Improvements to inclusive V_{cb}

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Outline

- Goal : Unitarity Triangle !

- Measurement Tools:
  - ✓ Inclusive B decays : Theory !!

- State of the Art : V_{cb} from B -> X_c l ν

- OUTLOOK !!

B-Physics: Goal

Quark Mixing

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \hat{V}_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM Phenomenology:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} .$$

Wolfenstein Parametrization

$$V \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix},$$

Unitarity Triangle

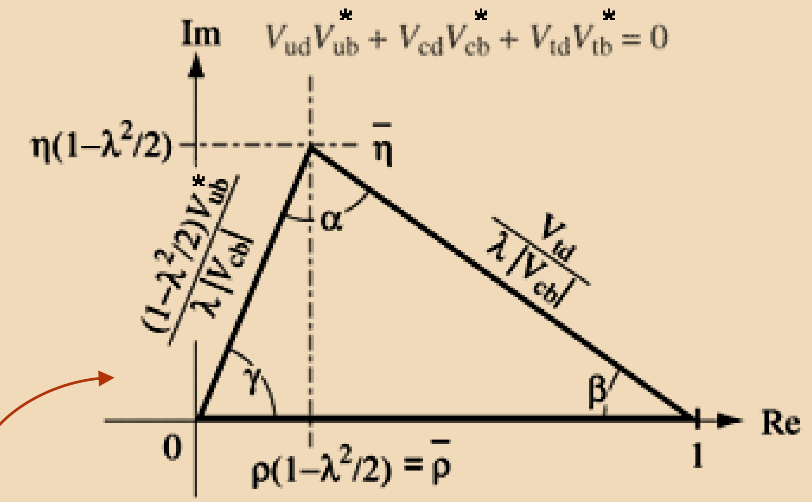


Figure 1. The unitarity relation $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ drawn in the complex $[\bar{\rho}, \bar{\eta}]$ plane.

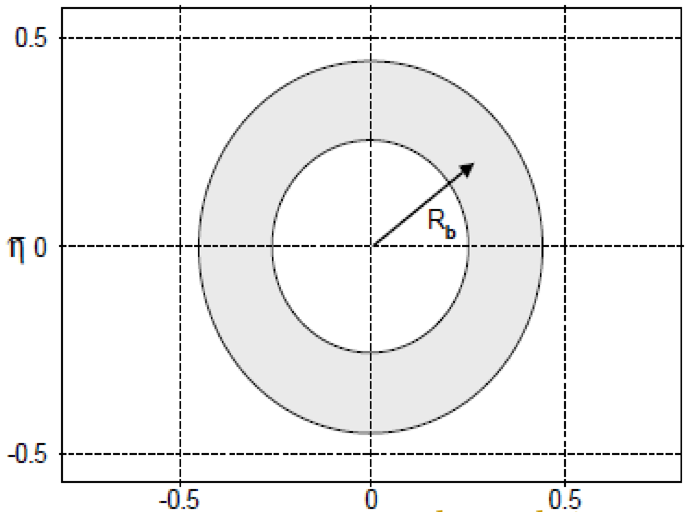
- Consistency check in the SM !!
- Searches for NP evidences !!

Role of $|V_{ub}|$ and $|V_{cb}|$

✓ $|V_{ub}|$,$|V_{cb}|$ hence $\bar{\eta}^2 + \bar{\rho}^2 = R_b^2 = \frac{(1-\lambda^2/2)^2}{\lambda^2} \left|\frac{V_{ub}}{V_{cb}}\right|^2$ are determined from tree level decays !

Expected to be free of NP effects !!

✓ They are universal fundamental constants valid in any extension of the SM! !



