$|V_{us}|$ from τ decays

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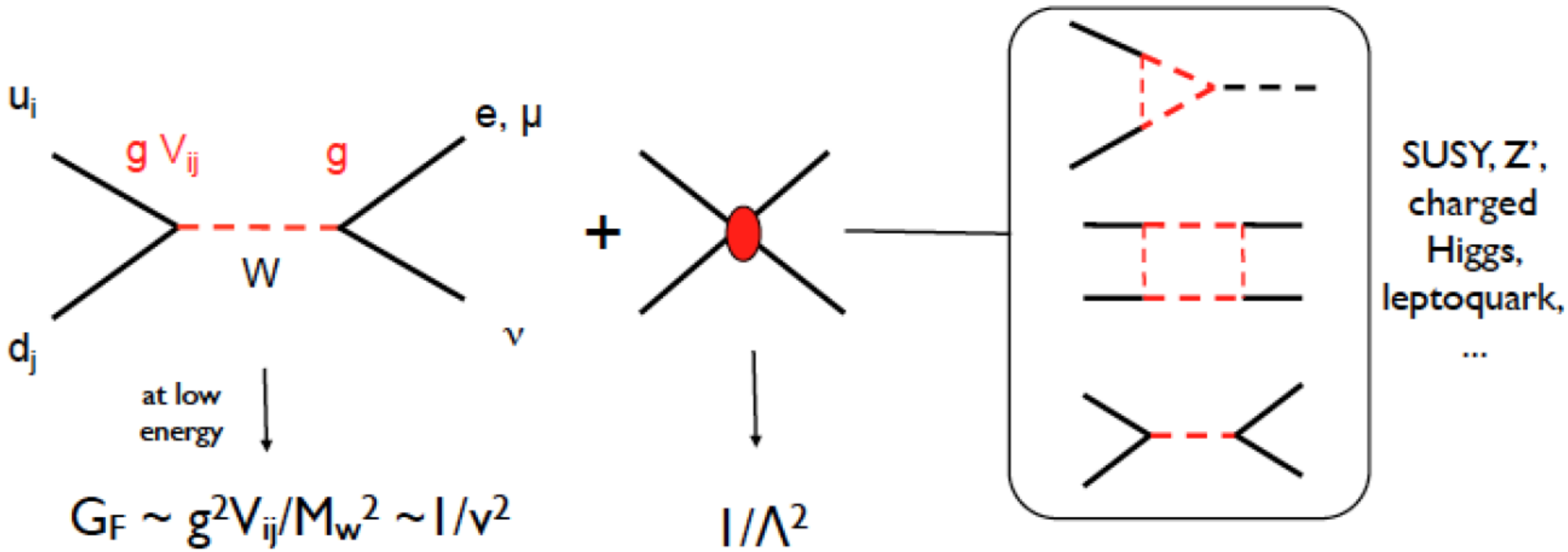
CKM Matrix

V-A interaction via W-exchange with quarks have V_{ij}



Standard Model

New Physics

$\Delta_{CKM} \sim (v/\Lambda)^2$ **sensitive to new physics in large class of models**

CKM Unitarity violation: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}$

The $|V_{us}|$ element of CKM Matrix

V_{ij}: Mixing between Weak and Mass Eigenstates

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

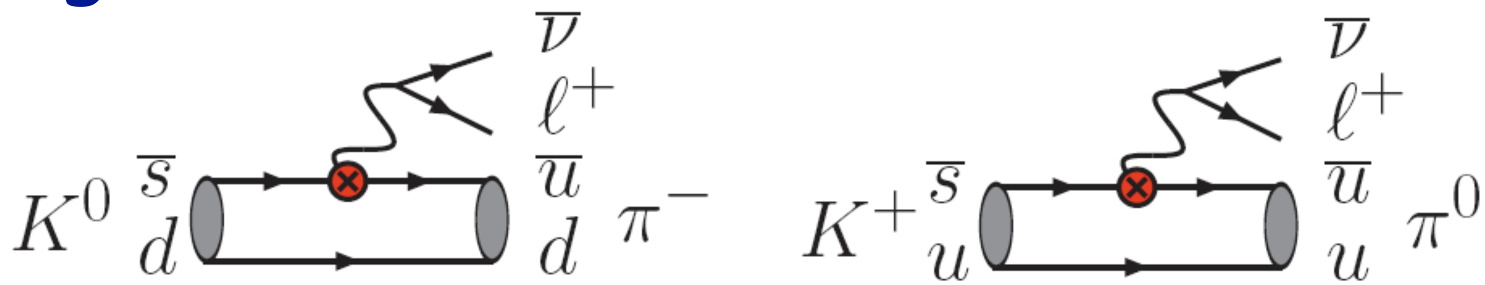
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- $|V_{ud}| = 0.97417 \pm 0.00021$ (from nuclear β decays)
  J.C.Hardy & I.S.Towner, PRC 91 (2015) 025501
- $|V_{ub}| = (4.09 \pm 0.39) \times 10^{-3}$ (from $B \to X_u \ell \nu$ decays)
  Particle Data Group 2016

$\Rightarrow |V_{us}|^{CKM} = 0.22582 \pm 0.00091$

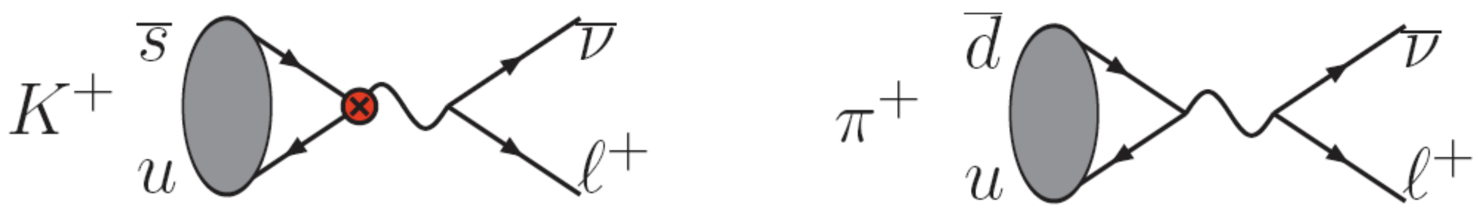
Precision measurement of $|V_{us}|$ is a test of CKM unitarity

Kl3 decays:



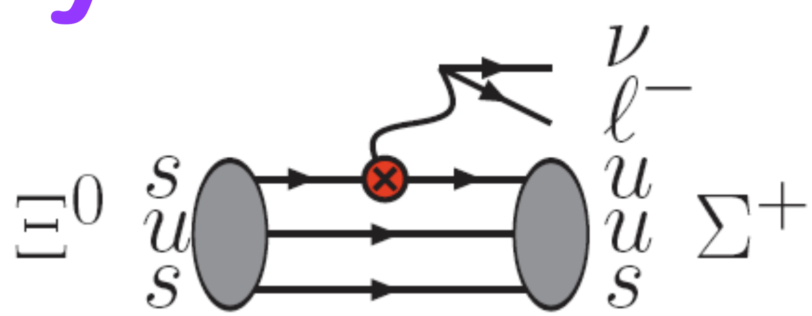
$\Rightarrow |V_{us}| f_+(0)$

Kl2 decays:



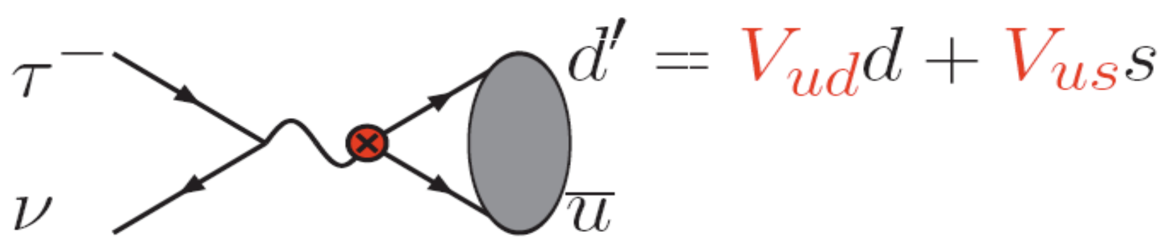
$\Rightarrow \frac{|V_{us}|}{|V_{ud}|} \frac{F_K}{F_\pi}$

Hyperon decays:



$\Rightarrow |V_{us}| f_1(0)$

τ decays:

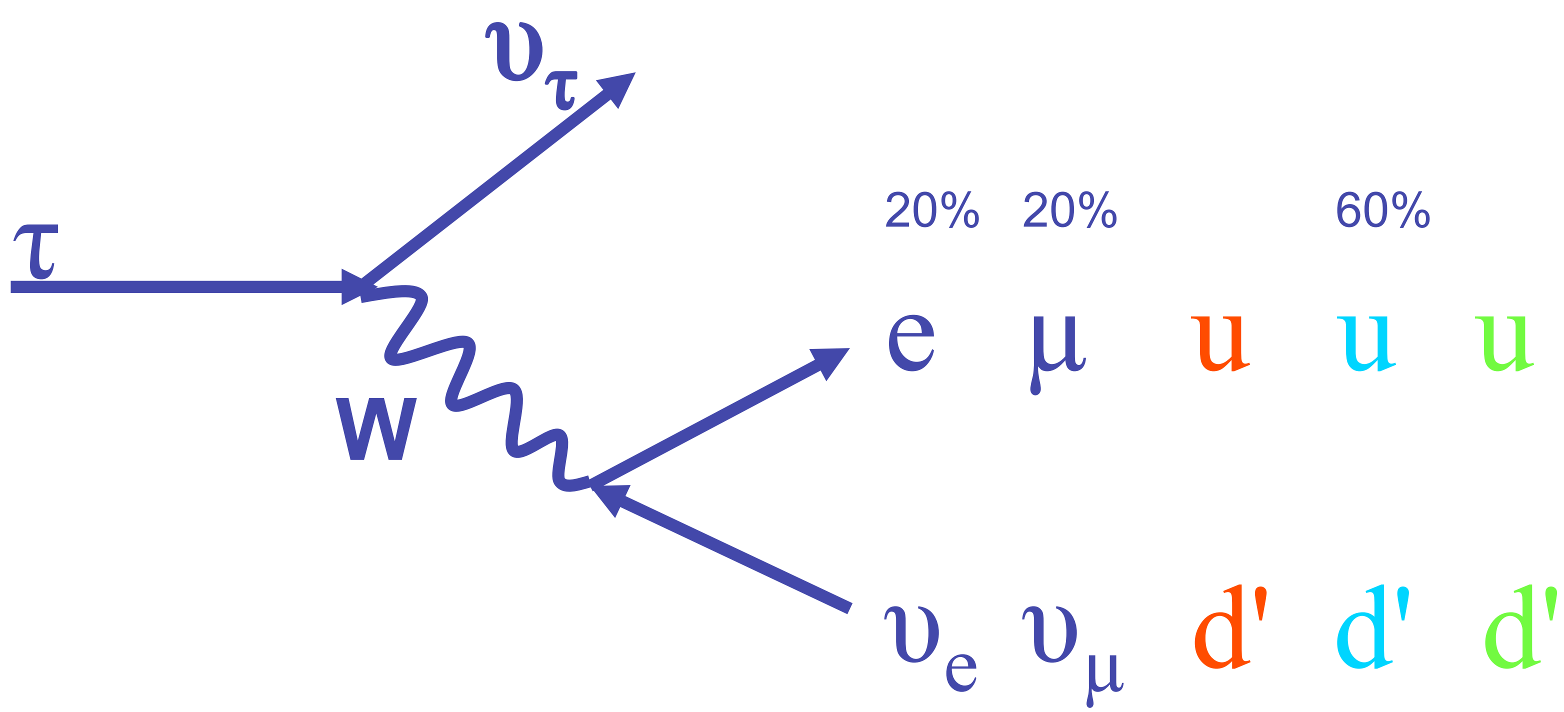


$\Rightarrow m_s, |V_{us}|$

τ decays

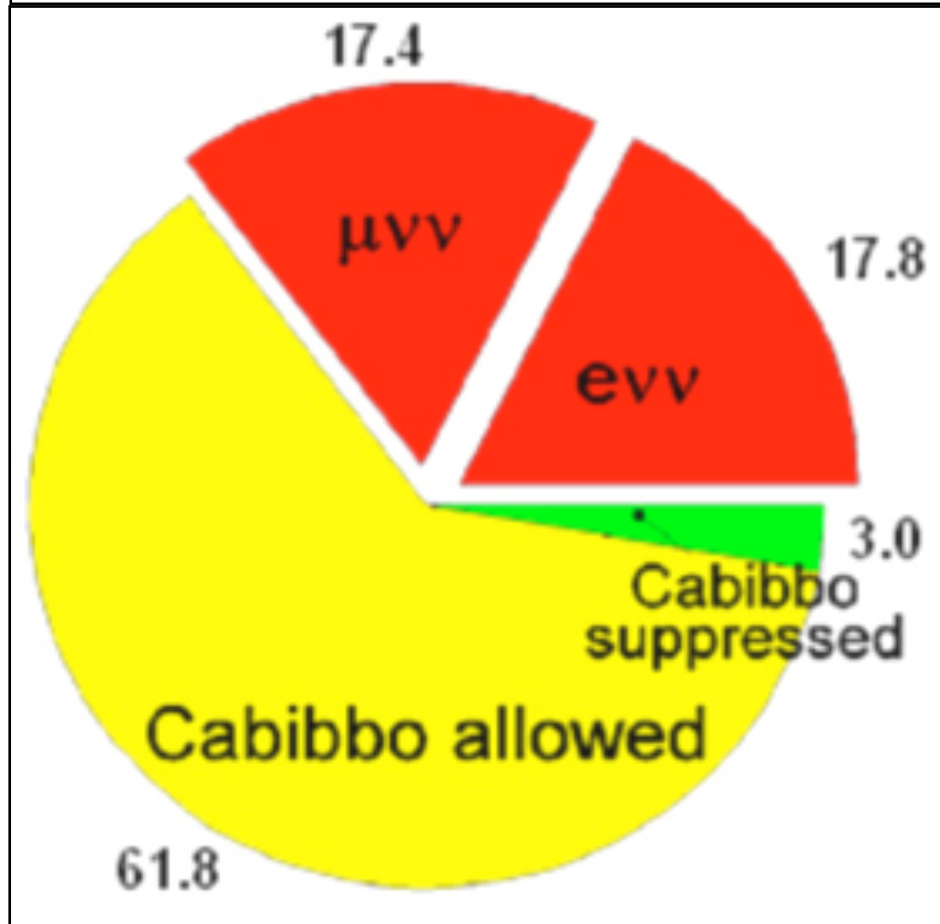
Experiment	Number of τ pairs
LEP	~3x10[5]
CLEO	~1x10[7]
BaBar	~5x10[8]
Belle	~9x10[8]

Naive prediction:



$$|d'\rangle = V_{ud}|d\rangle + V_{us}|s\rangle$$

Including QED & QCD corrections:



Hadronic Width of τ

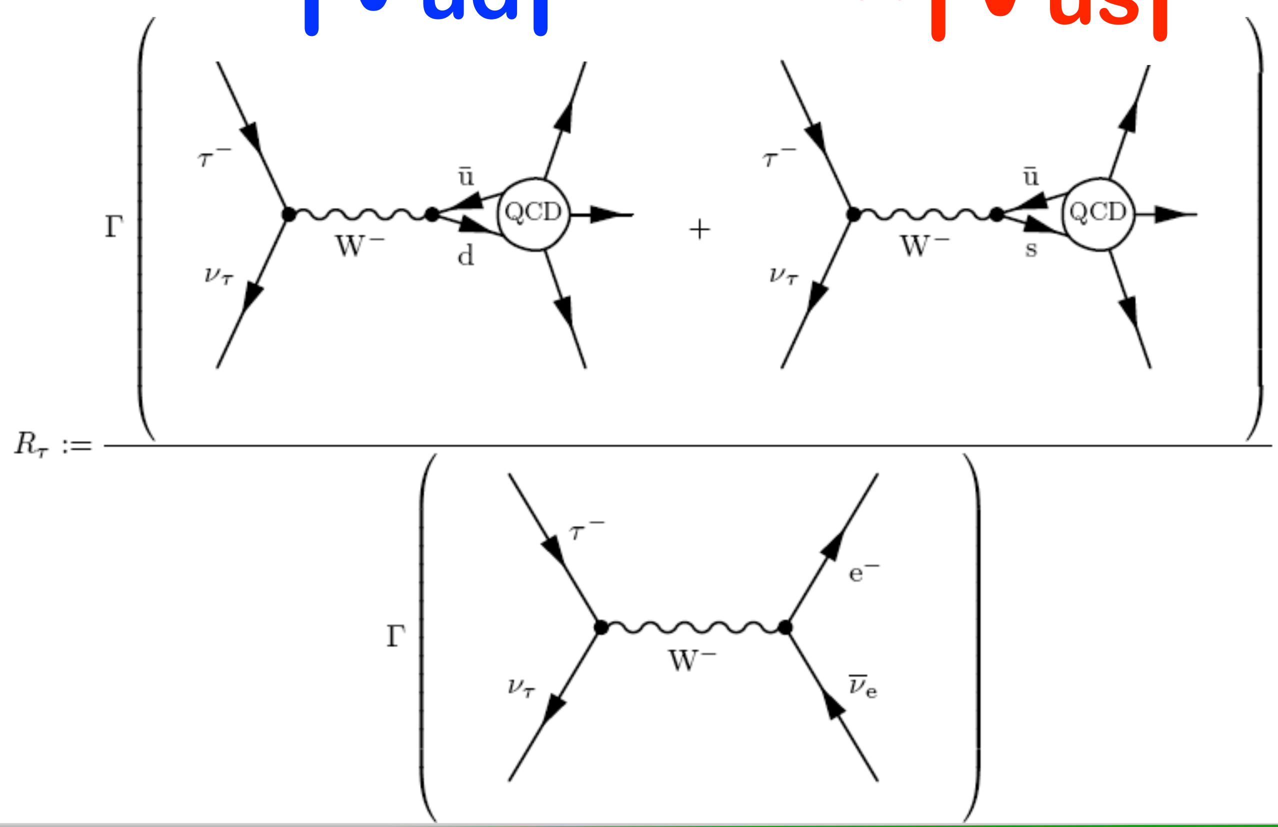
Parton model:

$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \textbf{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} \approx N_C$$

QCD:

$$R_\tau = R_\tau^{NS} + R_\tau^{S} \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C$$

$\propto |V_{ud}|^2$ $\propto |V_{us}|^2$



Hadronic Width of τ

QCD corrections : $R_\tau = |V_{ud}|^2 N_C + |V_{us}|^2 N_C + O(\alpha_s)$

Spectral Moments: $R_\tau^{kl} = \int_0^1 dz (1-z)^k z^l \frac{dR_\tau}{dz}, \;\; z = \frac{q^2}{m_\tau^2}$

Zeroth order moments are simply the τ branching fractions

Finite energy sum rules ⇒ SU(3) breaking sensitive to m_s :

$$\delta R_\tau^{kl} = \frac{R_{\tau,non-strange}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,strange}^{kl}}{|V_{us}|^2}$$

$$\delta R_{\tau,\mathrm{th}}^{00} = 0.1544\,(37) + 9.3\,(3.4)\, m_s^2 + 0.0034\,(28) = 0.242\,(32)$$

$m_s = 95.00 \pm 5.00$ MeV [PDG2015]

E.Gamiz, M.Jamin, A.Pich, J.Prades & F. Schwab, arXiv 0709.0282 [hep-ph]

Truncation errors studied with QCD lattice inputs in terms of weights:

$$|V_{us}| = \sqrt{R_{V+A;us}^w(s_0) / \left[\frac{R_{V+A;ud}^w(s_0)}{|V_{ud}|^2} - \delta R_{V+A}^{w,OPE}(s_0) \right]}$$

K.Maltman, R.J.Hudspith, R.Lewis, C.E.Wolfe, J.Zanotti, arXiv 1511.08514 [hep-ph]

