D-meson semileptonic decays with Lattice QCD

Elvira Gámiz

Universidad de Granada / CAFPE

· **9th International Workshop on the CKM Unitarity Triangle**,
TIFR, Mumbai, Nov 28-Dec 2 2016 ·

1. Introduction

Goals in the study of semileptonic D decays

* Precise determination of CKM matrix elements ($|V_{cd,cs}|$)

$$\text{Experiment} = (\text{known factors}) \times (V_{CKM}) \times \underbrace{(\text{hadronic matrix elements})}_{\text{lattice QCD}}$$

* Check Standard Model

  ** Consistency of different determinations of CKM matrix elements

  ** Test unitarity of CKM matrix in the second row.

  ** Comparison of shape of form factors with experimental data.

* Validate lattice QCD techniques to use in B physics

* Good candidate for New Physics searches (constraining NP models)

  ** Correlated signals of NP in leptonic and semileptonic decays.

See **S. Fajfer talk**

1. Introduction: Lattice QCD

Lattice QCD: Numerical evaluation of QCD path integral (rely only on first principles).

* Control over systematic errors:

** Unquenched calculations: $N_f = 2$, $\mathbf{N_f = 2+1}$ or $\mathbf{N_f = 2+1+1}$.

** Discretization: improved actions + simulations at several lattice spacings $a's$ $\rightarrow$ continuum limit.

** Chiral extrapolation: simulate at several m_π and extrapolate to $m_\pi^{\rm phys}$

Next step: configurations with **physical light quark masses**.

Five collaborations have generated sets of configurations with physical light quark masses (**PACS-CS, BMW, MILC, RBC/UKQCD, ETM**)

** Renormalization: non-perturbative, perturbative.

** Tuning lattice scale and quark masses $M_{H,lat} = M_{H,exp} \rightarrow m_f = m_{f,phys}$

** Finite volume, **isospin effects, electromagnetic effects**, ...

1. Introduction: Lattice QCD

Several extrapolations/interpolations: lattice spacing, lattice volume, quark masses.

Effective Field Theory: good tool for

* Provides functional form for extrapolation (or interpolation).
* Used to build improved lattice actions/methods
* Used to anticipate size of systematic effects (discretization, FV, chiral extrap. ...

Charm: heavy or light quark?

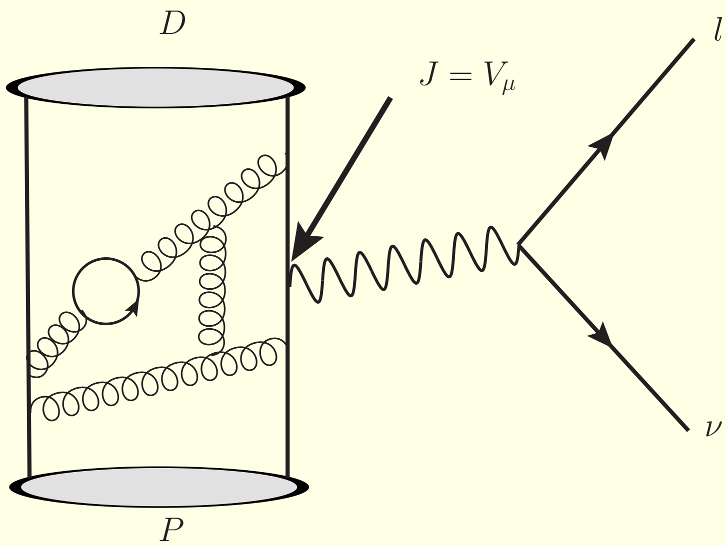
Dominant discretization errors for light quarks are $\mathcal{O}(\alpha_s^k (a\Lambda_{QCD})^n)$

Dominant discretization errors for heavy quarks are $\mathcal{O}(\alpha_s^k (am_h)^n)$

($am_c \sim 0.15 - 0.6$ in current lattice spacings)

For cham use light quark methods
with improved actions (HISQ, tmWilson, NP imp. Wilson ...)

2. Semileptonic D decays



$P = \pi, K$

$x = d, s$ daughter light quark

$q = (p_D - p_P)$ (momentum of lepton pair)

$$\underbrace{\frac{d\Gamma(D \to Pl\nu)}{dq^2}}_{experimental} = \frac{G_F^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_D^2} |V_{cx}|^2$$

$$\left[\left(1 + \frac{m_l^2}{2q^2}\right) m_D^2 (E_P^2 - m_P^2) \underbrace{|f_+(q^2)|^2}_{lattice\, QCD} + \frac{3m_l^2}{8q^2} (m_D^2 - m_P^2)^2 \underbrace{|f_0(q^2)|^2}_{lattice\, QCD} \right]$$

With vector and scalar form factors $f_+(q^2)$ and $f_0(q^2)$ defined by

$$\langle P(p_P)|V_\mu|D(p_D)\rangle = \left(p_{P\mu} + p_{D\mu} - \frac{m_D^2 - m_P^2}{q^2} q_\mu \right) f_+(q^2) + \frac{m_D^2 - m_P^2}{q^2} q_\mu \, f_0(q^2)$$

2. Semileptonic D decays

For $l = e, \mu$ the contribution from $f_0(q^2)$ can be neglected and

$$\underbrace{\frac{d\Gamma(D \to Pl\nu)}{dq^2}}_{experimental} = \frac{G_F^2}{24\pi^3} |\vec{p}_P|^3 \, |V_{cx}|^2 \, \underbrace{|f_+^{DP}(q^2)|^2}_{lattice\,QCD}$$

The errors on those studies are still dominated by errors in the calculation of the relevant form factors.

