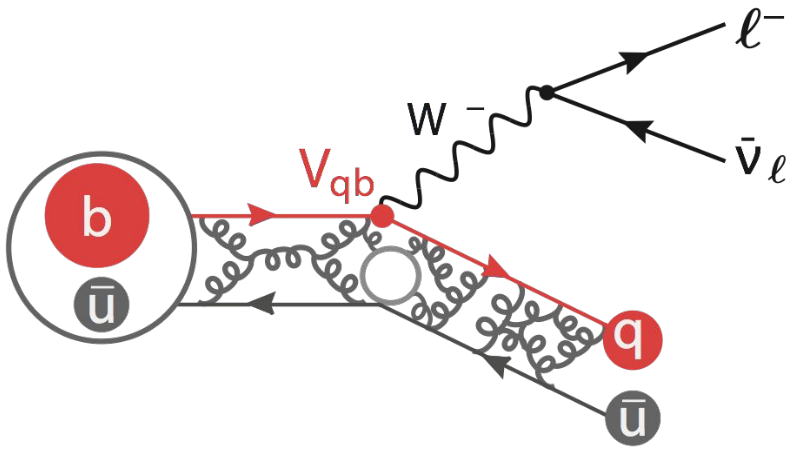
# Experimental mini-review: inclusive $|V_{cb}|$




Many thanks to Kerstin Tackmann and Christoph Schwanda

Florian Bernlochner

CKM 2016

# Talk Overview

Briefly recap existing meas. and most recent extractions in kinetic scheme

- No new experimental results since 2010!
  - Most results (besides very high moments) are systematically limited

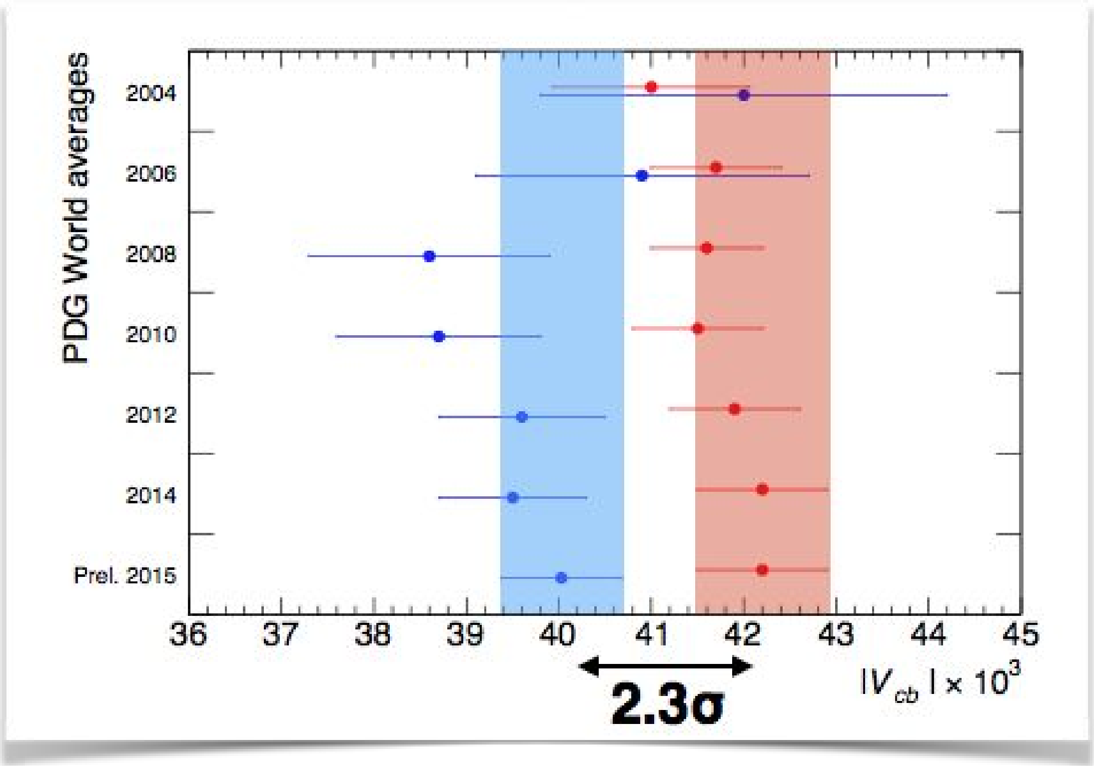
- Latest fits:
  - Phys.Rev.Lett. 114 (2015), 061802
  - Phys.Rev. D89 (2014) 014022
- Incorporate sum-rule results for charm-quark mass (Phys. Rev. D 80, 074010) and 2015 results includes corrections $O(\alpha_s \Lambda^2_{QCD}/m_b^2)$

| Experiment | Hadron moments $\langle M^n_X \rangle$ | Lepton moments $\langle E^n_l \rangle$ | References |
|---|---|---|---|
| BaBar | n=2 c=0.9,1.1,1.3,1.5<br>n=4 c=0.8,1.0,1.2,1.4<br>n=6 c=0.9,1.3 [1] | n=0 c=0.6,1.2,1.5<br>n=1 c=0.6,0.8,1.0,1.2,1.5<br>n=2 c=0.6,1.0,1.5<br>n=3 c=0.8,1.2 [1,2] | [1] Phys.Rev. D81 (2010) 032003<br>[2] Phys.Rev. D69 (2004) 111104 |
| Belle | n=2 c=0.7,1.1,1.3,1.5<br>n=4 c=0.7,0.9,1.3 [3] | n=0 c=0.6,1.4<br>n=1 c=1.0,1.4<br>n=2 c=0.6,1.4<br>n=3 c=0.8,1.2 [4] | [3] Phys.Rev. D75 (2007) 032005<br>[4] Phys.Rev. D75 (2007) 032001 |
| CDF | n=2 c=0.7<br>n=4 c=0.7 [5] | . | [5] Phys.Rev. D71 (2005) 051103 |
| CLEO | n=2 c=1.0,1.5<br>n=4 c=1.0,1.5 [6] | . | [6] Phys.Rev. D70 (2004) 032002 |
| DELPHI | n=2 c=0.0<br>n=4 c=0.0<br>n=6 c=0.0 [7] | n=1 c=0.0<br>n=2 c=0.0<br>n=3 c=0.0 [7] | [7] Eur.Phys.J. C45 (2006) 35-59 |

HFAG Measurement overview

# Premise

Inclusive and exclusive Vcb exhibit a slight tension (~ 2.3 standard deviations)




Recently moved closer, but still a bit unsettling as both methods measure same fundamental parameter and serve as input for SM predictions (e.g. for $B_s \to \mu\mu$)

# Inclusive $|V_{cb}|$: An experimentalist's guide

At parton level, the decay rate for $b \to c\, l\, \nu$ can be calculated accurately and is proportional to $|V_{cb}|^2$

- To relate measurements of semileptonic B-meson decays to $|V_{cb}|^2$ the parton-level expressions have to be corrected for the effects of non-perturbative effects.
- Heavy-Quark-Expansions (HQE) successful tool to incorporate perturbative and nonperturbative QCD corrections.
- E.g. total decay rate expanded in the kinetic scheme

$$\begin{aligned}\Gamma_{sl} = &\Gamma_0 \Big[ 1 + a^{(1)} \frac{\alpha_s(m_b)}{\pi} + a^{(2,\beta_0)} \beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 + a^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 \\ &+ \left(-\frac{1}{2} + p^{(1)} \frac{\alpha_s}{\pi}\right) \frac{\mu_\pi^2}{m_b^2} + \left(g^{(0)} + g^{(1)} \frac{\alpha_s}{\pi}\right) \frac{\mu_G^2(m_b)}{m_b^2} \\ &+ d^{(0)} \frac{\rho_D^3}{m_b^3} - g^{(0)} \frac{\rho_{LS}^3}{m_b^3} + \text{higher orders} \Big], \end{aligned} \qquad (3)$$

