Charmless non-leptonic B decays - Theory

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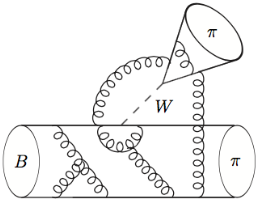
:: Motivations

- Huge multiplicity of final states (2-body + multi-body), large data sets
- Important input in CKM studies (mostly angles)
- CP violation (SM and new physics)
- Non-trivial hadronic dynamics $\Rightarrow$ Perturbative and non-perturbative QCD methods



:: Non-leptonic B-decay Amplitudes

- Effective Hamiltonian at the hadronic scale $\mu \sim m_B$

$$\mathcal{H}_{\text{eff}} = -\mathcal{L}_{QED+QCD} + \sum_i C_i(\mu)\, \mathcal{O}_i(\mu)$$

- C_i – Wilson coefficients (UV physics) $\to$ perturbation theory

  Known to NNLL: Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06.

- $\mathcal{O}_i$ – Effective operators (IR physics) [e.g. $\mathcal{O} = (\bar{b}\gamma^\mu u)(\bar{u}\gamma_\mu d)$]

- Amplitudes:

$$\mathcal{A}(B \to M_1 M_2 \cdots) = \sum_i C_i \, \langle M_1 M_2 \cdots | \mathcal{O}_i | B \rangle$$

The problem is to compute the **operator matrix elements**

$\longrightarrow$ non-perturbative, process dependent (non-universal)

:: Direct CP Violation

$$\mathcal{A}(\bar{B} \to f) \equiv \mathcal{A}_f = \underbrace{\lambda_u}_{\sim e^{i\gamma}} \underbrace{(T_f^u - P_f)}_{\mathcal{A}^u} + \underbrace{\lambda_c}_{\simeq \text{real}} \underbrace{(T_f^c - P_f)}_{\mathcal{A}^c} \qquad \lambda_p = V_{pb} V^\star_{p\{d,s\}}$$

$$T_f^p = \sum_{1,2} C_i^p \langle f | Q_i^p | \bar{B} \rangle \quad \text{(current-current operators)}$$

$$P_f = \sum_{3,\ldots,6} C_i \langle f | Q_i^p | \bar{B} \rangle \quad \text{(penguin operators)}$$

- ▷ In the SM, C_i contain no phases.
- ▷ We write $\mathcal{A}^p = |\mathcal{A}^p| e^{i\delta_p}$. Then:

$$\mathcal{A}_{\text{CP}} \equiv \frac{|\mathcal{A}_f| - |\bar{\mathcal{A}}_{\bar{f}}|}{|\mathcal{A}_f| + |\bar{\mathcal{A}}_{\bar{f}}|} \propto \left| \frac{\lambda_u \mathcal{A}^u}{\lambda_c \mathcal{A}^c} \right| \cdot \sin\gamma \cdot \sin(\delta_c - \delta_u)$$

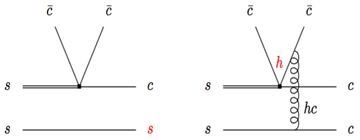
► Look for relative strong phases in interfering amplitudes

:: OUTLINE

:: Multiscale problem

▷ 3 scales: m_b, $\sqrt{m_b \Lambda_{\rm QCD}}$, $\Lambda_{\rm QCD}$.

▷ 4 modes: hard ($p_h^2 \sim m_b^2$)
hard-collinear ($p_{hc}^2 \sim m_b \Lambda_{\rm QCD}$)
collinear and soft ($p_{c,\bar c,s}^2 \sim \Lambda_{\rm QCD}^2$)



1. **QCD → SCET-1**: Integrate out hard modes

▷ $\mathcal{O} = \int dt\ \tilde T^I(t) O^I(t) + \int dt\, ds\ \tilde H^{II}(t,s) O^{II}(t,s)$

$$O^I(t) = \big[(\bar\chi W_{\bar c})(tn_-)\ldots(W_{\bar c}^\dagger \chi)(0)\big]\big[(\bar\xi W_c)(0)\ldots h_v(0)\big]$$

$$O^{II}(t,s) = \big[(\bar\chi W_{\bar c})(tn_-)\ldots(W_{\bar c}^\dagger \chi)(0)\big]\big[(\bar\xi W_c)(0)\ldots(W_c^\dagger i\not{D}_{\perp c} W_c)(sn_+)\ldots h_v(0)\big]$$

▷ decoupling of anti-collinear modes. $\langle M_2|\big[(\bar\chi W_{\bar c})(tn_-)\ldots(W_{\bar c}^\dagger \chi)(0)\big]|0\rangle \sim \phi_{M_2}$

2. **SCET-1 → SCET-2**: Integrate out hard-collinear modes

▷ $\langle M_1|\big[(\bar\xi W_c)(0)\ldots(W_c^\dagger i\not{D}_{\perp c} W_c)(sn_+)\ldots h_v(0)\big]|B\rangle \sim J(s) \otimes \phi_B \otimes \phi_{M_1}$

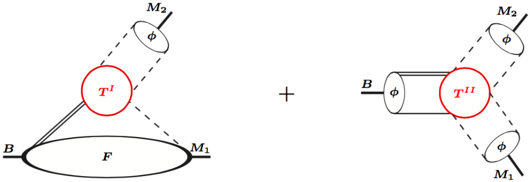
▷ Hard-collinear factorization fails for $O^I(t)$.

▷ End-point divergences can be absorbed into form factor F^{BM_1}.

:: Factorization formula for $B \to M_1 M_2$

To leading power in the heavy-quark expansion — Beneke, Buchalla, Neubert, Sachrajda '99

$$\langle M_1 M_2|\mathcal{O}|B\rangle = F^{BM_1}\int du\, T^I(u)\phi_{M_2}(u) + \int d\omega\, du\, dv\, T^{II}(\omega,u,v)\phi_B(\omega)\phi_{M_1}(u)\phi_{M_2}(v)$$



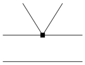
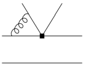
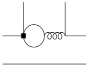
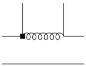
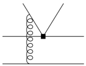
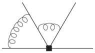
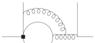
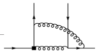
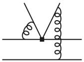
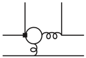
- Vertex corrections: $T^I(u) = 1 + \mathcal{O}(\alpha_s)$
- Spectator scattering: $T^{II}(\omega,u,v) = \underbrace{\mathcal{O}(\alpha_s)}_{real} + \mathcal{O}(\alpha_s^2/\pi)$ – (power supp. if M_1 heavy)
- Strong phases are perturbative [$\mathcal{O}(\alpha_s)$] or power suppressed [$\mathcal{O}(\Lambda/m_b)$].
- $A_{\rm CP} = \mathcal{O}(\alpha_s(m_b)/\pi) + \mathcal{O}(\Lambda/m_b)$ – But ... $\alpha_s(m_b)/\pi \sim \Lambda/m_b$!!

