Improvements to sum rules predictions for V_{ub}

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CKM workshop, Mumbai

28th November 2016

Introduction to exclusive V_{ub}

- Uncertainty on $|V_{ub}|^{\mathbf{incl}} \sim 7\%$ ($< 2\%$ on $|V_{cb}|^{\mathbf{incl}}$) due to large $b \to c\ell\nu$ background
- Competitive $|V_{ub}|^{\mathbf{excl}}$ from $B \to \pi\ell\nu$, depends on $f_+(q^2)$ (as $m_l \to 0$) from Lattice QCD ($q^2 \gtrsim 15\,\mathrm{GeV}^2$) or QCD sum rules on the light-cone (LCSR) ($q^2 \lesssim 6 - 7\,\mathrm{GeV}^2$)
- Also possible via other B decays, e.g. recent progress in $B \to \rho\ell\nu$, $\Lambda_b \to p\ell\nu$, $B_s \to K\ell\nu$

Obtaining the form factor in Light-cone sum rules:

$$\Pi_\mu = i\, m_b \int d^D x e^{-i p_B \cdot x} \langle \pi(p) | T\{\bar{u}(0)\gamma_\mu b(0) \bar{b}(x) i\gamma_5 d(x)\} | 0 \rangle ,$$
$$= (p_B + p)_\mu \Pi_+(p_B^2, q^2) + (p_B - p)_\mu \Pi_-(p_B^2, q^2).$$

into

$B \to \pi$ transition ($f_+(q^2)$)

$$\langle \pi(p) | \bar{u}\gamma_\mu b | B(p_B) \rangle = (p_B + p)_\mu f_+(q^2) + (p_B - p)_\mu f_-(q^2)$$

B meson decay (f_B)

$$m_b \langle 0 | \bar{d} i\gamma_5 b | B \rangle = m_B^2 f_B$$

Leading to:

$$\Pi_+(p_B^2, q^2) = f_B m_B^2 \frac{f_+(q^2)}{m_B^2 - p_B^2} + \int_{s>m_B^2} ds \frac{\rho_{\rm had}}{s - p_B^2},$$

($\rho_{\rm had}$ is spectral density of the higher-mass hadronic states)

On the other hand:

Light-cone expand about $x^2 = 0 \;\; \Rightarrow$

$\Pi_+(p_B^2, q^2) = \sum_n \int du\, \mathcal{T}_+{}^{(n)}(u, p_B^2, q^2, \mu^2)\phi^{(n)}(u, \mu^2) = \int ds \frac{\rho_{\rm LC}}{s - p_B^2},$

$\mathcal{T}_+^{(n)}(u, \mu^2)$: perturbatively calculable hard kernels

$\phi^{(n)}(u, \mu^2)$: non-perturbative LCDAs at twist n

e.g. n=2, $\langle \pi(p)|\bar{u}(0)\gamma_\mu\gamma_5\, d(x)|0\rangle = -i f_\pi p_\mu \int_0^1 du\, e^{i\bar{u}p\cdot x}\phi(u, \mu^2) + \ldots,$

where $\phi(u, \mu^2) = 6u(1-u)\sum_{n=0}^{\infty} a_n(\mu^2) C_n^{3/2}(2u-1)$

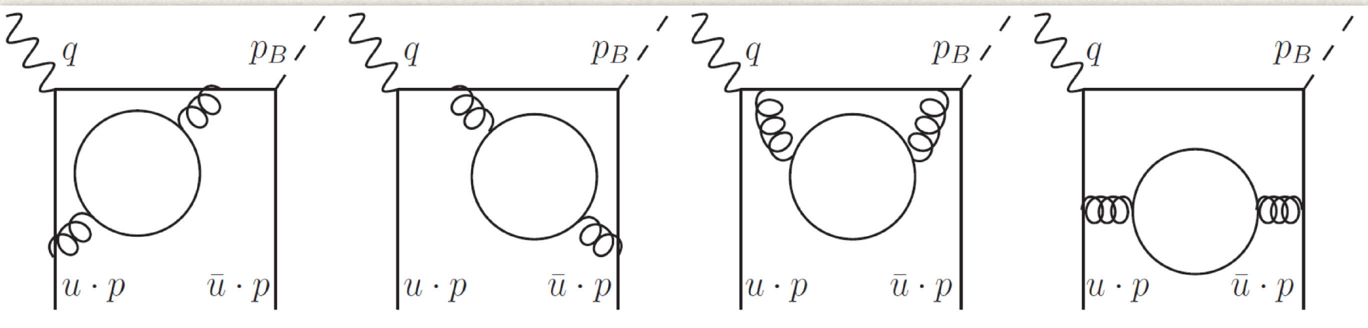
Sum rule for $f_+(q^2)$: $f_+(q^2) = \frac{1}{f_B m_B^2} \int_{m_b^2}^{s_0} ds\, \rho_{\rm LC}\, e^{-(s-m_B^2)/M^2}$

