$|V_{ud}|$ from nuclear beta decays

J.C. Hardy  
Cyclotron Institute  
Texas A&M University

with  
I.S. Towner
CURRENT STATUS OF $V_{ud}$

$$V_{ud} = 0.97420 \pm 0.00021$$
SUPERALLOWED $0^+ \rightarrow 0^+$ BETA DECAY

BASIC WEAK-DECAY EQUATION

$$ft = \frac{K}{G_v^2} \langle \tau \rangle^2$$

$f$ = statistical rate function: $f(Z, Q_{EC})$
$t$ = partial half-life: $f(t_{1/2}, BR)$

$G_v$ = vector coupling constant
$\langle \tau \rangle$ = Fermi matrix element

EXPERIMENT
**SUPERALLOWED $0^+ \rightarrow 0^+$ BETA DECAY**

**BASIC WEAK-DECAY EQUATION**

\[
ft = \frac{K}{G_v^2 \langle \tau \rangle^2}
\]

- $f = \text{statistical rate function: } f(Z, Q_{EC})$
- $t = \text{partial half-life: } f(t_{1/2}, BR)$
- $G_v = \text{vector coupling constant}$
- $\langle \tau \rangle = \text{Fermi matrix element}$

**INCLUDING RADIATIVE AND ISOSPIN-SYMMETRY-BREAKING CORRECTIONS**

\[
\mathcal{J}t = ft (1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)}
\]
SUPERAALLOWED $0^+ \rightarrow 0^+$ BETA DECAY

BASIC WEAK-DECAY EQUATION

$$ f t = \frac{K}{G_v^2 <\tau>^2} $$

$\mathbf{f} = \text{statistical rate function: } f(Z, Q_{EC})$

$\mathbf{t} = \text{partial half-life: } f(t_{1/2}, \text{BR})$

$G_v = \text{vector coupling constant}$

$<\tau> = \text{Fermi matrix element}$

INCLUDING RADIATIVE AND ISOSPIN-SYMMETRY-BREAKING CORRECTIONS

$$ ft = ft (1 + \delta'_R) \left[ 1 - (\delta_C - \delta_{NS}) \right] = \frac{K}{2G_v^2 (1 + \Delta_R)} $$

$\mathbf{f}(Z, Q_{EC})$

$\sim 1.5\%$

$\mathbf{f}(\text{nuclear structure})$

$0.3-1.5\%$

$\mathbf{f}(\text{interaction})$

$\sim 2.4\%$
BASIC WEAK-DECAY EQUATION

\[ f_t = \frac{K}{G_v^2 \langle \tau \rangle^2} \]

- \( f = \) statistical rate function: \( f(Z, Q_{EC}) \)
- \( t = \) partial half-life: \( f(t_{1/2}, BR) \)
- \( G_v = \) vector coupling constant
- \( \langle \tau \rangle = \) Fermi matrix element

INCLUDING RADIATIVE AND ISOSPIN-SYMMETRY-BREAKING CORRECTIONS

\[ \mathcal{J} t = f_t (1 + \delta'_R) [1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)} \]

- \( f(Z, Q_{EC}) \)
  \( \sim 1.5\% \)
- \( f(\text{nuclear structure}) \)
  \( 0.3-1.5\% \)
- \( f(\text{interaction}) \)
  \( \sim 2.4\% \)

THEORETICAL UNCERTAINTIES

\( 0.05 - 0.10\% \)
FROM A SINGLE TRANSITION

Experimentally determine \( G_v^2 (1 + \Delta_R) \)

\[
\mathcal{H}_t = ft (1 + \delta_R') \left[ 1 - (\delta_C - \delta_{NS}) \right] = \frac{K}{2G_v^2 (1 + \Delta_R)}
\]
FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

$$\mathcal{F}_t = ft (1 + \delta_R') \left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate the correction terms

Test for presence of a Scalar current

$\mathcal{F}_t$ values constant
THE PATH TO $V_{ud}$

FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

$$\not{t} = ft (1 + \delta_R') \left[ 1 - (\delta_c - \delta_{ns}) \right] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate the correction terms

