Hadronic multibody heavy meson decays

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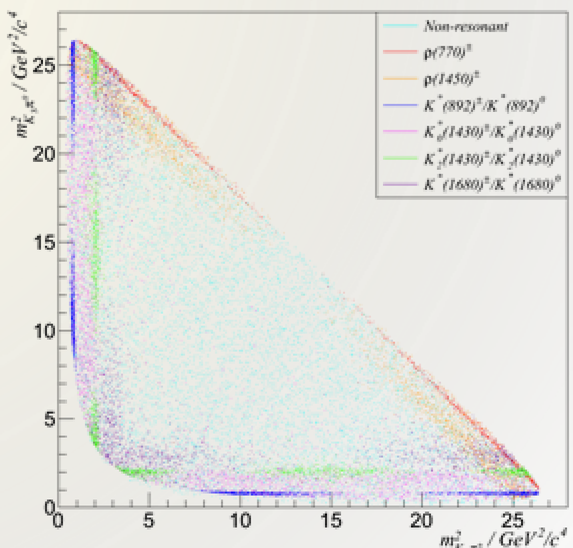


Courtesy: Matt Barrett

Why study three body decay modes?

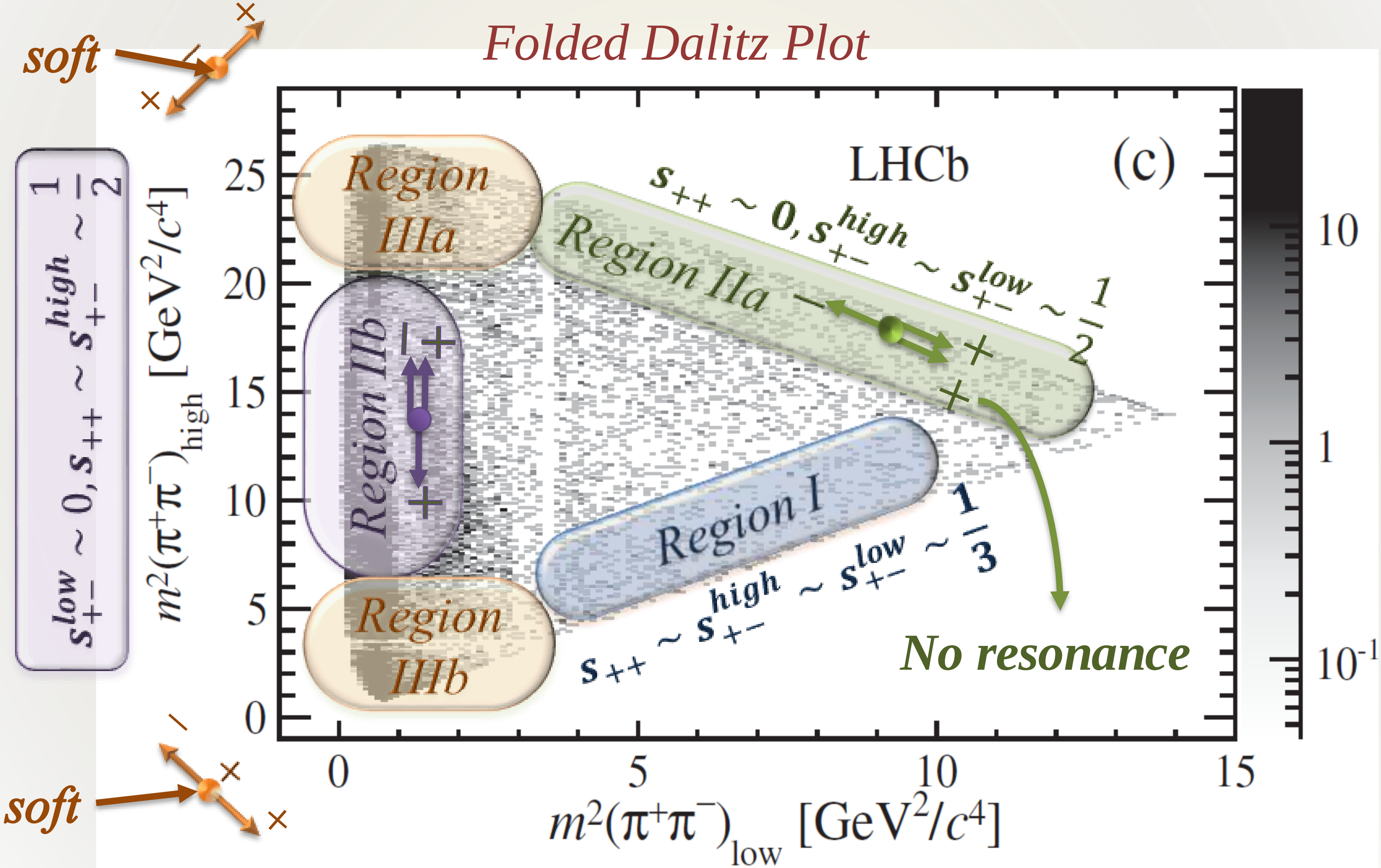
- *Experimentally established that charmless 3-body decays more abundant than 2-body decays*
- *3-body decays are more challenging to understand theoretically.*
- *Description is still at modelling stage. QCD based approach for various regions needed.*
- *Many phenomenological applications. Well known ones are study of CP violation and measurement of weak phase. Discuss some applications in detail later.*
- *CP violation helped by two additional sources for strong phase arising from long-distance effects involving hadron-hadron interactions in the final state:*

1. *Interference between intermediate states of the decay can introduce large strong-phase differences inducing local CP asymmetries in the phase space.*
2. *Another mechanism is final-state $KK \leftrightarrow \pi\pi$ rescattering – occur between decay channels having the same flavor quantum numbers.*

- *In general learn about role of hadronic long-distance effects and final-state interactions in unitarized description.*
- *Large non-resonant fractions in penguin-dominated B decay modes, where as, non-resonant signal is less than 10% in D decays.*
- *Significant effort in trying underway to understand 3-body charmless decays.*