Status of $f_+(q^2)$ for $B\to\pi$ < 2012

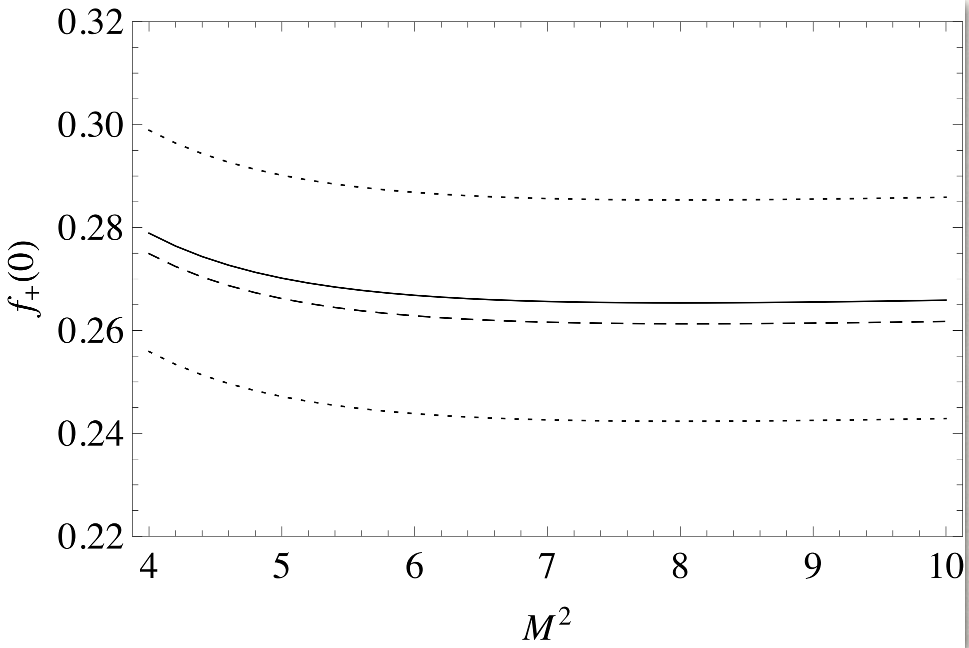
- 1997: NLO twist-2 corrections were calculated (A. Khodjamirian et al, [arXiv:hep-ph/9706303]; E. Bagan, P. Ball and V. M. Braun, [arXiv:hep-ph/9709243])
- 2000: LO corrections up to twist-4 were calculated (A. Khodjamirian et al, [arXiv:hep-ph/0001297])
- 2004: NLO twist-3 corrections (P. Ball and R. Zwicky, [arXiv:hep-ph/0406232])
- 2008: $\overline{\rm MS}$ m_b is used in place of the pole mass (G. Duplancic et al, 2008)
- 2011: Use a_2, a_4 from F_π, LCSR+new JLab, Extrapolate by fitting to BCL q^2 parameterisation (A. Khodjamirian, T. .Mannel, N. Offen, Y. -M. Wang, [arXiv:1103.2655])

