CKM2016

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Experimental determination of V_{us} from kaon decays

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V_{us}, CKM unitarity, gauge universality

Standard-model coupling of quarks and leptons to W:

$$\frac{g}{\sqrt{2}} W_{\alpha}^{+} \left(\overline{\mathbf{U}}_L \mathbf{V}_{\mathrm{CKM}} \gamma^{\alpha} \mathbf{D}_L + \overline{e}_L \gamma^{\alpha} \nu_{e\,L} + \overline{\mu}_L \gamma^{\alpha} \nu_{\mu\,L} + \overline{\tau}_L \gamma^{\alpha} \nu_{\tau\,L}\right) \quad + \quad \text{h.c.}$$

Single gauge coupling

Unitary matrix

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1$$

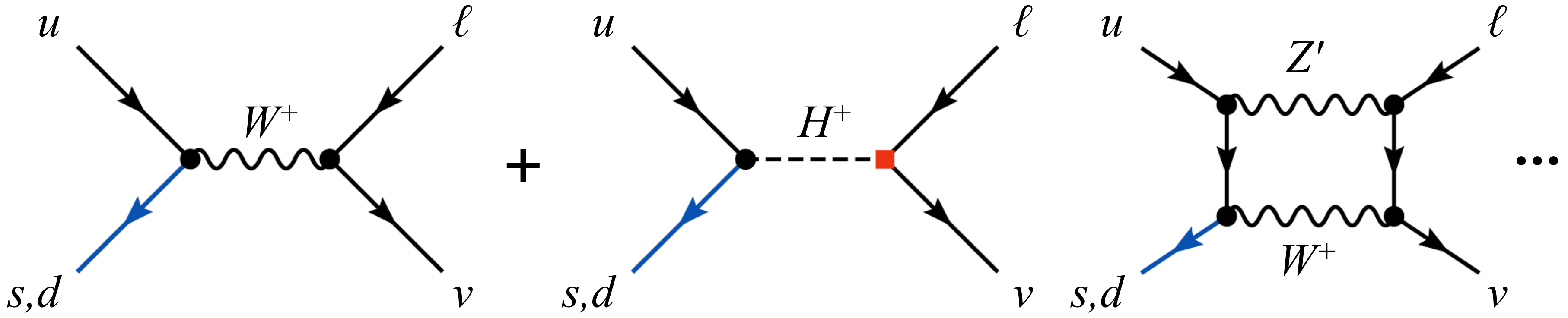
$\approx 2\times10^{-5}$

Most precise test of CKM unitarity

Universality: Is G_F from μ decay equal to G_F from π, K, nuclear β decay?

$$G_{\mu}^{2} = (g_{\mu} g_{e})^{2} / M_{W}^{4} \quad \overset{?}{=} \quad G_{\mathrm{CKM}}^{2} = (g_{q} g_{\ell})^{2} \left(|V_{ud}|^{2} + |V_{us}|^{2}\right) / M_{W}^{4}$$

Physics beyond the Standard Model can break gauge universality:



V_{us} from kaon decays: A modern history

→ 2002 (2004 PDG)	**Old $K_{\ell 3}$ data give 1 – $|V_{ud}|^2$ – $|V_{us}|^2$ = 0.0035(15)** A 2.3σ hint of unitarity violation?
2003	**BNL 865 measures BR($K^+ \to \pi^0 e^+ \nu$) = 5.13(10)%** Value for V_{us} consistent with unitarity
2004-2008 (mostly)	**Many new measurements from KTeV, ISTRA+, KLOE, NA48** • **BRs, lifetimes, form-factor slopes** • **Much higher statistics** than older measurements • Importance of **radiative corrections** • Proper reporting of **correlations** between measurements
2008- beyond	**Much progress on hadronic constants from lattice QCD** **Value of V_{us} used in precision tests of the Standard Model**
2016?	**New wave of $K_{\ell 3}$ measurements imminent?** **NA62, OKA, KLOE-2, LHCb, TREK…**

Experiment, theory, and evaluation

V_{us} from
$K_{\ell 3}$ & $K_{\ell 2}$

- ~100 measurements of ~10 experimental parameters
- 50+ (and counting!) lattice results for 2 hadronic matrix elements
- Radiative and SU(2)-breaking corrections, ChPT results, etc.

FlaviA net **Kaon WG**

2006-2010 (EU 6FP)

Experimental averages, fits, etc

Selection of results (experiments, corrections)

Evaluation, discussion and intepretation

Final report: EPJC 69 (2010) 399

This talk is my attempt at an update to 2016

Corresponding effort to synthesize results from **lattice QCD:**

Flavor Lattice Averaging Group (FLAG):

http://itpwiki.unibe.ch/flag

Participation by all major lattice collaborations

Biannual review of lattice results for π, K, B, D physics

2013 review: EPJC 74 (2014) 2890

2016 review: arXiv:1607.00299

Determination of V_{us} from $K_{\ell 3}$ data

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\mathrm{EW}} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \times I_{K\ell}(\lambda_{K\ell}) \left(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\mathrm{EM}}\right)$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

$C_K{}^2$	1/2 for K^+, 1 for K^0
S_{EW}	Universal SD EW correction (1.0232)

Inputs from experiment:

$\Gamma(K_{\ell 3(\gamma)})$	Rates with well-determined treatment of radiative decays: • Branching ratios • Kaon lifetimes
$I_{K\ell}(\{\lambda\}_{K\ell})$	Integral of form factor over phase space: λs parameterize evolution in t • K_{e3}: Only λ_+ (or λ_+', λ_+'') • $K_{\mu 3}$: Need λ_+ and λ_0

Inputs from theory:

$f_+^{K^0\pi^-}(0)$	Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)
$\Delta_K{}^{SU(2)}$	Form-factor correction for $SU(2)$ breaking
$\Delta_{K\ell}{}^{\mathrm{EM}}$	Form-factor correction for long-distance EM effects

Modern experimental data for V_{us} from $K_{\ell 3}$

Experiment	Measurement	Year
BNL865	BR($K^+ \to \pi^0{}_{\rm D} e^+ \nu$)/BR($K^+ \to \pi^0{}_{\rm D} X^+$)	2003
KTeV	$\tau(K_S)$	2003
	BR(K_{Le3}), BR($K_{L\mu3}$), $\lambda_+(K_{Le3})$, $\lambda_{+,0}(K_{L\mu3})$	2004
ISTRA+	$\lambda_+(K^-{}_{e3})$, $\lambda_{+,0}(K^-{}_{e3})$	2004
KLOE	$\tau(K_L)$	2005
	BR(K_{Le3}), BR($K_{L\mu3}$), BR(K_{Se3}), $\lambda_+(K_{Le3})$	2006
	$\lambda_{+,0}(K_{L\mu3})$	2007
	$\tau(K^\pm)$, BR(K_{Le3}), BR($K_{L\mu3}$)	2008
NA48	$\tau(K_S)$	2002
	BR(K_{Le3}/2 tracks), $\lambda_+(K_{Le3})$	2004
	$\Gamma(K_{Se3}/K_{Le3})$, $\lambda_{+,0}(K_{L\mu3})$	2007
NA48/2	BR($K^+{}_{e3}/\pi^+\pi^0$), BR($K+_{\mu3}/\pi^+\pi^0$)	2007

Above data set used for 2010 FlaviaNet review (fits, averages, etc.)

Updated fit to K_S rate data

6 input measurements:

KLOE BR $\pi^0\pi^0/\pi^+\pi^-$

KLOE BR $\pi e\nu/\pi^+\pi^-$

NA48 $\Gamma(K_S \to \pi e\nu)/\Gamma(K_L \to \pi e\nu)$, τ_S

KLOE '11 τ_S

KTeV '11 τ_S

2 constraints:

- Σ **BR = 1**
- **BR(K_{e3})/BR($K_{\mu3}$) = 0.66492(137)**

  From ratio of phase-space integrals from current fit to dispersive $K_{\ell3}$ form factor parameters

Parameter	**Value**
BR($\pi^+\pi^-(\gamma)$)	**69.20(5)%**
BR($\pi^0\pi^0$)	**30.69(5)%**
BR(K_{e3})	**$7.05(8) \times 10^{-4}$**
BR($K_{\mu3}$)	**$4.69(6) \times 10^{-4}$**
τ_S	**89.58(4) ps**

χ^2/ndf = 0.20/3 (Prob = 98%)

ρ(BR($\pi^+\pi^-$), BR($\pi^0\pi^0$)) = –0.998

Little freedom in fit

Largest effect of **2011 τ_S data:**

FlaviaNet 2010
τ_S = 89.59(6) ps

Update
τ_S = 89.58(4) ps

Updated fit to K_L rate data

21 input measurements:

- **5 KTeV** ratios
- **NA48** BR(K_{e3}/2 track)
- **4 KLOE** BRs
  with dependence on τ_L
- **KLOE**, **NA48** BR($\pi^+\pi^-/K_{\ell3}$)
- **KLOE**, **NA48** BR($\gamma\gamma/3\pi^0$)
- **BR($2\pi^0/\pi^+\pi^-$) from K_S fit, Re ε'/ε**
- **KLOE** τ_L from $3\pi^0$
- **Vosburgh '72** τ_L
- **KTeV** BR($\pi^+\pi^-\gamma/\pi^+\pi^-(\gamma)$)
- **E731, 2 KTeV** BR($\pi^+\pi^-\gamma_{\text{DE}}/\pi^+\pi^-\gamma$)

1 constraint: Σ BR = 1

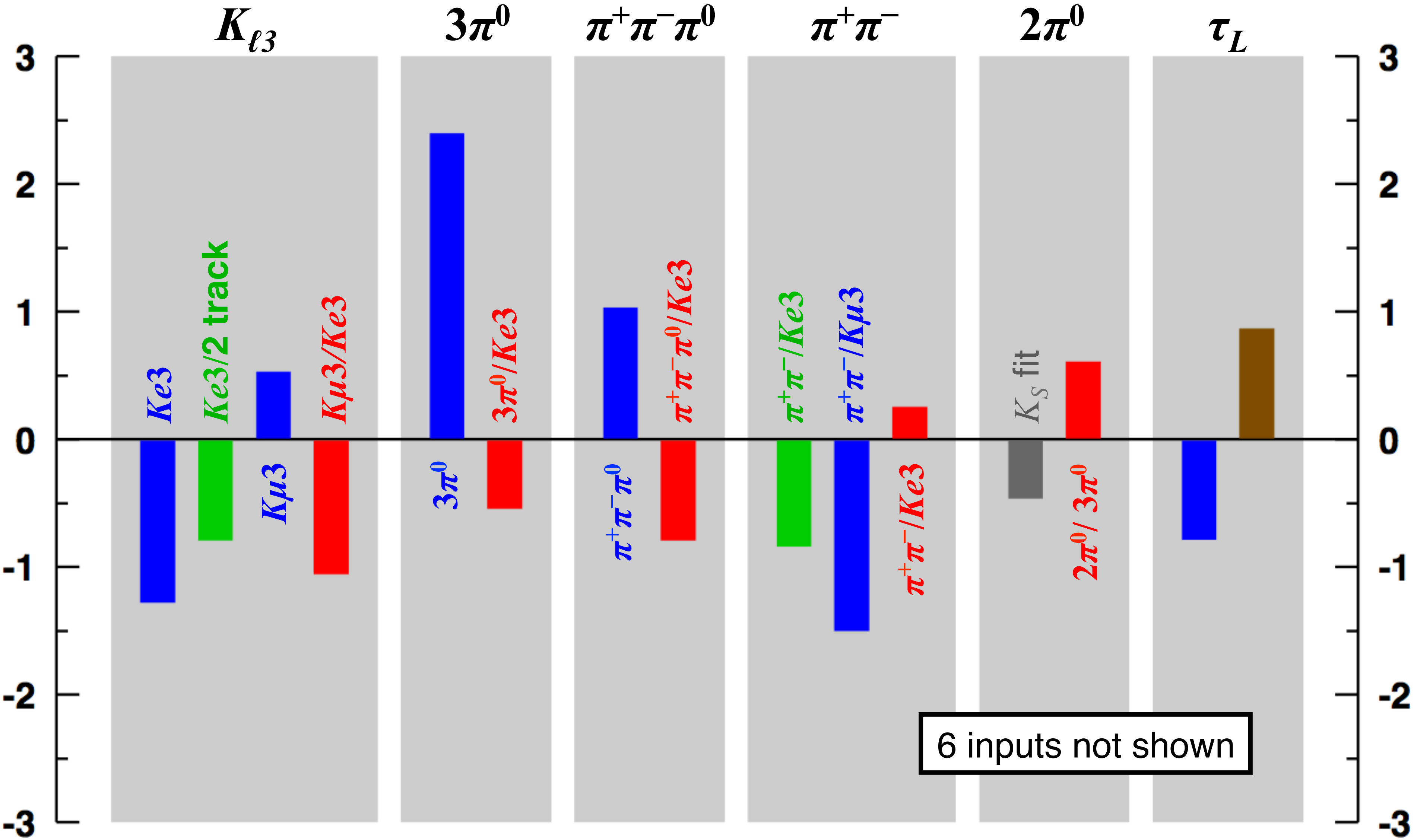
Parameter	Value	S
BR(K_{e3})	0.4056(9)	1.3
BR($K_{\mu3}$)	0.2704(10)	1.5
BR($3\pi^0$)	0.1952(9)	1.2
BR($\pi^+\pi^-\pi^0$)	0.1254(6)	1.3
BR($\pi^+\pi^-(\gamma_{\text{IB}})$)	$1.967(7) \times 10^{-3}$	1.1
BR($\pi^+\pi^-\gamma$)	$4.15(9) \times 10^{-5}$	1.6
BR($\pi^+\pi^-\gamma_{\text{DE}}$)	$2.84(8) \times 10^{-5}$	1.3
BR($2\pi^0$)	$8.65(4) \times 10^{-4}$	1.4
BR($\gamma\gamma$)	$5.47(4) \times 10^{-4}$	1.1
τ_L	51.16(21) ns	1.1

χ^2/ndf = 19.8/12 (Prob = 7.0%)

Essentially same result as 2010 fit

Current PDG ('09): **37.4/17** (0.30%)

Comparison: K_L fit result vs. input data



Measurement pulls for KTeV KLOE NA48 K_S fit Vosburgh '72

Updates: $K^\pm$ BRs and lifetimes

KLOE-2
PLB 738 (2014)

BR($\pi^+\pi^+\pi^-$) = 0.05565(31)(25) (0.7%)

- **No good measurements of BR($\pi^+\pi^+\pi^-$) in 2010 fit**
- Reconstruct 2 tracks in small fiducial volume near interaction region; evaluate missing mass for 3rd track
- Fully inclusive of radiation, but radiative corrections handled differently from other KLOE measurements
- Significant impact on value of BR($\mu\nu$) from fit
  Correlation between BR($\mu\nu$), BR($\pi^+\pi^+\pi^-$) = –0.75

ISTRA+
PAN 77 (2014)

BR($K^-_{e3}/\pi^-\pi^0$) = 0.2423(15)(37) (1.6%)

- Claimed to be fully inclusive for $K_{e3\gamma}$
  - No mention of radiative corrections
  - Many cuts, mainly topological
  - 3 different selections, at least 1 may be largely inclusive
- Included in PDG ’15 fit
- **Treated as preliminary here (not in $K^\pm$ BR fit)**

Updated fit to $K^{\pm}$ rate data

17 input measurements:

- 3 old τ values in PDG
- KLOE τ
- KLOE BR $\mu\nu$, $\pi\pi^0$
- KLOE BR K_{e3}, $K_{\mu3}$
  - with dependence on τ
- NA48/2 BR $K_{e3}/\pi\pi^0$, $K_{\mu3}/\pi\pi^0$
- E865 BR $K_{e3}/K\mathrm{Dal}$
- 3 old BR $\pi\pi^0/\mu\nu$
- KEK-246 $K_{\mu3}/K_{e3}$
- KLOE BR $\pi\pi\pi$, $\pi\pi^0\pi^0$
- (Bisi ’65 BR $\pi\pi^0\pi^0/\pi\pi\pi$ removed)