- Determine the five parameters + $|V_{cb}|$ from a simultaneous fit to moments

# A moment for moments

- Moments of the lepton spectrum ($<E_l^n>$) or invariant mass squared spectrum ($<M_x^{2n}>$) show small dependence on Wilson factorization scale of HQE

$$\langle E_\ell^n \rangle = \frac{1}{\Gamma_{E_\ell > E_{\rm cut}}} \int_{E_\ell > E_{\rm cut}} E_\ell^n \, \frac{d\Gamma}{dE_\ell} \, dE_\ell \ ,$$

$$\langle m_X^{2n} \rangle = \frac{1}{\Gamma_{E_\ell > E_{\rm cut}}} \int_{E_\ell > E_{\rm cut}} m_X^{2n} \, \frac{d\Gamma}{dm_X^2} \, dm_X^2 \ .$$

Moments are measured with incremental cuts on the lepton momentum

- Ideal observable to extract the 5 non-perturbative parameters + $|V_{cb}|$
    - Also some benefit in using combined mass and energy moments (Gambino, Uraltsev), but none of the recent fits include this (plus there is only one measurement providing those)

$$n_X^2 = m_X^2 c^4 - 2\tilde{\Lambda} E_X + \tilde{\Lambda}^2,$$

# Confronting the data

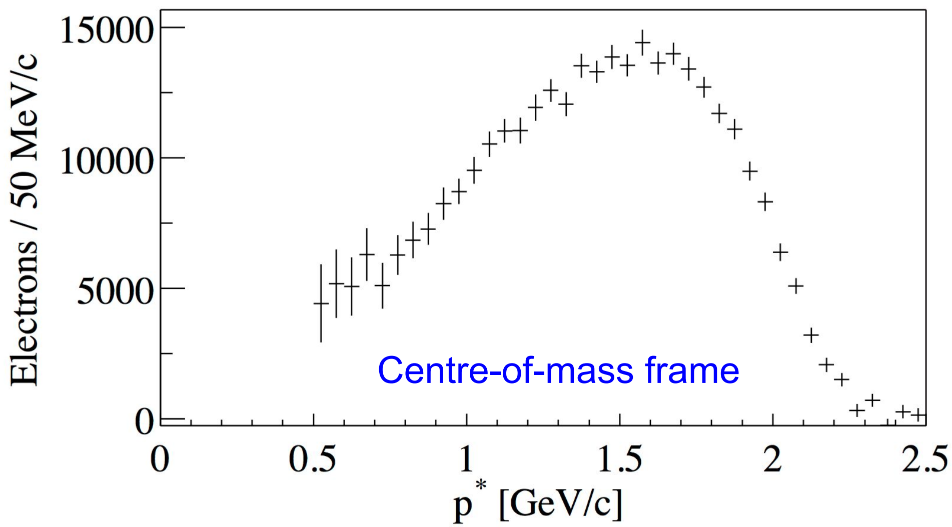
$$\chi^2 = \left(\vec{M}_{\rm exp} - \vec{M}_{\rm HQE}\right)^T \mathcal{C}_{\rm tot}^{-1} \left(\vec{M}_{\rm exp} - \vec{M}_{\rm HQE}\right).$$

- Fit measured moments with $\chi^2$ using the standard construction
- Moments mostly sensitive to ~ $m_b - 0.8\, m_c$; need to include information of at least one heavy quark
  - Uncertainty on $m_c$ much smaller than on $m_b$ thus preferably to constrain $m_c$
- Theory correlations important; two sources:
  - Missing higher order corrections → Taken into account in state-of-the-art fits
  - Terms that violate quark-hadron duality → not included
- Included by adding to experimental covariance in fit
  - $C_{tot} = C_{exp} + C_{theo}$
  - Would be interesting to see if a N(uisance) P(arameter) based implementation would yield identical results

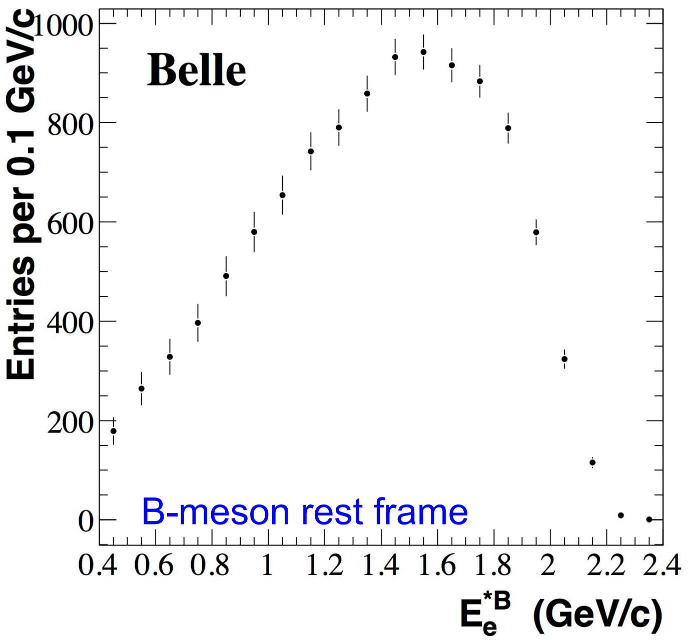
# Lepton spectrum measurements

## Lepton spectrum moments

- Can either be measured fully inclusive, hadronically tagged or via lepton tag of second B-meson decay




BaBar lepton tag with 47.4/fb [2] (cf. slide 2)




Belle hadronic tag with 140/fb [4]

$$M_n = \langle E_\ell^n \rangle = \frac{1}{\Gamma_{E_\ell > E_{\rm cut}}} \int_{E_\ell > E_{\rm cut}} E_\ell^n \frac{d\Gamma}{dE_\ell} dE_\ell ,$$

## Belle hadronic tag with 140/fb [4]

| $E_{\rm cut}$[GeV] | $M_1$ [MeV] | $M_2$ [$10^{-3}$GeV$^2$] | $M_3$ [$10^{-3}$GeV$^3$] | $M_4$ [$10^{-3}$GeV$^4$] | $\Delta\mathcal{B}$ [$10^{-2}$] |
|---|---|---|---|---|---|
| 0.4 | 1393.92 ± 6.73 ± 3.02 | 168.77 ± 3.68 ± 1.53 | -21.04 ± 1.93 ± 0.66 | 64.153 ± 1.813 ± 0.935 | 10.44 ± 0.19 ± 0.22 |
| 0.6 | 1427.82 ± 5.82 ± 2.55 | 146.15 ± 2.88 ± 1.08 | -11.04 ± 1.35 ± 0.49 | 45.366 ± 1.108 ± 0.548 | 10.07 ± 0.18 ± 0.21 |
| 0.8 | 1480.04 ± 4.81 ± 2.13 | 117.97 ± 2.05 ± 0.55 | -3.45 ± 0.83 ± 0.30 | 28.701 ± 0.585 ± 0.247 | 9.42 ± 0.16 ± 0.19 |
| 1.0 | 1547.76 ± 3.96 ± 1.45 | 88.17 ± 1.42 ± 0.36 | 0.83 ± 0.49 ± 0.20 | 15.962 ± 0.302 ± 0.142 | 8.41 ± 0.15 ± 0.17 |
| 1.2 | 1627.79 ± 3.26 ± 1.08 | 61.36 ± 1.02 ± 0.36 | 2.40 ± 0.30 ± 0.11 | 7.876 ± 0.162 ± 0.106 | 7.11 ± 0.13 ± 0.14 |
| 1.4 | 1719.96 ± 2.58 ± 1.10 | 38.99 ± 0.71 ± 0.24 | 2.33 ± 0.16 ± 0.07 | 3.314 ± 0.080 ± 0.055 | 5.52 ± 0.11 ± 0.11 |
| 1.6 | 1826.15 ± 1.80 ± 1.03 | 21.75 ± 0.47 ± 0.22 | 1.45 ± 0.08 ± 0.05 | 1.129 ± 0.033 ± 0.032 | 3.71 ± 0.09 ± 0.07 |
| 1.8 | 1943.18 ± 0.93 ± 1.16 | 10.14 ± 0.28 ± 0.18 | 0.68 ± 0.03 ± 0.04 | 0.283 ± 0.010 ± 0.017 | 1.93 ± 0.06 ± 0.04 |
| 2.0 | 2077.59 ± 0.21 ± 1.23 | 3.47 ± 0.13 ± 0.19 | 0.19 ± 0.01 ± 0.03 | 0.047 ± 0.002 ± 0.007 | 0.53 ± 0.02 ± 0.02 |

↑ Lepton momentum cut ↑ First error is stat., second syst.