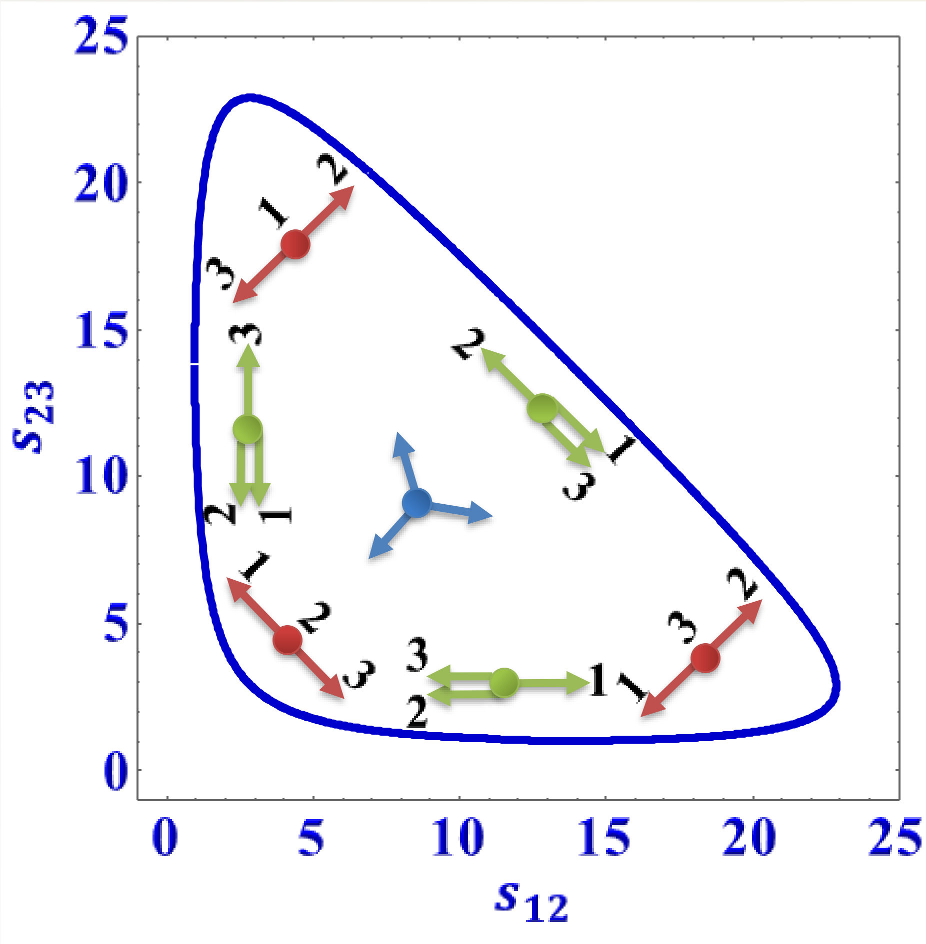
Regions on the Dalitz Plot

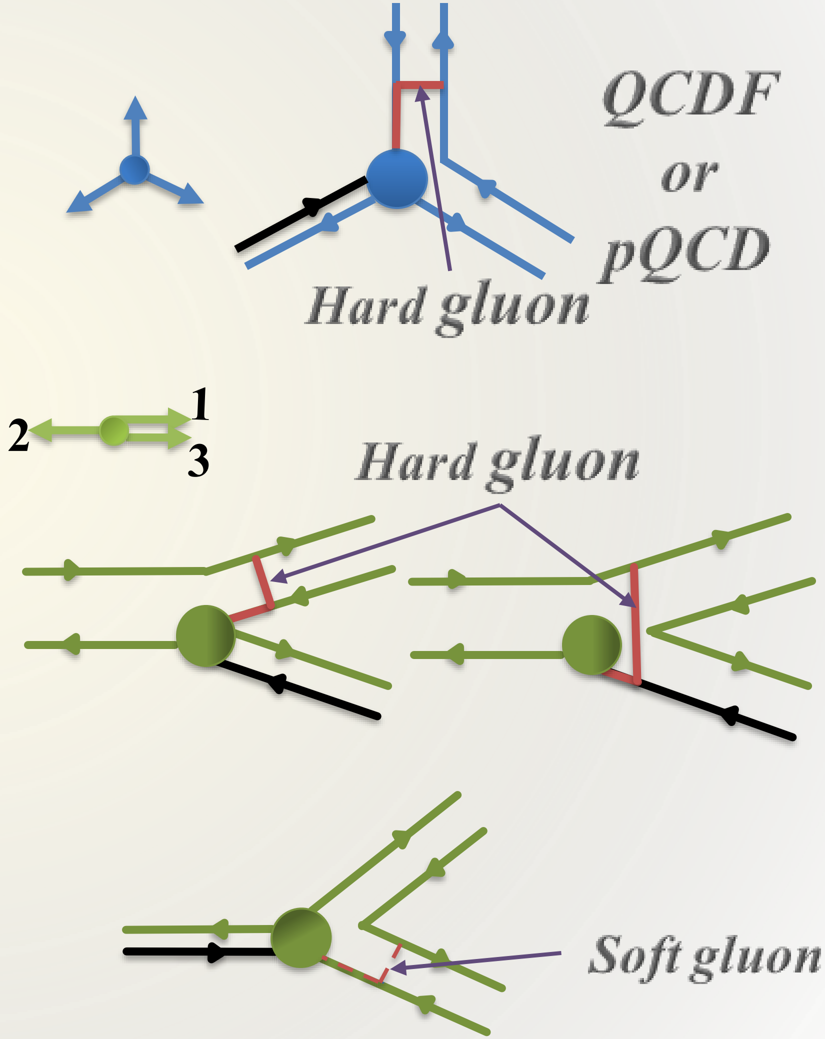
$B^+ \to \pi^+\pi^-\pi^+$



Regions on the Dalitz Plot

Hadronic three body decays of B





Region with soft meson emission can be explored using Heavy meson chiral perturbation theory (HMChPT). Particle 2 and 3 hard but 1 can be soft.

Description of 2 body charmless hadronic B mesons -several competing approaches-QCDF, pQCD, and SCET.

3-body decays much more complicated.

- *Decay described in terms of two invariants. Talk of differential decay rate.*
- *Three body decays of B mesons both resonant and non resonant contributions in general.*
- *Important to pin down the mechanism responsible for large local CP asymmetries.*

❑ *Correlation seen by* LHC*b*: $A_{CP}(K^-K^+K^-) \approx -A_{CP}(K^-\pi^+\pi^-)$, $A_{CP}(\pi K^+K^-) \approx -A_{CP}(\pi\pi^+\pi^-)$

Conjectured that CPT theorem & final-state rescattering of $\pi^+\pi^- \leftrightarrow K^+K^-$ *may play important roles*

I. Bediaga, T. Frederico, O. Lourenço, Phys. Rev. D 89, 094013 (2014)

CP violation in D on "a" Dalitz plot

The central idea

If CP is violated in neutral D mesons, it would exhibit its signature on the X→ $\overline{D}^0 D^0 Y$ Dalitz plots when the neutral D mesons are reconstructed from daughter particles of definite CP.

For conceptual clarity, we shall mostly focus on B → $\overline{D}^0 D^0 K$ decays.

- *Assume there is no direct CP violation in the $D^0 - \overline{D}^0$ system and the D mesons are reconstructed from daughter particles of definite CP f^{CP}. Let us say they are reconstructed in $f^+ = \{K^+K^-, \pi^+\pi^-\}$.*
- *Note that we reconstruct the D mesons on flavor insensitive modes, D^0 and $\overline{D}^0$ are indistinguishable. In the final state D^0 and $\overline{D}^0$ are entangled.*

- *Both the D mesons are reconstructed in the f^+state and must have been in same state D_1. The state $D_1 D_1$ ⇒two identical particles and Bose symmetry demands that the Dalitz plot must be fully symmetric under exchange of D mesons .*
- *Any difference in the Dalitz plot under the exchange of the two D mesons must be a signature of CP violation. If Bose symmetry is more fundamental one of the states must be D_2 which decayed to f^+ violating CP.*

The neutral D mesons can be described in terms of mass, flavor and CP eigenstates. Mass eigenstates D_1 and D_2:

$$\left|D_{1,2}\right\rangle = N_{1,2}\left(p\sqrt{1\mp z}\left|D^0\right\rangle \pm q\sqrt{1\pm z}\,|\bar{D}^0\rangle\right)$$

- *p, q (in general complex) lead to CP violation*
- *z (also complex) leads to CPT violation in mixing*
- *If no CPT violation* $z = 0 \implies |p|^2 + |q|^2 = 1 \implies N_{1,2} = 1$
- *Exact CP asymmetry* $\Rightarrow p = q \Rightarrow \left|D_{1,2}\right\rangle \equiv \left|D_\pm\right\rangle = \frac{1}{\sqrt{2}}\left(|D^0\rangle \pm |\bar{D}^0\rangle\right)$

The $|D^0\rangle$ and $|\overline{D}^0\rangle$ mesons can be expressed in terms of mass eigenstates and CP eigenstates as:

$$|D^0\rangle = \frac{1}{2p}(|D_1\rangle + |D_2\rangle) = \frac{1}{\sqrt{2}}(|D_+\rangle + |D_-\rangle)$$

$$|\overline{D}^0\rangle = \frac{1}{2q}(|D_1\rangle - |D_2\rangle) = \frac{1}{\sqrt{2}}(|D_+\rangle - |D_-\rangle)$$

The mass eigenstates can be written in terms of the CP eigenstates as:

$$\left|D_{1,2}\right\rangle = \frac{1}{\sqrt{2}}((p \pm q)\,|D_+\rangle + (p \mp q)|D_-\rangle)$$

The time evolution of the entangled state is easy to study if we write the state in terms of mass eigenstates and is given by,

