

Phenomenology of $B \rightarrow K\pi\pi$ modes and Prospects with LHCb and Belle-II data

Alejandro Pérez Pérez
IPHC – CNRS Strasbourg

On behalf of the CKMfitter Group



Outline

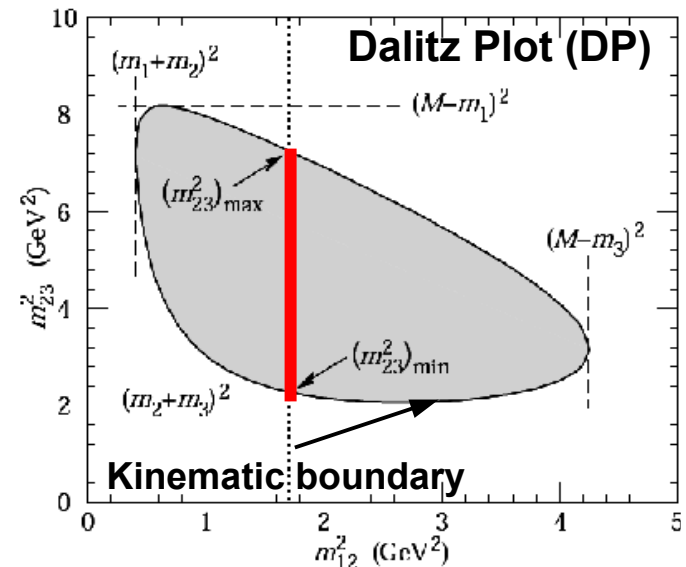
- Introduction and the $B \rightarrow K^* \pi$ phenomenological framework
- Some theoretical scenarios to constrain CKM
- Constrains on hadronic amplitudes with latest $B \rightarrow K^* \pi$ results
- Prospects with future LHCb and Belle-II data
- Summary and outlook

Introduction

B→Kππ modes: Motivation

- **Generally dominated by intermediate vector/scalar resonances (e.g. B→VP)**
 - Some of them receive sizeable contributions from tree and loop diagrams
 - If different weak phases ⇒ CP violation ⇒ constraints of CKM parameters
 - If sizeable loop contribution ⇒ sensitivity to NP
- **Several resonant states contribute to the same three-body final state**
 - Interferences along DP allow to disentangle strong and weak phases
 - Many observables: besides BF and A_{CP} , also access to interference phases
 - Rich phenomenology to be exploited

- **B→K*π system is particularly interesting**



Amplitude Analyses

Parametrizing Decay amplitude using Isobar Model:

Dalitz Plot
Isobar Model

$$\begin{cases} A(DP) = \sum_j a_j F_j(DP) \\ \bar{A}(DP) = \sum_j \bar{a}_j \bar{F}_j(DP) \end{cases}$$

Shapes of intermediate states over DP

Isobar amplitudes \Rightarrow Weak phases

Time-dependent DP PDF ($|q/p| = 1$)

$$f(\Delta t, DP, q_{\text{tag}}) \propto (|A|^2 + |\bar{A}|^2) \frac{e^{-|\Delta t|/\tau}}{4\tau} \left(1 + q_{\text{tag}} \frac{2\text{Im}[(q/p)\bar{A}A^*]}{|A|^2 + |\bar{A}|^2} \sin(\Delta m_d \Delta t) - q_{\text{tag}} \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \cos(\Delta m_d \Delta t) \right)$$

mixing and decay CPV

Direct CPV

Sensitivity to phase difference between amplitudes in the same DP plane (B or \bar{B})

Sensitivity to phase differences between a_j and \bar{a}_j amplitudes
Includes q/p mixing phase

Amplitude Analyses

Parametrizing Decay amplitude using Isobar Model:

Dalitz Plot
Isobar Model

$$\begin{cases} A(DP) = \sum_j a_j F_j(DP) \\ \bar{A}(DP) = \sum_j \bar{a}_j \bar{F}_j(DP) \end{cases}$$

Shapes of intermediate states over DP

Isobar amplitudes \Rightarrow Weak phases

Time-dependent DP PDF ($|q/p| = 1$)

$$f(\Delta t, DP, q_{\text{tag}}) \propto (|A|^2 + |\bar{A}|^2) \frac{e^{-|\Delta t|/\tau}}{4\tau} \left(1 + q_{\text{tag}} \frac{2\text{Im}[(q/p)\bar{A}A^*]}{|A|^2 + |\bar{A}|^2} \sin(\Delta m_d \Delta t) - q_{\text{tag}} \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \cos(\Delta m_d \Delta t) \right)$$

mixing and decay CPV

Direct CPV

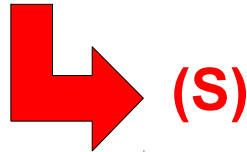
- Isobar amplitudes determine interference pattern in DP
- Any convention independent function of isobar amplitudes is a physical observable

B → K* π System: Isospin relations

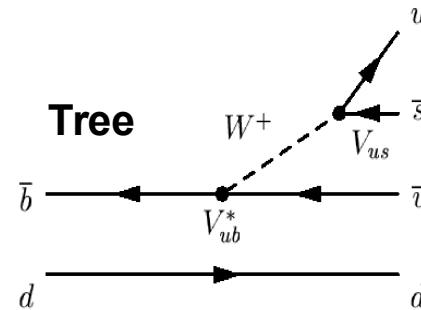
SU(2) Isospin relations:

$$A^{0+} + \sqrt{2}A^{+0} = \sqrt{2}A^{00} + A^{+-}$$

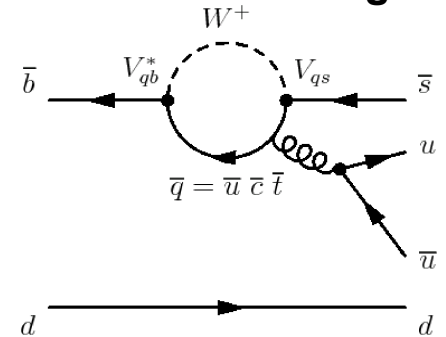
$$\bar{A}^{0+} + \sqrt{2}\bar{A}^{+0} = \sqrt{2}\bar{A}^{00} + \bar{A}^{+-}$$



B⁰ → K^{*+} π⁻



Penguin



$$A(B^0 \rightarrow K^{*+} \pi^-) = V_{us} V_{ub}^* T^{+-} + V_{ts} V_{tb}^* P^{+-}$$

$$A(B^+ \rightarrow K^{*0} \pi^+) = V_{us} V_{ub}^* N^{0+} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW}^C)$$

$$\sqrt{2}A(B^+ \rightarrow K^{*+} \pi^0) = V_{us} V_{ub}^* (T^{+-} + T_c^{00} - N^{0+}) + V_{ts} V_{tb}^* (P^{+-} - P_{EW}^C + P_{EW})$$

$$\sqrt{2}A(B^0 \rightarrow K^{*0} \pi^0) = V_{us} V_{ub}^* T_c^{00} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW})$$

- CKM unitarity ⇒ different topological contributions to hadronic amplitudes
- In above convention hadronic amplitudes referred by main contributions
 - T⁺⁻ & P⁺⁻: colour allowed tree & penguin
 - N⁰⁺ & T_c⁰⁰: annihilation & colour suppressed tree
 - P_{EW} & P_{EW}^C: colour allowed & colour suppressed electroweak penguins

B→K*π System: sensitivity to CKM (CPS/GPSZ)

CPS PRD74:051301
GPSZ PRD75:014002

$$A(B^0 \rightarrow K^{*+} \pi^-) = V_{us} V_{ub}^* T^{+-} + V_{ts} V_{tb}^* P^{+-} \quad (S)$$

$$A(B^+ \rightarrow K^{*0} \pi^+) = V_{us} V_{ub}^* N^{0+} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW}^C)$$

$$\sqrt{2}A(B^+ \rightarrow K^{*+} \pi^0) = V_{us} V_{ub}^* (T^{+-} + T_{EW}^{00} - N^{0+}) + V_{ts} V_{tb}^* (P^{+-} - P_{EW}^C + P_{EW})$$

$$\sqrt{2}A(B^0 \rightarrow K^{*0} \pi^0) = V_{us} V_{ub}^* T_{EW}^{00} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW})$$

Neglecting P_{EW} , the amplitude combinations

$$\bullet \quad 3A_{3/2} = A(B^0 \rightarrow K^{*+} \pi^-) + \sqrt{2}A(B^0 \rightarrow K^{*0} \pi^0) = V_{us} V_{ub}^* (T^{+-} + T_{EW}^{00})$$

$$\bullet \quad 3\bar{A}_{3/2} = \bar{A}(B^0 \rightarrow K^* \pi^+) + \sqrt{2}\bar{A}(B^0 \rightarrow \bar{K}^{*0} \pi^0) = V_{us}^* V_{ub} (T^{+-} + T_{EW}^{00})$$

$$\Rightarrow R'_{3/2} = (3\bar{A}_{3/2}) / (3A_{3/2}) = e^{-2i\gamma} \Rightarrow \text{access to } \gamma \text{ CKM (claimed by CPS/GPSZ)}$$

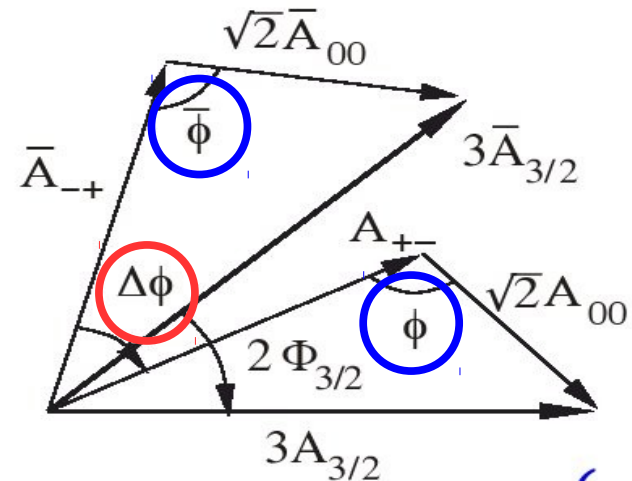
$B^0 \rightarrow K^+ \pi^- \pi^0$ Analysis

$$\phi = \arg(A(B^0 \rightarrow K^{*+} \pi^-) / A(B^0 \rightarrow K^{*0} \pi^0))$$

$$\bar{\phi} = \arg(\bar{A}(B^0 \rightarrow K^* \pi^+) / \bar{A}(B^0 \rightarrow \bar{K}^{*0} \pi^0))$$

$B^0 \rightarrow K^0_s \pi^+ \pi^-$ Analysis

$$\Delta\phi = \arg(\bar{A}(B^0 \rightarrow K^* \pi^+) / A(B^0 \rightarrow K^{*+} \pi^-))$$



From Experiment

B → K* π System: sensitivity to CKM (CPS/GPSZ)

CPS PRD74:051301
GPSZ PRD75:014002

$$A(B^0 \rightarrow K^{*+} \pi^-) = V_{us} V_{ub}^* T^{+-} + V_{ts} V_{tb}^* P^{+-} \quad (S)$$

$$A(B^+ \rightarrow K^{*0} \pi^+) = V_{us} V_{ub}^* N^{0+} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW}^C)$$

$$\sqrt{2}A(B^+ \rightarrow K^{*+} \pi^0) = V_{us} V_{ub}^* (T^{+-} + T_C^{00} - N^{0+}) + V_{ts} V_{tb}^* (P^{+-} - P_{EW}^C + P_{EW})$$

$$\sqrt{2}A(B^0 \rightarrow K^{*0} \pi^0) = V_{us} V_{ub}^* T_C^{00} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW})$$

Neglecting P_{EW} , the amplitude combinations

- $3A_{3/2} = A(B^0 \rightarrow K^{*+} \pi^-) + \sqrt{2}A(B^0 \rightarrow K^{*0} \pi^0) = V_{us} V_{ub}^* (T^{+-} + T^{00})$

- $3\bar{A}_{3/2} = \bar{A}(B^0 \rightarrow K^* \pi^+) + \sqrt{2}\bar{A}(B^0 \rightarrow \bar{K}^{*0} \pi^0) = V_{us}^* V_{ub} (T^{+-} + T^{00})$

$$\Rightarrow R'_{3/2} = (3\bar{A}_{3/2}) / (3A_{3/2}) = e^{-2i\gamma} \Rightarrow \text{access to } \gamma \text{ CKM (claimed by CPS/GPSZ)}$$

- But $B^0 \rightarrow K_s^0 \pi^+ \pi^-$ analysis actually measures $\Delta\phi = \arg((q/p)\bar{A}(B^0 \rightarrow K^* \pi^+) / A(B^0 \rightarrow K^{*+} \pi^-))$

- The actual physical observable (invariant under phase redefinitions) is

$$R_{3/2} = (q/p)R'_{3/2} = (q/p)(3\bar{A}_{3/2}) / (3A_{3/2}) = e^{-2i\beta} e^{-2i\gamma} = e^{-2i\alpha} \Rightarrow \text{access to } \alpha \text{ CKM}$$

In real life P_{EW} is not negligible \Rightarrow complicated constrain on CKM parameters

B → K* π System: counting unknowns and observables

$$\begin{aligned}
 A(B^0 \rightarrow K^{*+} \pi^-) &= V_{us} V_{ub}^* T^{+-} + V_{ts} V_{tb}^* P^{+-} \\
 A(B^+ \rightarrow K^{*0} \pi^+) &= V_{us} V_{ub}^* N^{0+} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW}^C) \\
 \sqrt{2} A(B^+ \rightarrow K^{*+} \pi^0) &= V_{us} V_{ub}^* (T^{+-} + T_C^{00} - N^{0+}) + V_{ts} V_{tb}^* (P^{+-} - P_{EW}^C + P_{EW}) \\
 \sqrt{2} A(B^0 \rightarrow K^{*0} \pi^0) &= V_{us} V_{ub}^* T_C^{00} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW})
 \end{aligned}$$

(S)

11 Had. + 2 CKM 13 unknowns

(A and λ well known)

