Global Fit Strategy for Inclusive $B \rightarrow XL$

Frank Tackmann

Deutsches Elektronen-Synchrotron

CKM 2016 workshop, TIFR Mumbai
November 30, 2016
**Status Overview.**

**Inclusive \(|V_{cb}|\) [see previous talk]**

- Current global moment fits are dominated by theory uncertainties, and in particular theory correlations.
- Goal for Belle II will really be to reduce the current uncertainty (50-100%) on the uncertainty (∼2%).

**Inclusive \(|V_{ub}|\) [see Bob’s and Paolo’s talks yesterday]**

- Current ∼5-7% uncertainties are probably underestimated (which contributes to the tension with excl. \(|V_{ub}|\)).
- Current methods do not extrapolate to 3% total uncertainty, need qualitative improvements to get there.

Both are (or will be) theory limited, but not in a way that more calculations alone will help.

- Overall only little room for improvement in perturbative inputs.
- Parametric uncertainties dominate, require coordinated effort between theory and experiment.
For a given moment $M_i(E_{\text{cut}})$, results with different lepton energy cut $E_\ell > E_{\text{cut}}$ are strongly correlated (most events are the same, so they have a high statistical correlation)

- The independent new information the fit sees is really in the differences $M_i(E_{\text{cut}} + 100 \text{ GeV}) - M_i(E_{\text{cut}})$
- The theory uncertainty on this difference is however never directly evaluated, but only follows indirectly from the assumed correlation for different $E_{\text{cut}}$
- Not surprising that resulting uncertainties (in particular for OPE parameters) strongly depend on theory correlation assumption [Gambino, Schwanda, 1307.4551]
Theory Uncertainties in Inclusive $|V_{cb}|$.

- Different lepton energy moments at different $E_{\text{cut}}$ are also not independent.
  - They all come from the same underlying lepton energy spectrum.
  - For example, higher $E_{\ell}$ moments are sensitive to in principle the same high-$E_{\ell}$ information as the rate with high $E_{\text{cut}}$.
  - Currently, different moments are assumed to be completely uncorrelated.

Various ways more data can help the theory:

- Measure mass moments in bins of $E_{\ell}$ instead of different $E_{\text{cut}}$.
- Directly measure the lepton energy spectrum as precisely as possible and all the way to the endpoint.
  - Potential avenue to provide important nontrivial constraint on shape function using precise $b \rightarrow c$ data (beyond $m_b$ and $\lambda_1$ constraints).
- Also measure the $q^2$ spectrum, $E_{\nu}$ spectrum, $q^0 = E_{\ell} + E_{\nu}$ spectrum.
- Could also think about performing helicity decomposition and directly measuring independent hadronic structure functions (e.g. mass moments).

$\Rightarrow$ Belle 2 can improve incl. $|V_{cb}|$ (perhaps contrary to common believe)
Measurements probe different phase-space regions
[see also Bob’s talk yesterday]

- **SF region:** $p_X^+ \ll p_X^-$ (large $E_\ell, E_\gamma$)
  - Experimentally clean(er) and highest sensitivity
  - Theory more difficult

- **Local OPE:** $p_X^+ \sim p_X^-$ ($q^2$ spec., small $E_\ell, E_\gamma$)
  - Large backgrounds, least sensitivity
  - Theoretically easier

- **Something in between:** $m_X \sim m_D$, moderately large $E_\ell (E_\gamma)$

There is no single “golden” region

- There are no “optimal” cuts
  - Measuring deep into $b \rightarrow c$ hides the issue
    (e.g. in the MC signal model), dominant sensitivity still comes from region with least
    $B \rightarrow X_c \ell \nu$ background

⇒  Do not choose, measure full spectrum, which gives most information (same for $B \rightarrow X_s \gamma$)
Global Fit Approach for $|V_{ub}|$.

Follow same basic strategy as for $|V_{cb}|$ (just more complicated now)

- Simultaneously determine from the data
  - Overall normalization: $|V_{ub}|$, $\mathcal{B}(B \to X_s\gamma)$
  - Input parameters and their uncertainties: $m_b$, shape function(s)

- Combine different decay modes and measurements
  - Different $B \to X_u\ell\nu$ and $B \to X_s\gamma$ spectra
  - Can eventually include/predict also $B \to X_s\ell^+\ell^-$
  - External constraints on $m_b$, $\mu_\pi^2 (\lambda_1)$, from $B \to X_c\ell\nu$ or elsewhere

What it achieves

- Minimize uncertainties by making maximal use of all available data
  - Fit automatically “chooses” most sensitive region given exp. and theory unc.