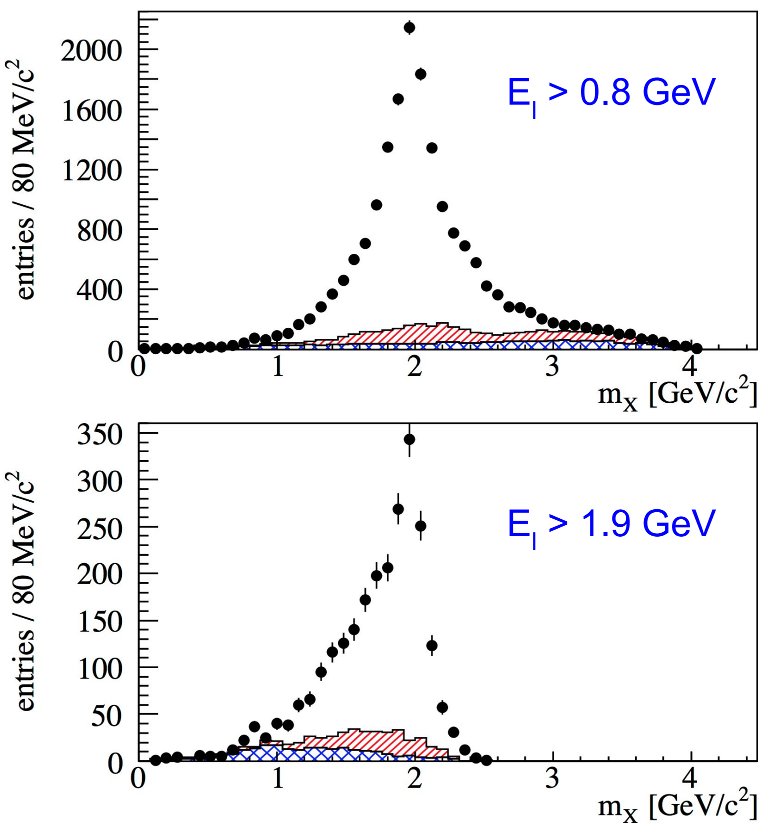
## BaBar lepton tag with 47.4/fb [2] (Moments unfolded to B-meson rest frame)

TABLE IV: Measured moments $M_1$, $M_2$, $M_3$, and $\mathcal{B}$ for five cut-off energies $E_0$ with first their statistical error and second their systematic error.The moments are given in the $B$-meson rest frame and are corrected for QED radiative effects.

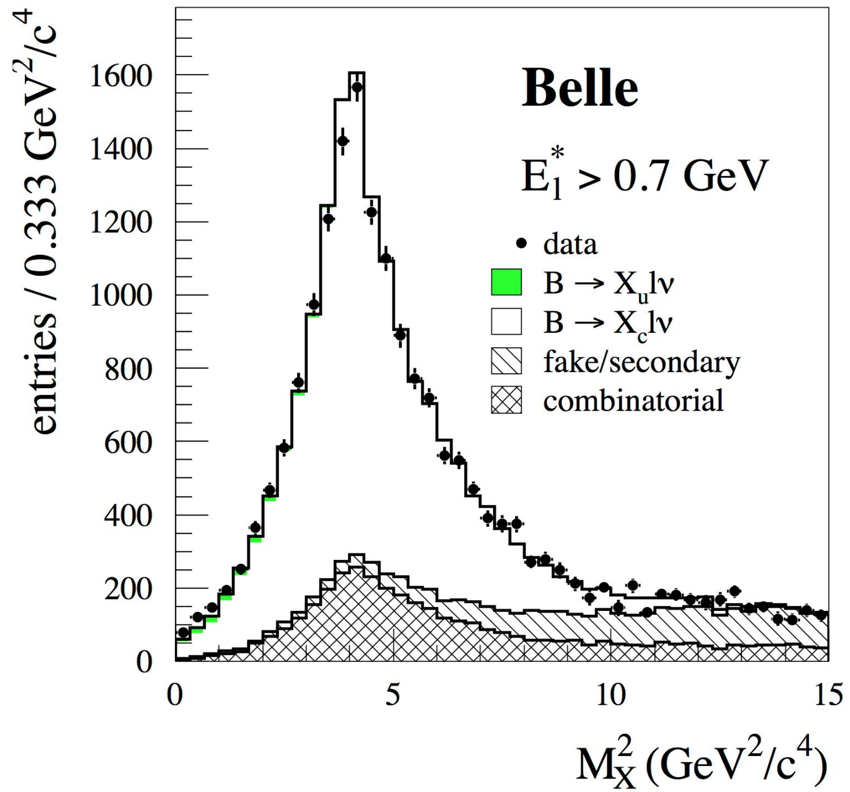
| $E_0$[GeV] | $\mathcal{B}[10^{-2}]$ | $M_1$[MeV] | $M_2[10^{-3}$ GeV$^2]$ | $M_3[10^{-3}$ GeV$^3]$ |
|---|---|---|---|---|
| 0.6 | 10.30 ± 0.06 ± 0.24 | 1432.8 ± 3.9 ± 7.8 | 148.0 ± 2.2 ± 3.1 | −12.05 ± 0.88 ± 0.46 |
| 0.8 | 9.61 ± 0.05 ± 0.20 | 1484.8 ± 2.6 ± 3.7 | 117.7 ± 1.3 ± 1.0 | −3.18 ± 0.64 ± 0.38 |
| 1.0 | 8.65 ± 0.04 ± 0.17 | 1548.7 ± 2.0 ± 2.2 | 89.1 ± 1.0 ± 0.6 | 1.19 ± 0.57 ± 0.30 |
| 1.2 | 7.31 ± 0.04 ± 0.14 | 1629.9 ± 1.9 ± 1.7 | 62.1 ± 0.9 ± 0.6 | 2.66 ± 0.52 ± 0.24 |
| 1.5 | 4.79 ± 0.03 ± 0.09 | 1774.3 ± 1.9 ± 1.4 | 30.3 ± 0.9 ± 0.5 | 2.12 ± 0.47 ± 0.20 |

# Hadronic mass spectrum measurements

Needs to be measured with a hadronic tag (otherwise cannot be reconstructed reliably)




BaBar hadronic tag with 210/fb [1]




Belle hadronic tag with 140/fb [3]

# Calibration / Unfolding is crucial

Non-trivial task to unfold measured moments; need $m_X$ dependent calibration

Example from BaBar:

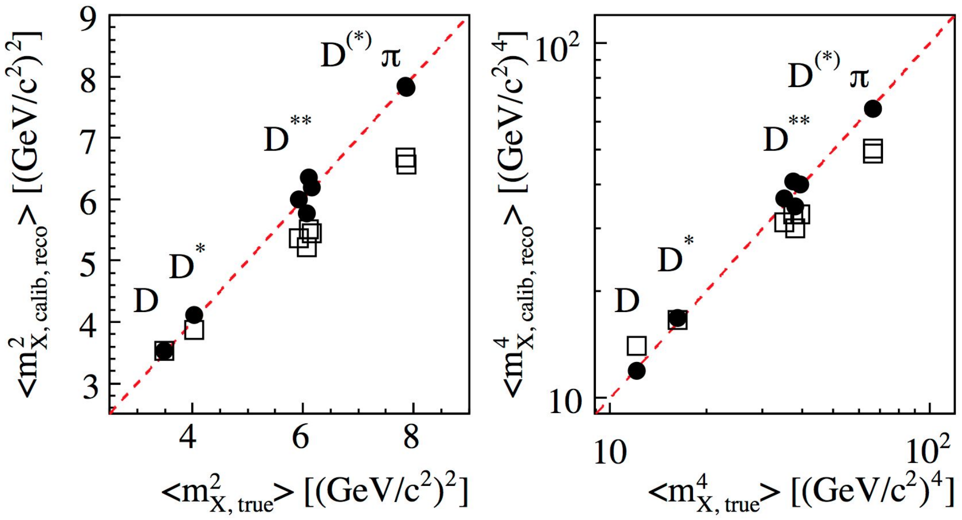



FIG. 4: Calibrated (•) and uncorrected (□) moments $\langle m_X^2 \rangle$ (left) and $\langle m_X^4 \rangle$ (right) of individual hadronic modes for lepton momenta $p_\ell^* \geq 0.8\,\mathrm{GeV}/c$. A reference line with $\langle m_{X,calib} \rangle = \langle m_{X,true} \rangle$ is superimposed.