$$\frac{d}{dq^2}\Gamma(D \to K(\pi) l\nu) \quad \propto \quad |V_{cs(cd)}|^2 \, |f_+^{D\to K(\pi)}(q^2)|^2$$

$$\text{HFAG2016 } 1.0(2.6)\% \,\text{error} \qquad\qquad 5(8.7)\% \,\text{error}$$

(in leptonic determinations of $|V_{cd(cs)}|$ the main error is experimental)

See **A. Soffer's talk**

2. Semileptonic D decays: Form factors

For $l = e, \mu$ the contribution from $f_0(q^2)$ can be neglected and

$$\underbrace{\frac{d\Gamma(D \to Pl\nu)}{dq^2}}_{experimental} = \frac{G_F^2}{24\pi^3} |\vec{p}_P|^3 \, |V_{cx}|^2 \, \underbrace{|f_+^{DP}(q^2)|^2}_{lattice\,QCD}$$

With lattice QCD we can:

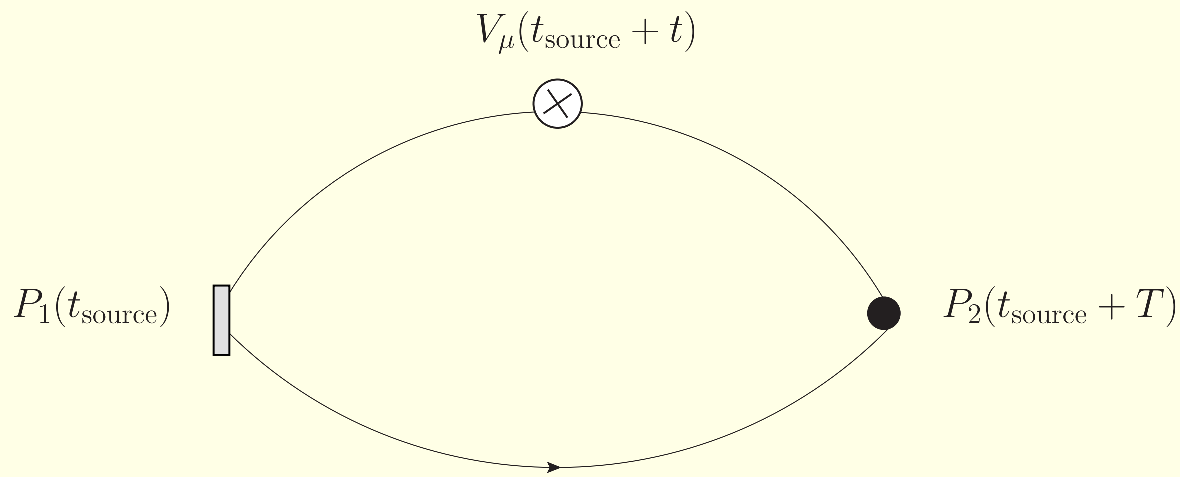
- * Calculate both $f_+^{DP}(q^2)$ and $f_0^{DP}(q^2)$ for the entire q^2 range.
- * Extend to rare semileptonic decay form factors ($f_T^{DP}(q^2)$).

Use `z-expansion` for model-independent parametrization of q^2 dependence

$\rightarrow$ compare shape of lattice form factor with experimental data

3. Semileptonic D decays with Lattice QCD

Two main strategies to eliminate the need of renormalize the lattice currents



Double ratios of 3-point correlators **Becirevic, Haas, Mescia** 0710.1741
(get the form factors from linear combinations of double ratios, with $P=\pi, K$)

$$R_\mu = \frac{C^{DP}_{3pt,\mu}(t,T,\vec{p}_D,\vec{p}_P)\,C^{PD}_{3pt,\mu}(t,T,\vec{p}_P,\vec{p}_D)}{C^{PP}_{3pt,\mu}(t,T,\vec{p}_P,\vec{p}_P)\,C^{DD}_{3pt,\mu}(t,T,\vec{p}_D,\vec{p}_D)}$$

$$\underbrace{\rightarrow}_{t+t_{\rm source}>>a,(T-t-t_{\rm source})>>a} \frac{1}{4}\left|< P(p_P)|V_\mu|D(p_D)>\right|^2 (p_D)_\mu (p_P)_\mu$$

3. Semileptonic D decays with Lattice QCD

Two main strategies to eliminate the need of renormalize the lattice currents

Use the Ward identity ($S = \bar{x}c$) **HPQCD**, Phys.Rev.D82:114506(2010)

$$q^\mu \langle P|V_\mu^{cont.}|D\rangle = (m_c - m_x)\langle P|S^{cont}|D\rangle$$

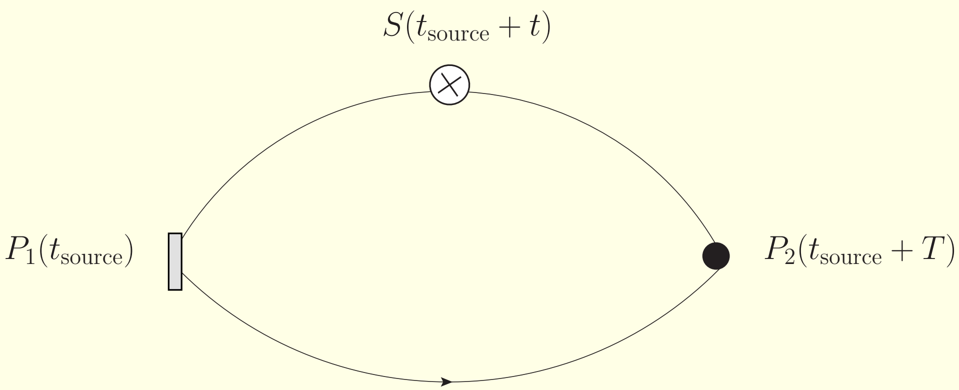
that relates matrix elements of vector and scalar currents. In the lattice

$$q^\mu \langle P|V_\mu^{lat.}|D\rangle Z = (m_c - m_x)\langle P|S^{lat.}|D\rangle$$