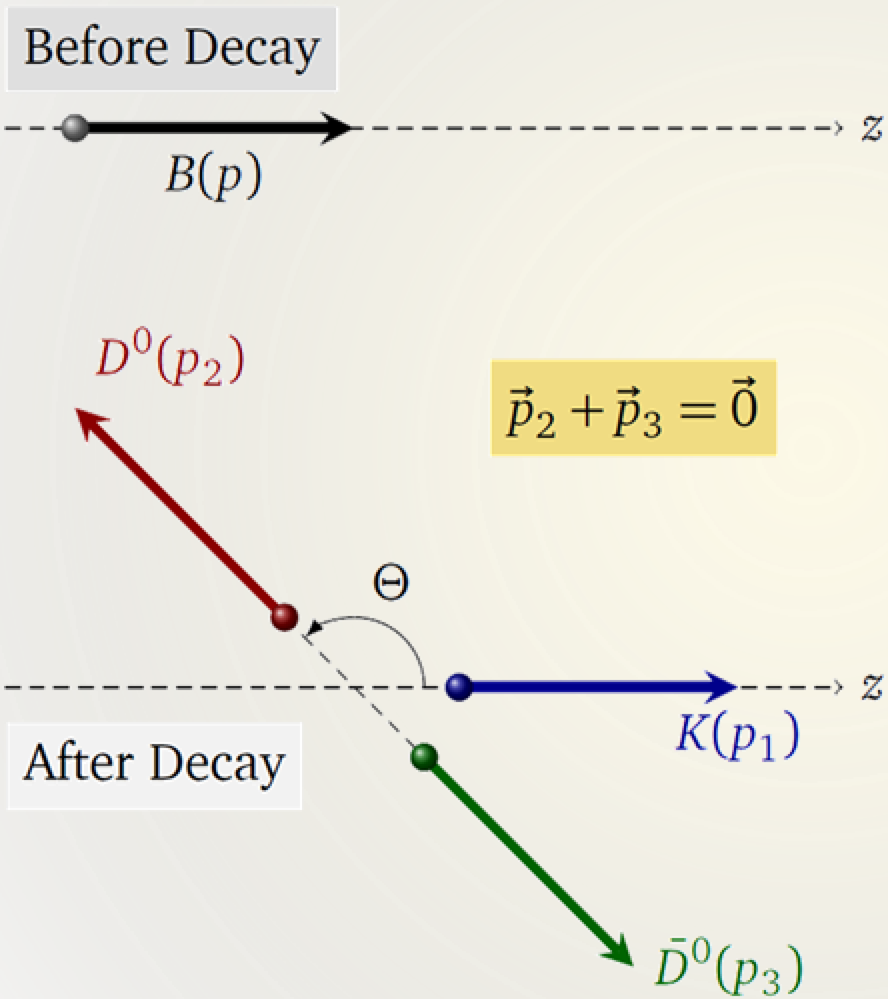
$$\left|D_{1,2}(t)\right\rangle = e^{-i\mu_{1,2}t}\left|D_{1,2}\right\rangle \equiv e^{-i(\mu\pm\Delta\mu)t}\left|D_{1,2}\right\rangle$$

$$\mu = M - \frac{i\Gamma}{2}$$

$$\Delta\mu = (x - iy)\,\frac{\Gamma}{2}$$

M, Γ are mass and decay width average; $x\Gamma, 2y\Gamma$ are mass and decay width difference.

The decay $B(p) \to K(p_1)D^0(p_2)\bar{D}^0(p_3)$ is best analysed in the Gottfried-Jackson frame.



Variables à la Mandelstam:

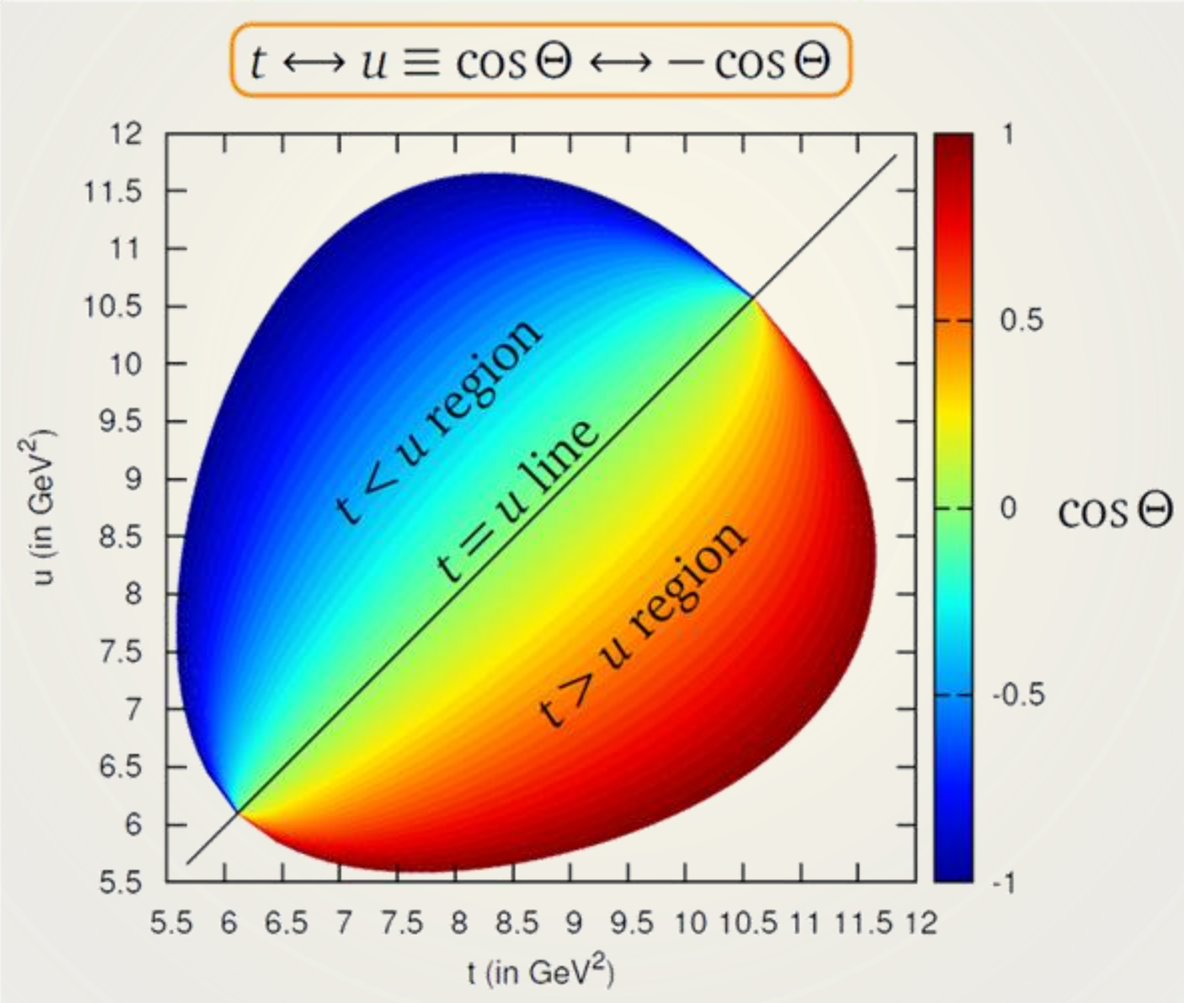
$$s = (p_2 + p_3)^2,$$
$$t = (p_1 + p_3)^2 = a + b\cos\Theta,$$
$$u = (p_1 + p_2)^2 = a - b\cos\Theta,$$