Test for presence of a Scalar current

WITH CVC VERIFIED

Obtain precise value of $G_v^2 (1 + \Delta_R)$

Determine $V_{ud}^2$

$$V_{ud}^2 = \frac{G_v^2}{G_\mu^2}$$
**THE PATH TO $V_{ud}$**

FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

$$\mathcal{F} t = ft (1 + \delta'_R) \left[ 1 - (\delta_c - \delta_{NS}) \right] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

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Test for presence of a Scalar current

WITH CVC VERIFIED

Obtain precise value of $G_v^2 (1 + \Delta_R)$

Determine $V_{ud}^2$

Test CKM unitarity

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1$$

$V_{ud} = G_v^2 / G_{\mu}^2$
THE PATH TO \( V_{ud} \)

FROM A SINGLE TRANSITION

Experimentally determine \( G_v^2 (1 + \Delta_R) \)

\[
\mathcal{F} t = ft (1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)}
\]

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)
Validate the correction terms
Test for presence of a Scalar current

WITH CVC VERIFIED

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

Obtain precise value of \( G_v (1 + \Delta_R) \)
Determine \( V_{ud} \)
Test Conservation of \( L^0 + L^+ + L^- = 0 \)

\( \mathcal{F} t \) values constant

\[
V_{ud} = \frac{G_v^2}{G_\mu^2}
\]

ONLY POSSIBLE IF PRIOR CONDITIONS SATISFIED

\[
V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1
\]
8 cases with $ft$-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.

~220 individual measurements with compatible precision

$$t = ft (1 + \delta_R') [1 - (\delta_c - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$
WORLD DATA FOR $0^+ \rightarrow 0^+$ DECAY, 2016

What’s new:
1) $^{10}$C $t_{1/2}$ – R. Dunlop et al., PRL 116, 172501 (2016).
2) $^{16}$O $Q_{EC}$ – A.A. Valverde et al., PRL 114, 232502 (2015).
3) $^{14}$O $br$ – P.A. Voytas et al., PRC 92, 065502 (2015).

Hardy & Towner

- 8 cases with $ft$-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.
- ~220 individual measurements with compatible precision

$$t = ft \left[ 1 + \delta_R' \right] \left[ 1 - (\delta_C - \delta_{NS}) \right] = \frac{K}{2Gv^2 (1 + \Delta_R)}$$
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\[ t = ft (1 + \delta'_R) \left[ 1 - (\delta_c - \delta_{NS}) \right] = \frac{K}{2G^2 (1 + \Delta_R)} \]
NUMBER OF PROTONS, $Z$

NUMBER OF NEUTRONS, $N$

$R_b$

$Q_{EC}$

$BR$

$0^+ 1$

$0^+ 1$

$\frac{1}{2}$

$WORLD\ DATA\ FOR\ 0^+ \rightarrow 0^+\ DECAY,\ 2016$

- 8 cases with $ft$-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.

- ~220 individual measurements with compatible precision

$ft = ft\ (1 + \delta_R)'\left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_v^2\ (1 + \Delta_R)}$

Hardy & Towner

8 cases with $ft$-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.

~220 individual measurements with compatible precision.

Critical test passed: $ft$ values consistent

$$ft = ft\left(1 + \delta'_{R}\right)\left[1 - (\delta_c - \delta_{NS})\right] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$
1. Radiative corrections

\[ \mathcal{F} t = ft (1 + \delta'_R) \left[ 1 - (\delta_C - \delta_{NS}) \right] = \frac{K}{2G_v^2 (1 + \Delta_R)} \]

\[ \delta'_R = \frac{\alpha}{2\pi} [g(E_m) + \delta_2 + \delta_3 + \ldots] \]

\[ \Delta_R = \frac{\alpha}{2\pi} \left[ 4 \ln(m_z/m_p) + \ln(m_p/m_A) + 2C_{\text{Born}} + \ldots \right] \]

\[ \delta_{NS} \quad \text{Order-$\alpha$ axial-vector universal photonic contributions} \]

2. Isospin symmetry-breaking corrections

\[ \delta_C \quad \text{Charge-dependent mismatch between parent and daughter analog states (members of the same isospin triplet).} \]
\[ \delta_C = \delta_{C1} + \delta_{C2} \]

Difference in configuration mixing between parent and daughter.

