QCD sum rules predictions for exclusive $b \to c$ transitions

Status 2016

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Motivation

\[
\frac{d\Gamma(B \to D^* \ell \nu)}{d\omega} \propto |V_{cb}|^2 |\mathcal{F}(\omega)|^2
\]

\[
|V_{cb}| \mathcal{F}(1) = 35.81 \pm 0.11|_{\text{stat}} \pm 0.44|_{\text{syst}} \quad \text{[HFAG 2014, averg. of ALEPH, BaBar, Belle, CLEO, DELPHI, OPAL meas.]} \\
\mathcal{F}(1) = 0.906 \pm 0.004|_{\text{stat}} \pm 0.012|_{\text{syst}} \quad \text{[Fermilab/MILC PRD 89 (2014) 114504]}
\]

\[
|V_{cb}| = (39.2 \pm 0.7) \cdot 10^{-3} \quad \text{[PDG 2014 (w/ 2015 partial update)]}
\]

Continuum methods are important to

- provide complementary information
  - $\mathcal{F}(\omega_{\text{max}} \approx 1.5)$
  - shape of $\mathcal{F}(\omega)$
- cross check existing lattice results
  [also for $B \to D \ell \nu$: $G(\omega)$]
Outline

Review two continuum methods that have been successfully used to infer knowledge on $B \rightarrow D^{(*)}$ form factors

- Zero Recoil Sum Rules
  inclusive constraints on combination of form factors in a single phase-space point
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  constraint on each form factor in small region around maximal hadronic recoil
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  inclusive constraints on combination of form factors in a single phase-space point

- Light-Cone Sum Rules with $B$-meson Light-Cone Distribution Amplitudes
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- brief comment on Dispersive Bounds
  contraints on the shape of the form factors as functions of the momentum transfer
Zero Recoil Sum Rules: Basic Idea

Consider an artificial two-point function

\[ T_J(\varepsilon) \equiv \frac{1}{N_J} \int d^4 x e^{i(v \cdot x)} \varepsilon \langle \overline{B}(M_B v) | T \{ J^{\dagger, \mu}(x), J_\mu(0) \} | \overline{B}(M_B v) \rangle \]

\[ J^\mu = \overline{c} \gamma^\mu (\gamma_5) b \]

\[ \varepsilon = M_{X_c} - M_D : \text{excitation energy above } M_D \]

– can be obtained in two representations

**OPE inclusive calculation:** express in terms of local operators, and expand in \(1/m_c, 1/m_b\) and \(\alpha_s\)

**Hadronic** express in terms of spectral densities involving hadronic matrix elements of exclusive processes (form factors)

– sum rule: equate moments of \(T(\varepsilon)\) in both representations, and infer knowledge on the form factors

\[ \int_{|\varepsilon| = \varepsilon_M} d\varepsilon \, T_A(\varepsilon) = |F(1)|^2 + \ldots \]

– hadronic representation is sum of strictly positive quantities

\[ \Rightarrow \text{upper bound on } B \to D^{(*)} \text{ form factors} \]
Limitations

– triple expansion in $\alpha_s, 1/m_b, 1/m_c$

– $\langle \overline{B} | \overline{B} \rangle$ matrix elements of operators comprise non-perturbative input
  – universal input: $\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3$
  – can be extracted from $B \to X_c \ell \nu$ data [see backups for definition]

– $1/m_c$ expansion might converge slowly or not at all

  $B \to D$ only $1/m_c^2$ corrections in BPS limit
  $B \to D^*$ reverse setup ($D^* \to B^*$ sum rules) suggests that the terms in the $1/m_c$ expansion alternate in sign

– continuum background can be estimated in the OPE, but involves matrix elements of nonlocal operators ($\rho_{\pi \pi}, \ldots$)
  – ZRZR provides reliable upper bound on sum of form factor terms [Gambino/Mannel/Uraltsev JHEP 1210 (2012) 169]
**Status 2016:** \( B \to D\mu\nu \)