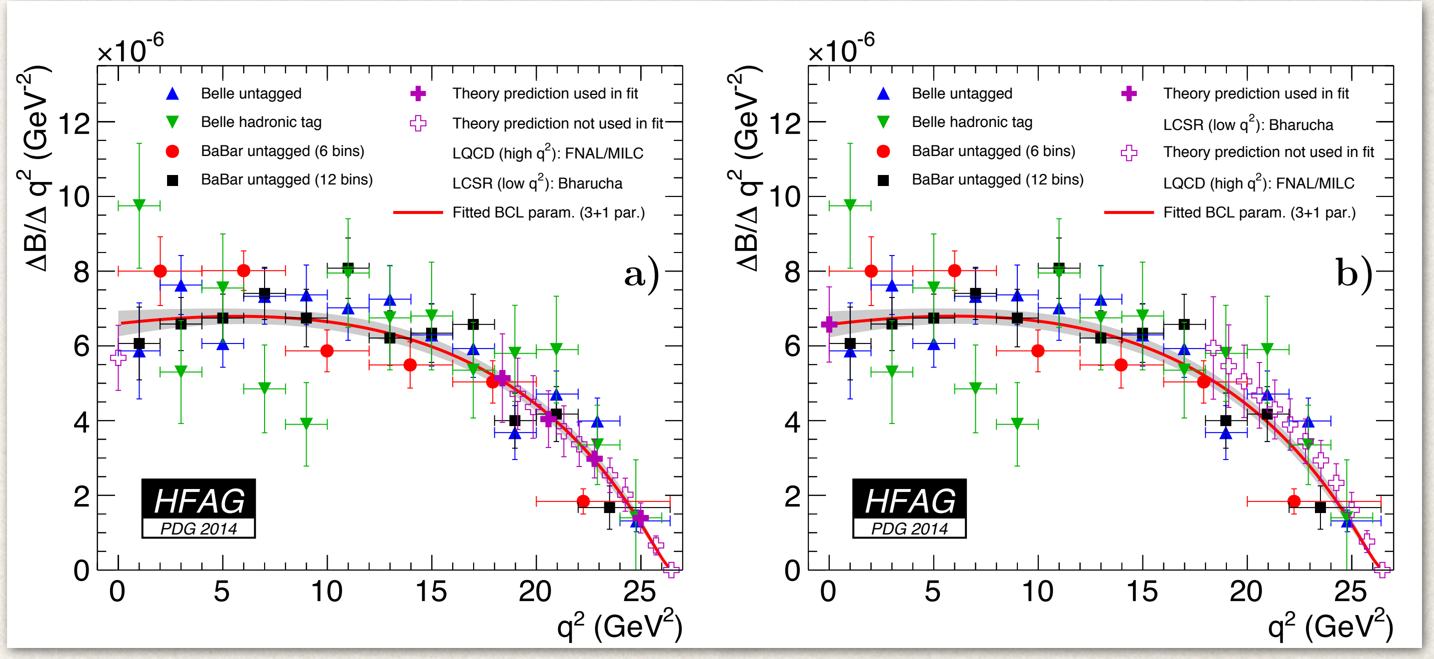
Two-loop corrections

(A. Bharucha 1203.1359)



- Test argument that radiative corrections to $f_+ f_B$ and f_B should cancel when both calculated in sum rules (2-loop contribution to f_B in QCDSR sizeable) $\Rightarrow$ *Calculate subset of two-loop radiative corrections for twist-2 contribution to* $f_+(0) \propto \beta_0$
- $f_+(0)$ $(0.262^{+0.020}_{-0.023})$ at $\mathcal{O}(\alpha_s^2 \beta_0)$ (solid) with uncertainties $\lesssim 9\%$ (dotted), compared to $\mathcal{O}(\alpha_s)$ result (dashed), as a function of Borel parameter M^2
- Despite $\sim 9\%$ $\mathcal{O}(\alpha_s^2 \beta_0)$ corrections to f_B, change in $f_+(0)$, only $\sim 2\%$