1 constraint: Σ BR = 1

Much more selective than PDG fit
PDG ’16: 35 inputs, 8 parameters

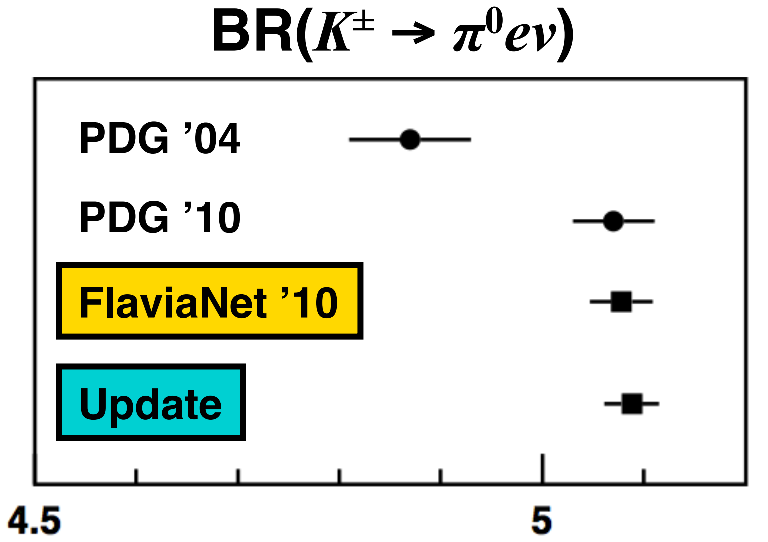
Parameter	Value	S
BR($\mu\nu$)	63.58(11)%	1.1
BR($\pi\pi^0$)	20.64(7)%	1.1
BR($\pi\pi\pi$)	5.56(4)%	1.0
BR(K_{e3})	5.088(27)%	1.2
BR($K_{\mu3}$)	3.366(30)%	1.9
BR($\pi\pi^0\pi^0$)	1.764(25)%	1.0
$\tau_{\pm}$	12.384(15) ns	1.2

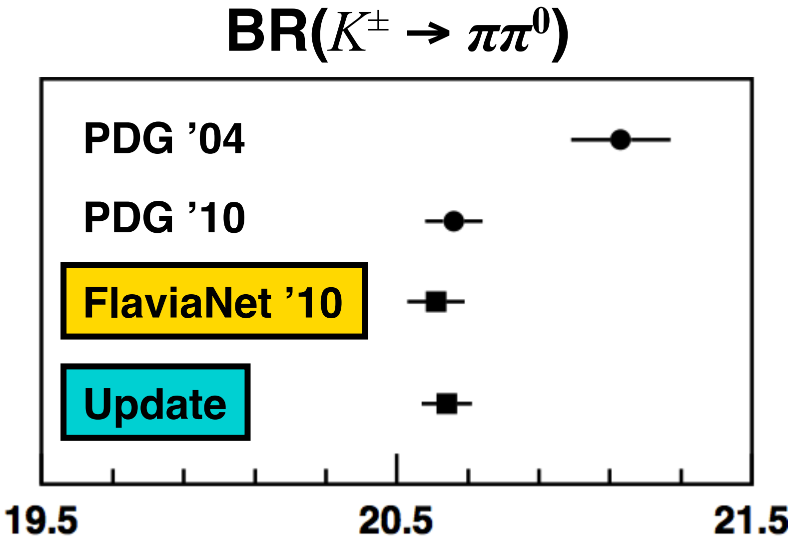
χ^2/ndf = 25.5/11 (Prob = 0.78%)
compare PDG ’16: 53/28 (0.26%)

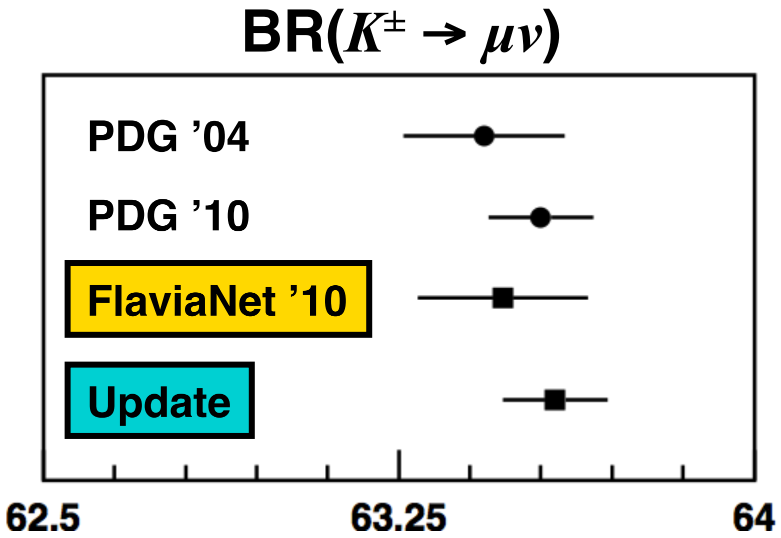
With **ISTRA+ ’14 BR($K^-_{e3}/\pi^-\pi^0$)**

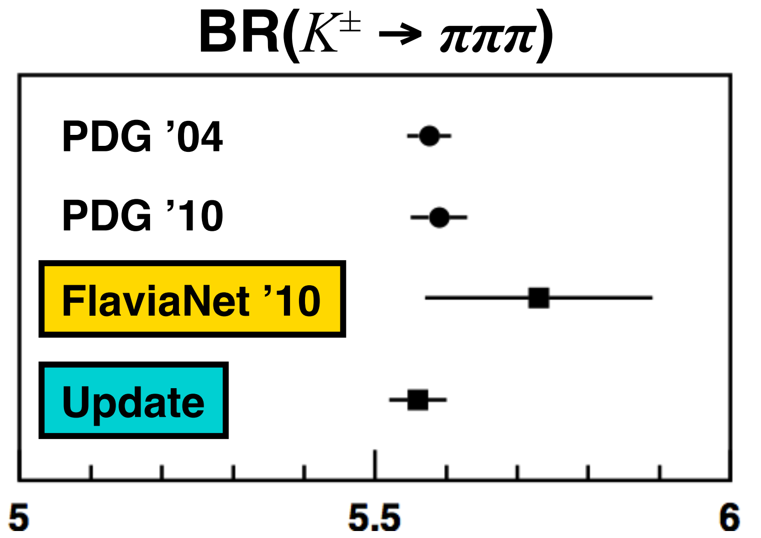
- **BR(K_{e3}) = 5.083(27)%**
- Negligible changes in other parameters, fit quality

Evolution of $K^{\pm}$ BRs

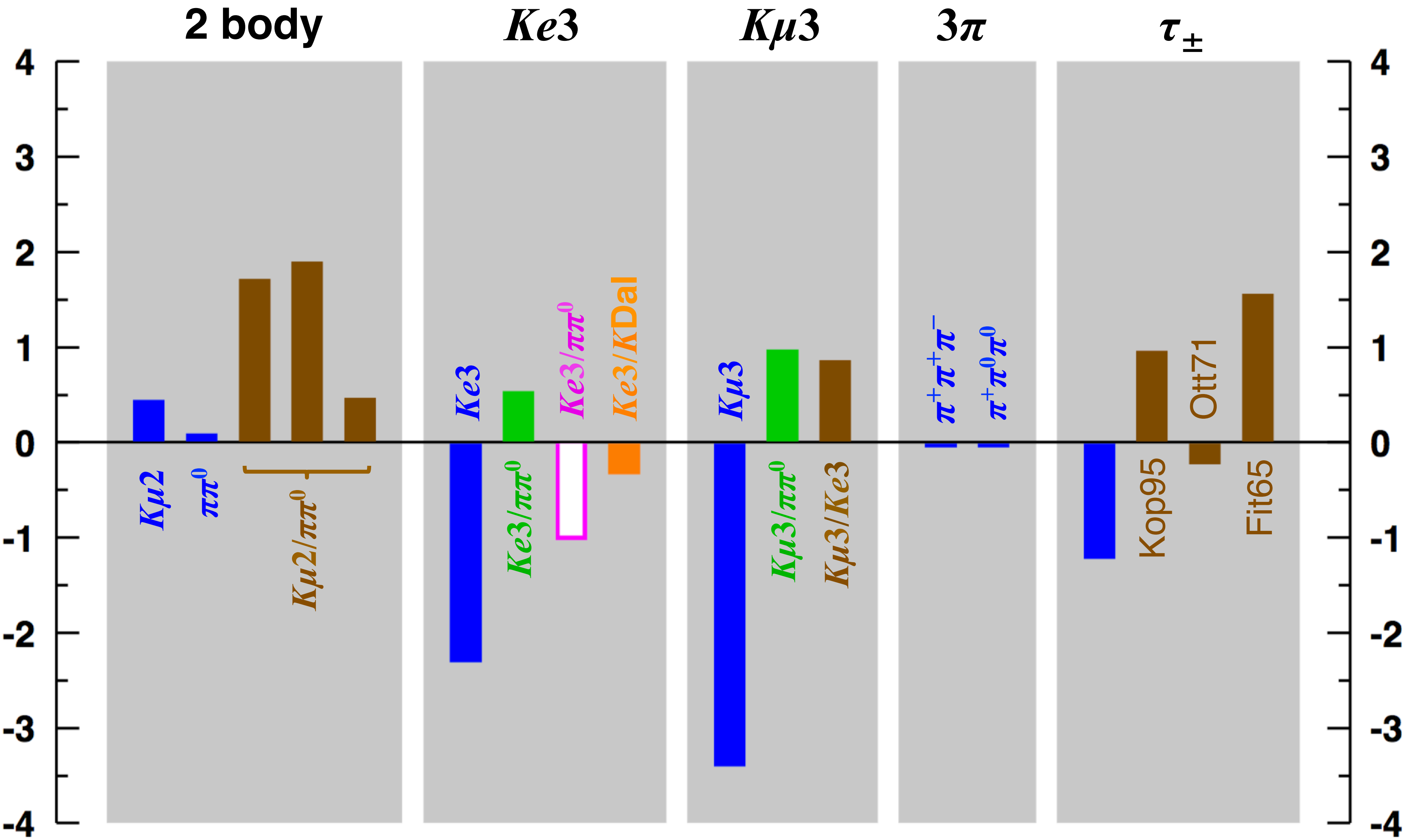






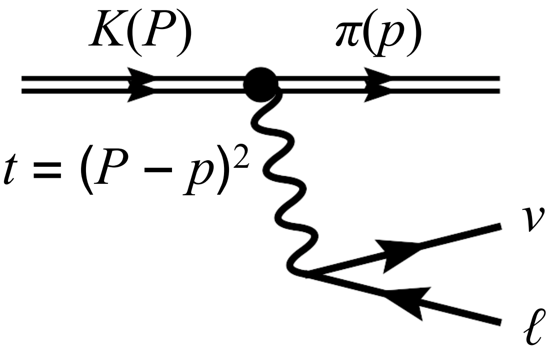


Comparison: $K^{\pm}$ fit result vs. input data



Measurement pulls for **KLOE** **E865** **NA48/2** **ISTRA+** **others**

$K_{\ell 3}$ form factors



Hadronic matrix element:

$$\langle \pi | J_\alpha | K \rangle = f(0) \times \left[\tilde{f}_+(t)(P+p)_\alpha + \tilde{f}_-(t)(P-p)_\alpha \right]$$

K_{e3} decays: Only **vector form factor:** $\tilde{f}_+(t)$

$K_{\mu 3}$ decays: Also need **scalar form factor:** $\tilde{f}_0(t) = \tilde{f}_+ + \tilde{f}_- \frac{t}{m_K^2 - m_\pi^2}$

For V_{us}, need integral over phase space of squared matrix element:

Parameterize form factors and fit distributions in t (or related variables)

Parameterizations based on systematic expansions

Taylor expansion:

$$\tilde{f}_{+,0}(t) = 1 + \lambda_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right)$$

$$\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right) + \lambda''_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right)^2$$

Notes:

Many parameters: λ_+', λ_+'', λ_0', λ_0''

Large correlations, unstable fits

Higher-order terms ignored

$K_{\ell 3}$ form-factor parameterizations

Parameterizations incorporating physical constraints

Pole dominance:

$$\tilde{f}_{+,0}(t) = \frac{M_{V,S}^2}{M_{V,S}^2 - t}$$

Notes:

What does M_S correspond to?

Dispersion relations:

$$\tilde{f}_{+}(t) = \exp\left[\frac{t}{m_\pi^2}(\Lambda_+ - H(t))\right]$$

$$\tilde{f}_{0}(t) = \exp\left[\frac{t}{m_K^2 - m_\pi^2}(\ln C - G(t))\right]$$

Notes:

Allows tests of ChPT & low-energy dynamics

$H(t)$, $G(t)$ evaluated from $K\pi$ scattering data and given as polynomials

Bernard et al., PRD 80 (2009)

Uncertainties from representations $H(t)$, $G(t)$ of $K\pi$ phase-shift data contribute to fit results for Λ_+, $\ln C$

- Small compared to other uncertainties for single measurements (so far)

2010 FlaviaNet analysis used average of FF parameters from dispersive fits

- Parameterization uncertainties beginning to dominate averages for Λ_+, $\ln C$

$K_{\ell 3}$ form factor data

Form-factor parameter measurements in FlaviaNet 2010 fit:

K_L: KTeV, KLOE, NA48

K^-: ISTRA+

Even if not in the original publications, all experiments have:

- Obtained results for Taylor, pole, and dispersive parameterizations
- Supplied parameter correlation coefficients

New preliminary measurements:

NA48/2
2012 preliminary

2.6 × 10^6 $K^{\pm}_{\mu 3}$
4.0 × 10^6 $K^{\pm}_{e3}$

K^+ and K^- simultaneously acquired in dedicated minimum-bias run

Taylor and pole fits only

No dispersive fits yet

OKA
2016 preliminary

3.2 × 10^6 K^{+}_{e3}

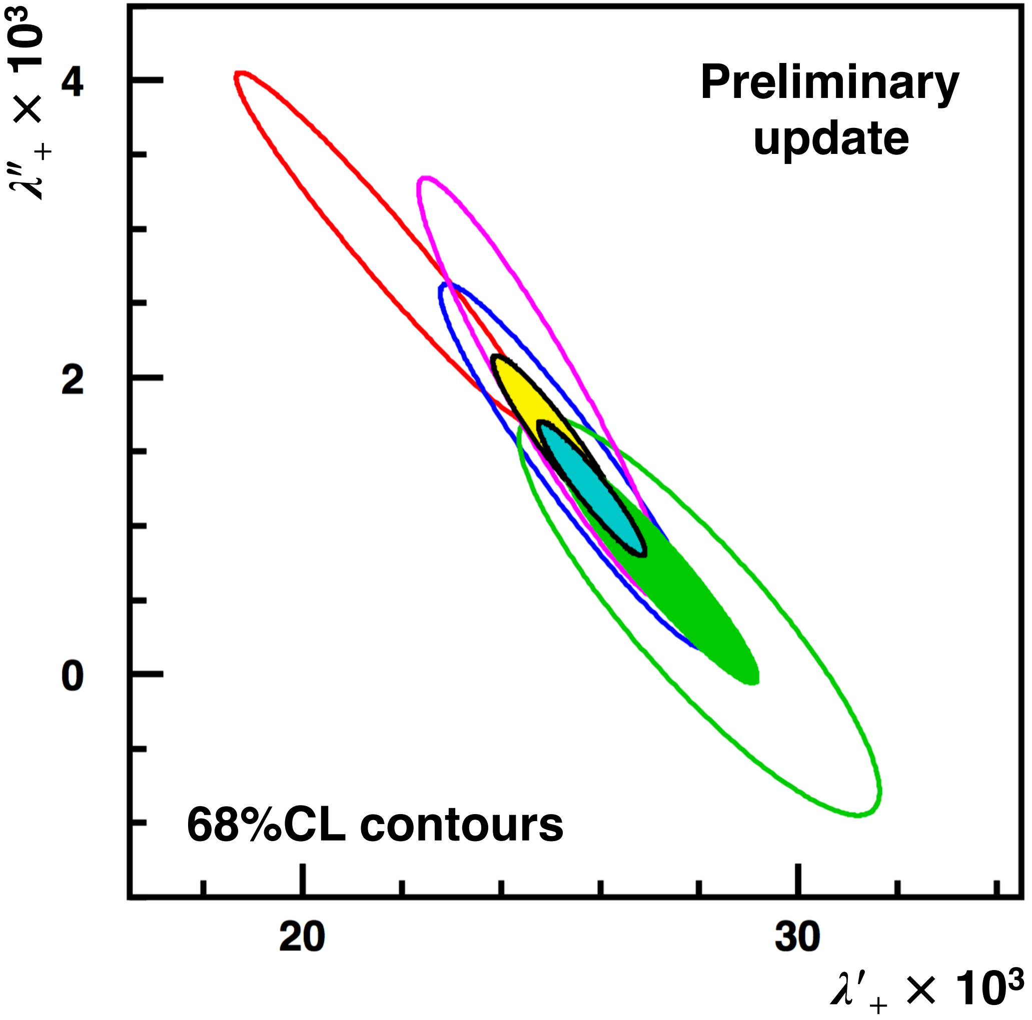
Taylor, pole, and dispersive fits

Systematics still under evaluation, e.g. from radiative corrections

Not yet included in updated K_{e3} fit

Fit to K_{e3} form-factor slopes: Update

KTeV | **KLOE** | **ISTRA+** | **NA48** | **NA48/2 ’12 prel** | **2010 fit** | **Update**