BaBar hadronic tag with 210/fb [1]

## Belle hadronic tag with 140/fb [3]

TABLE II: Measurements of $\langle M_X^2 \rangle$, $\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle$ and $\langle M_X^4 \rangle$ for different lepton energy thresholds. The results in this table are the averages of the four sub-samples, defined by the charge of $B_{\rm tag}$ ($B^+$, $B^0$) and the lepton type (electron, muon). The first error is statistical, the second is the estimated systematic uncertainty. The different measurements are highly correlated (Tables VI–X).

| $E^*_{\rm min}$ (GeV) | $\langle M_X^2 \rangle$ (GeV$^2/c^4$) | $\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle$ (GeV$^4/c^8$) | $\langle M_X^4 \rangle$ (GeV$^4/c^8$) |
|---|---|---|---|
| 0.7 | $4.403 \pm 0.036 \pm 0.052$ | $1.494 \pm 0.173 \pm 0.327$ | $20.88 \pm 0.48 \pm 0.77$ |
| 0.9 | $4.353 \pm 0.032 \pm 0.041$ | $1.229 \pm 0.138 \pm 0.244$ | $20.18 \pm 0.40 \pm 0.58$ |
| 1.1 | $4.293 \pm 0.028 \pm 0.029$ | $0.940 \pm 0.098 \pm 0.137$ | $19.37 \pm 0.33 \pm 0.36$ |
| 1.3 | $4.213 \pm 0.027 \pm 0.024$ | $0.641 \pm 0.071 \pm 0.080$ | $18.40 \pm 0.29 \pm 0.26$ |
| 1.5 | $4.144 \pm 0.028 \pm 0.022$ | $0.515 \pm 0.061 \pm 0.064$ | $17.69 \pm 0.28 \pm 0.23$ |
| 1.7 | $4.056 \pm 0.033 \pm 0.022$ | $0.322 \pm 0.058 \pm 0.040$ | $16.77 \pm 0.32 \pm 0.21$ |
| 1.9 | $3.996 \pm 0.041 \pm 0.021$ | $0.143 \pm 0.056 \pm 0.038$ | $16.11 \pm 0.38 \pm 0.20$ |

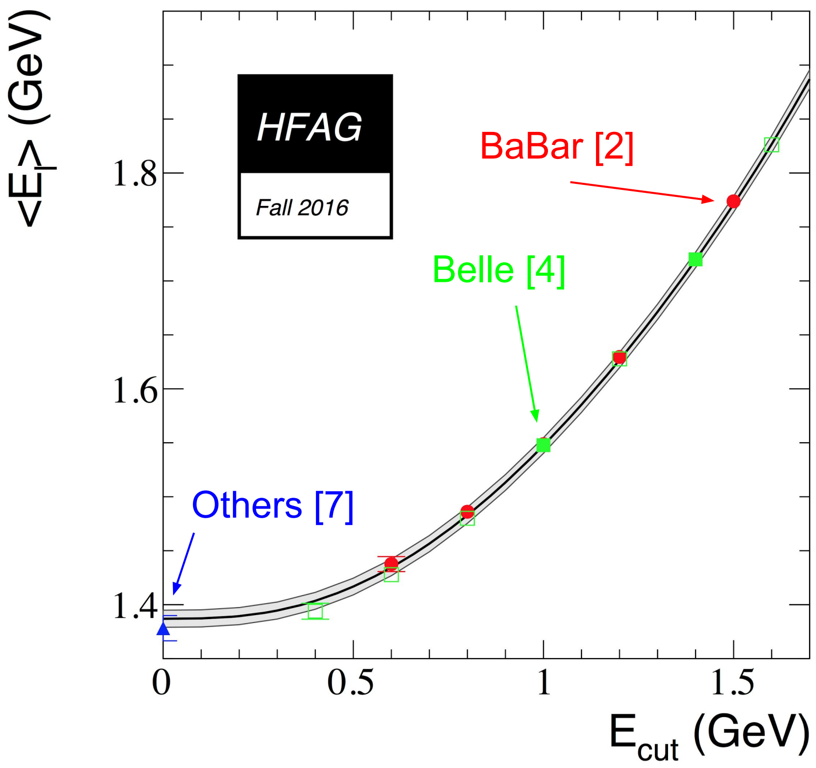
↑ Lepton momentum cut ↑ First error is stat., second syst.

## BaBar hadronic tag with 210/fb [1]; tables for k = 1,2,3,4,5,6

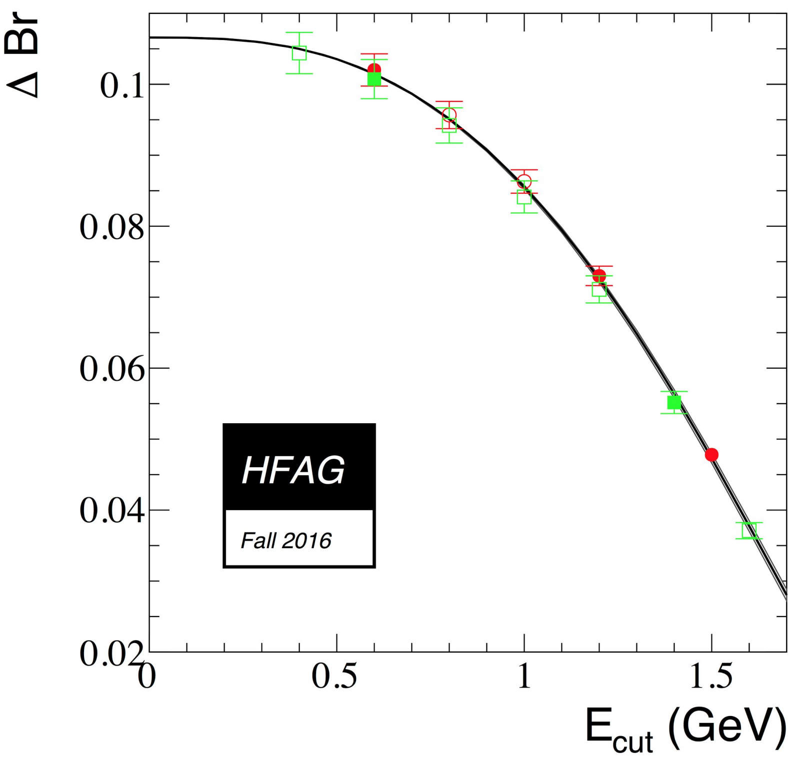
| $k$ | $p^*_{\ell,\rm min}$ [GeV/$c$] | $\langle m_X^k \rangle$ | $\sigma_{stat}$ | $\sigma_{sys}$ | MC statistics | simulation related | extraction method | back-groud | signal model |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.8 | 2.0906 | ±0.0063 | ±0.0166 | 0.0058 | 0.0099 | 0.0096 | 0.0047 | 0.0031 |
| | 0.9 | 2.0890 | ±0.0062 | ±0.0158 | 0.0048 | 0.0088 | 0.0103 | 0.0045 | 0.0028 |
| | 1.0 | 2.0843 | ±0.0061 | ±0.0153 | 0.0044 | 0.0076 | 0.0109 | 0.0044 | 0.0027 |
| | 1.1 | 2.0765 | ±0.0063 | ±0.0165 | 0.0044 | 0.0072 | 0.0127 | 0.0047 | 0.0026 |
| | 1.2 | 2.0671 | ±0.0064 | ±0.0160 | 0.0046 | 0.0073 | 0.0120 | 0.0045 | 0.0025 |
| | 1.3 | 2.0622 | ±0.0068 | ±0.0168 | 0.0048 | 0.0073 | 0.0131 | 0.0050 | 0.0023 |
| | 1.4 | 2.0566 | ±0.0073 | ±0.0183 | 0.0047 | 0.0069 | 0.0150 | 0.0054 | 0.0021 |
| | 1.5 | 2.0494 | ±0.0081 | ±0.0198 | 0.0036 | 0.0074 | 0.0168 | 0.0061 | 0.0019 |
| | 1.6 | 2.0430 | ±0.0092 | ±0.0221 | 0.0038 | 0.0082 | 0.0187 | 0.0070 | 0.0018 |
| | 1.7 | 2.0387 | ±0.0109 | ±0.0265 | 0.0047 | 0.0081 | 0.0232 | 0.0083 | 0.0015 |
| | 1.8 | 2.0370 | ±0.0143 | ±0.0337 | 0.0069 | 0.0097 | 0.0299 | 0.0098 | 0.0013 |
| | 1.9 | 2.0388 | ±0.0198 | ±0.0413 | 0.0082 | 0.0123 | 0.0355 | 0.0150 | 0.0008 |

