Supercomputing the properties of the quark-gluon plasma

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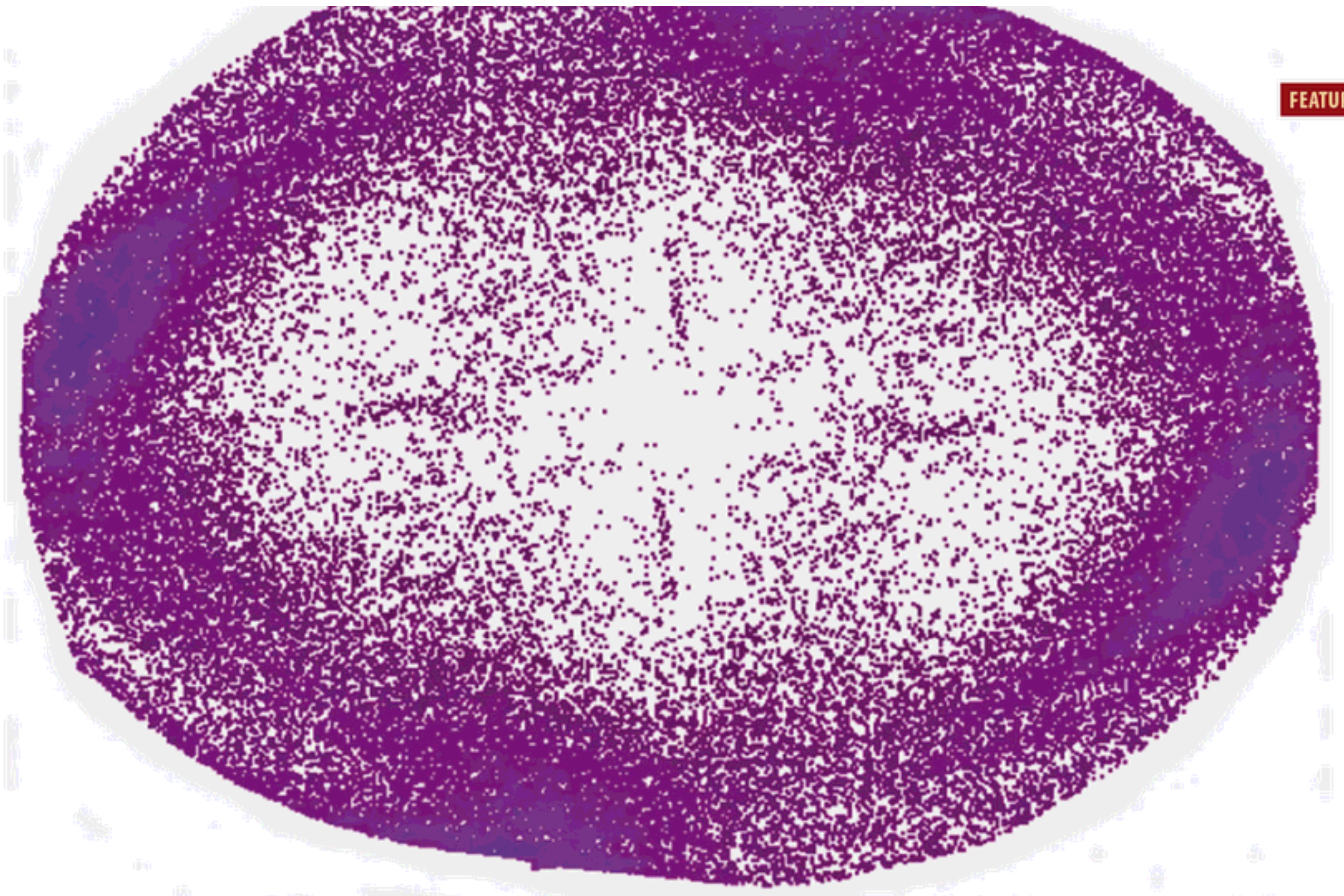
What is the quark-gluon plasma?

FEATURES



The Quark-Gluon Plasma, a nearly perfect fluid

L. Cifarelli[1], L.P. Csernai[2] and H. Stöcker[3] - DOI: 10.1051/epn/2012206
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We are living in interesting times, where the World's largest accelerator, the Large Hadron Collider, has its most dominant successes in Nuclear Physics: collective matter properties of the Quark-Gluon Plasma (QGP) are studied at a detail which is not even possible for conventional, macro scale materials.

At the early plans the only dedicated heavy ion detector was ALICE, but as the first results started to arise, also ATLAS and CMS started to invest increasingly more effort into heavy ion research. This change of interest has two aspects. Contrary to early expectations of very high hadron multiplicity the collective flow became more dominant and a larger part of the available energy (of 208 × 2.76 TeV = 574 TeV) is invested into the collective flow, which developed in the QGP. The number of produced final particles is not as high as expected. This made the ATLAS and CMS detectors fully adequate for heavy ion research, and these detectors provided even a more extended rapidity acceptance, complementing the possibilities of ALICE detector well.