- Shell-model calculation with well-established 2-body matrix elements.
- Charge dependence tuned to known single-particle energies and to measured IMME coefficients.
- Results also adjusted to measured non-analog 0\(^+\) state energies.

0.01 – 0.3 %

Mismatch in radial wave function between parent and daughter.

- Full-parentage Saxon-Woods wave function also matched to known binding energy and charge radius from electron scattering.
- Compared with Hartree-Fock calculation matched to known binding energy.
- Core states included based on measured spectroscopic factors.

0.4 – 1.5 %
### A. Agreement with CVC:

$
\mathcal{F}_t
$
values have been calculated with different models for $\delta_c$, then tested for consistency. Normalized $\chi^2$ and confidence levels are shown.

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$\chi^2 = 1.37$

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**Diagram Description:**

- **Title:** Shell-model, Saxon-Woods radial functions
- **Axes:** 
  - $Z$ of daughter
  - $\mathcal{F}_t$
- **Legend:**
  - Towner & Hardy
  - PRC 77, 025501 (2008)
- **Data Points:**
  - Several $\mathcal{F}_t$ values with error bars.
A. Agreement with CVC:

$\mathcal{F}_t$ values have been calculated with different models for $\delta_c$, then tested for consistency. Normalized $\chi^2$ and confidence levels are shown.

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Shell-model, Saxon-Woods radial functions

$\chi^2 = 1.37$

Towner & Hardy
PRC 77, 025501 (2008)

Shell-model, Hartree-Fock radial functions

$\chi^2 = 6.38$

Towner & Hardy
PRC 79, 055502 (2009)
A. Agreement with CVC:

$\mathcal{F} t$ values have been calculated with different models for $\delta_c$, then tested for consistency. Normalized $\chi^2$ and confidence levels are shown.

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$\chi^2 = 1.37$

$\chi^2 = 4.26$

$\chi^2 = 6.38$
**A. Agreement with CVC:**

\( \mathcal{F} \) values have been calculated with different models for \( \delta_c \), then tested for consistency. Normalized \( \chi^2 \) and confidence levels are shown.

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\( \chi^2 = 1.37 \) for Shell-model, Saxon-Woods radial functions.

\( \chi^2 = 4.26 \) for Nuclear density functional theory.

\( \chi^2 = 6.38 \) for Shell-model, Hartree-Fock radial functions.
B. Measurements of mirror superallowed transitions:

\[
\begin{align*}
\text{38}^{88}\text{Ar}^{20} & \rightarrow \text{38}^{88}\text{Ca}^{20} \\
& \text{444 ms} \\
\text{Q}_{\text{EC}} &= \text{6612} \\
\end{align*}
\]
B. Measurements of mirror superallowed transitions:

\[ t = ft \left( 1 + \delta'_R \right) \left[ 1 - (\delta_C - \delta_{\text{NS}}) \right] \]

\[
\frac{ft_A}{ft_B} = \frac{(1 + \delta'_B) \left[ 1 - (\delta^B_C - \delta^B_{\text{NS}}) \right]}{(1 + \delta'_A) \left[ 1 - (\delta^A_C - \delta^A_{\text{NS}}) \right]}
\]

\[
= 1 + (\delta^B_R - \delta^A_R) + (\delta^B_{\text{NS}} - \delta^A_{\text{NS}}) - (\delta^C_C - \delta^C_C)
\]
B. Measurements of mirror superallowed transitions:

$$\mathcal{H}t = ft (1 + \delta_R')\left[1 - (\delta_C - \delta_{NS})\right]$$

$$\frac{ft_A}{ft_B} = \frac{(1 + \delta_R^B)[1 - (\delta_C^B - \delta_{NS}^B)]}{(1 + \delta_R^A)[1 - (\delta_C^A - \delta_{NS}^A)]}$$

