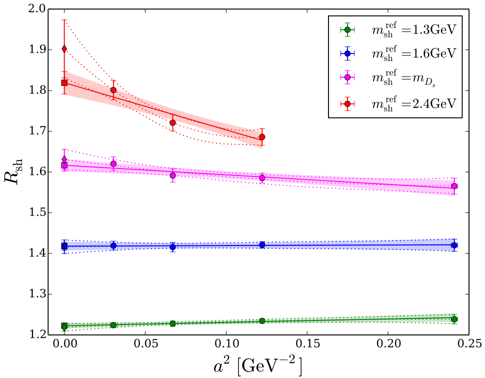
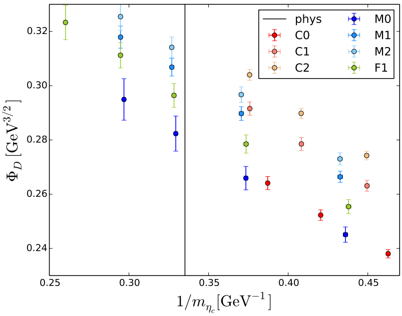
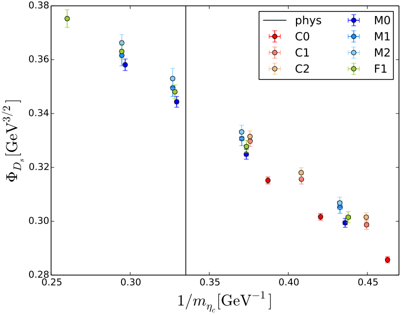


Tested in Pilot Study: arXiv:1602.04118

RBC/UKQCD - data

$$\Phi_P = f_P \sqrt{m_P}$$





$\Rightarrow$ mild a^2 behaviour $\Rightarrow$ mild m_π^2 behaviour

$\Rightarrow$ Slight extrapolation in am_h on coarse ensembles needed.

$\Rightarrow$ Linear in m_π^2, a^2 and $1/m_H$

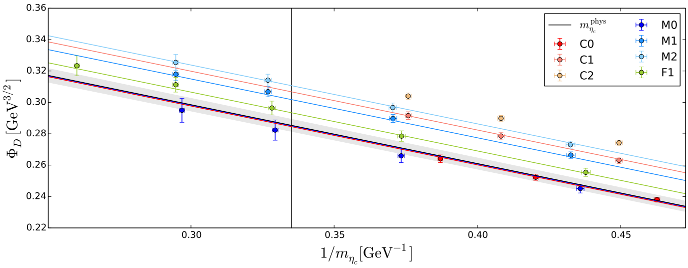
RBC/UKQCD - chiral-CL fits

$$\Phi_P^{\mathrm{phys}} + C_{CL}(\Delta m_H^{-1})a^2 + C_\chi(\Delta m_H^{-1})\Delta m_\pi^2 + C_h^0\,\Delta m_H^{-1}$$