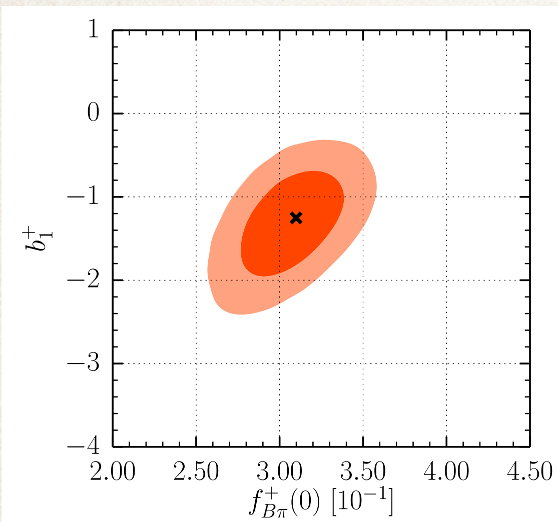


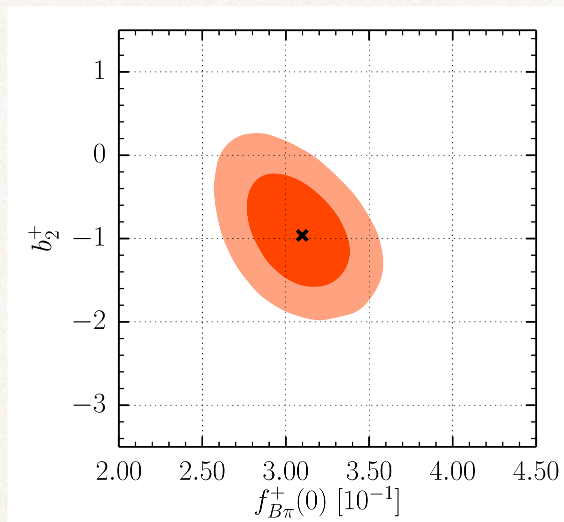


HFAG 2014 (lattice) [22]	3.28 ± 0.29
HFAG 2014 (LCSR) [138, 22]	3.53 ± 0.29

Extrapolation and unitarity bounds for the B → π form factor

(I. S. Imsong, A. Khodjamirian, T. Mannel, D. van Dyk, 1409.7816)





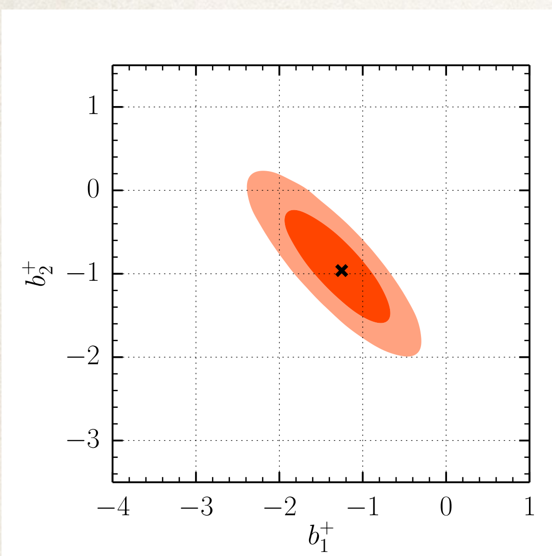


Figure 1. The regions with 68% probability (red) and 95% probability (orange) for all two-dimensional marginalisations of the posterior $P(\vec{\lambda}|\mathrm{LCSR})$. The cross marks the best-fit point.

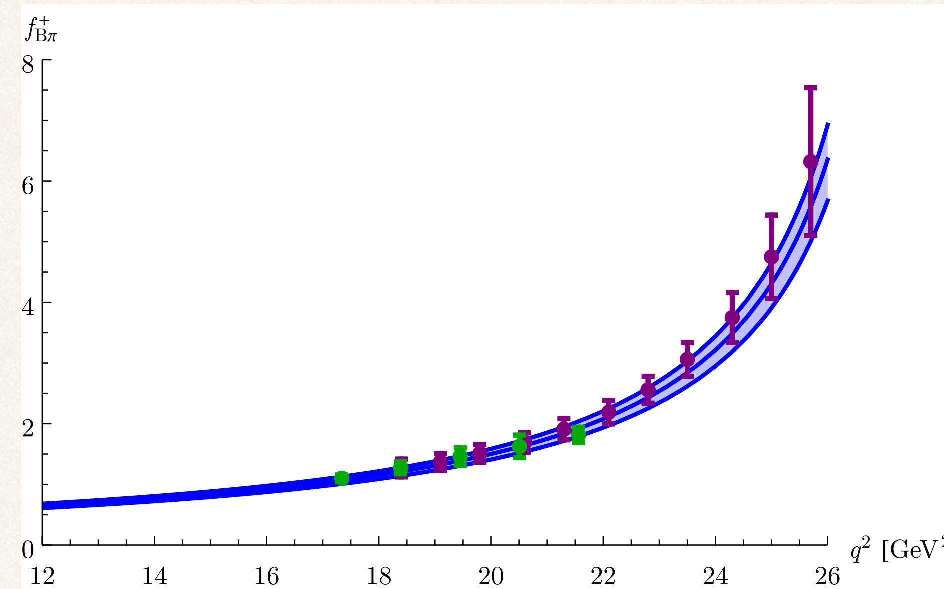


Figure 2. Form factor $f^+_{B\pi}(q^2)$ obtained at $q^2 < 12\mathrm{GeV}^2$ from the statistical analysis of LCSR, fitted to z-series representation and extrapolated to large q^2. The solid lines correspond to the 68% probability envelope and the best fit curve. The green (magenta) points are HPQCD [7] (Fermilab-MILC [8]) lattice QCD results.