Slope parameters × 10^3

λ'_+ = **25.83 ± 0.71**

λ''_+ = **1.25 ± 0.30**

$\rho(\lambda'_+, \lambda''_+)$ = −0.941

χ^2/ndf = 7.2/8 (52%)

Quality of fit unchanged

λ'_+ increased $0.8\sigma_{2010}$

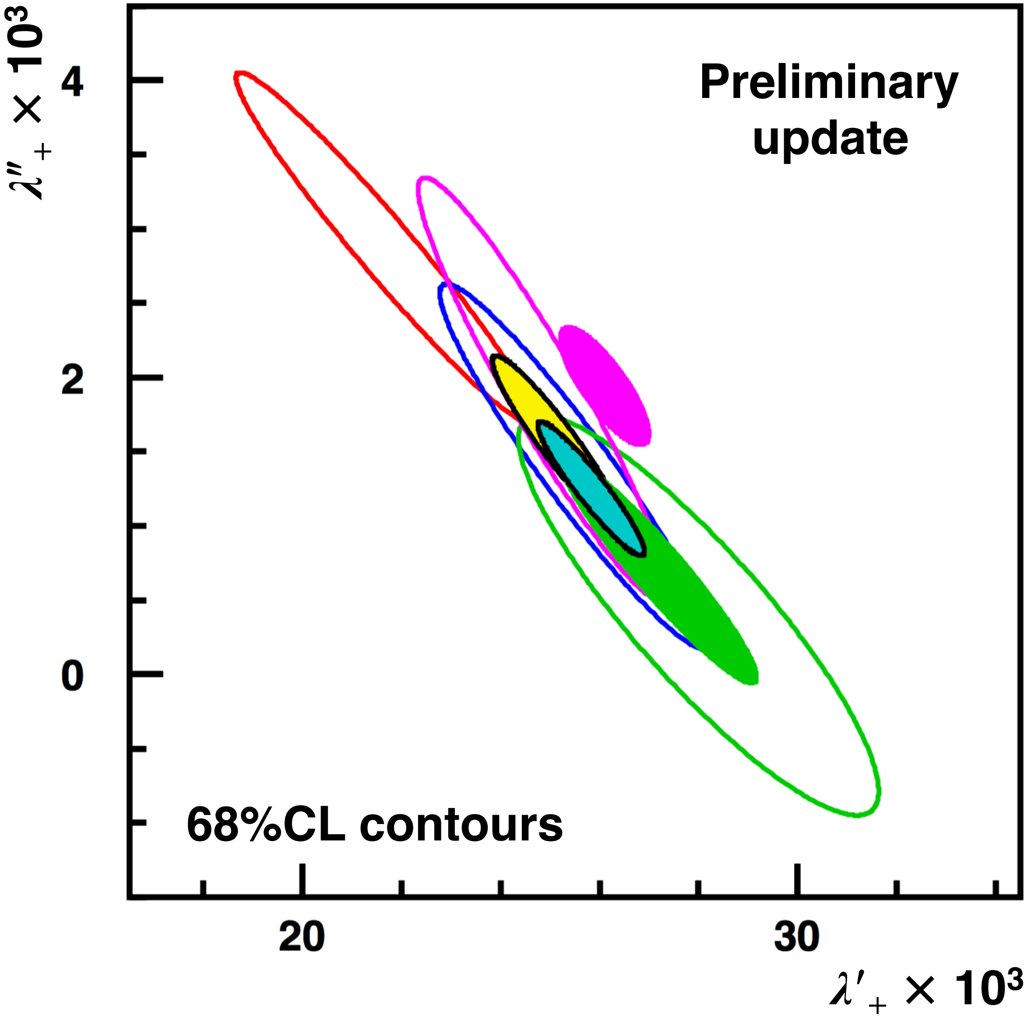
$I(K^0_{e3})$ = 0.15475(18)

$I(K^+_{e3})$ = 0.15912(19)

Increased by +0.08% (~$0.5\sigma_{2010}$)

Fit to K_{e3} form-factor slopes: Update

KTeV **KLOE** **ISTRA+** **NA48** **NA48/2 ’12 prel** **2010 fit** **Update**



Slope parameters × 10^3

$$\lambda'_+ = \mathbf{25.83 \pm 0.71}$$

$$\lambda''_+ = \mathbf{1.25 \pm 0.30}$$

$$\rho(\lambda'_+, \lambda''_+) = -0.941$$

$$\chi^2/\text{ndf} = 7.2/8\ (52\%)$$

OKA
2016 preliminary

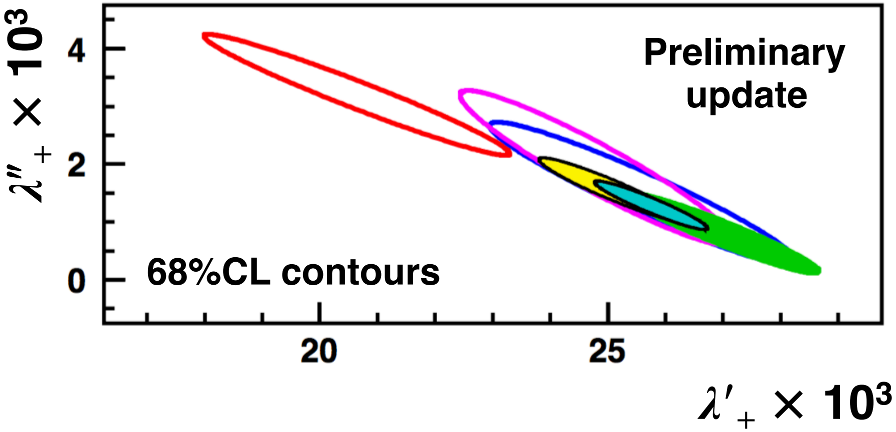
Not included in fit

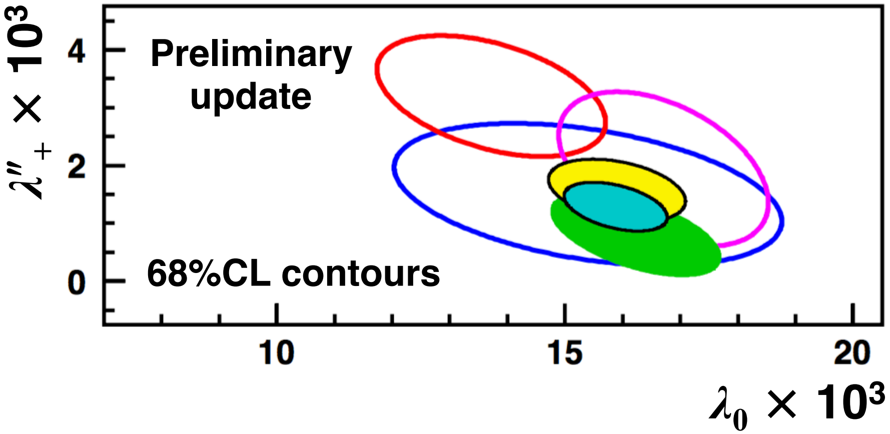
- Evaluation of systematic uncertainties in progress
- If included χ^2/ndf → 26/10 (0.36%)

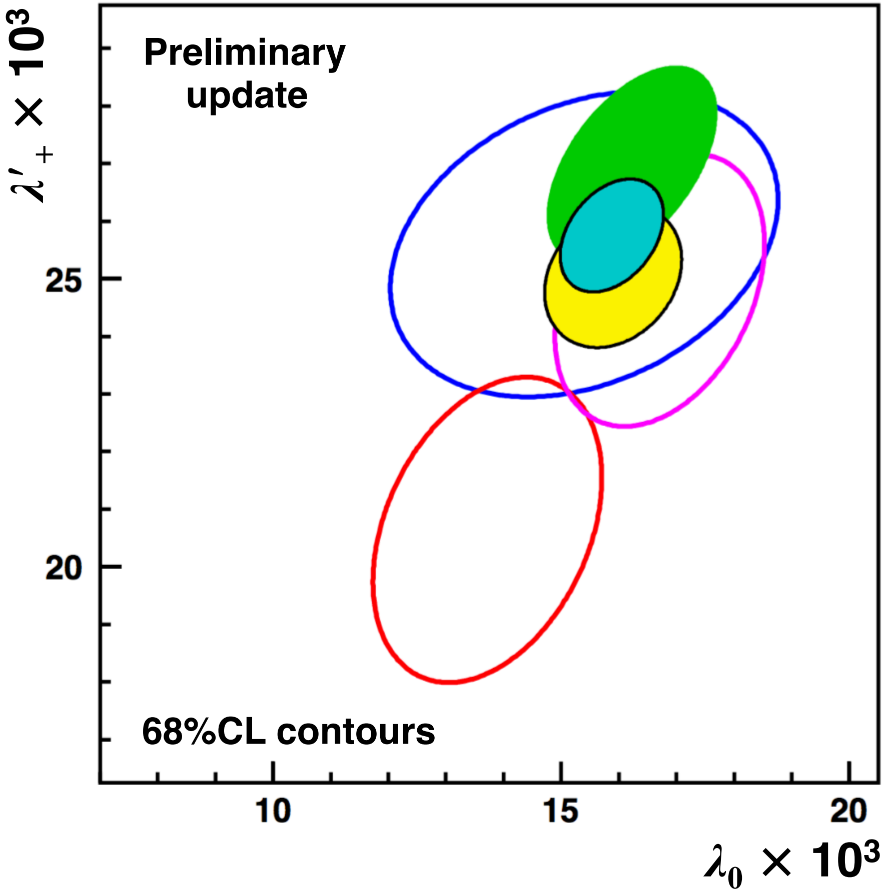
Fits to $K_{e3} + K_{\mu3}$ form-factor slopes: Update

KTeV | **KLOE** | **ISTRA+** | **NA48/2 ’12 prel** | **2010 fit** | **Update**

NA48 K_{e3} data included in fits but not shown





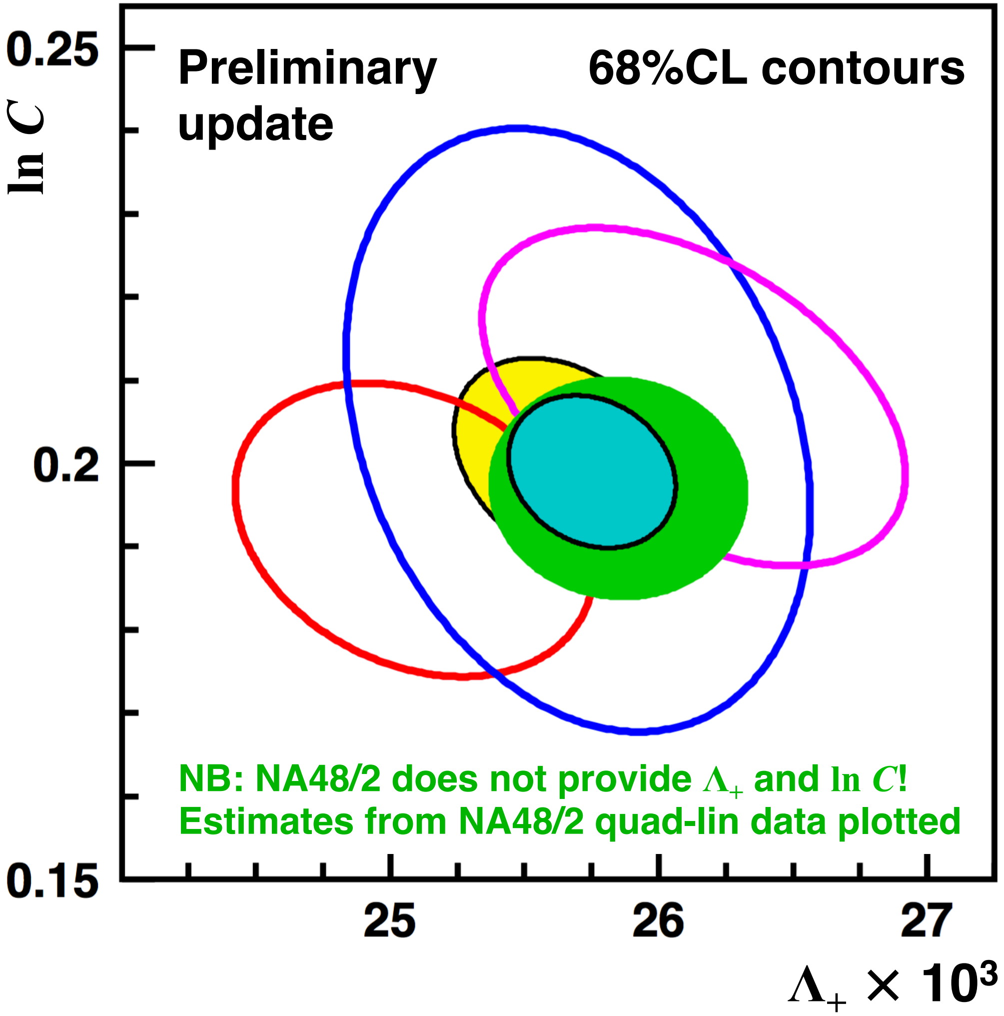


2010: χ^2 = 12.1/8 (P = 14.5%) | Update: χ^2 = 14.3/11 (P = 22.0%)

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2 '12 prel** **2010 fit** **Update**

NA48 K_{e3} data included in fits but not shown



NB: NA48/2 does not provide Λ_+ and $\ln C$!
Estimates from NA48/2 quad-lin data plotted

$\Lambda_+ \times 10^3 = \mathbf{25.75 \pm 0.36}$
$\ln C = \mathbf{0.1985(70)}$
$\rho(\Lambda_+, \ln C) = -0.202$
χ^2/ndf = 5.9/7 (55%)

Integrals

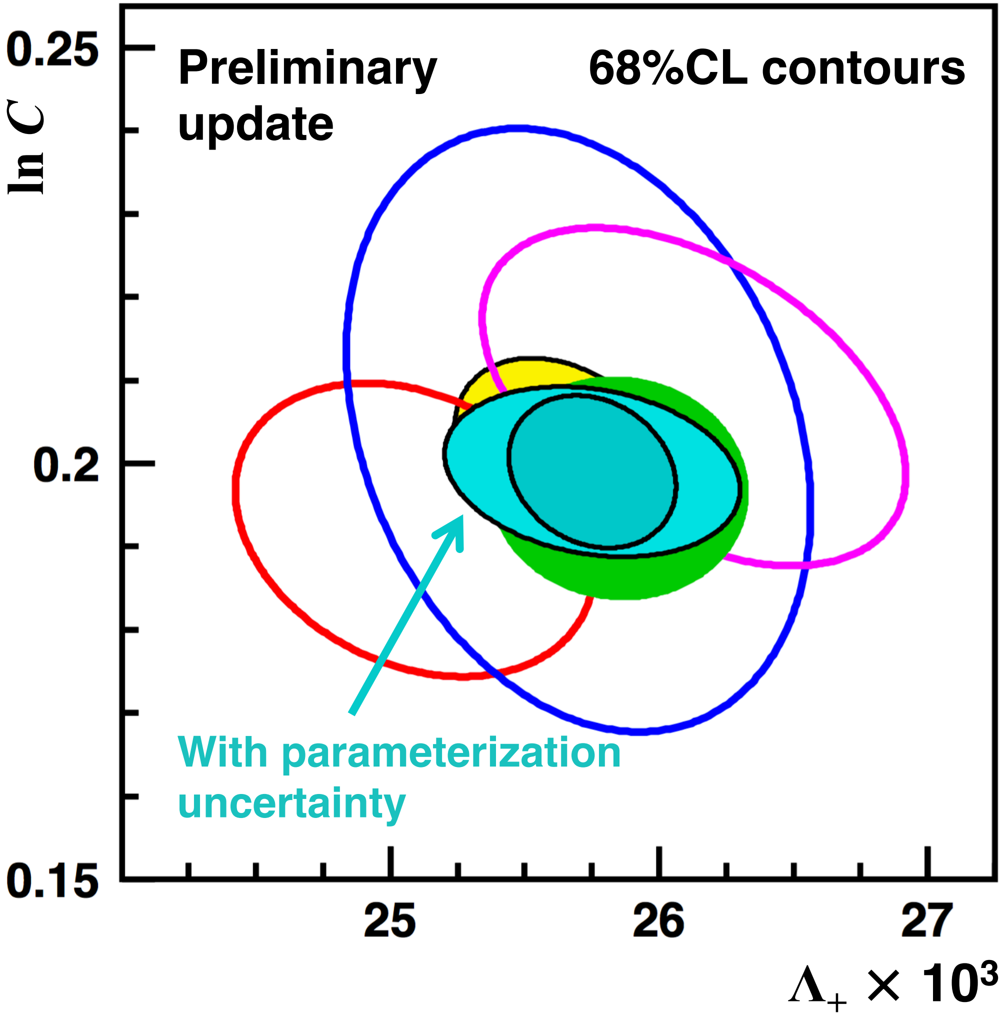
Mode	Update	2010
K^0_{e3}	**0.15481(14)**	0.15476(18)
K^+_{e3}	**0.15927(14)**	0.15922(18)
$K^0_{\mu 3}$	**0.10253(13)**	0.10253(16)
$K^+_{\mu 3}$	**0.10558(14)**	0.10559(17)