ZRSR upper bounds/estimates of \( G(1) \):
- \( O(\alpha_s) \) and partial \( O(\alpha_s^2) \) terms (for the unit operator)
- up to \( O(1/m^3) \) correction

\[
\begin{align*}
\text{U2004} & \quad G(1) < 1.04 \pm 0.02 \pm \delta_{\text{exp}} \\
\text{GMU2010} & \quad G(1) < 1.02 \pm 0.04
\end{align*}
\]


Comparison with lattice results

![Graph comparing lattice results](image)

**FNAL/MILC 2005** \( G(1) = 1.074(18)_{\text{stat}}(16)_{\text{syst}} \)


**MILC 2015** \( G(1) = 1.054(4)_{\text{stat}}(8)_{\text{syst}} \)


**HPQCD 2015** \( G(1) = 1.035(40) \)

Status 2016: $B \rightarrow D \mu \bar{\nu}$

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\text{LvD2016} & \quad \mathcal{G}(1) < 1.012 \pm 0.012 \quad \text{preliminary!}
\end{align*}

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[LvD2016 w.i.p.]
**Status 2016:** $B \rightarrow D^* \mu \bar{\nu}$

ZRSR estimates of $\mathcal{F}(1)$
- complete $O(\alpha_s^2)$ (for the unit operator)
- up to $O(1/m^3)$ correction

\[ \text{GMU2012 } \mathcal{F}(1) \approx 0.86 \]

[GMbino,Mannel,Uraltsev JHEP 1210 (2012) 169]

Comparison with lattice results

\[ F(1) = 0.906(4)_{\text{stat}}(12)_{\text{syst}} \]

[Фermilab Lattice and MILC Collaborations Phys.Rev. D89 (2014) no.11, 114504]

**HPQCD prel.** see talk by Christine Davies ($F(1) = h_{A_1}(1)$)

[HPQCD Collaboration preliminary result shown at CKM2016]
**Status 2016:** \( B \rightarrow D^* \mu \nu \)

ZRSR estimates of \( \mathcal{F}(1) \)
- complete \( O(\alpha_s^2) \) (for the unit operator)
- up to \( O(1/m^3) \) correction

\[ \text{GMU2012} \quad \mathcal{F}(1) \approx 0.86 \]
\[ \text{LvD2016} \quad \mathcal{F}(1) < 0.828 \pm 0.014 \]

Comparison with lattice results

<table>
<thead>
<tr>
<th>FNAL/MILC 2014</th>
<th>HPQCD prel.</th>
</tr>
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<tbody>
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<td>( F(1) = 0.906(4)<em>{\text{stat}}(12)</em>{\text{syst}} )</td>
<td>( F(1) = h_{A_1}(1) )</td>
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29.11.2016
What changed?

updated input values

2012

\[ m_b^{\text{kin}} (1 \text{ GeV}) = 4.6 \text{ GeV} \]
\[ m_c^{\text{kin}} (1 \text{ GeV}) = 1.2 \text{ GeV} \]
\[ \mu_\pi^2 (1 \text{ GeV}) = 0.4 \text{ GeV}^2 \]
\[ \mu_G^2 (1 \text{ GeV}) = 0.3 \text{ GeV}^2 \]
\[ \rho_D^3 (1 \text{ GeV}) = 0.15 \text{ GeV}^3 \]
\[ -\rho_{LS}^3 (1 \text{ GeV}) = 0.12 \text{ GeV}^3 \]

[Gambino, Mannel, Uraltsev JHEP 1210 (2012) 169]
## What changed?