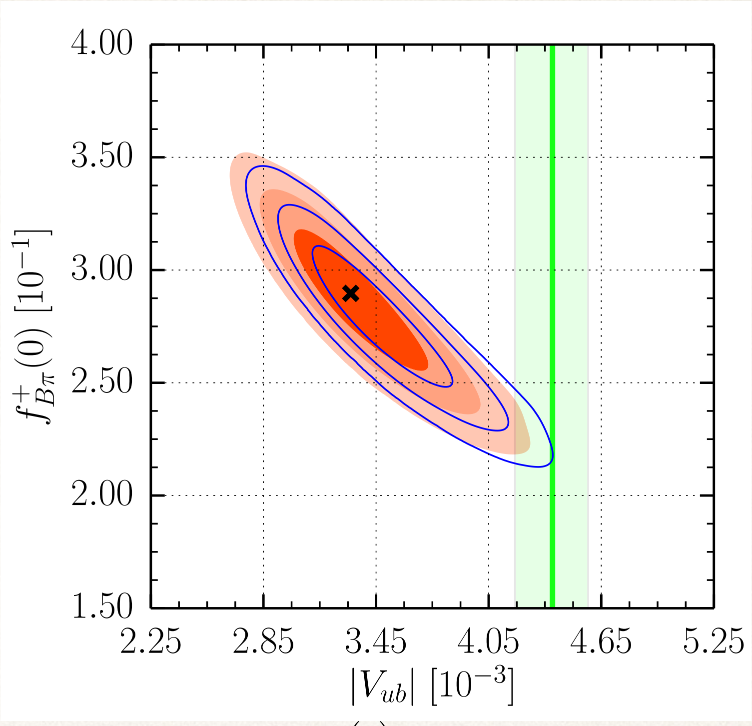
- Use Bayesian analysis: prior distributions for inputs, construct likelihood function based on SR fulfilling mB to 1%, obtain posterior distributions using Bayes theorem
- Posterior distributions of inputs only different for s_0 : (41±4) GeV^2(~gaussian)
- Fit to BCl exp, find central value of $f_+(0) = 0.31±0.02$: raised due to value m_b, s_0, μ
- Obtaining $f_+(q^2)$ and first two derivatives at 0 and 10 GeV^2 allowed extrapolation to high q2 using improved unitarity bounds

- Perform Bayesian analysis including experimental results to obtain $|V_{ub}|$

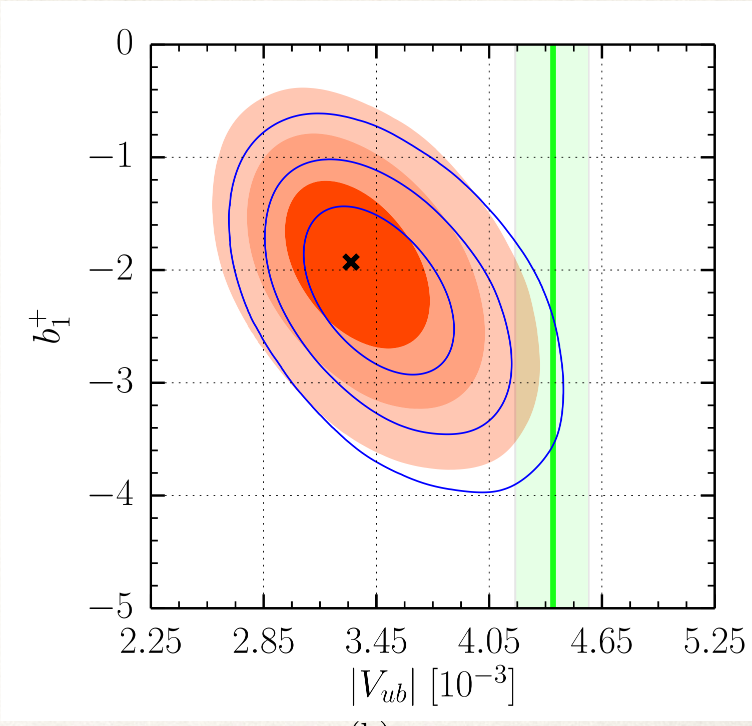
- Theory uncertainty on $|V_{ub}|$ obtained from analysis comparable to that of most accurate determinations from inclusive $b \to u$ transitions