Only tiny changes in central values

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2 ’12 prel** **2010 fit** **Update**

NA48 K_{e3} data included in fits but not shown



$\Lambda_+ \times 10^3$ = **25.75 ± 0.36**

$\ln C$ = **0.1985(70)**

$\rho(\Lambda_+, \ln C)$ = −0.202

χ^2/ndf = 5.9/7 (55%)

Includes common uncertainty from $H(t)$, $G(t)$:

$\sigma_{\text{param}}(\Lambda_+) = 0.3 \times 10^{-3}$

$\sigma_{\text{param}}(\ln C) = 0.0040$

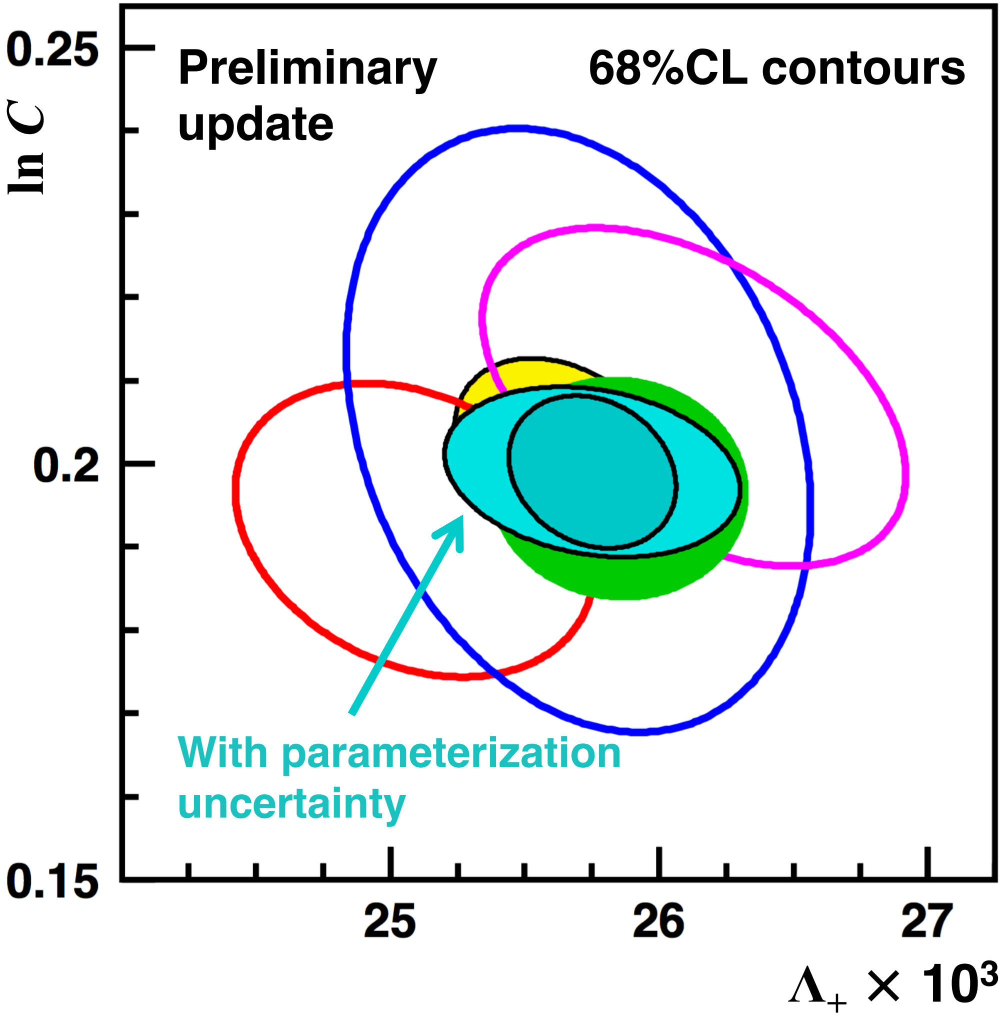
KTeV, Bernard et al. ’09

Confidence ellipses shown **without** common uncertainty

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2 ’12 prel** **2010 fit** **Update**

NA48 K_{e3} data included in fits but not shown



$$\Lambda_+ \times 10^3 = 25.75 \pm 0.36$$
$$\ln C = 0.1985(70)$$
$$\rho(\Lambda_+, \ln C) = -0.202$$
$$\chi^2/\text{ndf} = 5.9/7\ (55\%)$$

Includes common uncertainty from $H(t)$, $G(t)$

Without common uncertainty:

$\sigma(\Lambda_+)$	$(0.36 \to 0.21) \times 10^{-3}$
$\sigma(\ln C)$	$0.0070 \to 0.0058$
$\sigma(K_{e3}$ int)	$0.09\% \to 0.07\%$
$\sigma(K_{\mu 3}$ int)	$0.13\% \to 0.12\%$

$K_{\ell 3}$ data and lepton universality

For each state of kaon charge, evaluate:

$$r_{\mu e} = \frac{(R_{\mu e})_{\text{obs}}}{(R_{\mu e})_{\text{SM}}} = \frac{\Gamma_{\mu 3}}{\Gamma_{e3}} \cdot \frac{I_{e3}\,(1+\delta_{e3})}{I_{\mu 3}\,(1+\delta_{\mu 3})} = \frac{[|V_{us}|\, f_+(0)]^2_{\mu 3,\text{ obs}}}{[|V_{us}|\, f_+(0)]^2_{e3,\text{ obs}}} = \frac{g_\mu^2}{g_e^2}$$

Modes	2004 BRs*,†	Current†
K_L	1.054(14)	**1.003(5)**
$K^\pm$	1.014(12)	**0.999(9)**
Avg	1.030(9)	**1.002(5)**

← Was 0.998(9) for 2010

*Assuming current values for form-factor parameters and Δ^{EM} †K_S not included

As statement on lepton universality

Compare to other precise tests:

$\pi \to \ell\nu$ $(r_{\mu e})$ = **1.0020(19)**

PDG '16 with PIENU '15 result

$\tau \to \ell\nu\nu$ $(r_{\mu e})$ = **1.0038(28)**

HFAG '14 and Oct '16 web update

As statement on calculation of Δ^{EM}

Cirigliano et al. '08

Calculation at fixed order e^2p^2

Fully inclusive for real photons

Confirmed at per-mil level

Update LECs for SD terms?

Accuracy of $SU(2)$-breaking correction

$$\Delta^{SU(2)} \equiv \frac{f_+(0)^{K^+\pi^0}}{f_+(0)^{K^0\pi^-}} - 1$$

Strong isospin breaking

Quark mass differences, η-π^0 mixing in $K^+\pi^0$ channel

$$= \frac{3}{4}\frac{1}{Q^2}\left[\frac{\overline{M}_K^2}{\overline{M}_\pi^2} + \frac{\chi_{p^4}}{2}\left(1 + \frac{m_s}{\hat{m}}\right)\right] \qquad Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

$\chi_p{}^4$ = 0.219
NLO in strong interaction
O(e^2p^2) term $\varepsilon_{\mathrm{EM}}{}^{(4)}$ ~ 10^{-6}

= **+2.45(19)%** **Calculated using N_f = 2+1 FLAG ’16 averages:**

Q = 22.5(6)(6) M_K = 494.2(3)

$m_s/\hat{m}$ = 27.43(13)(27) M_π = 134.8(3)

Isopsin-limit meson masses

Value checked by **E. Passemar:**

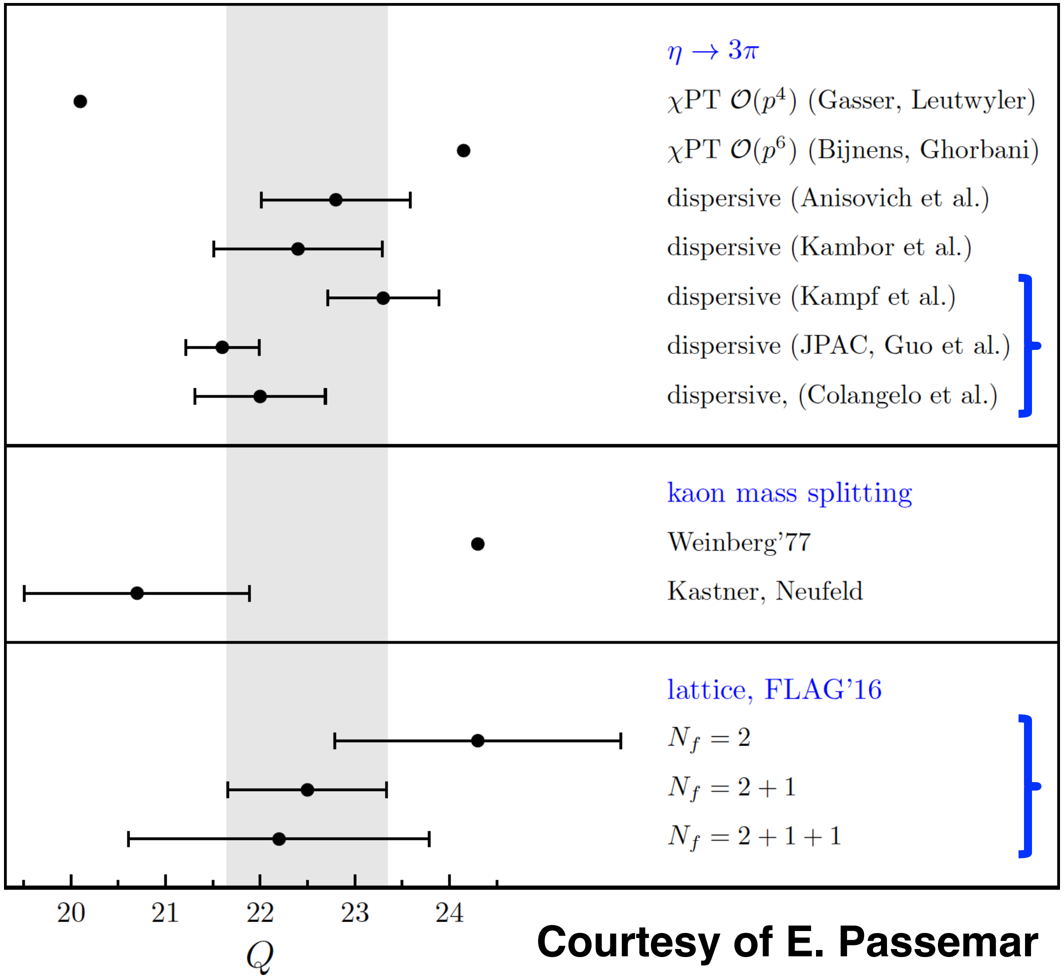
- Calculation scheme of Kastner & Neufeld ’08, Cirigliano et al. ’02
- LECs from Bijnens & Ecker ’14

Test by evaluating V_{us} from $K^\pm$ and K^0 data with **no** corrections:
Equality of V_{us} values would require **$\Delta^{SU(2)}$ = 2.82(38)%**

Accuracy of $SU(2)$-breaking correction

Previous to lattice results for Q, uncertainty on $\Delta^{SU(2)}$ was leading contributor to uncertainty on V_{us} from $K^{\pm}$ decays

Contribution to uncertainty still significant—can it be reduced?



Courtesy of E. Passemar

Continuing progress + systematic review of existing results for light-quark masses may help

Recent dispersion relation analyses of $\eta \to 3\pi$ Dalitz plot

E.g. **Colangelo et al.** 1610.03494

1.6 fb^{-1} KLOE '04-'05 data

Continuing progress on lattice

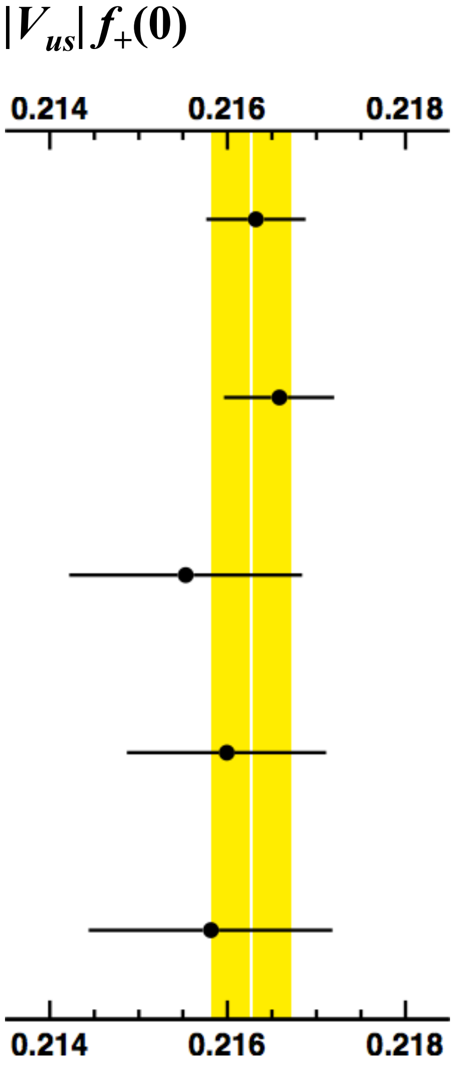
E.g. **BMW '16** PRL 117

N_f = 2+1 QCD, 5sp, m_π phys

Partially quenched QED

$Q = 23.4(4)_{\mathrm{st}}(3)_{\mathrm{sy}}(4)_{\mathrm{QED}}$

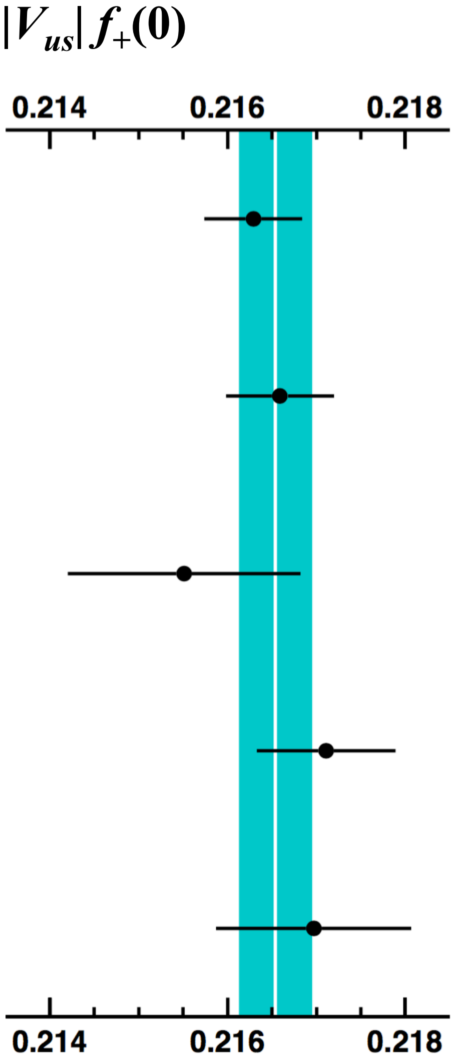
$|V_{us}| f_+(0)$ from world data: 2010



			Approx. contrib. to % err from:			
		% err	**BR**	τ	Δ	**Int**
K_Le3	**0.2163(6)**	**0.26**	0.09	0.20	0.11	0.06
$K_L\mu3$	**0.2166(6)**	**0.29**	0.15	0.18	0.11	0.08
K_Se3	**0.2155(13)**	**0.61**	0.60	0.03	0.11	0.06
$K^\pm e3$	**0.2160(11)**	**0.52**	0.31	0.09	0.40	0.06
$K^\pm\mu3$	**0.2158(14)**	**0.63**	0.47	0.08	0.39	0.08