Observables: a total of 13 observables

- 4 BF_s and 4 A_{CP} from DP and Q2B analyses ⇒ 8 observables
- Phase differences ⇒ 5 observables
 - $\Delta\phi = \arg((q/p)\bar{A}(\bar{B}^0 \rightarrow K^{*-} \pi^+) A^*(B^0 \rightarrow K^{*+} \pi^-)) \Rightarrow B^0 \rightarrow K_S^0 \pi^+ \pi^-$
 - $\phi = \arg(A(B^0 \rightarrow K^{*0} \pi^0) A^*(B^0 \rightarrow K^{*+} \pi^-))$ &
 $\bar{\phi} = \arg(\bar{A}(\bar{B}^0 \rightarrow \bar{K}^{*0} \pi^0) \bar{A}^*(\bar{B}^0 \rightarrow K^{*-} \pi^+)) \Rightarrow B^0 \rightarrow K^+ \pi^- \pi^0$
 - $\phi = \arg(A(B^+ \rightarrow K^{*0} \pi^+) A^*(B^+ \rightarrow K^{*+} \pi^0))$ &
 $\bar{\phi} = \arg(\bar{A}(\bar{B}^+ \rightarrow \bar{K}^{*0} \pi^-) \bar{A}^*(\bar{B}^+ \rightarrow K^{*-} \pi^0)) \Rightarrow B^+ \rightarrow K^0 \pi^+ \pi^0$

PRD71:094008 (2005)

No free-lunch Theorem
 Reparametrization Invariance (RI) prevents the simultaneous extraction of CKM and hadronic parameters without additional information

unknowns = # observables, but →

B→K*π System: two strategies

- **Scenario 1:** theoretical hypothesis on hadronic parameters and constrain CKM
 - If $\text{Had} \rightarrow \text{Had} + \delta\text{Had}$ gives $\text{CKM} \rightarrow \text{CKM} + \delta\text{CKM}$ 😊
E.g.: α from $B \rightarrow \pi\pi$
 - If $\text{Had} \rightarrow \text{Had} + \delta\text{Had}$ gives $\text{CKM} \rightarrow \text{CKM} + \Delta\text{CKM}$ 😞
 - **Goal:** test CPS/GPSZ method and/or find alternative methods
- **Scenario 2:** CKM from external input (global fit) and constrain hadronic parameters
 - Uncontroversial \Rightarrow assumes CKM unitarity + no large NP contributions
 - Inputs
 - CKM parameters from global fit
 - $B \rightarrow K^*\pi$ experimental measurements
 - Output
 - Prediction on unavailable observables
 - Constraint of hadronic amplitudes \Rightarrow test of QCD predictions

$B \rightarrow K^* \pi$ System: CPS/GPSZ theoretical hypothesis

CPS PRD74:051301
GPSZ PRD75:014002

■ **CPS/GPSZ:** relation between the P_{EW} and $T_{3/2} = T^{+-} + T^0_C$

- **$B \rightarrow \pi\pi$ system**

- $P_{EW} = R T_{3/2}$, with real and $R = 1.35\% \Rightarrow$ assumes SU(2) and Wilson coeff. $|c_{8,9}|$ small
- CKM accompanying P and T of same order
 $\Rightarrow P_{EW}$ contribution negligible even if high uncertainty

Eur. Phys. J. C. 11 (1999) 93
Phys. Lett. B 441 (1998) 403
PRL81 (1998) 5076

- **$B \rightarrow K\pi$ system**

- $P_{EW} = R T_{3/2} \Rightarrow$ same assumptions as in $B \rightarrow \pi\pi$ + SU(3)
- $|V_{ts} V_{tb}^* / V_{us} V_{ub}^*| \sim 55 \Rightarrow$ P/T CKM amplified $\Rightarrow P_{EW}$ cannot be neglected even if small

- **$B \rightarrow K^* \pi$ system**

- $P_{EW} = R_{eff} T_{3/2}$, with $R_{eff} = R (1 - r_{VP}) / (1 + r_{VP})$
- r_{VP} complex \Rightarrow vector-pseudoscalar phase space
- GPSZ estimation $|r_{VP}| < 5\%$

Scenarios to constrain CKM

Scenarios to constrain CKM: Exploration of Had. hypotheses

- **Benchmark of theoretical hypotheses to constrain CKM**
- **Assume ad-hoc values of CKM and hadronic amplitudes (Closure Test)**
 - CKM parameters from global fit
 - Had. amplitudes constrained to follow naïve hierarchy pattern
 - $T^{+-} > T^{00}_C > N^{0+}$ and $P^{+-} > P_{EW} > P_{EW}^C$
 - Furthermore, P_{EW} constrained to match CPS/GPSZ: $P_{EW}/(T^{+-} + T^{00}_C) = (1.35\%, 0)$
 - This ad-hoc choice of “true” values roughly reproduces current BF and A_{CP} (c.f. table)
- **Test theoretical hypotheses: explore constrains on CKM assuming very precise observables** ⇒ **test stability of theoretical method**

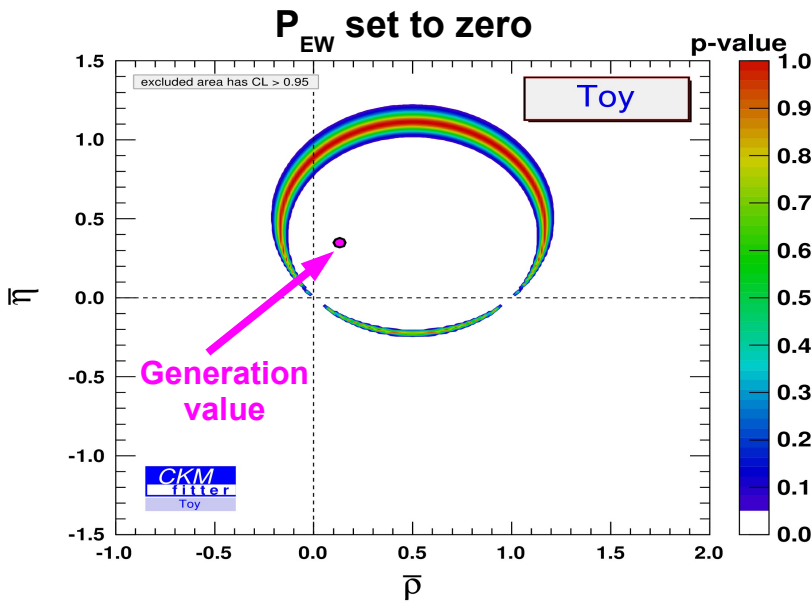
Hadronic par.	magnitude	phase (deg)	Physical observable	Measurement	Value
T^{+-}	2.540	0.00	$\mathcal{B}(B^0 \rightarrow K^{*+}\pi^-)$	8.2 ± 0.9	7.1
T^{00}	0.762	75.74	$\mathcal{B}(B^0 \rightarrow K^{*0}\pi^0)$	3.3 ± 0.6	1.6
N^{0+}	0.143	108.37	$\mathcal{B}(B^+ \rightarrow K^{*+}\pi^0)$	9.2 ± 1.5	8.5
P^{+-}	0.091	-6.48	$\mathcal{B}(B^+ \rightarrow K^{*0}\pi^+)$	11.6 ± 1.2	10.9
P_{EW}	0.038	15.15	$A_{CP}(B^0 \rightarrow K^{*+}\pi^-)$	-24.0 ± 7.0	-12.9
P_{EW}^C	0.029	101.90	$A_{CP}(B^0 \rightarrow K^{*0}\pi^0)$	-15.0 ± 13.0	-46.5
$\left \frac{V_{ts}V_{tb}^*P^{+-}}{V_{us}V_{ub}^*T^{+-}} \right $	1.801		$A_{CP}(B^+ \rightarrow K^{*+}\pi^0)$	-0.52 ± 15.0	-35.4
T^{00}/T^{+-}	0.300		$A_{CP}(B^+ \rightarrow K^{*0}\pi^+)$	$+5.0 \pm 5.0$	+3.9
N^{0+}/T^{00}	0.187				
$ P_{EW}/P^{+-} $	0.420				
$ P_{EW}/(T^{+-} + T^{00}) /R$	1.000				
$ P_{EW}/P_{EW}^C $	0.762				

Explored hypotheses

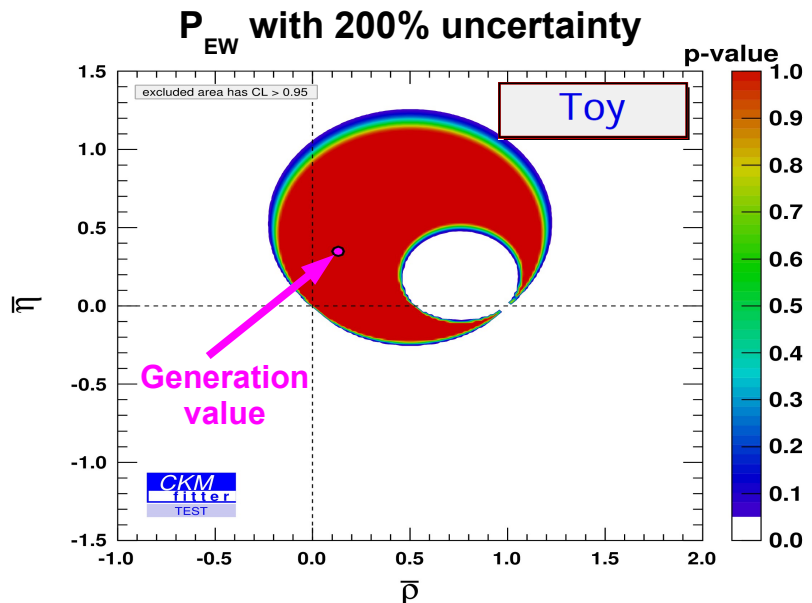
- **CPS/GPSZ-like assumption**
- **Constrains on annihilation**

Scenarios to constrain CKM: CPS/GPSZ-like

CPS PRD74:051301
GPSZ PRD75:014002



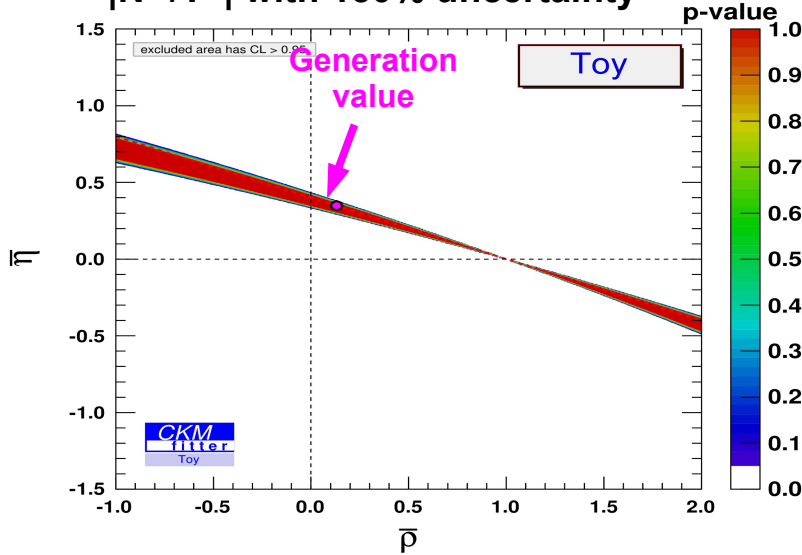
- **α -like constrain with negligible P_{EW}**
- $$3A_{3/2} = A(B^0 \rightarrow K^{*+}\pi^-) + \sqrt{2}A(B^0 \rightarrow K^{*0}\pi^0) = V_{us} V_{ub}^* (T^{+-} + T^{00})$$
- $$3\bar{A}_{3/2} = \bar{A}(\bar{B}^0 \rightarrow K^{*-}\pi^+) + \sqrt{2}\bar{A}(\bar{B}^0 \rightarrow \bar{K}^{*0}\pi^0) = V_{us}^* V_{ub} (T^{+-} + T^{00})$$
- $$\Rightarrow R_{3/2} = (q/p)(3\bar{A}_{3/2})/(3A_{3/2}) = e^{-2i\beta} e^{-2i\gamma} = e^{-2i\alpha}$$



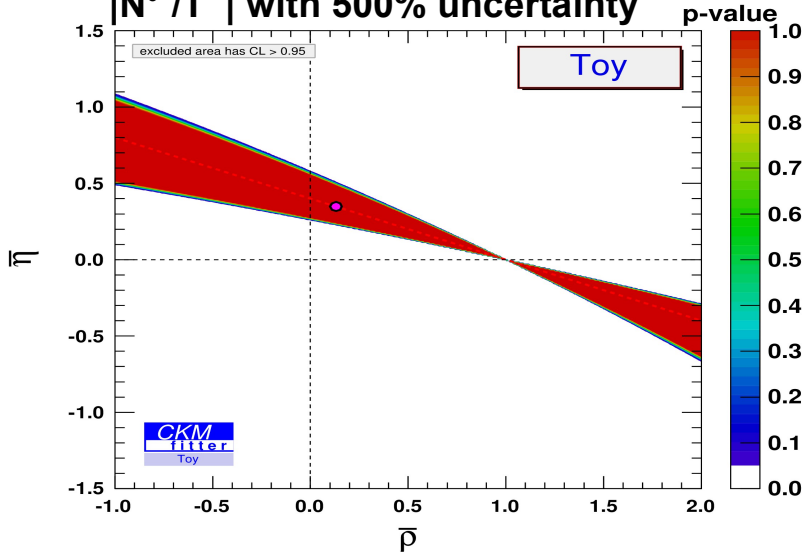
- **CPS/GPSZ method overly sensitive to assumed P_{EW} values**
 - Realistic uncertainty on P_{EW} spoils method predictability
 - Remember that “true” P_{EW} is 1.35% smaller than tree amplitude!
 - But its impact is strongly enhanced by the CKM factors on penguin terms

Scenarios to constrain CKM: hypothesis on $|N^{0+}/T^{+-}|$

$|N^{0+}/T^{+-}|$ with 150% uncertainty



$|N^{0+}/T^{+-}|$ with 500% uncertainty



- β -like constrain with negligible N_{0+}

$$A' = A(B^+ \rightarrow K^{*0} \pi^+) = V_{ts} V_{tb}^* (-P^+ + P_{EW}^C)$$

$$\bar{A}' = \bar{A}(B^- \rightarrow \bar{K}^{*0} \pi^-) = V_{ts}^* V_{tb} (-P^+ + P_{EW}^C)$$

$$\Rightarrow R' = (q/p)(\bar{A}')/(A') = e^{-2i\beta}$$

- CKM enhancement does not affect “tree” terms
- Furthermore, N^{0+} is naively expected to be small: in this example $|N^{0+}/T^{+-}| = \sim 6\%$
- May be constrained from theory and/or from annihilation-dominated modes
- **Relaxing the hypothesis on $|N^{0+}/T^{+-}|$ yields only a mild deterioration on the constraints**