M. Battaglia et al. arXiv:hep-ph/0304132v2

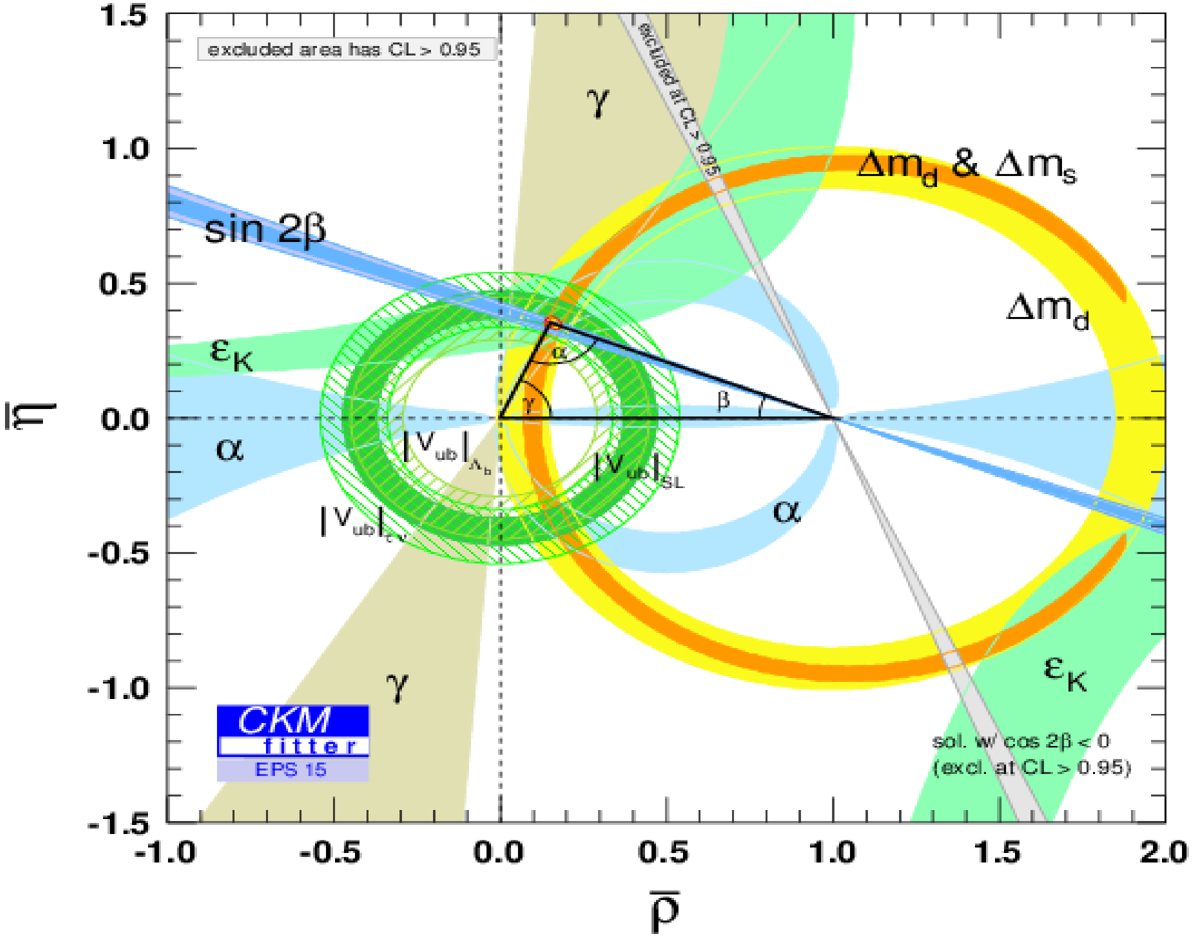
This tells us that the apex of the unitarity triangle lies in the band shown

To find where the apex lies on the UT we have to look at other decays !!

Most promising in this respect are the so-called loop induced decays and CP violating B-decays !!

✓**Precise determination of $|V_{ub}|$, $|V_{cb}|$ is of utmost importance !**

Unitarity Triangle: Fit result



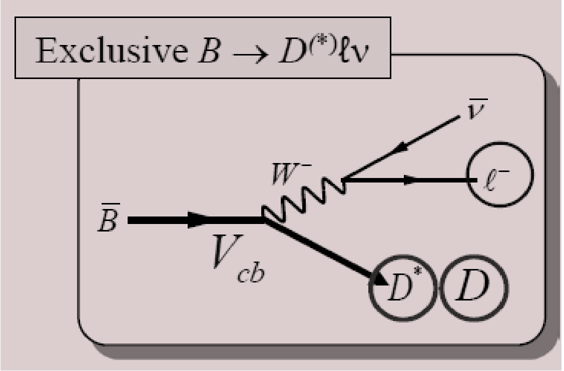
V_{cb}: Semileptonic decays

Measurement of $|V_{cb}|$

Semileptonic B-decays provide a clean environment !!