Average: $|V_{us}| f_+(0)$ = 0.2163(5) **χ^2/ndf = 0.77/4 (94%)**

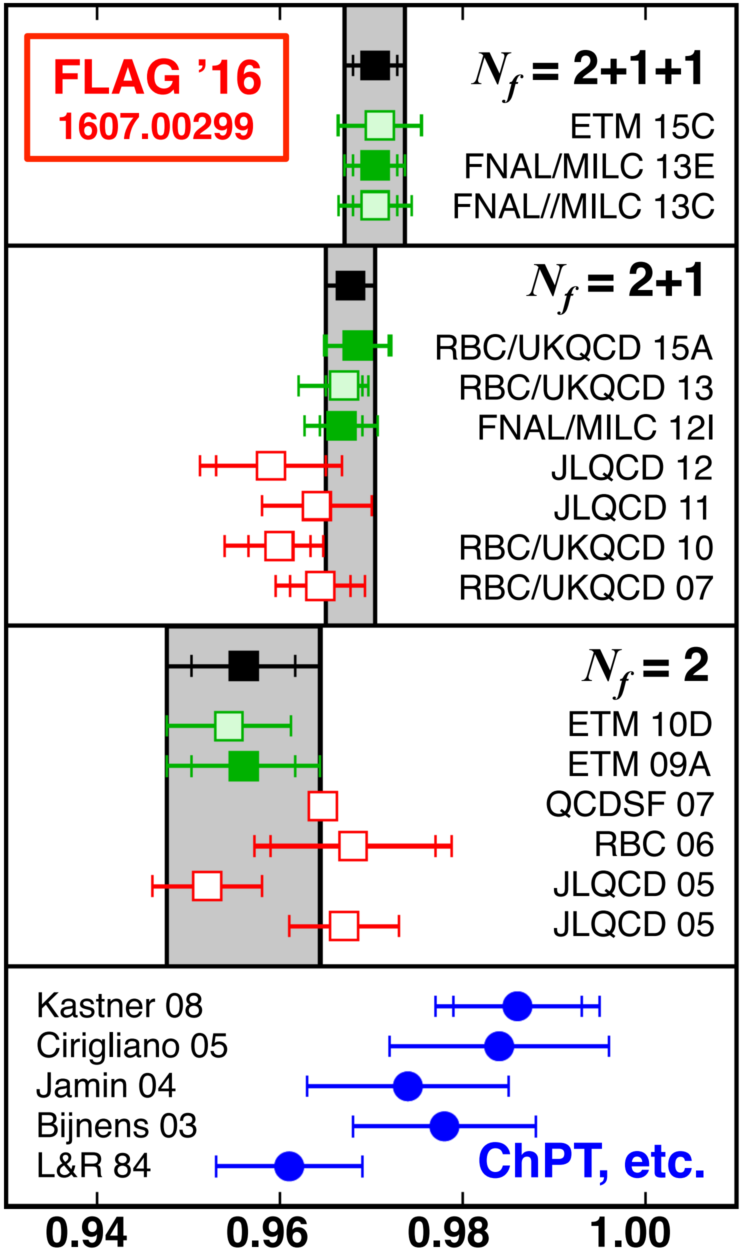
$|V_{us}| f_+(0)$ from world data: Update



			Approx. contrib. to % err from:			
		% err	**BR**	τ	Δ	**Int**
K_Le3	**0.2163(6)**	**0.25**	0.09	0.20	0.11	0.05
$K_L\mu3$	**0.2166(6)**	**0.28**	0.15	0.18	0.11	0.06
K_Se3	**0.2155(13)**	**0.61**	0.60	0.02	0.11	0.05
$K^\pm e3$	**0.2171(8)**	**0.36**	0.27	0.06	0.22	0.05
$K^\pm\mu3$	**0.2170(11)**	**0.51**	0.45	0.06	0.22	0.06

Average: $|V_{us}| f_+(0)$ = 0.21654(41) χ^2/ndf = 1.54/4 (82%)

Evaluations of $f_+(0)$



FLAG '16 averages:

N_f = 2+1 $f_+(0)$ = **0.9677(27)**
Uncorrelated average of:
RBC/UKQCD 15A: DWF, $m_\pi \rightarrow$ 139 MeV
FNAL/MILC 12I: HISQ, $m_\pi \sim$ 300 MeV

N_f = 2+1+1 $f_+(0)$ = **0.9704(32)**
FNAL/MILC 13E: HISQ, $m_\pi \rightarrow$ 135 MeV

Recent updates:

N_f = 2+1 $f_+(0)$ = **0.9636($^{+62}_{-65}$)** Lattice 15
JLQCD: Overlap, $m_\pi \rightarrow$ 300 MeV
Exact chiral symmetry

N_f = 2+1+1 $f_+(0)$ = **0.9709(46)** PRD 93 (2016)
ETM 16: TwMW, 3sp, $m_\pi \rightarrow$ 210 MeV
Full q^2 dependence of f_+, f_0

$f_+(0)$ = **0.9704(32)** Lattice 16
FNAL/MILC update 13E, expect 0.2% error

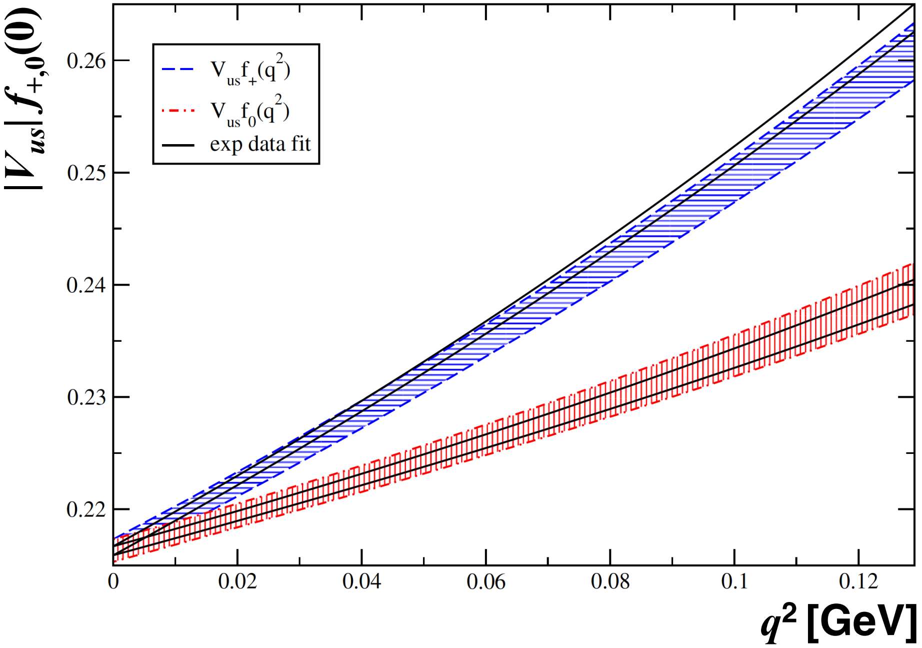
ChPT:

N_f = 2+1 $f_+(0)$ = **0.970(8)** Chiral Dynamics 15
Ecker 15: According to Bijnens 03
New LECs from Bijnens, Ecker 14

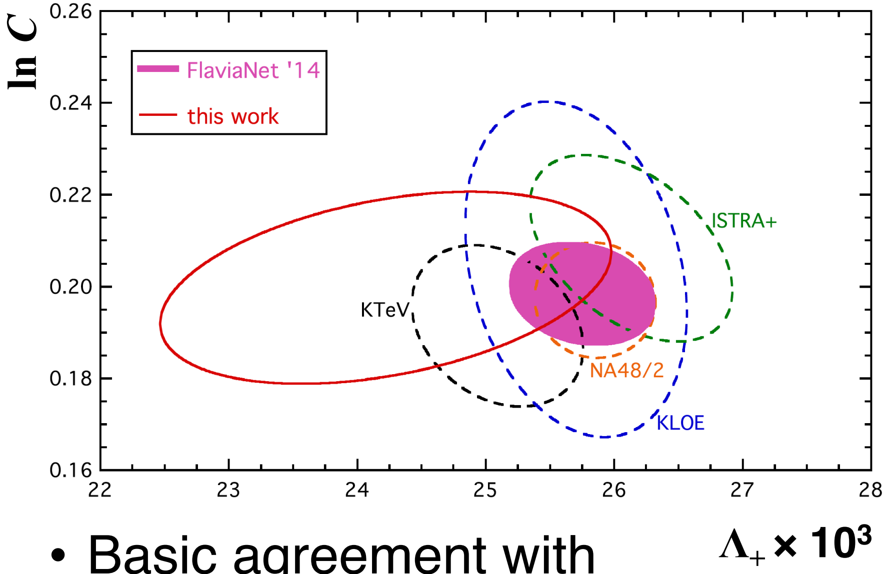
Evaluations of $f_+(0)$

ETM
PRD 93 (2016)

N_f **= 2+1+1** Twisted-mass Wilson fermions
3 lattice spacings, smallest $m_\pi \rightarrow$ 210 MeV
Results for full q^2 dependence of f_+, f_0



$f_+(0)$ = 0.9709(44)$_{\mathrm{st}}$(9)$_{\mathrm{sy}}$(11)$_{\mathrm{ext}}$



Fit synthetic data points with dispersive parameterization

Λ_+ **= 24.22(1.16) × 10^{-3}**

ln C = 0.1998(138)

$\rho(\Lambda_+, f_+(0)) = -0.228$
$\rho(\ln C, f_+(0)) = -0.719$
$\rho(\Lambda_+, \ln C) = +0.376$

- Basic agreement with experimental results
- Confirms basic correctness of lattice calculations for $f_+(0)$
- In the near future FF parameters will be obtained on lattice?

$|V_{us}|(K_{\ell 3})$ and $|V_{ud}|(0^+ \to 0^+)$: Update

Hardy, CKM ’16 preliminary
$|V_{ud}| = 0.97420(21)$

Several new measurements, including new BR, Q for ^{14}O

World data set very robust

- Small impact from new measurements

From FlaviaNet 2010 $K_{\ell 3}$ analysis

$|V_{us}| f_+(0) = 0.2163(5)$ **$|V_{us}| = 0.2254(13)$**

with $f_+(0) = 0.959(5)$ **with $|V_{ud}| = 0.97425(22)$**

$\Delta_{\text{CKM}} = +0.0000(8)$

Update with $|V_{us}| f_+(0) = 0.21654(41)$ and $|V_{ud}| = 0.97420(21)$

Choice of $f_+(0)$		V_{us}	$\Delta_{\text{CKM}} = V_{ud}^2 + V_{us}^2 - 1$	
$N_f = 2+1$	0.9677(27)	**0.2238(8)**	**–0.0009(5)**	$= -1.6\sigma$
$N_f = 2+1+1$	0.9704(32)	**0.2231(9)**	**–0.0011(6)**	$= -2.0\sigma$

Previously excellent consistency with unitarity no longer observed

Relative to 2014 slightly better agreement between $N_f = 2+1$ and $N_f = 2+1+1$

V_{us}/V_{ud} and $K_{\ell 2}$ decays

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{\mathrm{EM}}-\frac{1}{2}\delta_{SU(2)}\right)$$

Inputs from theory:

Cirigliano, Neufeld ’11

δ_{EM} = –0.0069(17)

Long-distance EM corrections

$\delta_{SU(2)}$ = –0.0043(5)(11)

Strong isospin breaking

$f_K/f_\pi \rightarrow f_{K\pm}/f_{\pi\pm}$

Lattice: f_K/f_π

Cancellation of lattice-scale uncertainties from ratio

NB: Most lattice results already corrected for $SU(2)$-breaking: $f_{K\pm}/f_{\pi\pm}$

Inputs from experiment:

Updated $K^\pm$ BR fit:

BR($K^\pm_{\mu 2(\gamma)}$) = 0.6358(11)

$\tau_{K\pm}$ = 12.384(15) ns

PDG:

BR($\pi^\pm_{\mu 2(\gamma)}$) = 0.9999

$\tau_{\pi\pm}$ = 26.033(5) ns

$|V_{us}/V_{ud}| \times f_{K\pm}/f_{\pi\pm}$ =
0.27599(37)

No $SU(2)$-breaking correction

V_{us}/V_{ud} and $K_{\ell 2}$ decays

ETM
1610.09668

Lattice ’16 preliminary
First lattice calculation of EM corrections to P_{l2} decays

- N_f = 2+1+1 Twisted-mass Wilson fermions
- 3 lattice spacings, smallest $m_\pi \to$ 220 MeV
- Quenched QED

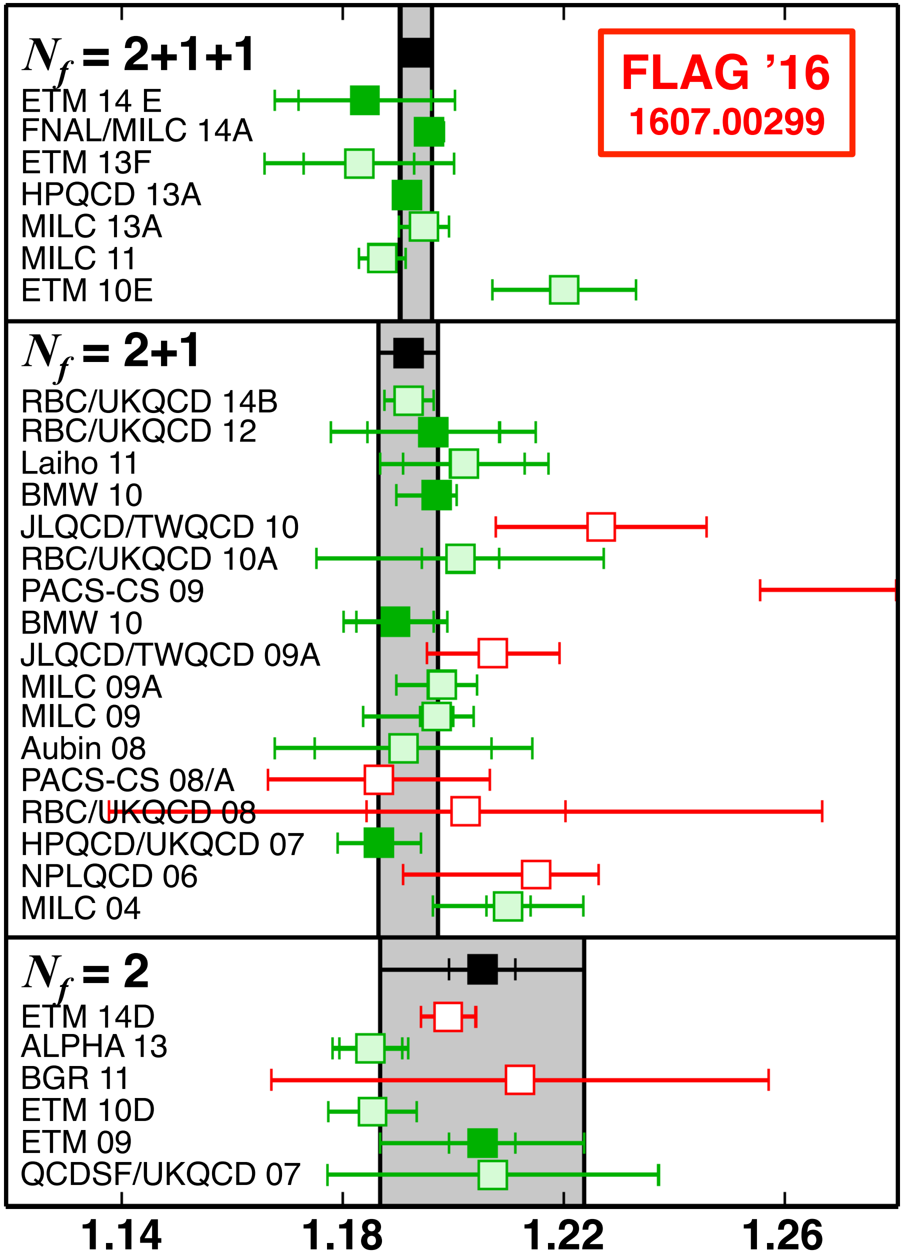
$$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0137(11)_{\mathrm{st}}(6)_{\chi}(1)_{\mathrm{FS}}(1)_{a2}$$

- Uncertainty from quenched QED not included

Compare to ChPT result from Cirigliano, Neufeld ’11:

$$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0112(21)$$

Lattice evaluations of f_K/f_π



FLAG '16 averages:

N_f = 2+1 $f_{K\pm}/f_{\pi\pm}$ = **1.192(5)**

Unchanged from FLAG '13 average

N_f = 2+1+1 $f_{K\pm}/f_{\pi\pm}$ = **1.1933(29)**

ETM 14E: TwM, 3sp, m_π = 210-450 MeV

FNAL/MILC 14A: HISQ, 4sp, m_π phys

Updates MILC 13A

HPQCD 13A: HISQ, 3sp, m_π phys,

Same ensembles as FNAL/MILC 14A

Recent updates:

N_f = 2+1 f_K/f_π = **1.1945(45)**

RBC/UKQCD '14: DWF, m_π = 139 MeV

f_K and f_π separately (isospin limit)

Recently published

$f_{K\pm}/f_{\pi\pm}$ = **1.1978(28)**

BMW '16: Clover, 5sp, m_π → 139 MeV

$|V_{us}|(K_{\ell 2})$ and $|V_{ud}|(0^+ \to 0^+)$: Update

$|V_{us}/V_{ud}| \times f_{K\pm}/f_{\pi\pm}$ = 0.27599(37) and $|V_{ud}|$ = 0.97420(21)

Choice of $f_{K\pm}/f_{\pi\pm}$		**V_{us}/V_{ud}**	**$\Delta_{\mathrm{CKM}} = V_{ud}^2[1 + (V_{us}/V_{ud})^2] - 1$**	
N_f = 2+1	1.192(5)	**0.2315(10)**	**–0.00004(59)**	= –0.06σ
N_f = 2+1+1	1.1933(29)	**0.2313(6)**	**–0.00015(48)**	= –0.3σ

$K_{\ell 2}$ results give rather better agreement with unitarity via V_{ud} than $K_{\ell 3}$ results (–2σ)

Exercise:

- Assume $|V_{ud}|$, $|V_{us}/V_{ud}| \times f_{K\pm}/f_{\pi\pm}$, and $f_{K\pm}/f_{\pi\pm}$ all correct
- In $K_{\ell 3}$ does the discrepancy arise from data or from lattice results for $f_+(0)$?

