Finite difference time domain methods as applied to electromagnetic waveguide problems and numerical stability analysis

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Finite difference time domain (FDTD) and related space-grid time domain techniques are direct solution methods for Maxwell's curl equations. These methods employ no potentials. Rather, they are based upon volumetric sampling of the unknown electric field ($\overrightarrow{E}$) and magnetic field ($\overrightarrow{H}$) within and surrounding the structure of interest, and over a period of time. The sampling resolution in time is selected to ensure numerical stability of the algorithm. In this project, I shall demonstrate the application of FDTD techniques to obtain the numerical solution for the propagation of electromagnetic waves in the waveguide structures. The various issues associated with stability of the solutions will also be described as a part of this project.

BRIEF INTRODUCTION

A general one dimensional scalar wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \tag{1}$$

with $u = u(x, t)$, can be written in the form

$$u_i^{n+1} = (c\Delta t)^2 \left[\frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{(\Delta x)^2}\right] + 2u_i^n - u_i^{n-1} + O\left[(\Delta t)^2\right] + O\left[(\Delta x)^2\right] \tag{2}$$

using the method of finite differences. Here i and n represent the space and time coordinate respectively. This is a fully explicit second-order expression for u_i^{n+1} in that all wave quantities on the right hand side are known, obtained during the previous time steps n and $n-1$. Using this idea, the Maxwell's curl equations for a given boundary value problem can be solved and the $E-$ and $B-$ field propagation in space-time can be simulated. The procedure involves the approximate representation of the dispersion relation. Therefore the accuracy of the method depends upon the sampling density of the finite difference grid. However, the numerical stability of the solution is determined by a parameter known as Courant's stability parameter.

In this project, I shall be discussing the numerical solutions of the Maxwell's equations for the case of electromagnetic (EM) waves propagating in one dimensional dielectric waveguide materials. The numerical analysis will involve the c programming language and mathematica codes written to describe the propagation of EM waves in the waveguide materials. The various accuracy issues such as error introduced into the solution because of finite sample size of the finite difference grid shall be discussed. Also, the c language routines will be written to discuss the numerical stability of the solutions in detail, based on the Neumann's stability criteria. I shall try to extend this approach to the case of two dimensional waveguide propagation case, if time permits.

[1] Allen Taflove, Computational Electrodynamics: The Finite-Difference Time-Domain Method, 2nd ed., Artech House, Inc., (2000)