- Qualitatively better inclusive $|V_{ub}|$ with consistent treatment of correlated uncertainties (experimental, theoretical, input parameters)

What it requires

- Experiment: Precise spectra, including full correlations
- Theory: Proper theory description across phase space, model-independent treatment of shape function
Global fit approach will become very powerful with high statistics

- Measure as many spectra as precisely as possible to maximize the available shape information
  - Detailed shape information is key to constraining subleading corrections
  - Analyses need input on shape in any case, e.g. to improve signal MC
  - $E_\ell$ spectrum (in bins of $m_X$), (high) $q^2$, $m_X$, $p_X^+$ (all in bins of $E_\ell$?)
  - Separate $B^+$ and $B^0$

- Take advantage of large datasets to maximize resolution and to aggressively reject backgrounds at the cost of efficiency
  - super-clean full reconstruction sample
Theory Side of Global $|V_{ub}|$ Fit.


- Global fit combining all available information
- Employs model-independent treatment for SF

$$d\Gamma_s = |V_{tb}V_{ts}^*|^2 m_b^2 |C_{7}^{incl}|^2 \int dk \, \hat{W}_{77}(E_\gamma; k) \, \hat{F}(m_B - 2E_\gamma - k) + \cdots$$

$$d\Gamma_u = |V_{ub}|^2 \int dk \, \hat{W}_{u}(p_X^-, p_X^+, E_\ell; k) \, \hat{F}(p_X^+ - k) + \cdots$$

- Fit parameters: $|V_{tb}V_{ts}^*|^2 m_b^2, |V_{ub}|^2, \hat{F}(\lambda x) = \frac{1}{\lambda} \left[ \sum_{n=0}^{\infty} c_n f_n(x) \right]^2$
- Theory input: $\hat{W}_i(\ldots; k)$ computed to (N)NLL’+(N)NLO in 1S scheme

NNVub [Healey, Mondino, Gambino, arXiv:1604.07598]

- Based on same idea, quite different approach [see Paolo’s talk yesterday]
Factorized Shape Function.

\[ S(\omega, \mu_\Lambda) = \int dk \, \hat{C}_0(\omega - k, \mu_\Lambda) \, \hat{F}(k) \]

\( \hat{F}(k) \) nonperturbative part
- Determines peak region
- Fit from data

\( \hat{C}_0(\omega, \mu_\Lambda) \) perturbative part
- Generates perturbative tail with correct \( \mu_\Lambda \) dependence

resulting \( S(\omega, 2.5 \text{ GeV}) \)
Factorized Shape Function.

\[ S(\omega, \mu_\Lambda) = \int dk \hat{C}_0(\omega - k, \mu_\Lambda) \hat{F}(k) \]

- \( \hat{F}(k) \) nonperturbative part
  - Determines peak region
  - Fit from data

- \( \hat{C}_0(\omega, \mu_\Lambda) \) perturbative part
  - Generates perturbative tail with correct \( \mu_\Lambda \) dependence

Given \( \hat{F}(k) \) we can compute \( S(\omega, \mu_\Lambda) \) in perturbation theory
  - Vary \( \mu_\Lambda \) to estimate perturbative uncertainty in SF
  - Instead of modeling \( S(\omega, \mu_\Lambda) \) we can fit for \( \hat{F}(k) \)
Basis Expansion for $\hat{F}(k)$.

Expand $\hat{F}(k)$ into suitable orthonormal basis

$$\hat{F}(\lambda x) = \frac{1}{\lambda} \left[ \sum_{n=0}^{\infty} c_n f_n(x) \right]^2$$

$$\int dk \hat{F}(k) = \sum_{n=0}^{\infty} c_n^2 = 1$$

- Provides model-independent description

Fit for $\hat{F}(k)$ by fitting basis coefficients $c_n$

- Experimental uncertainties and correlations can be properly captured in covariance matrix of fitted coefficients $c_n$

$\Rightarrow$ Allows for data-driven, reliable estimation of SF uncertainties
Residual Basis Dependence from Series Truncation.

\[ \hat{F}(\lambda x) = \frac{1}{\lambda} \left[ \sum_{n=0}^{N} c_n f_n(x) \right]^2 \]

In practice, series must be truncated

- Induces residual basis (model) dependence
- Truncation error scales as \( 1 - \sum_{n=0}^{N} c_n^2 \)

In practice most complications are in choosing good basis (\( \lambda \)) and \( N \)

- Want basis so series converges quickly but still unbiased (e.g. iterate)
- Choose \( N \) large enough so truncation error is smaller than to exp. uncertainties, but small enough to have stable fit and not waste statistical power
- Add coefficients with more precise data
Global Fit to $B \rightarrow X_{S}\gamma$.