Form factors & the Callan-Treiman relation

Callan-Treiman relation:

$$\tilde{f}_0(t_{\mathrm{CT}}) = \frac{f_K}{f_\pi}\frac{1}{f_+(0)} + \Delta_{\mathrm{CT}}$$

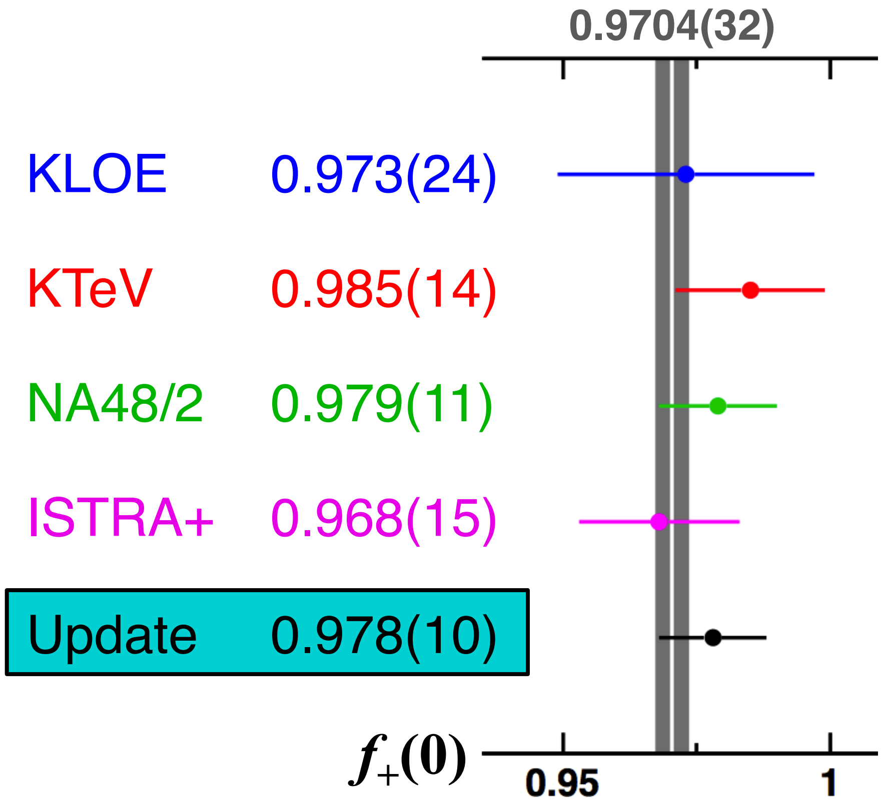
$t_{\mathrm{CT}} = m_K^2 - m_\pi^2$

$\Delta_{\mathrm{CT}} = (-3.5 \pm 0.8) \times 10^{-3}$ ~$O(m_u, m_d)$
Gasser, Leutwyler ’85

Dispersive representation: $f_0(t_{\mathrm{CT}}) \equiv C$

Use ChPT & form-factor data to test $N_f = 2+1+1$ lattice consistency:

- Use lattice reference value $f_K/f_\pi = \mathbf{1.1933(29)}$
- Obtain $f_+(0)$ corresponding to each result for $\ln C$
- Compare to lattice reference value $f_+(0) = \mathbf{0.9704(32)}$
- Basic consistency **(0.7σ)** between lattice values for f_K/f_π and $f_+(0)$ and measurements of $\ln C$
- **Uses no experimental information on decay widths**



V_{us} and CKM unitarity: All data

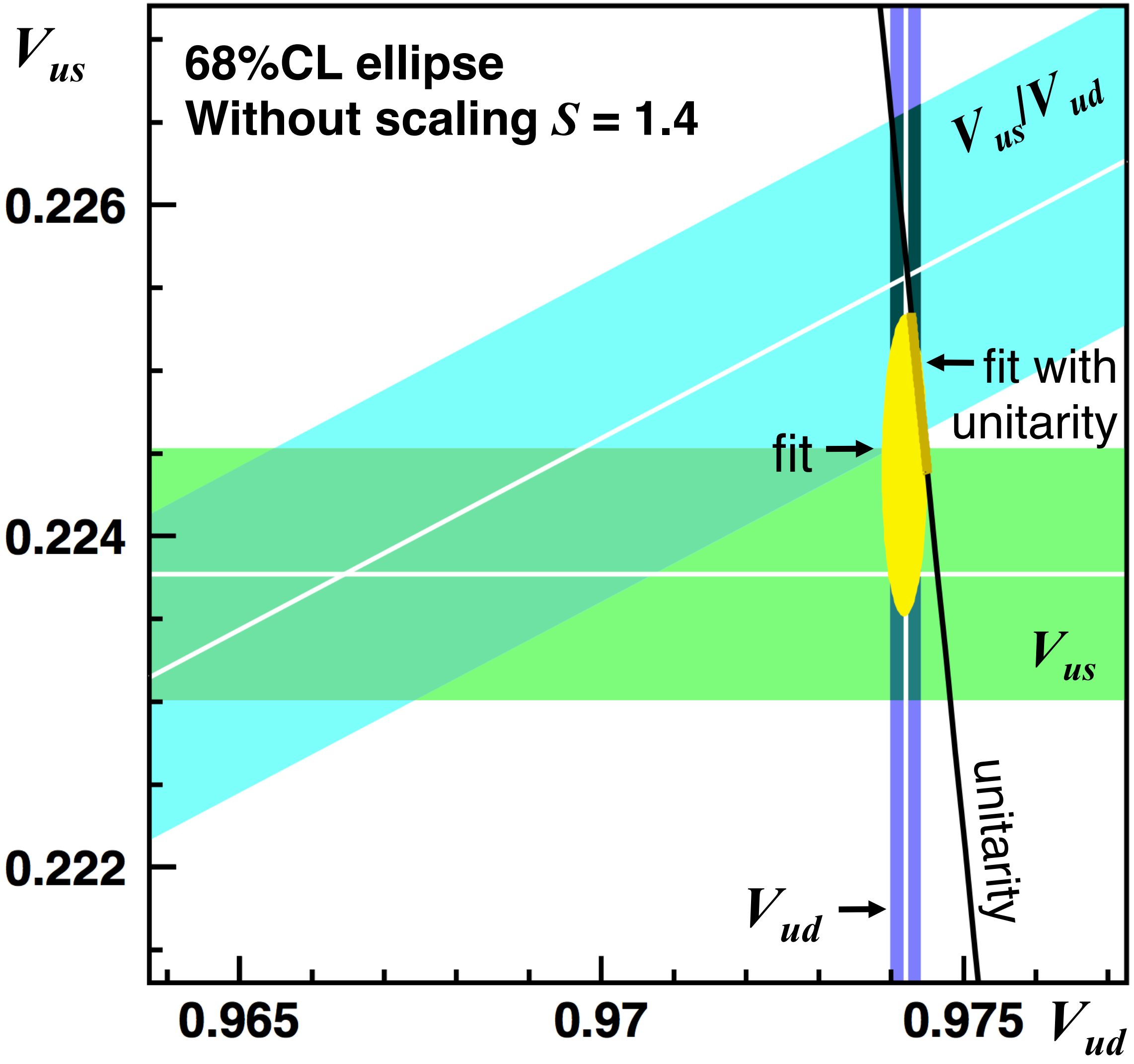
$N_f = 2+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$

$f_+(0) = 0.9677(27)$, $f_K/f_\pi = 1.192(5)$

$|V_{ud}| = 0.97420(21)$

$|V_{us}| = 0.2238(8)$

$|V_{us}|/|V_{ud}| = 0.2315(10)$



Fit results, no constraint

$V_{ud} = 0.97419(21)$

$V_{us} = 0.2244(6)$

χ^2/ndf = 2.07/1 (15.0%)

$\Delta_{CKM} = -0.0006(5)$

-1.2σ

With scale factor S = 1.4

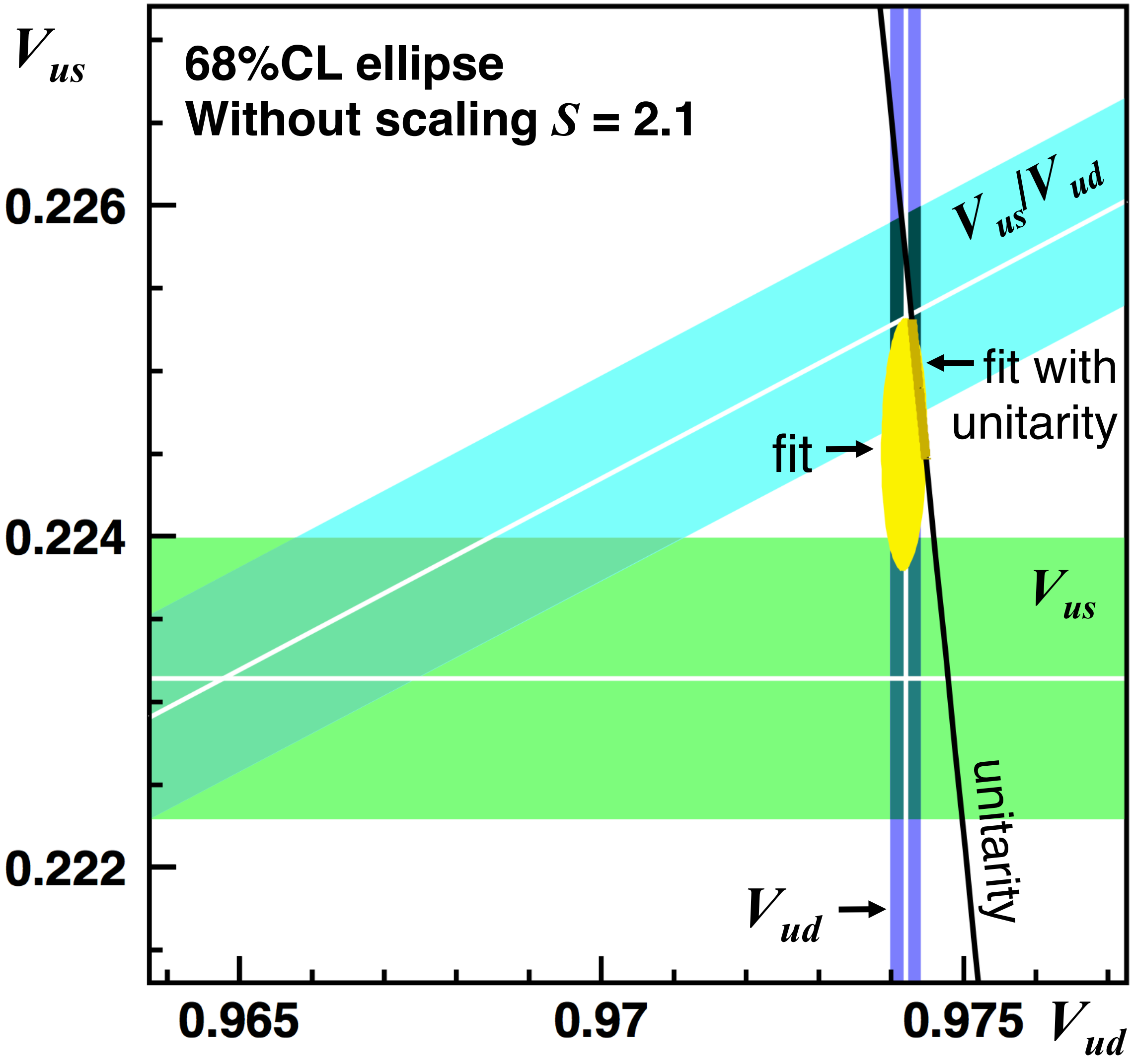
$V_{ud} = 0.97419(31)$

$V_{us} = 0.2244(9)$

V_{us} and CKM unitarity: All data

$N_f = 2+1+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
$f_+(0) = 0.9704(32)$, $f_K/f_\pi = 1.1933(27)$

$|V_{ud}| = 0.97420(21)$
$|V_{us}| = 0.2231(9)$
$|V_{us}|/|V_{ud}| = 0.2308(6)$



Fit results, no constraint

$V_{ud} = 0.97418(21)$
$V_{us} = 0.2246(5)$
$\chi^2/\text{ndf} = 4.2/1\ (3.9\%)$
$\Delta_{\text{CKM}} = -0.0005(5)$
-1.1σ

With scale factor $S = 2.1$

$V_{ud} = 0.97418(43)$
$V_{us} = 0.2246(10)$

Preliminary conclusions

Experimental results

$|V_{us}|\,f_+(0)$ **= 0.21654(41)**

$|V_{us}/V_{ud}| \times f_{K\pm}/f_{\pi\pm}$ **= 0.27599(37)**

With N_f = 2+1+1 lattice inputs

V_{ud} = **0.97418(21)** **±0.02%**

V_{us} = **0.2246(5)** **±0.22%**

Δ_{CKM} = **–0.0005(5)** = -1.1σ

Good agreement with unitarity for $K_{\ell 2}$

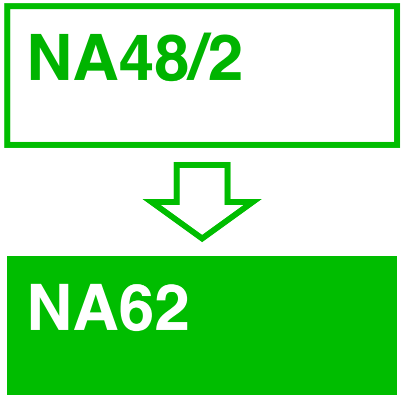
Previous excellent consistency for $K_{\ell 3}$ no longer observed

- Change occurred after 2014-era more precise evaluations of $f_+(0)$
- Experimental results for $K_{\ell 3}$ have changed little since 2010

Are residual systematics in the data and/or calculations becoming important as stated uncertainties shrink?

- Evaluation of $|V_{us}|\,f_+(0)$ from $K_{\ell 3}$ data set based on some creaky BR fits, but errors are scaled and consistency between modes is good (K_L, K_S, $K^\pm$)
- Lots of redundancy in $K_{\ell 3}$ data set. Adding or eliminating individual measurements doesn't change $|V_{us}|\,f_+(0)$ much.