$\rightarrow$ replace the V_μ with a scalar current in the 3-point function

$$f_0^{DP}(q^2) = \frac{m_c - m_x}{m_D^2 - m_P^2}\langle P|S|D\rangle_{q^2} \Longrightarrow$$

$$f_+^{DP}(0) = f_0^{DP}(0) = \frac{m_c - m_x}{m_D^2 - m_P^2}\langle P|S|D\rangle_{q^2=0}$$



3. Semileptonic D decays with Lattice QCD

2010/2011: Important reduction of errors in the lattice determination of the form factors $f_+^{D\pi(K)}(0)$ by $N_f = 2+1$ **HPQCD Collaboration** calculations, due mainly to

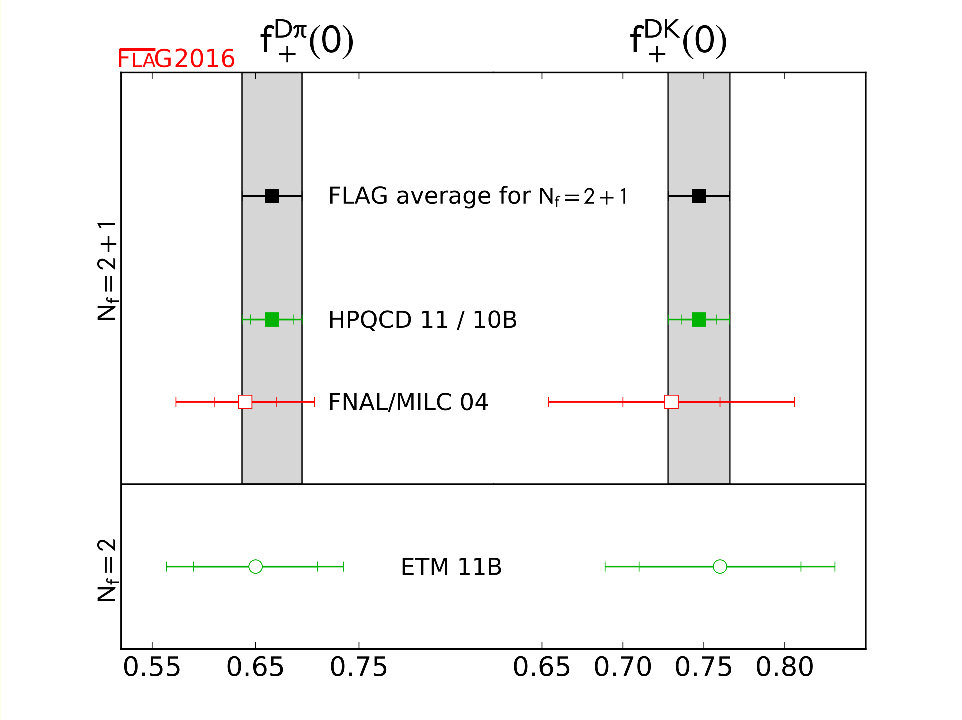
* Use a relativistic action, HISQ, to describe light and charm quarks.

* Used WI to relate scalar matrix elements to vector matrix element

HPQCD, 1008.4562, 1109.1501

$$f_+^{D\pi}(0) = 0.666(29)$$
$$f_+^{DK}(0) = 0.747(19)$$

FLAG 2016 averages 1607.00299,
http://itpwiki.unibe.ch/flag/



Using **HFGA2014** experimental averages and **HPQCD** form factors above:

$$|V_{cd}| = 0.2140(29)_{\rm exp}(93)_{\rm lat} \qquad |V_{cs}| = 0.975(7)_{\rm exp}(25)_{\rm lat}$$

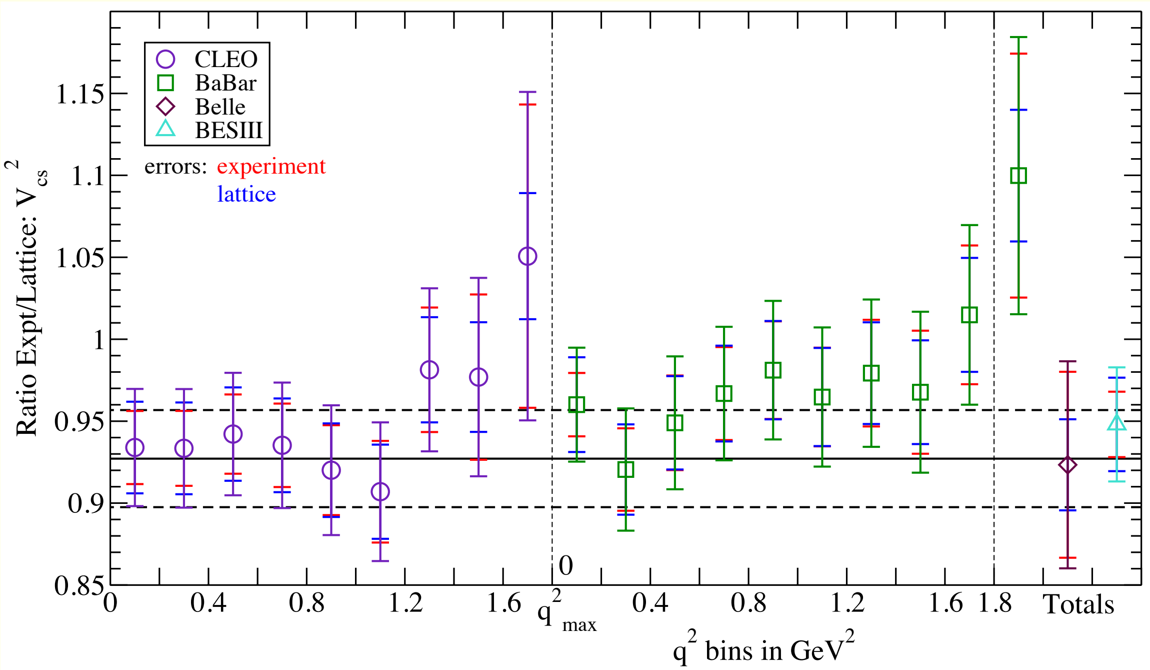
Error dominated by form factor (lattice) uncertainty. Main sources: statistics and am_c disc.