- 2010 data set agrees better with theory 2013 data set

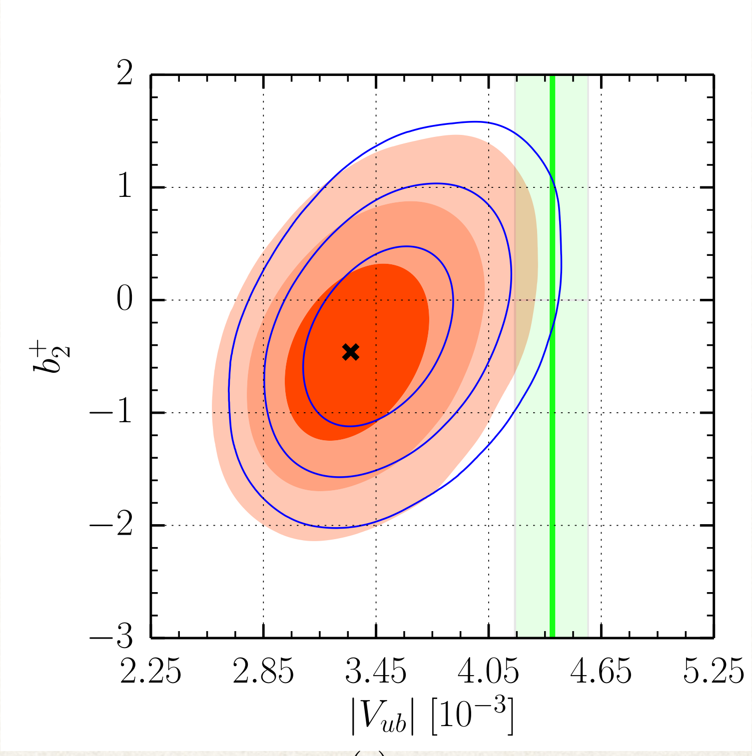
- Tension wrt GGOU determination seen beyond 99% C.L.



(a)



(b)



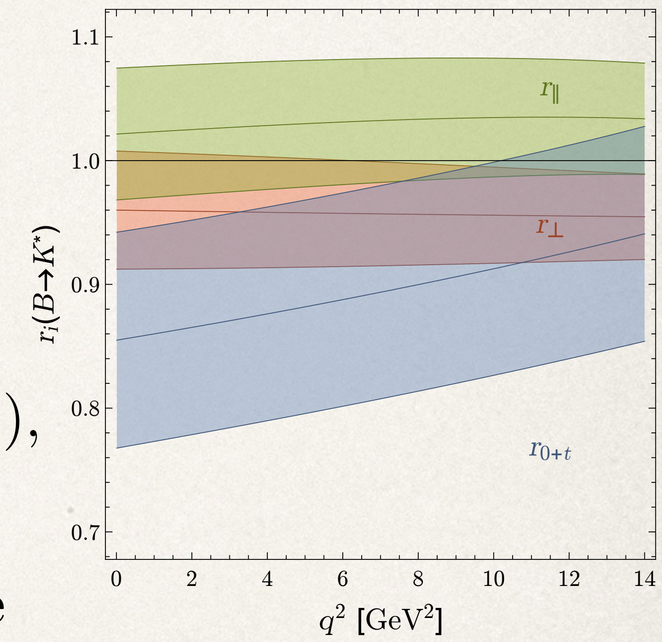
(c)

Figure 4. The two-dimensional marginal posteriors for $|V_{ub}|$ versus the BCL parameters (a) $f^+_{B\pi}(0)$, (b) b^+_1, and (c) b^+_2. The dark orange, orange, and light orange regions show, respectively, the 68%, 95% and 99% probability regions when using the "2013" data set. The blue contours delineate the corresponding probability regions of the "2010" data set. The green and light green vertical bands denote the central value and 68% CL interval of the HFAG world average [39] of the $|V_{ub}|$ determinations from inclusive decays $B \to X_u \ell \bar{\nu}$ according to the GGOU method [40].

(I. S. Imsong, A. Khodjamirian, T. Mannel, D. van Dyk, 1409.7816)

Update for V_{ub} from B to V
(A. Bharucha, D. Straub and R. Zwicky 1503.05534)