:: Perturbative calculation

Two hard-scattering kernels for each operator insertion: T^I (vertex), T^{II} (spectator)

$$\langle M_1 M_2 | \mathcal{O}_i | B \rangle \simeq F^{BM_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

and two classes of topological amplitudes: "Tree", "Penguin".

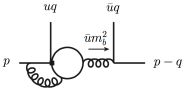
	T^I, tree	T^I, penguin	T^{II}, tree	T^{II}, penguin
LO: $\mathcal{O}(1)$				
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'04		 		
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07,'09 Beneke, Huber, Li '09	  Kim, Yoon '11, Bell Beneke, Huber, Li '15	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

:: Perturbative calculation

Motivation for NNLO: first correction to CP asymmetries

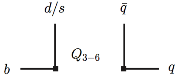
NNLO: non-trivial calculation Bell, Huber '14

- ▷ $\mathcal{O}(70)$ diagrams
- ▷ 2 loops, 3 scales (m_b, um_b, m_c), 4 legs
- ▷ charm contribution has non-trivial threshold at $\bar{u}m_b^2 \gtrsim 4m_c^2$

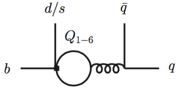


Missing NNLO pieces:

- ▷ 2-loop tree insertions of penguin operators $\mathcal{O}_{3-6}$
  Similar to $\mathcal{O}^u_{1,2}$ calculation, easier than $\mathcal{O}^c_{1,2}$



- ▷ 2-loop penguin insertions of penguin operators $\mathcal{O}_{3-6}$
  Additional topology with "closed" quark loop.



:: Tree decays

Beneke, Huber, Li '09

$$
\begin{aligned}
T \equiv a_1(\pi\pi) &= 1.009 + [0.023 + 0.010i]_{\text{NLO}} + [0.026 + 0.028i]_{\text{NNLO}} \\
&\quad - \left[\frac{r_{\text{sp}}}{0.485}\right]\left\{[0.015]_{\text{LOsp}} + [0.037 + 0.029i]_{\text{NLOsp}} + [0.009]_{\text{tw3}}\right\} \\
&= 1.00 + 0.01i \quad \rightarrow \quad 0.93 - 0.02i \quad (\text{if } 2 \times r_{\text{sp}})
\end{aligned}
$$

$$
r_{\text{sp}} = \frac{9 f_{M_1} \hat{f}_B}{m_b f_+^{B\pi}(0)\lambda_B}
$$

$$
\begin{aligned}
C \equiv a_2(\pi\pi) &= 0.220 - [0.179 + 0.077i]_{\text{NLO}} - [0.031 + 0.050i]_{\text{NNLO}} \\
&\quad + \left[\frac{r_{\text{sp}}}{0.485}\right]\left\{[0.123]_{\text{LOsp}} + [0.053 + 0.054i]_{\text{NLOsp}} + [0.072]_{\text{tw3}}\right\} \\
&= 0.26 - 0.07i \quad \rightarrow \quad 0.51 - 0.02i \quad (\text{if } 2 \times r_{\text{sp}})
\end{aligned}
$$

- ▷ Individual NNLO corrections large, but cancellations between FF and sp. terms.
- ▷ Perturbative expansion well behaved (remember color suppression).
- ▷ Color suppressed $a_2(\pi\pi)$ dominated by spectator scattering [larger uncertainty]
  Can be large if λ_B is small.
- ▷ Relative phase $\arg(C/T)$ remains small.

:: Tree decays

Beneke, Huber, Li '09

	Theory I		Theory II		Experiment
$B^- \to \pi^-\pi^0$	$5.43^{+0.06}_{-0.06}{}^{+1.45}_{-0.84}$	$(\star)$	$5.82^{+0.07}_{-0.06}{}^{+1.42}_{-1.35}$	$(\star)$	$5.59^{+0.41}_{-0.40}$
$\bar{B}^0_d \to \pi^+\pi^-$	$7.37^{+0.86}_{-0.69}{}^{+1.22}_{-0.97}$	$(\star)$	$5.70^{+0.70}_{-0.55}{}^{+1.16}_{-0.97}$	$(\star)$	5.16 ± 0.22
$\bar{B}^0_d \to \pi^0\pi^0$	$0.33^{+0.11}_{-0.08}{}^{+0.42}_{-0.17}$		$0.63^{+0.12}_{-0.10}{}^{+0.64}_{-0.42}$		1.55 ± 0.19
			BELLE CKM 14:		0.90 ± 0.16
$B^- \to \pi^-\rho^0$	$8.68^{+0.42}_{-0.41}{}^{+2.71}_{-1.56}$	$(\star\star)$	$9.84^{+0.41}_{-0.40}{}^{+2.54}_{-2.52}$	$(\star\star)$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$12.38^{+0.90}_{-0.77}{}^{+2.18}_{-1.41}$	$(\star)$	$12.13^{+0.85}_{-0.73}{}^{+2.23}_{-2.17}$	$(\star)$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^+\rho^-$	$17.80^{+0.62}_{-0.56}{}^{+1.76}_{-2.10}$	$(\star)$	$13.76^{+0.49}_{-0.44}{}^{+1.77}_{-2.18}$	$(\star)$	15.7 ± 1.8
$\bar{B}^0 \to \pi^-\rho^+$	$10.28^{+0.39}_{-0.39}{}^{+1.37}_{-1.42}$	$(\star\star)$	$8.14^{+0.34}_{-0.33}{}^{+1.35}_{-1.49}$	$(\star\star)$	7.3 ± 1.2
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$28.08^{+0.27}_{-0.19}{}^{+3.82}_{-3.50}$	$(\dagger)$	$21.90^{+0.20}_{-0.12}{}^{+3.06}_{-3.55}$	$(\dagger)$	23.0 ± 2.3
$\bar{B}^0 \to \pi^0\rho^0$	$0.52^{+0.04}_{-0.03}{}^{+1.11}_{-0.43}$		$1.49^{+0.07}_{-0.07}{}^{+1.77}_{-1.29}$		2.0 ± 0.5
$B^- \to \rho^-_L\rho^0_L$	$18.42^{+0.23}_{-0.21}{}^{+3.92}_{-2.55}$	$(\star\star)$	$19.06^{+0.24}_{-0.22}{}^{+4.59}_{-4.22}$	$(\star\star)$	$22.8^{+1.8}_{-1.9}$
$\bar{B}^0_d \to \rho^+_L\rho^-_L$	$25.98^{+0.85}_{-0.77}{}^{+2.93}_{-3.43}$	$(\star\star)$	$20.66^{+0.68}_{-0.62}{}^{+2.99}_{-3.75}$	$(\star\star)$	$23.7^{+3.1}_{-3.2}$
$\bar{B}^0_d \to \rho^0_L\rho^0_L$	$0.39^{+0.03}_{-0.03}{}^{+0.83}_{-0.36}$		$1.05^{+0.05}_{-0.04}{}^{+1.62}_{-1.04}$		$0.55^{+0.22}_{-0.24}$