Prospects for new measurements



Can measure BRs and form-factor parameters for K^+

NA62 has O(1M) K_{e3}/week of low-intensity running

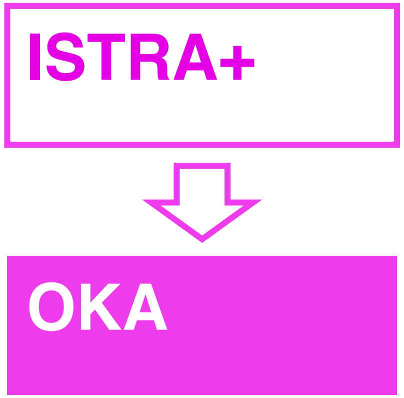
Relative to NA48/2:

- Better particle identification π/μ
- Better systematics for t reconstruction:
  - full beam tracking, better σ_p in spectrometer

NA48/2 (2003-2004) finalizing analysis of $K_{\ell 3}$ form factors

NA62-RK (2007) has O(10M) $K_{\ell 3}$ decays

- New results from 2007 (e.g. $K \to \pi\mu\mu$) coming out now
  - hope to see $K_{\ell 3}$ results soon



Fixed target experiment at U-70 (Protvino), like ISTRA+

- New beamline with RF-separated K^+ beam

Can measure BRs and form-factor parameters

- Analysis of systematics for K_{e3} form factors in progress

Runs from 2010-2013: ~17M K^+_{e3} events

- More data in 2016-2017

Prospects for new measurements

KLOE

KLOE-2

Can measure all observables: BRs, τs, FFs: $K^{\pm}$, K_L, K_S

2.4 fb^{-1} of data from first KLOE-2 physics run (2015-2016)

- +3 fb^{-1} expected in 2017
- +2 fb^{-1} of original KLOE data never analyzed for V_{us}
- All KLOE V_{us} results so far based on ~0.4 pb^{-1}

Measurements that can be improved with more statistics:

- K_S BRs ($K_S \to \pi e \nu$, but also $K_S \to \pi \mu \nu$)
- $K^{\pm}$, K_L form factors (particularly $K_{\ell 3}$)
- K_L mean life?

LHCb

Proven capability to measure K_S decays to muons

- 10^{13} K_S/fb^{-1} produced
- KAON '16 preliminary: BR($K_S \to \mu\mu$) < 6.9×10^{-9} 95%CL

  Limited by acceptance, trigger efficiency ($\varepsilon_{\text{trig}} \sim 2\%$)

Can LHCb measure BR($K_S \to \pi \mu \nu$) to < 1% in Run II?

- Would require dedicated HLT line

$K_S \to \pi \mu \nu$ never yet measured – a new channel for V_{us}

- τ_S known to 0.04% (vs 0.41% for τ_L, 0.12% for $\tau_{\pm}$)

Prospects for new measurements

KEK-246

TREK E36

Primary focus is BR($K_{e2}/K_{\mu2}$) to 0.25%

+ Invisible heavy neutrino searches

+ T violation in $K_{\mu3}$ (as E06)

Upgraded KEK-246 setup, moved to J-PARC

- Stopped K^+ in active target
- Toroidal spectrometer surrounding target
- e/μ particle ID by time of flight, Cerenkov counters, lead-glass calorimetry

KEK-246 measured BR($K_{\mu3}/K_{e3}$) and K_{e3} FF, so TREK could potentially measure at least some BRs and FFs of interest for V_{us}

Progress on V_{us} from kaons: Final notes

- **Dominant errors are systematic for most measurements**
  - New high-statistics data mainly allows detailed systematic studies
- **$K_{\ell 3}$ FFs do not directly contribute significantly to uncertainty on V_{us}**
  - However, uncertainties on high-statistics BR ratio measurements may be so low that FFs become a major systematic
    - e.g. BR($K_{\mu 3}/\pi\pi^0$), BR($K_{\mu 3}/K_{e3}$)
- **Uncertainties from parameterization of $K\pi$ phase shift data now limit precision for $K_{\ell 3}$ FFs and phase space integrals**
  - Better parameterization will require old data to be re-fit!
  - Imperative for future averages that experiments publish full FF data so that it can be re-fit as parameterizations improve
- **Direct lattice calculation of $K_{\ell 3}$ FFs may help**
- **For $K^{\pm}$, normalization BRs have significant uncertainties**
  - Effect of any precise new BR($K_{e3}/\pi\pi^0$) results will limited by uncertainty on BR($\pi\pi^0$)
  - Very important to measure absolute BRs or ratios involving BRs of other modes, e.g. $\pi\pi^0/\mu\nu$, $\pi\pi\pi/\pi\pi^0$, $\pi\pi\pi/\mu\nu$

Summary and conclusions

Experimental results

$|V_{us}|\,f_{K\pm}/f_{\pi\pm} = 0.21654(41)$

$|V_{us}/V_{ud}| \times f_{K\pm}/f_{\pi\pm} = 0.27599(37)$

With $N_f = 2+1+1$ lattice inputs

$V_{ud} = 0.97418(21)$ ±0.02%

$V_{us} = 0.2246(5)$ ±0.22%

$\Delta_{\mathrm{CKM}} = -0.0007(5) = -1.1\sigma$

1.1σ overall agreement with unitarity hides disagreement between $K_{\ell 3}$ and $K_{\ell 2}$

- **Good agreement with unitarity for $K_{\ell 2}$**
- **Previous excellent consistency for $K_{\ell 3}$ no longer observed**
  - Experimental results for $K_{\ell 3}$ have changed little since 2010

Continuing to see impressive progress on the lattice

- Not only $f_{K\pm}/f_{\pi\pm}$ and $f_{K\pm}/f_{\pi\pm}$, but also full t-dependence of FFs, EM corrections, etc.

Very good prospects for new round of experimental results to reduce uncertainty on $|V_{us}|\,f_+(0)$ from current 0.18% to ~0.12% within 5 years

NA62, OKA, KLOE-2, LHCb, TREK…

It will be interesting to see if Δ_{CKM} for $K_{\ell 3}$ decays persists or grows!

2016 summary of unitarity tests

With $|V_{ud}| = \mathbf{0.97420(21)}$: Hardy CKM '16 preliminary from $0^+ \rightarrow 0^+$ β decays

$K_{\ell 3}$ results

$N_f = 2+1$

$f_+(0) = 0.9677(27)$

$V_{us} = \mathbf{0.22377(43)_{exp}(62)_{lat}}$

$\Delta_{\mathrm{CKM}} = \mathbf{-0.00085(19)_{exp}(28)_{lat}(41)_{ud}}$ $= -1.6\sigma$

$N_f = 2+1+1$

$f_+(0) = 0.9704(32)$

$V_{us} = \mathbf{0.22314(43)_{exp}(74)_{lat}}$

$\Delta_{\mathrm{CKM}} = \mathbf{-0.00112(19)_{exp}(33)_{lat}(41)_{ud}}$ $= -2.0\sigma$

$K_{\mu 2}$ results

$N_f = 2+1$

$f_{K\pm}/f_{\pi\pm} = 1.192(5)$

$V_{us} = \mathbf{0.22557(30)_{exp}(95)_{lat}(05)_{ud}}$

$\Delta_{\mathrm{CKM}} = \mathbf{-0.00004(14)_{exp}(43)_{lat}(39)_{ud}}$ $= -0.06\sigma$

$N_f = 2+1+1$

$f_{K\pm}/f_{\pi\pm} = 1.1933(29)$

$V_{us} = \mathbf{0.22532(30)_{exp}(55)_{lat}(05)_{ud}}$

$\Delta_{\mathrm{CKM}} = \mathbf{-0.0015(14)_{exp}(25)_{lat}(39)_{ud}}$ $= -0.3\sigma$

Additional information

Updates: K_L and K_S BRs and lifetimes

KLOE
EPJC 71 (2011)

τ_S = 89.562(29)(43) ps

- 2 × 10^7 $K_S \rightarrow \pi^+\pi^-$ decays from 0.4 fb^{-1} ’04 data
- Tight track quality cuts & geometric fit
- $\sigma(L_K) \sim 0.22\text{-}0.27\ \lambda_S$ (1.3-1.6 mm)
  Measured for 180 bins in (θ_K, ϕ_K)

KTeV
PRD 83 (2011)

τ_S = 89.589(42)(56) ps

Re ε'/ε = (21.10 ± 3.43) × 10^{-6}

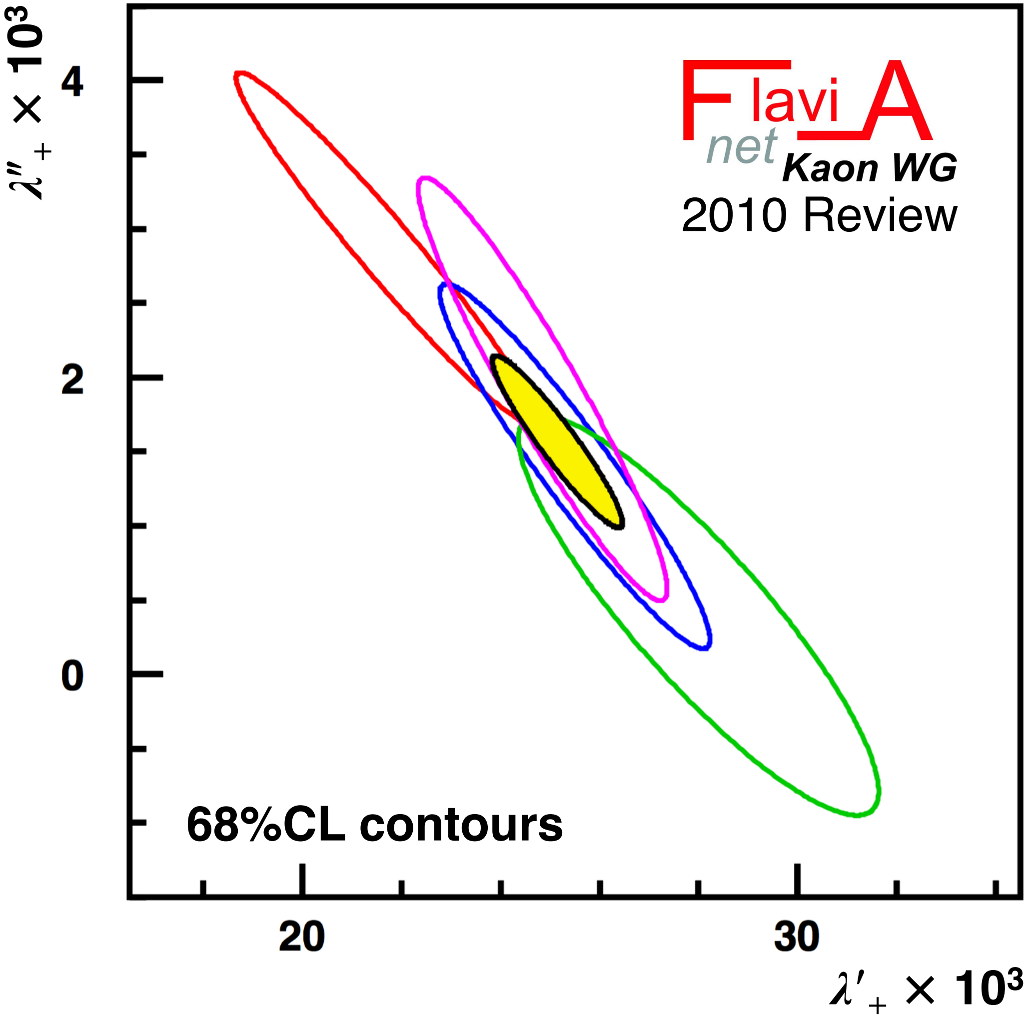
- New analysis of Re ε'/ε with improved Monte Carlo
- From fit to z_{vertex} distribution for regenerator beam without assuming CPT

Instead of PDG ETAFIT value for BR($K_L \rightarrow \pi^0\pi^0$)/BR($K_L \rightarrow \pi^+\pi^-$):

- Average **NA48 ’02** and **KTeV ’11** results for **Re ε'/ε**
- Obtain BR($K_L \rightarrow \pi^0\pi^0$)/BR($K_L \rightarrow \pi^+\pi^-$) using **Re ε'/ε** and results from K_S fit
- **Note:** K_S fit uses NA48 ’07 value of $\Gamma(K_S \rightarrow \pi e\nu)/\Gamma(K_L \rightarrow \pi e\nu)$
  Use results from K_L fit to convert this to BR($K_S \rightarrow \pi e\nu$)
  Would be better to do a K_S-K_L fit, but for now simply iterate process

Fit to K_{e3} form-factor slopes: 2010

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** **2010 fit**



Slope parameters × 10^3

λ'_+ = **25.15 ± 0.87**

λ''_+ = **1.57 ± 0.38**

$\rho(\lambda'_+, \lambda''_+)$ = −0.941

χ^2/ndf = 5.3/6 (51%)

Excellent compatibility

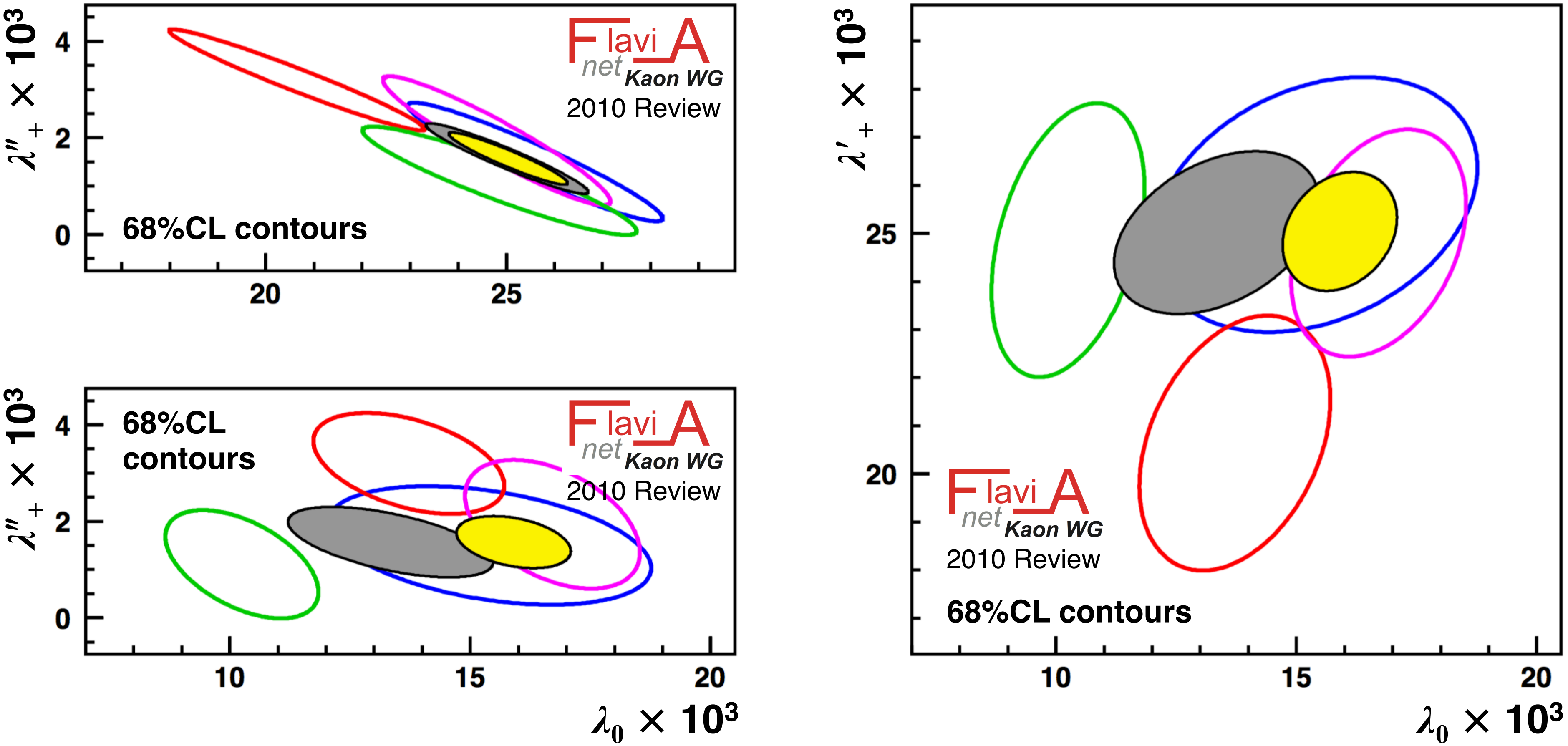
Significance of $\lambda''_+ > 4\sigma$

$I(K^0_{e3})$ **= 0.15463(21)**

$I(K^+_{e3})$ **= 0.15900(22)**

Fits to $K_{e3} + K_{\mu3}$ form-factor slopes: 2010

KTeV | **KLOE** | **ISTRA+** | **NA48** | **2010 fit** (all) | **2010 fit** (no $K_{\mu3}$ NA48)