# Fitting time (New HFAG Fall 2016 result)

Fitting all these inputs and the latest predictions from Phys.Rev.Lett. 114 (2015), 061802 we are ready to fit for $|V_{cb}|$

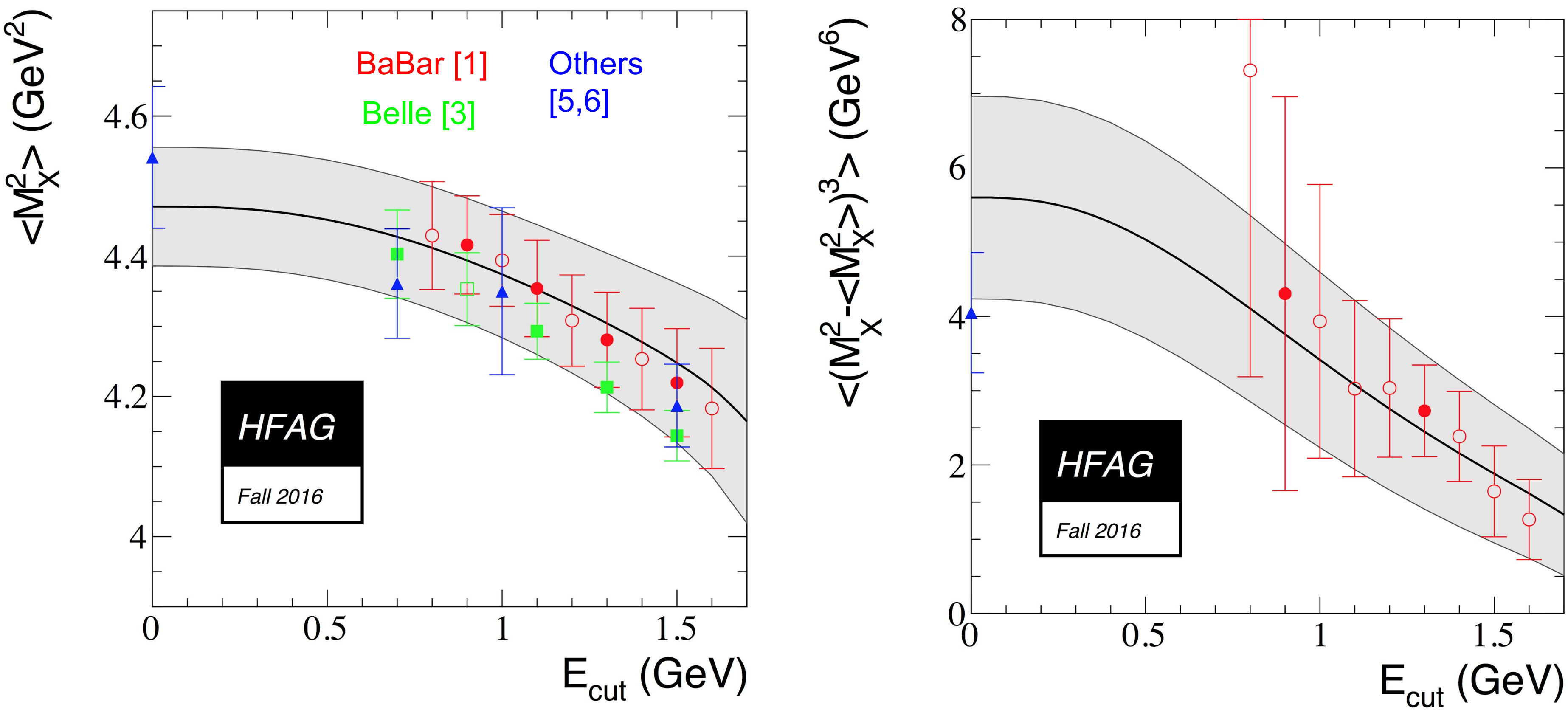



First Lepton Moment




Partial BF (zeroth order moments)

Example of hadronic moments: grey band: fit error (including theory errors)



# $|V_{cb}|$ and $m_b$ (New HFAG Fall 2016 result)

| Br(B -> $X_c$lnu) (%) | $\|V_{cb}\|$ ($10^{-3}$) | $m_b^{kin}$ (GeV) | $mu^2_{pi}$ ($GeV^2$) | |
|---|---|---|---|---|
| 10.65 +/- 0.16 | 42.19 +/- 0.78 | 4.554 +/- 0.018 | 0.464 +/- 0.076 | details |

Fit result: $\chi^2$ =14.6 over 43 ndf (P-Value: > 99%)

- Good p-value driven by conservative theory errors
- Without theory errors $\chi^2$ is of *O(80)* (P-Value < 0.05%)
  - Theory errors very important.
  - Inclusion of $m_c$ constraint reduces impact of different theory correlation assumptions (cf. next slide)

