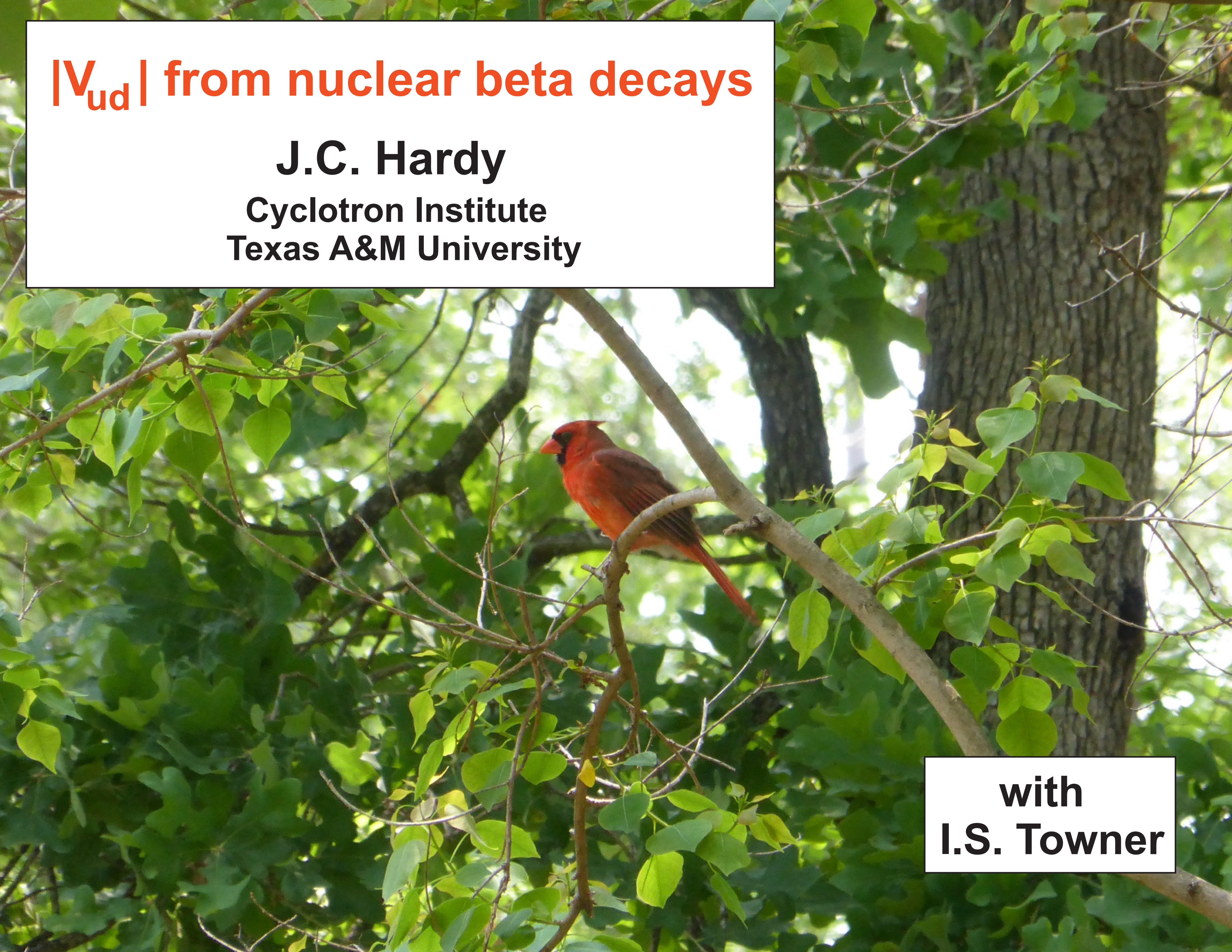
$|V_{ud}|$ from nuclear beta decays

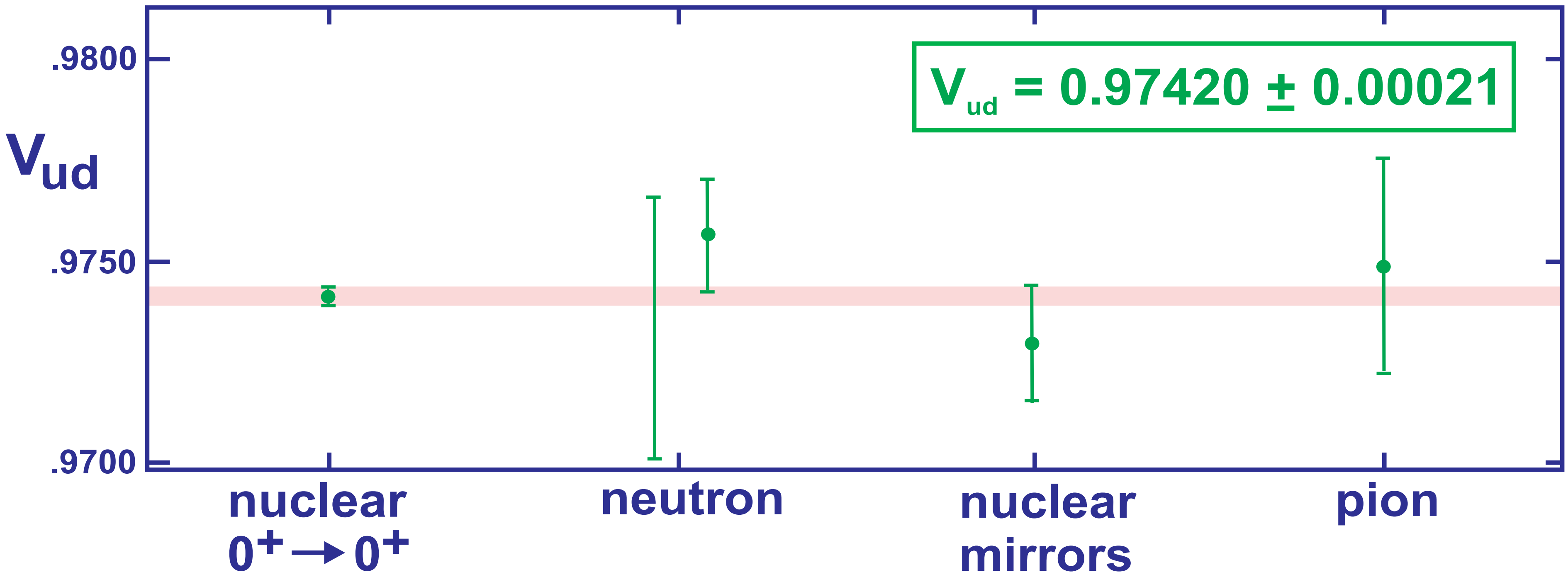
J.C. Hardy

Cyclotron Institute
Texas A&M University

with
I.S. Towner



CURRENT STATUS OF V_{ud}

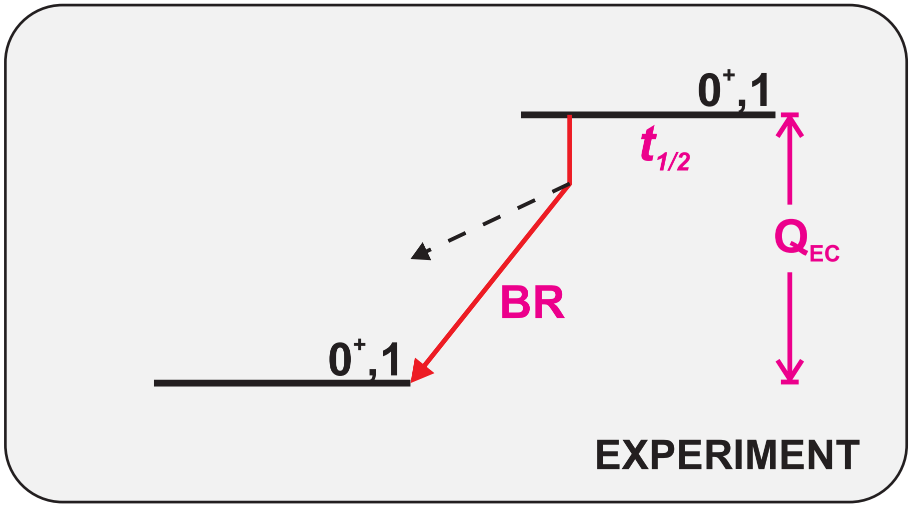


SUPERALLOWED $0^+ \rightarrow 0^+$ BETA DECAY

BASIC WEAK-DECAY EQUATION

$$\boxed{ft = \frac{K}{G_V^{\,2} \langle \tau \rangle^2}}$$

f = statistical rate function: $f(Z, Q_{EC})$
t = partial half-life: $f(t_{1/2}, BR)$
G_V = vector coupling constant
$\langle \tau \rangle$ = Fermi matrix element



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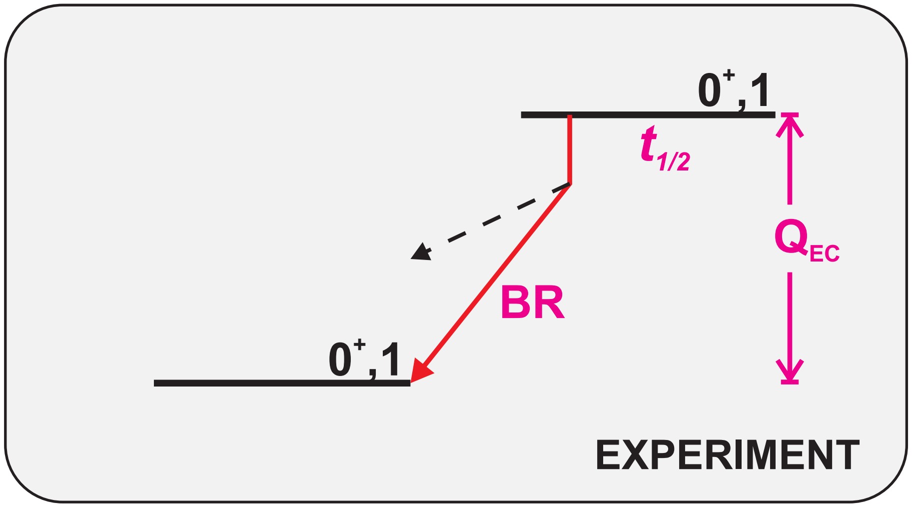

INCLUDING RADIATIVE AND ISOSPIN-SYMMETRY-BREAKING CORRECTIONS

$$\boxed{\mathcal{F}t = ft\,(1 + \delta_R')[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2\,(1 + \Delta_R)}}$$

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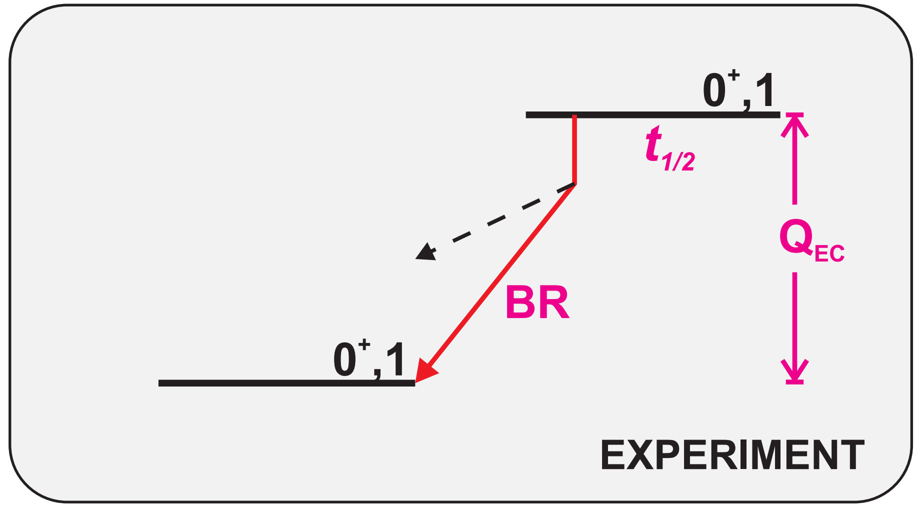

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$f(Z, Q_{EC})$
~1.5%

f(nuclear structure)
0.3-1.5%

f(interaction)
~2.4%

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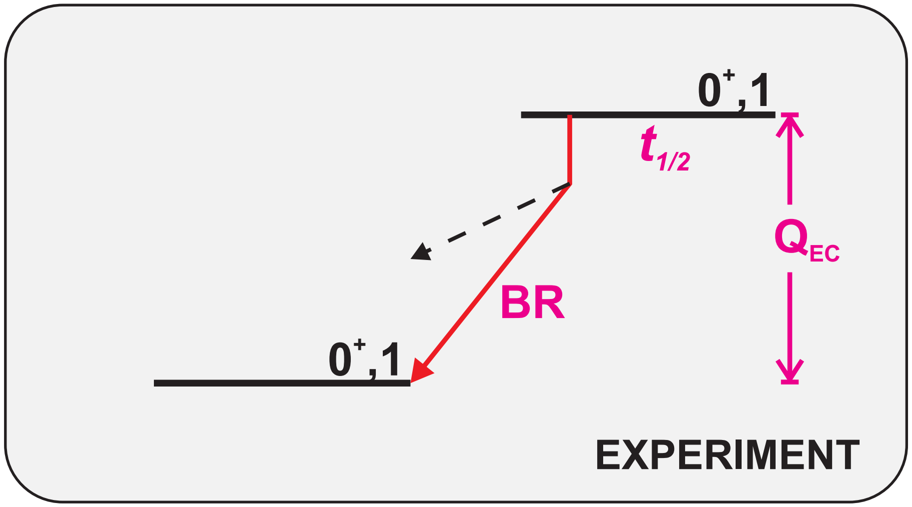

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THEORETICAL UNCERTAINTIES

0.05 – 0.10%

THE PATH TO V_{ud}

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta_R')\left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate the correction terms

Test for presence of a Scalar current

$\mathcal{F}t$ values constant

THE PATH TO V_{ud}

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$\boxed{\mathcal{F}t \text{ values constant}}$

WITH CVC VERIFIED

$$\underbrace{\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}}_{\text{weak eigenstates}} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{Cabibbo Kobayashi Maskawa (CKM) matrix}} \underbrace{\begin{pmatrix} d \\ s \\ b \end{pmatrix}}_{\text{mass eigenstates}}$$

Obtain precise value of $G_V^2(1 + \Delta_R)$

Determine V_{ud}^2

$$\boxed{V_{ud}^2 = G_V^2/G_\mu^2}$$

THE PATH TO V_{ud}

FROM A SINGLE TRANSITION

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weak eigenstates

Cabibbo Kobayashi Maskawa (CKM) matrix

mass eigenstates

Obtain precise value of $G_V^2(1 + \Delta_R)$

Determine V_{ud}^2

$$\boxed{V_{ud}^2 = G_V^2/G_\mu^2}$$

Test CKM unitarity

$$\boxed{V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1}$$

THE PATH TO V_{ud}

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \Delta_R)$

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weak eigenstates

Cabibbo Kobayashi Maskawa (CKM) matrix

mass eigenstates

Obtain precis... $(1 + \Delta_R)$

Determin...

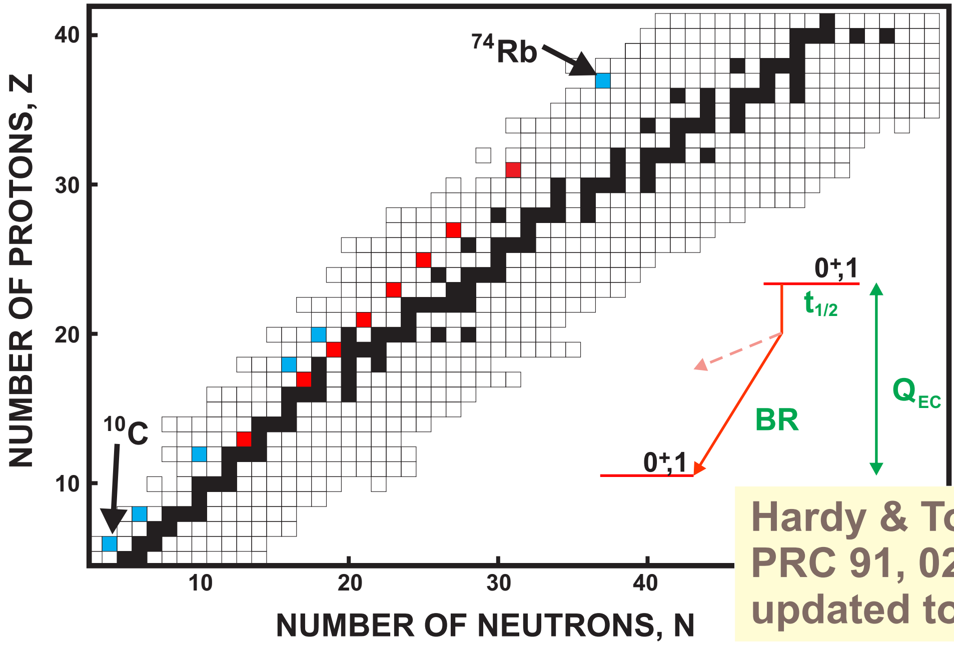
$$V_{ud}^2 = G_V^2/G_\mu^2$$

T... unitarity

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1$$

ONLY POSSIBLE IF PRIOR CONDITIONS SATISFIED

WORLD DATA FOR $0^+ \rightarrow 0^+$ DECAY, 2016



Hardy & Towner
PRC 91, 025501 (2015)
updated to 2016

- 8 cases with *ft*-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.

- ~220 individual measurements with compatible precision

$$\mathcal{F}t = ft\,(1 + \delta_R')[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^{\,2}\,(1 + \Delta_R)}$$

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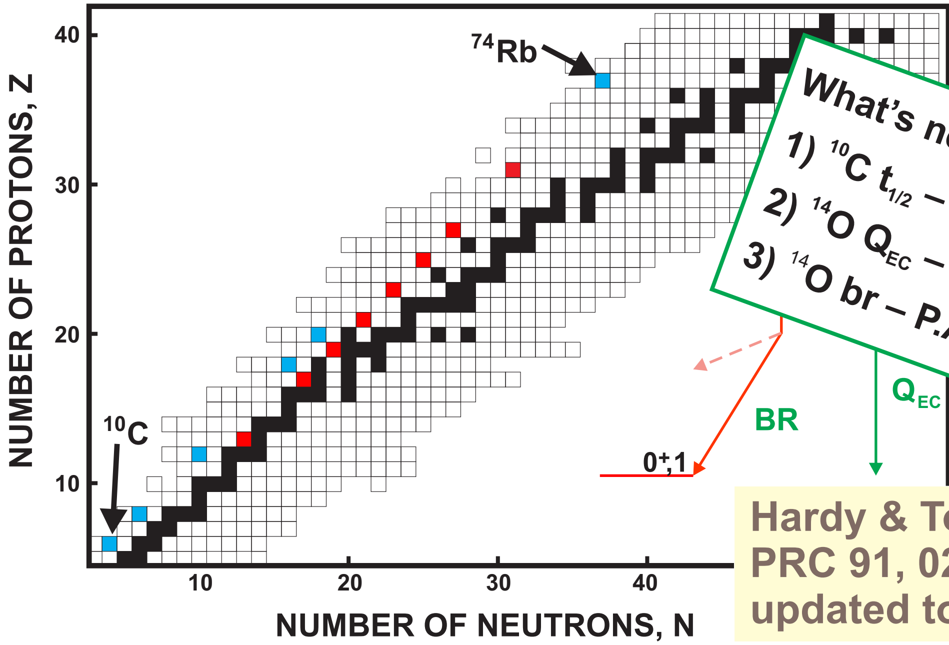

What's new:
1) ^{10}C $t_{1/2}$ – R. Dunlop *et al.*, PRL 116, 172501 (2016).
2) ^{14}O Q_{EC} – A.A. Valverde *et al.*, PRL 114, 232502 (2015).
3) ^{14}O br – P.A. Voytas *et al.*, PRC 92, 065502 (2015).