3. Done but not published: $q^2 \neq 0$

$N_f = 2+1$ determination of $|V_{cs}|$ from $D \to K l \nu$ at non-zero momentum transfer HPQCD, 1305.1462

Calculation of $f_0^{DK}(q^2)$ (using Ward identity method) and $f_+^{DK}(q^2)$ (using its definition)

* Global fit to available experimental data (using $z-$expansion) $\to$ extraction of $|V_{cs}|$ using all experimental q^2 bins $\to$ large reduction of error

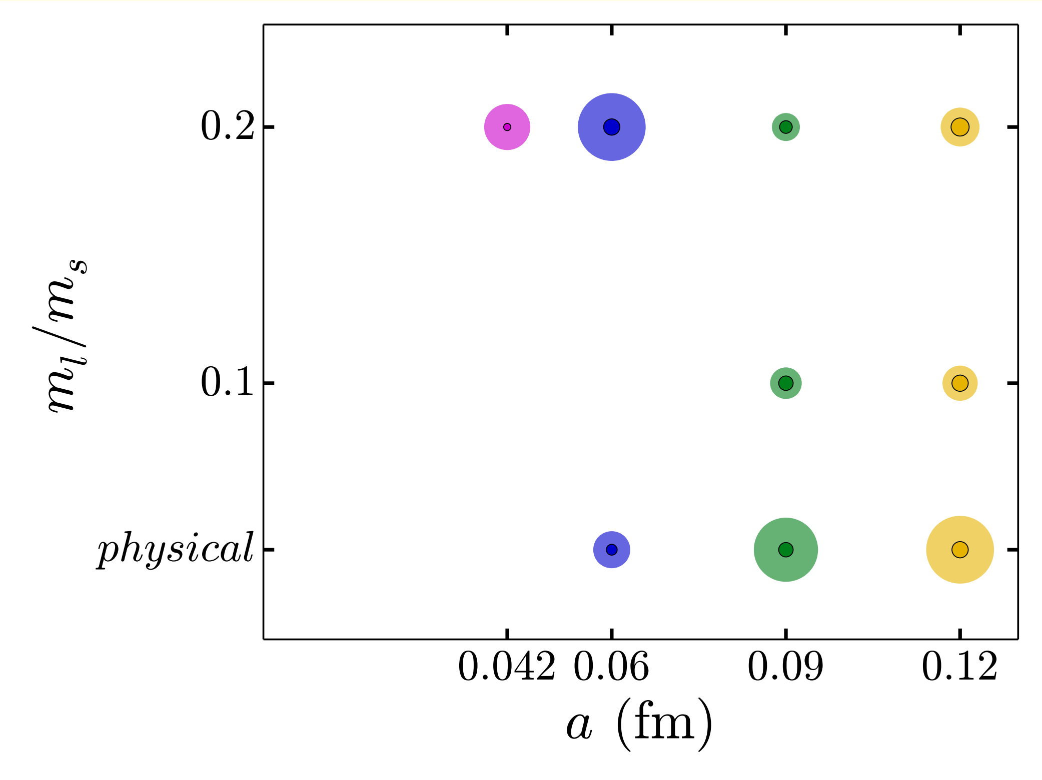


$$|V_{cs}| = 0.963(5)_{exp}(14)_{lat}$$

Lattice form factors are more precise at smaller external momenta (near/at $q^2_{\rm max}$)

4. Work in progress: $q^2 = 0$

FNAL/MILC, talk by S. Gottlieb (T. Primer) at Lattice 2016 $N_f = 2+1+1$



* **HISQ** ensembles including

** Charm quarks on the sea

** Three set of data with physical light quark masses

* Simulate directly at $q^2 = 0$

Use the relation $f_+^{DP}(0) = f_0^{DP}(0) = \frac{m_c - m_x}{m_D^2 - m_P^2} \langle P|S|D\rangle_{q^2=0}$

$\rightarrow$ $f_+^{D\pi}(0)$ and $f_+^{DK}(0)$

4. Work in progress: $q^2 = 0$

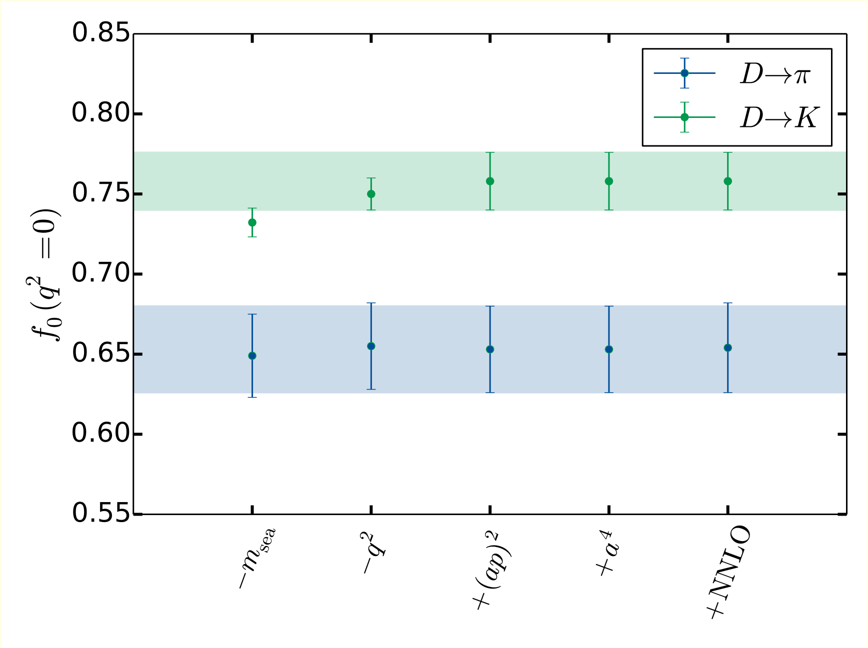
FNAL/MILC, talk by S. Gottlieb at Lattice 2016 $N_f = 2+1+1$