**Constraints on Hadronic amplitudes
with current data
(Fixing CKM params. from global fit)**

Experimental inputs

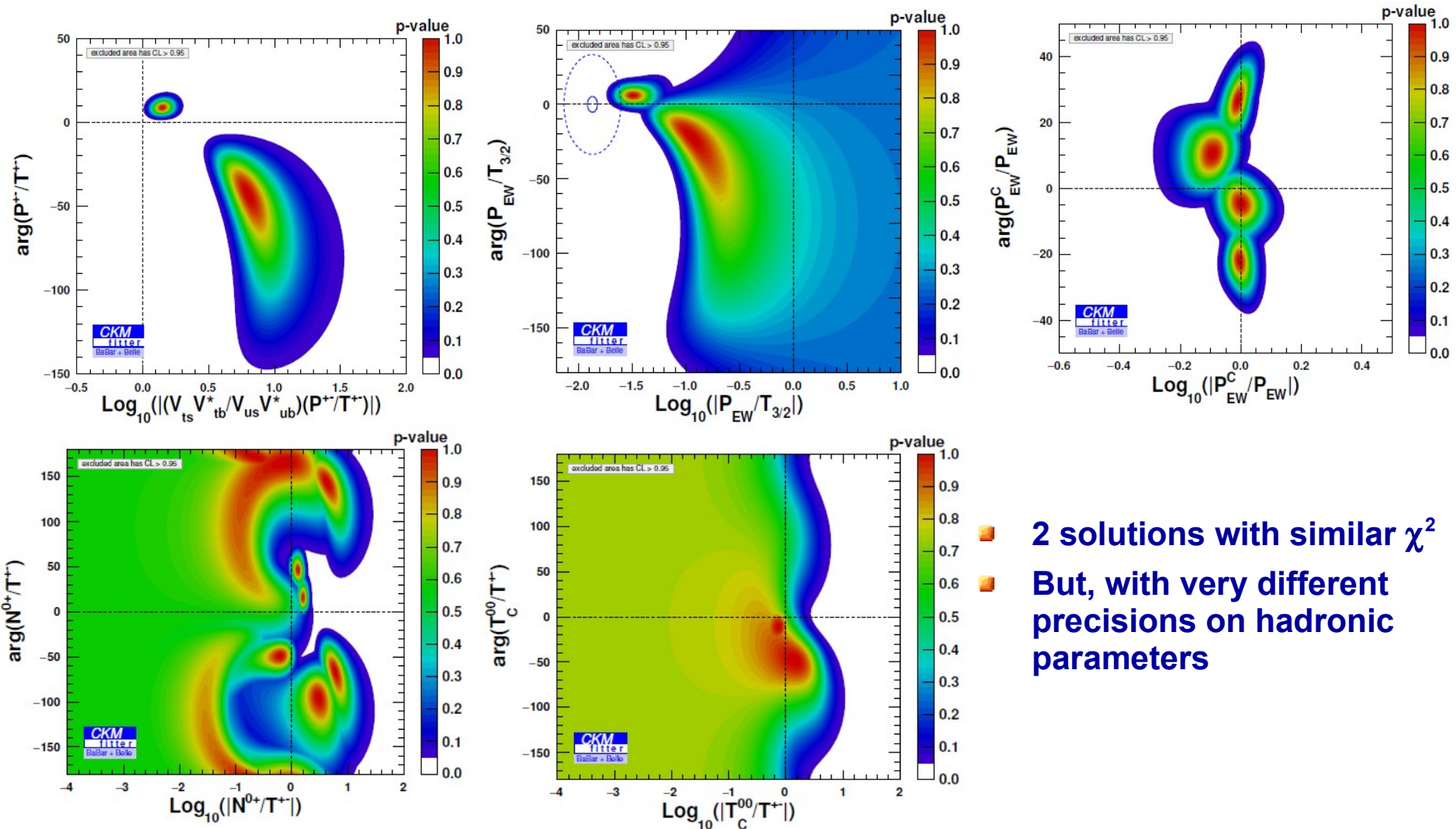
- $B^+ \rightarrow K^+ \pi^- \pi^+$ amplitude analysis: PRD78:012004 (2008)
- $B^0 \rightarrow K_S^0 \pi^+ \pi^-$ time-dependent amplitude analysis: PRD80:112001 (2009)
 - $A_{CP}(K^{*+} \pi^-) = (-20 \pm 10 \pm 2)\%$ ($\sim 2\sigma$ significance)
- $B^0 \rightarrow K^+ \pi^- \pi^0$ amplitude analysis: PRD83:112010 (2011)
 - $A_{CP}(K^{*+} \pi^-) = (-29 \pm 11 \pm 2)\%$ ($\sim 3\sigma$ significance)
- $B^+ \rightarrow K_S^0 \pi^+ \pi^0$ amplitude analysis: 1501.00705 [hep-ex] (2015)
 - $A_{CP}(K^{*+} \pi^0) = (+52 \pm 15)\%$ ($\sim 3.4\sigma$ significance)

- $B^+ \rightarrow K^+ \pi^- \pi^+$ amplitude analysis: PRL96: 251803 (2006)
- $B^0 \rightarrow K_S^0 \pi^+ \pi^-$ time-dependent amplitude analysis: PRD75:012006 (2007)
PRD79:072004 (2009)
 - $A_{CP}(K^{*+} \pi^-) = (-20 \pm 11 \pm 7)\%$ ($\sim 1.5\sigma$ significance)
- No amplitude analyses for $B^0 \rightarrow K^+ \pi^- \pi^0$ & $B^+ \rightarrow K_S^0 \pi^+ \pi^0$ modes



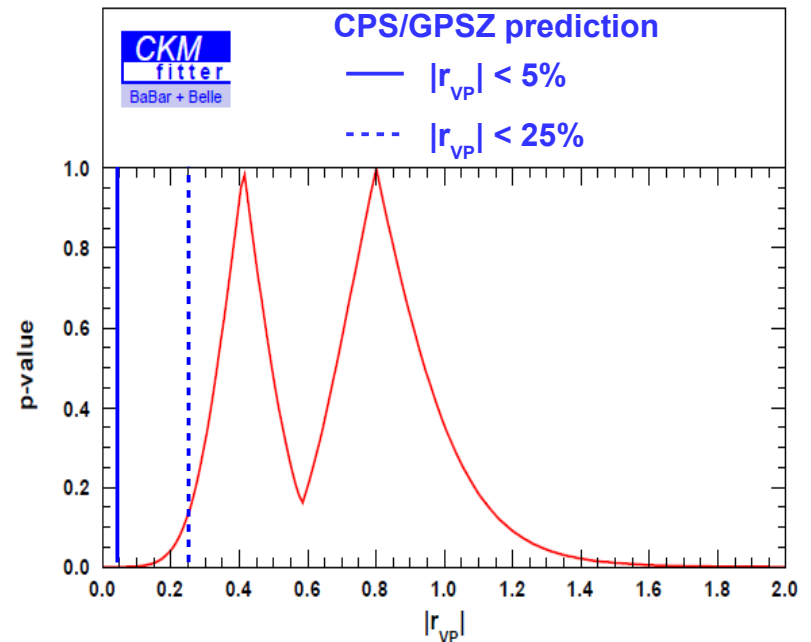
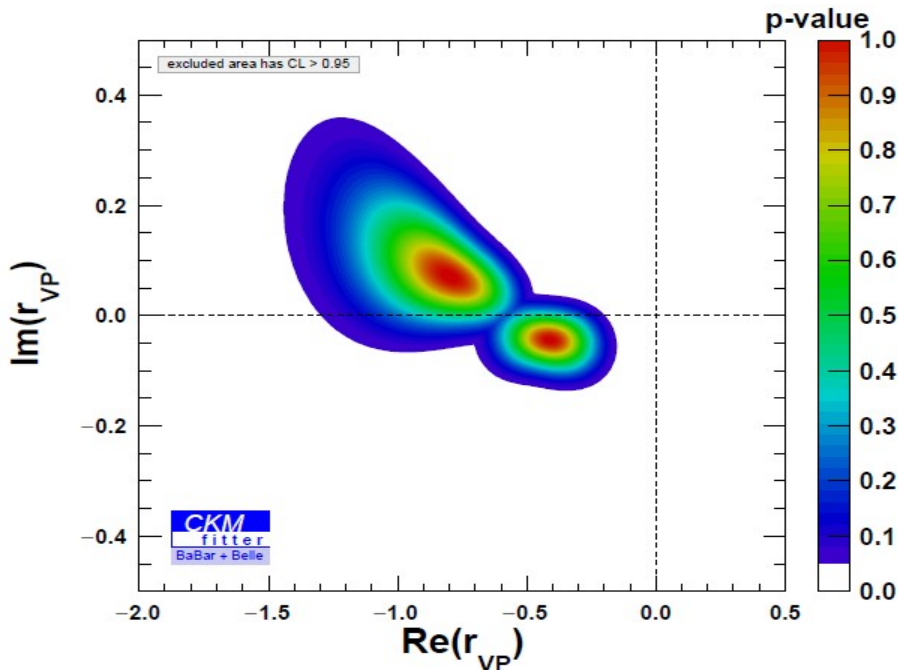
- **BABAR and BELLE have been combined and used for this analysis**

Results on Had. Amplitudes: all together



Results on Had. Amplitudes: agreement with CPS/GPSZ

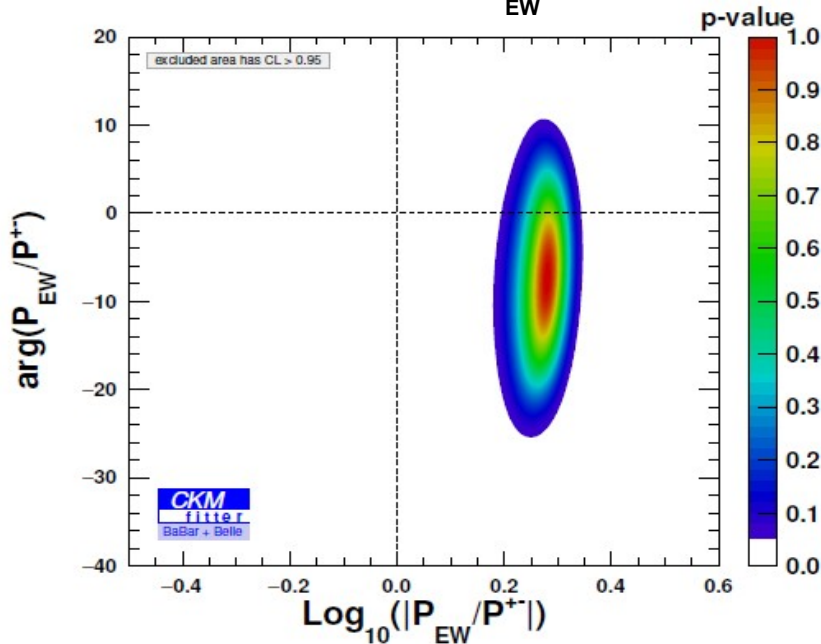
- CPS/GPSZ prediction: $P_{EW}/(T^{+-} + T^{00}) = R(1-r_{VP})/(1+r_{VP})$ with $R = 1.35\%$ and $|r_{VP}| < 5\%$
- The current experimental constraints in poor agreement with the CPS/GPSZ prediction
- Marginal comparison only reached by inflating the uncertainty on $|r_{VP}|$ up to 25%



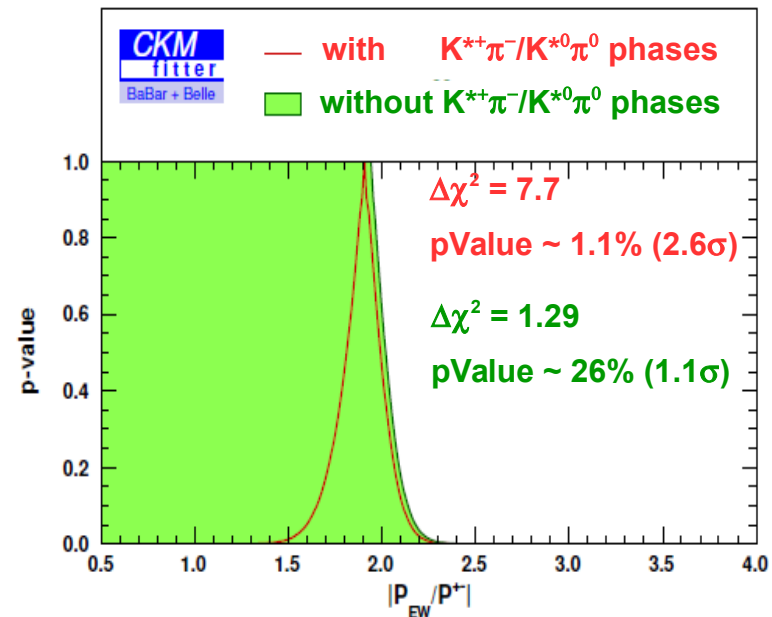
Results on Had. Amplitudes: Hierarchies (1/2)

- Current data favours a relatively high P_{EW}
- This result is mainly driven by $K^{*+}\pi^-/K^{*0}\pi^0$ phase differences from $B^0 \rightarrow K^+\pi^-\pi^0$
- Good agreement among measurements without these phases: $\Delta\chi^2 = 1.29 \Rightarrow \sim 1.1\sigma$
- Adding these phases brings slight tension: $\Delta\chi^2 = 7.7 \Rightarrow \sim 2.6\sigma$
- $B^0 \rightarrow K^+\pi^-\pi^0$ analysis performed only by *BABAR*
- Independent confirmation needed to claim non-zero (and large!) P_{EW} value