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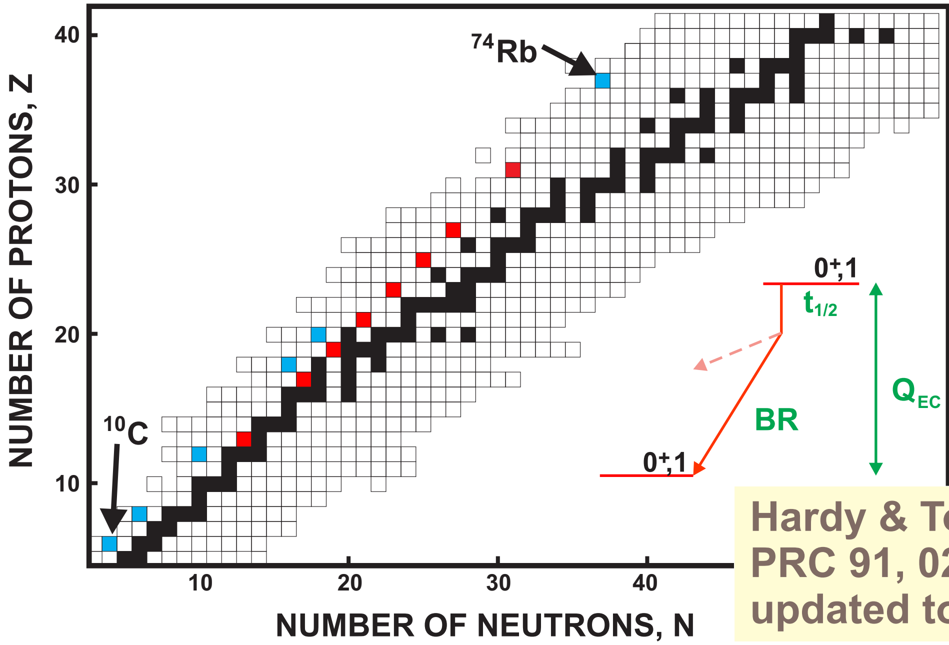

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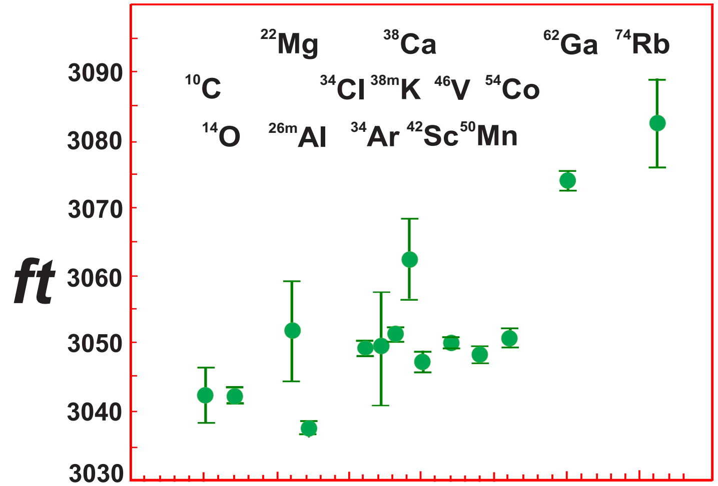

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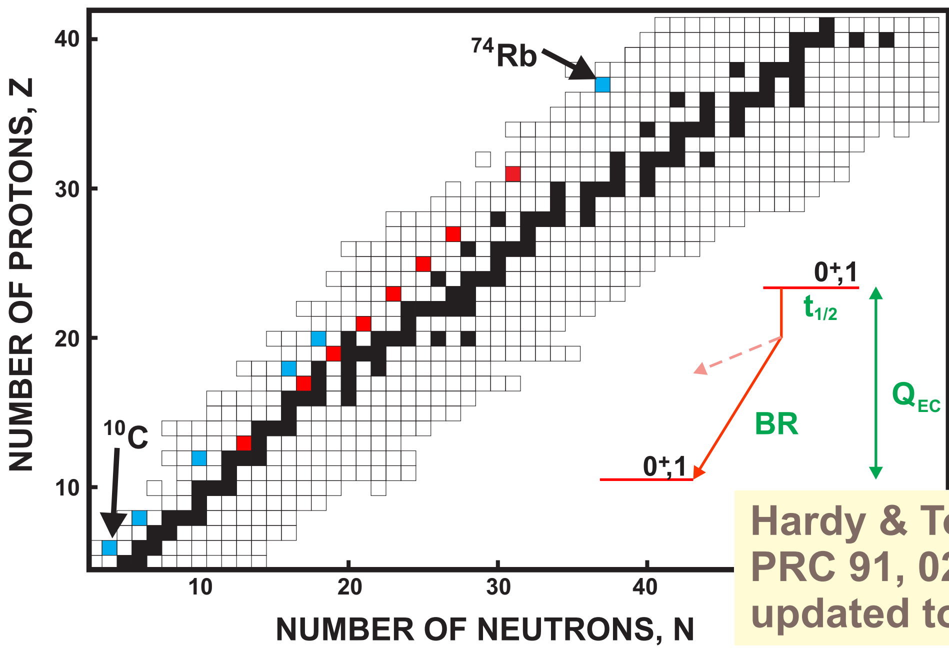

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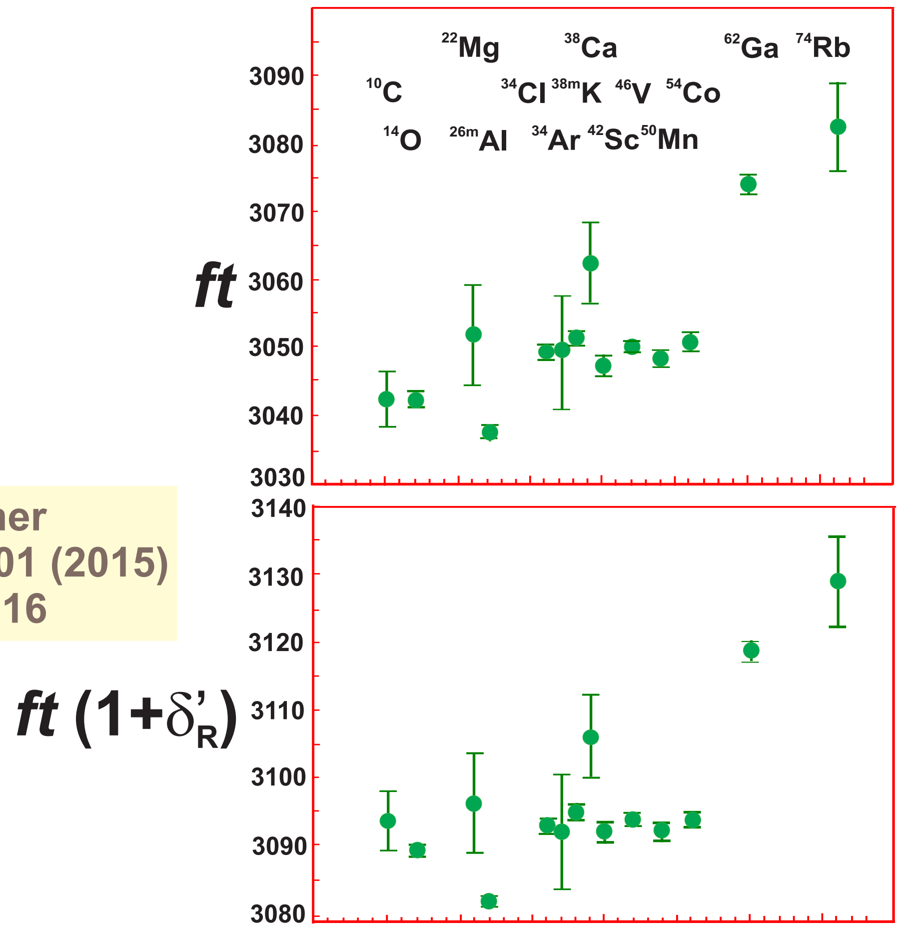

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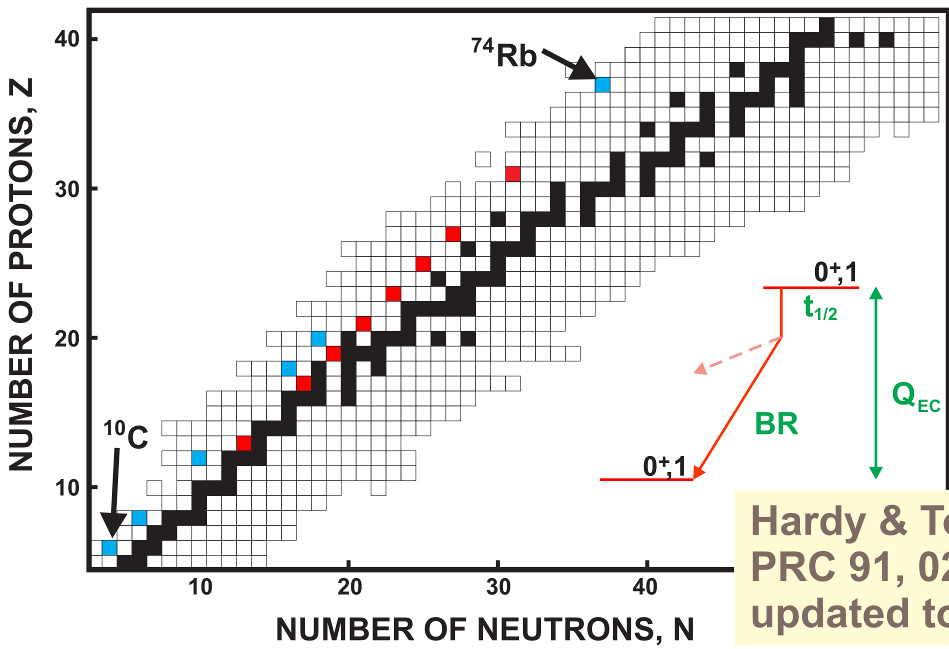

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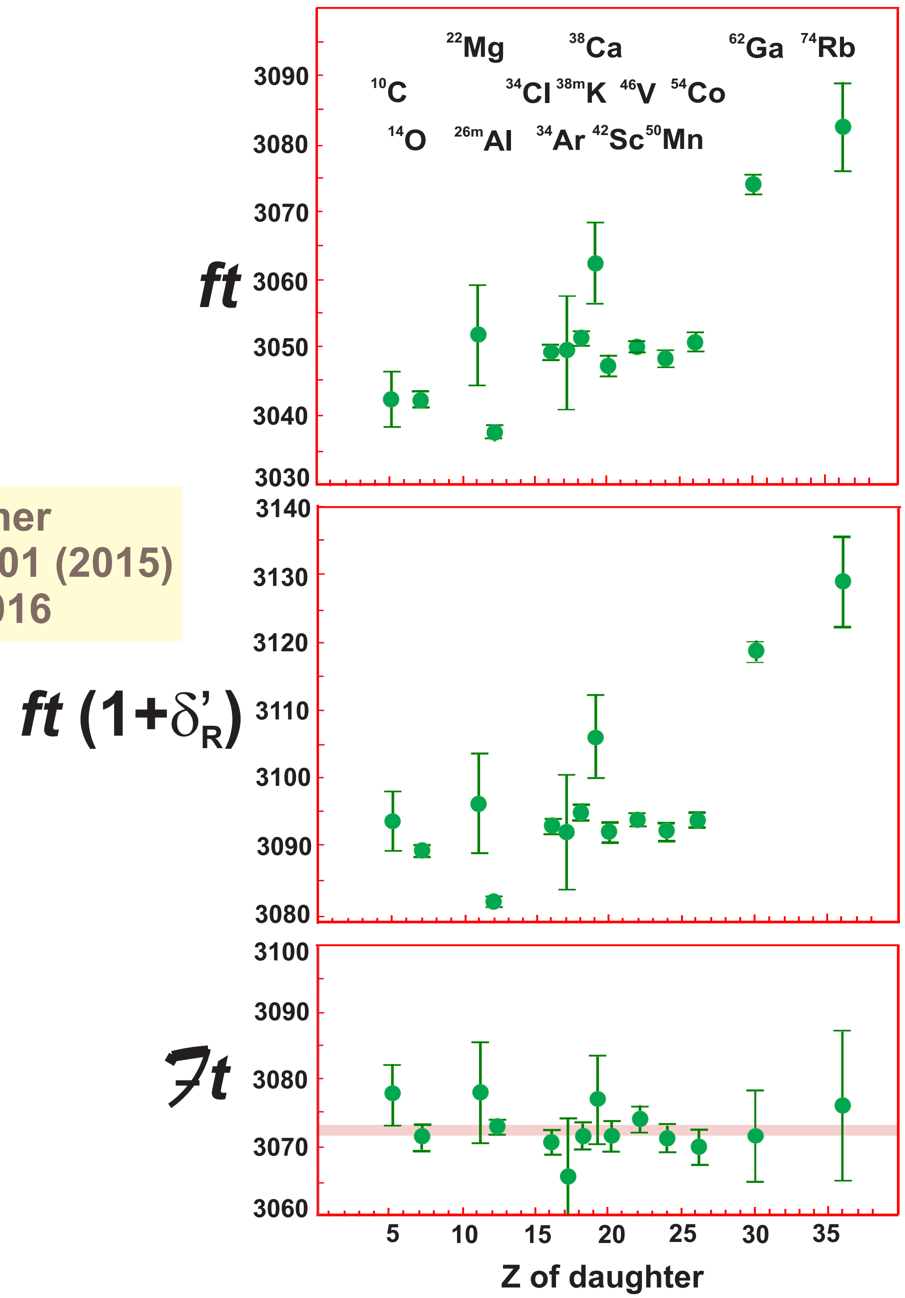

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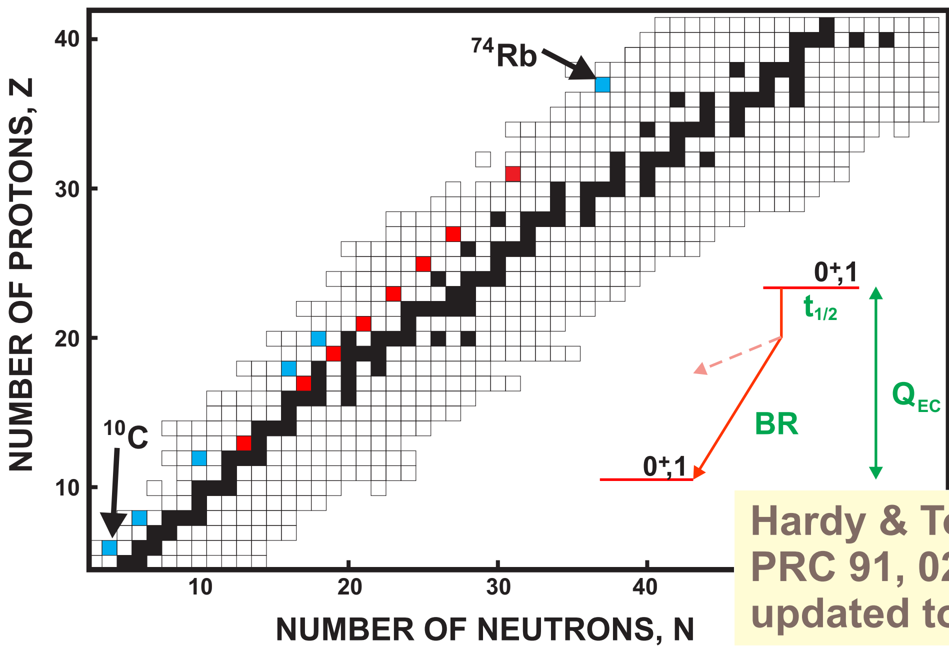

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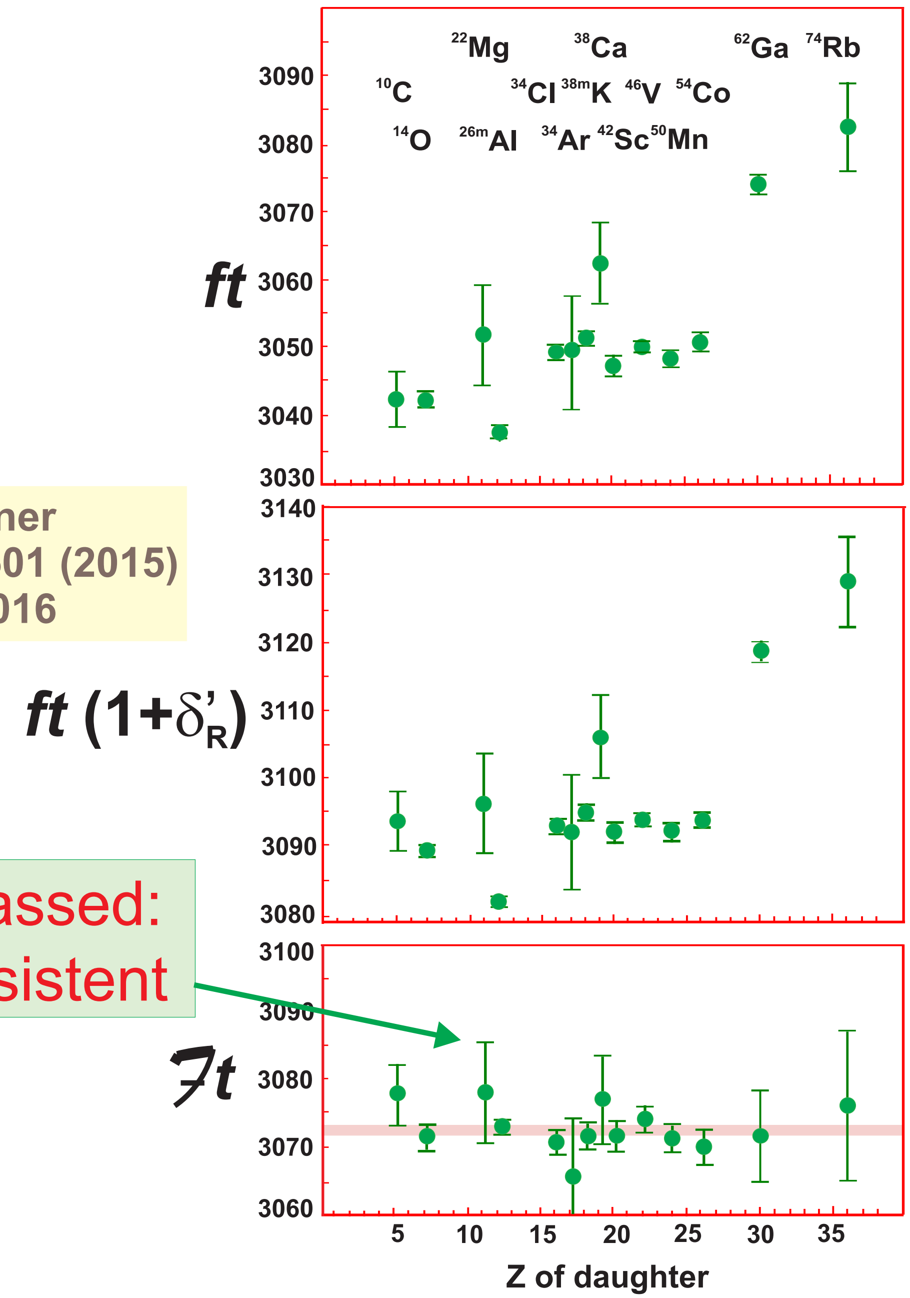

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Critical test passed:
$\mathcal{F}t$ values consistent

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CALCULATED CORRECTIONS TO $0^+ \rightarrow 0^+$ DECAYS

$$\mathcal{F}t = ft\,(1 + \;'_R)[1 - (\;_C - \;_{NS})] = \frac{K}{2G_V^2\,(1 + \;_R)}$$

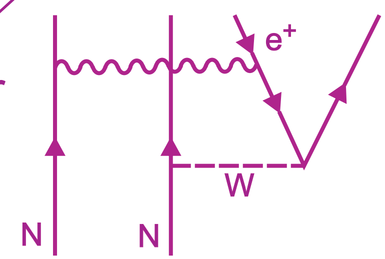
1. Radiative corrections

$$'_R = \frac{\;}{2}[g(E_m) + \;_2 + \;_3 + \ldots]$$

$$_R = \frac{\;}{2}[4\ \ln(m_Z/m_p) + \ln(m_p/m_A) + 2C_{Born} + \ldots]$$

$_{NS}$

Order- axial-vector universal photonic contributions



Dependent on nuclear structure

2. Isospin symmetry-breaking corrections

$_C$ Charge-dependent mismatch between parent and daughter analog states (members of the same isospin triplet).