# Theory correlations

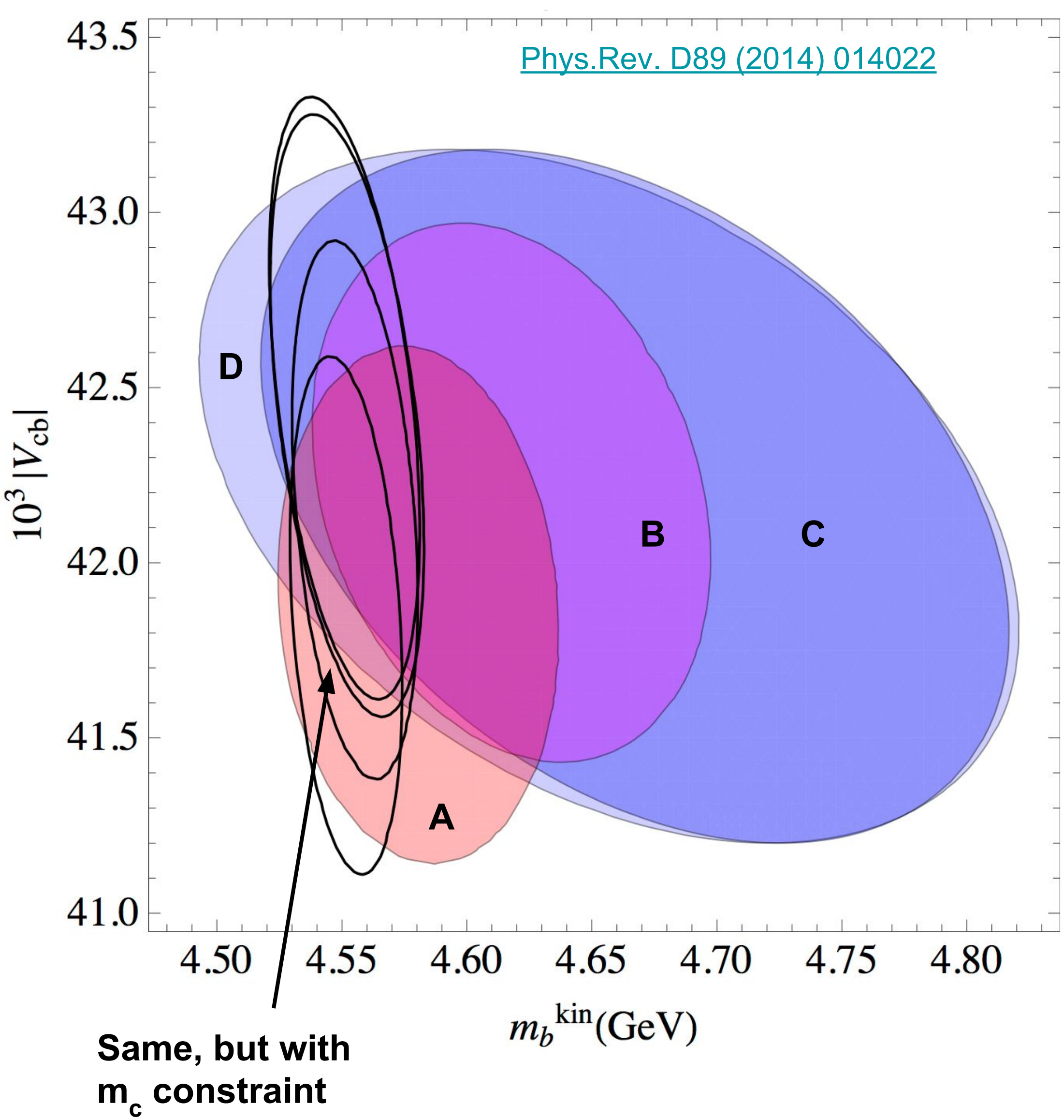
**A** 100% correlation between moments at different cuts;

**B** correlations computed from theory predictions, as discussed above;

**C** constant scale factor $0 < \xi < 1$, with $\xi = 0.97$ for 100 MeV steps;

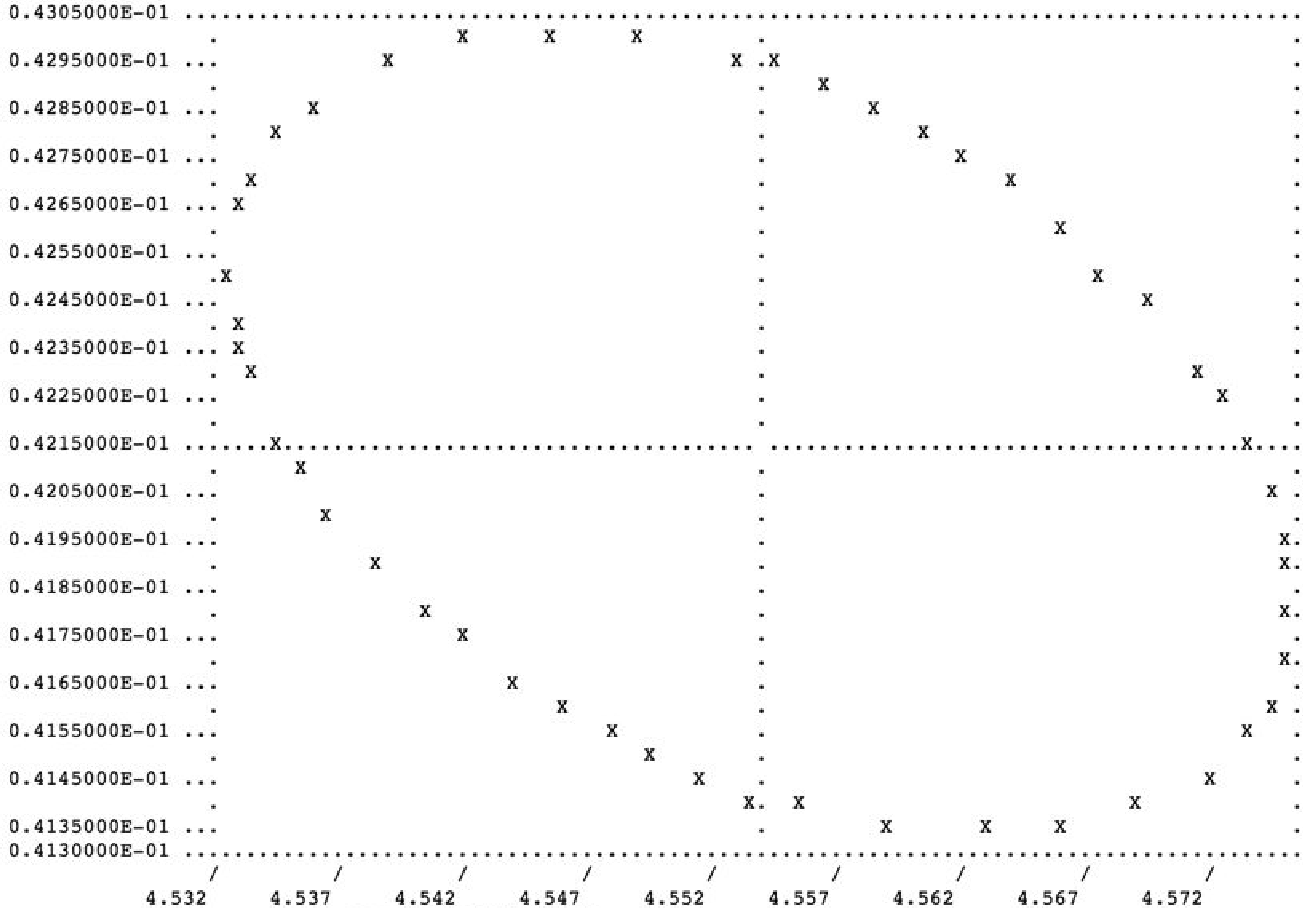
**D** a scale factor like in **C** that depends on the cut, $\xi = \xi(E_{cut})$, with