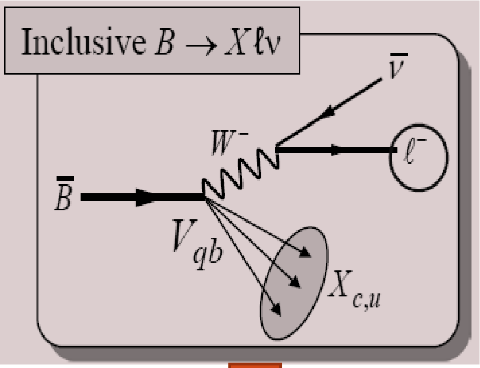
Exclusive Measurement

B-> D l ν and B-> D* l ν



Inclusive Measurement

B-> X_c ℓ ν



$$\frac{d\Gamma}{dq^2\, dE_e dE_\nu} = 2\, G_F^2\, V_{cb}^2\, W_{\mu\nu}\, L^{\mu\nu}$$

$$L^{\mu\nu} = 2\left(p_e^\mu p_{\bar{\nu}}^\nu + p_e^\nu p_{\bar{\nu}}^\mu - g^{\mu\nu} p_e \cdot p_{\bar{\nu}} - i\epsilon^{\eta\nu\lambda\mu}(p_e)_\eta (p_{\bar{\nu}})_\lambda\right)$$

$$W^{\mu\nu} = \frac{1}{2m_B}\sum_X (2\pi)^3 \delta^4(p_B - q - p_X)$$
$$\times <\bar{B}(p_B)|J_L^{\dagger\mu}|X_c(p_X)><X_c(p_X)|J_L^\nu|\bar{B}(p_B)>$$

Inclusive vs Exclusive

- Tree level semileptonic (s.l.) decays of B mesons are crucial for determining the $|V_{cb}|$ elements of the CKM matrix !
- Inclusive $b \rightarrow c l \nu$ decay rates have a solid description via OPE/HQE
- Exclusive s.l. decays have a similarly solid description in terms of heavy-quark effective theory (HQET) !

- Inclusive decays: Non perturbative unknowns can be extracted experimentally!

  ⟹ Experimentally Challenging !!

- Exclusive decays: Non perturbative unknowns have to be calculated !

  ⟹ Major theoretical challenges !!

Inclusive Semileptonic

Inclusive channels are relatively clean

- **Theoretical framework is OPE/HQE !**
- **Analysis of the final state lepton and hadron energy distribution yields:**
  - ✓ **b-quark mass !**
  - ✓ **Non-perturbative QCD parameters !**
  - ✓ **Consistency check of the OPE/HQE and other effective theory approaches !**
- **As per the measurement is concern : small statistical and systematic errors !**
  - ✓ **High sensitivity to the theoretical uncertainties !**

Precise predictions in the SM including reliable uncertainties is possible !!

OPE/HQE

- ✓ In the large m_b limit $M_W >> m_b >> \Lambda_{QCD}$ ➡ organize an expansion in Λ_{QCD}/m_b !
- ➢ The energy released in the decay is large ➡ The b quark decay mediated by weak interactions takes place on a time scale that is much shorter than the time it takes the quarks in the final state to form physical hadronic states.
  - ✓ The inclusive rate may be modelled simply by the decay of a free b quark
- ➢ Once the b quark has decayed on a time scale $t << (\Lambda_{QCD})^{-1}$, the probability that the final states will hadronize somehow is unity, and we need not know the probability of hadronization into specific final states.
- ✓ The energy release in the decay is much larger than the hadronic scale, the decay is largely insensitive to the details of the initial state hadronic structure.

This intuitive picture is formalized by the OPE, which expresses the inclusive rate as an expansion in inverse powers of the heavy quark mass, with the leading term corresponding to the free quark decay!!!

Matching

Forward scattering amplitude ➡ $W_{\mu\nu} \propto \mathrm{Im}(T_{\mu\nu})$

$$T^{\mu\nu} = -i\int d^4x\, e^{-iq\cdot x}\frac{<\bar{B}|T\left[J_L^{\mu\dagger}(x)J_L^{\nu}(0)\right]|\bar{B}>}{2m_B}$$

Using OPE

$$-i\int d^4x\, e^{-iq\cdot x}\, T\left[J_L^{\mu\dagger}(x)J_L^{\nu}(0)\right] = \sum_i c_i O_i$$

Projector

$$T^{\mu\nu} = -g^{\mu\nu}T_1 + v^{\mu}v^{\nu}T_2 - i\epsilon^{\mu\nu\alpha\beta}v_{\alpha}q_{\beta}T_3 + q^{\mu}q^{\nu}T_4 + (v^{\mu}q^{\nu} + v^{\nu}q^{\mu})T_5$$