where

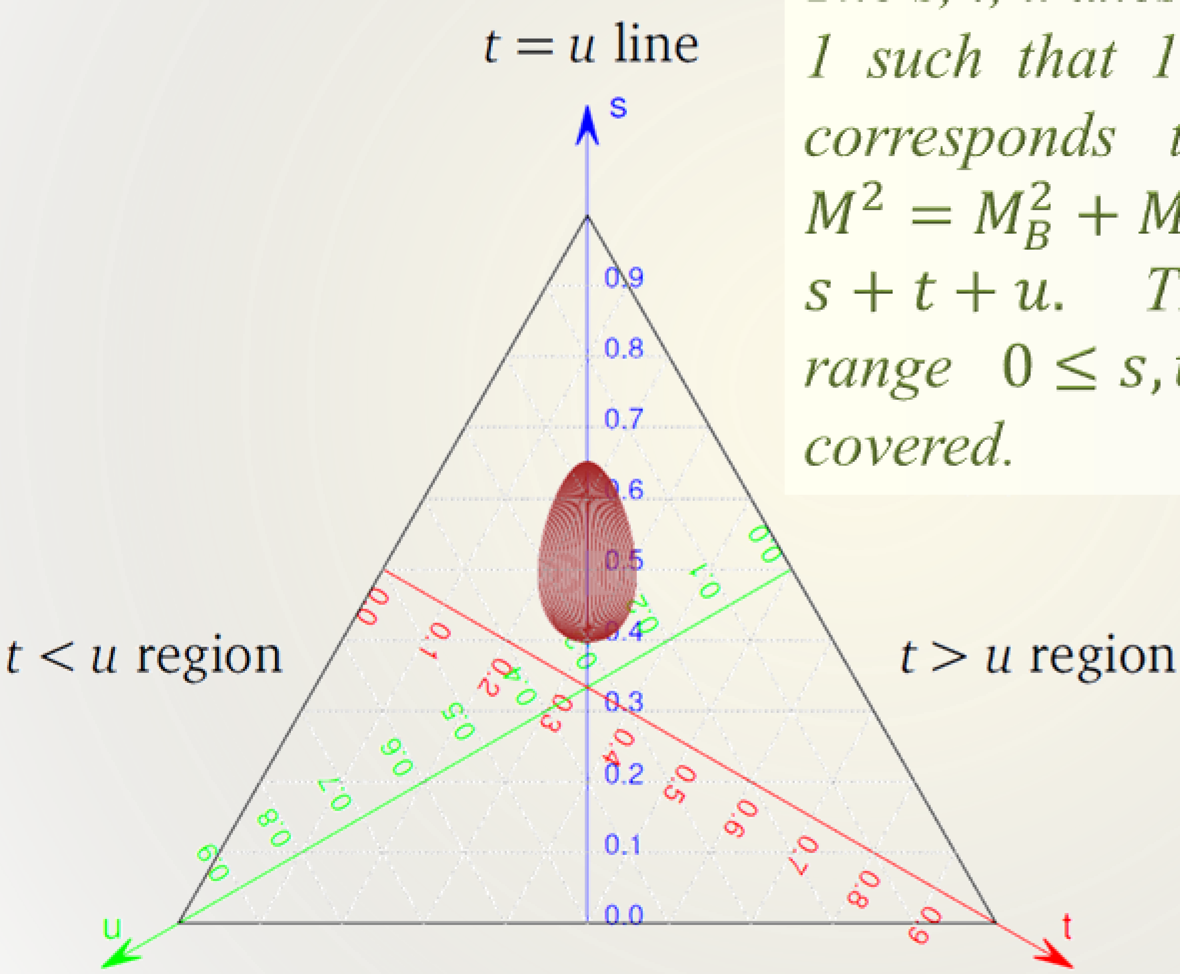
$$a = \frac{1}{2}\left(M_B^2 + M_K^2 + 2M_D^2 - s\right),$$
$$b = \frac{\sqrt{\left(s - 4M_D^2\right)\lambda(M_B^2, M_K^2, s)}}{2\sqrt{s}},$$

with $\lambda(x,y,z)$ being the Källén function $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$.

The kinematically allowed region for the traditional Dalitz plot in case of $B \to K\, D^0\, \bar{D}^0$ looks as follows.



The kinematically allowed region for the triangular Dalitz plot in case $B \to K\, D^0\, \bar{D}^0$ looks as follows.



The s, t, u axes run from 0 to 1 such that 1 on any axis corresponds to the value $M^2 = M_B^2 + M_K^2 + 2M_D^2 = s + t + u$. *Thus the full range* $0 \le s, t, u \le M^2$ *is covered.*

The state $|KD^0(p_2)\bar{D}^0(p_3)\rangle$ *in terms of the mass eigenstate is,*

$$|KD^0(p_2)\bar{D}^0(p_3)\rangle = \frac{1}{4pq}\Big(\{|KD_1(p_2)D_1(p_3)\rangle - |KD_2(p_2)D_2(p_3)\rangle\} - \{|KD_1(p_2)D_2(p_3)\rangle - |KD_2(p_2)D_1(p_3)\rangle\}\Big)$$

Odd $t \leftrightarrow u$ → Antisymmetric $p_2 \leftrightarrow p_3$

Even $t \leftrightarrow u$ → Symmetric $p_2 \leftrightarrow p_3$

Amplitude for the decay of $D_\pm$ *to a CP even final state* f_i^+:

$$Amp(D_+ \to f_i^+) = \langle f_i^+|D_+\rangle = A_i$$
$$Amp(D_- \to f_i^+) = \langle f_i^+|D_-\rangle = \epsilon_i A_i$$

ϵ_i *indicates CP violation.*

$$Amp\big(D_{1,2} \to f_i^+\big) = \frac{1}{\sqrt{2}}\big((p \pm q)A_i + (p \mp q)\epsilon_i A_i\big)$$

No direct CP violation $\Rightarrow$ *No asymmetry under* $p_2 \leftrightarrow p_3$

For the general case:

$$Amp\left(B \to \left\{(f_1^+)_{D_1}(f_2^+)_{D_1}K - (f_1^+)_{D_2}(f_2^+)_{D_2}K\right\}\right) = 2A_1A_2e^{-i\mu(t_1+t_2)}\left(A(t,u) + A(u,t)pq(1-\epsilon_1\epsilon_2)\right)$$

$\approx \mathbf{0}$

$$Amp\left(B \to \left\{(f_1^+)_{D_1}(f_2^+)_{D_2}K - (f_1^+)_{D_2}(f_2^+)_{D_1}K\right\}\right) = 2A_1A_2e^{-i\mu(t_1+t_2)}\left(A(t,u) - A(u,t)pq(\epsilon_2-\epsilon_1)\right)$$

$$D^{++} = \frac{d\Gamma(B \to (f_1^+)_D(f_2^+)_D K)}{dt\,du} \propto \left(|A_e|^2 - 2\,\mathrm{Re}\left((\epsilon_1^+ - \epsilon_2^+)\,A_e^*A_o\right)\,cos\,\theta\right)$$

$$A_e = \frac{\left(A(t,u)+A(u,t)\right)}{2},\ A_o = \frac{\left(A(t,u)-A(u,t)\right)}{2\,cos\,\theta}$$

$$D^{-+} = \frac{d\Gamma(B \to (f_1^-)_D(f_2^+)_D K)}{dt\,du} \propto \left(|A_o|^2\,cos^2\,\theta - 2\,\mathrm{Re}\left((\epsilon_1^- - \epsilon_2^+)\,A_e^*A_o\right)\,cos\,\theta\right)$$

Easy to extract $|A_e|, |A_o|$ *and* $2\mathrm{Re}(\epsilon_1^+ - \epsilon_2^+)\cos\delta$*;* δ *strong phase between* A_e *and* A_o*.*

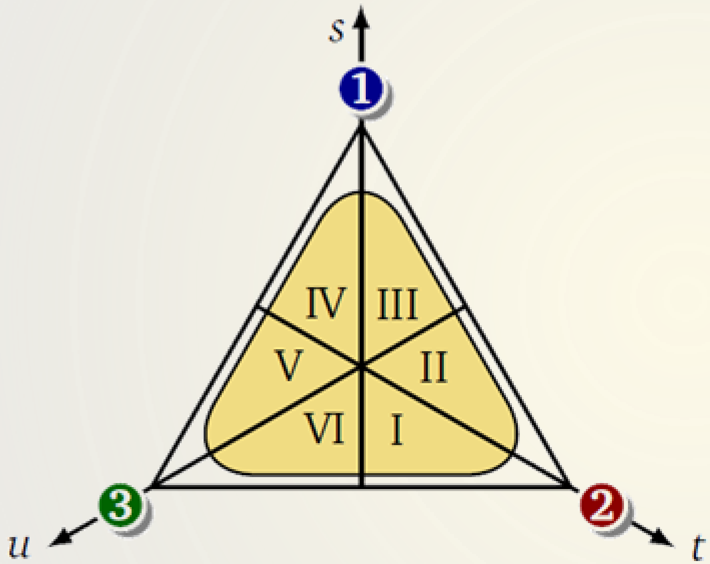
A $t \leftrightarrow u$ *exchange asymmetry in* $B \rightarrow K(K^+K^-)_D(\pi^+\pi^-)_D$ *Dalitz plot is a measure of* $\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-)$

It is easy to see that $\Delta A_{CP} = 2\mathrm{Re}(\epsilon_1^+ - \epsilon_2^+)$*. Hence the asymmetry measures a* ***lower bound*** *on* ΔA_{CP}

- *Note that parent particle the B meson or the nature of accompanying meson plays no role in the kind of direct CP violation under consideration.*
- *Only the momentum of the K meson was important. Therefore instead of concentrating on a single decay mode we could add other decay modes or even look at non-resonant processes such as* $e^+e^- \rightarrow Y\, D^0\, \overline{D}^0$*. Y could be a pseudoscalar, vector, … or even a multibody state.*
- *Generalizing we consider* $X \rightarrow Y\, D^0\, \overline{D}^0$*. Finally asymmetry can be observed in a superposition of many individual "Dalitz plots" or "Dalitz Prism"*

Bose symmetry

If two final particles are fully Bose symmetric, the Dalitz plot must be symmetric under their exchange.