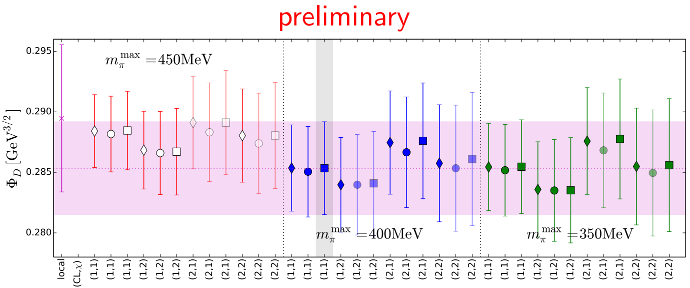
where

$$m_H^{-1} = 1/m_H - 1/m_H^{\mathrm{phys}} \qquad m_H = D, D_s, \eta_c$$

$$C_{CL,\chi}(\Delta m_H^{-1}) = C_{CL,\chi}^0 + C_{CL,\chi}^1 \Delta m_H^{-1}$$

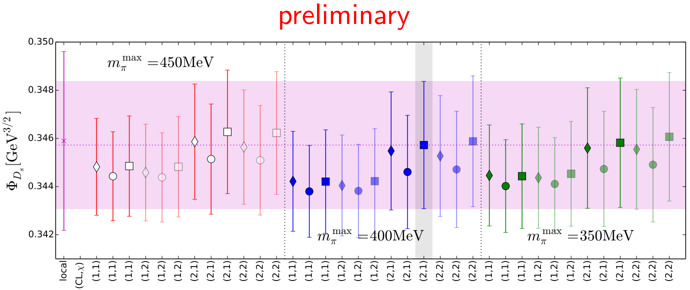


RBC/UKQCD - systematic errors analysis



- fix heavy quark with $H = D(\diamond)$, $D_s(\bigcirc)$, and $\eta_c^{\text{connected}}(\square)$
- Pion mass cuts (350, 400, 450 MeV)
- w and w/o mass dependent CL and m_π coefficients

RBC/UKQCD - systematic errors analysis



- fix heavy quark with $H = D(\Diamond)$, $D_s(\bigcirc)$, and $\eta_c^{\text{connected}}(\Box)$
- Pion mass cuts (350, 400, 450 MeV)
- w and w/o mass dependent CL and m_π coefficients

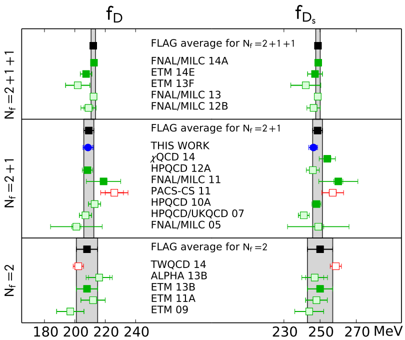
RBC/UKQCD - systematic errors analysis

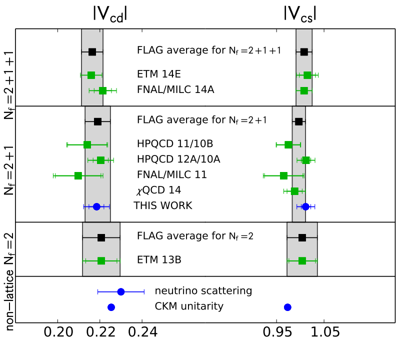
preliminary

observable	central	stat ($\times 10^4$)	total sys ($\times 10^4$)	fit sys ($\times 10^4$)	renormalisation ($\times 10^4$)	finite volume ($\times 10^4$)	$m_u \neq m_d$ ($\times 10^4$)	other ($\times 10^4$)
Φ_D [$\mathrm{GeV}^{3/2}$]	0.2853	38	$^{+29}_{-24}$	$^{+\mathbf{24}}_{-\mathbf{18}}$	11	10	4.7	-
Φ_{D_s} [$\mathrm{GeV}^{3/2}$]	0.3457	26	$^{+18}_{-26}$	$^{+\mathbf{3}}_{-\mathbf{19}}$	14	6	4.4	7.1

RBC/UKQCD - comparison with FLAG

preliminary





⇒ Good agreement with other lattice computations

RBC/UKQCD - Current Limitations

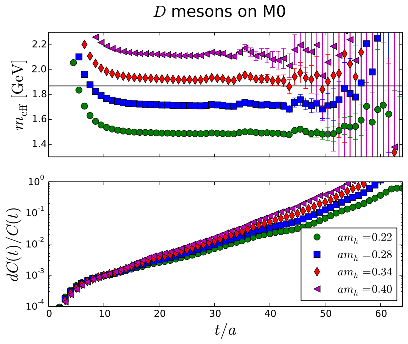
Problems

- $am_h^{\rm max} < am_c^{\rm phys}$ on Coarse
- Poor Signal-to-noise for $m_\pi = 139{\rm MeV}$ and $m_h \gtrsim m_c^{\rm phys}$

Solutions?