Theory I: $f_+^{B\pi}(0) = 0.25 \pm 0.05$, $A_0^{B\rho}(0) = 0.30 \pm 0.05$, $\lambda_B(1\,\text{GeV}) = 0.35 \pm 0.15$ GeV

Theory II: $f_+^{B\pi}(0) = 0.23 \pm 0.03$, $A_0^{B\rho}(0) = 0.28 \pm 0.03$, $\lambda_B(1\,\text{GeV}) = 0.20^{+0.05}_{-0.00}$ GeV

First error γ, $|V_{cb}|$. $|V_{ub}|$ uncertainty *not* included. Second error from hadronic inputs.
Brackets: form factor uncertainty not included.

:: Impact of λ_B

G. Bell

B-meson LCDA inverse moment: $\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\phi_B(\omega,\mu)$

Dominant parametric uncertainty in QCDF

- QCD sum rule estimate $\lambda_B(1\text{GeV}) \simeq (460 \pm 110)$ MeV [Braun, Ivanov, Korchemsky 03]
- $\pi\pi/\pi\rho/\rho\rho$ data seems to prefer $\sim$ 200 MeV ?

λ_B can be measured in $B \to \gamma\ell\nu$ decays

- state-of-the-art analysis (NLL, tree-level $1/m_b$) [Beneke, Rohrwild 11; Braun, Khodjamirian 12]
- Babar 09 data ($E_\gamma > 1$GeV) $\Rightarrow$ $\lambda_B(1\text{GeV}) > 115$ MeV
- Belle 15 data ($E_\gamma > 1$GeV) $\Rightarrow$ $\lambda_B(1\text{GeV}) > 238$ MeV
- good prospects to measure λ_B at Belle-II

:: Penguin decays

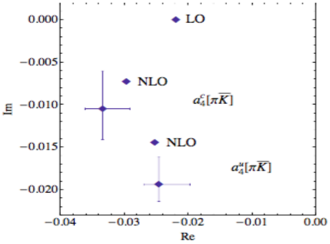
Bell, Beneke, Huber, Li '15

$$a_4^u(\pi\bar{K})/10^{-2} = -2.87 - [0.09+0.09i]_{V_1} + [0.49-1.32i]_{P_1} - [0.32+0.71i]_{P_2}$$
$$+ \left[\frac{r_{sp}}{0.434}\right]\left\{[0.13]_{LO} + [0.14+0.12i]_{HV} - [0.01-0.05i]_{HP} + [0.07]_{tw3}\right\}$$
$$= (-2.46^{+0.49}_{-0.24}) + (-1.94^{+0.32}_{-0.20})i$$

$$r_{sp} = \frac{9 f_{M_1}\hat{f}_B}{m_b f_+^{B\pi}(0)\lambda_B}$$

$$a_4^c(\pi\bar{K})/10^{-2} = -2.87 - [0.09+0.09i]_{V_1} + [0.05-0.62i]_{P_1} - [0.77+0.50i]_{P_2}$$
$$+ \left[\frac{r_{sp}}{0.434}\right]\left\{[0.13]_{LO} + [0.14+0.12i]_{HV} + [0.01+0.03i]_{HP} + [0.07]_{tw3}\right\}$$
$$= (-3.34^{+0.43}_{-0.27}) + (-1.05^{+0.45}_{-0.36})i$$

- Two-loop is 40% (15%) of the imaginary (real) part of $a_4^u(\pi\bar{K})$, and 50% (25%) in the case of $a_4^c(\pi\bar{K})$.
- Spectator-scattering not relevant.



M.Beneke, talk at *Future challenges in non-leptonic B decays* (2016)

:: Penguin decays (CPAs)

Bell, Beneke, Huber, Li '15

f	NLO	NNLO	NNLO + LD	Exp
$\pi^-\bar{K}^0$	$0.71\,^{+0.13+0.21}_{-0.14-0.19}$	$0.77\,^{+0.14+0.23}_{-0.15-0.22}$	$0.10\,^{+0.02+1.24}_{-0.02-0.27}$	-1.7 ± 1.6
π^0K^-	$9.42\,^{+1.77+1.87}_{-1.76-1.88}$	$10.18\,^{+1.91+2.03}_{-1.90-2.62}$	$-1.17\,^{+0.22+20.00}_{-0.22-\ 6.62}$	4.0 ± 2.1
π^+K^-	$7.25\,^{+1.36+2.13}_{-1.36-2.58}$	$8.08\,^{+1.52+2.52}_{-1.51-2.65}$	$-3.23\,^{+0.61+19.17}_{-0.61-\ 3.36}$	-8.2 ± 0.6
$\pi^0\bar{K}^0$	$-4.27\,^{+0.83+1.48}_{-0.77-2.23}$	$-4.33\,^{+0.84+3.29}_{-0.78-2.32}$	$-1.41\,^{+0.27+5.54}_{-0.25-6.10}$	1 ± 10
$\delta(\pi\bar{K})$	$2.17\,^{+0.40+1.39}_{-0.40-0.74}$	$2.10\,^{+0.39+1.40}_{-0.39-2.86}$	$2.07\,^{+0.39+2.76}_{-0.39-4.55}$	12.2 ± 2.2

Table 3: Direct CP asymmetries (in percent) for πK final states (from Ref. [28]).

