Higher dimensional HQET parameters

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[Ayesh Gunawardana, GP *to appear*]

Outline

- Motivation
- A little bit of history
- HQET (and NRQCD) operators at dimension 8 and above
- Conclusions

Motivation

Motivation

- Inclusive semileptonic B decays and
  $Q_{7\gamma} - Q_{7\gamma}$ contribution to $B \to X_s\gamma$
  can be described by local OPE

$$\Gamma = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_k c_{k,n} \langle O_{k,n} \rangle$$

- $c_{k,n}$ are Wilson coefficients: perturbative

- $\langle O_{k,n} \rangle$ are matrix elements of HQET operators: non-perturbative
  Often called HQET parameters
  $O_{k,n} \sim \bar{h}\, iD^{\mu_1} \ldots iD^{\mu_n}(s^\lambda)h \cdot T_{\mu_1 \ldots \mu_n \lambda}$, T is a Lorentz tensor

- $|V_{cb}|$ extraction from inclusive B decays
  uses dimension 7 and 8 HQET operators
  [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

Motivation

- $|V_{cb}|$ extraction from inclusive B decays
  uses dimension 7 and 8 HQET operators
  [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

- Question:
  Are these *all* possible HQET operators at dimension 7,8?

- Question: Can we go higher?

- Question:
  Since HQET and NRQCD are related,
  [Manohar PRD **56**, 230 (1997), Gunawardana, GP *to appear*]
  What are the NRQCD operators?

- Question:
  What can we learn about the structure of effective field theories?
  ["Higher dimension operators in the SM EFT"
  Henning, Lu, Melia, Murayama, arXiv:1512.03433]

A little bit of history

Prehistory

$$D_t = \frac{\partial}{\partial t} + ieA^0, \quad \mathbf{D} = \boldsymbol{\nabla} - ie\mathbf{A}$$

- Schrödinger equation: $iD_t + \frac{\mathbf{D}^2}{2M}$

- Hydrogen Fine Structure:
  - Spin-Orbit: $\boldsymbol{\sigma} \cdot \mathbf{B}$
  - Relativistic correction: $\mathbf{D}^4$
  - Darwin term: $\boldsymbol{\partial} \cdot \mathbf{E}$

- Organize operators in Lagrangian form

- The dim=5,6 were given in [Caswell, Lepage PLB **167**, 437 (1986)]

$$\mathcal{L}_{\text{NRQED}}^{\text{dim=5,6}} = \psi^\dagger \left\{ iD_t + \frac{\mathbf{D}^2}{2M} + \frac{\mathbf{D}^4}{8M^3} + c_F g \frac{\boldsymbol{\sigma} \cdot \mathbf{B}}{2M} + c_D g \frac{[\boldsymbol{\partial} \cdot \mathbf{E}]}{8M^2} + i c_S g \frac{\boldsymbol{\sigma} \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D})}{8M^2} + c_{W1} g \frac{\{\mathbf{D}^2, \boldsymbol{\sigma} \cdot \mathbf{B}\}}{8M^3} \right\} \psi$$

1994

- First systematic discussion of HQET parameters
  [Mannel, PRD 50, 428 (1994)]

- Between HQET fields $\bar{h}\ldots h$ the Dirac basis reduces to $\{1, \boldsymbol{\sigma}\} = \{1, s^\lambda\}$ with v.s=0

- $\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n}(s^\lambda)h$ is the general operator
  Since $iv\cdot Dh = 0$

- $v_{\mu_1}\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n}(s^\lambda)h = 0$
- $v_{\mu_n}\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n}(s^\lambda)h = 0$
- $v_\lambda\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n}(s^\lambda)h = 0$

- Dimension 3: $\bar{h}h$
- Dimension 4: $\bar{h}iD^\mu h = 0$
- Dimension 5: Two operators $\bar{h}\, iD^{\mu_1}iD^{\mu_2}h$, $\bar{h}iD^{\mu_1}iD^{\mu_2}s^\lambda h$
- Dimension 6: Two operators $\bar{h}\, iD^{\mu_1}iD^{\mu_2}iD^{\mu_3}h$, $\bar{h}iD^{\mu_1}iD^{\mu_2}iD^{\mu_3}s^\lambda h$
- Dimension 7: Wrong decomposition