ISOSPIN SYMMETRY BREAKING CORRECTIONS

$$\delta_C = \delta_{C1} + \delta_{C2}$$

Difference in configuration mixing between parent and daughter.

- Shell-model calculation with well-established 2-body matrix elements.
- Charge dependence tuned to known single-particle energies and to measured IMME coefficients.
- Results also adjusted to measured non-analog 0^+ state energies.

0.01 – 0.3 %

Mismatch in radial wave function between parent and daughter.

- Full-parentage Saxon-Woods wave function also matched to known binding energy and charge radius from electron scattering.
- Compared with Hartree-Fock calculation matched to known binding energy.
- Core states included based on measured spectroscopic factors.

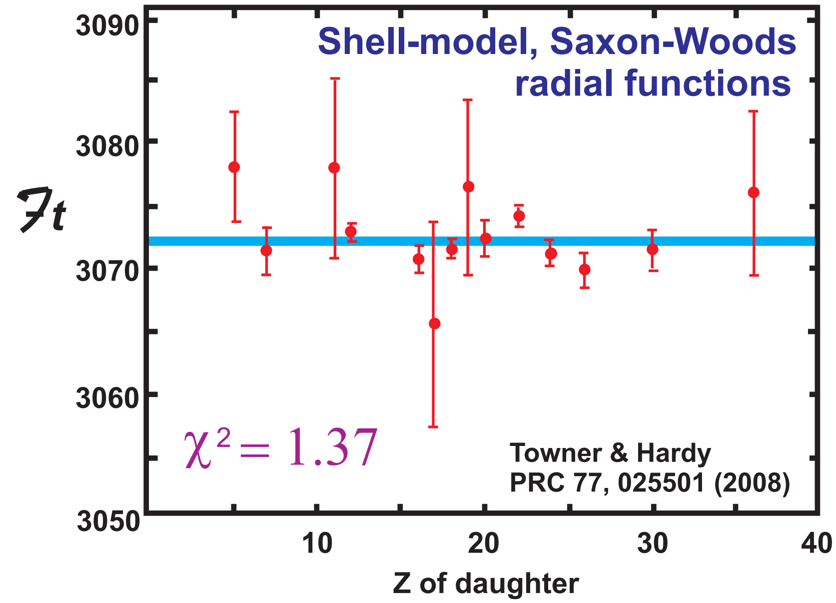
0.4 – 1.5 %

TESTS OF δ_C CALCULATIONS

A. Agreement with CVC:

$\mathcal{F}t$ values have been calculated with different models for δ_C, then tested for consistency. Normalized χ^2 and confidence levels are shown.

Model	χ^2/N	CL(%)
SM-SW	1.37	17



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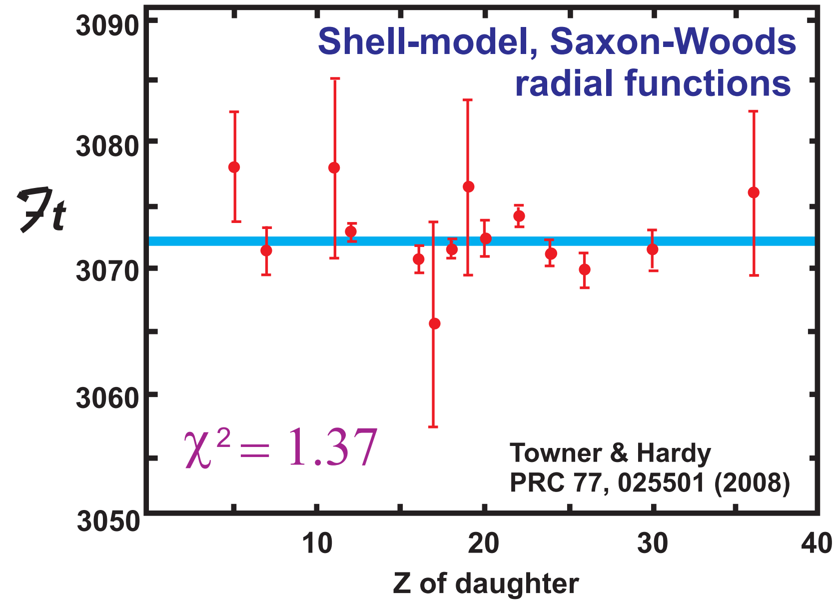

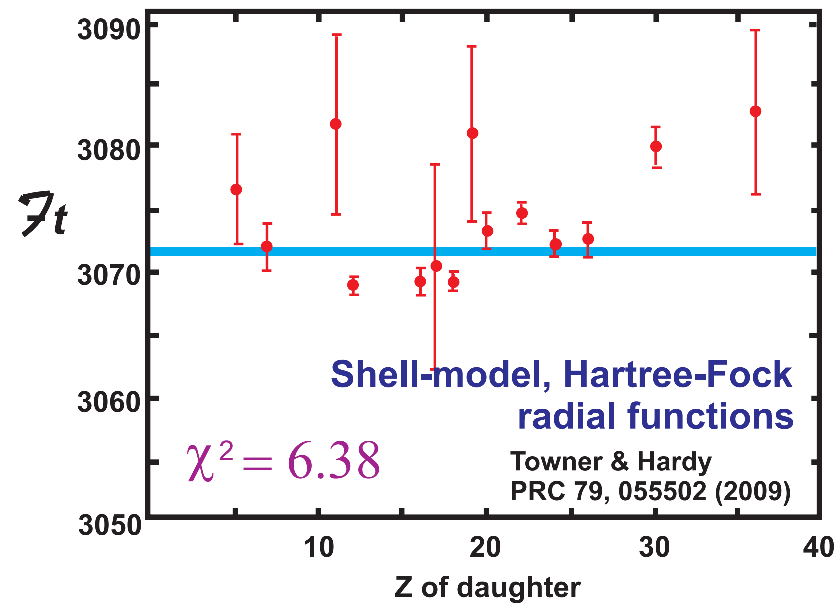

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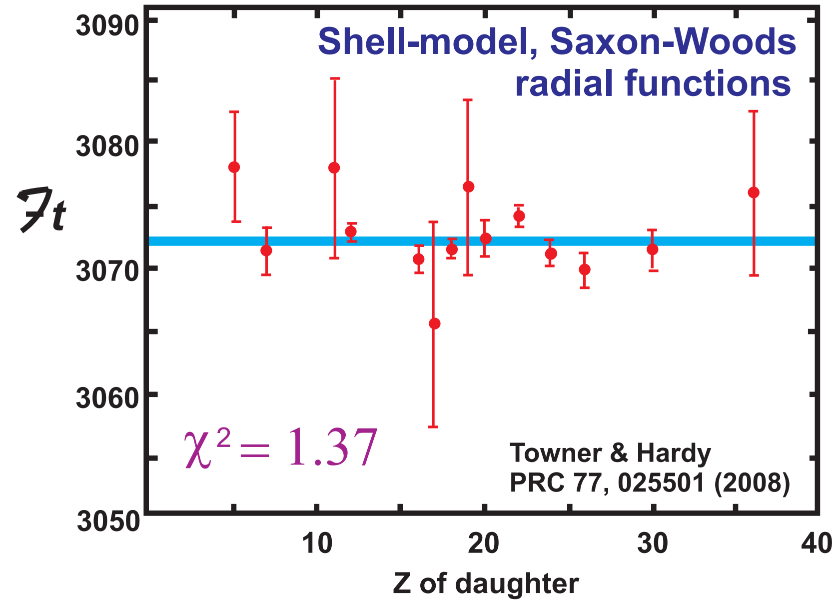

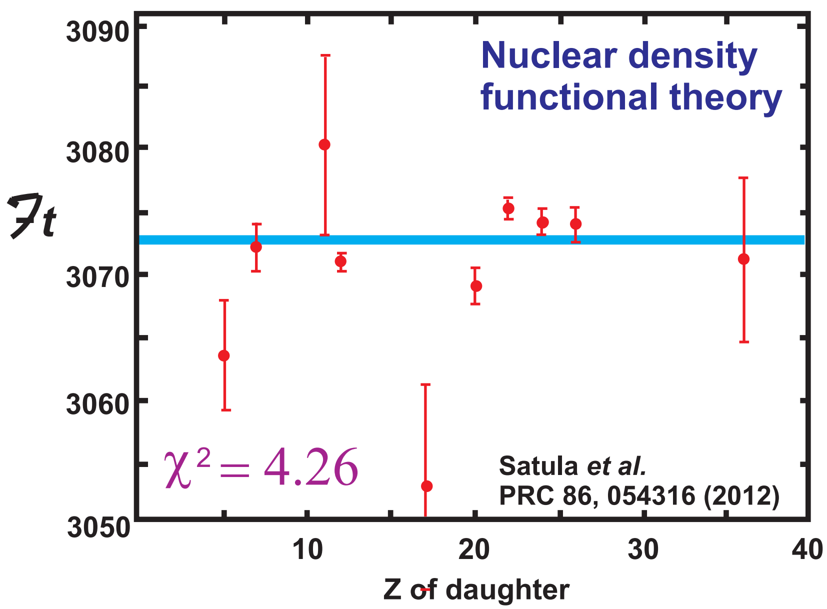

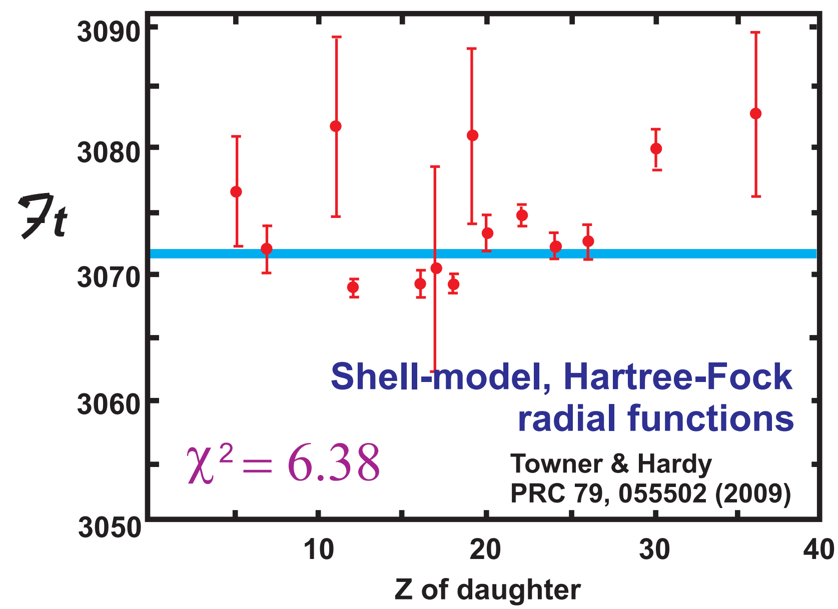

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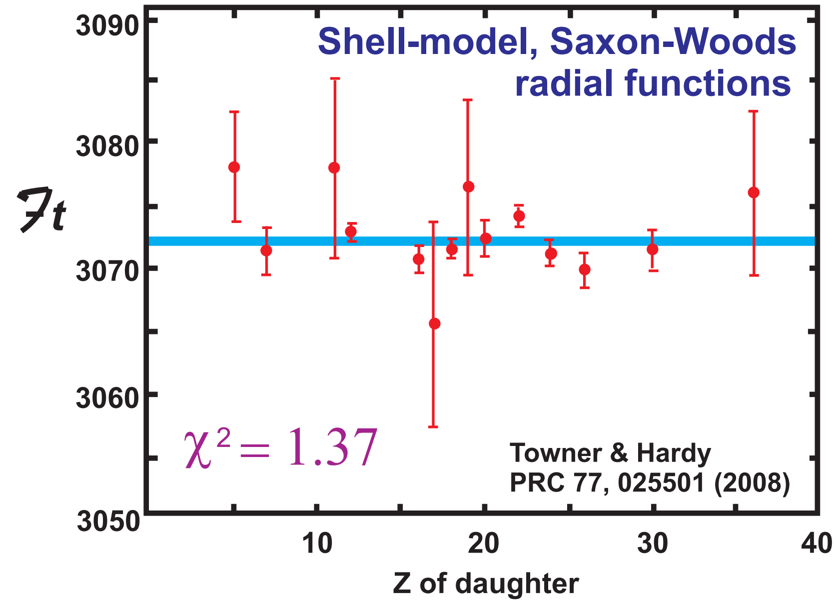
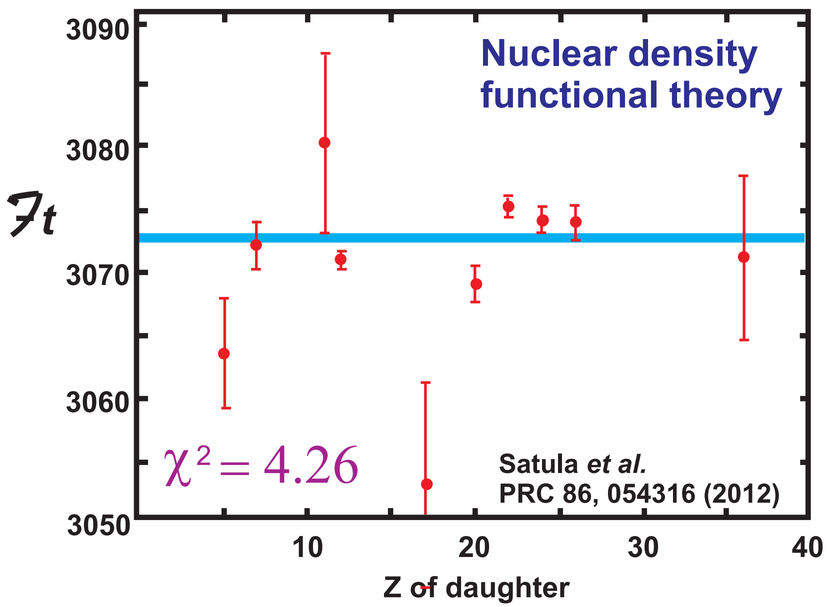
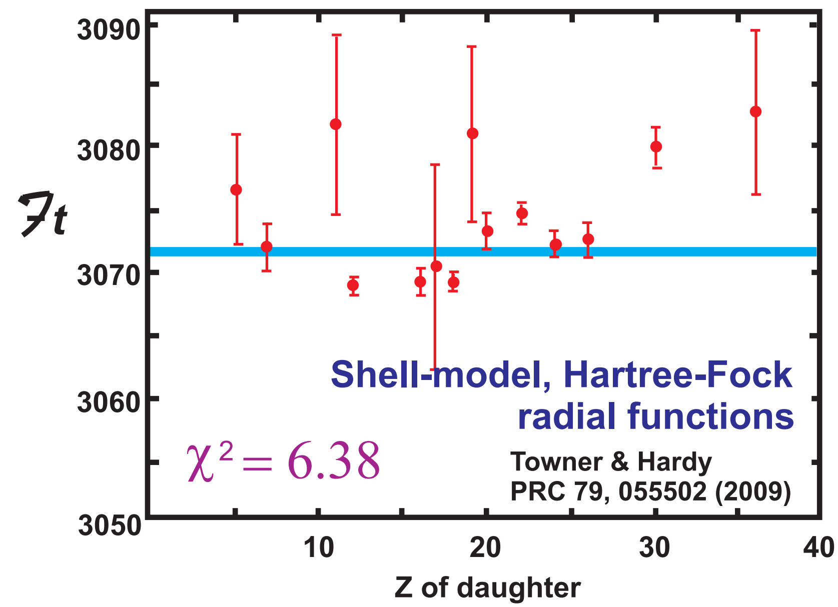
RHF-RPA	4.91	0
RH-RPA	3.68	0