- Overall, large experimental and/or theory uncertainties
- $\delta(\pi K)$ remains a puzzle.

:: Power Corrections

G. Bell

Main limitation of QCDF approach, e.g. weak annihilation

$$\sim \int d\omega\, du\, dv\; T(\omega,u,v)\; \phi_B(\omega)\; \phi_{M_1}(v)\; \phi_{M_2}(u) \quad ?$$

- convolutions diverge at endpoints $\Rightarrow$ non-factorisation in SCET-2
- currently modelled with arbitrary soft rescattering phase

Pure annihilation decays

$$10^6\,\mathrm{Br}(B_d \to K^+K^-) = 0.13 \pm 0.05 \qquad (\Delta D = 1, \text{exchange topology})$$

$$10^6\,\mathrm{Br}(B_s \to \pi^+\pi^-) = 0.76 \pm 0.13 \qquad (\Delta S = 1, \text{penguin annihilation})$$

$\Rightarrow$ extract weak annihilation amplitudes from data [Wang, Zhu 13; Bobeth, Gorbahn, Vickers 14; Chang, Sun, Yang, Li 14]

▷ Or use "clean" combinations, *e.g.* $\Delta = T - P$ in penguin mediated decays

[Descotes-Genon, Matias, JV '06,'07,'11]

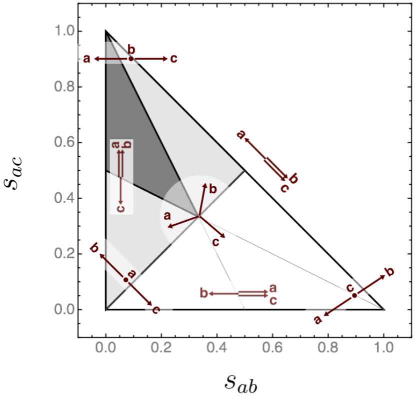
:: OUTLINE

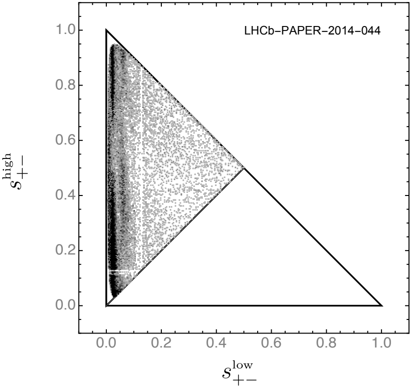
:: Three-body B decays

- ▷ Model-independent treatment of vector resonances:
  - $B \to \rho \ell \nu \quad \longrightarrow \quad B \to [\pi\pi] \ell \nu$
  - $B \to K^* \ell\ell \quad \longrightarrow \quad B \to [K\pi] \ell\ell$
  - Finite-width effects, interference (S-wave pollution, etc.)
- ▷ More complicated kinematics $\longrightarrow$ more observables
- ▷ Larger phase space: different kinematic regimes, different theory descriptions
- ▷ Kinematic distributions $\longrightarrow$ tests of EFT expansions & Factorization
- ▷ E-dependent rescattering effects $\longrightarrow$ large strong phases
  $\longrightarrow$ Large localized CP asymmetries
- ▷ Huge data sets
- ▷ Many applications: CKM parameters, tests of factorization, New Physics, spectroscopy, meson-meson scattering,...

:: Three-body decays – kinematics

- $\bar{B} \to M_a(p_a) M_b(p_b) M_c(p_c)$
- Two independent invariants, *e.g.* $s_{ab} = \frac{(p_a+p_b)^2}{m_B^2}$ and $s_{ac} = \frac{(p_a+p_c)^2}{m_B^2}$

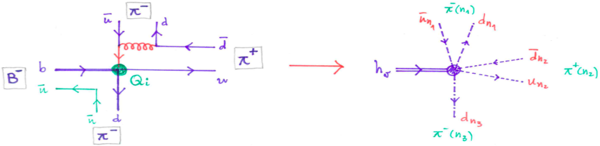




- Different kinematic regions with different factorization properties.

:: Central region

⋆ Three collinear directions n_1, n_2, n_3, disconnected at the leading power.



$$\langle \pi^- \pi^+ \pi^- | \mathcal{O}_i | \bar{B} \rangle = F^{B\to\pi} \int du\, dv\, T_i^I(u,v)\, \phi_\pi(u)\, \phi_\pi(v)$$
$$+ \int d\omega\, du\, dv\, dy\, T_i^{II}(\omega,u,v,y)\, \phi_B(\omega)\, \phi_\pi(u)\, \phi_\pi(v)\, \phi_\pi(y)$$

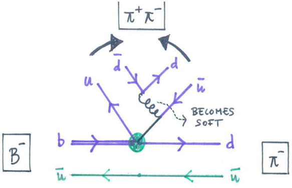
- ▷ Power $(1/m_b^2)$ & α_s suppressed with respect to two-body.
- ▷ At leading order/power/twist all convolutions are finite → factorization ✓
- ▷ Some pieces proven at NLO: Factorization of $B \to \pi\pi$ form factors [Böer, Feldmann, van Dyk '16] and 2π LCDAs [Diehl, Feldmann, Kroll, Vogt '99]

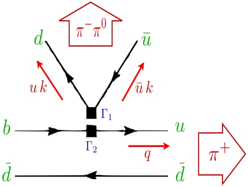
- ► $A_{\rm CP} = \mathcal{O}(\alpha_s(m_b)/\pi) + \mathcal{O}(\Lambda/m_b)$ – Like two-body !
- ► But this region might not exist for $m_B = 5$ GeV Krankl, Mannel, JV '15

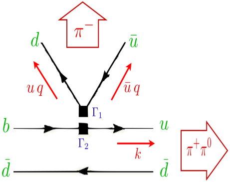
:: Edges

Krankl, Mannel, JV '15

- Breakdown of factorization at resonant edges requires new NP functions.
- 3-body decay remsembles 2-body, but with new $(\pi\pi)$ "compound object":