- Particles 2 and 3 identical to one another (but reconstructed from distinct final states), e.g. $(K^+, D^+, D_s^+) \to \underbrace{\pi^+(p_1)}_{\mu^+\nu_\mu}\ \underbrace{\pi^0(p_2)}_{e^+e^-\gamma}\ \underbrace{\pi^0(p_3)}_{\gamma\gamma}$ $\Longrightarrow$ the Dalitz plot must be left-right symmetric.

- All particles identical (but two are reconstructed from distinct final states), e.g. $B^0 \to \underbrace{K_S^0(p_1)}_{\pi^+\pi^-}\ \underbrace{K_S^0(p_2)}_{\pi^+\pi^-}\ \underbrace{K_S^0(p_3)}_{\pi^0\pi^0}$
$\Longrightarrow$ half of the Dalitz plot can be reconstructed. The three sextants of that half must be symmetrical to one another.

CPT violation

- *Let* $X \rightarrow 1+2+3$ *be a self conjugate process with no CP violation, i.e. it occurs via strong or electromagnetic interactions. Moreover 2 and 3 are CP conjugates of each other.*
- *Amplitude:*

$$A(r,\theta) = \sum_{n=0}^{\infty} (s_n(r)\ \sin n\theta + c_n(r)\ \cos n\theta)$$

Fourier coefficients $s_n(r)$ *and* $c_n(r)$ *are in general complex.*

- *Under CPT:* $\theta \rightarrow -\theta,\ \ s_n(r) \rightarrow s_n^*(r)$ *and* $c_n(r) \rightarrow c_n^*(r)$
- *When CPT is exact:* $A(r,\theta) = A^*(r,-\theta)$. *If both CP and CPT exact,* $s_n(r) = 0$ *and* $Im(c_n(r)) = 0$. *Hence, Dalitz plot symmetric under* $\theta \rightarrow -\theta$.
- *If CP is exact but not CPT, then there must be an observable asymmetry under* $\theta \rightarrow -\theta$.

The amplitude $\bar{A}(r,-\theta)$ *for the CP conjugate process, assuming CPT violation is:*

$$\bar{A}(r,-\theta) = \sum_{n=0}^{\infty} (-\bar{s}_n(r)\ \sin n\theta + \bar{c}_n(r)\ \cos n\theta)$$

where $\bar{s}_n(r)$ *and* $\bar{c}_n(r)$ *necessarily differ from* $s_n^*(r)$ *and* $c_n^*(r)$:

$$s_n(r) = (|s_n(r)| + \epsilon_n^s(r))e^{i\delta_n^s} \quad \bar{s}_n(r) = (|s_n(r)| - \epsilon_n^s(r))e^{i\delta_n^s}$$

$$c_n(r) = (|c_n(r)| + \epsilon_n^c(r))e^{i\delta_n^c} \quad \bar{c}_n(r) = (|c_n(r)| - \epsilon_n^c(r))e^{i\delta_n^c}$$

$\epsilon_n^{s,c}(r)$ *CPT violating parameters* $\delta_n^{s,c}(r)$ *strong phases*

No weak phases since CP is conserved.

Since process is CP conjugate amplitude is average of both $A(r,\theta)$ *and* $\bar{A}(r,-\theta)$:

$$A = \frac{1}{2}(A(r,\theta) + \bar{A}(r,-\theta))$$

$$\Rightarrow A = \sum_{n=0}^{\infty} \left(\epsilon_n^s(r) \sin n\theta \, e^{i\delta_n^s} + |c_n(r)| \cos n\theta \, e^{i\delta_n^c} \right)$$

In the Dalitz plot distribution, which is proportional to $|A|^2$, the term odd under $\theta \leftrightarrow -\theta$ is proportional to

$$\sum_{n,m=0}^{\infty} |c_n(r)| \epsilon_m^s(r) \cos(\delta_n^c - \delta_m^s) \cos n\theta \sin m\theta \, .$$

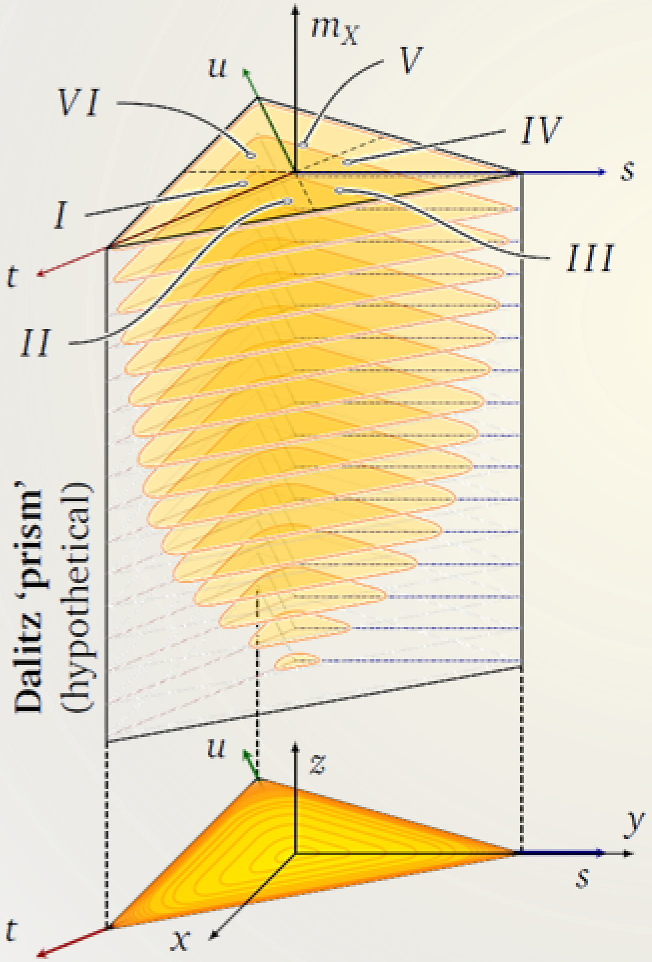
This term survives only if CPT is violated and leads to an asymmetry in Dalitz plot under $\theta \leftrightarrow -\theta \equiv t \leftrightarrow u$.

*Three-body decays via strong interaction are ideal for study of **CPT** violation. Decay mode: $J/\psi \rightarrow N\pi^+\pi^-$, where N could be $\pi^0, \omega, \eta, \phi \ldots$*

What about Dalitz prism?

Dalitz Prism

The Dalitz prism can handle gargantuan amount of data enabling precise measurements of the violations.



- Very precise measurements essential to study violations of **CP**, **CPT** and Bose symmetries require analysis of a huge number of events.
- Dalitz prism combines data from the continuum with data from many resonances. This enhances the statistics immensely.
- We just need the projection of the Dalitz prism at its base to do our analysis.

D. Sahoo, R. Sinha, N. G. Deshpande, Phys. Rev. D91, 051901(R) (2015)

- *The Dalitz prism helps in considering multi-body data. Treating a multi-body decay as an effective three-body decay we can construct a Dalitz prism, e.g.* $J/\psi \rightarrow N\pi^+\pi^-$, *where N could include* $K^+K^-, \pi^0K^+K^-, \eta K^+K^-, \omega\pi, p\bar{p}, p\bar{p}\pi^0,\ n\bar{n}, \ldots$

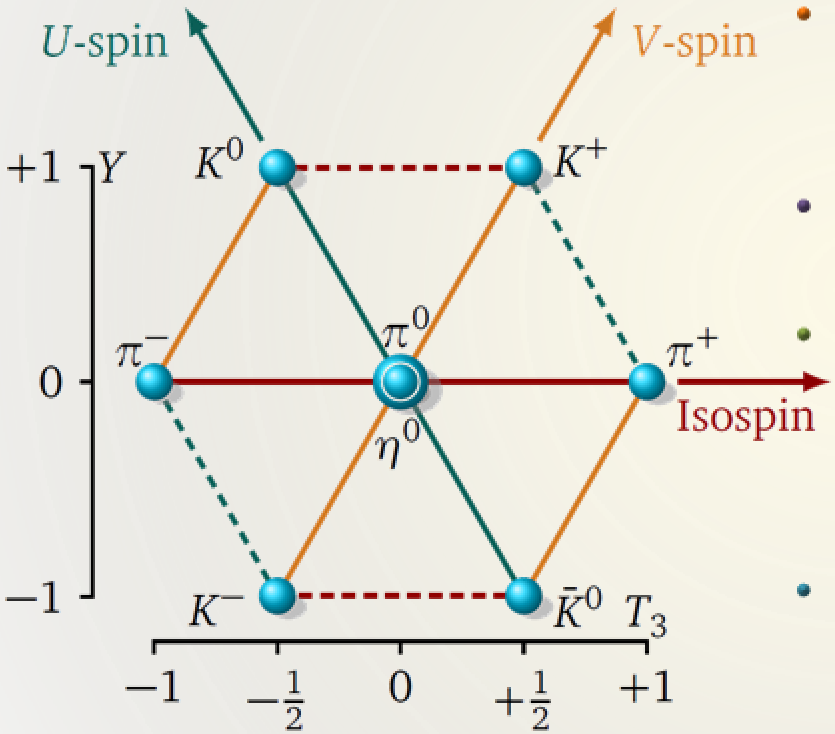
- *Dalitz prism is helpful even when initial state radiation (ISR) or final state radiation (FSR) are present. We only need initial* e^+e^- *energy and the* $4-$*momentum of the two specifically selected final state.*