Constraint on P_{EW}/P^{+-}



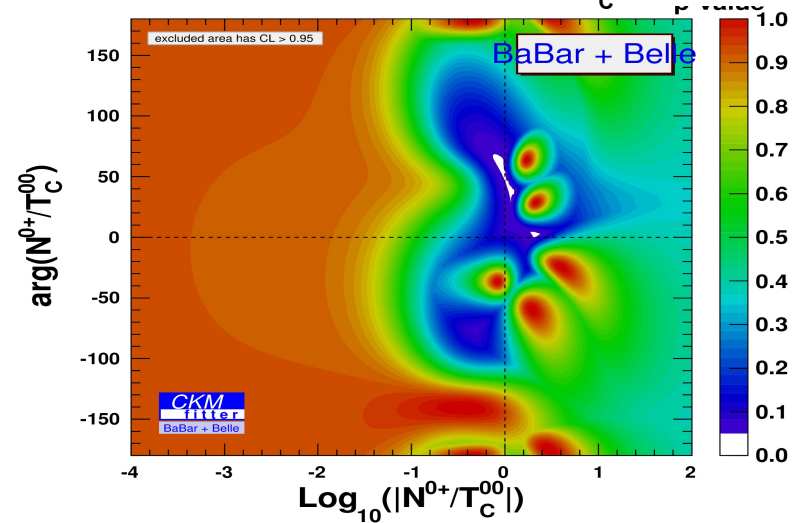
Constraint on $|P_{EW}/P^{+-}|$



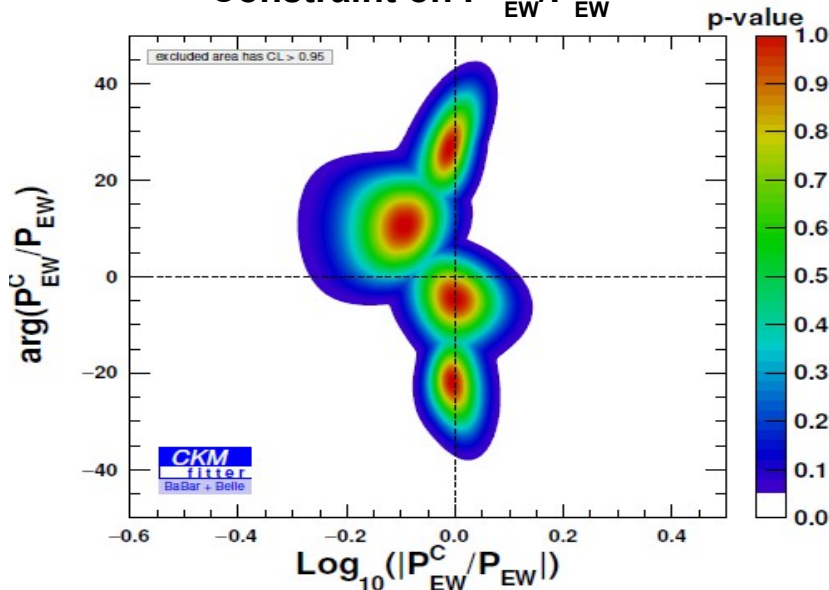
Results on Had. Amplitudes: Hierarchies (2/2)

- Essentially no constraint is obtained on N^{0+}/T_C^{00}
- Strong constrain on P_{EW}^C/P_{EW}
 - 2 solutions @ $|P_{EW}^C/P_{EW}| = 0.8$ & 1.0
 - Result consequence of large P_{EW}
 - Need independent confirmation of $B^0 \rightarrow K^+ \pi^- \pi^0$ analysis results

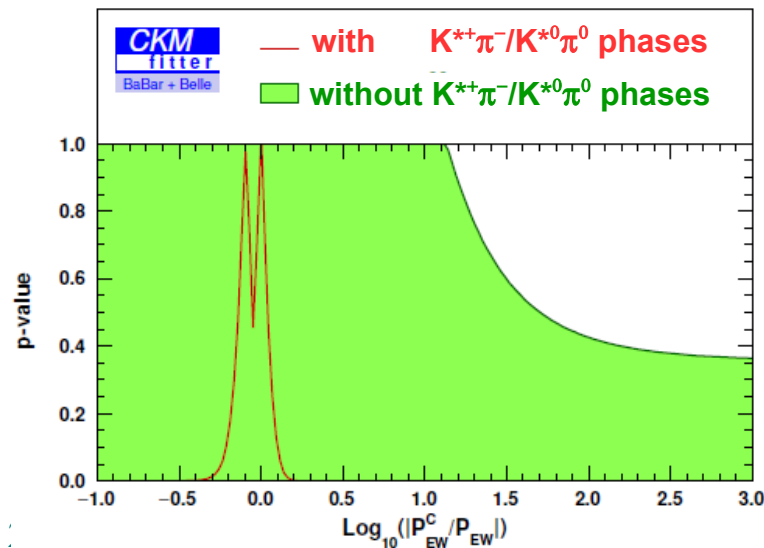
Constraint on N^{0+}/T_C^{00}



Constraint on P_{EW}^C/P_{EW}



Constraint on $\text{Log}_{10} |P_{EW}^C/P_{EW}|$



Prospects for future LHCb and Belle-II data

Prospects for LHCb and Belle-II

LHCb: high statistic for charged modes $B^0 \rightarrow K^0_S (\rightarrow \pi^+ \pi^-) \pi^+ \pi^-$ & $B^+ \rightarrow K^+ \pi^- \pi^+$

- Statistical error scaling considering different performances w.r.t *BABAR/Belle*
 - Better S/B \Rightarrow improvements on BF & A_{CP}
 - TDCPV error scales as $1/\sqrt{Q} \Rightarrow \text{sqrt}(Q_{BABAR}/Q_{LHCb}) = \text{sqrt}(30.5/3.02) \sim 3.2$
- Resolution in Dalitz Plot: negligible effect according to LHCb experts
- Sensitivity to $B^0 \rightarrow K^+ \pi^- \pi^0 / B^+ \rightarrow K^0_S \pi^+ \pi^0$, but difficult to anticipate performances



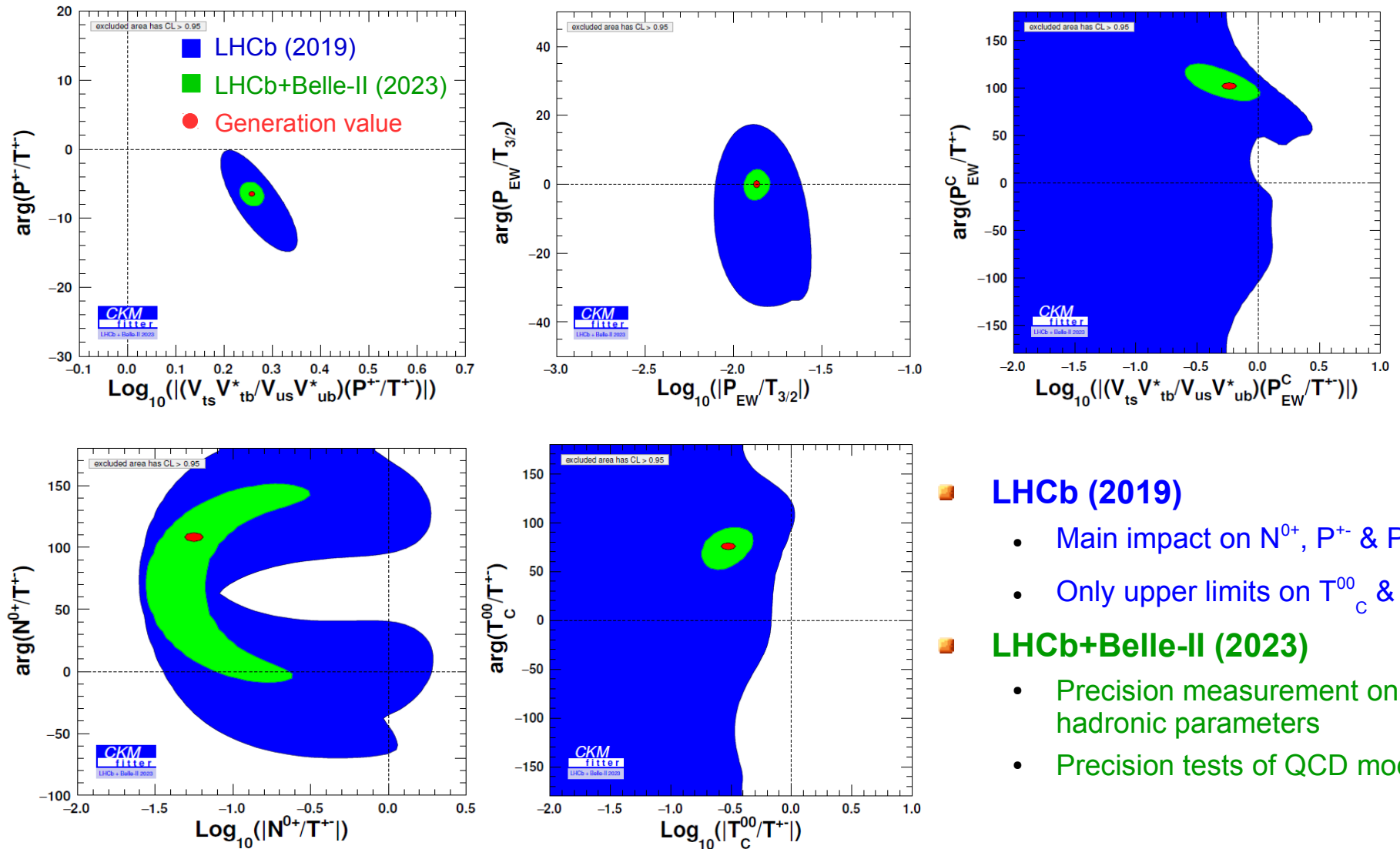
Belle-II: high statistics for all modes

- $B^0 \rightarrow K^0_S \pi^+ \pi^-$, $B^0 \rightarrow K^+ \pi^- \pi^0$, $B^+ \rightarrow K^+ \pi^- \pi^+$ & $B^+ \rightarrow K^0_S \pi^+ \pi^0$
- Similar environment to *BABAR/Belle*. Naive statistical error scaling $\Rightarrow \sqrt{(50 \text{ab}^{-1} / 1.0 \text{ab}^{-1})} \sim 7$



- Both LHCb and Belle-II will measure $B^+ \rightarrow K^+ \pi^- \pi^+$ mode with high precision
 - Will be able to well define the Signal Model \Rightarrow Model systematics significantly reduced
 - Assume negligible model uncertainties
- For this study assume same central values as in closure test

Had. Parameters: LHCb (2019) & LHCb+Belle-II (2023)



- **LHCb (2019)**
 - Main impact on N^{0+} , P^- & P_{EW}
 - Only upper limits on T_C^{00} & P_{EW}^C
- **LHCb+Belle-II (2023)**
 - Precision measurement on hadronic parameters
 - Precision tests of QCD models

Summary and Outlook

Summary and outlook

■ $B \rightarrow K^* \pi$ system: large phenomenology potential

- **Constraints on CKM limited by hadronic uncertainties**
 - α -like constraints (CPS/GPSZ): spoiled due to high sensitivity on P_{EW} theo. Prediction
 - β -like constraints (bounds on $|N^{0+}/T^{+}|$): less sensitive to theoretical input
- **Model-independent extraction of hadronic parameters (SU(2) & CKM from global fit)**
- **Constraints on hadronic amplitudes with current data**
 - Results don't favour colour suppression for P_{EW}^C/P_{EW} and to lesser extent T_C^{00}/T^{+}
 - Current data favours relatively large EWPs (mainly driven by *BABAR* $B^0 \rightarrow K^+ \pi^- \pi^0$ result)
 - If confirmed, would set evidence for EWPs in fully hadronic charmless B decays
- **Paper about $B \rightarrow K^* \pi$ results currently on preparation**

■ Outlook

- Future LHCb and Belle-II data will allow precision tests of QCD models
- Possible extension: **include $B \rightarrow K \rho$ modes**
 - Same isospin relations as $B \rightarrow K^* \pi$, but only 4 BF, 4 A_{CP} and 1 interference phase
 - Crossed observables: 7 $B \rightarrow K \rho / B \rightarrow K^* \pi$ interference phases
 - $B \rightarrow \rho K + B \rightarrow K^* \pi$ system: 25 unknowns & 29 observables \Rightarrow **rich phenomenology!**

Back up Slides

Amplitude Analyses: the signal model

- Isobar model needs predefined list of components w/ line-shapes: **Signal Model**
- No straightforward way to determine the Signal Model from theory
- Signal Model mainly determined from data
 - Previous experimental results (e.g. *quasy-two-body analyses*) to come out with a smart guess of Signal Model \Rightarrow “Raw Signal Model” (RSM)
 - Use data to test additional contributions which could eventually be added to RSM \Rightarrow building of the “Nominal Signal Model”
 - Minor contributions ignored and treated as systematics \Rightarrow **Model uncertainties**
 - Additional model errors: line-shapes uncertainties (e.g. non-resonant and $K\pi$ S-wave)
- SU(3) prediction: same components should contribute to SU(3) related final states
 - Final states with high efficiency and low background can be used to build the Signal Model
 - This model can then be used coherently among SU(3) related final states
 - This implies correlations of the model uncertainties of the SU(3) related final states which need to be evaluated \Rightarrow **currently no correlation is assumed**
- **We strongly recommend the analysts of all $B \rightarrow h'hh$ ($h/h' = \pi, K$) modes to work in coordination \Rightarrow coherent conventions used by all analyses and experiments**

Amplitude Analyses: What can be measured?