* Continuum-chiral extrapolation in `Heavy Meson ChPT` framework, including leading order discretization effects, in the hard pion/kaon limit.

Aubin, Bernard 0704.0795, **Becirevic, Prelovsek, Zupan** hep-lat/0305001, **Bijnens, Jemos** 1006.1197

$$f_0(q^2=0) = \frac{C_0}{f_\pi}\left[1 + \text{logs}(a, m_v, m_{\text{sea}}) + C_v m_v + C_s m_{sea} + C_a a^2 + C_q q^2\right]$$

Very stable (continuum-chiral) fits under variations of fit function



* Estimate of finite volume effects: negligible correction

Preliminary systematic error analysis: $\sim 4.1\%$ and $\sim 2.4\%$ for $f_+^{D\pi}(0)$ and $f_+^{DK}(0)$

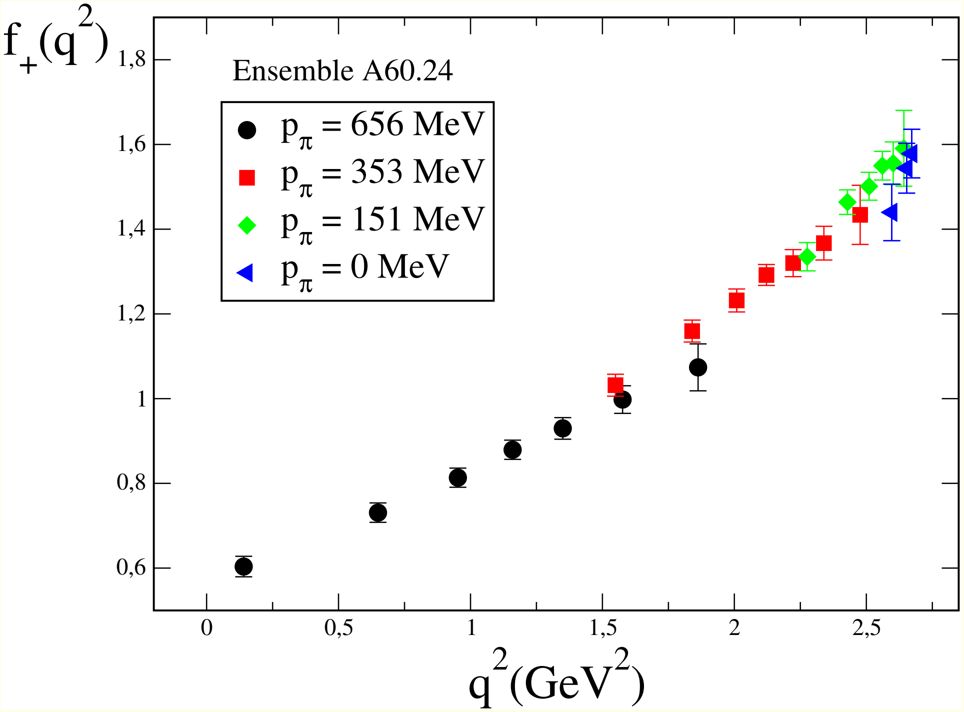
4. Work in progress: $q^2 = 0$

FNAL/MILC, talk by S. Gottlieb at Lattice 2016 $N_f = 2 + 1 + 1$

Future work: Scalar and vector form factors at multiple q^2
$\rightarrow$ combine with experiment to reduce errors. Compare shape.

4. Work in progress: $q^2 \neq 0$

ETM, talk by G. Salerno at Lattice 2016, 1611.00022 $N_f = 2+1+1$



* tmWilson ensembles including

** Charm quarks on the sea

** Pion masses in the range $[210 - 450]$ MeV

* Calculate $f_+^{D\pi}$ and $f_0^{D\pi}$ over whole q^2 range.

* Three different lattice spacings.

* Use double ratios to avoid renormalization.

Global fit to all $f_+(q^2)$, $f_0(q^2)$ data $\rightarrow$ study simultaenously q^2, m_l and a^2 dependence.

* Modified `z-expansion` **Bourrely, Caprini, Lellouch** 0807.2722 imposing $f_+(0) = f_0(0) = f(0)$ (parametrization of $f(0)$ inspired by hard pion $SU(2)$ `ChPT`)

* Add finite volume term in the fit $K_{FSE}^{+(0)} = 1 + C_{FSE}^{+(0)} \xi_l e^{-m_\pi L}/(m_\pi L)$