- Operators are the same as in 2-body, but final states different:

$$\begin{aligned}
\langle \pi^-_{\bar{n}}\pi^+_{\bar{n}}\pi^-_n|\mathcal{O}|B\rangle &= \langle \pi^-_n|\bar{h}_v\Gamma\xi_n|B\rangle \times \int dz\, T_1(z)\langle \pi^-_{\bar{n}}\pi^+_{\bar{n}}|\bar{\chi}_{\bar{n}}(z\bar{n})\Gamma'\chi_{\bar{n}}(0)|0\rangle \\
&+ \langle \pi^-_{\bar{n}}\pi^+_{\bar{n}}|\bar{h}_v\Gamma\xi_{\bar{n}}|B\rangle \times \int dz\, T_2(z)\langle \pi^-_n|\bar{\chi}_{\bar{n}}(zn)\Gamma'\chi_n(0)|0\rangle \\
&= F^{B\to\pi}\; T_1 \star \phi_{\pi\pi} + F^{B\to\pi\pi}\; T_2 \star \phi_\pi
\end{aligned}$$

- New non-perturbative input: (Contains NP strong phases!!)
  - Generalized Distribution Amplitudes (GDAs) [Diehl, Polyakov, Gousset, Pire, Grozin...]
  - Generalized Form Factors (GFFs) [Faller, Feldmann, Khodjamirian, Mannel, van Dyk...]

:: Main theory objects

PERTURBATIVE:

Hard-scattering kernels – T_I, T_{II} – Same as two-body!! (just matching coefficients)

NON-PERTURBATIVE:

Quasi-two-body		**Three-body**
$B \to \rho$ form factors	$\longleftarrow \cdots \longrightarrow$	$B \to \pi\pi$ form factors
$\langle \rho \vert \bar{q}(x) \Gamma b(0) \vert \bar{B} \rangle$		$\langle \pi\pi \vert \bar{q}(x) \Gamma b(0) \vert \bar{B} \rangle$
ρ-LCDAs...	$\longleftarrow \cdots \longrightarrow$	2π-LCDAs...
$\langle \rho \vert \bar{q}(x) \Gamma q(0) \vert 0 \rangle_{x^2 \to 0}$		$\langle \pi\pi \vert \bar{q}(x) \Gamma q(0) \vert 0 \rangle_{x^2 \to 0}$
...and its normalization f_ρ	$\longleftarrow \cdots \longrightarrow$	...and its normalization $F_\pi(s)$

:: 2π GDAs

Krankl, Mannel, JV '15

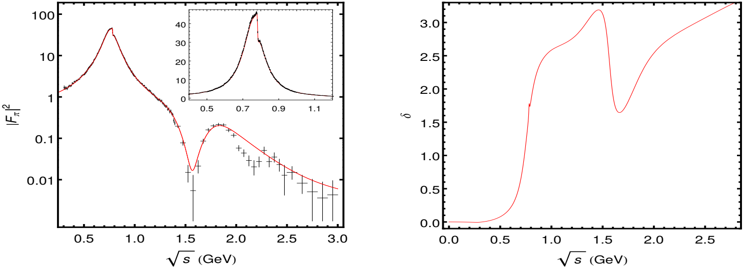
- Definition: $[s = (k_1 + k_2)^2,\ k_1 = \zeta k_{12},\ k_2 = (1-\zeta)k_{12}]$

$$\phi^q_{\pi\pi}(z,\zeta,s) = \int \frac{dx^-}{2\pi} e^{iz(k^+_{12}x^-)} \langle \pi^+(k_1)\pi^-(k_2)|\bar{q}(x^- n_-)\not{n}_+ q(0)|0\rangle$$

- Normalization (local correlator):

$$\int dz\, \phi_{\pi\pi}(z,\zeta,s) = (2\zeta - 1)F_\pi(s) \quad \text{(pion vector FF)}$$

- $F_\pi(s)$: Data ($e^+e^- \to \pi\pi(\gamma)$ [BaBar])



:: $B \to \pi\pi$ form factors from 2π-LCDAs

Hambrock, Khodjamirian, 2015; Cheng, Khodjamirian, JV w.i.p

▷ **Correlation function**

$$\Pi^5(p^2, k^2, q^2, q \cdot \bar{k}) = i \int d^4x\, e^{iq\cdot x} \langle \pi^+(k_1)\pi^0(k_2)| \mathrm{T}\{\bar{u}(x) i m_b \gamma_5 b(x), \bar{b}(0) i m_b \gamma_5 d(0)\}|0\rangle$$

▷ **Unitarity relation**

$$2\mathrm{Im}\Pi^5 = (2\pi)\delta(p^2 - m_B^2) \underbrace{\langle \pi^+(k_1)\pi^0(k_2)|\bar{u} i m_b \gamma_5 b|\bar{B}(p)\rangle}_{\sqrt{q^2} F_t(q^2,k^2,q\cdot k)} \underbrace{\langle \bar{B}(p)|\bar{b} i m_b \gamma_5 d|0\rangle}_{m_B^2 f_B} + \cdots$$

$$= (2\pi)\, \delta(p^2 - m_B^2)\, m_B^2\, f_B \sqrt{q^2}\, F_t(q^2, k^2, q \cdot k) + \cdots$$

▷ **Dispersion relation + LCOPE + Borel + duality**

$$m_B^2\, f_B \sqrt{q^2}\, F_t(q^2, k^2, q \cdot k)\, e^{-m_B^2/M^2} = \Pi^5_{OPE}(M^2, q^2, k^2, q \cdot \bar{k})$$

:: $B \to \pi\pi$ form factors from 2π-LCDAs

Hambrock, Khodjamirian, 2015; Cheng, Khodjamirian, JV w.i.p

- In this case:

$$\Pi^5_{OPE}(M^2, q^2, k^2, q\cdot\bar{k}) = \frac{m_b^2}{\sqrt{2}}\int_{u_0}^1 \frac{du}{u^2}\, e^{-s(u)/M^2}\,(m_b^2 - q^2 + u^2k^2)\,\Phi_\parallel(u, q\cdot\bar{k}, k^2)$$

- Where the 2π LCDA is defined as

$$\Phi^q_\parallel(z,\zeta,s) = \int \frac{dx^-}{2\pi}\, e^{iz(k^+_{12}x^-)}\langle\pi^+(k_1)\pi^-(k_2)|\bar{q}(x^- n_-)\not{n}_+ q(0)|0\rangle$$

- The 2π LCDA is normalized to the pion form factor:

$$\int dz\, \Phi_\parallel(z,\zeta,s) = (2\zeta - 1)\, F_\pi(s)$$

but for the sum rule we need higher moments.