- Stout Smeared charm
- Gaussian Smearing for source + sink

Noise growth



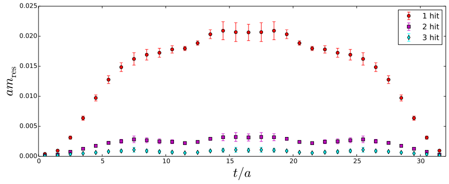
RBC/UKQCD - ongoing work

Smearing

Found sweetspot for

- $M_5 = 1.0$
- 3 hits of stout smearing
- Standard Stout parameter $\rho = 0.1$

hit comparison



Conclusion

⇒ Gaussian smearing of light and strange

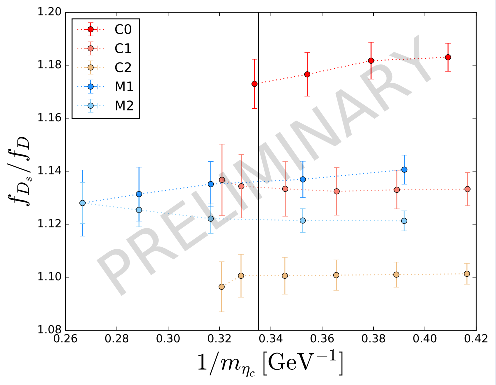
⇒ Stout smearing of heavy quarks

⇒ $am_h \lesssim 0.7$

RBC/UKQCD - ongoing work

WORK IN PROGRESS:

First Data (Limited statistic)

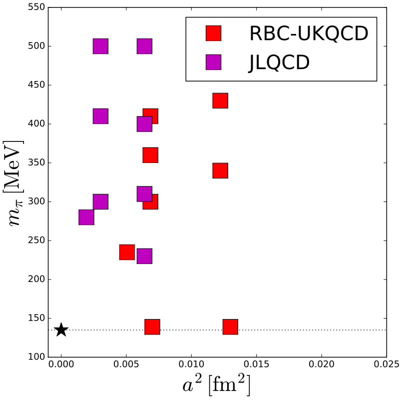


am_h reach

- **Can reach physical charm quark mass** on all ensembles
- **Physical program:** decay constants, semi-leptonics, bag parameters, ξ, HVP
- Ratio Method to reach b.

UKQCD/JLQCD outlook

Possible to combine data sets for well controlled chiral and continuum limit behaviour



Comparison to experimental decay constants

PDG: experimental decays constants

$$f_{D^+}^{\mathrm{exp.}} = 203.7(4.8)\mathrm{MeV} \qquad f_{D_s^+}^{\mathrm{exp.}} = 257.8(4.1)\mathrm{MeV}$$

FLAG: $N_f = 2+1+1$ result

$$f_{D^+}^{\mathrm{lat.}} = 212.15(1.45)\mathrm{MeV} \qquad f_{D_s^+}^{\mathrm{lat.}} = 248.83(1.27)\mathrm{MeV}$$

Leading Systematics in Lattice

- First calculations have reached half-percent precision

$\Rightarrow$ **Need to start to consider IB effects**
First estimate by FNAL/MILC $f_{D^+} - f_D = 0.47(1)(^{+25}_{-\ 6})\mathrm{MeV}$

- **Chiral and CL extrapolation** dominate error budget in most calculations.

Summary and Outlook

- Error on V_{cq} from leptonic decays **dominated by exp**.
- Lattice results mostly consistent but **few**
- More calculations with different fermions actions, set-ups, lattice spacings, number of flavours
  $\Rightarrow$ more are needed (and we are working on it)
- Only two published results for $N_f = 2 + 1 + 1$.
- Leading systematic error: chiral and continuum limit fits
- At present, only two calculations with $m_\pi \approx m_\pi^{\mathrm{phys}}$.