1994

- First systematic discussion of HQET parameters [Mannel, PRD 50, 428 (1994)]

- Dimension 5:

$$\langle B(v)|\bar{h}(iD_\alpha)(iD_\beta)h|B(v)\rangle = 2M_H[g_{\alpha\beta} - v_\alpha v_\beta]\frac{1}{3}\lambda_1$$
$$\langle B(v)|\bar{h}(iD_\alpha)(iD_\beta)s_\lambda h|B(v)\rangle = 2M_H d_H i\varepsilon_{\nu\alpha\beta\lambda}v^\nu\frac{1}{6}\lambda_2$$

- Dimension 6:

$$\langle B(v)|\bar{h}(iD_\alpha)(iD_\mu)(iD_\beta)h_v|B(v)\rangle = 2M_H[g_{\alpha\beta} - v_\alpha v_\beta]v_\mu\frac{1}{3}\rho_1$$
$$\langle B(v)|\bar{h}(iD_\alpha)(iD_\mu)(iD_\beta)s_\lambda h|B(v)\rangle = 2M_H d_H i\varepsilon_{\nu\alpha\beta\lambda}v^\nu v_\mu\frac{1}{6}\rho_2.$$

- Dimension 7: Wrong decomposition

HQET-NRQCD (NRQED) correspondence

- HQET: Heavy Quark Effective Theory and
  NRQCD: Non Relativistic Quantum Chromo Dynamics
  differ in the kinetic term

$$\mathcal{L}^{kinetic}_{HQET} = \bar{h} iv\cdot D\, h$$
$$\mathcal{L}^{kinetic}_{NRQCD} = \psi^\dagger iD_t + \frac{\mathbf{D}^2}{2M}\psi$$

  and power counting
- Correspondence between HQET and NRQCD (NRQED) operators
  [Manohar PRD **56**, 230 (1997); Gunawardana, GP *to appear*]

	NRQED (1920's-1980's)	HQET(1990's)
Dimension 5	$\mathbf{D}^2$	$(iD_\perp)^2$
	$\boldsymbol{\sigma}\cdot\mathbf{B}$	$(iD_\perp^\mu)(iD_\perp^\nu)(-i\sigma^{\mu\nu})$
Dimension 6	$[\partial\cdot\mathbf{E}]$	$(iD_{\perp\mu})(iv\cdot D)(iD_\perp^\mu)$
	$\boldsymbol{\sigma}\cdot(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})$	$(iD_\perp^\mu)(iv\cdot D)(iD_\perp^\nu)(-i\sigma^{\mu\nu})$

where $D_\perp^\mu = D^\mu - v\cdot D v^\mu$

1997

- The dim=7 NRQCD Lagrangian was given in [Manohar PRD **56**, 230 (1997)]

$$\mathcal{L}_{\text{NRQED}}^{\text{dim}=7} = \psi^\dagger \Big\{ \frac{\mathbf{D}^4}{8M^3} + i c_M g \frac{\{\mathbf{D}^i, [\partial \times \mathbf{B}]^i\}}{8M^3} + c_{A1} g^2 \frac{\mathbf{B}^2 - \mathbf{E}^2}{8M^3} - c_{A2} g^2 \frac{\mathbf{E}^2}{16M^3} + $$
$$+ c_{p'p} g \frac{\boldsymbol{\sigma} \cdot \mathbf{D} \mathbf{B} \cdot \mathbf{D} + \mathbf{D} \cdot \mathbf{B} \boldsymbol{\sigma} \cdot \mathbf{D}}{8M^3}$$
$$c_{W1} g \frac{\{\mathbf{D}^2, \boldsymbol{\sigma} \cdot \mathbf{B}\}}{8M^3} - c_{W2} g \frac{D^i \boldsymbol{\sigma} \cdot \mathbf{B} D^i}{4M^3} +$$
$$+ i c_{B1} g^2 \frac{\boldsymbol{\sigma} \cdot (\mathbf{B} \times \mathbf{B} - \mathbf{E} \times \mathbf{E})}{8M^3} - i c_{B2} g^2 \frac{\boldsymbol{\sigma} \cdot (\mathbf{E} \times \mathbf{E})}{8M^3} \Big\} \psi$$

- Last line is non-zero only for NRQCD
- The operators are listed, but not derived
- NRQCD dimension 7:
  - 4 SI (spin-independent) operators
  - 5 SD (spin-dependent) operators

Dimension 7 HQET operators

- How many HQET operators do we have at dimension 7?