- Narrow-ρ dominance on $\Phi_\parallel$ leads to $B \to \rho$ form factor from ρ-LCDA. ✓

[$\Phi_\parallel \longleftrightarrow \phi_\rho$ Polyakov '98]

:: $B \to \pi\pi$ form factors from B-meson LCDAs

Cheng, Khodjamirian, JV '16?

- **Correlation function**

$$F_\mu(k,q) = i\int d^4x e^{ik\cdot x}\langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), m_b\bar{u}(0)\gamma_5 b(0)\}|\bar{B}^0(q+k)\rangle$$

- **Unitarity relation**

$$2\mathrm{Im}F_\mu(k,q) = m_b\int d\tau_{2\pi}\underbrace{\langle 0|\bar{d}\gamma_\mu|\pi(k_1)\pi(k_2)\rangle}_{F_\pi^\star(s)}\underbrace{\langle\pi(k_1)\pi(k_2)|\bar{u}\gamma_5 b|\bar{B}^0(q+k)\rangle}_{F_t(s,q^2,\cos\theta_\pi)}+\cdots$$

$$= q_\mu\frac{s\sqrt{q^2}\beta_\pi(s)^2}{4\sqrt{6}\pi\sqrt{\lambda}}F_\pi^\star(s)\ F_t^{(\ell=1)}(s,q^2)+\cdots$$

Corollary: $F_\pi^\star(s)\ F_t^{(\ell=1)}(s,q^2)$ is real for all $s<16m_\pi^2\Rightarrow$

$$\text{Phase}(F^{B\to\pi\pi}) = \text{Phase(pion form factor)}$$

Important for CP violation!!!

[See also Kang, Kubis, Hanhart, Meissner '13]

:: $B \to \pi\pi$ form factors from B-meson LCDAs

Cheng, Khodjamirian, JV '16?

- **Dispersion relation + LCOPE + Borel + duality**

$$-\int_{4m_\pi^2}^{s_0^{2\pi}} ds\ e^{-s/M^2}\ \frac{s\ \sqrt{q^2}\ [\beta_\pi(s)]^2}{4\sqrt{6}\pi^2\sqrt{\lambda}}\ F_\pi^\star(s)\ F_t^{(1)}(s,q^2) = f_B m_B^2 m_b \left\{ \int_0^{\sigma_0^{2\pi}} d\sigma\ e^{-s(\sigma,q^2)/M^2} \times \right.$$

$$\left. \times \left[\frac{\sigma}{\bar\sigma}\phi_+^B(\sigma m_B) - \frac{\sigma}{\bar\sigma}\left[\phi_+^B(\sigma m_B) - \phi_-^B(\sigma m_B)\right] - \frac{1}{\bar\sigma m_B}\bar\Phi_\pm^B(\sigma m_B)\right] + \Delta A_0^{BV}(q^2, \sigma_0^{2\pi}, M^2) \right\}$$

- **ρ-dominance + zero-width limit:**

$$F_\pi^\star(s) \simeq \frac{f_\rho g_{\rho\pi\pi} m_\rho/\sqrt{2}}{m_\rho^2 - s + i\sqrt{2}\Gamma_\rho(s)} \quad , \quad F_t^{(1)}(s,q^2) \simeq -\frac{\beta_\pi(s)\sqrt{\lambda}}{\sqrt{3q^2}}\ \frac{m_\rho g_{\rho\pi\pi} A_0^{B\rho}(q^2)}{m_\rho^2 - s - i\sqrt{2}\Gamma_\rho(s)}$$

$$LHS = 2 f_\rho m_\rho A_0^{B\rho}(q^2) \int_{4m_\pi^2}^{s_0^{2\pi}} ds\ e^{-s/M^2} \underbrace{\left[\frac{\sqrt{s}\ \Gamma_\rho(s)/\pi}{(m_\rho^2 - s)^2 + s\Gamma_\rho^2(s)}\right]}_{\xrightarrow{\Gamma_\rho \to 0}\ \delta(s - m_\rho^2)} \xrightarrow{\Gamma_\rho \to 0} 2 f_\rho m_\rho A_0^{B\rho}(q^2)\ e^{-s/m_\rho^2}$$

hep-ph/0611193 ✓

:: Quasi-two-body limit

This is **always** an improvement w.r.t. quasi-two-body decays:

$$\mathcal{A}(B^- \to \pi^-[\pi^+\pi^-]) = F^{B\to\pi} \; T_1 \star \phi_{\pi\pi} + F^{B\to\pi\pi} \; T_2 \star \phi_\pi$$

$$\Big\downarrow \quad \rho \text{ dominance } + \text{ zero-width limit}$$

$$\mathcal{A}(B^- \to \pi^-\rho) = F^{B\to\pi} \; T_1 \star \phi_\rho + F^{B\to\rho} \; T_2 \star \phi_\pi$$

This limit can be checked analytically.

- ▷ Factorization is at the same level of theoretical rigour for quasi-two-body and 3-body.
- ▷ Any model for $\phi_{\pi\pi}$ and $F^{B\to\pi\pi}$ satisfying axiomatic constraints and compatible with data (e.g. $e^+e^- \to \pi\pi$) replaces any notion of "ρ".

$B \to (\pi\pi)_\rho \pi$

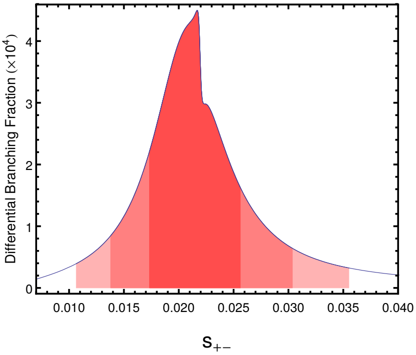
Krankl, Mannel, JV '15

- Leading order amplitude:

$$\mathcal{A}|_{s_{+-}\ll 1} = \frac{G_F}{\sqrt{2}}\left[4m_B^2 f_0(s_{+-})(2\zeta-1)F_\pi(s_{+-})(a_2+a_4) + f_\pi m_\pi (a_1-a_4)F_t(\zeta,s_{+-})\right]$$