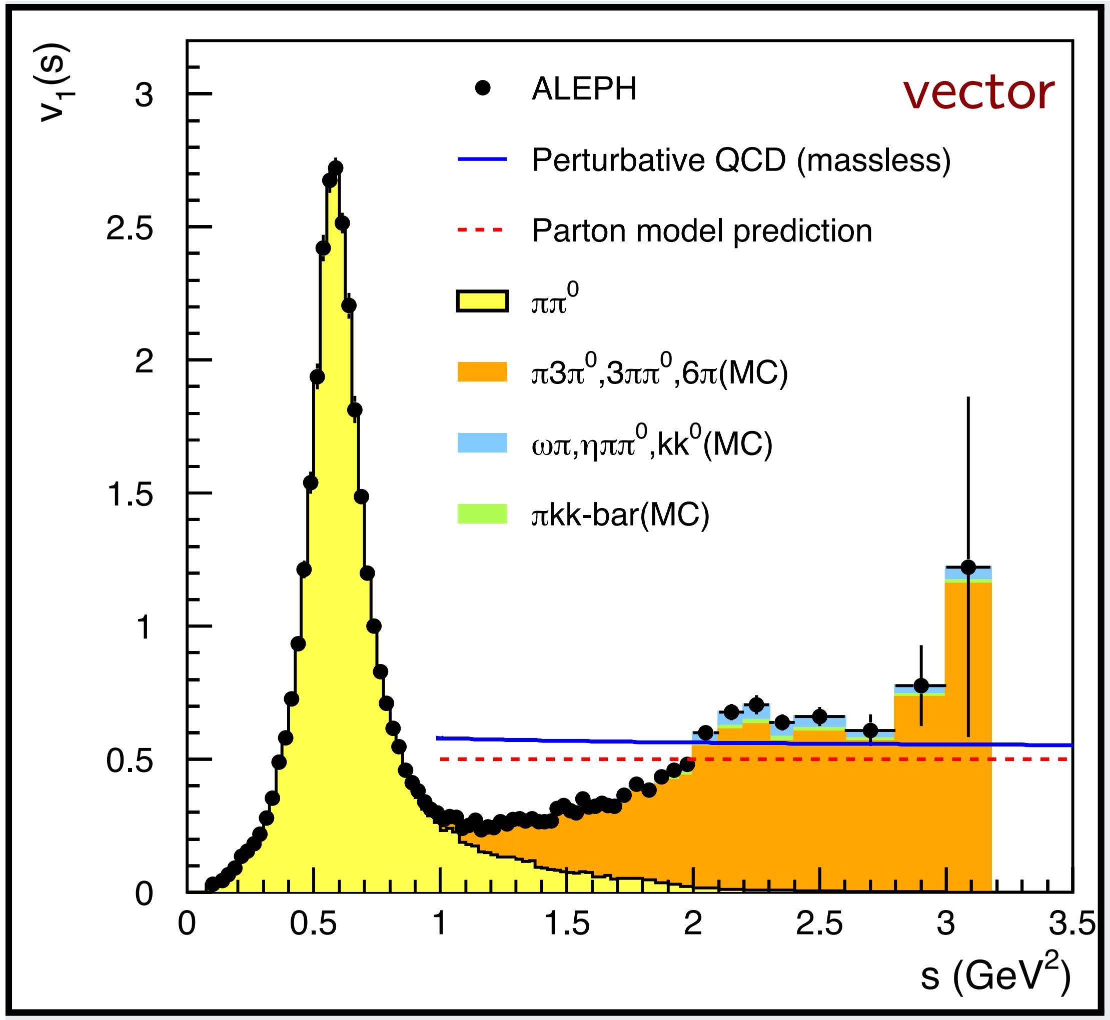
Non-Strange Spectral Functions

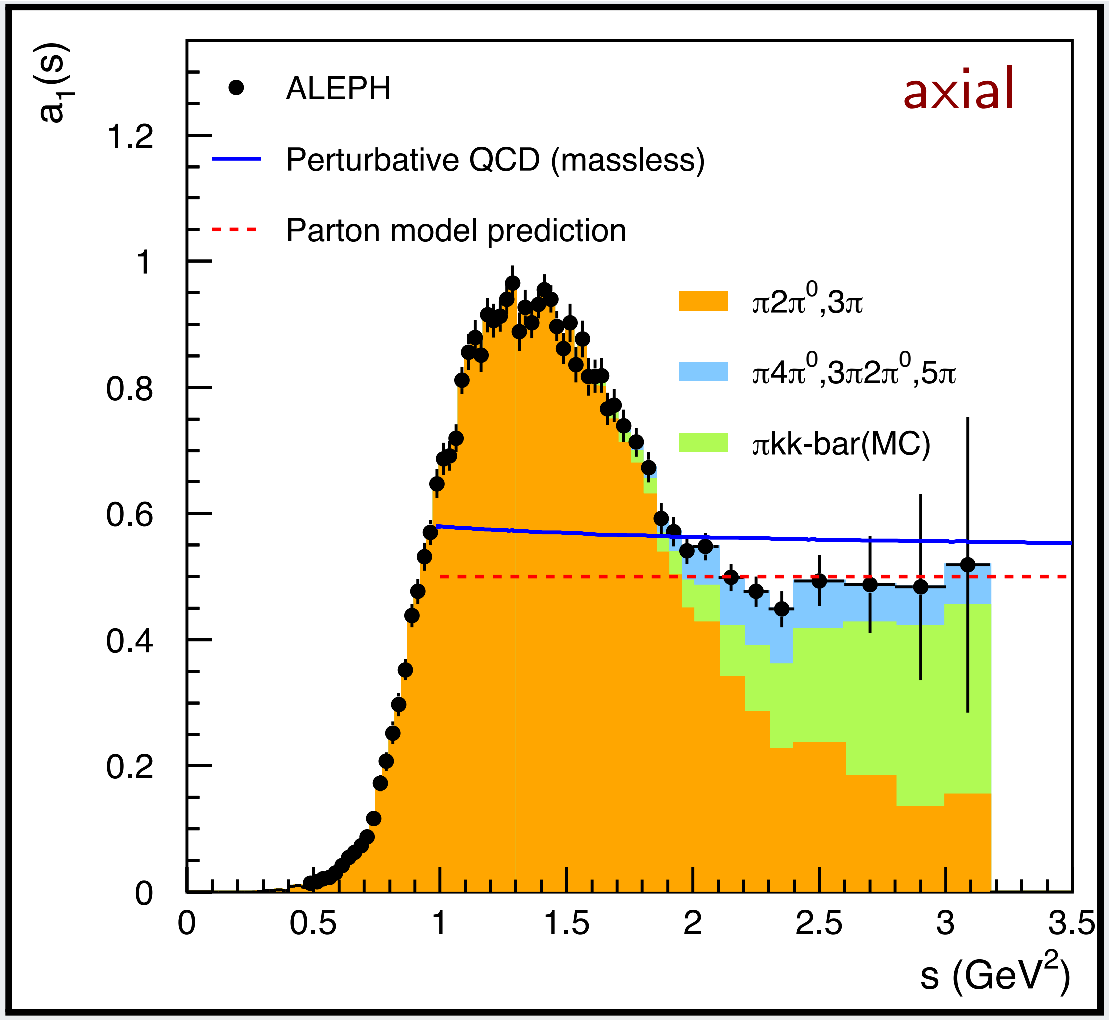
$R_{\tau,V} \Rightarrow \tau^- \to \nu_\tau + h_{v,s=0}$

(even number of pions)

$R_{\tau,A} \Rightarrow \tau^- \to \nu_\tau + h_{A,s=0}$

(odd number of pions)





Originally published in 2005, Revised calculations in 2014

M.Davier, A.Hoeker, B.Malaescu, C.Z.Yuan & Z.Zhang, EPCJ 74 (2014) 2803

Non-Strange Spectral Functions

$R_{\tau,V}$ ⇨ $\tau^- \rightarrow \nu_\tau + h_{v,s=0}$

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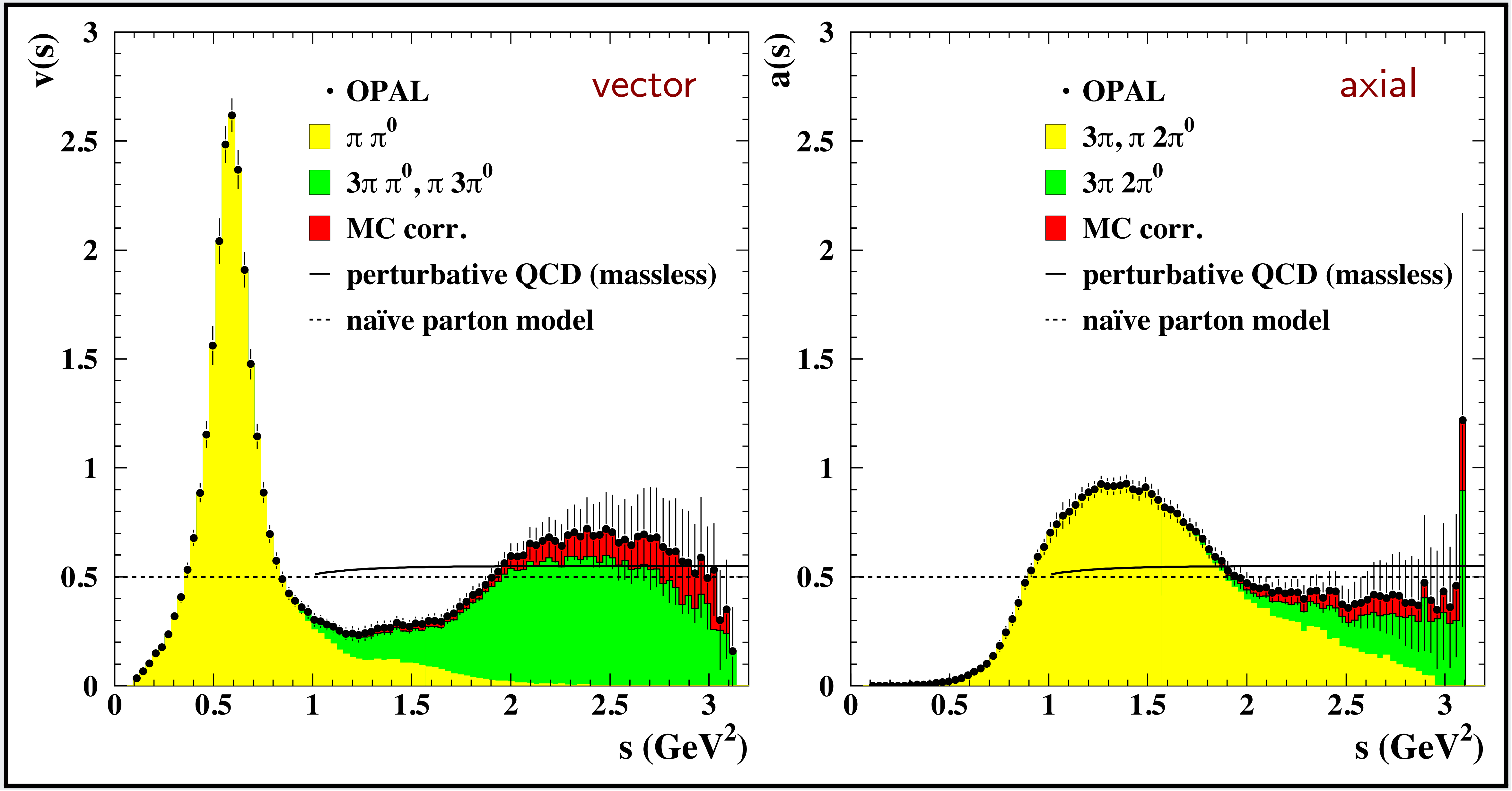
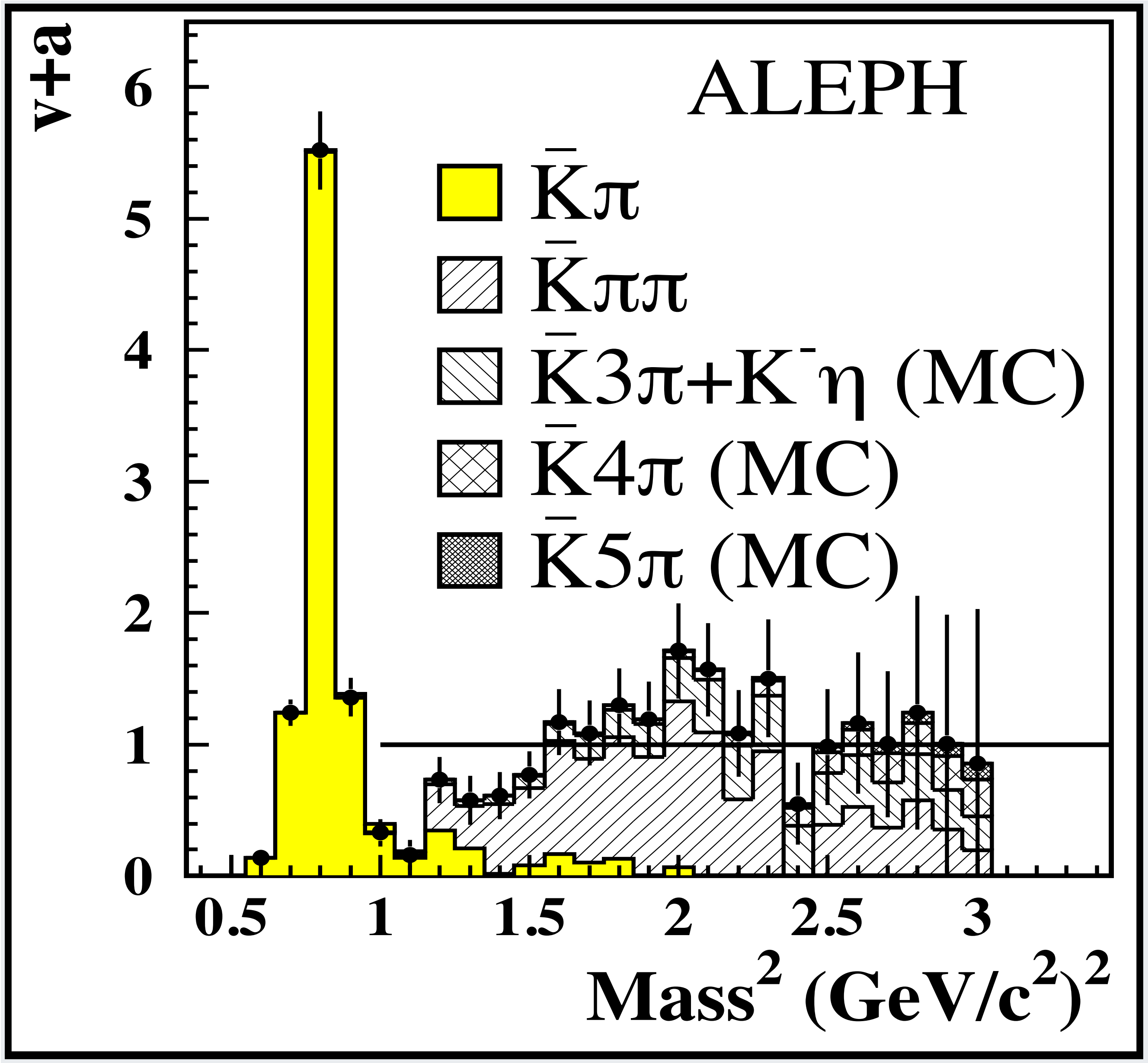