- *Easy generalization of the concept of Dalitz plot not realized earlier. Works only because we are only seeing asymmetries projected on the base*

Courtesy Dibyakrupa Sahoo

SU(3) symmetry

*The **SU(3)** flavor symmetry subsumes three non-commuting **SU(2)** symmetries: Isospin (or T-spin), U-spin, V-spin.*



- *SU(3) breaking effects cannot be calculated and must be estimated using experimental inputs.*
- *Mass differences between these mesons a measure of the extent of SU(3) breaking.*
- *Not possible to estimate binding energies from QCD calculations since resonances lie in the nonrelativistic low-energy regime.*
- *Up quark has different electric charge than down & strange, cannot be treated in the same way in these studies of loop contributions.*

Estimates of SU(3) breaking are currently empirical.

Physicist A chooses to apply isospin:

$$\pi^0 \leftrightarrow \pi^+ \Rightarrow t \leftrightarrow u$$

Concludes that the amplitude has two components: symmetric and anti-symmetric along the $t = u$ axis.

Physicist B chooses to apply U-spin:

$$K^0 \leftrightarrow \pi^0 \Rightarrow s \leftrightarrow t$$

Concludes that the amplitude has two components: symmetric and anti-symmetric along the $s = t$ axis.

Decay amplitude given by

$$\mathscr{A}(s,t,u) = \mathscr{A}_{SS}(s,t,u) + \mathscr{A}_{AA}(s,t,u)$$

Final state			Kind of $SU(2)$ exchange	
M_1	M_2	M_3	$M_1 \leftrightarrow M_2$	$M_2 \leftrightarrow M_3$
K^0	π^0	π^+	U-spin	Isospin
K^+	π^0	π^-	V-spin	Isospin
K^+	π^0	$\bar{K}^0$	V-spin	U-spin
π^+	π^0	$\bar{K}^0$	Isospin	U-spin

If both isospin and U-spin are individually good symmetries Dalitz Plot must obey the symmetries concluded by the two physicists. But that is impossible unless the amplitude is either

- symmetric under both
  $s \leftrightarrow t$ and $t \leftrightarrow u$,
- antisymmetric under both
  $s \leftrightarrow t$ and $t \leftrightarrow u$

Only the fully symmetric and the fully anti-symmetric amplitudes are allowed.

- $\mathscr{A}_{SS}(s,t,u)$ is fully symmetric under $s \leftrightarrow t \leftrightarrow u$:

$$\mathscr{A}_{SS}(s,t,u) \overset{s\leftrightarrow t}{=\!=} \mathscr{A}_{SS}(t,s,u) \overset{t\leftrightarrow u}{=\!=} \mathscr{A}_{SS}(u,s,t) \overset{s\leftrightarrow t}{=\!=} \mathscr{A}_{SS}(u,t,s).$$

- $\mathscr{A}_{AA}(s,t,u)$ is fully anti-symmetric under $s \leftrightarrow t \leftrightarrow u$:

$$\mathscr{A}_{AA}(s,t,u) \overset{s\leftrightarrow t}{=\!=} -\mathscr{A}_{AA}(t,s,u) \overset{t\leftrightarrow u}{=\!=} +\mathscr{A}_{AA}(u,s,t) \overset{s\leftrightarrow t}{=\!=} -\mathscr{A}_{AA}(u,t,s).$$

- $\mathscr{A}_{SA}(s,t,u)$ is identically zero:

$$\begin{aligned}\mathscr{A}_{SA}(s,t,u) &\overset{s\leftrightarrow t}{=\!=} \mathscr{A}_{SA}(t,s,u) \overset{t\leftrightarrow u}{=\!=} -\mathscr{A}_{SA}(u,s,t) \overset{s\leftrightarrow t}{=\!=} -\mathscr{A}_{SA}(u,t,s) \\ &\overset{t\leftrightarrow u}{=\!=} +\mathscr{A}_{SA}(t,u,s) \overset{s\leftrightarrow t}{=\!=} +\mathscr{A}_{SA}(s,u,t) \overset{t\leftrightarrow u}{=\!=} -\mathscr{A}_{SA}(s,t,u) = 0.\end{aligned}$$

- $\mathscr{A}_{AS}(s,t,u)$ is identically zero:

$$\begin{aligned}\mathscr{A}_{AS}(s,t,u) &\overset{s\leftrightarrow t}{=\!=} -\mathscr{A}_{AS}(t,s,u) \overset{t\leftrightarrow u}{=\!=} -\mathscr{A}_{AS}(u,s,t) \overset{s\leftrightarrow t}{=\!=} +\mathscr{A}_{AS}(u,t,s) \\ &\overset{t\leftrightarrow u}{=\!=} +\mathscr{A}_{AS}(t,u,s) \overset{s\leftrightarrow t}{=\!=} -\mathscr{A}_{AS}(s,u,t) \overset{t\leftrightarrow u}{=\!=} -\mathscr{A}_{AS}(s,t,u) = 0.\end{aligned}$$

Distribution function has two parts f_S and f_A

$$f_S(s,t,u) \propto |\mathscr{A}_{SS}(s,t,u)|^2 + |\mathscr{A}_{AA}(s,t,u)|^2$$

$$f_A(s,t,u) \propto 2\,\mathrm{Re}\Big(\mathscr{A}_{SS}(s,t,u)\cdot \mathscr{A}^*_{AA}(s,t,u)\Big)$$

The various sextants of the Dalitz plot have a characteristic alternate distribution pattern

$$f_I = f_{III} = f_V = f_S(s,t,u) + f_A(s,t,u) \qquad \Sigma^i_j(r,\theta) = f_i + f_j$$

$$f_{II} = f_{IV} = f_{VI} = f_S(s,t,u) - f_A(s,t,u) \qquad \Delta^i_j(r,\theta) = f_i - f_j$$

Probe the nature of SU(3) *breaking and quantitatively measure* SU(3) *breaking*

$$\mathbb{A}_1 = \left|\frac{\Sigma^I_{VI} - \Sigma^{III}_{IV}}{\Sigma^I_{VI} + \Sigma^{III}_{IV}}\right| + \left|\frac{\Sigma^{III}_{IV} - \Sigma^V_{II}}{\Sigma^{III}_{IV} + \Sigma^V_{II}}\right| + \left|\frac{\Sigma^V_{II} - \Sigma^I_{VI}}{\Sigma^V_{II} + \Sigma^I_{VI}}\right| + \left|\frac{\Delta^I_{VI} - \Delta^{III}_{IV}}{\Delta^I_{VI} + \Delta^{III}_{IV}}\right| + \left|\frac{\Delta^{III}_{IV} - \Delta^V_{II}}{\Delta^{III}_{IV} + \Delta^V_{II}}\right| + \left|\frac{\Delta^V_{II} - \Delta^I_{VI}}{\Delta^V_{II} + \Delta^I_{VI}}\right|,$$

$$\mathbb{A}_2 = \left|\frac{\Sigma^V_{IV} - \Sigma^I_{II}}{\Sigma^V_{IV} + \Sigma^I_{II}}\right| + \left|\frac{\Sigma^I_{II} - \Sigma^{III}_{VI}}{\Sigma^I_{II} + \Sigma^{III}_{VI}}\right| + \left|\frac{\Sigma^{III}_{VI} - \Sigma^V_{IV}}{\Sigma^{III}_{VI} + \Sigma^V_{IV}}\right| + \left|\frac{\Delta^V_{IV} - \Delta^I_{II}}{\Delta^V_{IV} + \Delta^I_{II}}\right| + \left|\frac{\Delta^I_{II} - \Delta^{III}_{VI}}{\Delta^I_{II} + \Delta^{III}_{VI}}\right| + \left|\frac{\Delta^{III}_{VI} - \Delta^V_{IV}}{\Delta^{III}_{VI} + \Delta^V_{IV}}\right|,$$

$$\mathbb{A}_3 = \left|\frac{\Sigma^I_{IV} - \Sigma^{III}_{II}}{\Sigma^I_{IV} + \Sigma^{III}_{II}}\right| + \left|\frac{\Sigma^{III}_{II} - \Sigma^V_{VI}}{\Sigma^{III}_{II} + \Sigma^V_{VI}}\right| + \left|\frac{\Sigma^V_{VI} - \Sigma^I_{IV}}{\Sigma^V_{VI} + \Sigma^I_{IV}}\right| + \left|\frac{\Delta^I_{IV} - \Delta^{III}_{II}}{\Delta^I_{IV} + \Delta^{III}_{II}}\right| + \left|\frac{\Delta^{III}_{II} - \Delta^V_{VI}}{\Delta^{III}_{II} + \Delta^V_{VI}}\right| + \left|\frac{\Delta^V_{VI} - \Delta^I_{IV}}{\Delta^V_{VI} + \Delta^I_{IV}}\right|.$$