- Any convention-independent function of the isobar parameters is a physical observable

Examples

- Direct CP-asymmetries:

$$A_{CP}^j = \frac{|\bar{a}_j|^2 - |a_j|^2}{|\bar{a}_j|^2 + |a_j|^2}$$

- Branching Fractions:

$$B_j \propto \iint (|a_j|^2 + |\bar{a}_j|^2) F_j(DP) dDP$$

- Phase differences in the same B or \bar{B} DP: $\varphi_{ij} = \arg(a_i/a_j)$ $\bar{\varphi}_{ij} = \arg(\bar{a}_i/\bar{a}_j)$

- Phase differences between B and \bar{B} DP: $\Delta\varphi_j = \arg((q/p)\bar{a}_j/a_j)$

- All amplitude analyses should provide the complete set of isobar parameters together with the full statistical and systematic covariance matrices
- This allows to properly use all the available experimental information and to correctly interpret the results

B → K* π System: Observables re-parametrization

$$B^0 \rightarrow K^0_S \pi^+ \pi^-$$

$$\begin{aligned} & B(B^0 \rightarrow K^{*+} \pi^-) \\ & A_{CP}(B^0 \rightarrow K^{*+} \pi^-) \\ & \Delta\phi(B^0 \rightarrow K^{*+} \pi^-) \end{aligned}$$



$$\begin{aligned} & \text{Re}(A(K^{*-} \pi^+)/A(K^{*+} \pi^-)) \\ & \text{Im}(A(K^{*-} \pi^+)/A(K^{*+} \pi^-)) \\ & B(B^0 \rightarrow K^{*+} \pi^-) \end{aligned}$$

$$B^0 \rightarrow K^+ \pi^- \pi^0$$

$$\begin{aligned} & B(B^0 \rightarrow K^{*+} \pi^-) \\ & A_{CP}(B^0 \rightarrow K^{*+} \pi^-) \\ & B(B^0 \rightarrow K^{*0} \pi^0) \\ & A_{CP}(B^0 \rightarrow K^{*0} \pi^0) \\ & \phi(K^{*0} \pi^0 / K^{*+} \pi^-) \\ & \phi(\bar{K}^{*0} \pi^0 / K^{*-} \pi^+) \end{aligned}$$



$$\begin{aligned} & |\bar{A}(K^{*-} \pi^+)/A(K^{*+} \pi^-)| \\ & \text{Re}(A(K^{*0} \pi^0)/A(K^{*+} \pi^-)) \\ & \text{Im}(A(K^{*0} \pi^0)/A(K^{*+} \pi^-)) \\ & \text{Re}(A(\bar{K}^{*0} \pi^0)/A(K^{*-} \pi^+)) \\ & \text{Im}(A(\bar{K}^{*0} \pi^0)/A(K^{*-} \pi^+)) \\ & B(B^0 \rightarrow K^{*0} \pi^0) \end{aligned}$$

$$B^+ \rightarrow K^+ \pi^- \pi^+$$

$$\begin{aligned} & B(B^+ \rightarrow K^{*0} \pi^+) \\ & A_{CP}(B^+ \rightarrow K^{*0} \pi^+) \end{aligned}$$



$$\begin{aligned} & |A(\bar{K}^{*0} \pi^-)/A(K^{*0} \pi^-)| \\ & B(B^+ \rightarrow K^{*0} \pi^+) \end{aligned}$$

$$B^+ \rightarrow K^0_S \pi^+ \pi^0$$

$$\begin{aligned} & B(B^+ \rightarrow K^{*0} \pi^+) \\ & A_{CP}(B^+ \rightarrow K^{*0} \pi^+) \\ & B(B^+ \rightarrow K^{*+} \pi^0) \\ & A_{CP}(B^+ \rightarrow K^{*+} \pi^0) \\ & \phi(K^{*+} \pi^0 / K^{*0} \pi^+) \\ & \phi(K^{*-} \pi^0 / \bar{K}^{*0} \pi^-) \end{aligned}$$



$$\begin{aligned} & |A(K^{*-} \pi^0)/A(K^{*+} \pi^0)| \\ & \text{Re}(A(K^{*+} \pi^0)/A(K^{*0} \pi^+)) \\ & \text{Im}(A(K^{*+} \pi^0)/A(K^{*0} \pi^+)) \\ & \text{Re}(A(K^{*-} \pi^0)/A(\bar{K}^{*0} \pi^-)) \\ & \text{Im}(A(K^{*-} \pi^0)/A(\bar{K}^{*0} \pi^-)) \\ & B(B^+ \rightarrow K^{*+} \pi^0) \end{aligned}$$

Prospects for LHCb and Belle-II: numerical inputs

Expected evolution of the uncertainties on the observables

Observable	Analysis	Current	LHCb (run1+run2)	Belle-II
$\text{Re}(A(K^* \pi^+)/A(K^{*+} \pi^-))$	$B^0 \rightarrow K^0 \pi^+ \pi^-$	0.11	0.04	0.01
$\text{Im}(A(K^* \pi^+)/A(K^{*+} \pi^-))$	$B^0 \rightarrow K^0 \pi^+ \pi^-$	0.16	0.11	0.02
$B(K^* \pi^+) \times 10^6$	$B^0 \rightarrow K^0 \pi^+ \pi^-$	0.69	0.32	0.09
$ A(K^* \pi^+)/A(K^{*+} \pi^-) $	$B^0 \rightarrow K^+ \pi^- \pi^0$	0.06	0.06	0.01
$\text{Re}(A(K^{*0} \pi^0)/A(K^{*+} \pi^-))$	$B^0 \rightarrow K^+ \pi^- \pi^0$	0.11	0.11	0.02
$\text{Im}(A(K^{*0} \pi^0)/A(K^{*+} \pi^-))$	$B^0 \rightarrow K^+ \pi^- \pi^0$	0.23	0.23	0.03
$\text{Re}(A(\overline{K}^{*0} \pi^0)/A(K^{*+} \pi^-))$	$B^0 \rightarrow K^+ \pi^- \pi^0$	0.10	0.10	0.01
$\text{Im}(A(\overline{K}^{*0} \pi^0)/A(K^{*+} \pi^-))$	$B^0 \rightarrow K^+ \pi^- \pi^0$	0.30	0.30	0.04
$B(K^{*0} \pi^0) \times 10^6$	$B^0 \rightarrow K^+ \pi^- \pi^0$	0.35	0.35	0.05
$ A(\overline{K}^{*0} \pi^-)/A(K^{*0} \pi^+) $	$B^+ \rightarrow K^+ \pi^- \pi^+$	0.04	0.005	0.004
$B(K^{*0} \pi^+) \times 10^6$	$B^+ \rightarrow K^+ \pi^- \pi^+$	0.81	0.50	0.11
$ A(K^* \pi^0)/A(K^{*+} \pi^0) $	$B^+ \rightarrow K^0 \pi^+ \pi^0$	0.15	0.15	0.02
$\text{Re}(A(K^{*+} \pi^0)/A(K^{*0} \pi^+))$	$B^+ \rightarrow K^0 \pi^+ \pi^0$	0.16	0.16	0.02
$\text{Im}(A(K^{*+} \pi^0)/A(K^{*0} \pi^+))$	$B^+ \rightarrow K^0 \pi^+ \pi^0$	0.30	0.30	0.04
$\text{Re}(A(K^{*+} \pi^0)/A(\overline{K}^{*0} \pi^+))$	$B^+ \rightarrow K^0 \pi^+ \pi^0$	0.21	0.21	0.03
$\text{Im}(A(K^{*+} \pi^0)/A(\overline{K}^{*0} \pi^+))$	$B^+ \rightarrow K^0 \pi^+ \pi^0$	0.13	0.13	0.02
$B(K^{*+} \pi^0) \times 10^6$	$B^+ \rightarrow K^0 \pi^+ \pi^0$	0.92	0.92	0.13

- LHCb cannot perform B-counting like in B-factories
- BF are normalized w.r.t modes measured somewhere else (mainly @ B-factories)
- Error contribution from norm. modes not scaling with stat.
- $B(B^0 \rightarrow K^{*+} \pi^-)$ norm. mode:
 $B(B^0 \rightarrow K^0 \pi^+ \pi^-)$ ($\sigma_{\text{rel}} \sim 4\%$)
- $B(B^+ \rightarrow K^{*0} \pi^+)$ norm. mode:
 $B(B^+ \rightarrow K^+ \pi^- \pi^+)$ ($\sigma_{\text{rel}} \sim 5\%$)

B → ρK System: Physical Observables

$$\begin{aligned}
 A(B^0 \rightarrow \rho^+ K^-) &= V_{us} V_{ub}^* t^{+-} + V_{ts} V_{tb}^* p^{+-} \\
 A(B^+ \rightarrow \rho^0 K^+) &= V_{us} V_{ub}^* n^{0+} + V_{ts} V_{tb}^* (-p^{+-} + p_{EW}^C) \\
 \sqrt{2} A(B^+ \rightarrow \rho^+ K^0) &= V_{us} V_{ub}^* (t^{+-} + t_C^{00} - n^{0+}) + V_{ts} V_{tb}^* (p^{+-} - p_{EW}^C + p_{EW}) \\
 \sqrt{2} A(B^0 \rightarrow \rho^0 K^0) &= V_{us} V_{ub}^* t_C^{00} + V_{ts} V_{tb}^* (-p^{+-} + p_{EW})
 \end{aligned}$$

11 QCD and 2 CKM = 13 unknowns

Same Isospin relations as $K^* \pi$

Observables:

- 4 BFs and 4 A_{CP} from DP and Q2B analyses.
- 1 phase differences:

$$* 2\beta_{\text{eff}} = \arg\left(\frac{q}{p} \overline{A}(B^0 \rightarrow \rho^0 \overline{K}^0) A^*(B^0 \rightarrow \rho^0 K^0)\right) \text{ from } B^0 \rightarrow K_S^0 \pi^+ \pi^-$$

A total of 9 observables

Under constraint system. Still some constraints possible

$\rho K + K^* \pi$ system: Physical Observables

Global phase between $K^* \pi$ and ρK now accessible:

- $K^* \pi$: 11 hadronic parameters (1 global phase fixed)
- ρK : 12 parameters
- CKM: 2 parameter

A total of = 25 unknowns

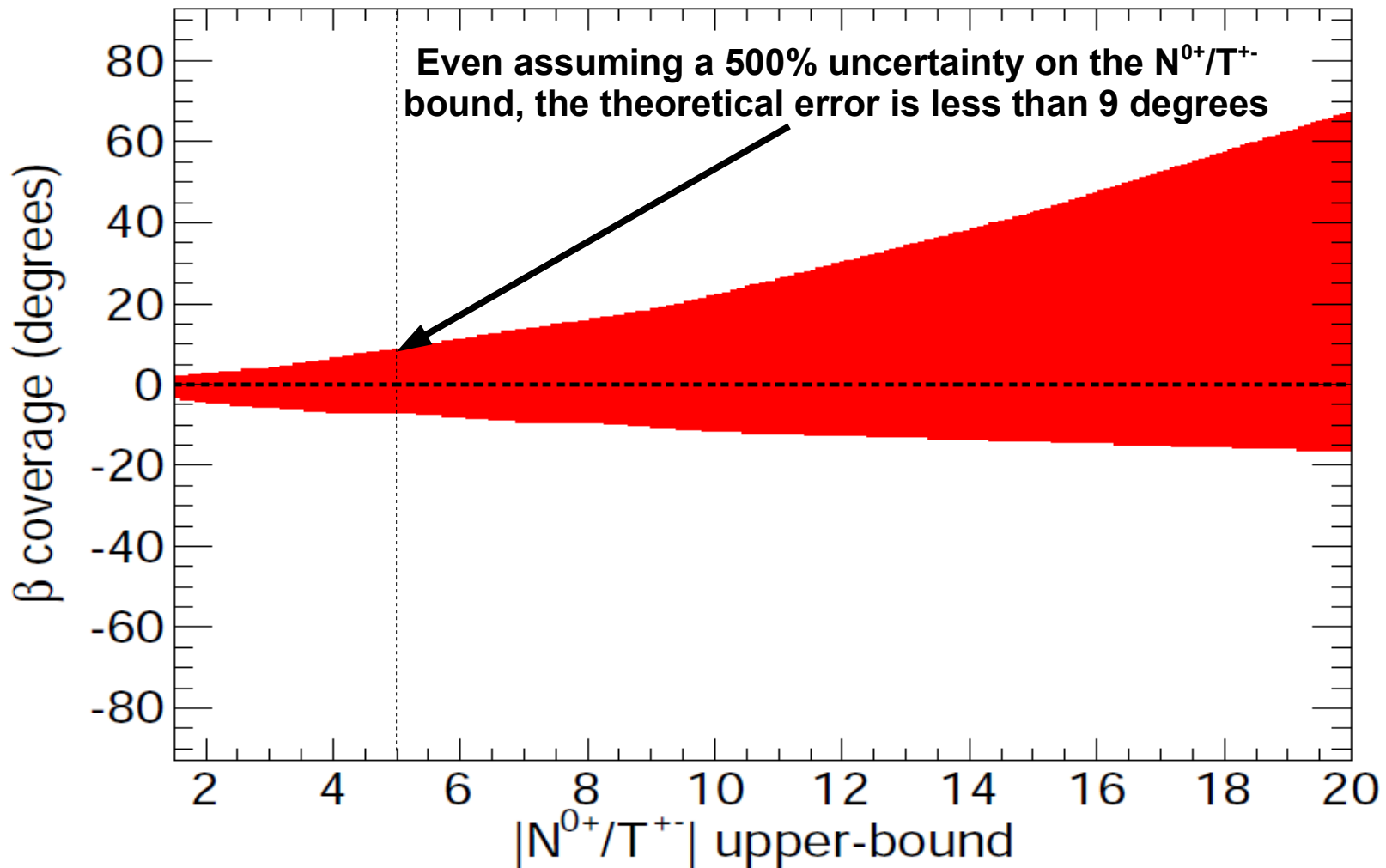
Observables:

- $K^* \pi$ only: 13 observables
- ρK only: 9 observables
- **7 phase differences from:** interference between $K^* \pi$ and ρK resonances contributing to the same DP
 - $\phi = \arg(A(B^0 \rightarrow \rho^0 K^0) A^*(B^0 \rightarrow K^{*+} \pi^-))$ from $B^0 \rightarrow K^0_S \pi^+ \pi^-$
 - $\phi = \arg(A(B^0 \rightarrow \rho^- K^+) A^*(B^0 \rightarrow K^{*+} \pi^-))$ and CP conjugated from $B^0 \rightarrow K^+ \pi^- \pi^0$
 - $\phi = \arg(A(B^0 \rightarrow \rho^0 K^+) A^*(B^0 \rightarrow K^{*0} \pi^+))$ and CP conjugated from $B^+ \rightarrow K^+ \pi^- \pi^+$
 - $\phi = \arg(A(B^0 \rightarrow \rho^+ K^0) A^*(B^0 \rightarrow K^{*+} \pi^0))$ and CP conjugated from $B^+ \rightarrow K^0 \pi^+ \pi^0$