The second aspect was recognized after the very heavy-ion first results [1], where the heavy-ion studies provided new and important insight into the features of QGP. QGP turned out a strongly coupled liquid, with small viscosity, especially at the threshold of the quark/hadron phase transition. The small viscosity, the related fluctuations, and the flow properties arising from these fluctuations enable us to gain insight into the properties of the matter of the early universe, and also the fluctuations observed in the early universe. These new results raised more interest in the ATLAS and CMS collaborations also and their progress in the heavy ion research activity is becoming more important. In recent months the CERN Courier has more and more news about new heavy-ion results.

▲ view of the expanding and rotating Quark-Gluon Plasma from a fluid dynamical calculation (discussed in: L.P. Csernai, V.K. Magas, H. Stöcker, and D.D. Strottman, *Phys. Rev.* C84, 02914 (2011).)

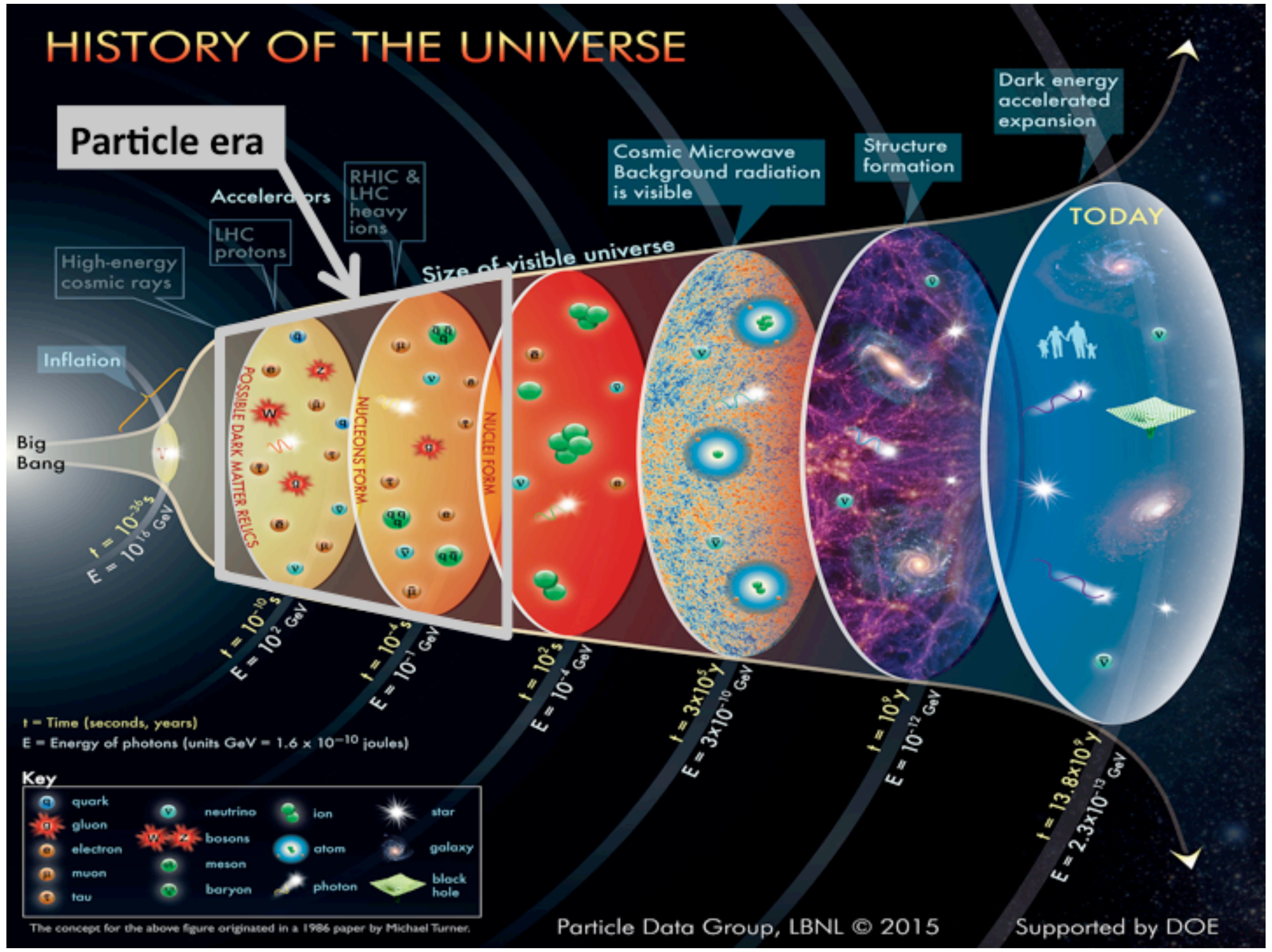
Article available at http://www.europhysicsnews.org or http://dx.doi.org/10.1051/epn/2012206

New state of matter formed at high temperatures and/or densities.