- FLAG $N_f = 2 + 1 + 1$:
  precision of 0.68% and 0.51% for f_D and f_{D_s}

- Strong IB becomes relevant

References

Reviews

PDG + FLAG

$N_f = 2+1+1$

Abbreviation	arXiv
FNAL/MILC 14A	1407.3772
ETM 14 E	1411.7908

$N_f = 2$

Abbreviation	arXiv
TWQCD 14	1404.3648

$N_f = 2+1$

Abbreviation	arXiv
χQCD 14	1410.3343
JLQCD 15	1512.08599[a]
RBC/UKQCD 16	1611.06804[b]

[a] and Proceedings of Lattice2016
[b] and in prep.

ADDITIONAL SLIDES

Motivation: Domain Wall Fermions

DWFs provide a method to simulate (approximately) chiral fermions on the lattice

PROS:

- Used for light and strange sea quarks ($N_f = 2 + 1$)
- Automatic $\mathcal{O}(a)$ improvement
- No operator mixing:
  $\Rightarrow$ easier renormalisation

CONS:

- More expensive due to fifth dimension

$\Rightarrow$ Introduces additional tunable parameters: L_s, M_5

Domain Wall Fermions I

- $4+1d$ model of free DWFs.
- s dependent mass term

$$M_5(s) = M_5\epsilon(s) = M_5\frac{|s|}{s} = \pm M_5$$

- Dirac equation

$$(\partial\!\!\!/ + \gamma_5\partial_s + M_5(s))\Psi(x,s) = 0$$

- Separation of variables $\Psi(x,s) = \psi(x)b_n(s)$
  $\Rightarrow$ Tower of massive modes propagating in the 5th dim.
  $\Rightarrow$ one **chiral massless** mode exponentially localised at $s=0$

$$b_0(s) \propto e^{-M_5|s|}$$

Domain Wall Fermions II

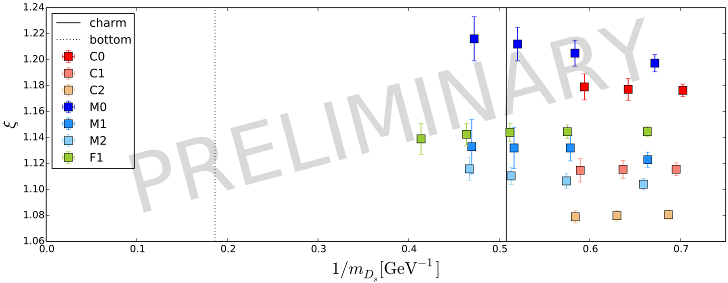
- add Wilson term

$$b_0(s,k) \propto e^{-\int_0^s (M_5(s')+\Delta m(k))\mathrm{d}s'}$$

- remains normalisable as long as $|\Delta m(k)| < M_5$. Otherwise chiral mode vanishes
- a quark mass term also changes the localisation of the physical mode at the boundary
- Add a 2nd *wall*. $\Rightarrow$ LH and RH modes
- L_s determines the **residual chiral symmetry breaking**, quantified by m_{res}.
- M_5 impacts UV behaviour and localisation of physical modes to DW boundary.

RBC/UKQCD - Ratio of Bag parameters ξ (1511.09328)

$$B_P = \frac{\langle P^0 | O_{VV+AA} | \bar{P}^0 \rangle}{\frac{8}{3} f_P^2 m_P^2}, \qquad \xi = \frac{f_{hs}\sqrt{B_{hs}}}{f_{hl}\sqrt{B_{hl}}}$$



PLAN:

- Renormalisation of mixed action for Bag parameters.
- Extrapolate to B via ratio method. arXiv:0909.3187

RBC/UKQCD - "Smeared Runs"- residual mass

$$am_{\rm res}(t) = \frac{\sum_{\mathbf{x}} \langle J_{5q}(x) P(0) \rangle}{\sum_{\mathbf{x}} \langle P(x) P(0) \rangle}$$

(where J_{5q} is the pseudoscalar density in 5^{th} dimension)