1)	[Mannel, PRD 50, 428 (1994)]	2 SI operators	5 SD operators
2)	[Manohar PRD **56**, 230 (1997)]	4 SI operators	5 SD operators
3)	[Dassinger, Mannel, Turczyk JHEP **0703**, 087 (2007)]	3 SI operators	2 SD operators
4)	[Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]	4 SI operators	5 SD operators

- HQET-NRQCD correspondance implies 4 SI and 5 SD operators

- No systematic derivation in either source

Dimension 8 HQET operators

- What about dimension 8?
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]
  7 SI operators and 11 SD operators
  No systematic derivation
- The dim=8 NRQED Lagrangian was given in
  [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

$$\mathcal{L}_{\text{NRQED}}^{\text{dim=8}} = \psi^\dagger \left\{ c_{X1} g \frac{[\mathbf{D}^2, \mathbf{D}\cdot\mathbf{E} + \mathbf{E}\cdot\mathbf{D}]}{M^4} + c_{X2} g \frac{\{\mathbf{D}^2, [\boldsymbol{\partial}\cdot\mathbf{E}]\}}{M^4} \right.$$
$$+ c_{X3} g \frac{[\boldsymbol{\partial}^2 \boldsymbol{\partial}\cdot\mathbf{E}]}{M^4} + i c_{X4} g^2 \frac{\{\mathbf{D}^i, [\mathbf{E}\times\mathbf{B}]^i\}}{M^4}$$
$$+ i c_{X5} g \frac{D^i \boldsymbol{\sigma}\cdot(\mathbf{D}\times\mathbf{E} - \mathbf{E}\times\mathbf{D}) D^i}{M^4} + i c_{X6} g \frac{\epsilon^{ijk}\sigma^i D^j [\boldsymbol{\partial}\cdot\mathbf{E}] D^k}{M^4}$$
$$+ c_{X7} g^2 \frac{\boldsymbol{\sigma}\cdot\mathbf{B}[\boldsymbol{\partial}\cdot\mathbf{E}]}{M^4} + c_{X8} g^2 \frac{[\mathbf{E}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\mathbf{B}]}{M^4} + c_{X9} g^2 \frac{[\mathbf{B}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\mathbf{E}]}{M^4}$$
$$\left. + c_{X10} g^2 \frac{[E^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} B^i]}{M^4} + c_{X11} g^2 \frac{[B^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} E^i]}{M^4} + c_{X12} g^2 \frac{\boldsymbol{\sigma}\cdot\mathbf{E}\times[\partial_t \mathbf{E} - \boldsymbol{\partial}\times\mathbf{B}]}{M^4} \right\} \psi$$

4 SI operators and 8 SD operators
Missing operators are presumably NRQCD operators
No systematic derivation

Lesson from History

- Finding all of the HQET operators at a given dimension is not easy
- Is there a systematic way to do that?

HQET (and NRQCD) operators at dimension 8 and above

[Ayesh Gunawardana, GP *to appear*]

General method

- We consider matrix elements of the form
  $\langle B(v)|\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n} h|B(v)\rangle$
  $\langle B(v)|\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n} s^\lambda h|B(v)\rangle$

- We use
- Parity and Time reversal symmetry
- Hermitian conjugation
- Four dimensions

To decompose them in terms of tensors

- v^{μ_i}
- $g^{\mu_i \mu_j}$
- $\epsilon^{\alpha\beta\rho\sigma}$

General method: PT symmetry

- We consider matrix elements of the form
  $\langle B(v)|\bar{h}\, iD^{\mu_1}\ldots iD^{\mu_n} h|B(v)\rangle$
  $\langle B(v)|\bar{h}\, iD^{\mu_1}\ldots iD^{\mu_n} s^\lambda h|B(v)\rangle$

- Parity and Time reversal are symmetries of HQET
  In particular under PT:

- $p=(p^0,\vec{p}\,)\overset{PT}{\to}(p^0,\vec{p}\,)=p \Rightarrow v=p/m \overset{PT}{\to} v$
- $iD^\mu \overset{PT}{\to} iD^\mu$
- $\bar{h}h \overset{PT}{\to} \bar{h}h$
- $\bar{h}s^\lambda h \overset{PT}{\to} -\bar{h}s^\lambda h$

- Since T is anti-linear

$$\langle B(v)|\bar{h}\, iD^{\mu_1}\ldots iD^{\mu_n} h|B(v)\rangle \overset{PT}{=} \langle B(v)|\bar{h}\, iD^{\mu_1}\ldots iD^{\mu_n} h|B(v)\rangle^*$$

$$\langle B(v)|\bar{h}\, iD^{\mu_1}\ldots iD^{\mu_n} s^\lambda h|B(v)\rangle \overset{PT}{=} -\langle B(v)|\bar{h}\, iD^{\mu_1}\ldots iD^{\mu_n} s^\lambda h|B(v)\rangle^*$$

- SI matrix elements are real, SD matrix elements are complex

General method: Hermitian conjugation

- We consider matrix elements of the form
  $\langle B(v)|\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n} h|B(v)\rangle$
  $\langle B(v)|\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n} s^\lambda h|B(v)\rangle$

- $\bar{h}h$, $\bar{h}s^\lambda h$, iD^μ are hermitian
  using Hermitian conjugation

$$\begin{aligned}\langle B|\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n}(s^\lambda)h|B\rangle &= \langle B|\left(\bar{h}\, iD^{\mu_1} \dots iD^{\mu_n}(s^\lambda)h\right)^\dagger|B\rangle^* \\ &= \langle B|\bar{h}\, iD^{\mu_n} \dots iD^{\mu_1}(s^\lambda)h|B\rangle^*\end{aligned}$$

- Combining with the PT constraints
  Under inversion of the indices:
- SI matrix elements are symmetric
- SD matrix elements are anti-symmetric

General method: Tensor decomposition

- We consider matrix elements of the form
  $\langle B(v)|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|B(v)\rangle$
  $\langle B(v)|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|B(v)\rangle$

- B is a pseudo-scalar $\Rightarrow$ matrix element can only depend on $v^{\mu_i}, g^{\mu_i\mu_j}$, and $\epsilon^{\alpha\beta\rho\sigma}$

- Alternatively following
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]
  Define $\Pi^{\mu\nu} = g^{\mu\nu} - v^{\mu}v^{\nu}$
  For the standard choice of $v = (1,0,0,0)$: $\Pi^{00} = 0$ and $\Pi^{ij} = -\delta^{ij}$

- Since the indices in $\epsilon^{\alpha\beta\rho\sigma}$ cannot be independent of v^{μ} replace $\epsilon^{\alpha\beta\rho\sigma}$ by $\epsilon^{\alpha\beta\rho\sigma}v_{\sigma}$

- Matrix element depend on
  $v^{\mu_i}, \Pi^{\mu_i\mu_j}$, and $\epsilon^{\alpha\beta\rho\sigma}v_{\sigma}$

General method: Four dimensions

- We consider matrix elements of the form
  $\langle B(v)|\bar{h}\, iD^{\mu_1} \ldots iD^{\mu_n} h|B(v)\rangle$
  $\langle B(v)|\bar{h}\, iD^{\mu_1} \ldots iD^{\mu_n} s^\lambda h|B(v)\rangle$
- Four dimensions $\Rightarrow$ only four independent directions
- Not all tensors with more than four indices are independent
- Example: for dimension 7 SD HQET operators
  need $\Pi^{\mu\nu}\epsilon^{\alpha\beta\rho\sigma}v_\sigma$: three indices are the same
  Tensors obtained by permuting indices are not linearly independent
- Example: for dimension 11 SI HQET operators
  need $\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}\Pi^{\mu_5\mu_6}\Pi^{\mu_7\mu_8}$: four indices are the same