D. Sahoo, R. Sinha and N. G. Deshpande, Phys. Rev. D 91, 076013 (2015)

Two & Three body decays using SU(3)

Study of $B \to PP,\ VP,\ PPP$ *decays in the framework of flavor symmetry*

- *Study fully symmetric final states in* $B \to PPP$, $P = \pi, K$. *Relations between fully symmetric final states in the* SU(3) *limit.*

$$\sqrt{2}\mathcal{A}(B^+ \to K^+\pi^+\pi^-)_{FS} = \mathcal{A}(B^+ \to K^+K^+K^-)_{FS}$$

$$\sqrt{2}\mathcal{A}(B^+ \to \pi^+K^+K^-)_{FS} = \mathcal{A}(B^+ \to \pi^+\pi^+\pi^-)_{FS}$$

Bhattacharya, Gronau, Imbeault, London, and Rosner, Phys. Rev. D 89, 074043 (2014).

- *Update on* $B \to PP, VP$ *using* SU(3).
  - *Extraction of W-exchange and penguin-annihilation amplitudes for the first time.*
  - *Larger than expected color suppressed tree and strong phases*
  - *Predict large BR* $\sim 10^{-6}$ *for* $B_s^0 \to \phi\pi^0$.
  - *Identify few observables to be determined experimentally in order to discriminate among theory calculations*

H. Y. Cheng, C.W. Chiang, and A. L. Kuo, Phys. Rev. D 91, 014011 (2015).

Extra information on $A_{3/2}$ amplitude using Dalitz plot

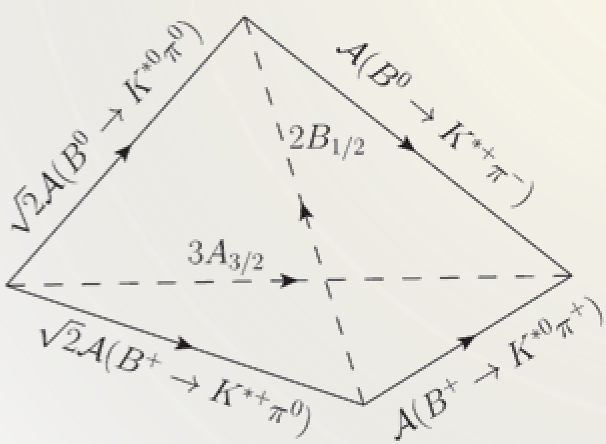
$I = \frac{1}{2}$ *initial state decays to a final state that is either* $I = \frac{1}{2}$ *or* $I = \frac{3}{2}$ *via a transition that allows* $\Delta I = \frac{1}{2}$ *or* $\Delta I = \frac{3}{2}$ *transitions*

$$\mathcal{A}^{-+} = \mathcal{A}(B^0 \to K^{*+}\pi^-) = A_{3/2} + A_{1/2} - B_{1/2},$$

$$\mathcal{A}^{+0} = \mathcal{A}(B^+ \to K^{*0}\pi^+) = A_{3/2} + A_{1/2} + B_{1/2},$$

$$\mathcal{A}^{00} = \sqrt{2}\mathcal{A}(B^0 \to K^{*0}\pi^0) = 2A_{3/2} - A_{1/2} + B_{1/2},$$

$$\mathcal{A}^{0+} = \sqrt{2}\mathcal{A}(B^+ \to K^{*+}\pi^0) = 2A_{3/2} - A_{1/2} - B_{1/2}.$$



$$A_{1/2} = \pm\sqrt{\frac{2}{3}}\left\langle\frac{1}{2}, \pm\frac{1}{2}\,\middle|\,\mathcal{H}_{\Delta I=1}\,\middle|\,\frac{1}{2}, \pm\frac{1}{2}\right\rangle,$$

$$A_{3/2} = \sqrt{\frac{1}{3}}\left\langle\frac{3}{2}, \pm\frac{1}{2}\,\middle|\,\mathcal{H}_{\Delta I=1}\,\middle|\,\frac{1}{2}, \pm\frac{1}{2}\right\rangle,$$

$$B_{1/2} = \sqrt{\frac{2}{3}}\left\langle\frac{1}{2}, \pm\frac{1}{2}\,\middle|\,\mathcal{H}_{\Delta I=0}\,\middle|\,\frac{1}{2}, \pm\frac{1}{2}\right\rangle.$$

$$t \equiv a + b\cos\theta \qquad a = \frac{M^2 + m_K^2 + 2m_\pi^2 - s}{2}$$

$$u \equiv a - b\cos\theta \qquad b = \frac{\sqrt{s - 4m_\pi^2}}{2\sqrt{s}}\,\lambda^{1/2}(M^2, m_K^2, s)$$

$$|K^0\pi^0\pi^+\rangle = \left(\frac{1}{\sqrt{5}}\left|\frac{5}{2},\frac{1}{2}\right\rangle_e + \sqrt{\frac{3}{10}}\left|\frac{3}{2},\frac{1}{2}\right\rangle_e\right)X - \left(\frac{1}{\sqrt{6}}\left|\frac{3}{2},\frac{1}{2}\right\rangle_o + \frac{1}{\sqrt{3}}\left|\frac{1}{2},\frac{1}{2}\right\rangle_o\right)Y\cos\theta$$

X and $Y\cos\theta$ are introduced to take care of the spatial and kinematic contributions

$$|K^{*0}\pi^+\rangle = \sqrt{\frac{1}{3}}|[K^0\pi^0]\pi^+\rangle - \sqrt{\frac{2}{3}}|[K^+\pi^-]\pi^+\rangle,$$

$$|K^{*+}\pi^0\rangle = -\sqrt{\frac{1}{3}}|[K^+\pi^0]\pi^0\rangle + \sqrt{\frac{2}{3}}|[K^0\pi^+]\pi^0\rangle.$$

$$\mathcal{M}(B^+ \to [K^0\pi^0]\pi^+)$$
$$= \frac{g_{K^*K\pi}}{\sqrt{3}}(A_{3/2} + A_{1/2} + B_{1/2}) \times (P + p_3)^\mu (p_1 - p_2)^\nu \frac{(-g_{\mu\nu} + \frac{(p_1+p_2)_\mu (p_1+p_2)_\nu}{m_{K^*}^2})}{u - m_{K^*}^2 + i m_{K^*}\Gamma_{K^*}}$$

$$\mathcal{M}(B^+ \to [K^0\pi^0]\pi^+) = \frac{g_{K^*K\pi}}{\sqrt{3}}(A_{3/2} + A_{1/2} + B_{1/2}) \times \left(\frac{s - t + c}{u - m_{K^*}^2 + i m_{K^*}\Gamma_{K^*}}\right).$$

$$c = \frac{(M^2 - m_\pi^2)(m_K^2 - m_\pi^2)}{m_{K^*}^2}$$

$$\mathcal{M}(B^+ \to [K^0\pi^+]\pi^0) = \frac{g_{K^*K\pi}}{\sqrt{3}}(2A_{3/2} - A_{1/2} - B_{1/2}) \times \left(\frac{s - u + c}{t - m_{K^*}^2 + i m_{K^*}\Gamma_{K^*}}\right)$$

$$A_e = \frac{g_{K^*K\pi}}{\sqrt{3}} \frac{3}{2} A_{3/2} \qquad A_o = \frac{g_{K^*K\pi}}{\sqrt{3}} \frac{1}{2}(-A_{3/2} + 2A_{1/2} + 2B_{1/2})$$