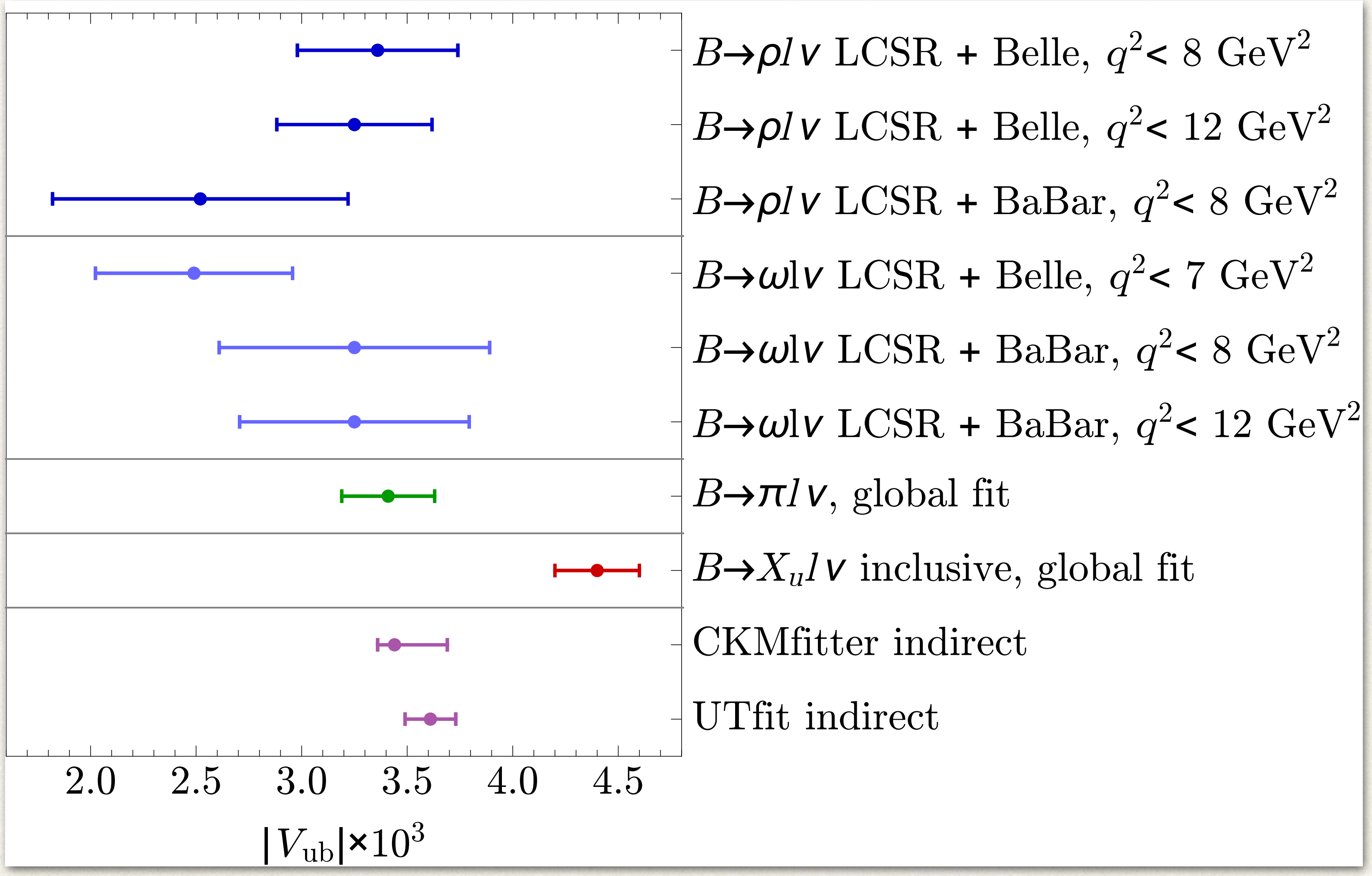
- Largest uncertainty in calculation is from form factors
- Best coverage in q^2: fit to LCSR/Lattice using series expansion, coefficients satisfy dispersive bounds.(AB, T. Feldmann, M. Wick, arXiv:1004.3249)
- **Our Aim:** improve uncertainty by making correlations available
- We obtain the four equation of motion relations:
  e.g. $T_1(q^2) + (m_b + m_s)\mathcal{V}_1(q^2) + \mathcal{D}_1(q^2) = 0$
- Isgur-Wise relations at low recoil follow from $\mathcal{D}_\iota/(\mathcal{V}_\iota \text{ or } T_\iota) \sim \mathcal{O}(\Lambda_{\rm QCD}/m_b)$, $\mathcal{D}_\iota$ is derivative FF, breaking of I-W relations.
- Certain combinations of $\mathcal{D}_\iota$'s may be small at large recoil: $\iota = 1, 2$ are direct candidates, and combinations of $\iota = 3, P$ result in potentially small ratio of $\mathcal{D}/T$



See that equation of motion works well in most cases, apply same values of sum rules parameters for related FFs with correlations, less correlated for 0+t case