updated input values

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<td>$m_b^{\text{kin}} (1 \text{ GeV}) = 4.6 \text{ GeV}$</td>
<td>$m_b^{\text{kin}} (1 \text{ GeV}) = 4.561 \pm 0.021 \text{ GeV}$</td>
</tr>
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<td>$m_c^{\text{kin}} (1 \text{ GeV}) = 1.2 \text{ GeV}$</td>
<td>$m_c^{\text{kin}} (1 \text{ GeV}) = 1.092 \pm 0.020 \text{ GeV}$</td>
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<td>$\mu_\pi^2 (1 \text{ GeV}) = 0.4 \text{ GeV}^2$</td>
<td>$\mu_\pi^2 (1 \text{ GeV}) = 0.464 \pm 0.067 \text{ GeV}^2$</td>
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<tr>
<td>$\mu_G^2 (1 \text{ GeV}) = 0.3 \text{ GeV}^2$</td>
<td>$\mu_G^2 (1 \text{ GeV}) = 0.333 \pm 0.061 \text{ GeV}^2$</td>
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<td>$\rho_D^3 (1 \text{ GeV}) = 0.15 \text{ GeV}^3$</td>
<td>$\rho_D^3 (1 \text{ GeV}) = 0.175 \pm 0.040 \text{ GeV}^3$</td>
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<td>$-\rho_{LS}^3 (1 \text{ GeV}) = 0.12 \text{ GeV}^3$</td>
<td>$-\rho_{LS}^3 (1 \text{ GeV}) = 0.146 \pm 0.096 \text{ GeV}^3$</td>
</tr>
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old values exhibit $> 3\sigma$ tension, $\approx 2\sigma$ deviation, $< 1\sigma$ agreement
Light-Cone Sum Rules: Basic Idea

– construct an artificial correlator

\[
F_{\alpha \mu}(p, q) = i \int d^4 x \ e^{ip \cdot x} \langle 0| T \{\overline{d}\Gamma_\alpha c(x), \overline{c}\gamma_\mu (1 - \gamma_5) b(0)|B\rangle
\]

\[
= \frac{\langle 0|\overline{d}\Gamma_\alpha |D(*)\rangle \langle D(*)|\overline{c}\gamma_\mu (1 - \gamma_5) b|B\rangle}{M^2_{D(*)} - p^2} + \text{multi-body contributions}
\]

\[
p: \text{momentum of } D(*) \quad q: \text{momentum of leptons}
\]

– for \( q^2 \) “sufficiently far away from zero recoil” the integral is dominated by light-like distances \( x^2 \)

– apply Operator Product Expansion on the light cone

  – input: universal non-perturbative non-local matrix elements \( \langle 0|\overline{d}(x)\Gamma h_v(0)|B\rangle \)
  
  – parametrized as \( B \)-meson light-cone distribution amplitudes
  
  – note: defined in HQET, subject to power corrections

– relate to \( B \rightarrow D(*) \) form factors and \( D(*) \) decay constants
Complementary Information

– complementary to ZRSR
  – each form factor can be obtained individually

– complementary to Lattice and ZRSR
  – by construction the LCSRs apply at/close to maximum hadronic recoil
  – can be used to anchor parametrization of the FFs for arbitrary momentum transfer
  – so far not used in experimental analyses
Status 2016-2008

\( \overline{B} \rightarrow D \mu \bar{\nu} : \)

\( G(\omega_{\text{max}}) = 0.61 \pm 0.11 \, \text{SR} \pm 0.10 \, f_B \pm 0.07 \, f_D \)

\( \overline{B} \rightarrow D^* \mu \bar{\nu} : \)

\( h_{A_1}(\omega_{\text{max}}) = 0.65 \pm 0.12 \, \text{SR} \pm 0.11 \, f_B \pm 0.07 \, f_{D^*} \)

\( R_1(\omega_{\text{max}}) = 1.32 \pm 0.04 \, \text{SR} \quad \text{[CLN: } R_1 = 1.22] \)

\( R_2(\omega_{\text{max}}) = 0.91 \pm 0.17 \, \text{SR} \quad \text{[CLN: } R_1 = 0.84] \)

Uncertainty budgets:

\( f_B \) due to normalization of \( B \)-meson LCDA

\( f_{D^*} \) due to decay constant in dispersion relation

\( \text{SR} \) due to \textbf{S}um \textbf{R}ule parameters \( (\lambda_B, M^2, s_0, \ldots) \)

[see backups for details]

Briefly: Dispersive Bounds

- dispersively relate hadronic matrix elements to vacuum-to-vacuum matrix elements

\[ \Pi^{\mu\nu} \equiv i \int d^4 x \, e^{i q \cdot x} \langle 0 \mid T \{ J^\dagger,\mu (x) , J^\nu (0) \} \mid 0 \rangle \]

\( q \): momentum of \( B D \) pair

- relates \( \overline{B} \rightarrow D(\ast) \), \( \overline{B}^* \rightarrow D(\ast) \), and further exclusive matrix elements with each other
- use of analytic structure of \( \Pi^{\mu\nu} \) in plane of complex-valued momentum transfer
- can be used to infer knowledge of the shape of the \( \overline{B} \rightarrow D(\ast) \) form factors as functions of momentum transfer
  - inspired CLN parametrization

- crucial input: normalization of form factors at one kinematical point
  - last results date from 1998, in dire need of update

Summary

– few new developments

– zero recoil sum rules still at odds with (some) lattice inputs

\[ B \to D \] MILC 2015 at \( \sim 3\sigma \) tension, HPQCD 2014 compatible

\[ B \to D^* \] FNAL/MILC 2015 at \( \sim 5\sigma \) tension, HPQCD prel. compatible

– light-cone sum rules provide information complementary to lattice results
  – so far, not used in fits to \( \overline{B} \to D(\ast) \mu \nu \) spectra as functions of recoil \( \omega \)

– dispersive bounds used to guide CLN parametrization
  – input parameters from 1998
  – in desperate need of an update
Definition of the Hadronic Matrix Elements

at order $1/m^2$:

$$
\mu_\pi^2 = - \frac{1}{2M_B} \langle \bar{B} | h_v (iD_\perp)^2 h_v | B \rangle, \quad \mu_G^2 = - \frac{i}{4M_B} \langle \bar{B} | h_v \sigma^{\mu\nu}[iD_\perp, iD_\perp] h_v | B \rangle
$$

at order $1/m^3$:

$$
\rho_D^3 = + \frac{1}{4M_B} \langle \bar{B} | [iD_\perp, [i(v \cdot D), iD_\perp]] | B \rangle, \quad \rho_{LS}^3 = - \frac{i}{4M_B} \langle \bar{B} | \sigma^{\mu\nu}[iD_\perp, [i(v \cdot D), iD_\perp]] | B \rangle
$$

[see e.g. Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109]
Zero-Recoil Sum Rule

\[
|\varepsilon| = \varepsilon_M
\]

OPE Representation

- pole(s) at \( \varepsilon = 0 \)
- parasitic branch cut from \(-2m_c \to -\infty\)
- parasitic branch cut from \(+2m_b \to +\infty\)
- separation scale \( \mu \approx \varepsilon_M = 0.75 \text{GeV} \)
  chosen for large distance from parasitic branch cuts, while still large enough to separate hard from soft modes in the OPE

Hadronic Representation

- pole for \( D \) at \( \varepsilon = 0 \)
- pole for \( D^* \) at \( \varepsilon = M_{D^*} - M_D - i\ldots \)
- branch cuts \((D + n \times \pi, \ldots)\) from \(n \times M_\pi \to +\infty\)
Light-Cone Sum Rules with $B$-meson LCDAs

Input parameters

- $\lambda_B = 460 \pm 110$ MeV: inverse moment of the two-particle LCDA
- $M^2 = 3 \ldots 6 \text{ GeV}^2$: Borel parameter window
- $s_0^{D(*)} = 6.0(8.0) \text{ GeV}^2$: hadronic threshold