**Theory**
- NNLL$'$+NNLO
- non-$C_{7}$ contributions from SM

**Experimental Inputs**
- Belle inclusive (in $\Upsilon(4S')$ frame)
  [arXiv:0907.1384]
- BaBar hadronic tag (in $B$ frame)
  [arXiv:0711.4889]
- (old) BaBar sum-over-exclusive (in $B$ frame)
  [hep-ex/0508004]
- BaBar inclusive (in $\Upsilon(4S')$ frame)
  [arXiv:1207.5772]
Global Fit to $B \to X_s \gamma$.

- Too few coefficients lead to clear bias and underestimated uncertainties
- Extracted $|C_7^{incl} V_{tb} V_{ts}^*|$ consistent with SM
Global Fit Strategy for Inclusive $B \rightarrow XL$

- Too few coefficients lead to clear bias and underestimated uncertainties
- Extracted $|C_7^{incl}V_{tb}V_{ts}^*|$ consistent with SM
Perturbative uncertainties

- Dominant source of theory uncertainties
- Important to take into account correlations across spectrum
  - Integrating resummed NNLL’+NNLO spectrum should reproduce smaller uncertainties in total NNLO rate (quite nontrivial)
- Evaluated via large set of profile scale variations
- Expect theory uncertainties of comparable size to fit uncertainties
Subleading shape functions

- $|C_7|^2$-like ones can be absorbed into leading SF
  - Irrelevant for fit
  - Important for interpretation: Cause substantial shift in $m_b$ given by their total 1st moment

$$\frac{-\lambda_1 + 3\lambda_2}{2m_b} \sim 70 \text{ MeV}$$

- Four-quark shape functions:
  - Formally $\alpha_s/m_b$ suppressed
  - Do not find large ($\sim 5\%$) effects as in [Benzke, Lee, Neubert, Paz, arXiv:1003.5012]
  - Dominant effect from $O_{1,2}O_7$ interference ($c\bar{c}$ loops) can be included via single subleading shape function $\rightarrow$ only minor effect on fit
Consistent treatment of charm contributions

- Integrate out charm loops ($n_f = 3$) vs. keeping charm dynamic ($n_f = 4$)
- Include known massive results
- In the end small effect (most $m_c$ dependence is absorbed into $C_7^{incl}$)

(Indirect) dependence on $\lambda_2$ and $\rho_2$

- Only small effect on uncertainties

Fit strategy

- Study systematically sensitivity to basis choice and truncation
- Study dependence on experimental inputs
Theory
- NLL’ + NLO
- ignoring subleading SFs

Experimental Inputs
- $B \rightarrow X_u \ell \nu$ partial branching fractions
  - picked measurements for which we are sure enough that they have negligible (SF) model dependence
  - BaBar and Belle hadronic tag
  - BaBar and Belle lepton endpoint
- $B \rightarrow X_s \gamma$
  - Belle inclusive (shown)
  - (old) BaBar sum-over-exclusive (not shown)
  - BaBar hadronic tag (not shown)
- $B \rightarrow X_c \ell \nu$
  - $m_{b_s}^{1S}$, $\lambda_1$ from moment fits
Parametric unc. (SF, $m_b$) are part of fit, no pert. unc. included yet

Without full $B \rightarrow X_s \gamma$: $\sim 10\%$ uncertainties on $|V_{ub}|$

Including $B \rightarrow X_s \gamma$: halves uncertainties but also shifts $|V_{ub}|$
Projections for Belle 2.

Theory
- NLL’ + NLO
- ignoring subleading SFs

Toy study
- Generated $m_X$, $E_\ell$, and $E_\gamma$ spectra from theory
- Smeared with uncertainties and correlations inspired by BaBar hadronic tag analysis, Belle 2 hadronic tagging efficiency is much better by now
- Originally for 75/ab, scaled down to 1/ab, 5/ab, 15/ab
- Caveats:
  - No resolution effects considered
  - Should be done more thoroughly by Belle 2
Projections for Belle 2.

