

# How Many Angels Can Dance on the Head of a Black Hole?

Tom Banks (work with W.Fischler)

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## Introduction

Hawking Radiation and Entropy

Black Hole Entropy - Bekenstein Hawking, Gibbons and Unruh

General Relativity as Hydrodynamics of the Area Law -  
Jacobson

The Covariant Entropy/Holographic Principle - 't Hooft,  
Fischler, Susskind and Bousso

There Are More Things in Heaven and Earth Than Are  
Dreamed of in Your Quantum Field Theory

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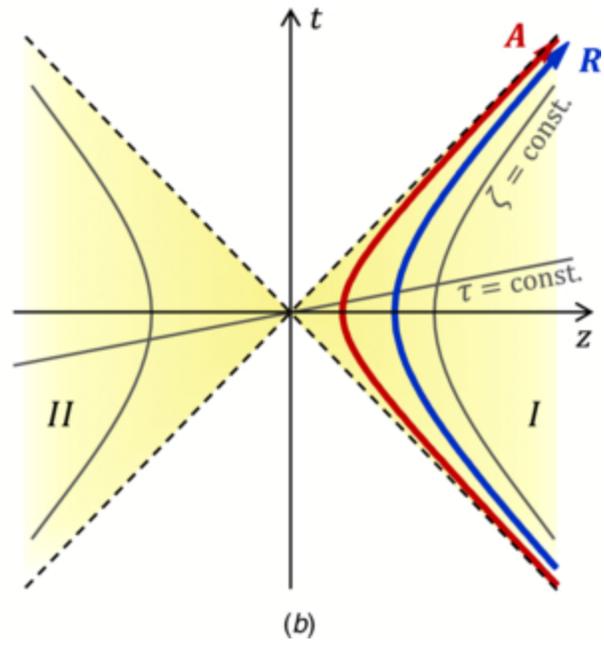
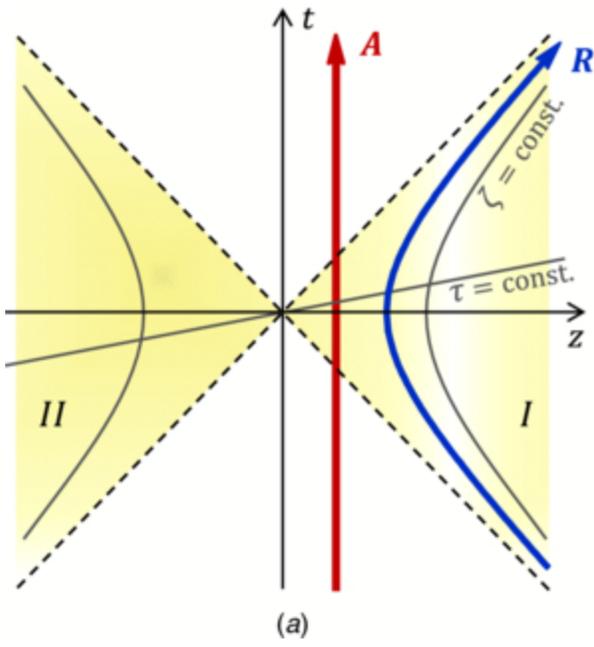
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- ▶ Bekenstein: Classical Black Hole Would Violate 2nd Law: But dropping minimal energy qubit (photon of energy  $1/R_S$ ) on it increases  $R_S$  by 1 Planck unit of area. Area = Entropy !!?? . Implies non-zero temp.  $\sim 1/R_S$  by  $dE = TdS$ , but Bekenstein doesn't mention this.

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- ▶ Geometric (but not quantum) understanding via Unruh, Gibbons and Hawking: GH show similar phenomenon for de Sitter horizon.



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- ▶ Condition for no Black Hole formation  $S \leq (R/L_P)^{3/2}$
- ▶ QFT/particle physics can't account for black hole entropy.