- Integrating around the ρ:

$$BR(B^- \to \rho\,\pi^-) \simeq \int_0^1 ds_{++} \int_{s_\rho^-}^{s_\rho^+} ds_{+-}\ \frac{\tau_B\, m_B |\mathcal{A}|^2}{32(2\pi)^3}$$



with $s_\rho^\pm = (m_\rho \pm n\Gamma_\rho)^2/m_B^2$

$BR(B^+ \to \rho\pi^+) \simeq 9.4 \cdot 10^{-6} \quad (n = 0.5)$

$BR(B^+ \to \rho\pi^+) \simeq 12.8 \cdot 10^{-6} \quad (n = 1)$

$BR(B^+ \to \rho\pi^+) \simeq 14.1 \cdot 10^{-6} \quad (n = 1.5)$

$BR(B^+ \to \rho\pi^+)_{\rm EXP} = (8.3 \pm 1.2) \cdot 10^{-6}$

$BR(B^+ \to \rho\pi^+)_{\rm QCDF} = (11.9^{+7.8}_{-6.1}) \cdot 10^{-6}$

:: Edges – implications for CP violation

⋆ Leading order amplitude: Krankl, Mannel, JV '15

$$\mathcal{A}|_{s_{+-}\ll 1} = \frac{G_F}{\sqrt{2}}\left[4m_B^2 f_0(s_{+-})(2\zeta - 1)F_\pi(s_{+-})(a_2 + a_4) + f_\pi m_\pi (a_1 - a_4)F_t(\zeta, s_{+-})\right]$$

- ▷ The Wilson coefficients a_1, a_2 have weak phase $\sim \lambda_u$, and a_4 has weak phase $\sim \lambda_c$.
- ▷ Everything here is LO, so all perturbative strong phases are ignored.
- ▷ $F_\pi(s_{+-})$ and the P-wave contribution to $F_t(\zeta, s_{+-})$ have the same strong phase.
- ▷ S-wave contributions to $F_t(\zeta, s_{+-})$ can generate a strong phase (S- and P-wave interference).
- ▷ The corresponding "scalar-penguin" amplitude (power-suppressed but chirally enhanced) is in this case proportional to the scalar pion form factor. Its interference with the P-wave contribution to the F_t part may also potentially contribute a large strong phase.
- ▷ All these issues are under study.

:: OUTLINE

:: Summary and Challenges

Two body decays

- NNLO: End of the road for perturbative calculations
- Mostly ok, except for a few cases (color-suppressed tree, $\delta_{\pi K}, \ldots$)
  Large uncertainty from λ_B and power corrections.
- Challenge: Precise determination of λ_B from $B \to \gamma \ell \nu$ (Belle-II).
- Challenge: Power corrections. Factorization in SCET-2.

Three body decays

- Lots of data, great potential.
- Can be studied within QCDF. Need 2π LCDA's and $B \to MM$ form factors.
- $B \to VP$: include finite-width effects and contributions from excited resonances.
- Challenge: Full analysis at NLO, including CPV.
- Challenge: Soft corners need alternative treatment. These regions include interferences from "crossed" resonances, potentially interesting for localized CP asymmetries.
- $B \to \pi\pi$ Form factors: the same approach can be applied to $B \to K\pi$ form factors:
  Important for $B \to K^* \ell\ell$!!!

Backup Slides

Kinematics: $B^-(p) \to \pi^+(k_1)\pi^-(k_2)\ell^-(q_1)\bar{\nu}(q_2)$

$$k^2 = (k_1 + k_2)^2\,,\ q^2 = (q_1 + q_2)^2\,,\ 2q \cdot (k_1 - k_2) = \beta_\pi \sqrt{\lambda} \cos\theta_\pi$$

$$\mathcal{L} = \mathcal{L}_{QED+QCD} + \mathcal{C}_{LL}\,[\bar{u}\gamma^\mu P_L b]\,[\bar{\ell}\gamma_\mu P_L \nu_\ell] + \cdots$$

$$\mathcal{A} = \mathcal{C}_{LL}\ \langle \ell\bar{\nu}|\bar{\ell}\gamma_\mu P_L \nu_\ell|0\rangle\ \langle \pi^+\pi^-|\bar{u}\gamma^\mu P_L b|B^-\rangle = \mathcal{C}_{LL}\ \mathcal{F}^\mu\ \bar{u}_\ell \gamma_\mu v_\nu$$

$\to \mathcal{F}^\mu$: $B \to \pi\pi$ **form factor** (one axial, three vector invariant FFs):

$$\varepsilon(q,0)^*_\mu \langle \pi\pi|\bar{u}\gamma^\mu P_L b|B\rangle = F_0$$

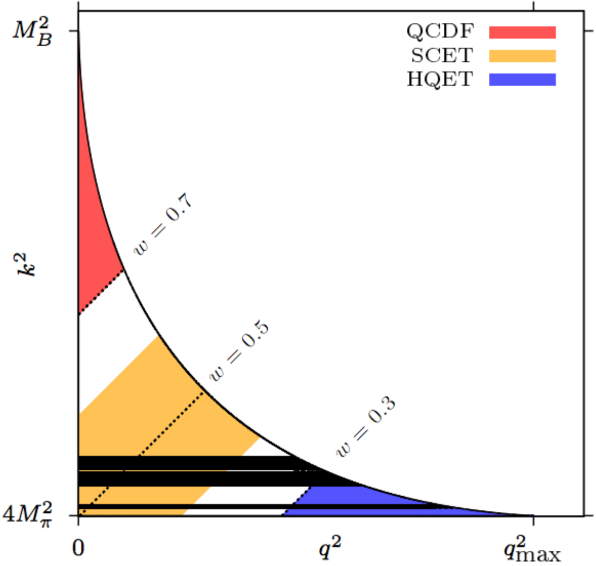
$$\varepsilon(q,t)^*_\mu \langle \pi\pi|\bar{u}\gamma^\mu P_L b|B\rangle = F_t$$

$$\varepsilon(q,\pm)^*_\mu \langle \pi\pi|\bar{u}\gamma^\mu P_L b|B\rangle = \beta_\pi \sin\theta_\pi e^{\pm i\phi}(F_\perp + F_\parallel)/\sqrt{2}$$

where $F_i = F_i(q^2, k^2, \theta_\pi) = \sum_\ell F_i^{(\ell)}(q^2, k^2)\, P_\ell(\cos\theta_\pi)$ [partial waves in $\pi\pi$]