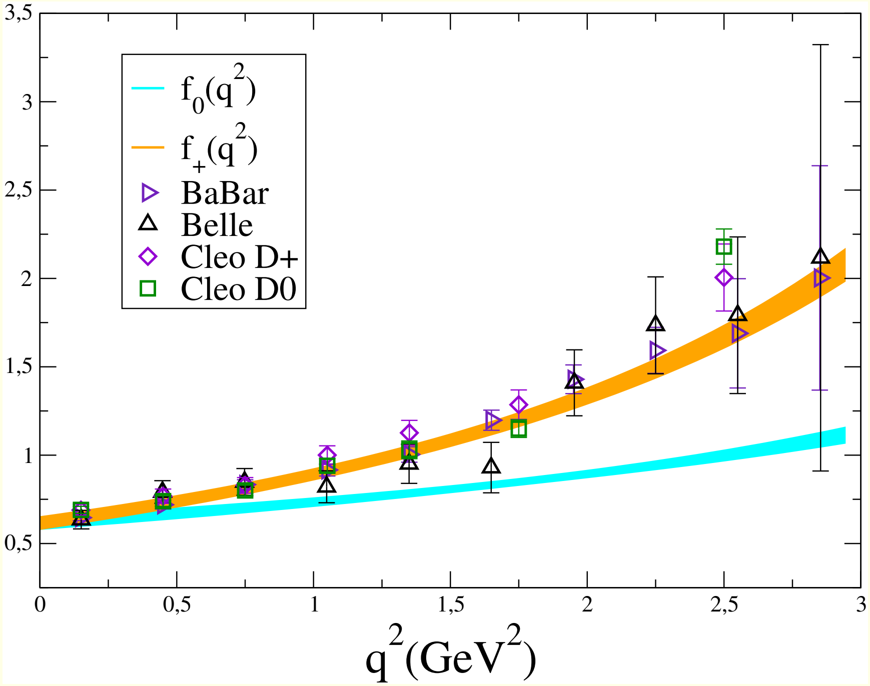
4. Work in progress: $q^2 \neq 0$

ETM, talk by G. Salerno at Lattice 2016, 1611.00022 $N_f = 2+1+1$

* $z_0 \equiv z(q^2 = 0)$

* parametrization of $f(0)$ inspired by hard pion $SU(2)$ ChPT (with $\xi_l \propto m_l$)

$$f(0) = b_0 \left[1 - \frac{3}{4}(1+3g^2)\xi_l \log \xi_l + b_1 \xi_l + b_2 a^2\right]$$



$$f_{+(0)}(q^2) = \frac{f(0) + c_{+(0)}(a^2)(z - z_0)[1 + (z - z_0)/2]}{1 - K_{FSE}^{+(0)} q^2 / M_{V(S)}^2}$$

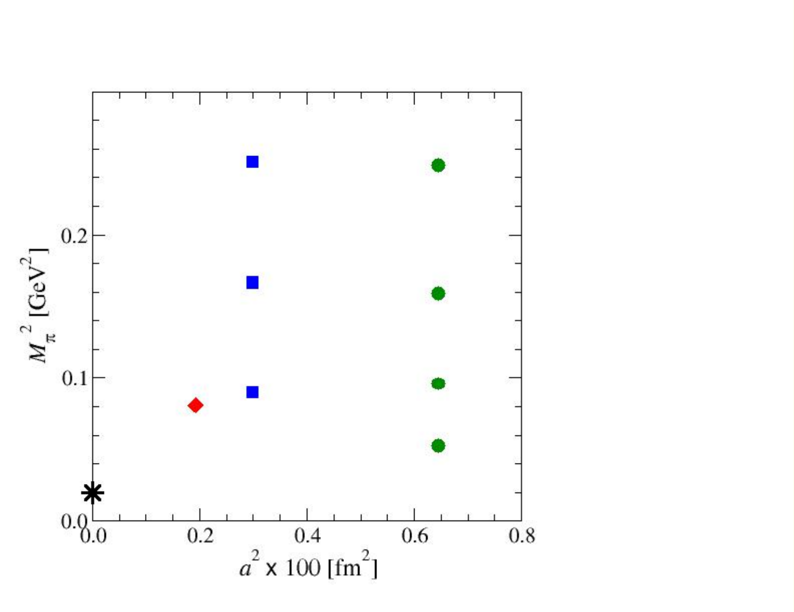
Preliminary systematic error analysis:

$$f_+^{D\pi}(0) = 0.631(37)_{stat}(14)_{chiral}(08)_{disc} = 0.631(40)$$

Future work: Improve statistics and finish systematic error analysis.
Extension to $D \to K$.

4. Work in progress: $q^2 \neq 0$

JLQCD, talk by T. Kaneko at Lattice 2016 $N_f = 2 + 1$



* **domain wall** ensembles including (on-going)

** Pion masses in the range $[230 - 500]$ MeV

** Large volumes $m_\pi L \geq 4$

* Calculate $f_+^{D\pi}$ and f_0^{DK} for several q^2 values

* Three different lattice spacings.

* Calculate renormalization non-perturbatively.

* Still adding ensembles to the analysis.

Use `z-expansion` to extrapolate results to $q^2 = 0$ **preliminary**

$$f_{+,0}^{D\pi}(t) = \frac{1}{1 - t/M_{\rm pole}^2} \sum_k a_k z^k$$

* $f_+(t)$: Include measured $M_{D(s)^*}$, $k = 1$ fit ($k = 2$ for systematic error)

* $f_0(t)$: $k = 1$ fit with no pole ($k = 1$ with pole for systematic error)