$$= 1 + (\delta_R^B - \delta_R^A) + (\delta_{NS}^B - \delta_{NS}^A) - (\delta_C^B - \delta_C^A)$$
B. Measurements of mirror superallowed transitions:

\[ \tau t = ft (1 + \delta'_R) [1 - (\delta_C - \delta_{NS})] \]

\[ \frac{ft_A}{ft_B} = \frac{(1 + \delta^B_R) [1 - (\delta^B_C - \delta^B_{NS})]}{(1 + \delta^A_R) [1 - (\delta^A_C - \delta^A_{NS})]} \]

\[ = 1 + (\delta^B_R - \delta^A_R) + (\delta^B_{NS} - \delta^A_{NS}) - (\delta^B_C - \delta^A_C) \]
B. Measurements of mirror superallowed transitions:

\[ \mathcal{A} t = ft \left( 1 + \delta_R^{'B} \right) \left[ 1 - (\delta_C - \delta_{NS}) \right] \]

\[ \frac{ft_A}{ft_B} = \frac{\left( 1 + \delta_R^{'B} \right) \left[ 1 - (\delta_C - \delta_{NS}) \right]}{\left( 1 + \delta_R^{'A} \right) \left[ 1 - (\delta_C - \delta_{NS}) \right]} \]

\[ = 1 + (\delta_R^{'B} - \delta_R^{'A}) + (\delta_{NS} - \delta_{NS}) - (\delta_C - \delta_C) \]
B. Measurements of mirror superallowed transitions:

\[ t = ft (1 + \delta_R) [1 - (\delta_C - \delta_{NS})] \]

\[ \frac{ft_A}{ft_B} = \frac{(1 + \delta_R^B)[1 - (\delta_C^B - \delta_{NS}^B)]}{(1 + \delta_R^A)[1 - (\delta_C^A - \delta_{NS}^A)]} = 1 + (\delta_R^B - \delta_R^A) + (\delta_{NS}^B - \delta_{NS}^A) - (\delta_C^B - \delta_C^A) \]

H.I. Park et al.
PRL 112, 102502 (2014)
PRC 92, 015502 (2015)
RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally
determine $G_V^2 (1 + \Delta_R)$

$\mathcal{F} t = ft (1 + \delta'_R) [1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2 (1 + \Delta_R)}$

FROM MANY TRANSITIONS

Test Conservation of
the Vector current (CVC)
FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \Delta_R)$

$$\mathcal{I}t = ft (1 + \delta'_R)\left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_V^2 (1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

$G_V$ constant to $\pm 0.011\%$

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

$\mathcal{I}t = 3072.1(7)$

$G_V(1+\Delta_R)^{1/2}/(hc)^3$

$= 1.14962(13)$

$\times 10^{-5}$ GeV$^{-2}$

$\chi^2/\nu = 0.6$
RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

$$\mathcal{F}t = ft (1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms

$G_v$ constant to $\pm 0.011\%$
FROM A SINGLE TRANSITION

\[ \mathcal{F}t = ft (1 + \delta_R^2)[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2 (1 + \Delta_R)} \]

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)
Validate correction terms

\[ G_V \text{ constant to } \pm 0.011\% \]
RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

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Experimentally determine $G_v^2 (1 + \Delta_R)$

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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

$G_v$ constant to $\pm 0.011\%$

Validate correction terms

### Table: Model Results

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### Graph: Comparison of ft for mirror pairs

- SW
- HF

### Graph: ft vs. Z of daughter

- Elements: $^{14}O$, $^{28m}Al$, $^{34}Ar$, $^{42}Sc$, $^{50}Mn$, $^{23}Mg$, $^{30}Ca$, $^{38m}K$, $^{46}V$, $^{44}Co$, $^{62}Ga$, $^{74}Rb$
RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2 (1 + \Delta_R)$

$$\mathcal{F} t = ft (1 + \delta_R') [1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2 (1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms

Test for Scalar current

$G_V$ constant to $\pm 0.011\%$
RESULTS FROM 0⁺→0⁺ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

$$\mathcal{Z}t = ft (1 + \delta_R') \left[ 1 - (\delta_C - \delta_{NS}) \right] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)
Validate correction terms
Test for Scalar current