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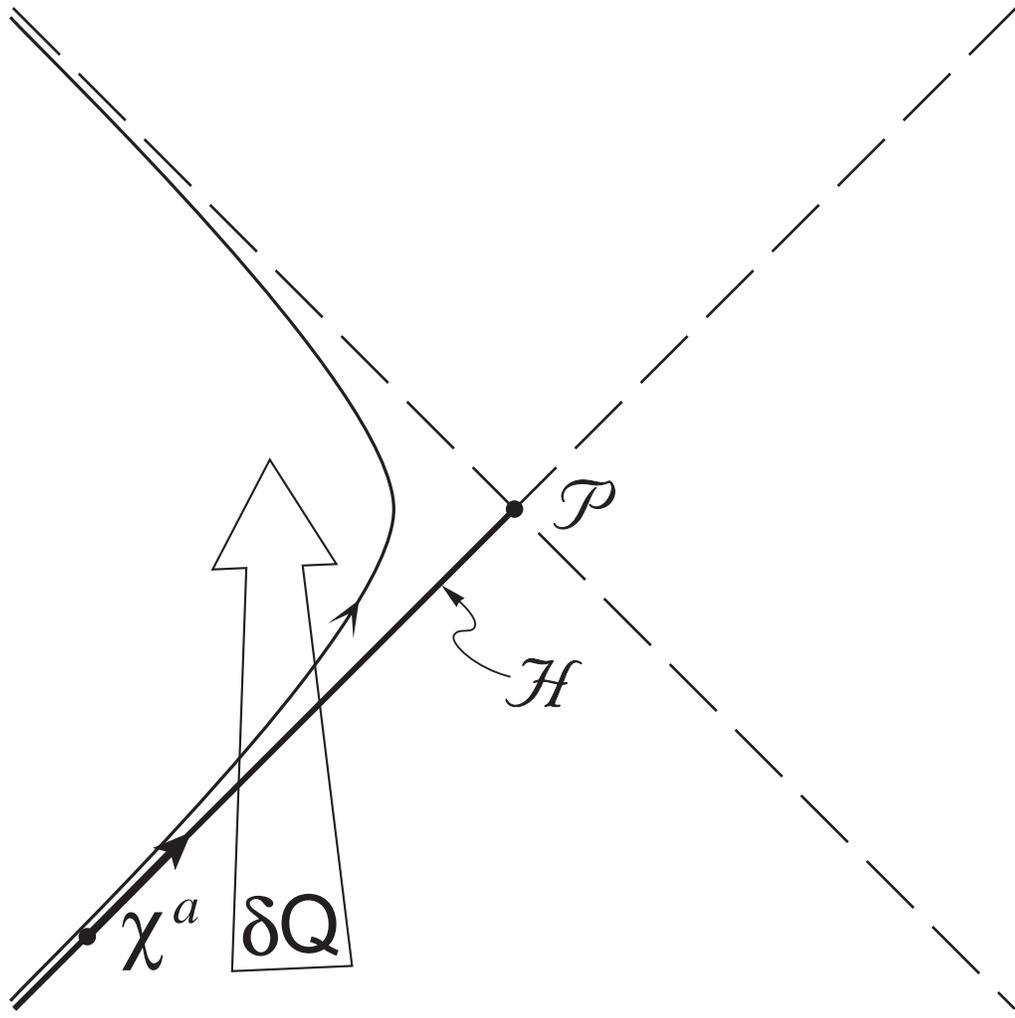
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 $dE = TdS + Unruh + Raychauduri \rightarrow$   
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- ▶ That is *Einstein's Equations (with cosmological constant undetermined) are the hydrodynamics of any quantum system obeying the area law!*
- ▶ Implies QFT (in string theory we learn that all of QFT follows from the supersymmetric generalization of Einstein's equations in 11 dimensions) should only be quantized when discussing small fluctuations around the ground state.



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- ▶ Much of this was anticipated in Jacobson's paper.

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- ▶ Black Hole Metric

$$ds^2 = -\left(1 - \frac{R_S}{r} \pm \frac{r^2}{R^2}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{R_S}{r} \pm \frac{r^2}{R^2}\right)} + r^2 d\Omega^2.$$

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- ▶ Leads to asymptotic energy conservation law in Minkowski ( $R \rightarrow \infty$ ) limit.

# Holographic Cosmology

- ▶ Principle that local excitations are constrained states of variables on the horizon, with a number of constraints  $\sim N = R/L_P \ll N^2$  has profound implications for early universe cosmology. Explains Boltzmann-Penrose question of why the universe began in a low entropy state.

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- ▶ Leads to a finite, quantum mechanical theory of inflation, more constrained than QFT models, and with no conceptual ("trans-Planckian mode") problem.
- ▶ Holographic theory explains current data as well as QFT models, but gives different results for tensor (B mode) correlation functions. Unfortunately these are not yet measured and theory predicts them to be small.

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- ▶ Leads to  $m_{3/2} \sim (M_U L_P)^{-1/4} \Lambda^{1/4}$
- ▶ Splitting in super multiplets  $\sim \sqrt{m_{3/2} M_P} \sim a \text{ few TeV}$ .

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- ▶ Space-time is not a fluctuating quantum variable, but instead a representation of the hydrodynamics of the underlying quantum system.
- ▶ Localized excitations are constrained low entropy states of that system.
- ▶ Implications for the early universe, tensor fluctuations in the CMB, as well as TeV scale physics and supersymmetry.