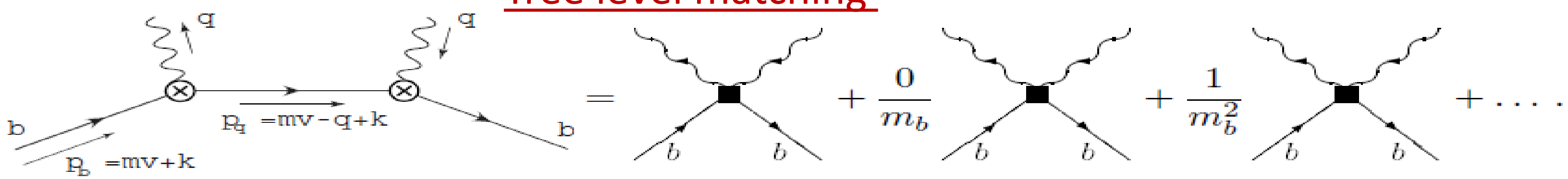
Dimension

$$T_i = \sum_{n\geq 3}\sum_{j\geq 0}\left(\frac{\Lambda_{QCD}}{m_b}\right)^{n-3}\left(\frac{\alpha_S}{\pi}\right)^{j} c_{ij}^{(n)} < \bar{B}|O^{(n)}|\bar{B}>$$

Double series expansion: perturbative in α_s and nonperturbative in Λ_{QCD}/m_b

✓ The coefficient in front of each operator are calculable in perturbation theory

Tree level matching



Decay Width

OPE relates parton to meson decay rate: $1/m_b$ and $\alpha_s(m_b)$

$$\Gamma_{SL} = |V_{cb}|^2 \frac{G_F^2 m_b^5}{192\pi^3}(1+A_{EW})A_{pert} \times \left[c_0(r) + \frac{0}{m_b} + c_2\left(r, \frac{\mu_\pi^2}{m_b^2}, \frac{\mu_G^2}{m_b^2}\right) + c_3\left(r, \frac{\rho_D^3}{m_b^3}, \frac{\rho_{LS}^3}{m_b^3}\right) + \ldots\right]$$

free quark decay — perturbative corrections — Non-perturbative power corrections

$r = m_c/m_b$

At $(\Lambda/m_b)^2$:

$$\langle \bar{B}(\vec{p})|\bar{b}_v D^\mu D_\mu b_v|\bar{B}(\vec{p})\rangle \sim m_B\,\mu_\pi^2,$$
$$\langle \bar{B}(\vec{p})|\bar{b}_v g_s G_{\mu\nu}\sigma^{\mu\nu} b_v|\bar{B}(\vec{p})\rangle \sim m_B\,\mu_G^2$$

$$\rho_D^3 = \langle B|\bar{b}(iD_{\perp\mu})(ivD)(iD_\perp^\nu)b|B\rangle\,,$$
$$\rho_{LS}^3 = \langle B|\bar{b}(iD_\perp^\mu)(ivD)(iD_\perp^\nu)\sigma_{\mu\nu}b|B\rangle$$

Main sources of uncertainties :

(1) Mass of the b-quark and the mass ratio ‘r’
(2) Higher order QED and QCD radiative corr.
(3) Higher order of the $1/m_b$ corrections !
(4) Extraction of HQE parameters !
(5) Parton Hadron Duality !!

Moments

OPE parameters can be extracted from the moments of the differential distributions

Leptonic Energy Moments: $M_1^\ell = \frac{1}{\Gamma}\int dE_\ell\, E_\ell \frac{d\Gamma}{dE_\ell}; \qquad M_n^\ell = \frac{1}{\Gamma}\int dE_\ell \left(E_\ell - M_1^\ell\right)^n \frac{d\Gamma}{dE_\ell} \quad (n > 1),$

$$M_n^\ell = \left(\frac{m_b}{2}\right)^n \left[\varphi_n(r) + \bar{a}_n(r)\frac{\alpha_s}{\pi} + \bar{b}_n(r)\frac{\mu_\pi^2}{m_b^2} + \bar{c}_n(r)\frac{\mu_G^2}{m_b^2} + \bar{d}_n(r)\frac{\rho_D^3}{m_b^3} + \bar{s}_n(r)\frac{\rho_{LS}^3}{m_b^3} + \ldots\right]$$

Moments of Invariant Hadronic Mass:

$$M_1^X = \frac{1}{\Gamma}\int dM_X^2\,(M_X^2 - \bar{M}_D^2)\frac{d\Gamma}{dM_X^2}; \qquad M_n^X = \frac{1}{\Gamma}\int dM_X^2\,(M_X^2 - \langle M_X^2\rangle)^n \frac{d\Gamma}{dM_X^2} \quad (n > 1),$$

$$M_n^X = m_b^{2n}\sum_{l=0}\left[\frac{M_B - m_b}{m_b}\right]^l \left(E_{nl}(r) + a_{nl}(r)\frac{\alpha_s}{\pi} + b_{nl}(r)\frac{\mu_\pi^2}{m_b^2} + c_{nl}(r)\frac{\mu_G^2}{m_b^2} + d_{nl}(r)\frac{\rho_D^3}{m_b^3} + s_{nl}(r)\frac{\rho_{LS}^3}{m_b^3} + \ldots\right).$$

arXiv:hep-ph/0304132v2

$\mathbf{M}_n^l$ and $\mathbf{M}_n^x$ are highly sensitive to the quark masses and OPE parameters !

