# PWA Method to Evaluate Photonic Band Structure

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Plane wave expansion method in frequency domains will be used to understand the band structure of photonic crystal. The goal is find the allowable energies and EM field configuration. The master equation to be solved is an eigen value problem with eigen function $\mathbf{E}(\mathbf{r})$ and eigen value $(\frac{\omega}{c})^2$. For a given configuration of $\epsilon(\mathbf{r})$ in a photonic crystal, we can evaluate the modes $\mathbf{E}(\mathbf{r})$ for the given frequency.

$$\frac{1}{\epsilon(\mathbf{r})}\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) = (\frac{\omega}{c})^2\mathbf{E}(\mathbf{r}) \tag{1}$$

In one dimensional case the above equation reduces to

$$\frac{1}{\epsilon(\mathbf{r})}[\frac{\partial^2}{\partial x^2}E_y - \frac{\partial^2}{\partial y^2}E_y] = (\frac{\omega}{c})^2 E_y \tag{2}$$

Fourier expansion of inverse dielectric function $\frac{1}{\epsilon(\mathbf{r})}$ and $E_y$ will be used.

$$\frac{1}{\epsilon(\mathbf{r})} = \sum_{m=-\infty}^{\infty} \kappa_m^{\epsilon(r)} \exp^{-j\frac{2\pi m}{a}z} \tag{3}$$

$$E_y = \sum_{n=-\infty}^{\infty} \kappa_n^{E_y} \exp^{-j\frac{2\pi n}{a}z} \exp^{-jk_z z} \tag{4}$$

Inserting above in the equation 2 and simplifying, we get:

$$\sum_n (\frac{2\pi n}{a} + k_z)^2 \kappa_{m-n}^{\epsilon(\mathbf{r})} \kappa_n^{E_y} = \frac{\omega^2}{c^2}\kappa_m^{E_y} \tag{5}$$

This is an ordinary eigen value problem where m and n are truncated around zero. This truncation of the series will define the accuracy of the obtained band structure. Using $Q_{m,n} = \sum_n (\frac{2\pi n}{a} + k_z)^2 \kappa_{m-n}^{\epsilon(\mathbf{r})}$, and diagonalization of the matrix provides us the $\frac{\omega^2}{c^2}$ (eigen value) and $\kappa_n^{E_y}$ (eigenfunctions). The eigenvalues give the dispersion diagram (band structure) and

eigenfunctions can be substituted back in equation 4 to get the field distribution. This method defined to study the band structure is plane wave expansion method. Here, plane waves forms the basis to represent fields in periodic media (as in our case). Dielectric constant and electric field is expanded in Fourier series to solve the master equation.

In this project, a code will be developed to evaluate the eigenvalues (band structure) and eigenfunctions (electric field distribution) using PWA (plane wave expansion) formulation in one dimensional photonic crystal. Next, we will study the variation of the obtained eigenvalues (band structure) as number of plane waves is changed to study the accuracy. Computing time will be minimized using better coding techniques. This computing time can be studied for each configuration (variation in order of the matrix and also variation in the initial condition provided). We will evaluate the influence of the number of expansion terms on the error and computational time. If time permits, this method will be expanded for 2 and 3 dimensional structures. Lastly, we will study the modification in PWA method to incorporate artificially introduced defect mode.