$G_v$ constant to $\pm$ 0.011%

limit, $C_s/C_v = 0.0012$ (10)

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY
FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

$$\mathcal{F}t = ft (1 + \delta_R') [1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)
Validate correction terms
Test for Scalar current

$G_v$ constant to ± 0.011%

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RESULTS FROM $0^+ \rightarrow 0^+$ DECAY
FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)
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WITH CVC VERIFIED

Obtain precise value of $G_v^2 (1 + \Delta_R)$

Determine $V_{ud}^2$

$$V_{ud}^2 = \frac{G_v^2}{G_{\mu}^2} = 0.94907 \pm 0.00041$$

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY
FROM A SINGLE TRANSITION

Experimentally determine \( G_v^2 (1 + \Delta_R) \)

\[
\mathcal{F} t = ft (1 + \delta'_R) [1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_v^2 (1 + \Delta_R)}
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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)
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Determine \( V_{ud}^2 \)

\[
V_{ud}^2 = \frac{G_v^2}{G_{\mu}^2} = 0.94907 \pm 0.00041
\]

RESULTS FROM \( 0^+ \rightarrow 0^+ \) DECAY

Cabibbo-Kobayashi-Maskawa matrix

weak eigenstates

mass eigenstates
FROM A SINGLE TRANSITION

Experimentally determine $G_v^2 (1 + \Delta_R)$

$$\mathcal{F} = f t \left( 1 + \delta_R' \right) \left[ 1 - (\delta_C - \delta_{NS}) \right] = \frac{K}{2G_v^2 (1 + \Delta_R)}$$

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$V_{ud}^2 = G_v^2/G_{\mu}^2 = 0.94907 \pm 0.00041$

Test CKM unitarity

$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99963 \pm 0.00049$

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY
**T=1/2 SUPERALLOWED BETA DECAY**

**BASIC WEAK-DECAY EQUATION**

\[ f_t = \frac{K}{G_V^2 \langle \tau \rangle^2 + G_A^2 \langle \sigma \tau \rangle^2} \]

\( f = \) statistical rate function: \( f(Z, Q_{EC}) \)

\( t = \) partial half-life: \( t_{1/2}, BR \)

\( G_{V,A} = \) coupling constants

\( \langle \rangle = \) Fermi, Gamow-Teller matrix elements

[Diagram showing the relationship between various parameters in the context of beta decay, including statistical rate functions and experimental measurements.]
**T=1/2 SUPERALLOWED BETA DECAY**

**BASIC WEAK-DECAY EQUATION**

\[
ft = \frac{K}{G_V^2 \langle \tau \rangle^2 + G_A^2 \langle \sigma \tau \rangle^2}
\]

- \( f \) = statistical rate function: \( f(Z, Q_{EC}) \)
- \( t \) = partial half-life: \( f(t_{1/2}, \text{BR}) \)
- \( G_{V,A} \) = coupling constants
- \( <> \) = Fermi, Gamow-Teller matrix elements

**INCLUDING RADIATIVE CORRECTIONS**

\[
\mathcal{F}t = ft (1 + \delta'_{R})[1 - (\delta_C - \delta_{NS})] = \frac{K}{G_V^2 (1 + \Delta_{R})(1 + \lambda^2 \langle \sigma \tau \rangle^2)}
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\( \lambda = G_A / G_V \)
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**NEUTRON DECAY**
Mean life:

\[ \tau = 880.2 \pm 1.0 \text{ s} \]

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Beam: 888.1 ± 2.0 s
Bottle: 879.6 ± 0.7 s
Mean life:
\[ \tau = 880.2 \pm 1.0 \text{ s} \]
\[ \chi^2 / N = 3.7 \]

\( g / g^V \) asymmetry:
\[ \lambda = -1.2725 \pm 0.0020 \]
\[ \chi^2 / N = 4.1 \]
Mean life:

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\[ V_{ud} = 0.9757 \pm 0.0014 \]
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Beam-bottle span
0.9701 ≤ \( V_{ud} \) ≤ 0.9767
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Beam: 888.1 \pm 2.0 \text{ s}
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Beam-bottle span