Results: Dimension 7 HQET operators

- Using the general method

$$\frac{1}{2M_B}\langle B|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h|B\rangle = a_{12}^{(7)}\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4} + a_{13}^{(7)}\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4} + \\ + a_{14}^{(7)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3} + b^{(7)}\Pi^{\mu_1\mu_4}v^{\mu_2}v^{\mu_3}$$

$$\frac{1}{2M_B}\langle B|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|B\rangle = i\tilde{b}^{(7)}v^{\mu_2}v^{\mu_3}\epsilon^{\mu_1\mu_4\lambda\rho}v_\rho + \\ + i\tilde{a}_{12}^{(7)}\left(\Pi^{\mu_1\mu_2}\epsilon^{\mu_3\mu_4\lambda\rho}v_\rho - \Pi^{\mu_4\mu_3}\epsilon^{\mu_2\mu_1\lambda\rho}v_\rho\right) + \\ + i\tilde{a}_{13}^{(7)}\left(\Pi^{\mu_1\mu_3}\epsilon^{\mu_2\mu_4\lambda\rho}v_\rho - \Pi^{\mu_4\mu_2}\epsilon^{\mu_3\mu_1\lambda\rho}v_\rho\right) + \\ + i\tilde{a}_{14}^{(7)}\Pi^{\mu_1\mu_4}\epsilon^{\mu_2\mu_3\lambda\rho}v_\rho + i\tilde{a}_{23}^{(7)}\Pi^{\mu_2\mu_3}\epsilon^{\mu_1\mu_4\lambda\rho}v_\rho$$

- How many HQET operators do we have at dimension 7?
  4 SI operators and 5 SD operators confirms
  [Manohar PRD **56**, 230 (1997)]
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

Results: Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_B}\langle B|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, h|B\rangle = a_{12}^{(8)}\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}v^{\mu_4} + \Pi^{\mu_1\mu_3}\Pi^{\mu_5\mu_4}v^{\mu_2}\right) +$$
$$a_{13}^{(8)}\left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}v^{\mu_4} + \Pi^{\mu_5\mu_3}\Pi^{\mu_4\mu_1}v^{\mu_2}\right) + a_{15}^{(8)}\left(\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_4}v^{\mu_2} + \Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_2}v^{\mu_4}\right) +$$
$$b_{12}^{(8)}\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_5}v^{\mu_3} + b_{14}^{(8)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}v^{\mu_3} + b_{15}^{(8)}\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}v^{\mu_3} +$$
$$c^{(8)}\Pi^{\mu_1\mu_5}v^{\mu_2}v^{\mu_3}v^{\mu_4}$$

$$\frac{1}{2M_B}\langle B|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, s^{\lambda}\, h|B\rangle =$$
$$i\tilde{a}_{12}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_2}\epsilon^{\mu_4\mu_5\lambda\rho}v_\rho - v^{\mu_3}\Pi^{\mu_4\mu_5}\epsilon^{\mu_2\mu_1\lambda\rho}v_\rho\right) + i\tilde{a}_{14}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_4}\epsilon^{\mu_2\mu_5\lambda\rho}v_\rho - v^{\mu_3}\Pi^{\mu_5\mu_2}\epsilon^{\mu_4\mu_1\lambda\rho}v_\rho\right)$$
$$+i\tilde{a}_{15}^{(8)}v^{\mu_3}\Pi^{\mu_1\mu_5}\epsilon^{\mu_2\mu_4\lambda\rho}v_\rho + i\tilde{a}_{24}^{(8)}v^{\mu_3}\Pi^{\mu_2\mu_4}\epsilon^{\mu_1\mu_5\lambda\rho}v_\rho +$$
$$+i\tilde{b}_{13}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_3}\epsilon^{\mu_4\mu_5\lambda\rho}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_3}\epsilon^{\mu_2\mu_1\lambda\rho}v_\rho\right) + i\tilde{b}_{14}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_4}\epsilon^{\mu_3\mu_5\lambda\rho}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_2}\epsilon^{\mu_3\mu_1\lambda\rho}v_\rho\right)$$
$$+i\tilde{b}_{15}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_5}\epsilon^{\mu_3\mu_4\lambda\rho}v_\rho - v^{\mu_4}\Pi^{\mu_1\mu_5}\epsilon^{\mu_3\mu_2\lambda\rho}v_\rho\right) + i\tilde{b}_{34}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_4}\epsilon^{\mu_1\mu_5\lambda\rho}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_2}\epsilon^{\mu_1\mu_5\lambda\rho}v_\rho\right)$$
$$+i\tilde{b}_{35}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_5}\epsilon^{\mu_1\mu_4\lambda\rho}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_1}\epsilon^{\mu_5\mu_2\lambda\rho}v_\rho\right) + i\tilde{b}_{45}^{(8)}\left(v^{\mu_2}\Pi^{\mu_4\mu_5}\epsilon^{\mu_1\mu_3\lambda\rho}v_\rho - v^{\mu_4}\Pi^{\mu_2\mu_1}\epsilon^{\mu_5\mu_3\lambda\rho}v_\rho\right)$$
$$+i\tilde{c}^{(8)}v^{\mu_2}v^{\mu_3}v^{\mu_4}\epsilon^{\mu_1\mu_5\lambda\rho}v_\rho.$$

