Project Proposal: Chaotic Behavior in Restricted Three Body Problem
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A circular restricted three body problem (CR3BP) assumes a system of three masses of which two are in circular motion around each other and another one moving in the field of these two, has negligible effect on the total potential of the system. I consider earth-moon system along with a rocket thrown at an angle ϕ with the x-axis (figure) towards moon in the plane of

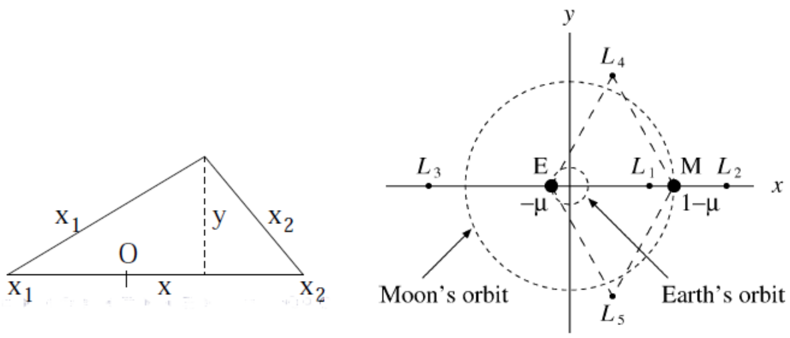

FIG. 1: Setting up the coordinate system.

rotation of the earth and the moon. Hence, this is a planar circular restricted three-body problem (PCR3BP). Let C be the sum of K.E. and an effective potential, r_i be the distance of the rocket from the two bodies and v be its velocity given as follows:

$$r_i^2 = (x - x_i)^2 + y^2 + z^2 \qquad and \qquad v^2 = (\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 + (\frac{dz}{dt})^2 \tag{1}$$

$$C = \frac{1}{2}v^2 + V \qquad where \qquad V(r) = -\frac{1}{2}(x^2 + y^2) - \frac{1-\mu}{r_1} - \frac{\mu}{r_2} \tag{2}$$

For each pair of (C,ϕ) I follow the trajectory

$$\frac{d^2\vec{r}}{dt^2} = -\nabla V \tag{3}$$

$$\Rightarrow \ddot{x} - 2\dot{y} = -\frac{\partial V}{\partial x} \qquad and \qquad \ddot{y} + 2\dot{x} = -\frac{\partial V}{\partial y} \tag{4}$$

Eqns.(4) have five equilibra called Lagrange points (denoted by L_i in figure). We want to construct a transfer trajectory of the spacecraft from a circular Earth parking orbit (radius r_s) to a circular Moon parking orbit (radius r_e). The total velocity change is δv=$|\delta v_s|$+$|\delta v_e|$. The terms in the RHS denote changes in velocity for transfering from earth orbit and putting around moon respectively. We shall get the following equations:

$$[x(0) + \mu]^2 + [y(0)]^2 = r_s^2 \qquad and \qquad [x(T) - 1 + \mu]^2 + [y(T)]^2 = r_e^2 \tag{5}$$

$$[x(0)+\mu][\dot{x}(0) - y(0)] + y(0)[\dot{y}(0) + x(0) + \mu] = 0 \quad , \quad [x(T)-1+\mu][\dot{x}(T) - y(T)] + y(T)[\dot{y}(T) + x(T) - 1 + \mu] = 0 \tag{6}$$

$$\delta v_s = \sqrt{(\dot{x}(0) - y(0))^2 + (\dot{y}(0) + x(0) + \mu)^2} - \sqrt{\frac{1-\mu}{r_s}} \ , \ \delta v_e = \sqrt{(\dot{x}(T) - y(T))^2 + (\dot{y}(T) + x(T) - 1 + \mu)^2} - \sqrt{\frac{\mu}{r_e}} \tag{7}$$

Problem 1: Set of trajectories which reach a distance D from the center of mass after a time T can be found.
Problem 2: Minimising δv by numerically solving (4), (5) and (6).
Problem 3: Finding those initial conditions (C,ϕ) which give rise to chaotic trajectories.

Approach

I shall use Fortran and/or C code to solve the differential equation by different methods viz. RK4 and Leap-frog. For different values of C and ϕ I shall get either chaotic or closed trajectories. Both these cases will be analyzed seperately. The nonchaotic trajectories are then picked for the calculation of δv minimization. Also we need to check the numerical stability in each case.