:: $B \to \pi\pi\ell\nu$

Faller et al 2013

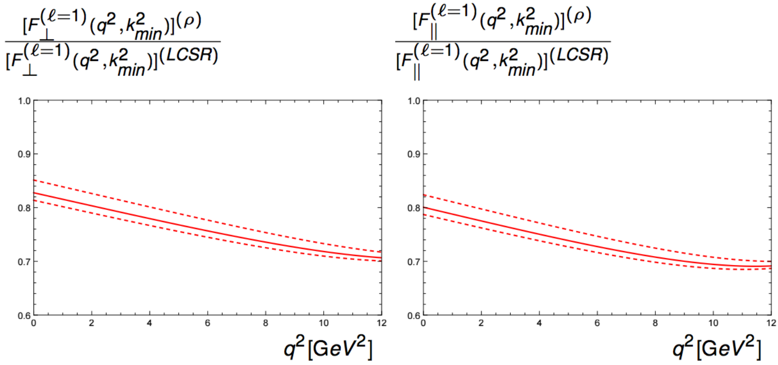


:: $B \to \pi\pi$ form factors from LCSRs

Hambrock, Khodjamirian 2015

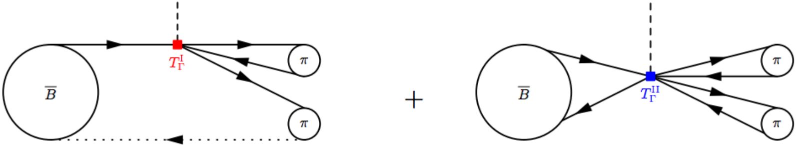
$\to$ Light-cone sum rule with 2π distribution amplitudes:

- ▷ **Sample result**: ρ contribution to the total vector form factor



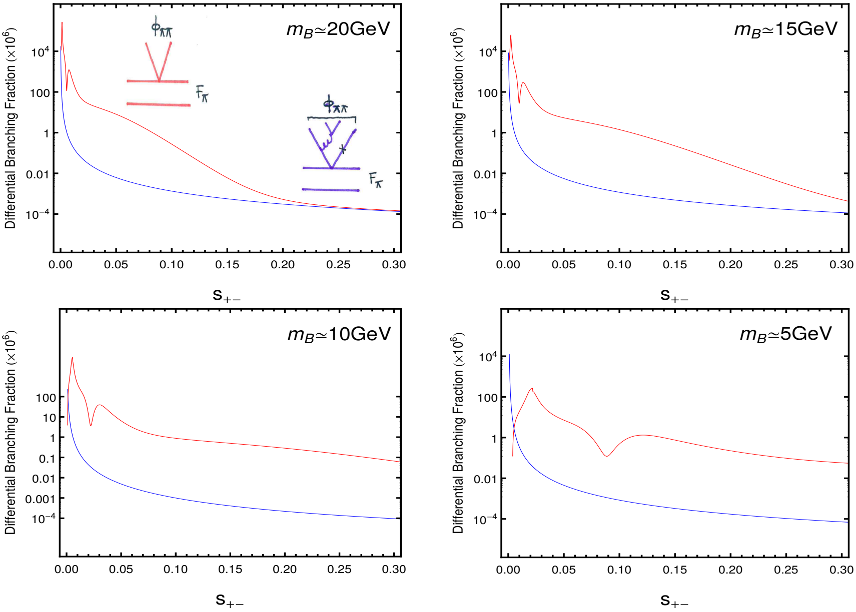
Factorization at large k^2 : $F_i = f_{B\pi}\ T_i^I \otimes \phi_\pi + T_i^{II} \otimes \phi_\pi \otimes \phi_\pi \otimes \phi_B$

$$\begin{aligned}
&\langle \pi^+(k_1)\pi^-(k_2)|\overline{\psi}_u\ \Gamma\ \psi_b|B^-(p)\rangle \\
&= \frac{2\pi f_\pi \xi_\pi(E_2;\mu)}{k^2} \int_0^1 du\, \phi_\pi(u,\mu)\, T_\Gamma^{\rm I}(u,\ldots;\mu) \\
&\quad + \int_0^1 du \int_0^1 dv \int_0^\infty \frac{d\omega}{\omega}\, \phi_\pi(u;\mu)\, \phi_\pi(v;\mu)\, \phi_B^+(\omega;\mu)\, T_\Gamma^{\rm II}(u,v,\omega,\ldots;\mu) \\
&\quad + \text{power corrections}\,.
\end{aligned}$$

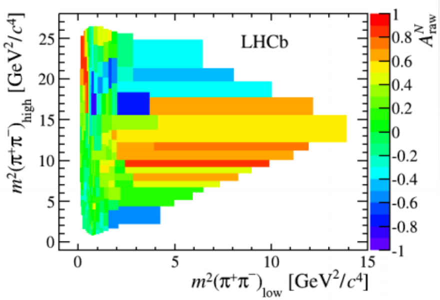


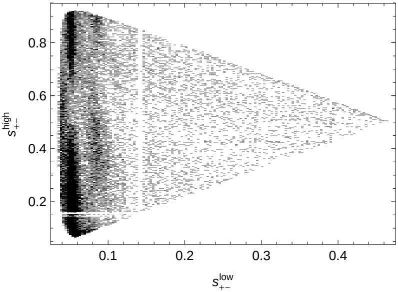
ξ_π denotes the universal non-factorizable $B \to \pi$ form factor in SCET

Merging Regions: How large should m_B be? ($\phi_{\pi\pi}$ term)



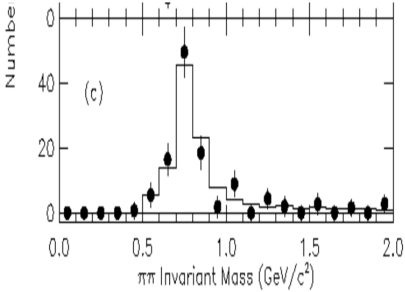
:: Local CP asymmetries

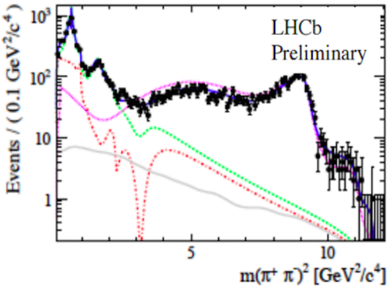




- We **do not** want just *a model* that fits well.
- Instead we want to **know** if CKM+QCD is compatible with the data.

:: $B \to D\pi\pi$





Left: the current source for $B^- \to D^0\rho^-$ (CLEO). Right: $B \to D\pi^+\pi^-$ (LHCb).