- How many HQET operators do we have at dimension 8?
  7 SI operators and 11 SD operators, confirms
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

Results: Dimension 9 HQET operators

- Using the general method:
  24 possible SI Dimension 9 HQET operators (New!)

$$
\begin{aligned}
&\langle B|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} iD^{\mu_6} h|B\rangle = \\
&\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}\Pi^{\mu_5\mu_6} + (\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}\Pi^{\mu_4\mu_6} + \Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}\Pi^{\mu_5\mu_6}) + \\
&(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}\Pi^{\mu_4\mu_5} + \Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}\Pi^{\mu_5\mu_6}) + \Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}\Pi^{\mu_4\mu_6} + \\
&(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}\Pi^{\mu_4\mu_5} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_6}) + \Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_6} + \\
&(\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_5} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_6}) + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_4} + \\
&\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_5} + \Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_5} + \\
&\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_4} + (\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_6} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1\mu_3}\Pi^{\mu_5\mu_6} v^{\mu_2} v^{\mu_4}) + \\
&(\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_6} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1\mu_3}\Pi^{\mu_5\mu_6} v^{\mu_2} v^{\mu_4}) + \Pi^{\mu_1\mu_2}\Pi^{\mu_5\mu_6} v^{\mu_3} v^{\mu_4} + \\
&(\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_6} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1\mu_3}\Pi^{\mu_5\mu_6} v^{\mu_2} v^{\mu_4}) + \Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_6} v^{\mu_2} v^{\mu_5} + \\
&\Pi^{\mu_1\mu_4}\Pi^{\mu_3\mu_6} v^{\mu_2} v^{\mu_5} + (\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_6} v^{\mu_2} v^{\mu_4}) + \\
&\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6} v^{\mu_3} v^{\mu_4} + (\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3} v^{\mu_4} v^{\mu_5} + \Pi^{\mu_1\mu_6}\Pi^{\mu_4\mu_5} v^{\mu_2} v^{\mu_3}) + \\
&(\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_5} v^{\mu_2} v^{\mu_4}) + \Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5} v^{\mu_3} v^{\mu_4} + \\
&\Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_4} v^{\mu_2} v^{\mu_5} + \Pi^{\mu_1\mu_6} v^{\mu_2} v^{\mu_3} v^{\mu_4} v^{\mu_5}
\end{aligned}
$$

Conclusions

Conclusions

- We presented a general method to construct HQET operators: using tensor decomposition of HQET matrix elements
  Several applications:

- Easily relate different bases
  - Dimension 7: Manohar '97 to Mannel-Turczyk-Uraltsev '10
  - Dimension 8: Mannel-Turczyk-Uraltsev '10 to Hill, Lee, GP, Solon '12

- SI dimension 9 HQET operators

- Moments of the leading order shape function up to and including dimension 9 HQET operators

- We will present the dimension 8 NRQCD Lagrangian

Conclusions

- [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]
  lists dimension 7 and 8 HQET operators

- Question: Are these *all* possible HQET operators at dimension 7,8?
- Answer: Yes

- Question: Can we go higher?
- Answer: Yes, we presented the general method for that

- Question: Since HQET and NRQCD are related,
  what are the NRQCD operators?
- Answer will be given in [Gunawardana, GP *to appear*]

- Question:
  What can we learn about the structure of effective field theories?
  [Henning, Lu, Melia, Murayama, arXiv:1512.03433]
- Answer: Simpler than we think...