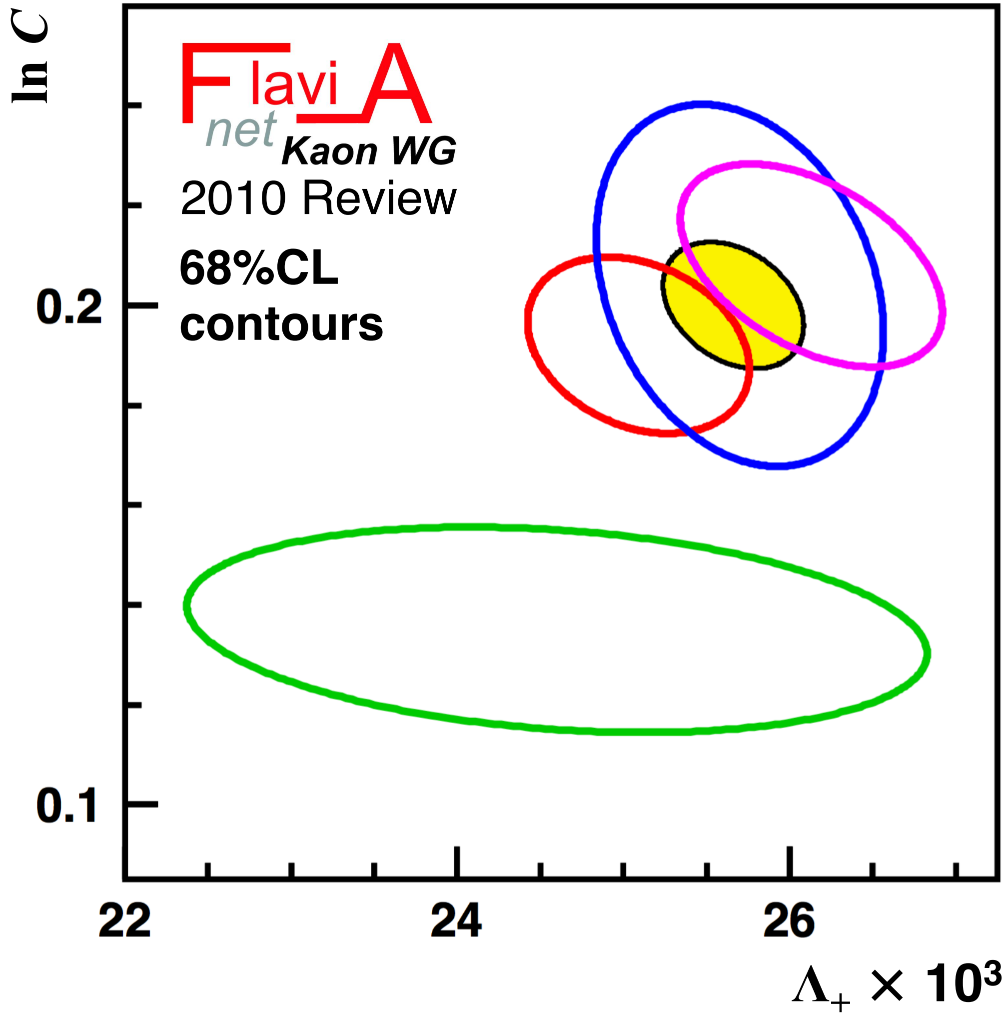


All data: $\chi^2 = 48/9$ ($P = 3 \times 10^{-7}$) | No NA48 $K_{\mu3}$: $\chi^2 = 12.1/8$ ($P = 14.5\%$)

Dispersive parameters for $K_{\ell 3}$ form-factors

$K_{e3} + K_{\mu 3}$ averages from **KTeV** **KLOE** **ISTRA+** **NA48** **2010 fit**

For NA48, only K_{e3} data included in 2010 fit (not shown)



$$\Lambda_+ \times 10^3 = \mathbf{25.66 \pm 0.41}$$

$$\ln C = \mathbf{0.2004(91)}$$

$$\rho(\Lambda_+, \ln C) = -0.328$$

$$\chi^2/\text{ndf} = 5.6/5\ (34\%)$$

Integrals

Mode	Quad-lin	Disp
$K^0{}_{e3}$	0.15457(20)	**0.15476(18)**
$K^+{}_{e3}$	0.15894(21)	**0.15922(18)**
$K^0{}_{\mu 3}$	0.10266(20)	**0.10253(16)**
$K^+{}_{\mu 3}$	0.10564(20)	**0.10559(17)**

Maximum change 0.2% if same data used as for quad-lin fits

Dispersive parameters for $K_{\ell 3}$ form-factors

$K_{e3} + K_{\mu 3}$ averages from **KTeV** **KLOE** **ISTRA+** **NA48** **2010 fit**

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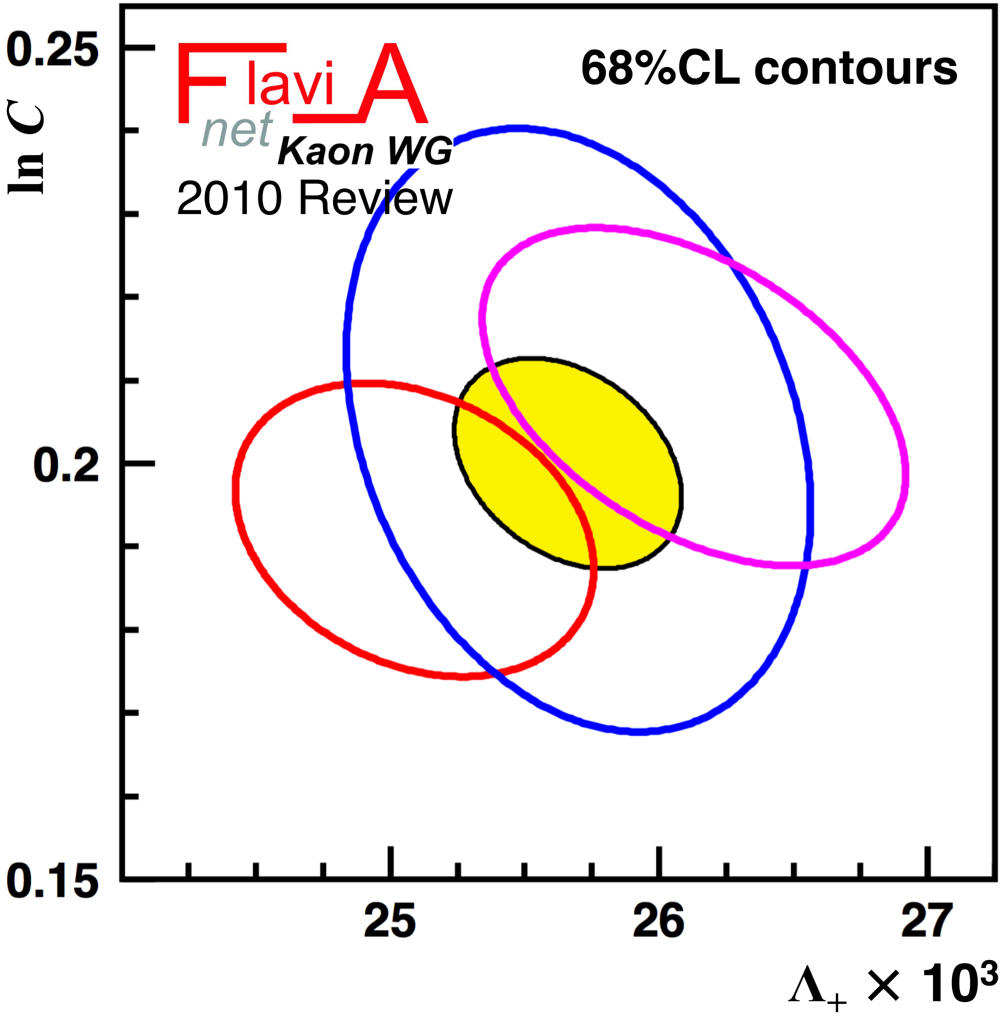

$\Lambda_+ \times 10^3$ = **25.66 ± 0.41**

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V_{us} and CKM unitarity: All data

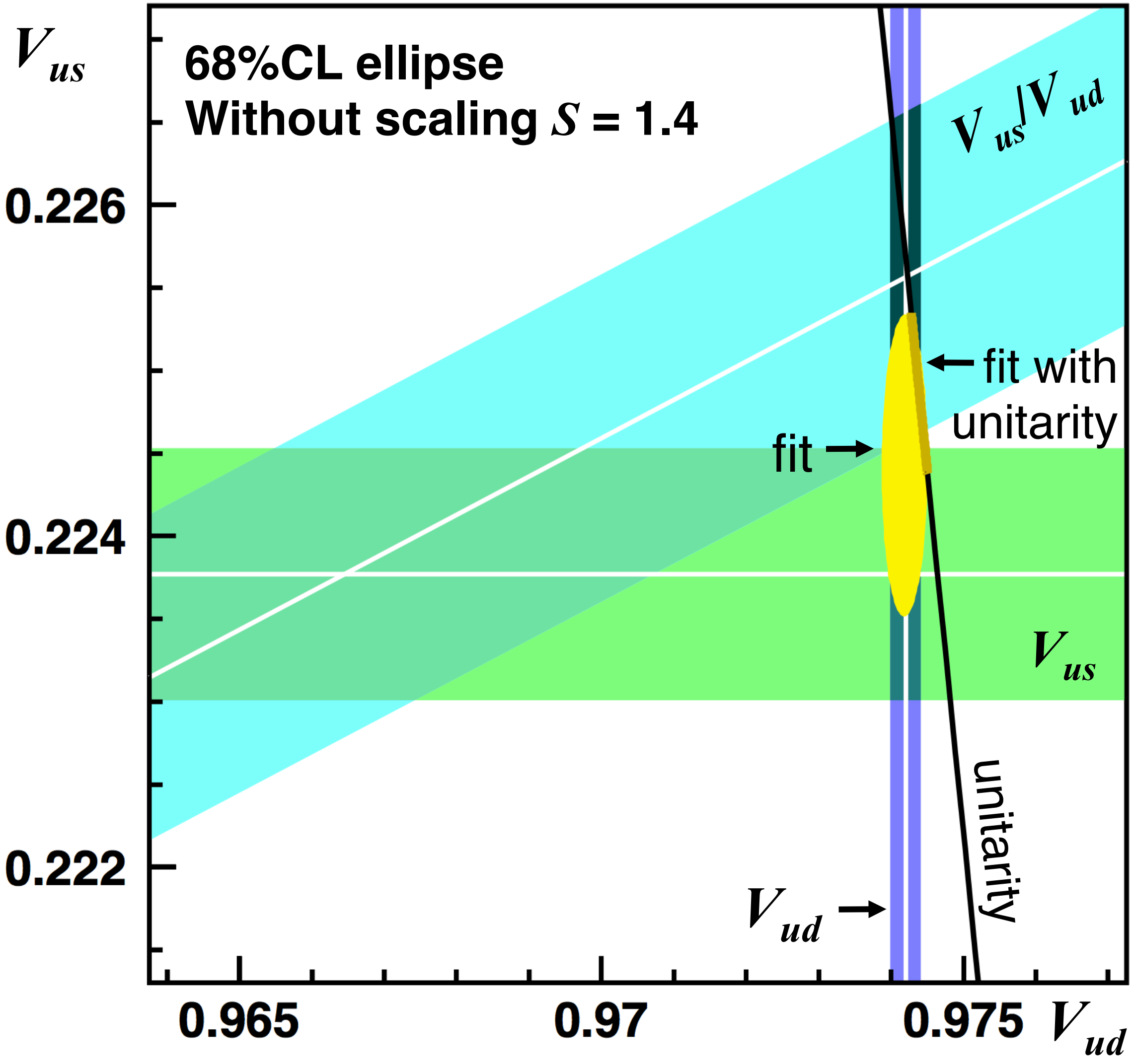
N_f = 2+1: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$

$f_+(0)$ = 0.9677(27), f_K/f_π = 1.192(5)

$|V_{ud}|$ = 0.97420(21)

$|V_{us}|$ = 0.2238(8)

$|V_{us}|/|V_{ud}|$ = 0.2315(10)



Fit results, no constraint

V_{ud} = **0.97419(21)**

V_{us} = **0.2244(6)**

χ^2/ndf = 2.07/1 (15.0%)

Δ_{CKM} = **−0.0006(5)**

-1.2σ

Fit results, unitarity constraint

V_{ud} = **0.97439(11)**

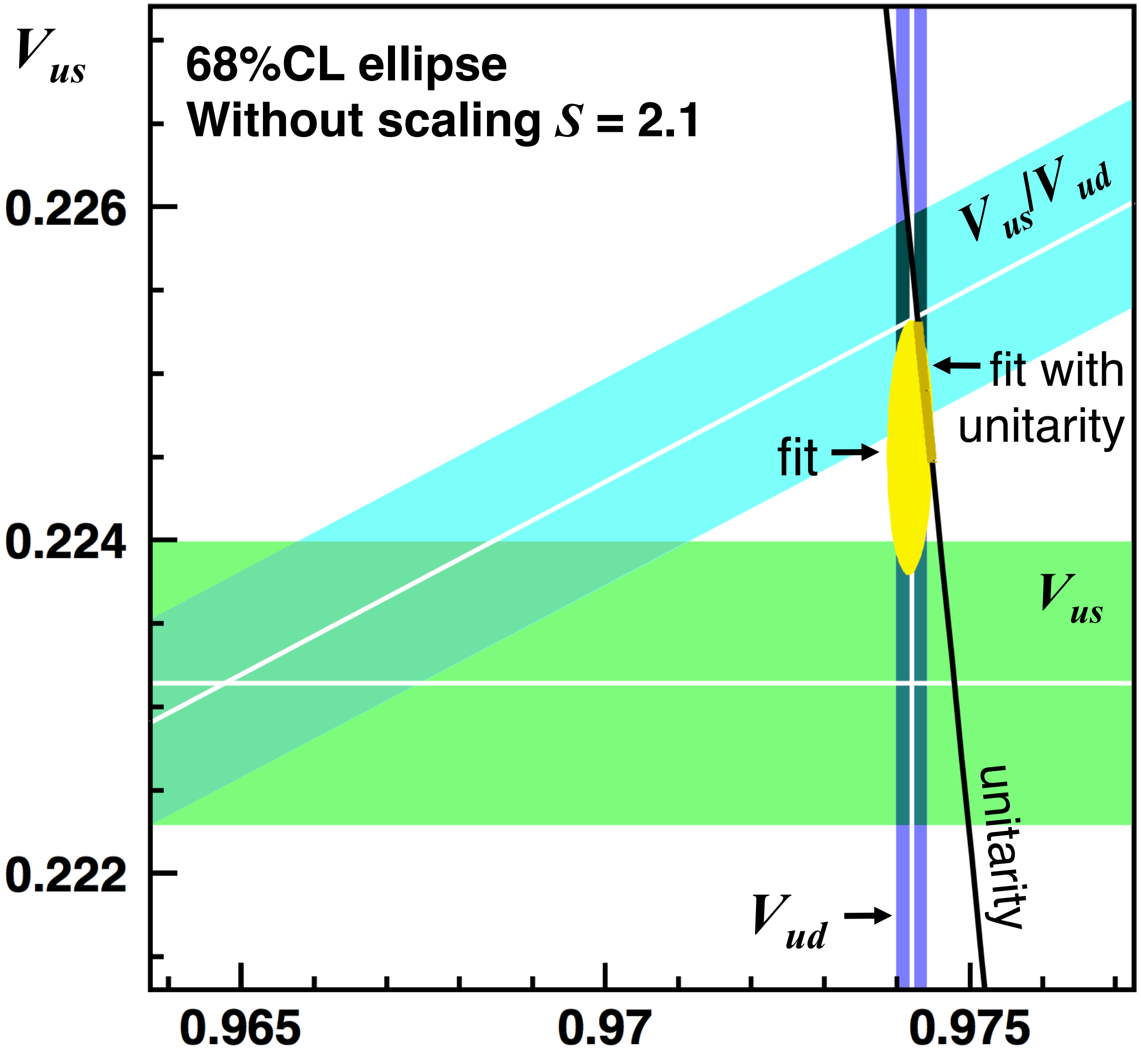
V_{us} = **0.2248(5)**

χ^2/ndf = 3.40/2 (18%)

V_{us} and CKM unitarity: All data

$N_f = 2+1+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
$f_+(0) = 0.9704(32)$, $f_K/f_\pi = 1.1933(27)$

$|V_{ud}| = 0.97420(21)$
$|V_{us}| = 0.2231(9)$
$|V_{us}|/|V_{ud}| = 0.2308(6)$



Fit results, no constraint

$V_{ud} = 0.97418(21)$
$V_{us} = 0.2246(5)$
$\chi^2/\text{ndf} = 4.2/1$ (3.9%)
$\Delta_{\text{CKM}} = -0.0007(5)$
-1.1σ

Fit results, unitarity constraint

$V_{ud} = 0.97439(10)$
$V_{us} = 0.2249(4)$
$\chi^2/\text{ndf} = 5.5/2$ (6%)