0.9701 \leq V_{ud} \leq 0.9767

nuclear 0^+ \rightarrow 0^+

\[ V_{ud} = 0.9742 \pm 0.0002 \]
\[ T = \frac{G_v^2 (1 + \Delta_R)(1 + \lambda ^2 \langle \sigma \tau \rangle^2)}{f t (1 + \delta'_R)[1 - (\delta_C - \delta_{NS})]} \]

Naviliat-Cuncic & Severijns
PRL 102, 142302 (2009)
+ B. Fenker, Phd Thesis TAMU
$$\mathcal{F} t = ft \left(1 + \delta'_R \right) \left[1 - (\delta_C - \delta_{NS}) \right] = \frac{K}{G_V^2 \left(1 + \Delta_R \right) \left(1 + \lambda^2 <\sigma \tau>_2 \right)}$$

$V_{ud} = 0.9730 \pm 0.0014$

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\[ \pi^+ \longrightarrow \pi^0 \ e^+ \ \nu_e \]
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Experimental data:

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\tau = 2.6033 \pm 0.0005 \times 10^{-8} \text{ s} \quad \text{(PDG 2009)}
$$

$$
\text{BR} = 1.036 \pm 0.007 \times 10^{-8}
$$

Pocanic et al, PRL 93, 181803 (2004)

Result:

$$
V_{ud} = 0.9749 \pm 0.0026
$$
PION BETA DECAY

Decay process:

\[ \pi^+ \longrightarrow \pi^0 e^+ \nu_e \]

\[ 0^-,1 \longrightarrow 0^-,1 \]

Experimental data:

\[ \tau = 2.6033 \pm 0.0005 \times 10^{-8} \text{ s} \quad \text{(PDG 2009)} \]

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Result:

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nuclear \( 0^+ \rightarrow 0^+ \)

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CURRENT STATUS OF $V_{ud}$ AND CKM UNITARITY

$V_{ud} = 0.97420 \pm 0.00021$

Graph showing the status of $V_{ud}$ with uncertainties for different processes: nuclear, neutron, nuclear mirrors, and pion. The values are as follows:

- Nuclear: $0.9700 \pm 0.0002$
- Neutron: $0.9800 \pm 0.0002$
- Nuclear Mirrors: $0.9750 \pm 0.0002$
- Pion: $0.9742 \pm 0.0002$

Legend:
- Yellow: Experiment
- Red: Radiative correction
- Blue: Nuclear correction
\[ V_{ud} + V_{us}^2 + V_{ub}^2 = 0.99963 \pm 0.00049 \]

- **$V_{ud}$** nuclear decays: $0.94907 \pm 0.00041$
- **$V_{us}$** PDG kaon decays: $0.05054 \pm 0.00027$
- **$V_{ub}$** B decays: $0.00002 \pm 0.00001$
1. Analysis of superallowed $0^+ \rightarrow 0^+$ nuclear $\beta$ decay is shown to confirm CVC and thus yield $V_{ud} = 0.97417(21)$. 

2. The three other experimental methods for determining $V_{ud}$ yield consistent results, but are less precise by a factor of 7 or more. 

3. The current value for $V_{ud}$, when combined with the PDG values for $V_{us}$ and $V_{ub}$, satisfies CKM unitarity to 0.06%.
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3. The current value for $V_{ud}$, when combined with the PDG values for $V_{us}$ and $V_{ub}$, satisfies CKM unitarity to 0.06%.

4. The largest contribution to $V_{ud}$ uncertainty is from the inner radiative correction, $\Delta_R$. Very little reduction in $V_{ud}$ uncertainty is possible without improved calculation of $\Delta_R$.

5. Isospin symmetry-breaking correction, $\delta_C$, has been tested by requiring consistency among the 14 known transitions (CVC), and agreement with mirror-transition pairs. It contributes much less to $V_{ud}$ uncertainty than does $\Delta_R$.

6. Significant improvement in neutron decay measurements would provide a valuable consistency check.