$$\xi(E_{cut}) = 1 - \frac{1}{2}\, e^{-\frac{(E_0 - E_{cut})}{\Delta}}$$



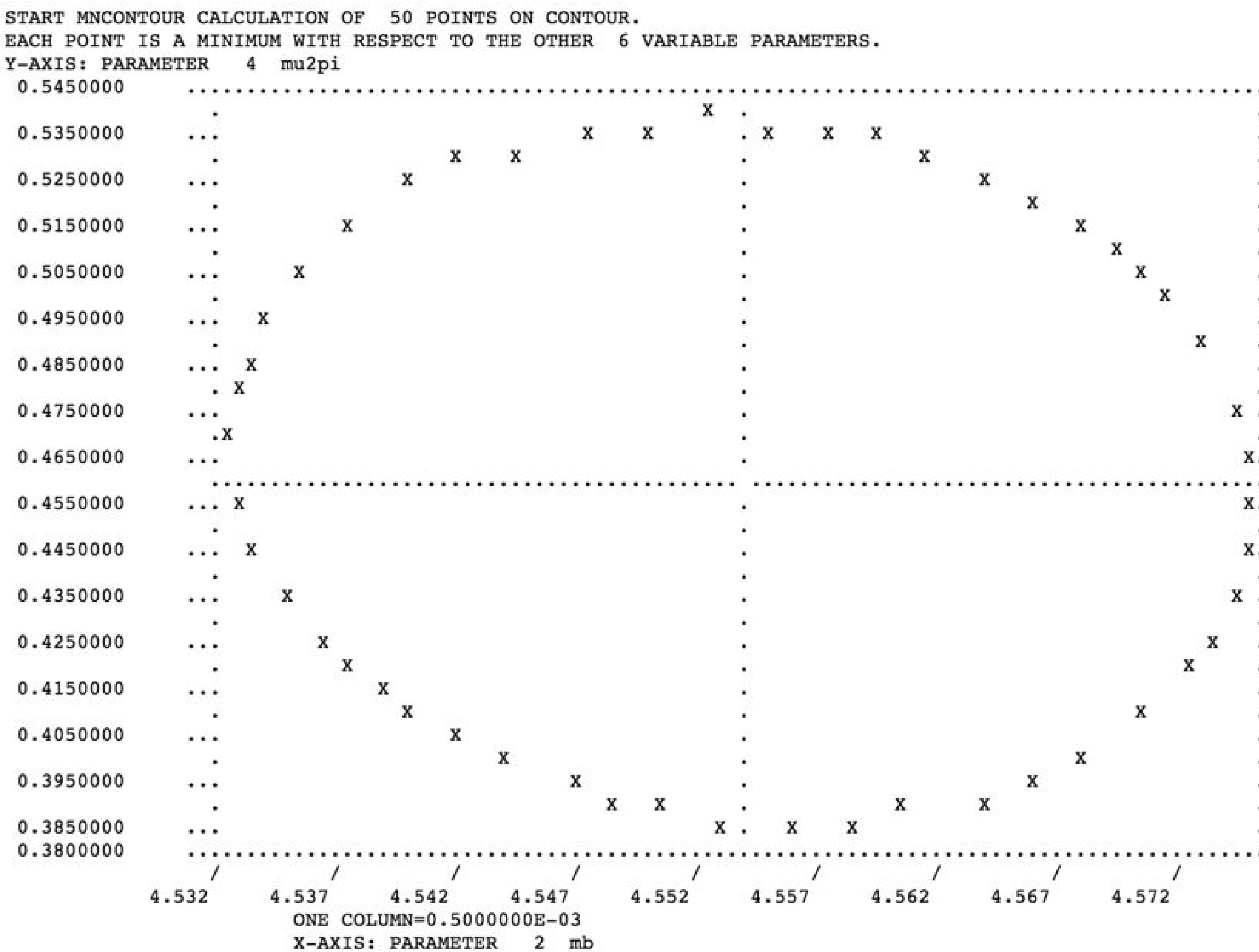
# Correlations: $|V_{cb}|$ versus $m_b$

```
**********
**    5 **SET ERR    1.000
**********
START MNCONTOUR CALCULATION OF  50 POINTS ON CONTOUR.
EACH POINT IS A MINIMUM WITH RESPECT TO THE OTHER  6 VARIABLE PARAMETERS.
Y-AXIS: PARAMETER   8  vcb
```




```
ONE COLUMN=0.5000000E-03
X-AXIS: PARAMETER   2  mb
```

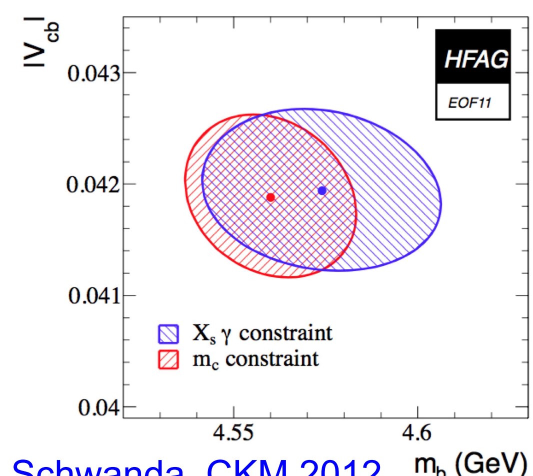
# Correlations: $\mu_{\pi}^2$ versus $m_b$



# Conclusion

Belle II likely will not significantly improve the experimental picture on inclusive $|V_{cb}|$ immediately

- Current errors driven by systematic limitations and by theory limitations
- Would be interesting to include external $m_b$ mass into fit to see how compatible things are or move theory errors to a Nuisance Parameter scheme
- Also would be interesting to include and compare to $B \rightarrow X_s \gamma$ again to test compatibility between semileptonic and radiative constraints on HQE parameters
- Also would be interesting to update 1S fit




Christoph Schwanda, CKM 2012

# Backup

# Systematic uncertainties on moment measurements

BaBar hadronic tag with 210/fb [1]

| $k$ | $p^*_{\ell,\min}$ [GeV/$c$] | $\langle m_X^k \rangle$ | $\sigma_{stat}$ | $\sigma_{sys}$ | MC statistics | simulation related | extraction method | back-groud | signal model |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.8 | 2.0906 | ±0.0063 | ±0.0166 | 0.0058 | 0.0099 | 0.0096 | 0.0047 | 0.0031 |
| | 0.9 | 2.0890 | ±0.0062 | ±0.0158 | 0.0048 | 0.0088 | 0.0103 | 0.0045 | 0.0028 |
| | 1.0 | 2.0843 | ±0.0061 | ±0.0153 | 0.0044 | 0.0076 | 0.0109 | 0.0044 | 0.0027 |
| | 1.1 | 2.0765 | ±0.0063 | ±0.0165 | 0.0044 | 0.0072 | 0.0127 | 0.0047 | 0.0026 |
| | 1.2 | 2.0671 | ±0.0064 | ±0.0160 | 0.0046 | 0.0073 | 0.0120 | 0.0045 | 0.0025 |
| | 1.3 | 2.0622 | ±0.0068 | ±0.0168 | 0.0048 | 0.0073 | 0.0131 | 0.0050 | 0.0023 |
| | 1.4 | 2.0566 | ±0.0073 | ±0.0183 | 0.0047 | 0.0069 | 0.0150 | 0.0054 | 0.0021 |
| | 1.5 | 2.0494 | ±0.0081 | ±0.0198 | 0.0036 | 0.0074 | 0.0168 | 0.0061 | 0.0019 |
| | 1.6 | 2.0430 | ±0.0092 | ±0.0221 | 0.0038 | 0.0082 | 0.0187 | 0.0070 | 0.0018 |
| | 1.7 | 2.0387 | ±0.0109 | ±0.0265 | 0.0047 | 0.0081 | 0.0232 | 0.0083 | 0.0015 |
| | 1.8 | 2.0370 | ±0.0143 | ±0.0337 | 0.0069 | 0.0097 | 0.0299 | 0.0098 | 0.0013 |
| | 1.9 | 2.0388 | ±0.0198 | ±0.0413 | 0.0082 | 0.0123 | 0.0355 | 0.0150 | 0.0008 |

# Systematic uncertainties on moment measurements

BaBar lepton tag with 47.4/fb [2]

TABLE II: Results and breakdown of the systematic errors for the partial branching fraction $\mathcal{B} = \tau_B \int_{E_0}^{\infty} (d\Gamma/dE_e)\, dE_e$, and the moments $M_1$, $M_2$, and $M_3$ for $B \to X_c e\nu$ and $B \to X_c e\nu(\gamma)$ in the $B$-meson rest frame for two values of $E_0$. Changes wrt. the previously published results [9] are due to updated background branching fractions [21, 41] and indicated by (†).