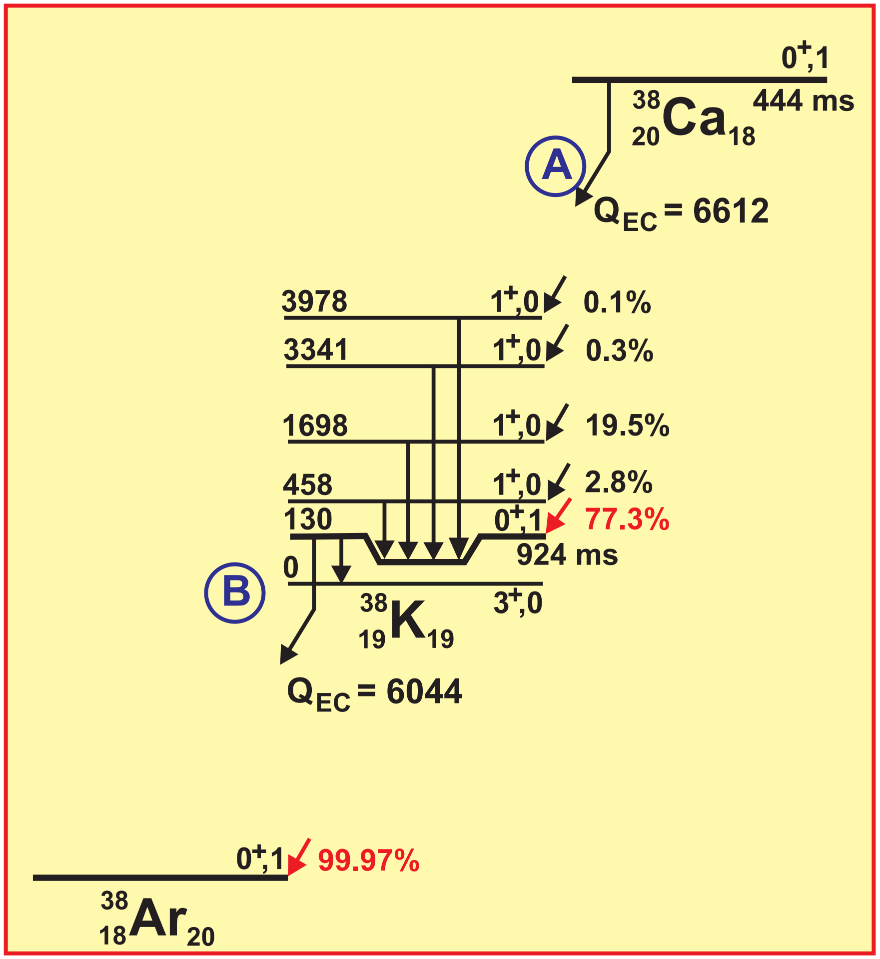






TESTS OF δ_c CALCULATIONS

B. Measurements of mirror superallowed transitions:



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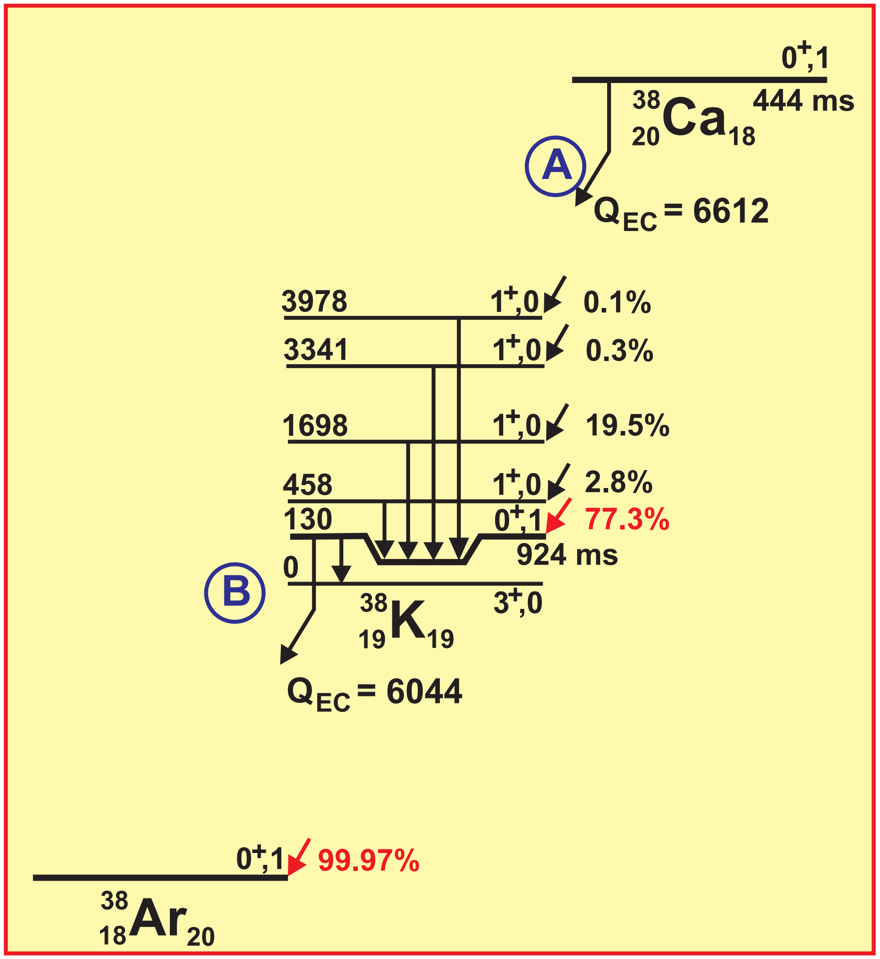
$$\mathcal{F}t = ft\,(1+\delta_R')[1-(\delta_C-\delta_{NS})]$$



$$\frac{ft_A}{ft_B}=\frac{(1+\delta_R'^B)[1-(\delta_C^B-\delta_{NS}^B)]}{(1+\delta_R'^A)[1-(\delta_C^A-\delta_{NS}^A)]}$$

$$=1+(\delta_R'^B-\delta_R'^A)+(\delta_{NS}^B-\delta_{NS}^A)-(\delta_C^B-\delta_C^A)$$

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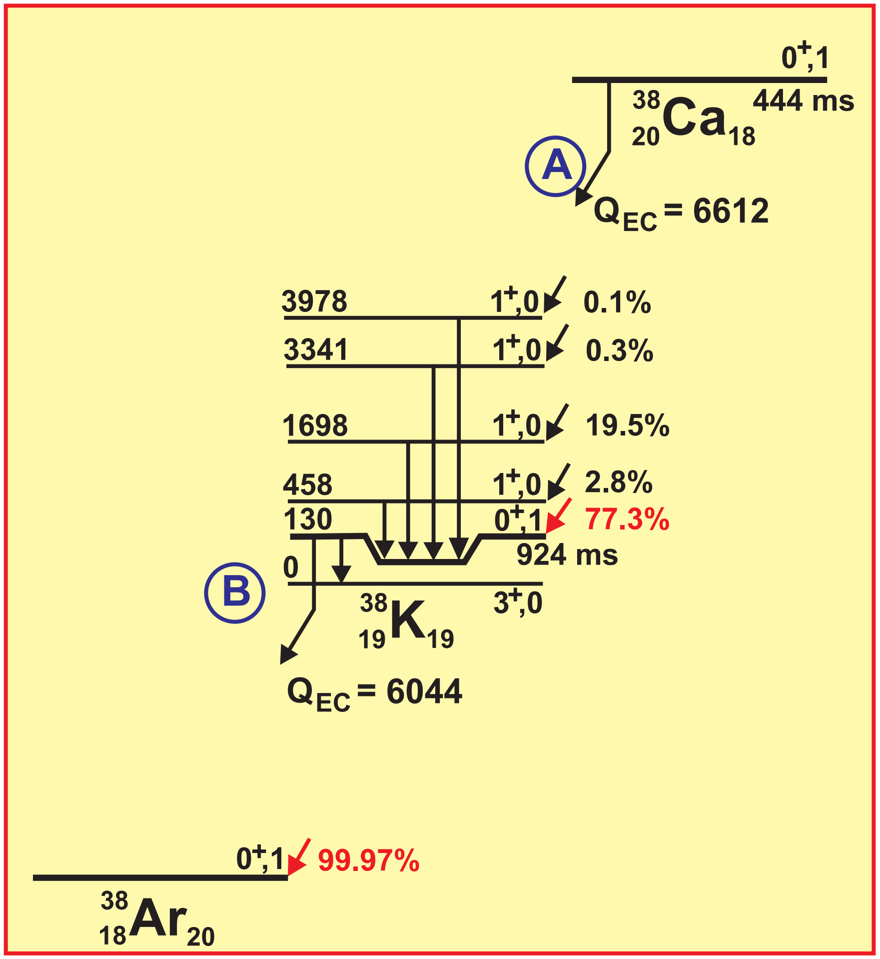

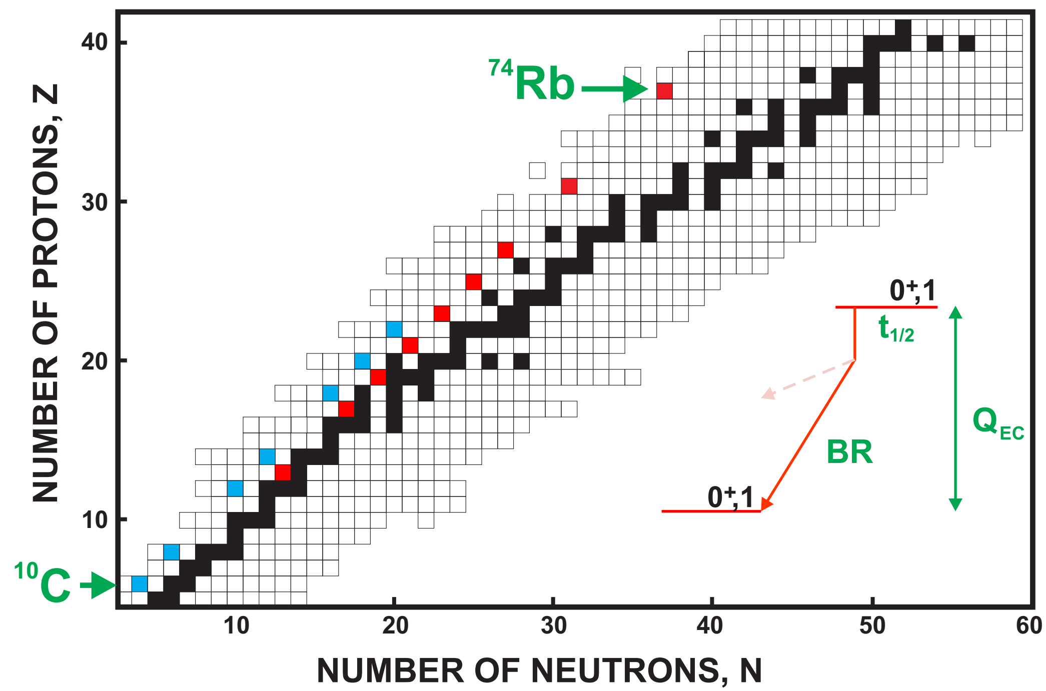

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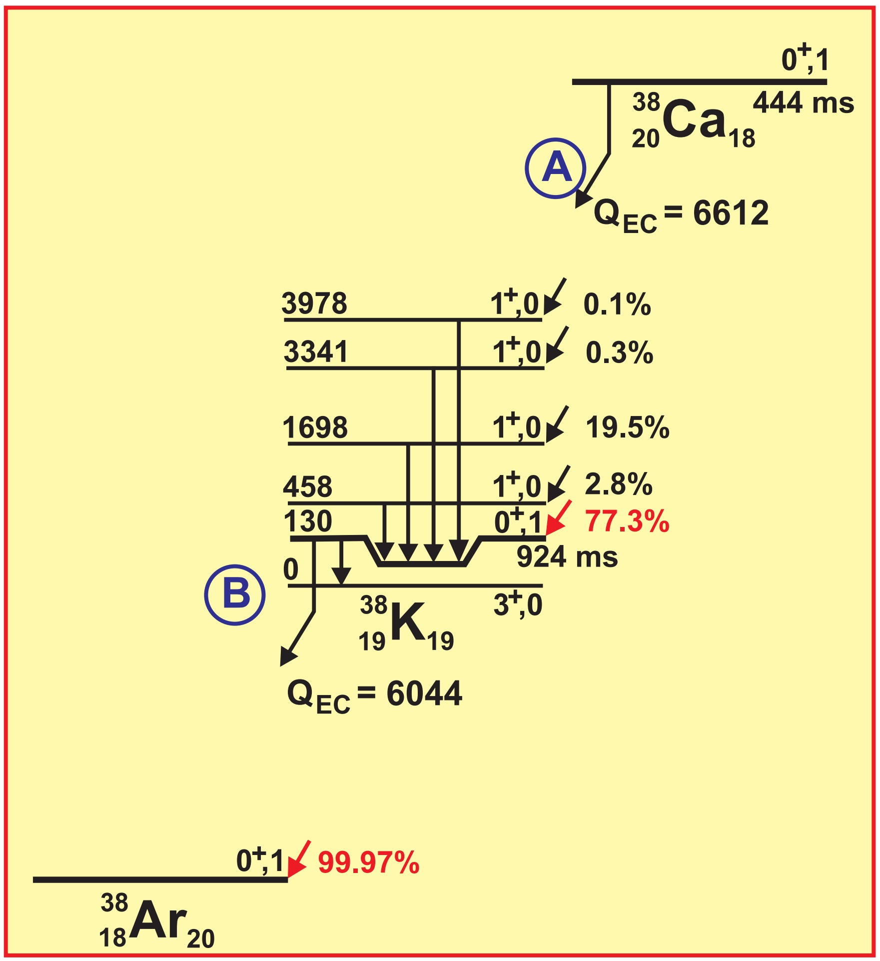

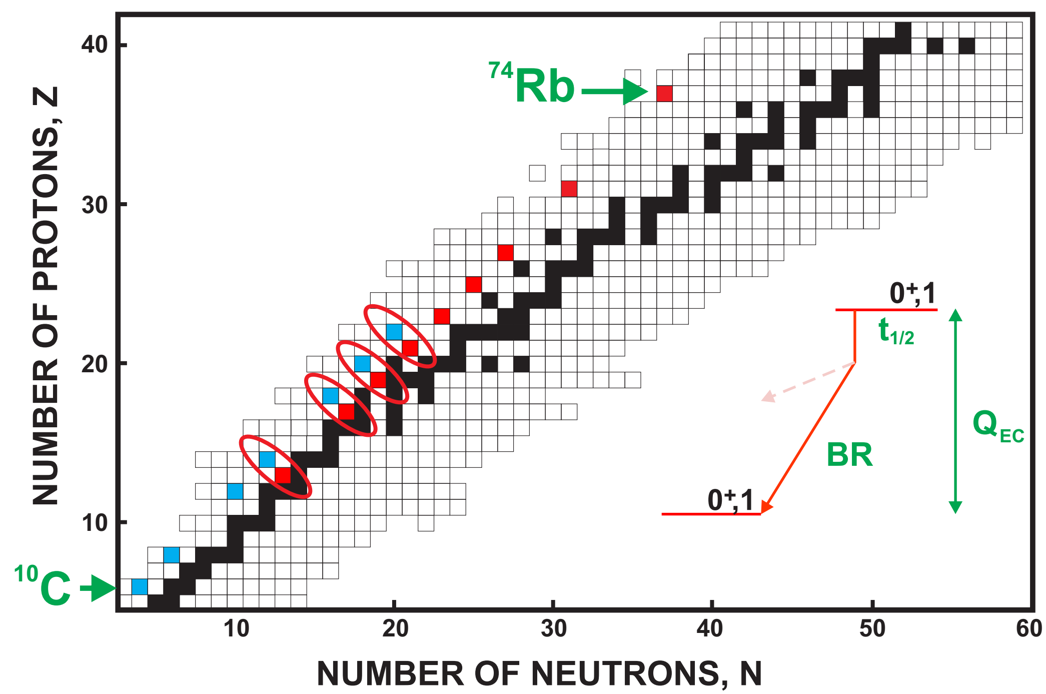

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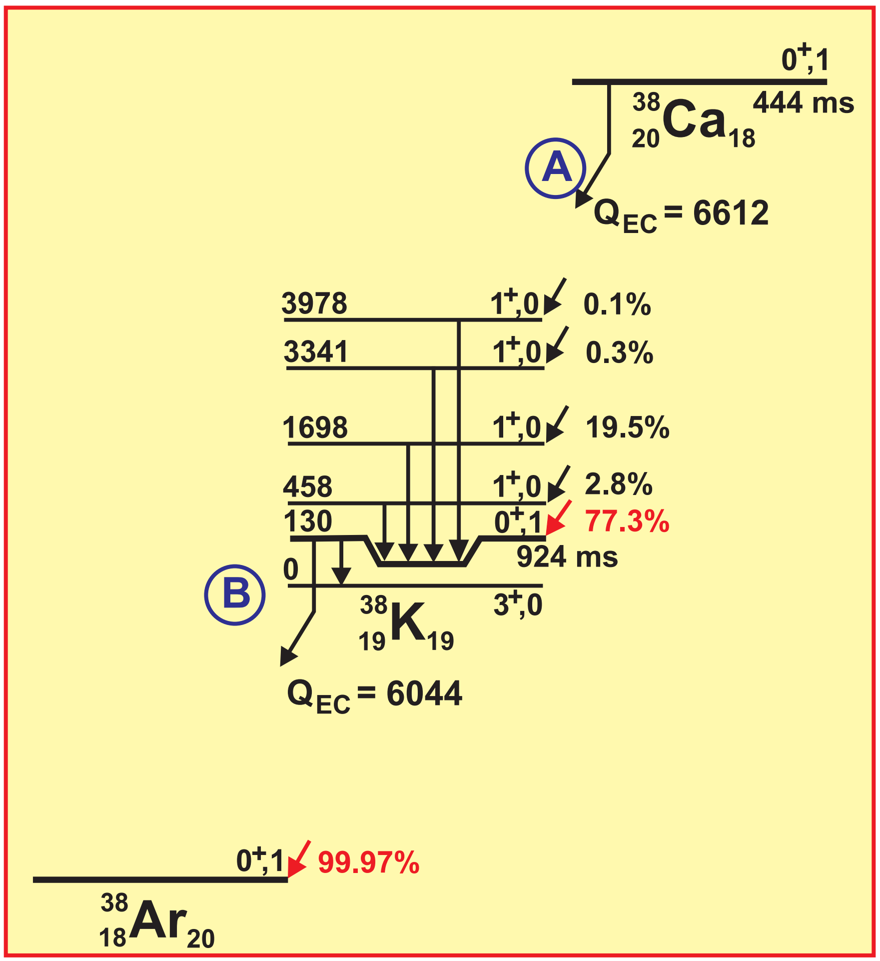
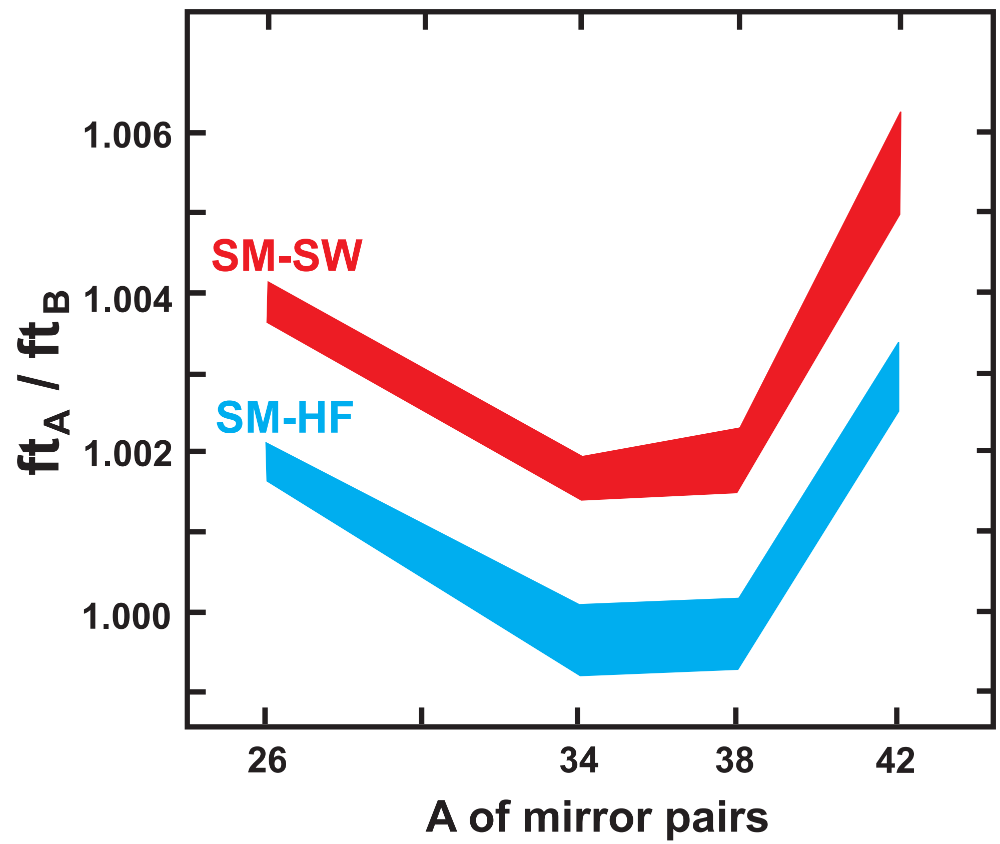


$$\frac{ft_A}{ft_B} = \frac{(1+\delta'^{B}_{R})[1-(\delta^{B}_{C}-\delta^{B}_{NS})]}{(1+\delta'^{A}_{R})[1-(\delta^{A}_{C}-\delta^{A}_{NS})]}$$

$$= 1 + (\delta'^{B}_{R}-\delta'^{A}_{R}) + (\delta^{B}_{NS}-\delta^{A}_{NS}) - (\delta^{B}_{C}-\delta^{A}_{C})$$



