Project Proposal

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Comparison of various methods for numerical solutions of Laplace and Poisson's equations.

The Laplace and Poisson's equations are two of the most ubiquitous equations in physics with applications ranging from fluid flow to electrostatics to the descriptions of heat transfers. It is therefore of fundamental importance to not only develop an efficient numerical algorithm for their solution but also to determine the errors and the ease of various methods vis-a-vis different grades of problems, each exhibiting different constraints (boundary or otherwise).

To this end, I would like to examine the following methods and study the various features associated with each to come up with an optimum algorithm for the sundry problems :

Laplace's Equation :

1. 2D Finite Difference method(Relaxation method) :
   - Illustration by solving for the "Temperature distribution in a 2D rectangular plate".
   - Illustration of "Successive Over-Relaxation" (SOR) method as a way of improving the solution.
   - Alternative direction successive over relaxation method(ADSOR)
   - Thomas (or double sweep) algorithm
   - Solving in non-rectangular domains
   - Solution using Dirichlet boundary conditions
2. Finite Element method

Poisson's Equation :

1. Solution to the Poisson's equation representing a steady, laminar flow through a square duct with no-slip boundary conditions at the walls, using the following methods :
- Jacobi Iteration
- Gauss-Siedel Method
- S.O.R. (Successive Over Relaxation)
- S.L.O.R. (Successive Line Over Relaxation)
- A.D.I (Alternate Direction Implicit)

Here, a comparison can be made with regard to the convergence, time taken and accuracy of the methods. Also, as an analytical solution can be found here, a comparison to that effect can also be made.

2. Numerical solution of poisson's equation in cylindrical coordinates.
3. Multigrid solution of poisson's equation (in 2D and 3D)
4. Using Adaptive mesh refinement (AMR)

An important part of this will be to study the error analysis associated with each method and their ramifications.[2]

If time permits, I would also like to include the 'Complex Polynomial Method' (CPM) and the 'Complex Variable Boundary Element Method' (CVBEM) for solving the boundary problems of the laplace equation.[1]

As an extrapolation of the project, a study of spherical harmonics related to the "Associated Legendre polynomials" can also be done.[3]

References :

1. Comparison of complex methods for numerical solutions of boundary problems of the Laplace equation
   -Engineering Analysis with Boundary Elements 28 (2004) 615–622
2. Error Analysis When Solving Laplace's Equation Numerically by Iteration
   -IEEE Transactions on Education, Vol. 31, No. 1
3. Numerical Recipes in C++ : Section 6.8