| | $\mathcal{B}[10^{-2}]$ | | $M_1$[MeV] | | $M_2[10^{-3}\ \mathrm{GeV}^2]$ | | $M_3[10^{-3}\ \mathrm{GeV}^3]$ | |
|---|---|---|---|---|---|---|---|---|
| $E_0$[GeV] | 0.6 | 1.5 | 0.6 | 1.5 | 0.6 | 1.5 | 0.6 | 1.5 |
| **Breakdown of systematic errors** | | | | | | | | |
| Conversion and Dalitz pairs | 0.028 | 0.001 | 1.5 | 0.02 | 0.6 | 0.00 | 0.04 | 0.00 |
| $e$ identification efficiency | 0.150 | 0.044 | 2.5 | 0.30 | 0.6 | 0.07 | 0.27 | 0.08 |
| $e$ from same $B$ | 0.019 | 0.000 | 1.3 | 0.00 | 0.6 | 0.00 | 0.05 | 0.00 |
| $B \to D_s \to e$ (†) | 0.024 | 0.000 | 1.4 | 0.01 | 0.5 | 0.00 | 0.03 | 0.00 |
| $B \to D \to e$ (†) | 0.035 | 0.000 | 2.2 | 0.00 | 1.0 | 0.00 | 0.03 | 0.00 |
| $B \to \tau \to e$ (†) | 0.027 | 0.001 | 1.2 | 0.04 | 0.3 | 0.00 | 0.10 | 0.00 |
| $e$ from $J/\psi$ or $\psi(2S)$ (†) | 0.002 | 0.001 | 0.0 | 0.01 | 0.0 | 0.01 | 0.00 | 0.00 |
| Secondary tags | 0.052 | 0.011 | 1.6 | 0.06 | 0.6 | 0.00 | 0.05 | 0.00 |
| $\chi$ | 0.022 | 0.011 | 0.9 | 0.01 | 0.3 | 0.00 | 0.03 | 0.00 |
| Tracking efficiency | 0.083 | 0.033 | 1.0 | 0.06 | 0.3 | 0.02 | 0.06 | 0.00 |
| Bremsstrahlung correction | 0.011 | 0.028 | 1.9 | 0.43 | 0.0 | 0.05 | 0.18 | 0.00 |
| Event selection | 0.052 | 0.024 | 0.6 | 0.14 | 0.0 | 0.03 | 0.07 | 0.01 |
| $b \to u$ subtraction (†) | 0.031 | 0.020 | 0.8 | 0.83 | 0.4 | 0.32 | 0.14 | 0.12 |
| $B$ momentum correction | 0.000 | 0.005 | 0.0 | 0.19 | 0.1 | 0.10 | 0.04 | 0.02 |
| $N_{tag}$ normalization | 0.068 | 0.030 | | | | | | |
| **Results** | | | | | | | | |
| Results for $B \to X_c e\nu(\gamma)$ | 10.08 | 4.53 | 1418.8 | 1768.7 | 146.1 | 29.6 | −10.08 | 2.04 |
| ±(stat.) | 0.06 | 0.03 | 3.8 | 1.9 | 2.0 | 0.8 | 0.81 | 0.44 |
| ±(syst.) | 0.21 | 0.08 | 5.4 | 1.0 | 1.9 | 0.4 | 0.40 | 0.15 |
| Results for $B \to X_c e\nu$ | 10.20 | 4.78 | 1437.6 | 1773.7 | 145.4 | 30.1 | −12.04 | 2.04 |
| ±(stat.) | 0.06 | 0.03 | 4.0 | 1.9 | 2.3 | 0.9 | 0.91 | 0.47 |
| ±(syst.) | 0.22 | 0.08 | 5.7 | 1.1 | 2.1 | 0.4 | 0.40 | 0.15 |

# Systematic uncertainties on moment measurements

Belle hadronic tag with 140/fb [3]

TABLE III: Breakup of the systematic error on $\langle M_X^2 \rangle$. Refer to the text for details.

| | $\Delta\langle M_X^2\rangle$ (GeV$^2/c^4$) | | | | | | |
|---|---|---|---|---|---|---|---|
| $E^*_{\rm min}$ (GeV) | 0.7 | 0.9 | 1.1 | 1.3 | 1.5 | 1.7 | 1.9 |
| secondary/fake leptons | 0.033 | 0.023 | 0.013 | 0.007 | 0.004 | 0.002 | 0.000 |
| combinatorial background | 0.006 | 0.004 | 0.003 | 0.002 | 0.002 | 0.002 | 0.000 |
| continuum | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $B \to X_u \ell\nu$ background | 0.004 | 0.004 | 0.004 | 0.004 | 0.006 | 0.007 | 0.009 |
| $\mathcal{B}(D^{(*)}\ell\nu)$ | 0.008 | 0.007 | 0.007 | 0.007 | 0.006 | 0.005 | 0.003 |
| $\mathcal{B}(D^{**}\ell\nu)$ | 0.022 | 0.014 | 0.006 | 0.000 | 0.000 | 0.008 | 0.006 |
| $\mathcal{B}((D^{(*)}\pi)_{\rm non-res.}\ell\nu)$ | 0.024 | 0.017 | 0.007 | 0.004 | 0.004 | 0.004 | 0.004 |
| $D^{(*)}\ell\nu$ form factors | 0.013 | 0.013 | 0.012 | 0.011 | 0.010 | 0.008 | 0.006 |
| $D^{**}\ell\nu$ form factors | 0.003 | 0.002 | 0.002 | 0.001 | 0.001 | 0.001 | 0.004 |
| unfolding | 0.015 | 0.015 | 0.015 | 0.015 | 0.015 | 0.015 | 0.015 |
| binning | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.000 | 0.001 |
| efficiency | 0.008 | 0.011 | 0.012 | 0.009 | 0.008 | 0.005 | 0.004 |
| total | 0.052 | 0.041 | 0.029 | 0.024 | 0.022 | 0.022 | 0.021 |

# Systematic uncertainties on moment measurements

Belle hadronic tag with 140/fb [4]

TABLE II: Breakdown of the systematic errors for the first moment, $M_1$, for $B \to X_c e\nu$ in the $B$ meson rest frame for nine values of the electron energy threshold $E_{\rm cut}$.

| | $M_1$ [MeV] | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $E_{\rm cut}$[GeV] | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
| Electron Detection | 0.81 | 0.77 | 0.42 | 0.16 | 0.05 | 0.09 | 0.02 | 0.02 | 0.01 |
| $(D^{(*)}e\nu)$ form factors | 0.59 | 0.62 | 0.61 | 0.47 | 0.33 | 0.75 | 0.80 | 0.99 | 1.20 |
| $\mathcal{B}(D^{(*)}e\nu)$ | 0.22 | 0.17 | 0.11 | 0.11 | 0.09 | 0.09 | 0.05 | 0.10 | 0.07 |
| $(D^{**}e\nu)$ form factors | 1.71 | 1.03 | 0.47 | 0.10 | 0.09 | 0.08 | 0.10 | 0.12 | 0.02 |
| $\mathcal{B}(D^{(*)}_{\rm non-res}\pi e\nu/D^{**}e\nu)$ | 1.15 | 1.37 | 1.50 | 0.96 | 0.66 | 0.38 | 0.28 | 0.30 | 0.16 |
| Continuum | 0.02 | 0.00 | 0.02 | 0.02 | 0.01 | 0.01 | 0.01 | 0.00 | 0.00 |
| $M_{\rm bc}$ | 1.14 | 0.72 | 0.24 | 0.02 | 0.03 | 0.05 | 0.08 | 0.05 | 0.04 |
| $X_u e\nu$ | 0.79 | 0.78 | 0.77 | 0.75 | 0.72 | 0.67 | 0.56 | 0.36 | 0.14 |
| Hadron Fakes | 0.65 | 0.56 | 0.42 | 0.24 | 0.11 | 0.04 | 0.01 | 0.00 | 0.00 |
| $B \to D^{(*)} \to e$ | 0.91 | 0.79 | 0.60 | 0.39 | 0.21 | 0.10 | 0.04 | 0.01 | 0.00 |
| Secondaries | 0.82 | 0.68 | 0.49 | 0.27 | 0.12 | 0.04 | 0.01 | 0.00 | 0.00 |
| Unfolding | 0.02 | 0.23 | 0.28 | 0.24 | 0.04 | 0.06 | 0.10 | 0.33 | 0.04 |
| Total Systematics | 3.02 | 2.55 | 2.13 | 1.45 | 1.08 | 1.10 | 1.03 | 1.16 | 1.23 |