Photonic Band Structure

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Photonic crystals are periodic dielectric structures engineered to allow or forbid propagation of electromagnetic waves in certain frequency range resulting in photonic band. Given any periodic dielectric structure, the allowable frequencies (eigenfrequencies) for light propagation in all crystal directions and the field distributions in the crystal for any frequency of light can be evaluated. There are several capable techniques, but one of the most studied and reliable methods is the plane wave expansion method.

The master equation represents an eigen value problem with eigen function $\mathbf{E}(\mathbf{r})$ and Eigen value $(\frac{\omega}{c})^2$. For a given configuration of $\epsilon(\mathbf{r})$ in a photonic crystal, we can evaluate the modes $\mathbf{E}(\mathbf{r})$ for the given frequency.

$$\frac{1}{\epsilon(\mathbf{r})}\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) = (\frac{\omega}{c})^2\mathbf{E}(\mathbf{r}) \tag{1}$$

In one dimensional case the above equation reduces to

$$\frac{1}{\epsilon(\mathbf{r})}[\frac{\partial^2}{\partial x^2}E_y - \frac{\partial^2}{\partial y^2}E_y] = (\frac{\omega}{c})^2 E_y \tag{2}$$

Fourier expansion for the inverse dielectric function $\frac{1}{\epsilon(\mathbf{r})}$ will be used. Then, the constant can be moved to the right side of the equation and can be expanded. This forms a generalized Hermitian eigenvalue problem, or an ordinary eigenvalue problem if an additional matrix inversion is carried out in the subsequent step. The summation of the fourier series is infinite, but needs to be truncated for computation.

Firstly, a code will be developed to find eigenvalues and respective electric fields using above formulation in one dimensional photonic crystal. Next, we will study the variation of the obtained eigenvalues as number of plane waves used is changed to study convergence problem.