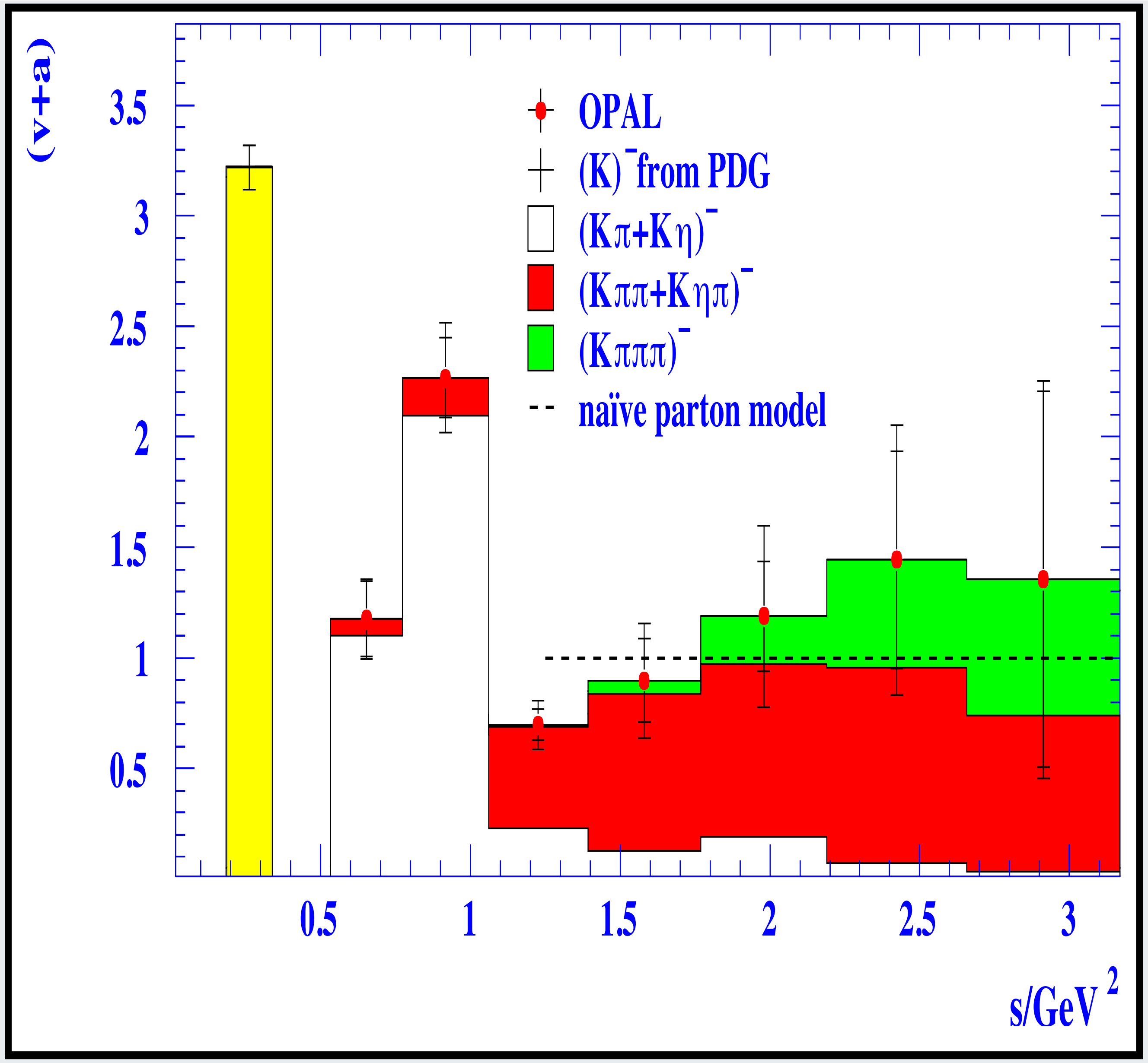
OPAL Collaboration, EPCJ 7 (1999) xxxx

Strange Spectral Functions

Strange Spectral Functions from ALEPH & OPAL are not so precise



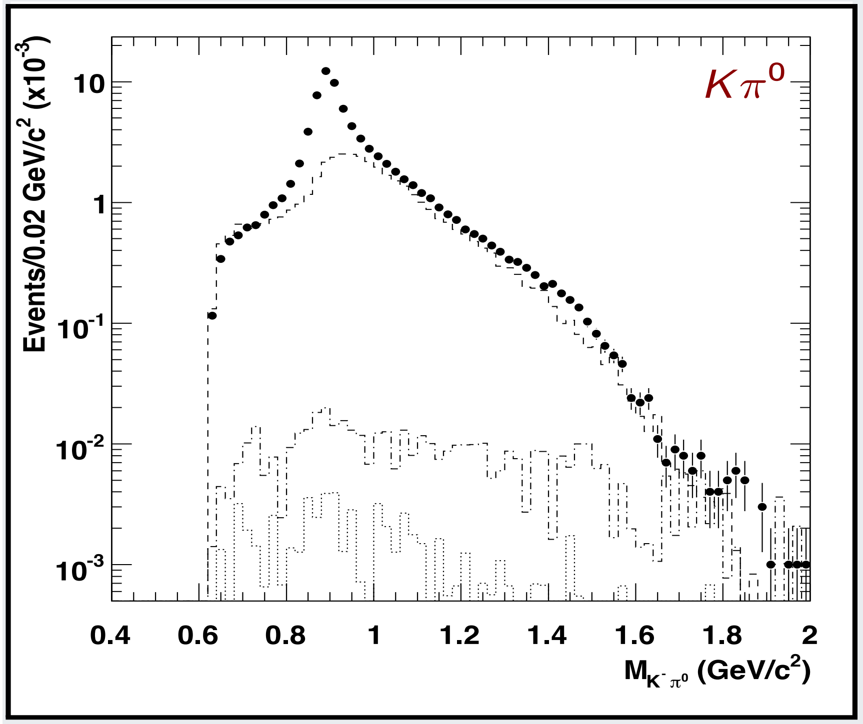
ALEPH, Phys. Rep. 421 (2005) 191



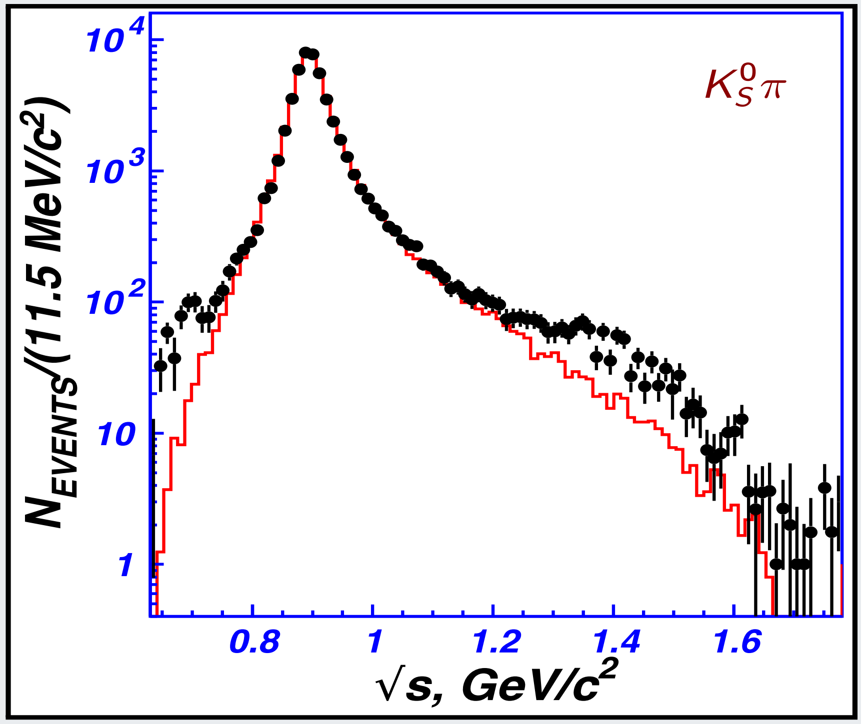
OPAL, EPJC 7 (1999)

Strange Spectral Functions

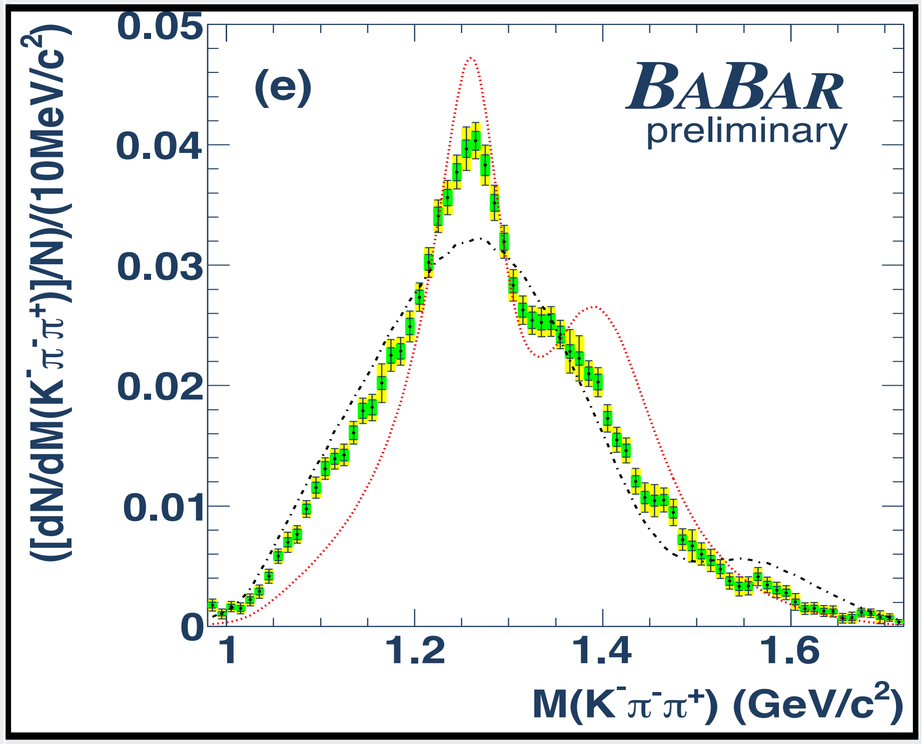
Some [preliminary] measurements from B-Factories are available

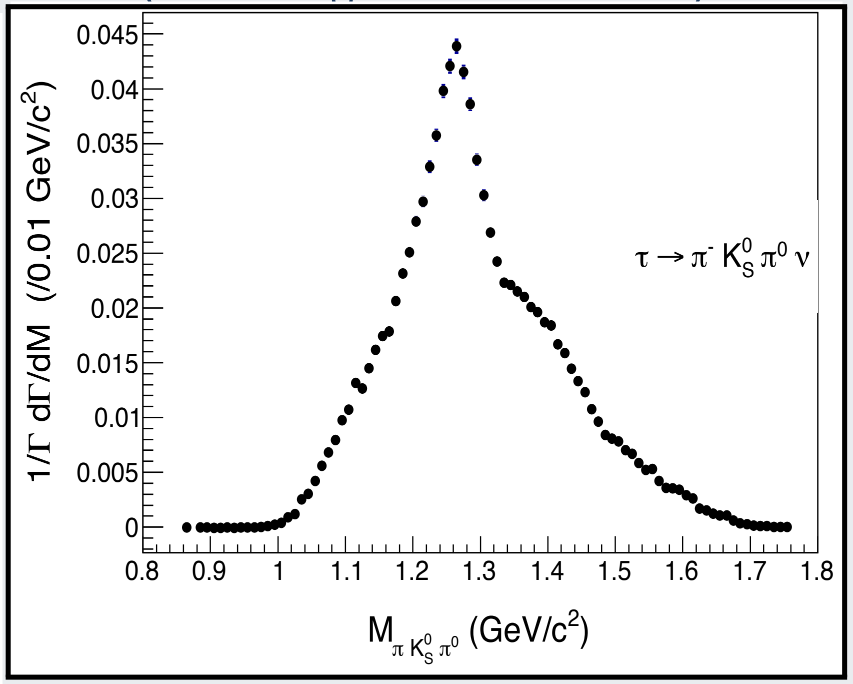


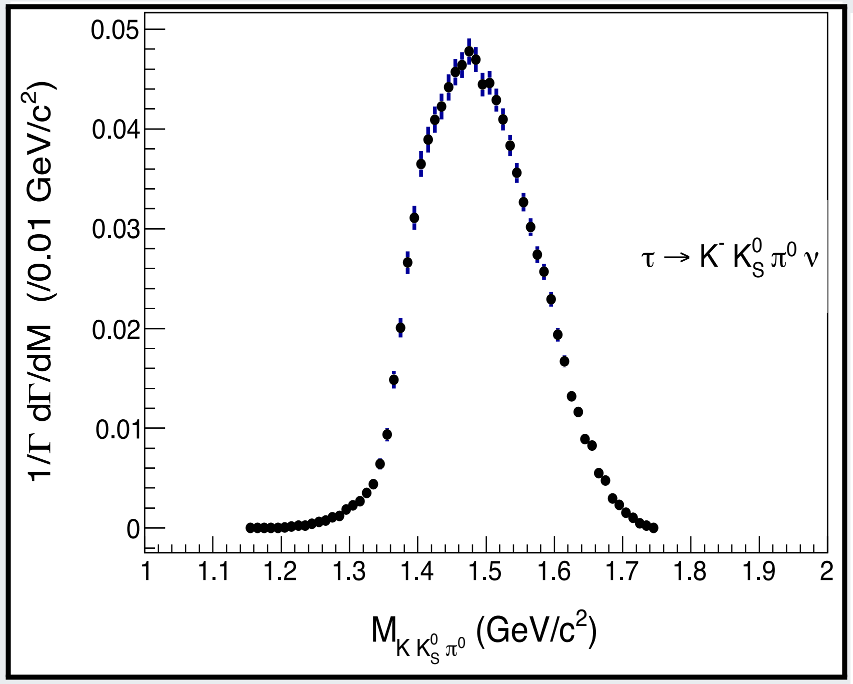
BABAR, PRD 76 (2007) 051104
(not background-subtracted)