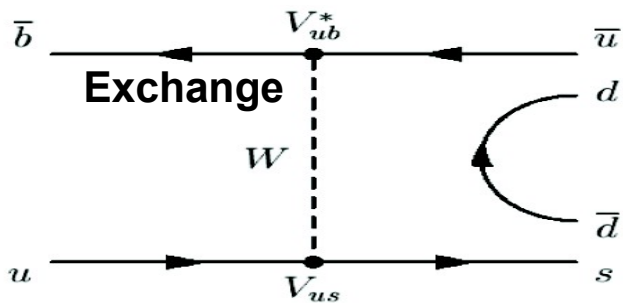
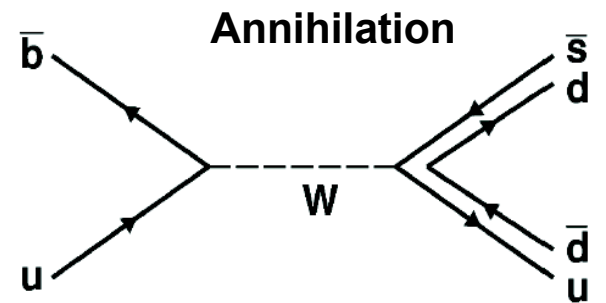
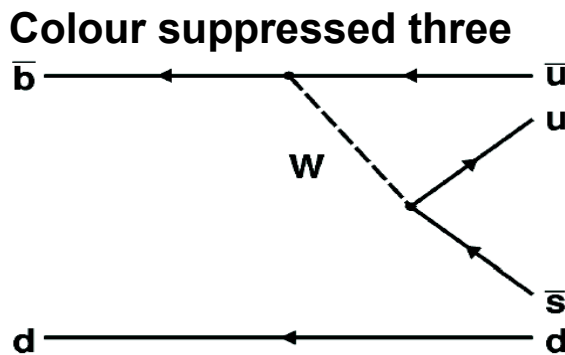
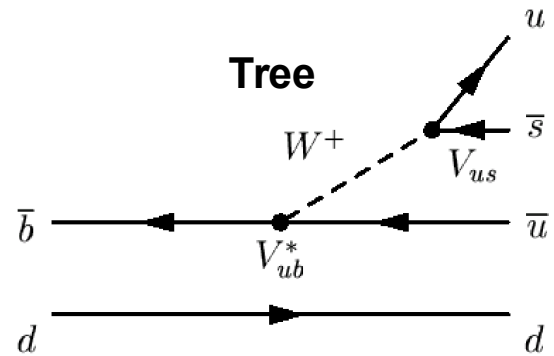
A total of 29 experimentally independent observables

Scenarios to constrain CKM: hypothesis on $|N^{0+}/T^{+-}|$

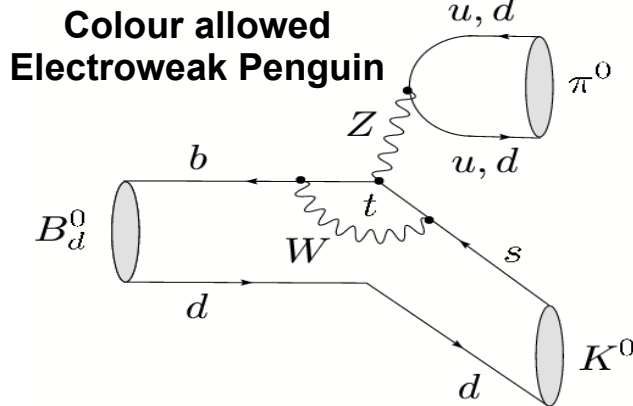
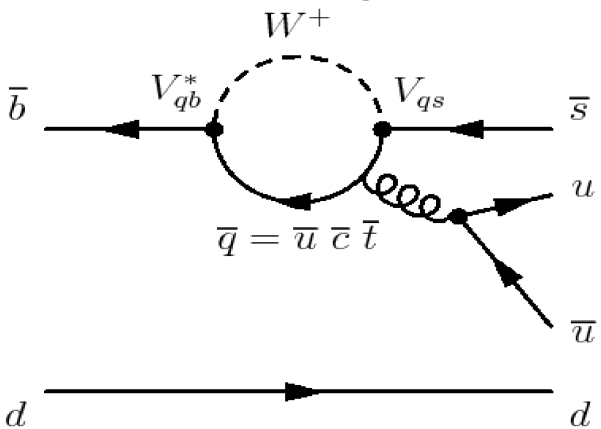
β coverage vs Upper bound on $|N^{0+}/T^{+-}|$ (in units of the generation value)



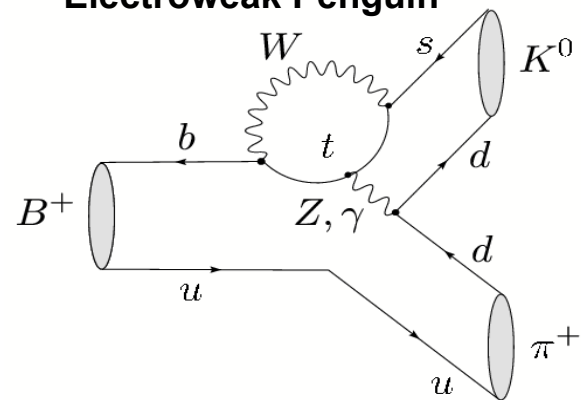
Feynman Diagrams



Gluonic Penguin



Colour suppressed Electroweak Penguin



CPS/GPSZ theoretical prediction

- Effective Hamiltonian of $B \rightarrow K^* \pi$

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1,2} c_i (\Omega_u Q_i^u + \Omega_c Q_i^c) - \Omega_t \sum_{i=3}^{10} c_i Q_i \right\} + \text{h.c.}, \text{ with } \Omega_q = V_{qs} V_{qb}^*$$

- Hierarchy of Wilson coefficients for electro-weak operators $|c_{9,10}| \gg |c_{7,8}|$

$$[\mathcal{H}_{EWP}]_{\Delta I=1} = \frac{3}{2} \frac{c_9 + c_{10}}{2} [Q_1^u + Q_2^u]_{\Delta I=1} + \frac{3}{2} \frac{c_9 - c_{10}}{2} [Q_1^u - Q_2^u]_{\Delta I=1} \quad \text{Electro-weak Hamiltonian}$$

$$[\mathcal{H}_{CC}]_{\Delta I=1} = \frac{c_1 + c_2}{2} [Q_1^u + Q_2^u]_{\Delta I=1} + \frac{c_1 - c_2}{2} [Q_1^u - Q_2^u]_{\Delta I=1} \quad \text{Current-current Hamiltonian}$$

- Using $\left(\frac{c_9 + c_{10}}{c_1 + c_2} \simeq -0.0084 \right) \simeq \left(\frac{c_9 - c_{10}}{c_1 - c_2} \simeq +0.0084 \right)$

$$[\mathcal{H}_{EWP}]_{\Delta I=1} = R \frac{c_1 + c_2}{2} [Q_1^u + Q_2^u]_{\Delta I=1} - R \frac{c_1 - c_2}{2} [Q_1^u - Q_2^u]_{\Delta I=1}$$

$$R = (3/2)(c_9 + c_{10}) / (c_1 + c_2)$$

$$R = (1.35 \pm 0.12)\%$$

- Obtain the relation $P_{EW} = R_{\text{eff}} (T^{+-} + T^{00})$,

$$\text{with } R_{\text{eff}} = R(1 + r_{VP}) / (1 - r_{VP}) \quad r_{VP} = \frac{\langle K^* \pi(I = 3/2) | Q_- | B \rangle}{\langle K^* \pi(I = 3/2) | Q_+ | B \rangle}, \quad Q_{\pm} = (Q_1 \pm Q_2) / 2.$$

$$r_{VP} = \left| \frac{f_{K^*} F_0^{B \rightarrow \pi} - f_{\pi} A_0^{B \rightarrow K^*}}{f_{K^*} F_0^{B \rightarrow \pi} + f_{\pi} A_0^{B \rightarrow K^*}} \right| \lesssim 0.05$$

Experimental inputs: BABAR (1/2)

BABAR $B^0 \rightarrow K^+ \pi^- \pi^0$ analysis

PRD83:112010 (2011)

$$A_{CP}(K^{*+}\pi^-) = -0.29 \pm 0.11 \pm 0.02$$

3σ significance

$$|A(K^{*-}\pi^+)/A(K^{*+}\pi^-)| = 0.74 \pm 0.09$$

$$\text{Re}(K^{*0}\pi^0/K^{*+}\pi^-) = 0.80 \pm 0.20$$

$$\text{Im}(K^{*0}\pi^0/K^{*+}\pi^-) = -0.32 \pm 0.42$$

$$\text{Re}(\overline{K^{*0}}\pi^0/K^{*-}\pi^+) = 1.00 \pm 0.15$$

$$\text{Im}(\overline{K^{*0}}\pi^0/K^{*-}\pi^+) = -0.07 \pm 0.53$$

$$B(K^{*0}\pi^0) \times 10^6 = 3.30 \pm 0.64$$

Full Correlation matrix

$$\begin{pmatrix} 1.0 & 0.06 & 0.02 & -0.35 & -0.11 & -0.06 \\ & 1.0 & 0.78 & 0.30 & -0.01 & 0.29 \\ & & 1.0 & -0.06 & 0.00 & -0.10 \\ & & & 1.0 & 0.30 & 0.42 \\ & & & & 1.0 & -0.02 \\ & & & & & 1.00 \end{pmatrix}$$

BABAR $B^0 \rightarrow K_s^0 \pi^+ \pi^-$ analysis

PRD80:112001 (2009)

two minima differing by 0.16 2NLL units

Global minimum

$$\text{Re}(K^{*-}\pi^+/K^{*+}\pi^-) = 0.43 \pm 0.41$$

$$\text{Im}(K^{*-}\pi^+/K^{*+}\pi^-) = -0.69 \pm 0.26$$

$$B(K^{*+}\pi^-) \times 10^6 = 8.3 \pm 1.2$$

Full Correlation matrix

$$\begin{pmatrix} 1.0 & 0.93 & 0.02 \\ & 1.0 & -0.08 \\ & & 1.0 \end{pmatrix}$$

Local Minimum

$$\text{Re}(K^{*-}\pi^+/K^{*+}\pi^-) = -0.82 \pm 0.09$$

$$\text{Im}(K^{*-}\pi^+/K^{*+}\pi^-) = -0.05 \pm 0.43$$

$$B(K^{*+}\pi^-) \times 10^6 = 8.3 \pm 1.2$$

Full Correlation matrix

$$\begin{pmatrix} 1.0 & -0.20 & 0.22 \\ & 1.0 & -0.01 \\ & & 1.0 \end{pmatrix}$$

$$A_{CP}(K^{*+}\pi^-) = -0.20 \pm 0.10 \pm 0.02$$

$\sim 2\sigma$ significance

Experimental inputs:



BABAR (2/2)

BABAR $B^+ \rightarrow K^+ \pi^- \pi^+$ analysis

PRD78:012004 (2008)

$$|A(K^{*0} \pi^-) / A(K^{*0} \pi^+)| = 1.033 \pm 0.047$$

$$B(K^{*0} \pi^+) \times 10^6 = 10.8 \pm 1.4$$

Full Correlation matrix

$$\begin{pmatrix} 1.0 & 0.02 \\ & 1.0 \end{pmatrix}$$

BABAR $B^+ \rightarrow K^0_s \pi^+ \pi^0$ analysis

ArXiv : 1501.00705 [hep-ex] (2015)

New Result!

- In communication with authors to get full set of observables and correlation matrices
- Results shown in next slides just use (no correlation between observables is assumed)

$$B(K^{*+} \pi^0) \times 10^6 = 9.2 \pm 1.5$$

$$\text{C}(K^{*+} \pi^0) = -0.52 \pm 0.15 \Rightarrow \sim 3.4\sigma \text{ significance}$$

$$\phi = \arg(A(B^+ \rightarrow K^{*0} \pi^+) A^*(B^+ \rightarrow K^{*+} \pi^0)) = (10 \pm 112)^\circ$$

$$\bar{\phi} = \arg(\bar{A}(B^- \rightarrow \bar{K}^{*0} \pi^-) \bar{A}^*(B^- \rightarrow \bar{K}^{*-} \pi^0)) = (98 \pm 97)^\circ$$

Huge uncertainty on phases

Experimental inputs:



Belle $B^0 \rightarrow K^0_S \pi^+ \pi^-$ analysis

PRD75:012006 (2007) and PRD79:072004 (2009)

two minima differing by 7.5 2NLL units

Global minimum

$$\text{Re}(K^{*-}\pi^+/K^{*+}\pi^-) = 0.79 \pm 0.14$$

$$\text{Im}(K^{*-}\pi^+/K^{*+}\pi^-) = -0.21 \pm 0.40$$

$$B(K^{*+}\pi^-) \times 10^6 = 8.4 \pm 1.5$$

Full Correlation matrix

$$\begin{pmatrix} 1.0 & 0.62 & 0.0 \\ & 1.0 & 0.0 \\ & & 1.0 \end{pmatrix}$$

Local Minimum

$$\text{Re}(K^{*-}\pi^+/K^{*+}\pi^-) = 0.81 \pm 0.11$$

$$\text{Im}(K^{*-}\pi^+/K^{*+}\pi^-) = 0.01 \pm 0.44$$

$$B(K^{*+}\pi^-) \times 10^6 = 8.4 \pm 1.5$$

Full Correlation matrix

$$\begin{pmatrix} 1.0 & 0.01 & 0.0 \\ & 1.0 & 0.0 \\ & & 1.0 \end{pmatrix}$$

$$A_{\text{CP}}(K^{*+}\pi^-) = -0.20 \pm 0.11 \pm 0.07$$

$\sim 1.5\sigma$ significance

Belle $B^+ \rightarrow K^+ \pi^- \pi^+$ analysis

PRL96:251803 (2006)