- Global fit to decay rate and moments extracts: $|V_{cb}|$, m_b , m_c , μ_π^2, μ_G^2, ρ_D^3, ρ_{LS}^3

Theory : State of the art !

- ✓ Tree level upto $(1/m_b)^5$ is known ….Mannel, Turczyk, Uraltsev, Gremm, Kapustin, Gambino, Healy…
- ✓ $\mathcal{O}(\alpha_s)$ **corrections** for the partonic rate are fully known !
- ✓ $\mathcal{O}(\alpha_s^2)$ **known** ⟶ Technically challenging ⟶ Numerical calculation,

  Pak and Czarnecki PRL 2008 , Melinkov PLB 2008 analytic results for limiting cases
- ✓ $\mathcal{O}(\alpha_s^2 \beta_0)$ are known !! …Aquila, Gambino, Ridolfi and Uraltsev NPB 2005
- ✓ $\alpha_s \mu_\pi^2 / m_b^2$ only numerically ! ….Becher, Boss and Lunghi, JHEP, 2007
- ✓ $\alpha_s \mu_\pi^2 / m_b^2$ and $\alpha_s \mu_G^2 / m_b^2$ corrections are calculated with analytical expressions!

  Aberti, Ewerth, Gambino, Nandi , NPB(2013)

  Alberti, Gambino, Nandi, JHEP(2014)
  Mannel, Pivovarov, Rosenthal, PRD(2015)

α_s / m_b^3 In progress….Alberti, Gambino, Healy , Nandi

V_{cb} : Inclusive decays

Alberti, Gambino , Healy and Nandi, PRL 2015; Gambino, Healy , Turczyk, PLB 2016

$$\Gamma_{sl} = \Gamma_0 \left[1 + a^{(1)} \frac{\alpha_s(m_b)}{\pi} + a^{(2,\beta_0)} \beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 + a^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \left(-\frac{1}{2} + p^{(1)} \frac{\alpha_s}{\pi}\right) \frac{\mu_\pi^2}{m_b^2} + \left(g^{(0)} + g^{(1)} \frac{\alpha_s}{\pi}\right) \frac{\mu_G^2(m_b)}{m_b^2} + d^{(0)} \frac{\rho_D^3}{m_b^3} - g^{(0)} \frac{\rho_{LS}^3}{m_b^3} + \text{higher orders} \right]$$

$\overline{m}_c(3\text{ GeV}) = 0.986(13)\text{ GeV}$

$\rho_{LS}^3 = -0.15(10) GeV^3$

$\mu_G^2(4.6\,GeV) = 0.35(7) GeV^2$

After fitting the parameters with the available data on width and moments :

$$\frac{\Gamma}{z(r)\Gamma_0} = 1 - 0.116_{\alpha_s} - 0.030_{\alpha_s^2} - 0.042_{1/m^2} - 0.002_{\alpha_s/m^2} - 0.030_{1/m^3} + 0.005_{1/m^4} + 0.005_{1/m^5}$$

$1 - 8r + 8r^3 - r^4 - 12r^2 \ln r$

$A_{ew}|V_{cb}^2|G_F^2 m_b^5/192\pi^3$

1.014

$|V_{cb}| = (42.42 \pm 0.86) \times 10^{-3}$ → Fit without (α_s/m_b^2) and ($1/m_b^{4,5}$) and h.o. contributions ,

Gambino and Schwanda, PRD 2014

$|V_{cb}| = (42.21 \pm 0.78) \times 10^{-3}$ → Fit without ($1/m_b^{4,5}$) and h.o. contributions ,

Alberti, Gambino , Healy and Nandi, PRL 2015

$|V_{cb}| = (42.11 \pm 0.74) \times 10^{-3}$ → Fit includes all the known h.o. corrections,

Gambino, Healy , Turczyk, PLB 2016

OUT LOOK

The onset of SUPER-B (BELLE-II) factory will bring us to a high precision era

- A more precise extraction of the CKM elements are necessary in order to understand SM, QCD, and for an implicit search of NP !
- Considerable progress has been made !!

  ⇒ Much more to do in order to improve precision !!
- Stay tuned for more results !!