$$\mathcal{M}(B^+ \to [K\pi]\pi) = \Bigg[A_e\left(\frac{s - t + c}{u - m_{K^*}^2 + i m_{K^*}\Gamma_{K^*}} + \frac{s - u + c}{t - m_{K^*}^2 + i m_{K^*}\Gamma_{K^*}}\right) + A_o\left(\frac{s - t + c}{u - m_{K^*}^2 + i m_{K^*}\Gamma_{K^*}} - \frac{s - u + c}{t - m_{K^*}^2 + i m_{K^*}\Gamma_{K^*}}\right)\Bigg]$$

$$|\mathcal{M}(B^+ \to [K\pi]\pi)|^2 = \frac{f_1|A_e|^2 + f_2\,\mathrm{Re}\,(A_eA_o^*) + f_3\,\mathrm{Im}\,(A_eA_o^*) + f_4|A_o|^2}{[(m_{K^*}^2 - t)^2 + m_{K^*}^2\Gamma_{K^*}^2][(m_{K^*}^2 - u)^2 + m_{K^*}^2\Gamma_{K^*}^2]}$$

$$f_1 = 4(-3a + c + Q)^2[(a - m_{K^*}^2)^2 + m_{K^*}^2\Gamma_{K^*}^2] + 8b^2(a - m_{K^*}^2)(3a - c - Q)\cos^2\theta + 4b^4\cos^4\theta,$$

$$f_2 = 8b\{(3a - c - Q)[m_{K^*}^2\Gamma_{K^*}^2 + (m_{K^*}^2 - a)(-4a + c + m_{K^*}^2 + Q)] - b^2(-4a + c + m_{K^*}^2 + Q)\cos^2\theta\}\cos\theta,$$

$$f_3 = 8bm_{K^*}\Gamma_{K^*}[-(3a - c - Q)^2 + b^2\cos^2\theta]\cos\theta,$$

$$f_4 = b^2\cos^2\theta[(-4a + c + m_{K^*}^2 + Q)^2 + m_{K^*}^2\Gamma_{K^*}^2],$$

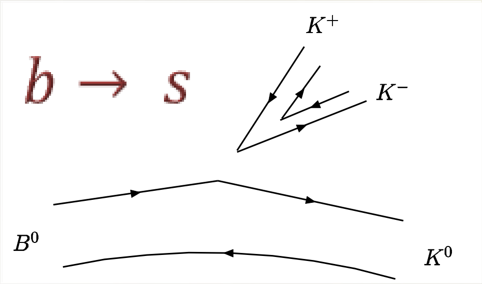
$$Q = M^2 + m_K^2 + 2\,m_\pi^2$$

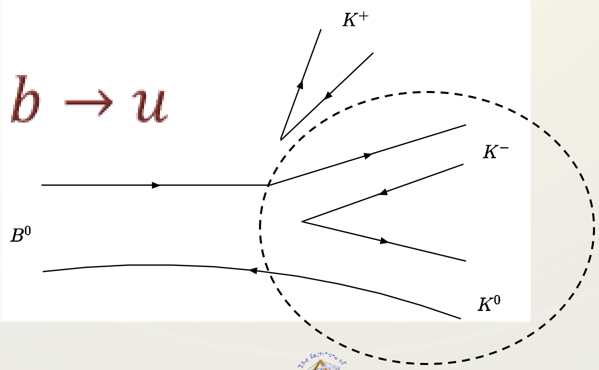
HMChPT approach to 3-body decay

Assuming factoriziation the resulting local correlators are computed in the framework of Heavy-Meson Chiral Perturbation Theory (HMChPT)

Under factorization approximation, three factorizable amplitudes for $B^0 \to K^+ K^- K^0$

- *current-induced process:* $\langle B^0 \to K^0 \rangle \langle 0 \to K^+ K^- \rangle$
- *transition process:* $\langle B^0 \to K^- K^0 \rangle \langle 0 \to K^+ \rangle$
- *annihilation process:* $\langle B^0 \to 0 \rangle \langle 0 \to K^+ K^- K^0 \rangle$





- *H.-Y. Cheng, C.-K.Chua, Phys. Rev. D 88, 114014 (2013) Phys. Rev. D 89, 074025 (2014)*

- *NR rates for tree-dominated $B \to KK\pi$, $\pi\pi\pi$ will become too large ,e.g.,* $Br(B^- \to K^+K^-\pi^-)_{NR} = 33\times 10^{-6}$ *larger than total BF,* 5×10^{-6} $\Rightarrow$ *HMChPT is applicable only to soft mesons !*

- *Ways of improving the use of HMChPT have been suggested before. Fajfer et al; Yang, HYC,…*

- *Write tree-induced NR amplitude as* $A_{transition}^{HMChPT}\ e^{-\alpha_{NR}\, p_B\cdot(p_1-p_2)}\ e^{i\varphi_{12}}$
  - *HMChPT is recovered in soft meson limit,* p_1, $p_2 \to 0$
  - *The parameter* $\alpha_{NR} \gg \frac{1}{2m_B\Lambda_\chi}$ *is constrained from* $B^- \to \pi^+\pi^-\pi^-$
  - *NR rates: mostly from* $b \to s$ *(via* $\langle \bar{K}K|\bar{s}s|0\rangle$*) and a few percentages from* $b \to u$ *transitions*
- Resonant:
  - $B^0 \to f_0 K^0 \to K^+K^-K^0$, $f_0 = f_0(980), f_0(1500), f_0(1710), \ldots$
  - $B^0 \to V K^0 \to K^+K^-K^0$, $V = \rho, \omega, \phi \ldots$

- *Three-body B decays receive sizable NR contributions governed by the matrix elements of scalar densities.*

- *U-spin symmetry relating* $\langle K\pi|\bar{s}d|0\rangle$ *to* $\langle \bar{K}K|\bar{s}s|0\rangle$ *badly broken.*

pQCD approach to 3-body decays

- *Approach to 3-body B decays based on kT factorization theorem with two-hadron distribution amplitude (TDA) for dominant region*

two hard-gluon power suppressed

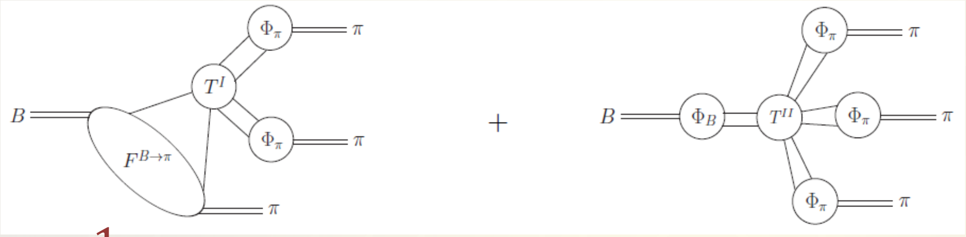
one hard, one soft gluon, two hadrons collimate, dominant

- *Short-distance and rescattering P-wave phases are equally important for predicting A_{CP}.*
- *Can explain and predict direct CP asymmetries of 3π and $K\pi\pi$ in various localized regions of phase space.*

W.-F. Wang, H.-C Hu, H.-n Li and C.-D Lü, Phys. Rev. D 89, 074031 (2014)

QCDF approach to $B \rightarrow \pi^+\pi^-\pi^+$ decays

- *In the limit of very heavy b-quark, Region-I of the Dalitz plot can be described in terms of the $B \rightarrow \pi$ form factor and the B and π light-cone distribution amplitudes*



Factorization formula

- *Power $(\frac{1}{m_b^2})$ & α_s suppressed with respect to two-body.*
- *At leading order/power/twist all convolutions finite ⇒ factorization*
- *The edges of the Dalitz plot, on the other hand, require different non-perturbative input: the $B \rightarrow \pi\pi$ form factor and the two-pion distribution amplitude.*
- *$(\pi^-\pi^-)$ edge -No resonances ⇒ perturbative result reasonable.*
- *For realistic B-meson masses no perturbative centre in the Dalitz plot, but systematic description might be possible in the context of two-pion states.*

Conclusions

- *The Dalitz plot and the new concept of Dalitz prism, provide a unified and powerful method to study violations of* ***CP****,* ***CPT*** *and* ***Bose*** *symmetries.*

- *Dalitz plots can, also, be used profitably for better estimation of the extent of breaking of the* ***SU(3)*** *flavor symmetry.*