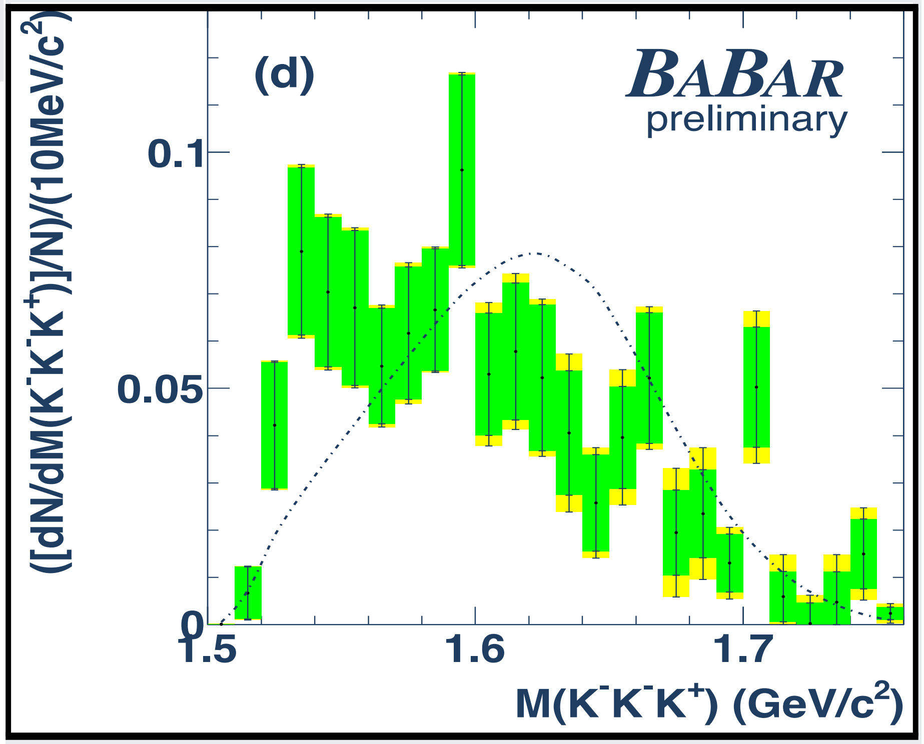
Belle, PLB 654 (2007) 65







Ryu [Belle] Nucl.Phys.Proc.Suppl. 253-255 (2014) 33



Nugent, Nucl.Phys.Proc.Suppl. 253-255 (2014) 38.

τ Branching Fractions are well measured

- Most of the branching fractions are highly correlated.
- Sources of correlation between the same experiment:
  - Track reconstruction ~ 1% for 1-vs-1 topology
  - Secondary vertex reconstruction ~ 1.5% for K_S
  - Calorimeter bump reconstruction ~ 3% for π^0
  - Particle identification ~ 2-4 %
  - Luminosity uncertainty ~ 1%

Sources of correlation between different experiments:

- Tau-pair cross-section uncertainty ~ 0.36%
- Uncertainty on Branching Fractions of backgrounds

➡ Simultaneous averaging of all branching fractions

Heavy Flavor Averaging Group (HFAG)

- Global fit to 170 measurements of τ Branching Fractions:
  - 39 from ALEPH
  - 35 from CLEO
  - 23 from BaBar
  - 19 from OPAL
  - 15 from Belle
  - 14 from DELPHI
  - 11 from L3
  - 6 from CLEO3
  - 3 from TPC
  - 2 from ARGUS
  - 2 from HRS
  - 1 from CELLO

HFAG tries to take into account correlations between measurements, as well as dependence on common external parameters such as tau-pair cross-section and background normalization errors between experiments.

As much as possible, HFAG tries to avoid inflating measured uncertainties using old PDG-style scale factors to account for spread between the different measurements. Instead, a confidence level (CL) for the average is quoted.

Special handling of ALEPH inputs

242 *ALEPH Collaboration / Physics Reports 421 (2005) 191–284*

Table 15
Correlation matrix of the statistical errors on the branching fractions

	μ	h	$h\pi^0$	$h2\pi^0$	$h3\pi^0$	$h4\pi^0$	$3h$	$3h\pi^0$	$3h2\pi^0$	$3h3\pi^0$	$5h$	$5h\pi^0$
e	−0.21	−0.15	−0.25	−0.09	−0.01	0.00	−0.15	−0.10	0.03	−0.06	0.00	0.01
μ	1.00	−0.13	−0.21	−0.07	−0.06	0.00	−0.09	−0.07	0.00	−0.02	0.00	−0.04
h		1.00	−0.31	−0.02	0.01	−0.06	−0.12	−0.06	−0.02	0.01	−0.01	0.02
$h\pi^0$			1.00	−0.40	0.05	0.00	−0.11	−0.06	−0.02	0.00	−0.04	−0.04
$h2\pi^0$				1.00	−0.51	0.26	−0.09	0.01	−0.07	0.06	−0.01	0.03
$h3\pi^0$					1.00	−0.75	0.01	−0.03	0.05	−0.02	−0.01	0.01
$h4\pi^0$						1.00	−0.02	−0.02	−0.03	0.01	0.02	−0.03
$3h$							1.00	−0.33	0.08	−0.05	−0.04	0.00
$3h\pi^0$								1.00	−0.45	0.19	−0.02	−0.02
$3h2\pi^0$									1.00	−0.65	0.03	0.02
$3h3\pi^0$										1.00	−0.01	−0.04
$5h$											1.00	−0.24
$5h\pi^0$												1.00

ALEPH quotes the correlation matrix for hadronic modes, but PDG used to translate the matrix into pion modes, which were obtained by subtracting the kaon contribution. HFAG uses hadronic branching ratios from ALEPH paper.

HFAG tau fits in PDG

From 2016, HFAG-style fits have been adopted by PDG.
Chin.Phys. C40 (2016) no.10, 100001.

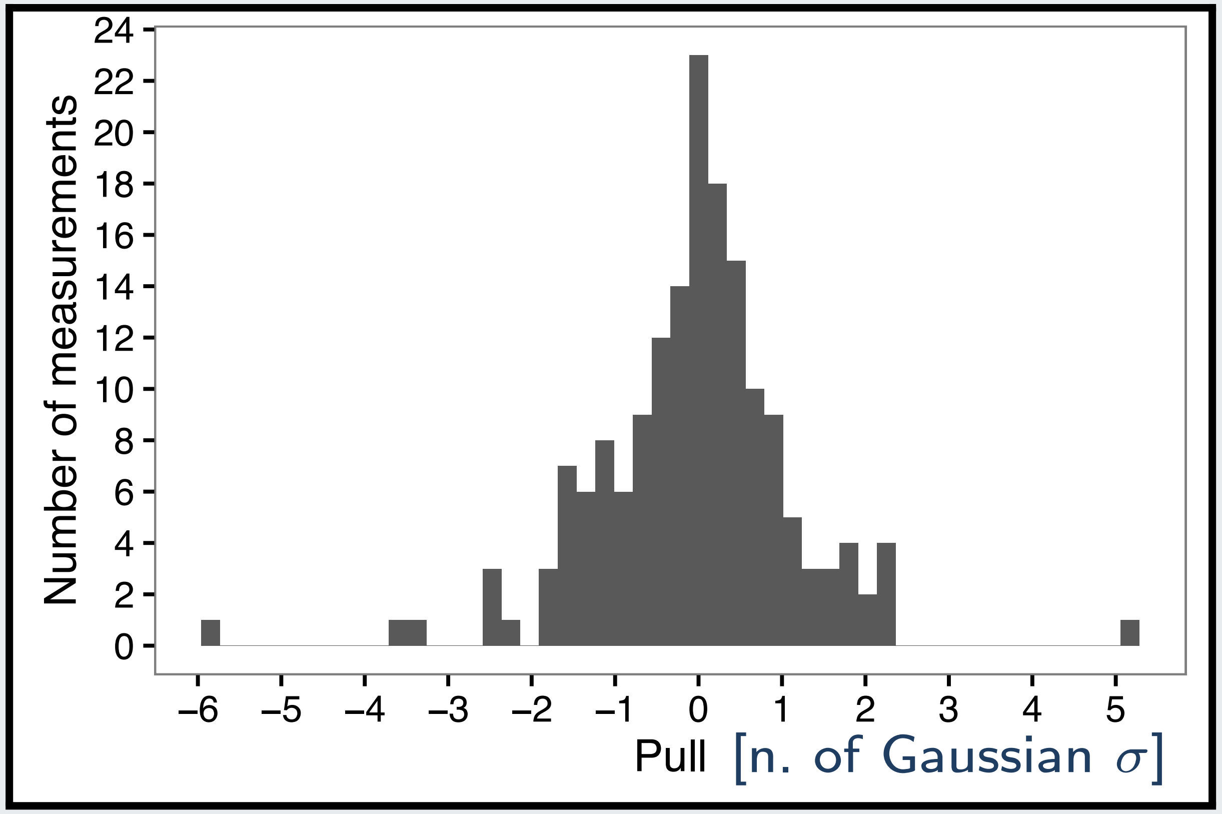
According to PDG naming convention,
47 basis nodes are fitted to 170 measurement
with constraint that linear sum of basis nodes add up to unity
$\Rightarrow$ 170 - 47 + 1 = 124 degrees of freedom

In HFAG notation, 135 quantities consisting of 47 basis nodes
and 88 linear combinations or ratios of linear combinations
are expressed as constraints.
Both the methods are equivalent.