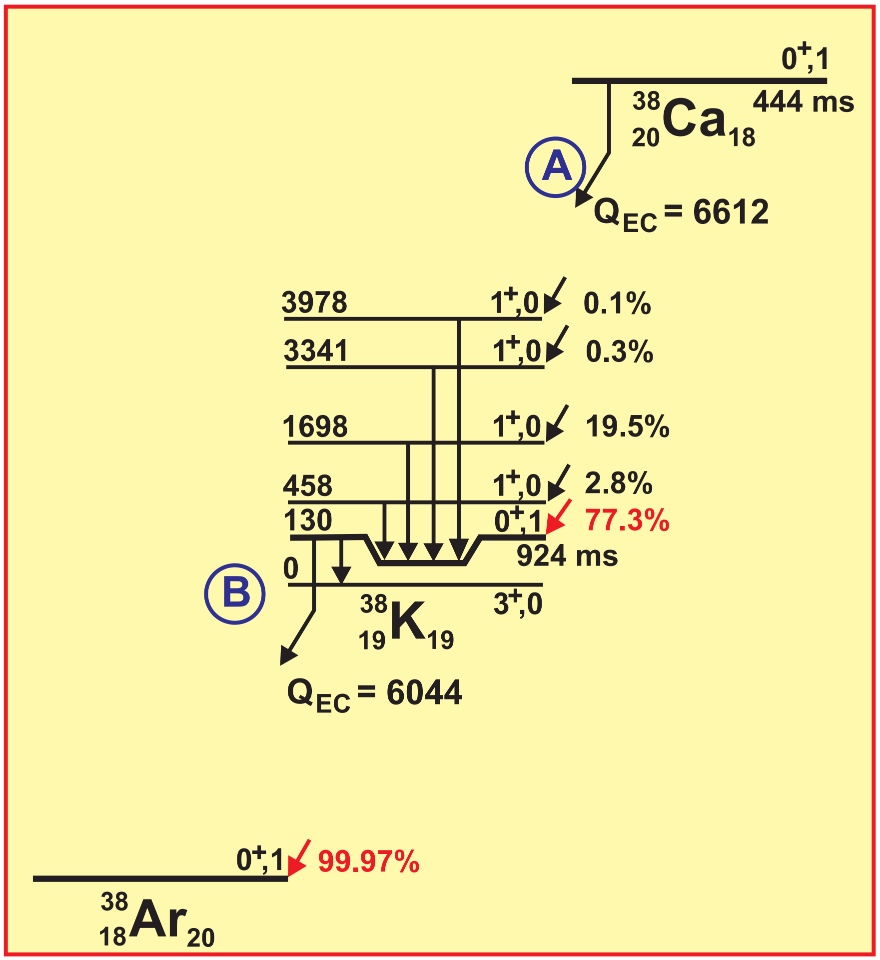
TESTS OF δ_C CALCULATIONS

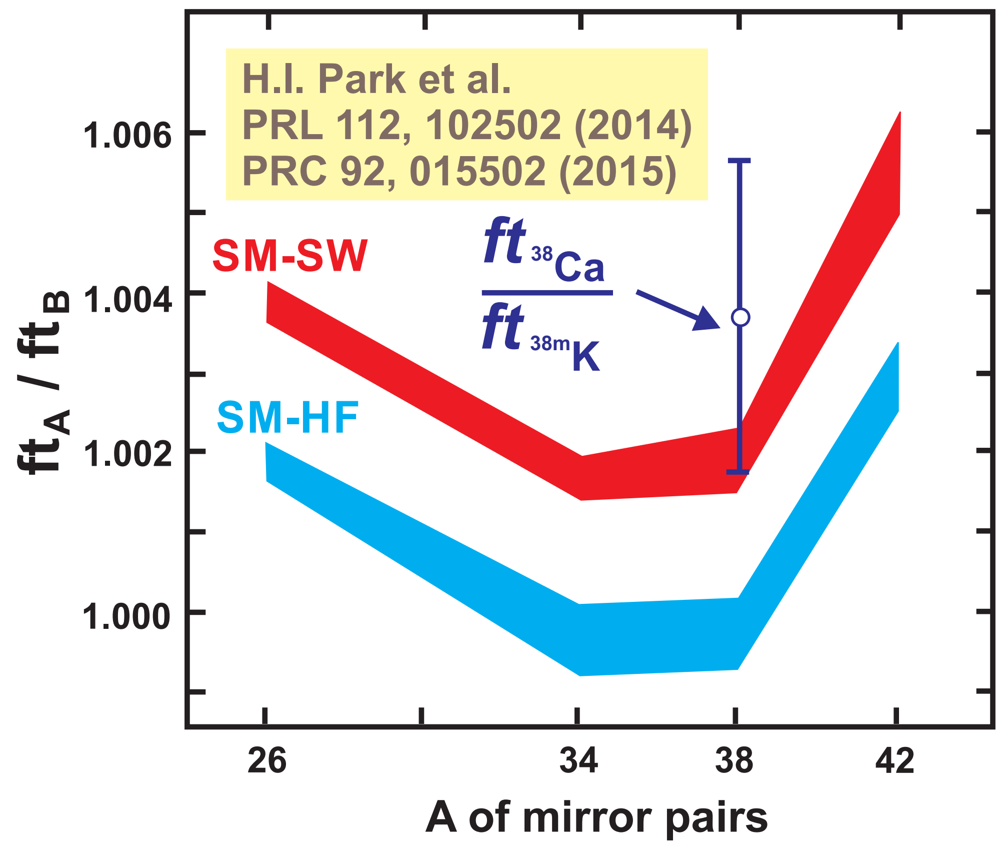
B. Measurements of mirror superallowed transitions:

$$\mathcal{F}t = ft\,(1 + \delta_R')[1 - (\delta_C - \delta_{NS})]$$

$$\frac{ft_A}{ft_B} = \frac{(1 + \delta_R'^B)[1 - (\delta_C^B - \delta_{NS}^B)]}{(1 + \delta_R'^A)[1 - (\delta_C^A - \delta_{NS}^A)]}$$

$$= 1 + (\delta_R'^B - \delta_R'^A) + (\delta_{NS}^B - \delta_{NS}^A) - (\delta_C^B - \delta_C^A)$$





RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2 (1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta'_R)\left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

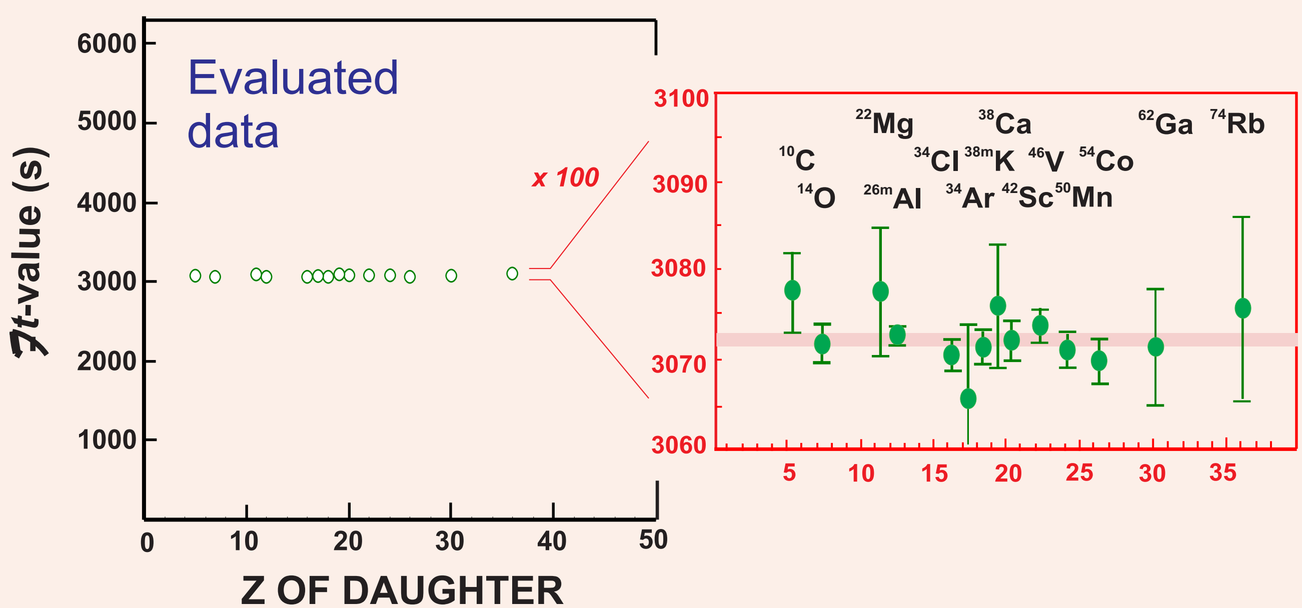
Experimentally determine $G_V^{\,2}(1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^{\,2}\,(1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

G_V constant to ± 0.011%



$\overline{\mathcal{F}t}$ = 3072.1(7)

$G_V(1+\Delta_R)^{1/2}/(\hbar c)^3$ = 1.14962(13) $\times 10^{-5}$ GeV^{-2}

$\chi^2/\nu = 0.6$

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_V^{\;2}(1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta_R')\left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_V^{\;2}\,(1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms

G_V constant to ± 0.011%

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta_R')[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

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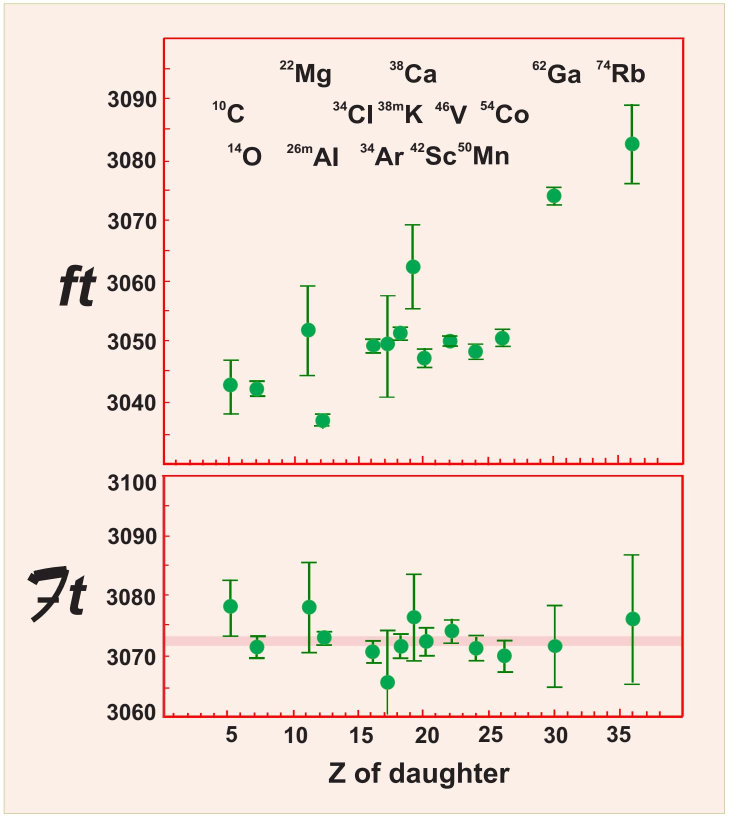

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

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Experimentally determine $G_V^2(1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

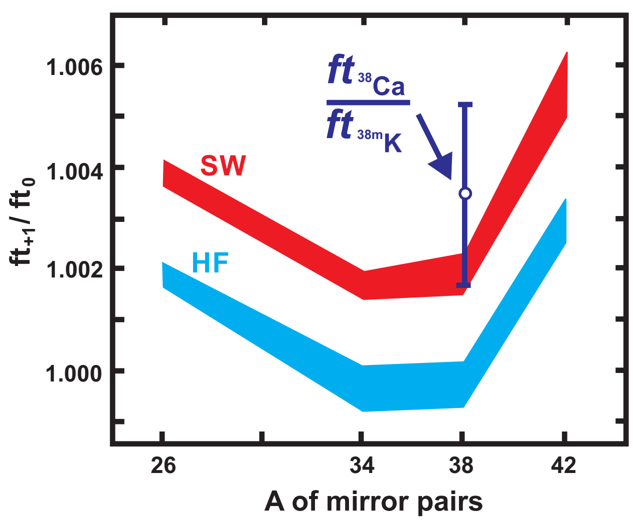
FROM MANY TRANSITIONS

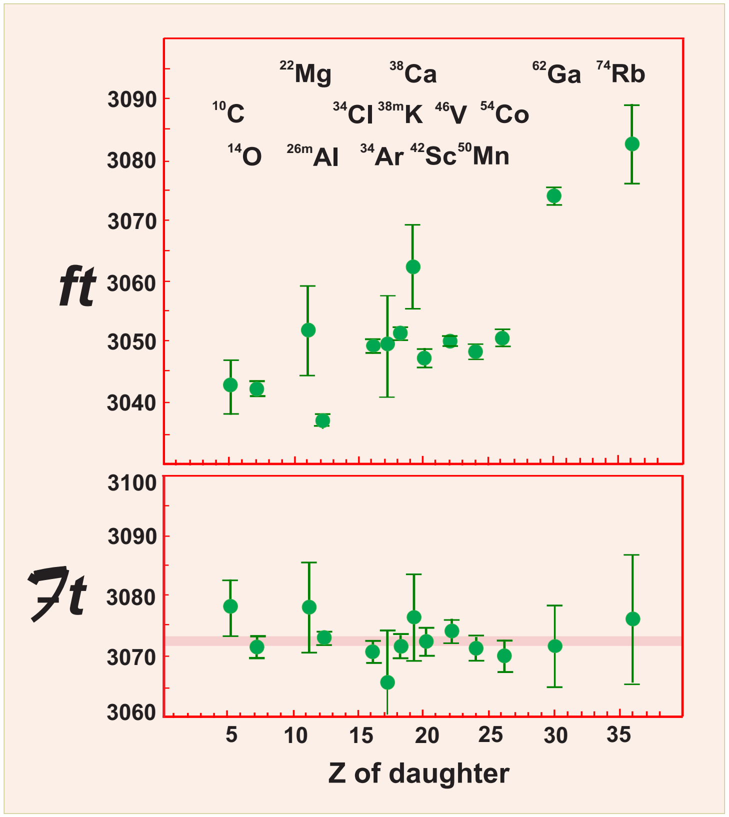
Test Conservation of the Vector current (CVC)

G_V constant to ± 0.011%

Validate correction terms ✔

Model	χ^2/N	CL(%)
SM-SW	1.37	17
SM-HF	6.38	0
DFT	4.26	0
RHF-RPA	4.91	0
RH-RPA	3.68	0





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FROM A SINGLE TRANSITION

Experimentally determine $G_V^{\,2}(1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta_R')\left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_V^{\,2}\,(1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms ✔

Test for Scalar current

G_V constant to ± 0.011%

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta_R')[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

FROM MANY TRANSITIONS

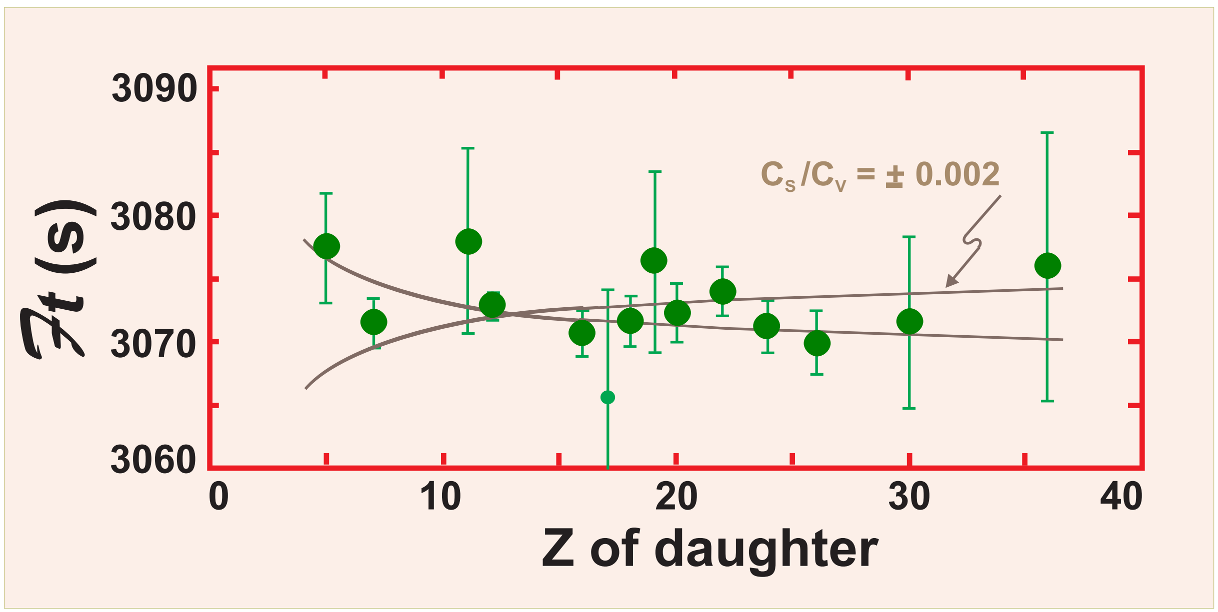
Test Conservation of the Vector current (CVC)