4. Work in progress: $q^2 \neq 0$

JLQCD, talk by T. Kaneko at Lattice 2016 $N_f = 2+1$

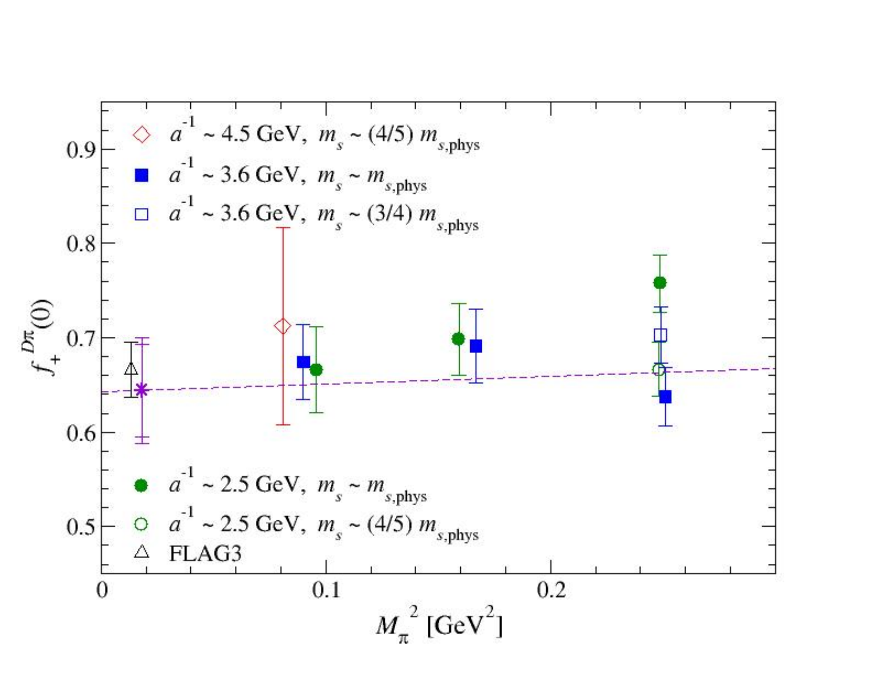
Test a simple linear extrapolation (similar for $f_{+,0}^{DK}$)

$$f_{+,0}^{D\pi}(0, m_\pi^2, m_{\eta_s}^2, a) =$$
$$c_0 + c_\pi m_\pi^2 + c_{\eta_s} m_{\eta_s}^2 + c_a a^2$$

Preliminary estimate

$$f_+^{D\pi}(0) = 0.644(49)_{stat}(36)_{q^2\to 0}(27)_{cont+chiral}$$

$$f_+^{DK}(0) = 0.701(46)_{stat}(12)_{q^2\to 0}(33)_{cont+chiral}$$

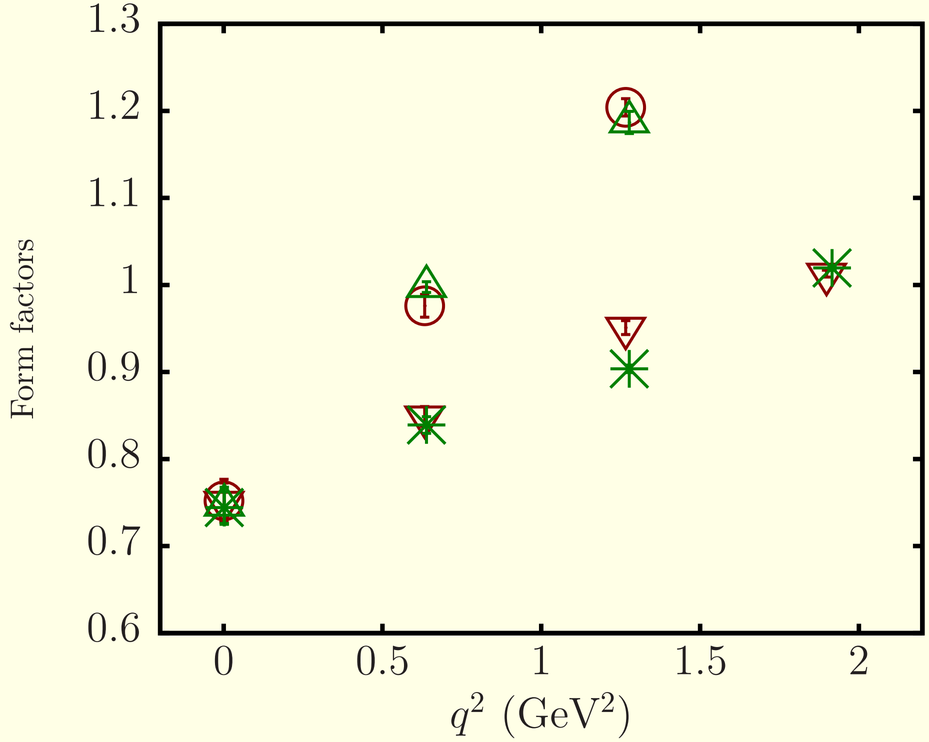


Dominated by statistical error

Future work: Add more ensembles to the analysis, more sophisticated cont+chiral extrapolation $\rightarrow$ significant reduction of errors in the near future. Extension to B physics.