$$|A(K^{*0}\pi^-)/A(K^{*0}\pi^+)| = 0.86 \pm 0.09$$

$$B(K^{*0}\pi^+) \times 10^6 = 9.7 \pm 1.1$$

Full Correlation matrix

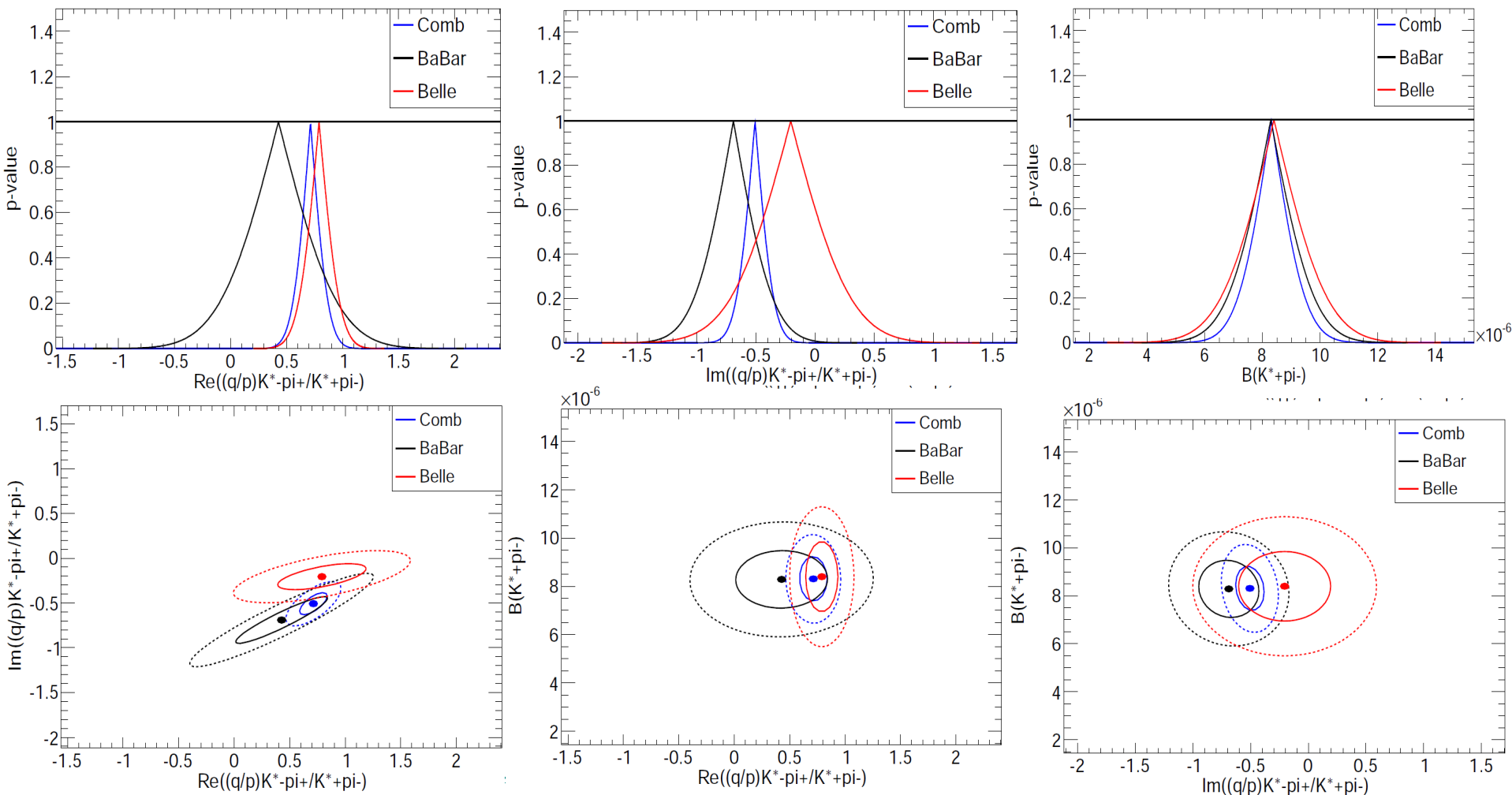
$$\begin{pmatrix} 1.0 & 0.0 \\ & 1.0 \end{pmatrix}$$

No Belle results on: $B^0 \rightarrow K^+ \pi^+ \pi^0$ and $B^+ \rightarrow K^0_S \pi^+ \pi^0$

Combining $BABAR + Belle: B^0 \rightarrow K^0_s \pi^+ \pi^-$

Two solutions for both $BABAR$ and $Belle$ analyses

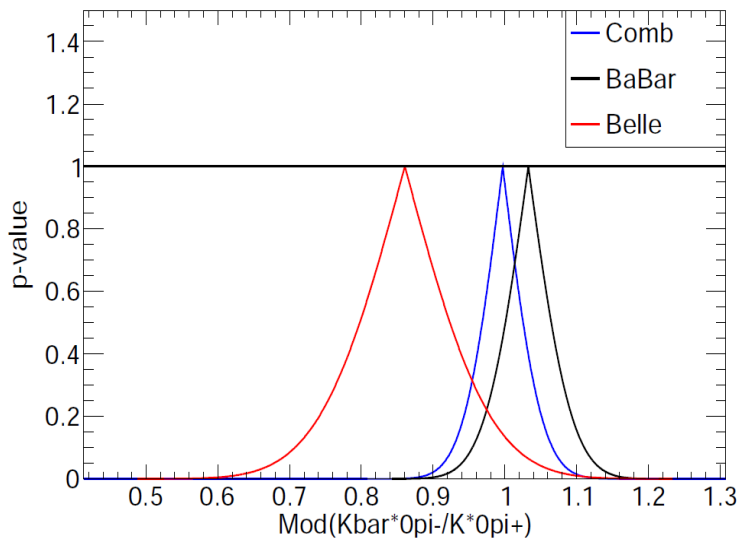
- Combine all possible combinations of $BABAR$ and $Belle$ solutions taking into account the difference in 2NLL
- Results: 4 solutions differing in χ^2 : 0, 7.7, 8.4 and 97.2. Consider only the global minimum



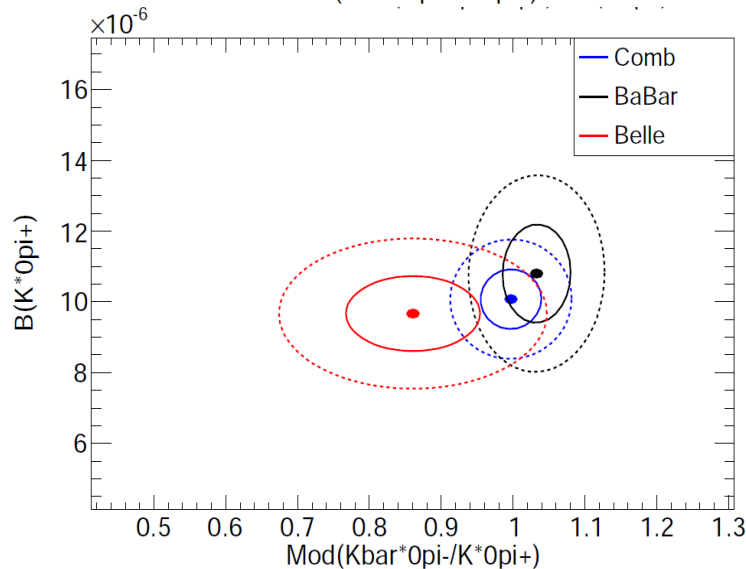
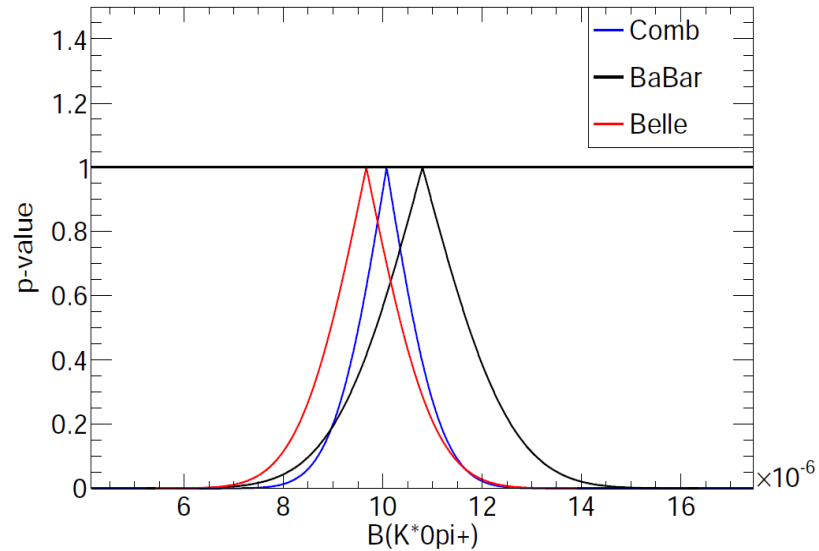
Combining BABAR + Belle: $B^+ \rightarrow K^+ \pi^- \pi^+$

Single solution for both BABAR and Belle

Likelihood vs $\text{Mod}(K\bar{0}\pi^-/K^0\pi^+)$



Likelihood vs $B(K^0\pi^+)$



Freedom in writing decay amplitudes in terms of weak and strong phases

$$\begin{aligned} A &= M_1 e^{+i\phi_1} e^{i\delta_1} + M_2 e^{+i\phi_2} e^{i\delta_2} , \\ \bar{A} &= M_1 e^{-i\phi_1} e^{i\delta_1} + M_2 e^{-i\phi_2} e^{i\delta_2} , \end{aligned}$$

Consider two basic sets of weak phases $\{\phi_1, \phi_2\}$ and $\{\phi_1, \varphi_2\}$ with $\phi_2 \neq \varphi_2$; if an algorithm allows us to write ϕ_2 as a function of physical observables then, owing to the functional similarity of equation (1) and (5), we would extract φ_2 with exactly the same function, leading to $\phi_2 = \varphi_2$, in contradiction with the assumptions; then, a priori, the weak phases in the parametrization of the decay amplitudes have no physical meaning, or cannot be extracted without hadronic input.

It is not possible to extract at the same time hadronic and CKM parameters without additional input

Results on Had. Amplitudes: CP violation (1/3)

- Decay amplitudes (δ_i and φ_i are weak/strong phases)

$$A = M_1 \exp(i\delta_1) \exp(i\varphi_1) + M_2 \exp(i\delta_2) \exp(i\varphi_2)$$

$$\bar{A} = M_1 \exp(i\delta_1) \exp(-i\varphi_1) + M_2 \exp(i\delta_2) \exp(-i\varphi_2)$$

$$A_{CP} = 2 \frac{\sin(\Delta\delta) \sin(\Delta\varphi)}{(M_1/M_2) + (M_2/M_1) + 2 \cos(\Delta\delta) \cos(\Delta\varphi)}$$

- In our case $\Delta\varphi = \arg(V_{ts} V_{tb}^* / V_{us} V_{ub}^*) = 2\gamma \neq 0$

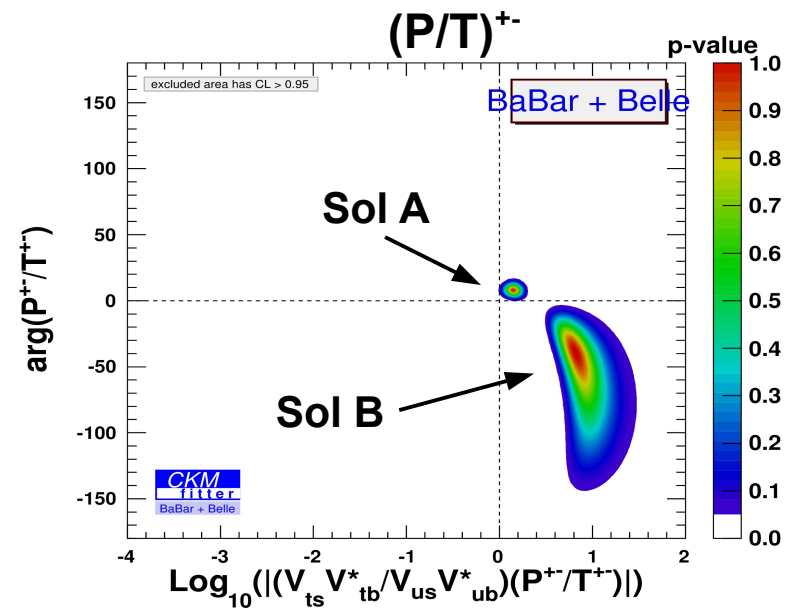
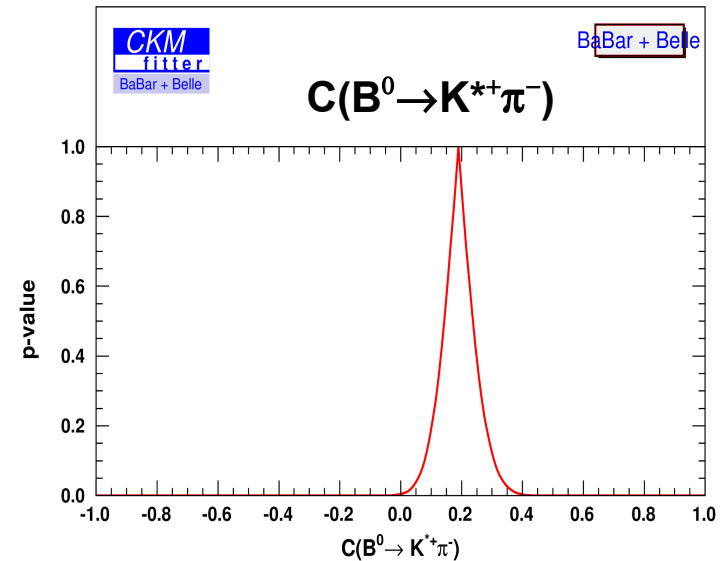
- If A_{CP} is significantly different from zero then

- $|\text{CKM}^*(P/T)| \sim 1$
- $\arg(\Delta\varphi) \neq 0$

- 3.0 σ significance for $C(B^0 \rightarrow K^{*+} \pi^-)$

$$A(B^0 \rightarrow K^{*+} \pi^-) = V_{us} V_{ub}^* T^{*+} + V_{ts} V_{tb}^* P^{*+}$$

- Two solutions with same χ^2 (Sol A and B)
- Both inconsistent with $\arg(\Delta\varphi) = 0/\pi$
- Only solution A has $|\text{CKM}^*(P/T)| \sim 1$



Results on Had. Amplitudes: CP violation (2/3)

- Decay amplitudes (δ_i and φ_i are weak/strong phases)

$$A = M_1 \exp(i\delta_1) \exp(i\varphi_1) + M_2 \exp(i\delta_2) \exp(i\varphi_2)$$

$$\bar{A} = M_1 \exp(i\delta_1) \exp(-i\varphi_1) + M_2 \exp(i\delta_2) \exp(-i\varphi_2)$$

$$A_{CP} = 2 \frac{\sin(\Delta\delta) \sin(\Delta\varphi)}{(M_1/M_2) + (M_2/M_1) + 2 \cos(\Delta\delta) \cos(\Delta\varphi)}$$

- In our case $\Delta\varphi = \arg(V_{ts} V_{tb}^* / V_{us} V_{ub}^*) = 2\gamma \neq 0$