- No perturbative uncertainties included (but they clearly won’t scale with statistics)
No perturbative uncertainties included (but they clearly won’t scale with statistics)
Projections for Belle 2.

- **Belle II Projection**
  - Global Analysis
  - $B \rightarrow X_u \ell \nu$ only

- **Caveats:**
  - No perturbative uncertainties included (but they clearly won’t scale with statistics)
  - At Belle 2 can use $B \rightarrow X_u \ell \nu$ alone to determine SF, $m_b$, and $|V_{ub}|$
Combined analysis of $B \rightarrow X_s\ell\ell$ and $B \rightarrow X\ell\nu$.

Experimental kinematic cuts for $B \rightarrow X_s\ell\ell$

- $1 < q^2 < 6 \text{ GeV}^2$, $m_X < m_X^{\text{cut}} \sim 2 \text{ GeV}$
- Unavoidable to suppress huge $b \rightarrow c\ell^-\bar{\nu} \rightarrow s\ell^+\ell^-\nu\bar{\nu}$ background
- Shape function effects must be taken into account to retain NP sensitivity

Helicity decomposition for inclusive rate

[Lee, Ligeti, Stewart, FT (2008)]

\[
\frac{d^3\Gamma}{dp_X^+dp_X^-dz} = \frac{3}{8} \left[ (1 + z^2)H_T(p_X^\pm) + 2zH_A(p_X^\pm) 
+ 2(1 - z^2)H_L(p_X^\pm) \right]
\]

- $z = \cos \theta = 2 \frac{E_\ell - E_\ell^\pm}{p_X^- - p_X^+}$ is angle between lepton and $B$ meson in $W$ rest frame
Combined analysis of $B \rightarrow Xs\ell\ell$ and $B \rightarrow X\ell\nu$.

Experimental kinematic cuts for $B \rightarrow Xs\ell\ell$

- $1 < q^2 < 6 \text{ GeV}^2$, $m_X < m_X^{\text{cut}} \sim 2 \text{ GeV}$
- Unavoidable to suppress huge $b \rightarrow c\ell^-\bar{\nu} \rightarrow s\ell^+\ell^-\nu\bar{\nu}$ background
- Shape function effects must be taken into account to retain NP sensitivity

Same basic structure

$$dH_{T,A,L} = \sum_{i,j} C_{i}^{\text{incl}} C_{j}^{\text{incl}} \int dk \hat{W}_{ij}^{A,T,L}(p_X^+, E_\ell, E_{\bar{\ell}}; k) \hat{F}(p_X^+ - k) + \cdots$$

Combined fit of $B \rightarrow Xs\ell\ell$ and $B \rightarrow X\ell\nu$

- Best (perhaps only) way to get clean extraction of $C_9^{\text{incl}}, C_{10}^{\text{incl}}$
- Using the same inclusive helicity decomposition for $B \rightarrow X\ell\nu$ would allow to fully disentangle dominant subleading SF effects
Combination with $B \to X_c \ell \nu$.

Combined analysis of $B \to X_u \ell \nu$ and $B \to X_c \ell \nu$

- Measure very precise lepton energy spectrum
- Allows for fully consistent and correlated treatment of both channels
  - Moves separation between the modes from analysis level to interpretation step
  - Can constrain leading SF from $b \to c$
- Combined fit to directly extract $|V_{ub}/V_{cb}|$

$B \to X_\tau \nu$ and $R(X)$

- Belle 2 should obviously measure $R(X)$
  - If $R(D(\ast))$ is due to new physics it must show up in $R(X)$
- Theory for inclusive decay is as clean
- Combined analysis of $B \to X \ell \nu$ and $B \to X_\tau \nu$ to measure $R(X)(q^2)$

[Ligeti, FT (2014)]
Summary.

- Inclusive $|V_{cb}|$ and $|V_{ub}|$ with current approaches are theory limited, but not in a way that more calculations alone will help.

- Strategy for Belle 2 should be to exploit increased data sets to help theory by providing maximal amount of information in the form of differential and as model-independent as possible measurements.

- Global fit to inclusive rare and semileptonic data with model-independent treatment of shape function will be key to reach ultimate precision for inclusive $|V_{ub}|$.

- Global analysis will also be essential to fully exploit new-physics sensitivity of inclusive $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell\ell$. 

Frank Tackmann (DESY)