4. Work in progress: $q^2 \neq 0$

HPQCD, B. Chakraborty $N_f = 2+1+1$

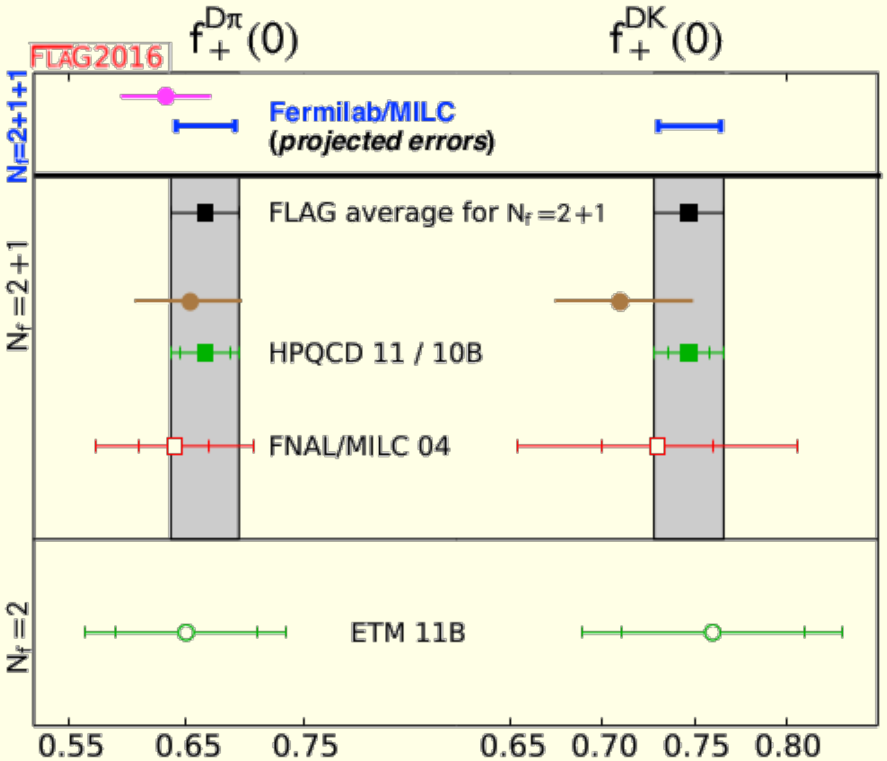


Physical light quark masses

* red circles: f_+ at $a \sim 0.09$ fm
* red triangles: f_0 at $a \sim 0.09$ fm
* green triangles: f_+ at $a \sim 0.12$ fm
* green stars: f_0 at $a \sim 0.12$ fm

HISQ lattice action: Very small discretization errors.

4. Work in progress: Summary



Preliminary results

blue: FNAL/MILC (assuming central values do not change from average)

magenta: ETM

brown: JLQCD

Plot by A. El-Khadra

$f_+^{D\pi}(0)/f_+^{DK}(0)$: **BESIII**, PRD92, 072012(2015) $0.8649 \pm 0.0112 \pm 0.0073$

LCSR, P. Ball, PLB641,50(2006) 0.84 ± 0.04

FLAG averages: 0.89 ± 0.04

Further reduction of error if new calculations take correlations into account in the ratio

5. Correlations with leptonic decays

Cancel CKM matrix elements building ratios of semileptonic and leptonic decay widths

* Experimental averages from **PDG16** (leptonic) and **F. Porter** 1604.04940 (semil.)

$$\left[\frac{f_+^{D\pi}(0)}{f_{D+}}\right]_{exp} = (3.12 \pm 0.08)\,\text{GeV}^{-1} \qquad \left[\frac{f_+^{DK}(0)}{f_{D_s}}\right]_{exp} = (2.87 \pm 0.05)\,\text{GeV}^{-1}$$

* Theoretical (lattice) averages from **PDG16** (leptonic including $N_f = 2+1$ and $N_f = 2+1+1$ calculations) and **FLAG** 1607.00299 (semileptonic)

$$\left[\frac{f_+^{D\pi}(0)}{f_{D+}}\right]_{lat} = (3.14 \pm 0.14)\,\text{GeV}^{-1} \qquad \left[\frac{f_+^{DK}(0)}{f_{D_s}}\right]_{lat} = (3.00 \pm 0.08)\,\text{GeV}^{-1}$$

Good agreement experiment-theory for $|V_{cd}|$-ratios, not so good for $|V_{cs}|$-ratios $\rightarrow$ slight tension between leptonic and semileptonic determinations of $|V_{cs}|$

6. Semileptonic D decays: beyond gold-platted quantities

Alternative determination of $|V_{cs}|$: $\boxed{D_s \to \phi l \nu}$ **HPQCD**, 1311.6669

More challenging: five form factors (vector meson), unstable meson ...

* Treat ϕ as stable and estimate the error.

* q^2 and angular distributions agree with **BaBar** data.

$$|V_{cs}| = 1.017(44)_{lat}(35)_{exp}(30)_{K\bar{K}}$$

* Expected reduction of exper. errors at **BESIII** $\to$ need improvement of theor. calculation (lattice error dominated by statistical error)

Exploratory $N_f = 2+1$ calculation of $D \to \eta^{(\prime)} l \nu$ **G. Bali et al**, 1406.5449

* Calculate $\eta - \eta'$ mixing angles and disconnected contributions

7. Conclusions and outlook

Relativistic description of charm → important reduction of lattice QCD errors in decay constants and semileptonic form factors ...

$$\texttt{Error } f_+^{DK(\pi)} \sim 2.5 - 4.3\%$$

... still theory errors are dominant in $|V_{cd(cs)}|$ extractions from semileptonic decays.

Goal: $\sim 1\%$ error in the form factors.

Not new results since 2013 but several on-going calculations (no statistically correlated with current calculations) will further reduce the error in the following 1-2 years

* $N_f = 2+1+1$ FNAL/MILC: $f_+^{D\pi(K)}(q^2=0)$
* $N_f = 2+1+1$ ETM and HPQCD, and $N_f = 2+1$ JLQCD: shape of $f_{+(0)}^{D\pi(K)}(q^2)$ over entire recoil range

Extensions to FCNC form factors (f_T) straightforward

7. Conclusions and outlook

What do we need to achieve the targetted errors?

* Form factors over the entire recoil range.

* Physical quark masses, especially for $D \to \pi$ quantities.

* Small lattice spacings and statistical errors (straightforward, but expensive)

* And will eventually need to include subdominant effects:

** Include charm in the sea (already started).

** EM effects $\to$ Eventually will do QCD+QED simulations.

** Strong isospin breaking effects: leading order corrections included via tuning light valence quarks (effects of degenerate sea are NNLO in CHPT).

Shed light over current tensions in the unitarity of the second row, and between leptonic and semileptonic $|V_{cs}|$ determinations. See **A. Soffer** talk

* Interesting to improve theory error in $D_s \to \phi l \nu$ (upcoming improvement of experimental error by **BESIII**)

Validate lattice techniques $\to$ extend to B physics