Computational Physics Project Proposal

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Chaotic Non-linear Pendulum

An oscillator which is non-linear, damped and driven is called a physical pendulum. In this pendulum, we also include damping term and the driving force. Hence the equation of such a system is given by

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l} sin\theta - q\frac{d\theta}{dt} + F_D sin(\Omega_D t) \tag{1}$$

where θ is the angle of displacement, and Ω_D and F_D are respectively the frequency and amplitude of the driving force. To solve the equation numerically one can use various methods like Euler-Cromers, RK4 or Verlet method. As the solution to this problem, we obtain θ and ω, where ω is the angular velocity, given by

$$\frac{d\theta}{dt} = \omega \tag{2}$$

The angle θ, in this problem can change from -π to π. The problem is deterministic in θ and ω for all future times. For large values of F_D, the motion becomes chaotic because of the vertical jumps in the value of θ.

The Problem

The problem I wish to study here is the stability of various numerical approaches, like Euler-Cromers, Verlet method or RK4 method, in solving this problem. The second thing I would like to see is the sensitivity of initial conditions, i.e θ_o on these methods.

As the value of F_D changes from a small value to a greater value, the motion becomes chaotic, from the above algorithms I wish to find the critical value of F_D, where the motion goes from non-chaotic to chaotic regime. The critical value obtained can be compared for the various methods used. Further, the phase-space plot of θ vs ω can be sketched for both chaotic and non-chaotic regimes. The phase-space plot can be used to study the random behaviour in the chaotic regime. The chaotic motion of two or more no. of pendulums with same set of parameters with slightly different θ_o can be and the change in $\Delta\theta$ can be seen.Or one can keep all the parameters and initial conditions same and vary the driving force amplitude for different pendulums. If time permits I would like to extend the same analysis for double pendulum with and without the damping force.