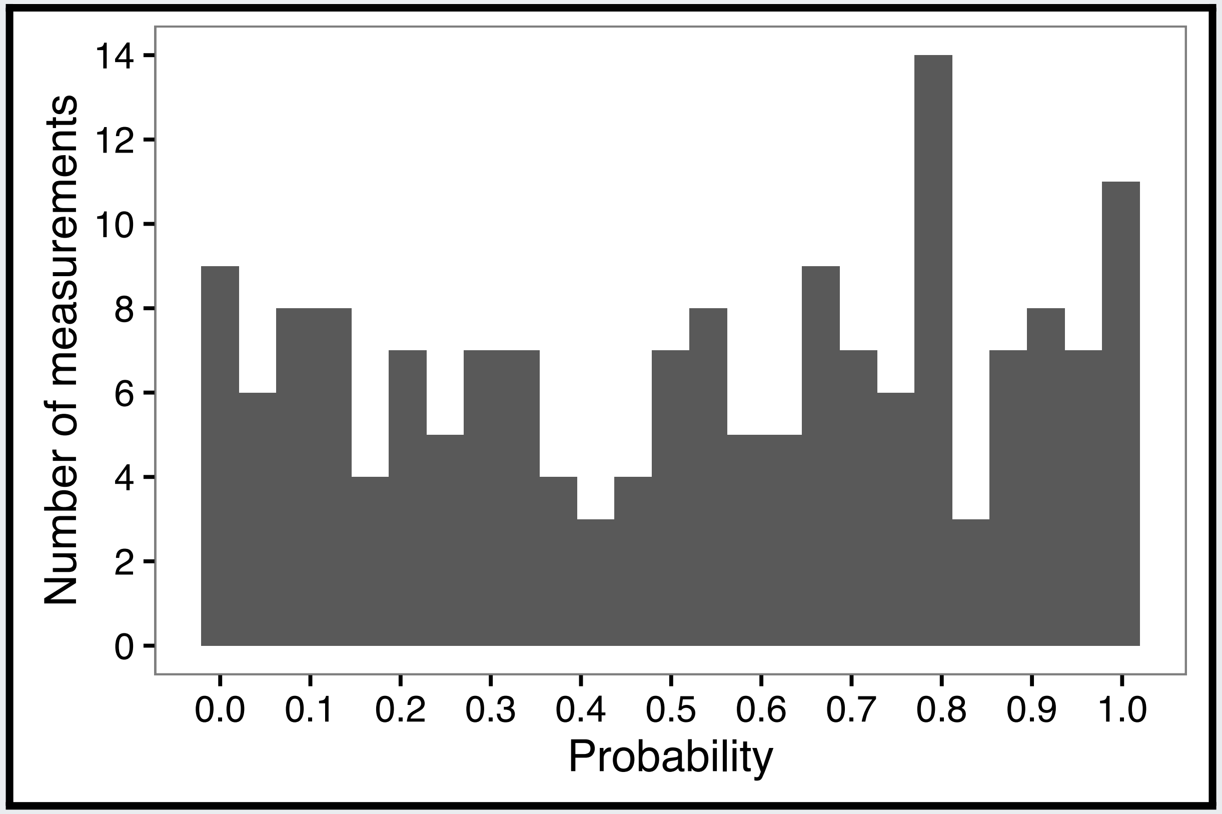
Quality of fit:
Unity-constrained fit: χ^2 / dof = 137.4/124, CL = 19.3%
Non-Unity-constrained fit: χ^2 / dof = 137.3/123, CL = 17.8%
Residual from unity in un-constrained fit = (0.035 ± 0.103)%

Error Scaling

Measurements' pulls



Measurements' pulls probability



- two outliers: *BABAR* and Belle $\mathcal{B}(\tau \to K^- K^- K^+ \nu_\tau)$ results

$\Gamma_{96} = K^- K^- K^+ \nu_\tau$	$(2.174 \pm 0.800) \cdot 10^{-5}$	HFAG Summer 2016 fit
$(1.578 \pm 0.130 \pm 0.123) \cdot 10^{-5}$	BaBar	PRL 100 (2008) 011801
$(3.290 \pm 0.170^{+0.190}_{-0.200}) \cdot 10^{-5}$	Belle	PRD 81 (2010) 113007

No automatic error scaling, but only in this case, a scale factor of 5.44 is applied, which improves CL on non-unity constrained fit from 1.1% to 17.8%.

HFAG basis modes

B ($\tau \to \ldots$)	HFAG 2016 prelim.
$\mu^- \bar{\nu}_\mu \nu_\tau$	17.3917 ± 0.0396
$e^- \bar{\nu}_e \nu_\tau$	17.8162 ± 0.0410
$\pi^- \nu_\tau$	10.8103 ± 0.0526
$K^- \nu_\tau$	0.6960 ± 0.0096
$\pi^- \pi^0 \nu_\tau$	25.5023 ± 0.0918
$K^- \pi^0 \nu_\tau$	0.4327 ± 0.0149
$\pi^- 2\pi^0 \nu_\tau$ (ex. K^0)	9.2424 ± 0.0997
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.0640 ± 0.0220
$\pi^- 3\pi^0 \nu_\tau$ (ex. K^0)	1.0287 ± 0.0749
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.0428 ± 0.0216
$h^- 4\pi^0 \nu_\tau$ (ex. K^0, η)	0.1099 ± 0.0391
$\pi^- \bar{K}^0 \nu_\tau$	0.8386 ± 0.0141
$K^- K^0 \nu_\tau$	0.1479 ± 0.0053
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.3812 ± 0.0129
$K^- \pi^0 K^0 \nu_\tau$	0.1502 ± 0.0071
$\pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. K^0)	0.0234 ± 0.0231
$\pi^- K_S^0 K_S^0 \nu_\tau$	0.0233 ± 0.0007
$\pi^- K_S^0 K_L^0 \nu_\tau$	0.1047 ± 0.0247
$\pi^- \pi^0 K_S^0 K_S^0 \nu_\tau$	0.0018 ± 0.0002
$\pi^- \pi^0 K_S^0 K_L^0 \nu_\tau$	0.0318 ± 0.0119
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.0222 ± 0.0202
$\pi^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	8.9704 ± 0.0515
$\pi^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω)	2.7694 ± 0.0711
$h^- h^- h^+ 2\pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.0976 ± 0.0355

B ($\tau \to \ldots$)	HFAG 2016 prelim.
$\pi^- K^- K^+ \nu_\tau$	0.1434 ± 0.0027
$\pi^- K^- K^+ \pi^0 \nu_\tau$	0.0061 ± 0.0018
$\pi^- \pi^0 \eta \nu_\tau$	0.1386 ± 0.0072
$K^- \eta \nu_\tau$	0.0155 ± 0.0008
$K^- \pi^0 \eta \nu_\tau$	0.0048 ± 0.0012
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0094 ± 0.0015
$\pi^- \pi^+ \pi^- \eta \nu_\tau$ (ex. K^0)	0.0218 ± 0.0013
$K^- \omega \nu_\tau$	0.0410 ± 0.0092
$h^- \pi^0 \omega \nu_\tau$	0.4058 ± 0.0419
$K^- \phi \nu_\tau$	0.0044 ± 0.0016
$\pi^- \omega \nu_\tau$	1.9544 ± 0.0647
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.2923 ± 0.0067
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.0410 ± 0.0143
$a_1^- (\to \pi^- \gamma) \nu_\tau$	0.0400 ± 0.0200
$\pi^- 2\pi^0 \omega \nu_\tau$ (ex. K^0)	0.0071 ± 0.0016
$2\pi^- \pi^+ 3\pi^0 \nu_\tau$ (ex. K^0, η, ω, f_1)	0.0013 ± 0.0027
$3\pi^- 2\pi^+ \nu_\tau$ (ex. K^0, ω, f_1)	0.0768 ± 0.0030
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$2\pi^- \pi^+ \omega \nu_\tau$ (ex. K^0)	0.0084 ± 0.0006
$3\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0, η, ω, f_1)	0.0038 ± 0.0009
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$\pi^- f_1 \nu_\tau$ ($f_1 \to 2\pi^- 2\pi^+$)	0.0052 ± 0.0004
$\pi^- 2\pi^0 \eta \nu_\tau$	0.0193 ± 0.0038
$1 - \Gamma_{\text{All}}$	0.0355 ± 0.1031

note: a linear combination sums up to 1

|Vus| from inclusive strange decays

[Preliminary]

Branching fraction	HFAG Summer 2016 fit (%)
$K^- \nu_\tau$	0.6960 ± 0.0096
$K^- \pi^0 \nu_\tau$	0.4327 ± 0.0149
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.0640 ± 0.0220
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.0428 ± 0.0216
$\pi^- \overline{K}^0 \nu_\tau$	0.8386 ± 0.0141
$\pi^- \overline{K}^0 \pi^0 \nu_\tau$	0.3812 ± 0.0129
$\pi^- \overline{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. K^0)	0.0234 ± 0.0231
$\overline{K}^0 h^- h^- h^+ \nu_\tau$	0.0222 ± 0.0202
$K^- \eta \nu_\tau$	0.0155 ± 0.0008
$K^- \pi^0 \eta \nu_\tau$	0.0048 ± 0.0012
$\pi^- \overline{K}^0 \eta \nu_\tau$	0.0094 ± 0.0015
$K^- \omega \nu_\tau$	0.0410 ± 0.0092
$K^- \phi \nu_\tau$ ($\phi \to K^+ K^-$)	0.0022 ± 0.0008
$K^- \phi \nu_\tau$ ($\phi \to K^0_S K^0_L$)	0.0015 ± 0.0006
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.2923 ± 0.0067
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.0410 ± 0.0143
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$X_s^- \nu_\tau$	2.9087 ± 0.0482

$$|V_{us}|_{\tau s} = \sqrt{R_s / \left[\frac{R_{VA}}{|V_{ud}|^2} - \delta R_{\text{theory}}\right]}$$

$$B_s = (2.909 \pm 0.048)\%$$

$$B_{\text{hadrons}} = B_{\text{all}} - B_e - B_\mu = (64.76 \pm 0.10)\%$$

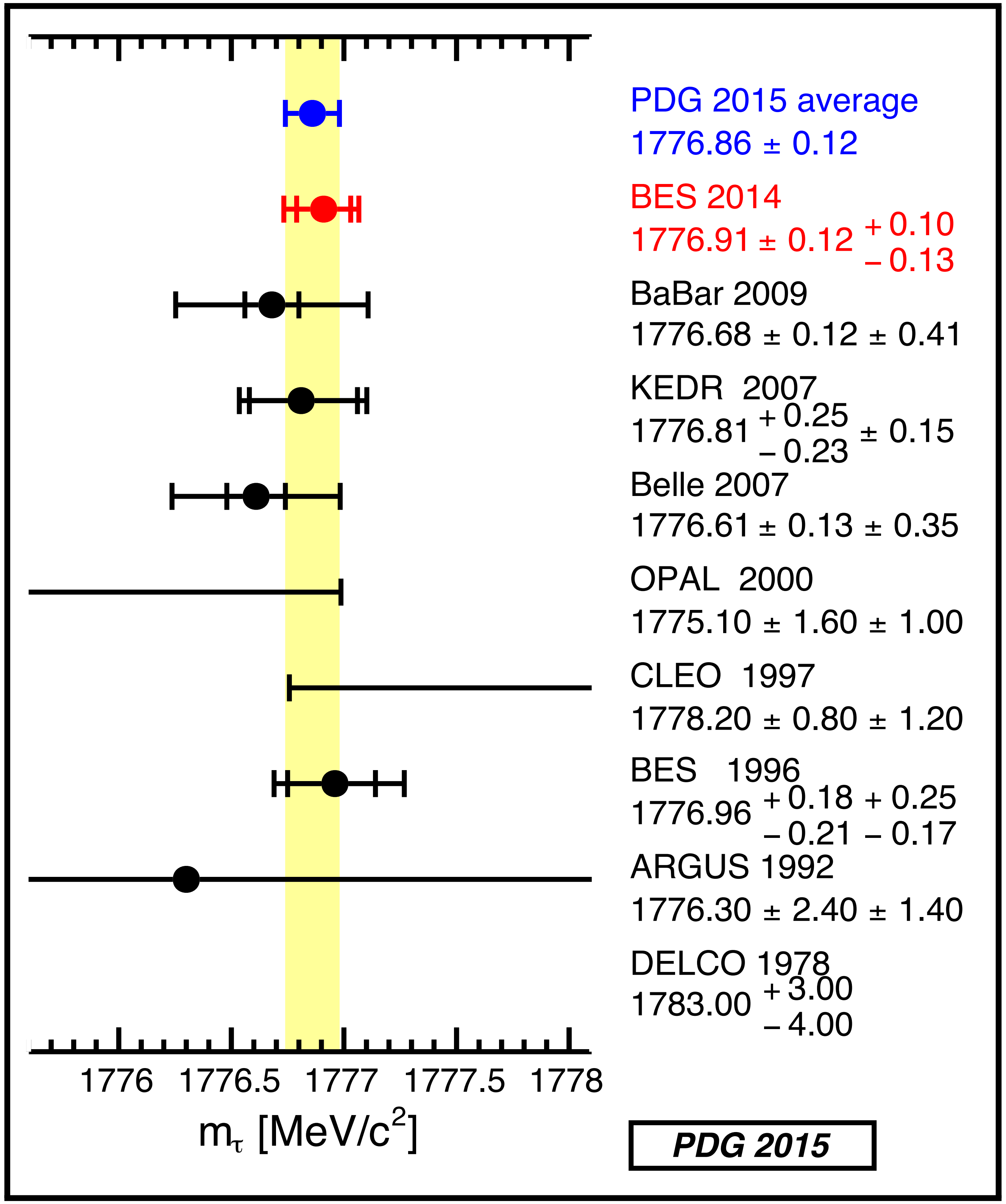
$$B_{VA} = B_{\text{hadrons}} - B_s = (61.85 \pm 0.10)\%$$