Comparison of results for Vub

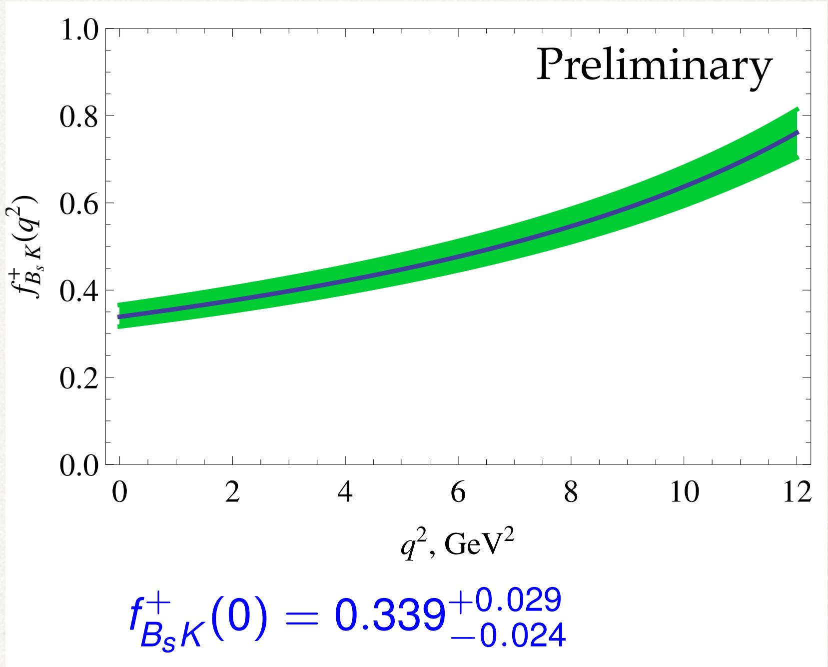
Good agreement with B to π and results from global fits



Results slightly below but larger uncertainties Note for B to ρ the S-wave contribution needs to be systematically subtracted by Belle and BaBar, which could lead to a 6% upward shift in V_{ub} (see Meissner and Wang, 1305.1311.5420).

Future Prospects

- $B_s \to K l \nu$: Preliminary uncorrelated results for $f_+(0)$ from Khodjamirian, Rusov '16, measurement at LHCb(?)/Belle II



Khodjamirian, Rusov (in prep)

- Twist 5,6 term in factorizable approximation [A. Rusov, in prep] are small, however still missing twist-3 $O(\alpha_s)$ corrections-not so easy!

- π, K DAs from LCSRs: BESS and Belle-2 data on $\gamma * \gamma \to \pi_0$; JLab data on $F_{\pi/K}$

- Future Belle-2 data on the q2-shape of $B \to \pi l \nu$ will provide additional constraints on the DA parameters

Summary: exclusive $|V_{ub}|$ in 2016

The latest HFAG simultaneous fit uses two-loop $f_+(0)$ from LCSR (AB 1203.1359)

TABLE 2. Status of exclusive $|V_{ub}|$ determinations and indirect fits

Exclusive decays	$\lvert V_{ub}\rvert \times 10^3$
$\bar{B} \to \pi l \bar{\nu}_l$	
FLAG 2016 [21]	3.62 ± 0.14
Fermilab/MILC 2015 [131]	3.72 ± 0.16
RBC/UKQCD 2015 [132]	3.61 ± 0.32
HFAG 2014 (lattice) [22]	3.28 ± 0.29
HFAG 2014 (LCSR) [138, 22]	3.53 ± 0.29
Imsong et al. 2014 (LCSR, Bayes an.) [143]	$3.32^{+0.26}_{-0.22}$
Belle 2013 (lattice + LCSR) [126]	3.52 ± 0.29
$\bar{B} \to \omega l \bar{\nu}_l$	
Bharucha et al. 2015 (LCSR) [146]	$3.31 \pm 0.19_{\rm exp} \pm 0.30_{\rm th}$
$\bar{B} \to \rho l \bar{\nu}_l$	
Bharucha et al. 2015 (LCSR) [146]	$3.29 \pm 0.09_{\rm exp} \pm 0.20_{\rm th}$
$\Lambda_b \to p \mu \nu_\mu$	
LHCb (PDG) [147]	3.27 ± 0.23
Indirect fits	
UTfit (2016) [94]	3.74 ± 0.21
CKMfitter (2015, 3σ) [95]	$3.71^{+0.17}_{-0.20}$

New LCSR result for V_{ub} from $B \to \rho l \nu$ with comparable errors (AB, Straub, Zwicky 1503.05534)

Lower result for V_{ub} from the Bayesian analysis (Imsong, Khodjamirian, Mannel, van Dyck1409.7816)