Validate correction terms ✔

Test for Scalar current

G_V constant to ± 0.011%

limit, C_S/C_V = 0.0012 (10)



RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms ✔

Test for Scalar current

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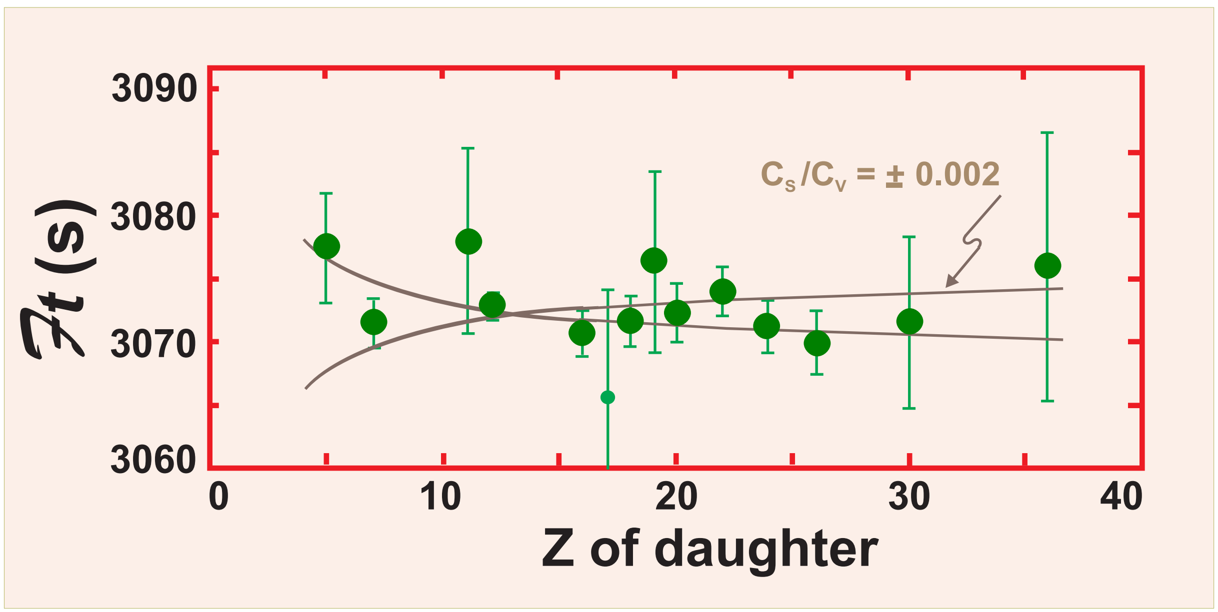

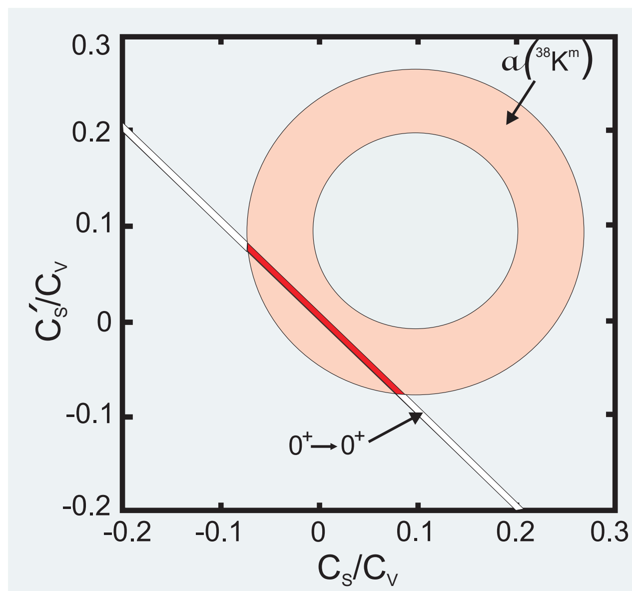

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

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$$\mathcal{F}t = ft\,(1+\delta_R')\left[1-(\delta_C-\delta_{NS})\right] = \frac{K}{2G_V^2\,(1+\Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

G_V constant to ± 0.011%

Validate correction terms ✔

Test for Scalar current

limit, $C_S/C_V = 0.0012\ (10)$

WITH CVC VERIFIED

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates

mass eigenstates

Cabibbo-Kobayashi-Maskawa matrix

Obtain precise value of $G_V^2(1+\Delta_R)$

Determine V_{ud}^2

$$V_{ud}^2 = G_V^2/G_\mu^2 = 0.94907 \pm 0.00041$$

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

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FROM MANY TRANSITIONS

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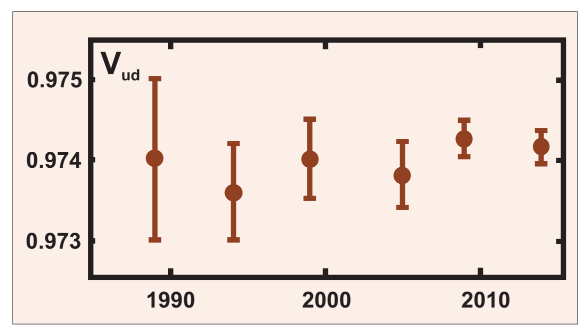

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

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Experimentally determine $G_V^2(1+\Delta_R)$

$$\mathcal{F}t = ft\,(1+\delta'_R)[1-(\delta_C-\delta_{NS})] = \frac{K}{2G_V^2\,(1+\Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

G_V constant to ± 0.011%

Validate correction terms ✔

Test for Scalar current

limit, $C_S/C_V = 0.0012\,(10)$

WITH CVC VERIFIED

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates

mass eigenstates

Cabibbo-Kobayashi-Maskawa matrix

Obtain precise value of $G_V^2(1+\Delta_R)$

Determine V_{ud}^2

$$V_{ud}^2 = G_V^2/G_\mu^2 = 0.94907 \pm 0.00041$$

Test CKM unitarity

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99963 \pm 0.00049$$

T=1/2 SUPERALLOWED BETA DECAY

BASIC WEAK-DECAY EQUATION

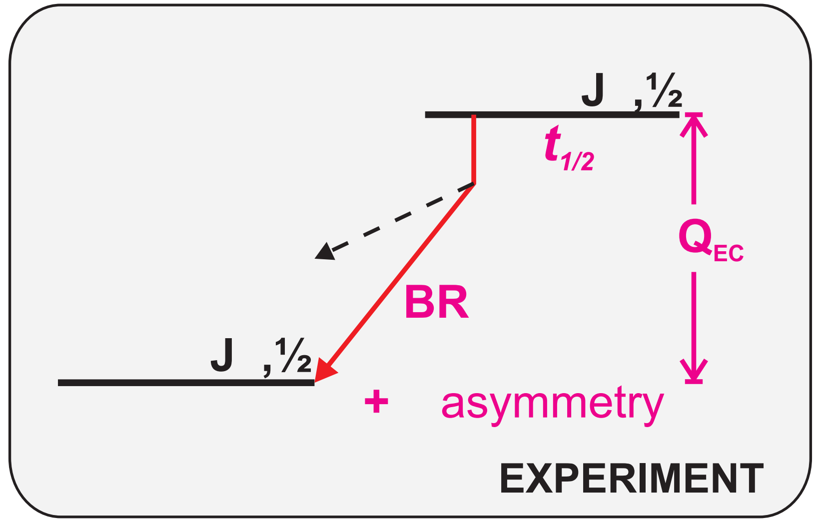
$$\boxed{ft = \frac{K}{G_V^2\langle\quad\rangle^2 + G_A^2\langle\quad\rangle^2}}$$

f **= statistical rate function: $f(Z, Q_{EC})$**

t **= partial half-life: $f(t_{1/2}, BR)$**

$G_{V,A}$ = coupling constants

< > = Fermi, Gamow-Teller matrix elements



T=1/2 SUPERALLOWED BETA DECAY

BASIC WEAK-DECAY EQUATION

$$ft = \frac{K}{G_V^2 < \quad >^2 + G_A^2 < \quad\quad >^2}$$

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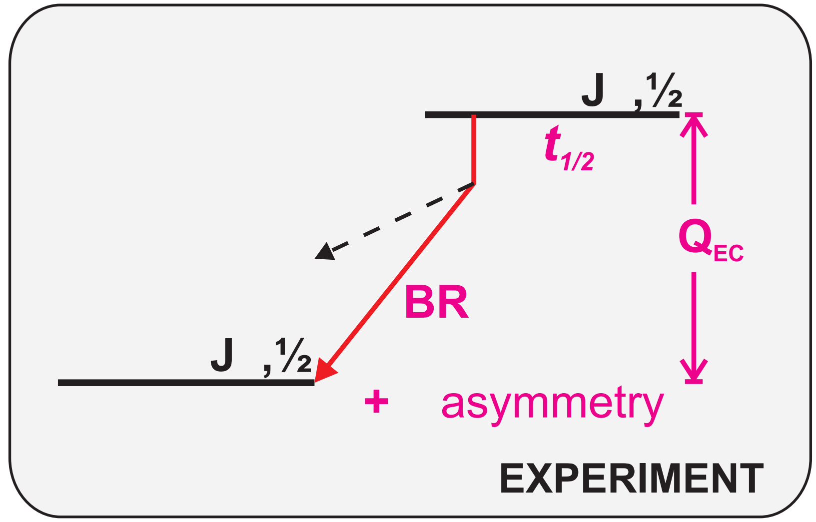

INCLUDING RADIATIVE CORRECTIONS

$$\mathcal{F}t = ft\,(1 + \ '_R)\left[1 - (\ _C - \ _{NS})\right] = \frac{K}{G_V^2\,(1 + \ _R)(1 + \ ^2 < \quad >^2)}$$

$= G_A/G_V$

T=1/2 SUPERALLOWED BETA DECAY

BASIC WEAK-DECAY EQUATION

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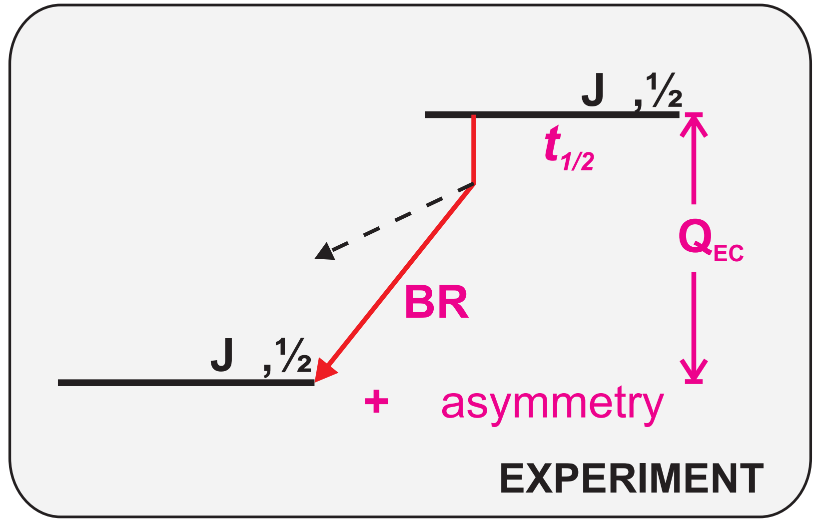

INCLUDING RADIATIVE CORRECTIONS

$$\mathcal{F}t = ft\,(1 + \ '_R)[1 - (\ _C - \ _{NS})] = \frac{K}{G_V^2\,(1 + \ _R)(1 + \ ^2 < \quad >^2)}$$

$= G_A/G_V$

Requires additional experiment:
for example, asymmetry (*A*)

T=1/2 SUPERALLOWED BETA DECAY

BASIC WEAK-DECAY EQUATION

$$ft = \frac{K}{G_V^2 < \;\; >^2 + G_A^2 < \;\;\;\; >^2}$$

f = statistical rate function: f(Z, Q_{EC})

t = partial half-life: f($t_{1/2}$, BR)

$G_{V,A}$ = coupling constants

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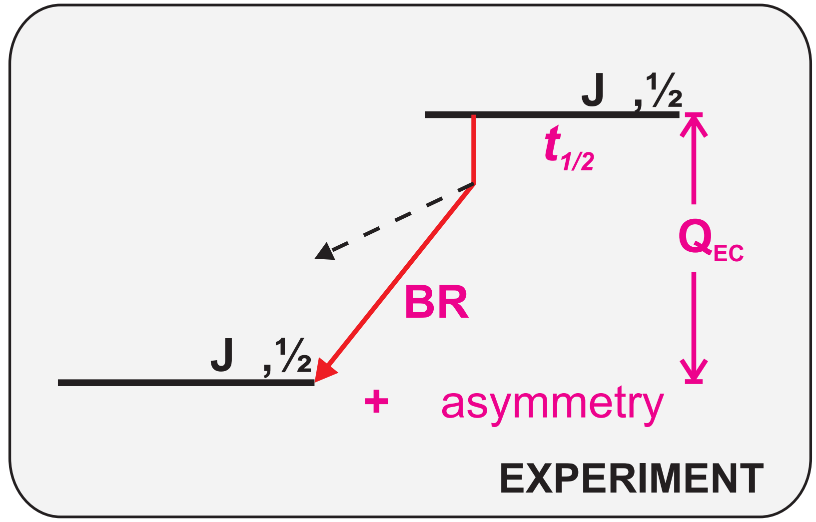


INCLUDING RADIATIVE CORRECTIONS

$$\mathcal{F}t = ft\,(1 + {}'_R)[1 - (\cancel{{}_C - {}_{NS}})] = \frac{K}{G_V^2\,(1 + {}_R)(1 + {}^2 < \;\;\;\; >^2)}$$

$= G_A/G_V$

NEUTRON DECAY

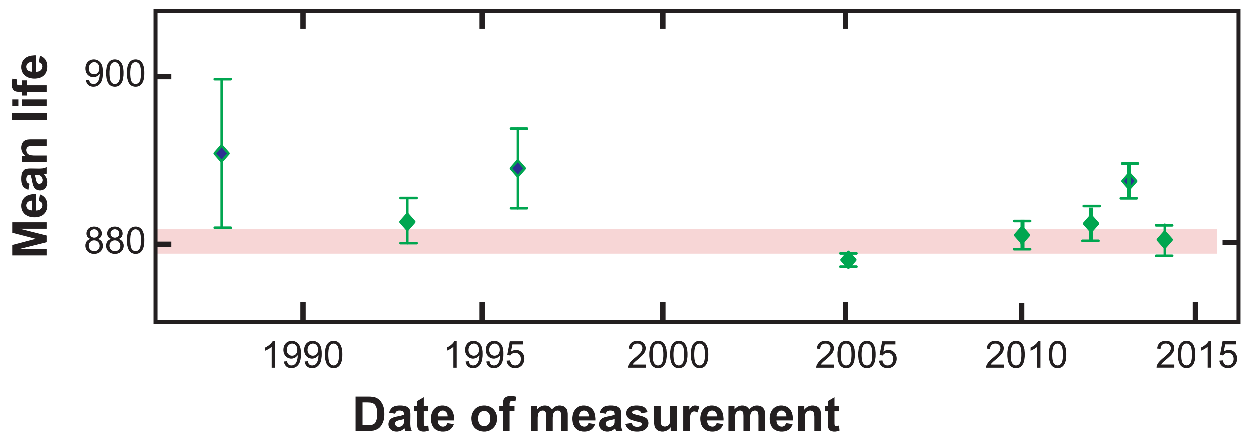
Requires additional experiment:
for example, asymmetry (A)

NEUTRON DECAY DATA 2016

Mean life:

τ **= 880.2 ± 1.0 s**

χ^2/N **= 3.7**

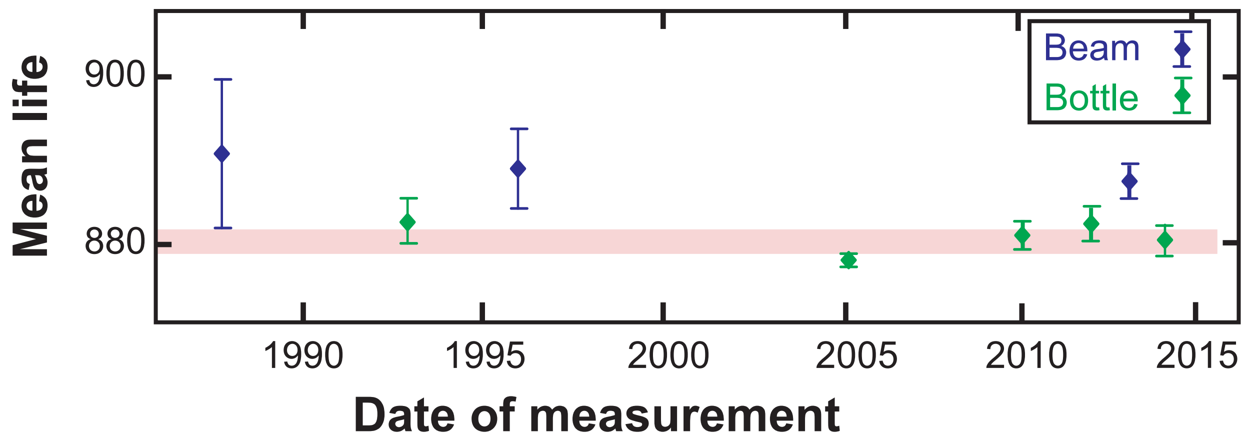


NEUTRON DECAY DATA 2016

Mean life:

τ = **880.2 ± 1.0 s**

χ^2/N = **3.7**



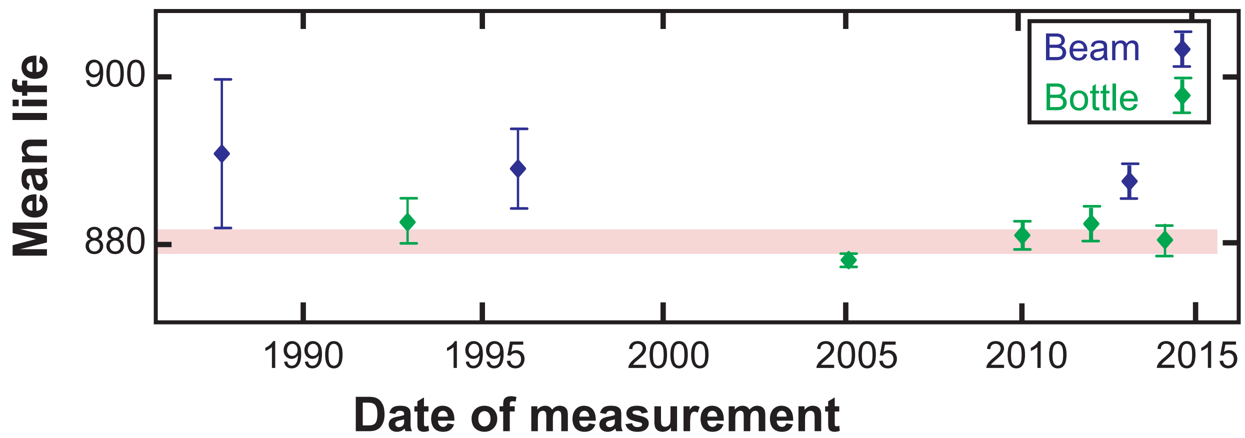
NEUTRON DECAY DATA 2016

Mean life:

τ = **880.2 ± 1.0 s**

χ^2/N = **3.7**

Beam: 888.1 ± 2.0 s
Bottle: 879.6 ± 0.7 s



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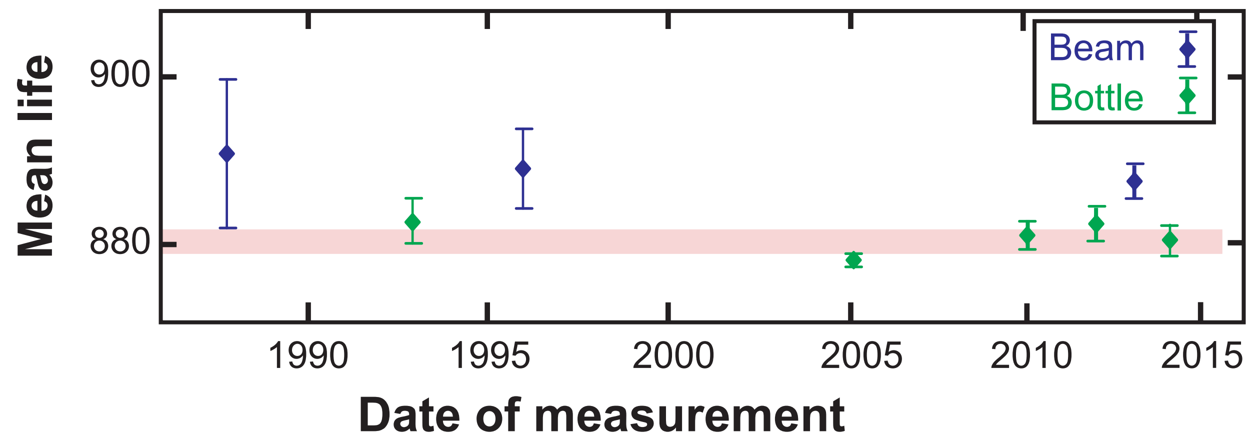
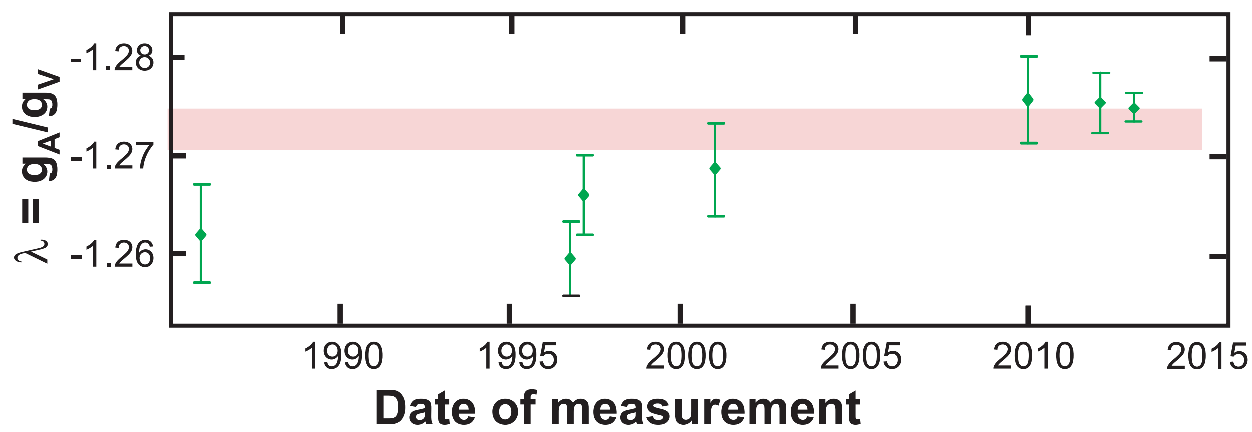


β asymmetry:

λ **= -1.2725 ± 0.0020**

$\chi^2/N = 4.1$



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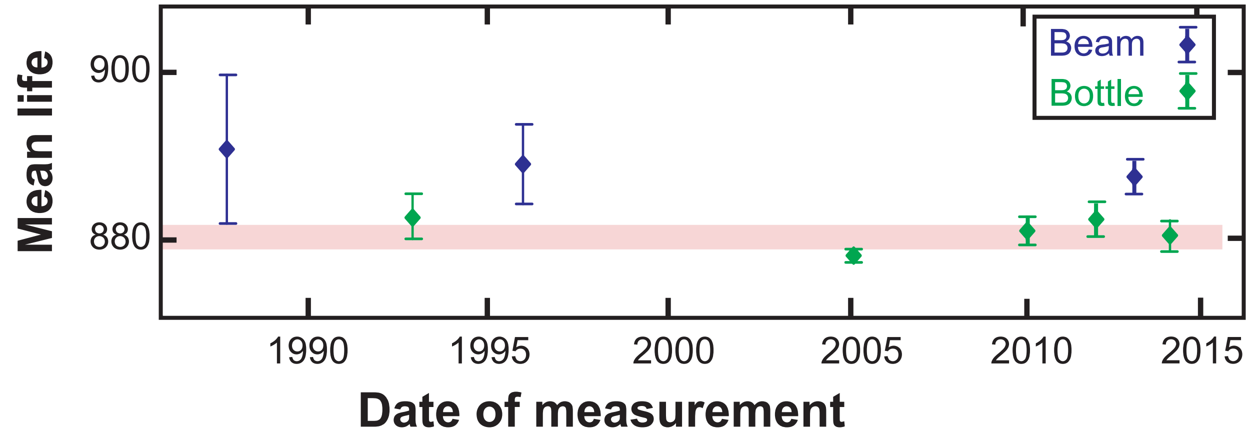

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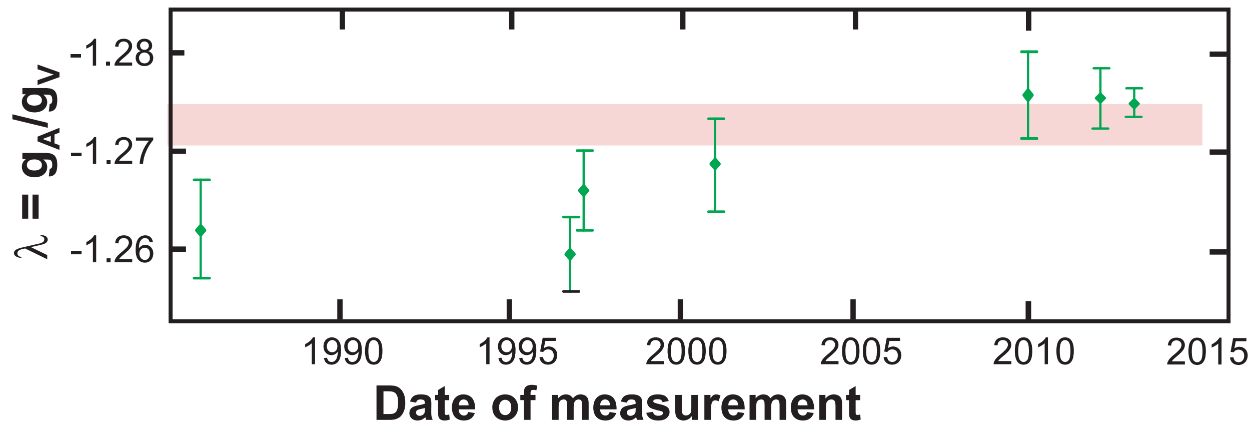


$\boxed{V_{ud} = 0.9757 \pm 0.0014}$

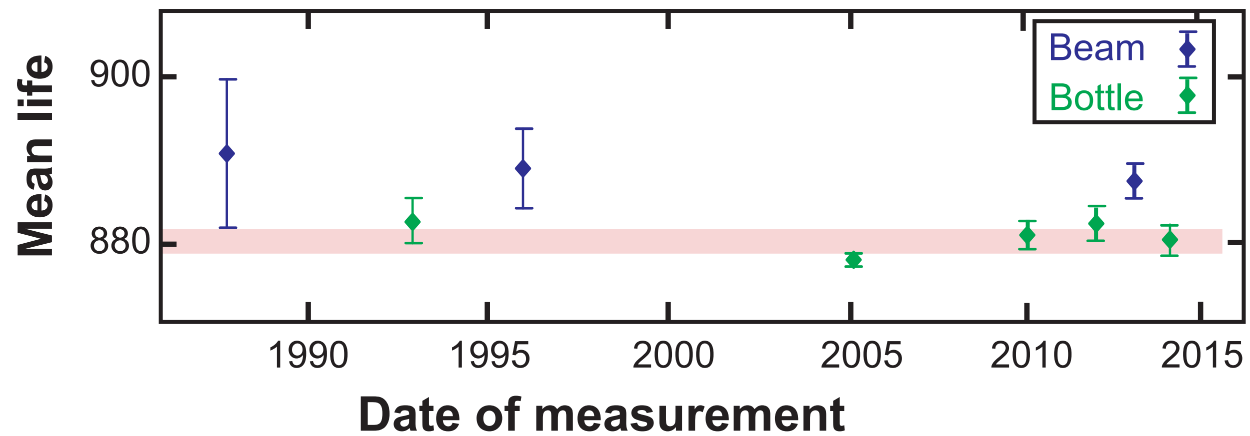
NEUTRON DECAY DATA 2016

Mean life:

$\tau = 880.2 \pm 1.0$ s

$\chi^2/N = 3.7$

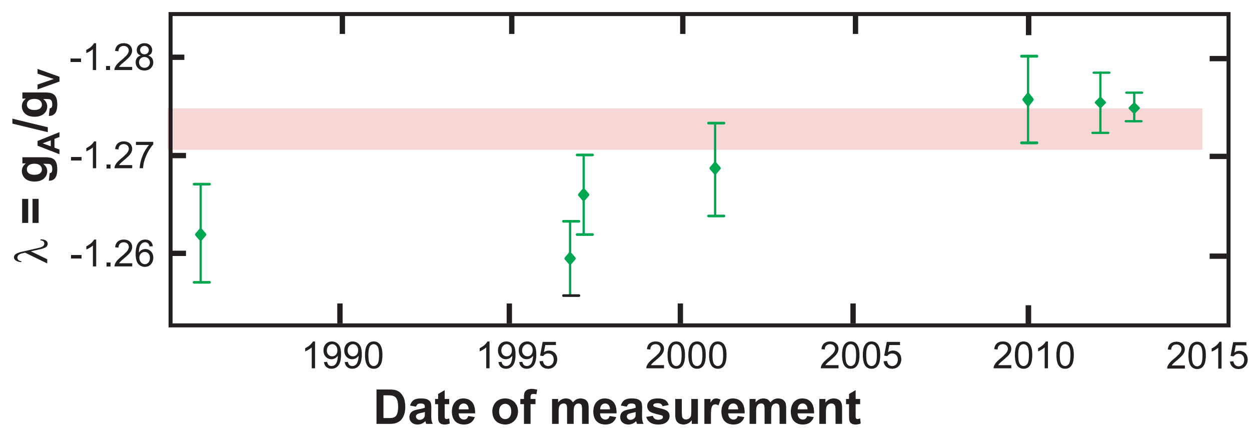
Beam: 888.1 ± 2.0 s
Bottle: 879.6 ± 0.7 s



β asymmetry:

$\lambda = -1.2725 \pm 0.0020$

$\chi^2/N = 4.1$



$V_{ud} = 0.9757 \pm 0.0014$

Beam-bottle span
$0.9701 \leq V_{ud} \leq 0.9767$

NEUTRON DECAY DATA 2016

Mean life:

$\tau = 880.2 \pm 1.0$ s

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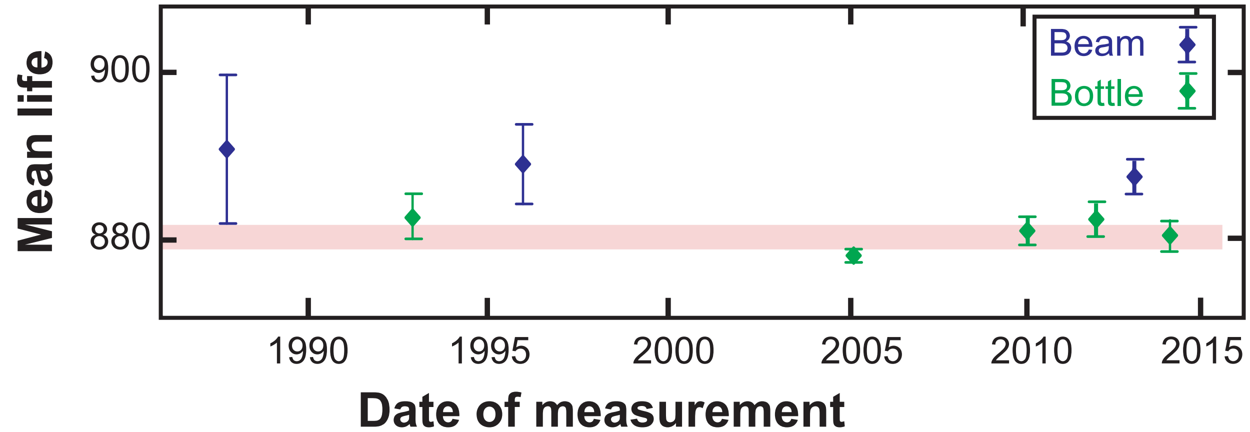

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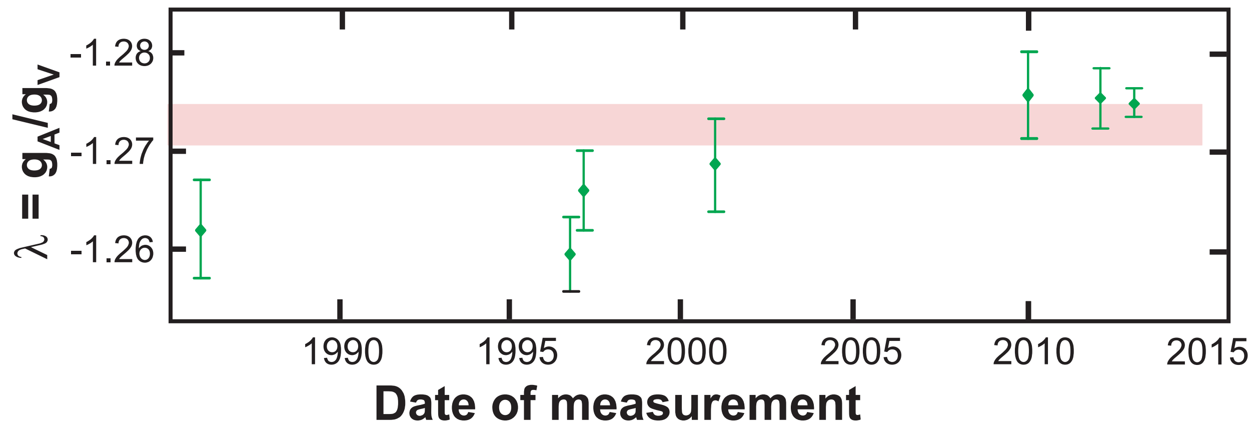


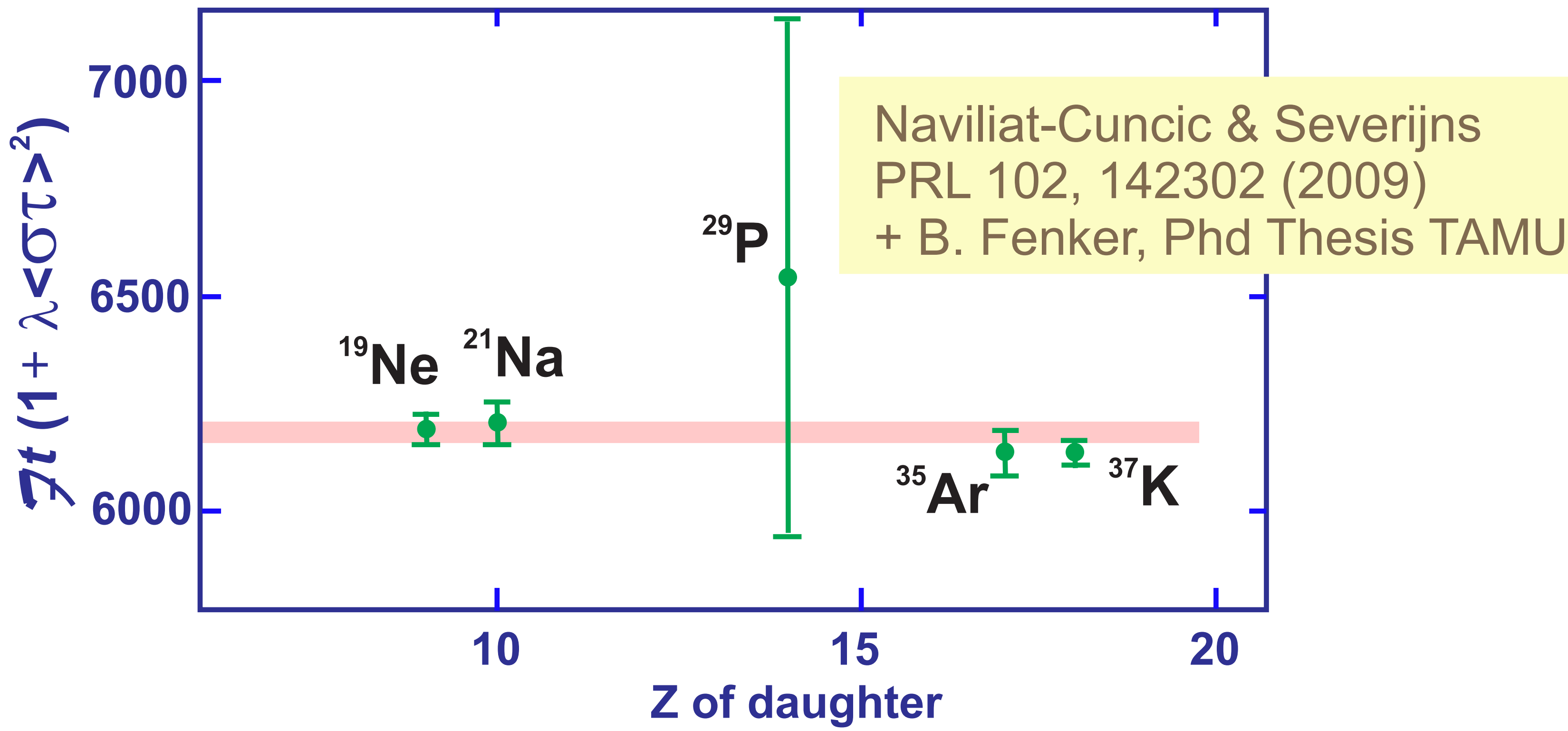
$$\boxed{V_{ud} = 0.9757 \pm 0.0014}$$

Beam-bottle span
$0.9701 \leq V_{ud} \leq 0.9767$

nuclear $0^+ \rightarrow 0^+$
$V_{ud} = 0.9742 \pm 0.0002$

NUCLEAR T=1/2 MIRROR DECAY DATA 2009

$$\mathcal{F}t = ft\,(1+\delta_R')[1-(\delta_C-\delta_{NS})] = \frac{K}{G_V^{\,2}\,(1+\Delta_R)(1+\lambda^2\langle\sigma\tau\rangle^2)}$$