To get R, we normalize by

$$B_e = (17.816 \pm 0.041)\%$$

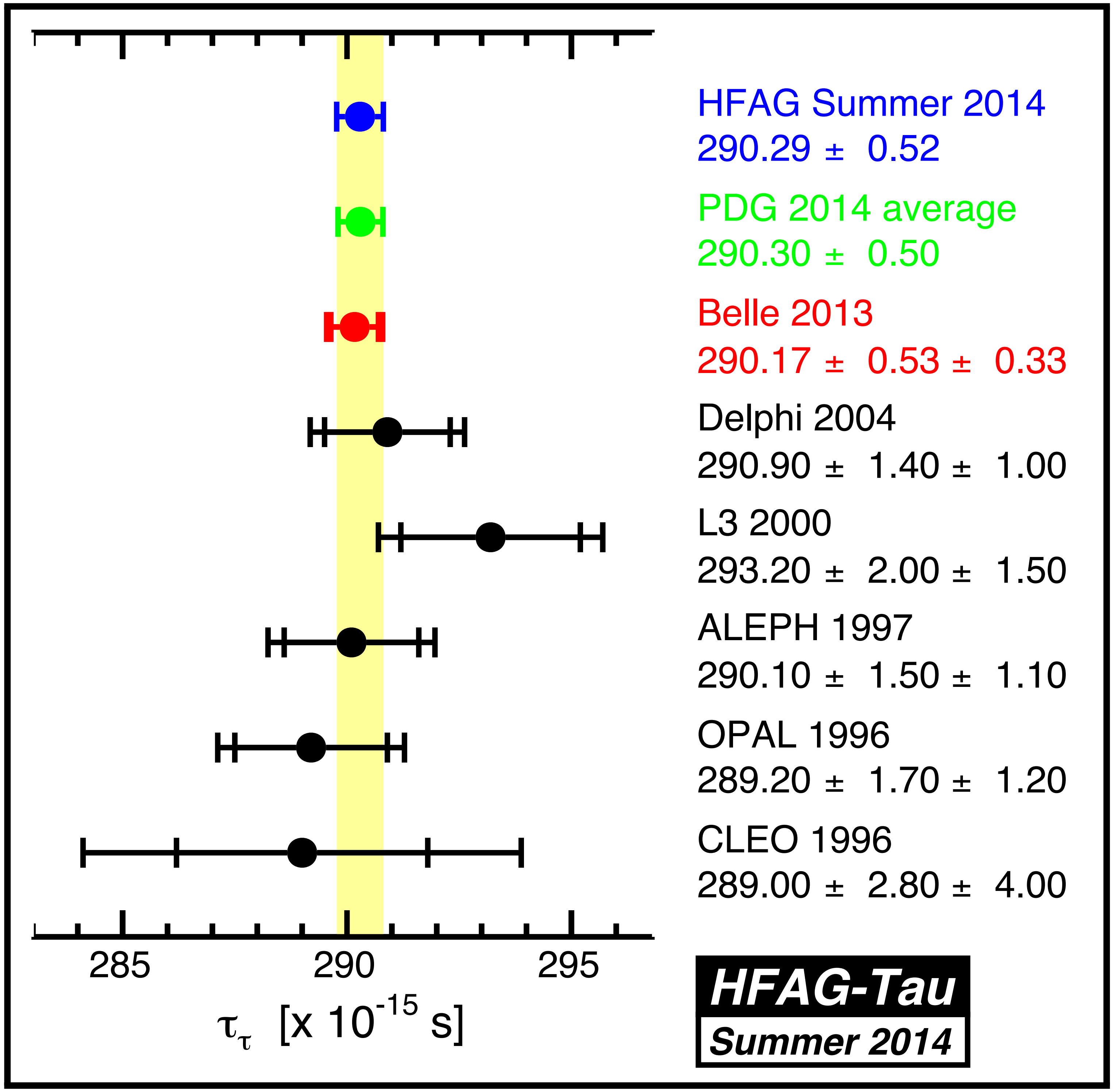
However, the error on B_e can be improved using lepton universality and improved measurements of mass (m_τ) and lifetime (τ_τ).

Tau Mass



- most precise measurements by e^+e^- **colliders at** $\tau^+\tau^-$ **threshold**
  - few events but very significant

Tau Lifetime



- LEP experiments, many methods
  - impact parameter sum (IPS)
  - momentum dependent impact parameter sum (MIPS
  - 3D impact parameter sum (3DIP)
  - impact parameter difference (IPD)
  - decay length (DL)
- Belle
  - 3-prong vs. 3-prong decay length
  - largest syst. error: alignment

Lepton Universality from leptonic decays

Standard Model for leptons $\lambda,\ \rho = e, \mu, \tau$ (Marciano 1988)

$$\Gamma[\lambda \to \nu_\lambda \rho \overline{\nu}_\rho(\gamma)] = \Gamma_{\lambda\rho} = \Gamma_\lambda \mathcal{B}_{\lambda\rho} = \frac{\mathcal{B}_{\lambda\rho}}{\tau_\lambda} = \frac{G_\lambda G_\rho m_\lambda^5}{192\pi^3} f\left(\frac{m_\rho^2}{m_\lambda^2}\right) r_W^\lambda r_\gamma^\lambda ,$$

$$G_\lambda = \frac{g_\lambda^2}{4\sqrt{2}M_W^2} \qquad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x \qquad f_{\lambda\rho} = f\left(\frac{m_\rho^2}{m_\lambda^2}\right)$$

where

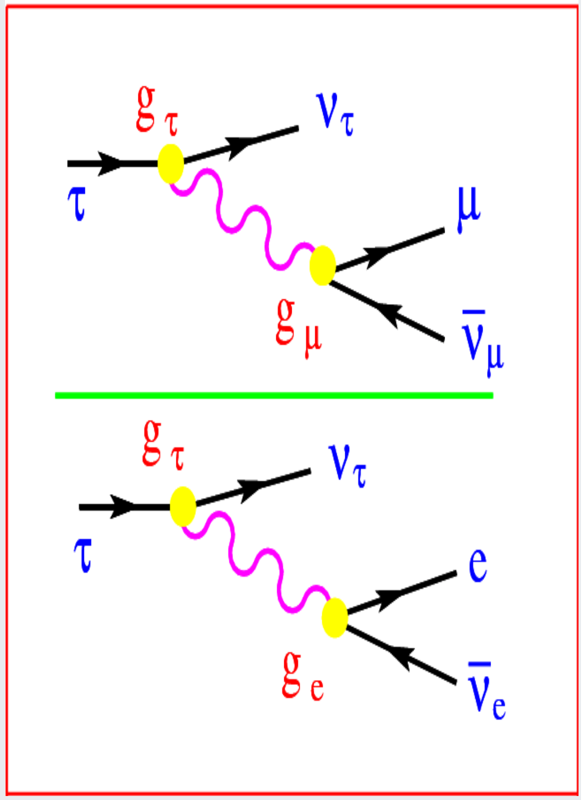
$$r_W^\lambda = 1 + \frac{3}{5}\frac{m_\lambda^2}{M_W^2} \qquad r_\gamma^\lambda = 1 + \frac{\alpha(m_\lambda)}{2\pi}\left(\frac{25}{4} - \pi^2\right)$$

Tests of lepton universality from ratios of above partial widths:

$$\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f_{\mu e} r_W^\mu r_\gamma^\mu}{\tau_\tau m_\tau^5 f_{\tau e} r_W^\tau r_\gamma^\tau}} = \mathbf{1.0010 \pm 0.0015} = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\tau e}^{SM}}}$$

$$\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f_{\mu e} r_W^\mu r_\gamma^\mu}{\tau_\tau m_\tau^5 f_{\tau \mu} r_W^\tau r_\gamma^\tau}} = \mathbf{1.0029 \pm 0.0015} = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\tau \mu}^{SM}}}$$

$$\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\tau e}} \frac{f_{\tau e}}{f_{\tau \mu}}} = \mathbf{1.0019 \pm 0.0014}$$

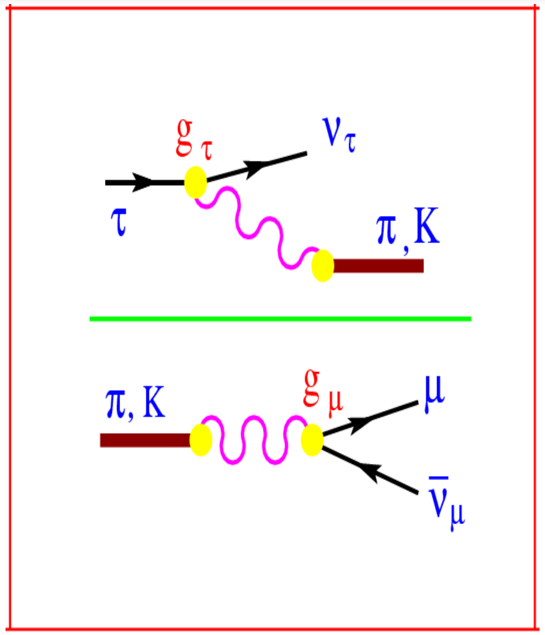


- precision: **0.20—0.23%** pre-*B*-Factories ⇒ **0.14—0.15%** today
  thanks essentially to the Belle tau lifetime measurement, PRL 112 (2014) 031801

Lepton universality tests limited by precision of $B_{e/\mu}$, not any more by τ_τ

Lepton Universality from hadronic decays

$$\frac{\mathcal{B}(\tau^- \to \pi^- \nu_\tau)}{\mathcal{B}(\pi^- \to \mu^- \overline{\nu}_\mu)}$$



$$\frac{\mathcal{B}(\tau^- \to K^- \nu_\tau)}{\mathcal{B}(K^- \to \mu^- \overline{\nu}_\mu)}$$

Standard Model:

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\mathcal{B}(\tau \to h\nu_\tau)}{\mathcal{B}(h \to \mu\bar{\nu}_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1+\delta_h) m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2}\right)^2 \quad (h = \pi \text{ or } K)$$

rad. corr. $\delta_\pi = (0.16 \pm 0.12)\%$, $\delta_K = (0.90 \pm 0.22)\%$ (Decker 1994)

$$\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9961 \pm 0.0027 \ , \qquad \left(\frac{g_\tau}{g_\mu}\right)_K = 0.9860 \pm 0.0070 \ .$$

(electron tests less precise because hadron two body decays to electrons are helicity-suppressed)

Averaging the three g_τ/g_μ ratios:

$$\left(\frac{g_\tau}{g_\mu}\right)_{\tau+\pi+K} = 1.0000 \pm 0.0014 \ , \quad \text{(accounting for statistical correlations)}$$

[recent useful contribution from *BABAR* $\frac{K^-\nu_\tau}{e^-\bar{\nu}_e\nu_\tau}$ measurement, PRL 105 (2010) 051602]

Lepton Universality improved B_e^{univ}

Universality improved $\mathcal{B}(\tau \to e\nu\bar{\nu})$

- (M. Davier, 2005): assume SM lepton universality to improve $\mathcal{B}_e = \mathcal{B}(\tau \to e\bar{\nu}_e\nu_\tau)$ fit $\mathcal{B}_e$ using three determinations:
  - $\mathcal{B}_e = \mathcal{B}_e$
  - $\mathcal{B}_e = \mathcal{B}_\mu \cdot f(m_e^2/m_\tau^2)/f(m_\mu^2/m_\tau^2)$
  - $\mathcal{B}_e = \mathcal{B}(\mu \to e\bar{\nu}_e\nu_\mu) \cdot (\tau_\tau/\tau_\mu) \cdot (m_\tau/m_\mu)^5 \cdot f(m_e^2/m_\tau^2)/f(m_e^2/m_\mu^2) \cdot (\delta_\gamma^\tau \delta_W^\tau)/(\delta_\gamma^\mu \delta_W^\mu)$
    [above we have: $\mathcal{B}(\mu \to e\bar{\nu}_e\nu_\mu) = 1$]
- $\boldsymbol{\mathcal{B}_e^{\mathrm{univ}} = (17.815 \pm 0.023)\%}$ HFAG-PDG 2016 prelim. fit