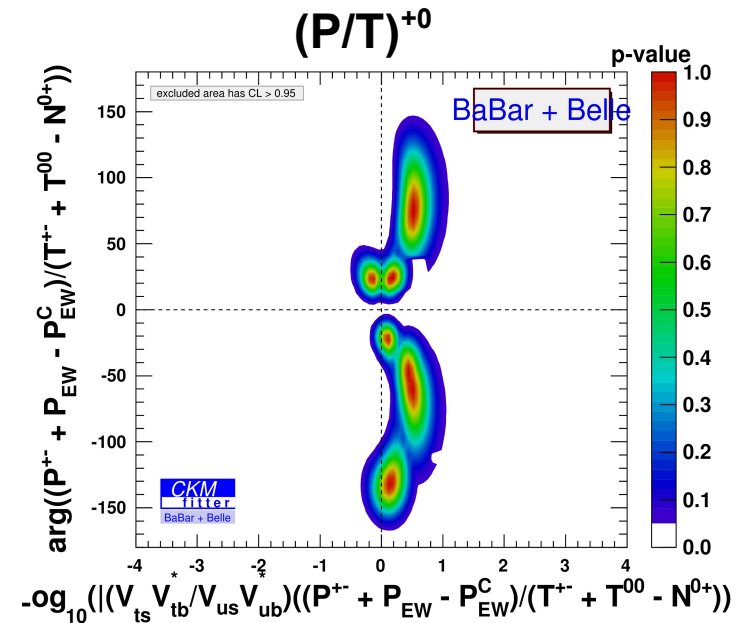
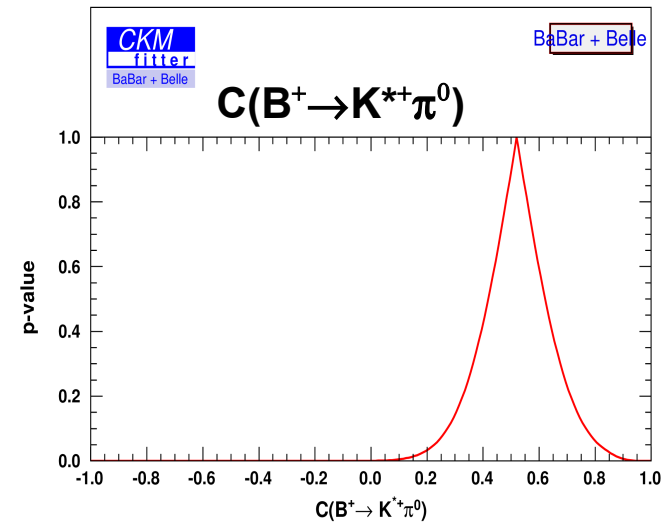
- If A_{CP} is significantly different from zero then

- $|\text{CKM}^*(P/T)| \sim 1$
- $\arg(\Delta\varphi) \neq 0$

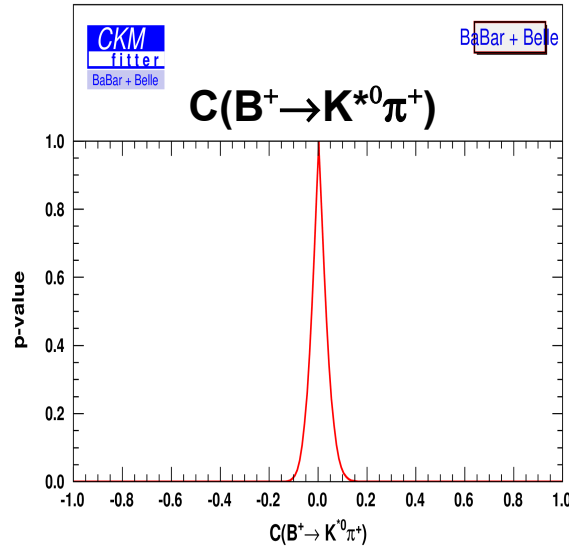
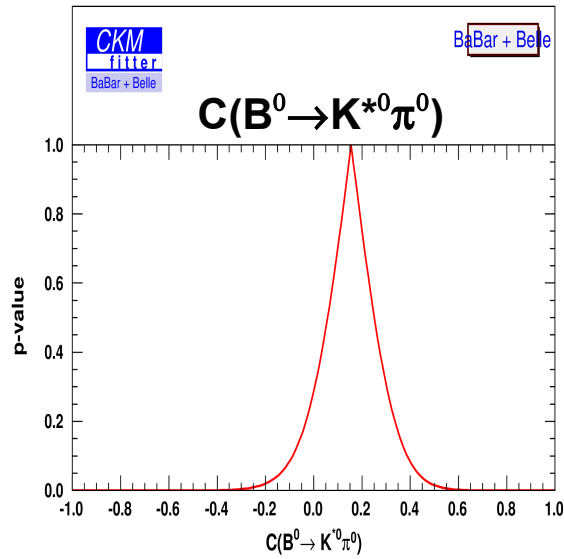
- 3.5 σ significance for $C(B^+ \rightarrow K^{*+} \pi^0)$

$$\sqrt{2}A(B^+ \rightarrow K^{*+} \pi^0) = V_{us} V_{ub}^* (T^{+-} + T^{00} - N^{0+}) + V_{ts} V_{tb}^* (P^{+-} - P_{EW}^C + P_{EW})$$

- Both solutions inconsistent with $\arg(\Delta\varphi) = 0/\pi$ and with $|\text{CKM}^*(P/T)| \sim 1$
- Appearance of other local minima



Results on Had. Amplitudes: CP violation (3/3)

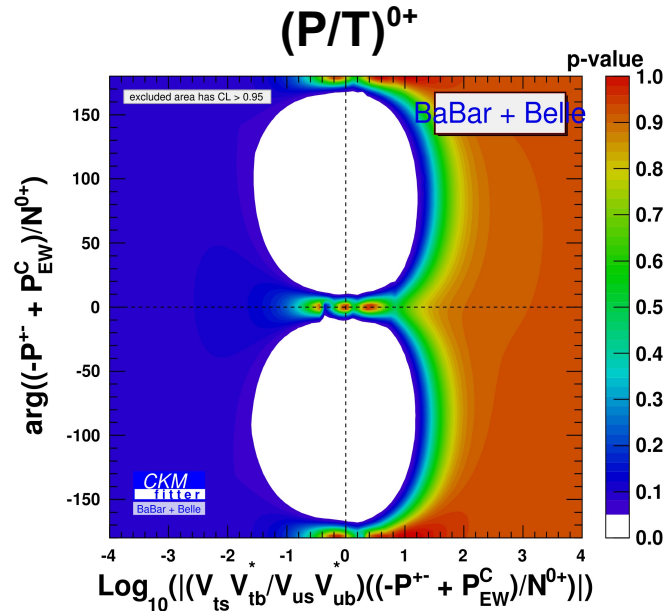
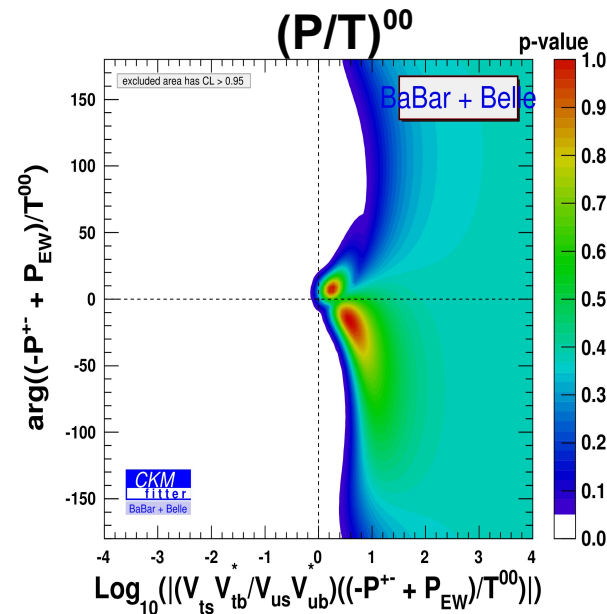


$$A(B^+ \rightarrow K^{*0} \pi^+) = V_{us} V_{ub}^* N^{0+} + V_{ts} V_{tb}^* (-P^{+} + P_{EW}^C)$$

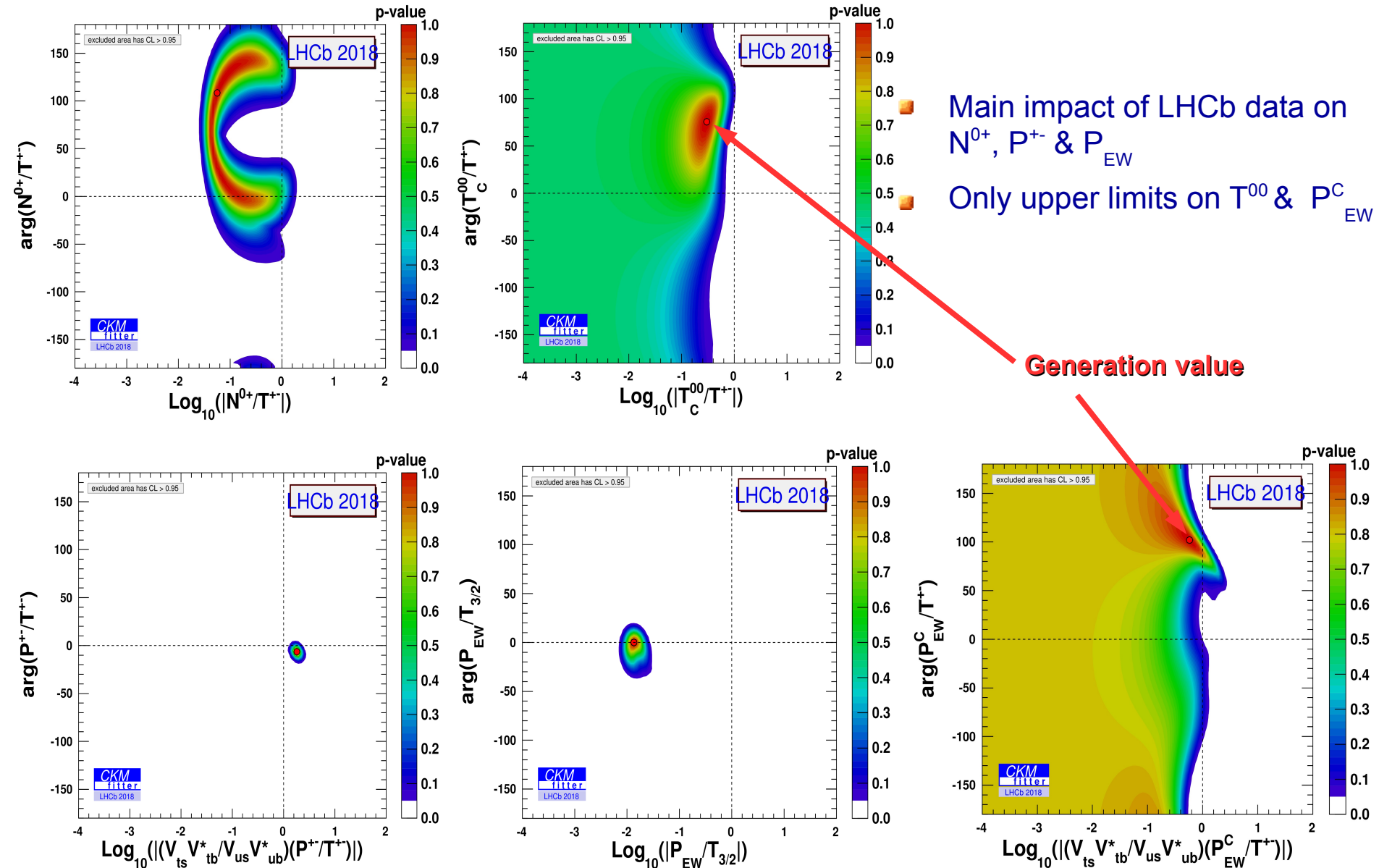
$$A(B^0 \rightarrow K^{*0} \pi^0) = V_{us} V_{ub}^* T^{00} + V_{ts} V_{tb}^* (-P^{+} + P_{EW}^C)$$

- $A_{CP}(K^{*0} \pi^0)$ and $A_{CP}(K^{*0} \pi^+)$ consistent with zero @ 1σ
- P/T constraints are consistent either with

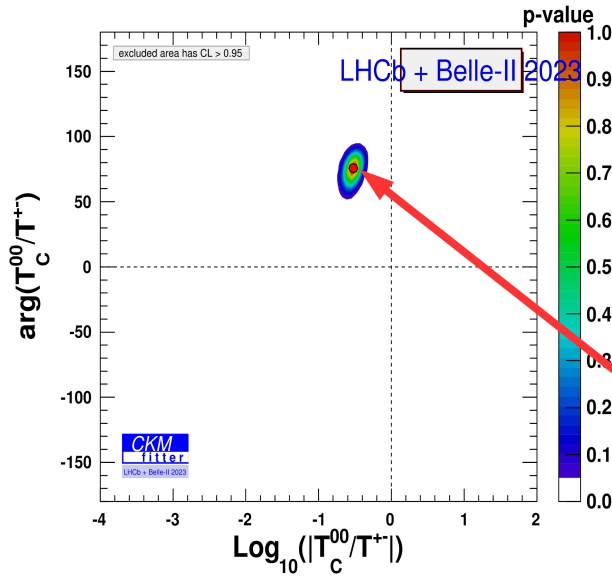
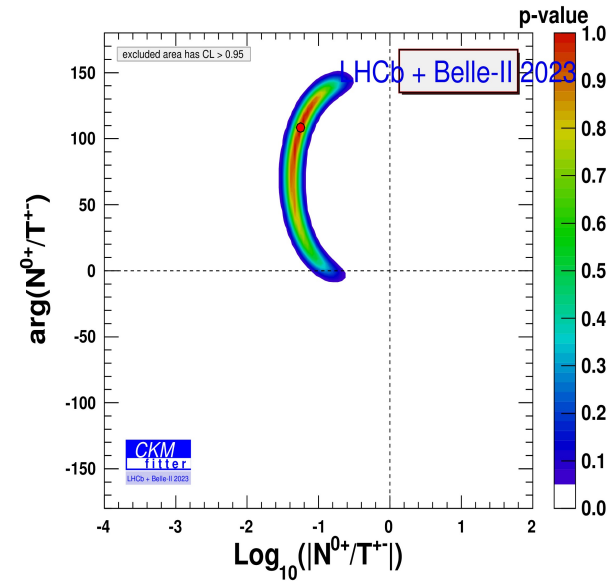
- $|CKM^*(P/T)| \gg 1$ or $\ll 1$
- $\arg(P/T) = 0$ or $\pm\pi$



Had. Parameters: LHCb (run1+run2) 2019



Had. Parameters: LHCb + Belle-II 2023



- With LHCb + Belle-II data will be able to make precision measurements on the hadronic parameters
- Precision test of theory predictions

Generation value