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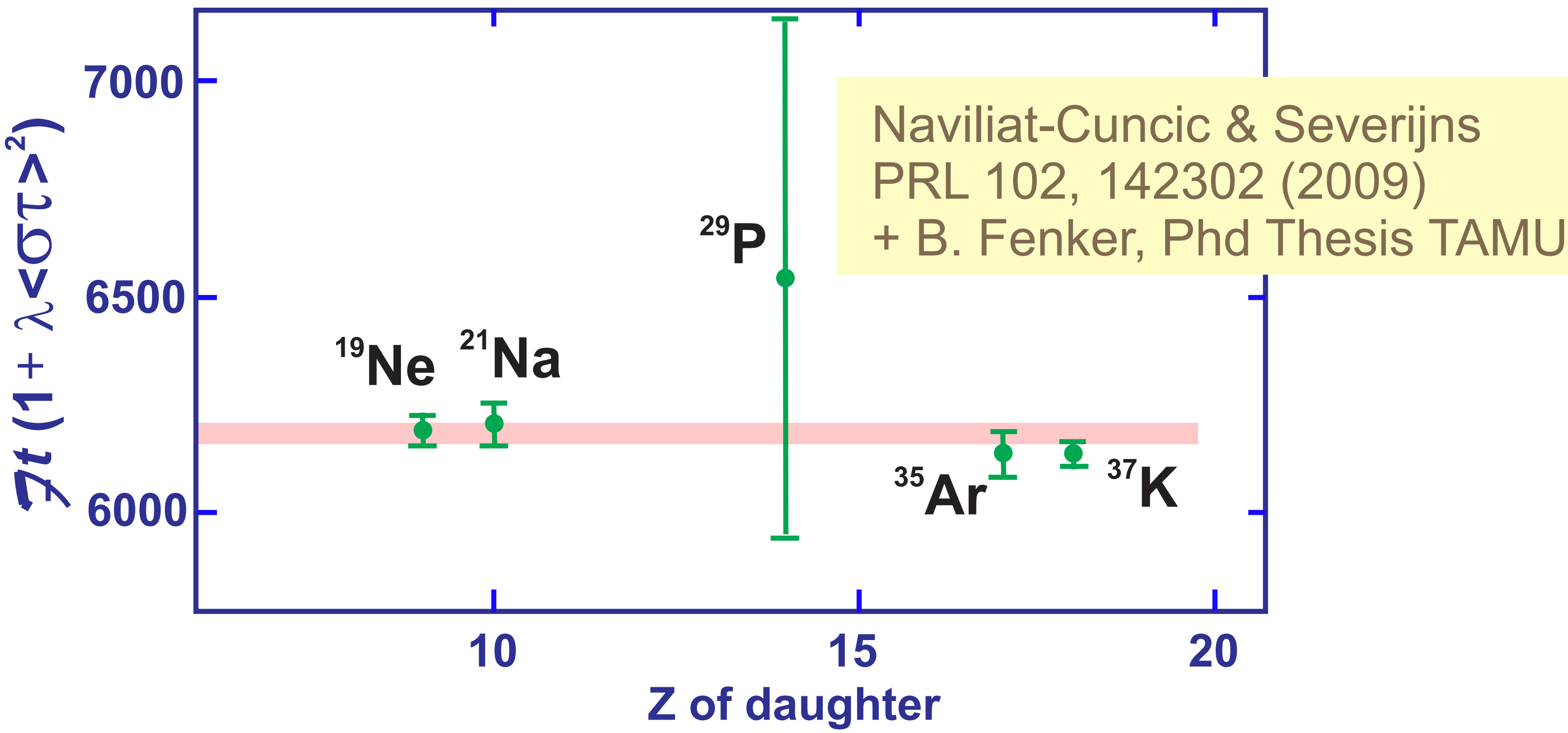
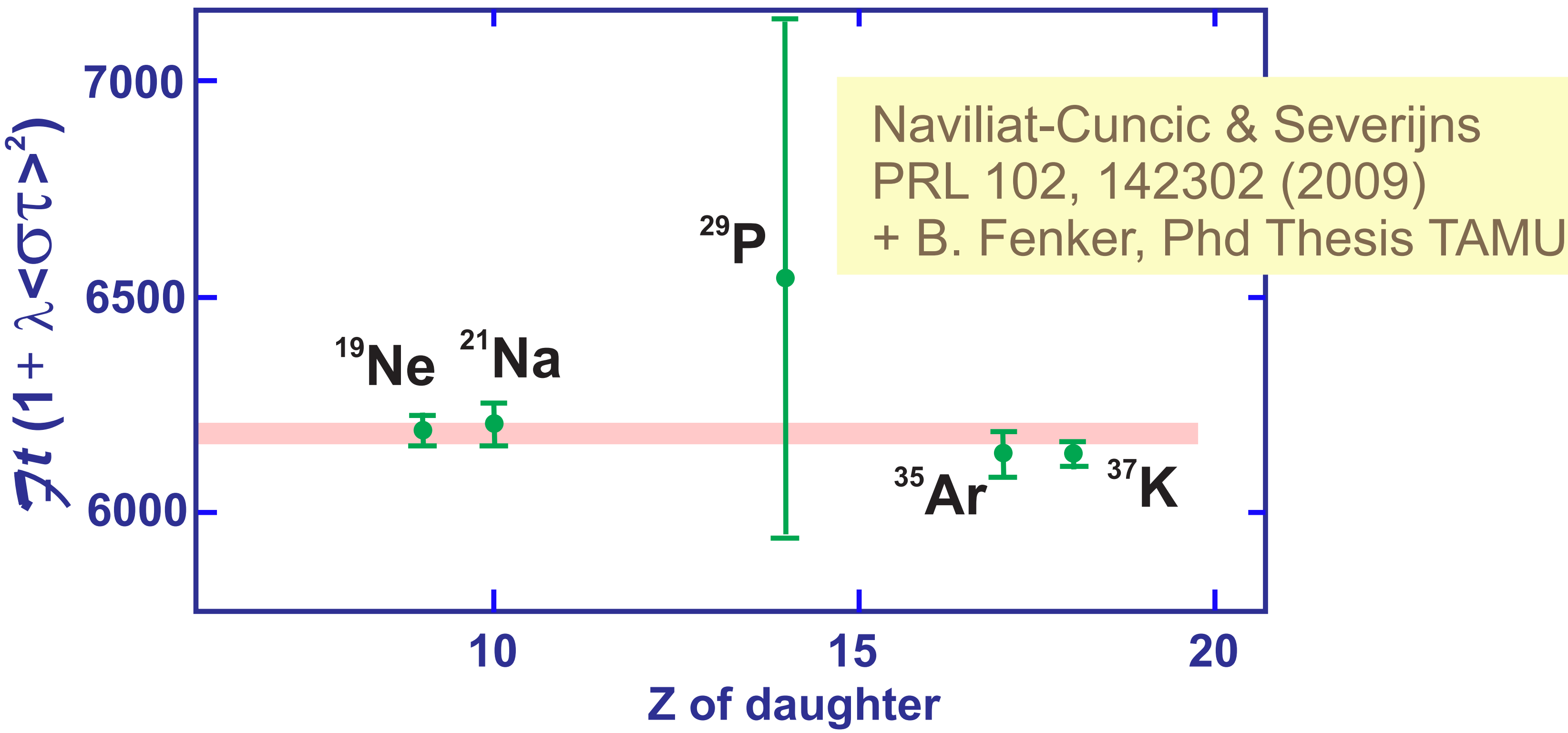


$V_{ud} = 0.9730 \pm 0.0014$

NUCLEAR T=1/2 MIRROR DECAY DATA 2009

$$\mathcal{F}t = ft\,(1 + \delta_R')[1 - (\delta_C - \delta_{NS})] = \frac{K}{G_V^2\,(1 + \Delta_R)(1 + \lambda^2 \langle\sigma\tau\rangle^2)}$$



$V_{ud} = 0.9730 \pm 0.0014$

nuclear $0^+ \rightarrow 0^+$

$V_{ud} = 0.9742 \pm 0.0002$

PION BETA DECAY

Decay process:

$$^{+} \longrightarrow \; ^{0} \, e^{+} \; _{e}$$

$$0^{-},1 \longrightarrow 0^{-},1$$

PION BETA DECAY

Decay process:

$$^{+} \longrightarrow \quad ^{0}\, e^{+} \quad _{e}$$

$$0^{-},1 \longrightarrow 0^{-},1$$

Experimental data:

$$= 2.6033 \pm 0.0005 \times 10^{-8}\ \mathrm{s}$$ (PDG 2009)

$$\mathrm{BR} = 1.036 \pm 0.007 \times 10^{-8}$$

Pocanic *et al*,
PRL 93, 181803 (2004)

Result:

$$\boxed{V_{ud} = 0.9749 \pm 0.0026}$$

PION BETA DECAY

Decay process:

$$\pi^+ \longrightarrow \pi^0\, e^+\, \nu_e$$

$$0^-,1 \longrightarrow 0^-,1$$

Experimental data:

$\tau = 2.6033 \pm 0.0005 \times 10^{-8}$ s (PDG 2009)

$BR = 1.036 \pm 0.007 \times 10^{-8}$

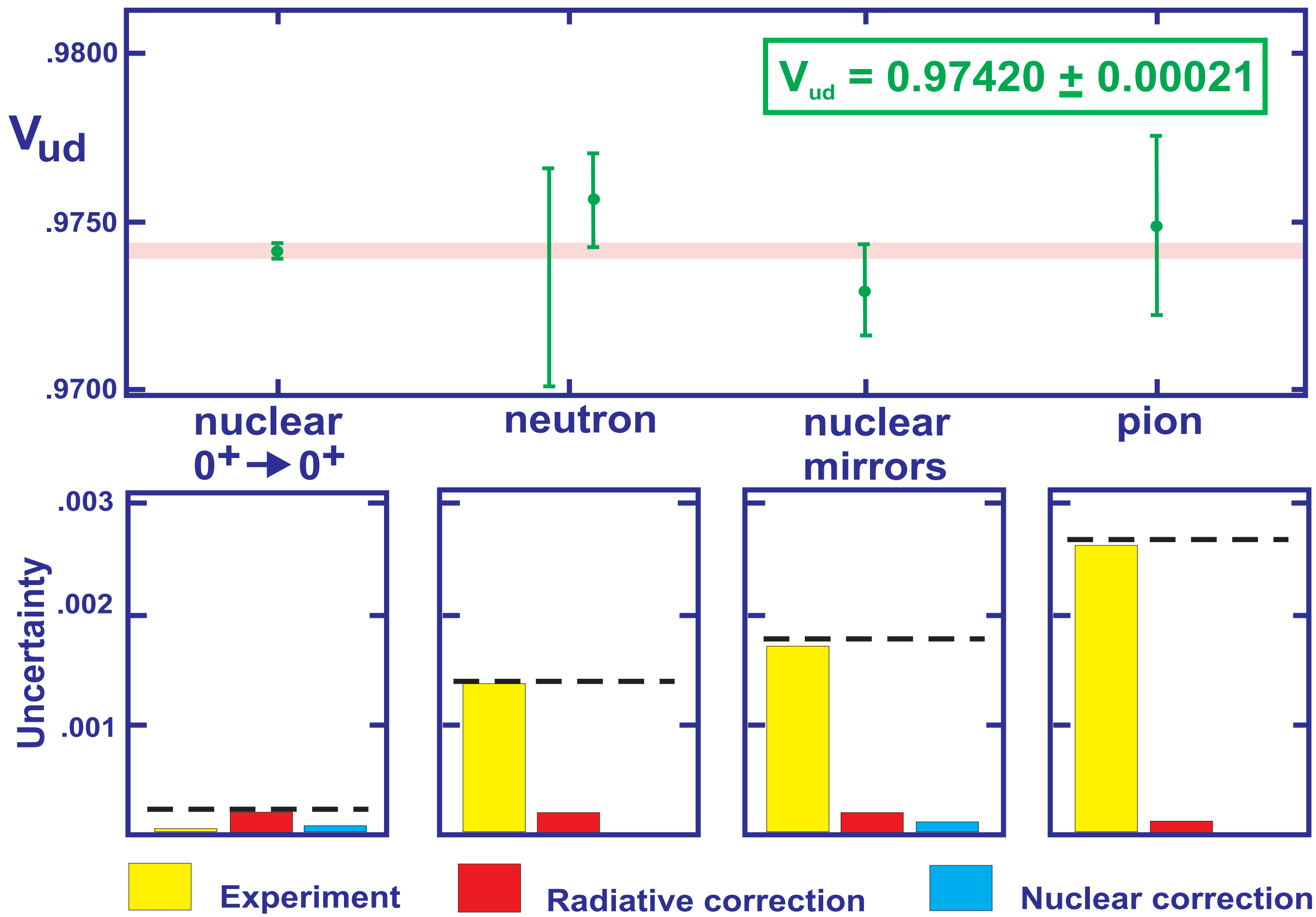
Pocanic *et al,*
PRL 93, 181803 (2004)

Result:

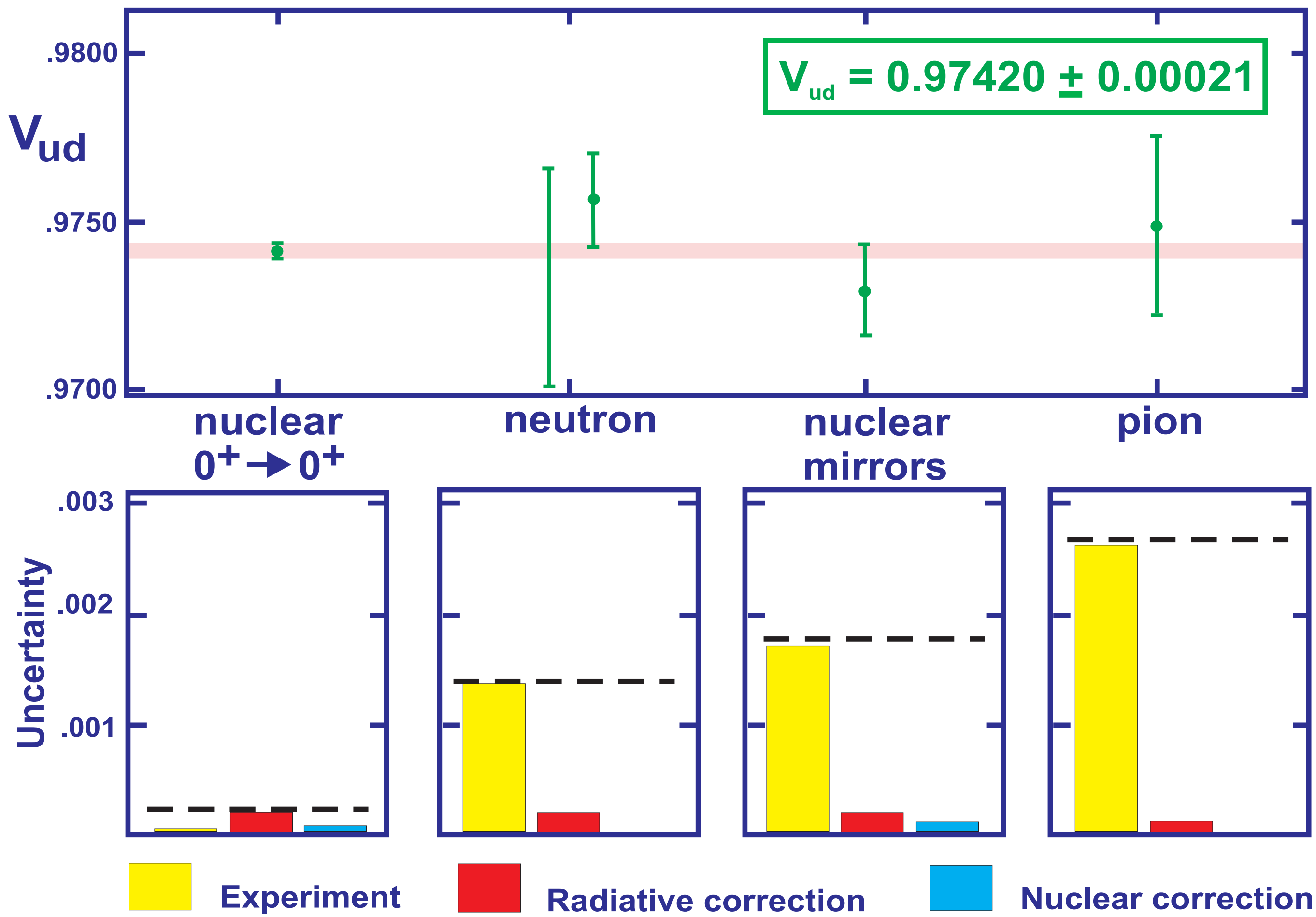
$$\boxed{V_{ud} = 0.9749 \pm 0.0026}$$

nuclear $0^+ \rightarrow 0^+$
$V_{ud} = 0.9742 \pm 0.0002$

CURRENT STATUS OF V_{ud} AND CKM UNITARITY



CURRENT STATUS OF V_{ud} AND CKM UNITARITY



$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99963 \pm 0.00049$$

V_{ud}^2 nuclear decays, muon decay: 0.94907 ± 0.00041

V_{us}^2 PDG kaon decays: 0.05054 ± 0.00027

V_{ub}^2 B decays: 0.00002 ± 0.00001

SUMMARY AND OUTLOOK

1. Analysis of superallowed $0^+ \rightarrow 0^+$ nuclear β decay is shown to confirm CVC and thus yield V_{ud} = 0.97417(21).
2. The three other experimental methods for determining V_{ud} yield consistent results, but are less precise by a factor of 7 or more.
3. The current value for V_{ud}, when combined with the PDG values for V_{us} and V_{ub}, satisfies CKM unitarity to 0.06%.

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4. The largest contribution to V_{ud} uncertainty is from the inner radiative correction, Δ_R. Very little reduction in V_{ud} uncertainty is possible without improved calculation of Δ_R.
5. Isospin symmetry-breaking correction, δ_C, has been tested by requiring consistency among the 14 known transitions (CVC), and agreement with mirror-transition pairs. It contributes much less to V_{ud} uncertainty than does Δ_R.
6. Significant improvement in neutron decay measurements would provide a valuable consistency check.