$\Rightarrow$ improvement by almost a factor of 2 from the value of $B_e = (17.816 \pm 0.041)\%$

Lepton Universality improved $|V_{us}|$

$R_{had} = \Gamma(\tau \to \text{hadrons})/\Gamma_{univ}(\tau \to e\nu\bar{\nu})$

- $R_{had} = \dfrac{\Gamma(\tau \to \text{hadrons})}{\Gamma_{univ}(\tau \to e\nu\bar{\nu})} = \dfrac{\mathcal{B}_{hadrons}}{\mathcal{B}_e^{univ}} = \dfrac{1 - \mathcal{B}_e^{univ} - f(m_\mu^2/m_\tau^2)/f(m_e^2/m_\tau^2) \cdot \mathcal{B}_e^{univ}}{\mathcal{B}_e^{univ}}$
  - two different determinations, second one not "contaminated" by hadronic BFs
- $R_{had} = 3.6349 \pm 0.0082$ HFAG-PDG 2016 prelim. fit
- R_{had}(leptonic BFs only) $= 3.6397 \pm 0.0070$ HFAG-PDG 2016 prelim. fit

$\Rightarrow |V_{us}| = (0.2186 \pm 0.0021)$ [Preliminary]

The measured $|V_{us}|$ values & errors are numerically almost identical using

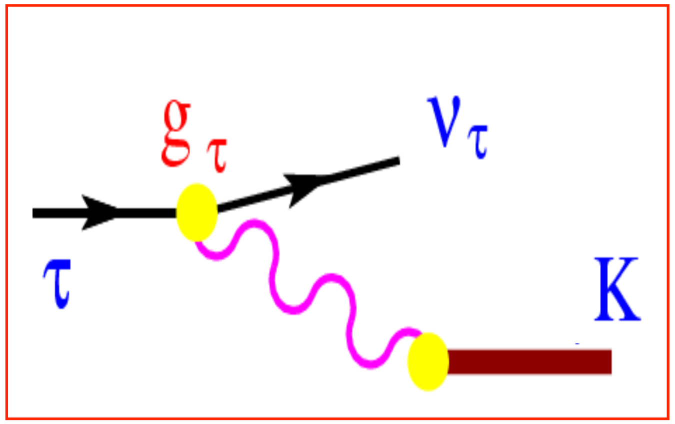
- measured $B_{had} = B_{non\text{-}strange} + B_s$ from unity non-constrained τ BR fit, OR
- $B_{had} = 1 - (1 + f_\mu/f_e)\, B_e^{univ}$ from unity constrained τ BR fit

This is because error on R_{had} feeds to error on $R_{non\text{-}strange}$ in calculation of $|V_{us}|$

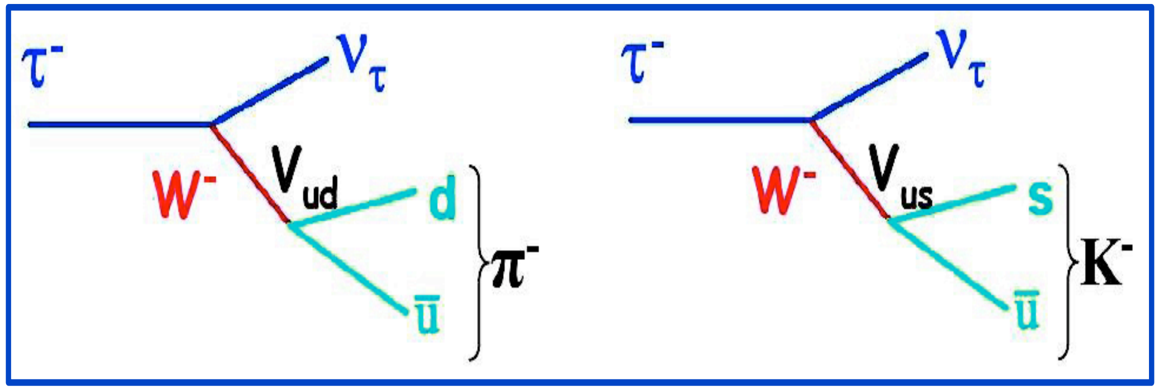
In both cases, R_{had} is normalized using B_e^{univ}

Dominant contribution to error on $|V_{us}|$ comes from error on the measured $R_{strange}$.
δR_{theory} contributes to 47% of the relative error on $|V_{us}|$.

$|V_{us}|$ from exclusive τ decays



$$B(\tau^- \to K^- \nu_\tau) = \frac{G_F^2 f_K^2 |V_{us}|^2 m_\tau^3 \tau_\tau}{16\pi\hbar} \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}$$



$$\frac{B(\tau \to K^- \nu_\tau)}{B(\tau \to \pi^- \nu_\tau)} = \frac{f_K^2 |V_{us}|^2}{f_\pi^2 |V_{ud}|^2} \frac{(1 - m_K^2/m_\tau^2)^2}{(1 - m_\pi^2/m_\tau^2)^2} R_{\tau K/\tau\pi}$$

- Independent of convergence of OPE, as electroweak corrections cancel
- Radiative corrections $S_{EW} = 1.02010 \pm 0.00030$ [Erler 2004]
- Long Distance effects ($R_{\tau K/\tau\pi}$) known [Decker & Finkmeier 1995, Marciano 2004]
- All non-perturbative QCD effects encapsulated as ratio of meson decay constants:

$f_K/f_\pi = 1.193 \pm 0.003$, $f_K = 155.6 \pm 0.4$ MeV [FLAG 2016 Lattice Averages]

Summary of |Vus| determinations

from kaon decays

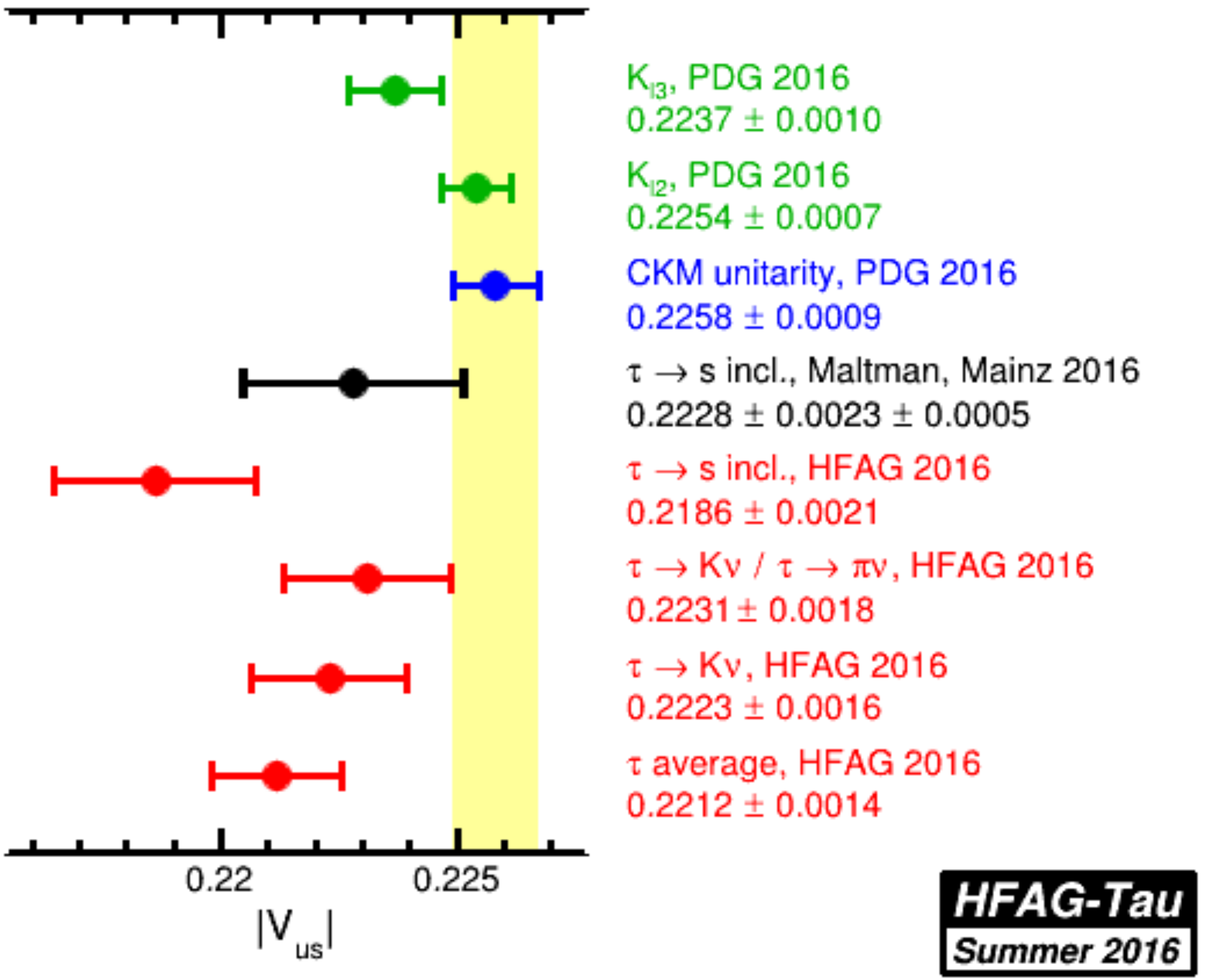
- $\Gamma(K \to \pi \ell \bar{\nu}_\ell[\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{\rm EW}^K \left(|V_{us}| f_+^{K\pi}(0)\right)^2 I_K^\ell \left(1 + \delta_{\rm EM}^{K\ell} + \delta_{\rm SU(2)}^{K\pi}\right)^2$
- $\frac{\Gamma(K^\pm \to \ell^\pm \nu)}{\Gamma(\pi^\pm \to \ell^\pm \nu)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_K^2}{f_\pi^2} \frac{m_K(1 - m_\ell^2/m_K^2)^2}{m_\pi(1 - m_\ell^2/m_\pi^2)^2} (1 + \delta_{\rm EM})$

from tau decays

- $\frac{R(\tau \to X_{\textbf{strange}})}{|V_{us}|^2} - \frac{R(\tau \to X_{\textbf{non-strange}})}{|V_{ud}|^2} = \delta R_{\tau,\textbf{SU3 breaking}},$ "tau inclusive"

  $[R(\tau \to X) = \Gamma(\tau \to X)/\Gamma(\tau \to e\nu\bar{\nu})]$
- $\frac{\mathcal{B}(\tau^- \to K^- \nu_\tau)}{\mathcal{B}(\tau^- \to \pi^- \nu_\tau)} = \frac{f_K^2 |V_{us}|^2}{f_\pi^2 |V_{ud}|^2} \frac{\left(1 - m_K^2/m_\tau^2\right)^2}{\left(1 - m_\pi^2/m_\tau^2\right)^2} \frac{r_{\textbf{LD}}(\tau^- \to K^- \nu_\tau)}{r_{\textbf{LD}}(\tau^- \to \pi^- \nu_\tau)}$
- $\mathcal{B}(\tau^- \to K^- \nu_\tau) = \frac{G_F^2 f_K^2 |V_{us}|^2 m_\tau^3 \tau_\tau}{16\pi\hbar} \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}^{\tau K}$

Summary of $|V_{us}|$ results



[Preliminary]

Summary

- $|V_{us}|$ has been measured using inclusive and exclusive tau decays.
- HFAG-PDG fit to branching ratio are being finalized for 2016.
- Improved $|V_{us}|$ will require strange spectral functions.
- Lattice calculations providing stable QCD results.
- Long standing discrepancy from CKM unitarity resolved using Lattice calculations in formulation using weighted strange spectral moments. Updated $|V_{us}|$ from this method using HFAG 2016 fit inputs are currently under progress.