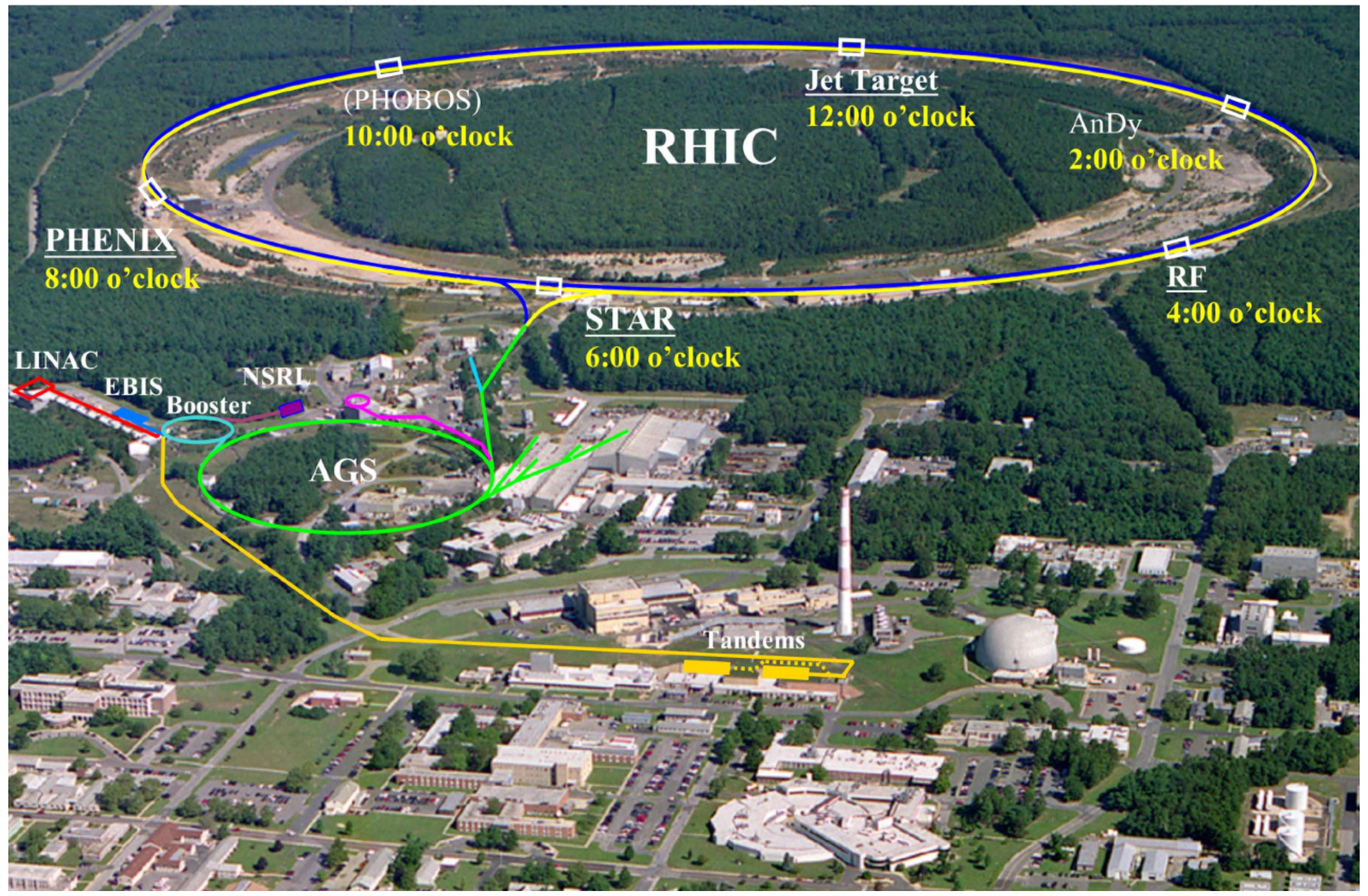
Regarded as a nearly perfect fluid: Very low shear viscosity to entropy density ratio.

Europhysics News, 2012

Creating and studying the Quark-Gluon Plasma



Existed in the very early universe. Today it must be created by colliding heavy nuclei (Cu, Ag, Pb, U, …) at energies of roughly 10-1000 times their rest mass.



Plasma created in heavy-ion collisions expands and cools, finally re-freezing into various hadrons. Its properties are inferred from the nature, distribution of and correlations among these hadrons.

Creating and studying the Quark-Gluon Plasma

Theoretical studies of the QGP fall into four broad categories:

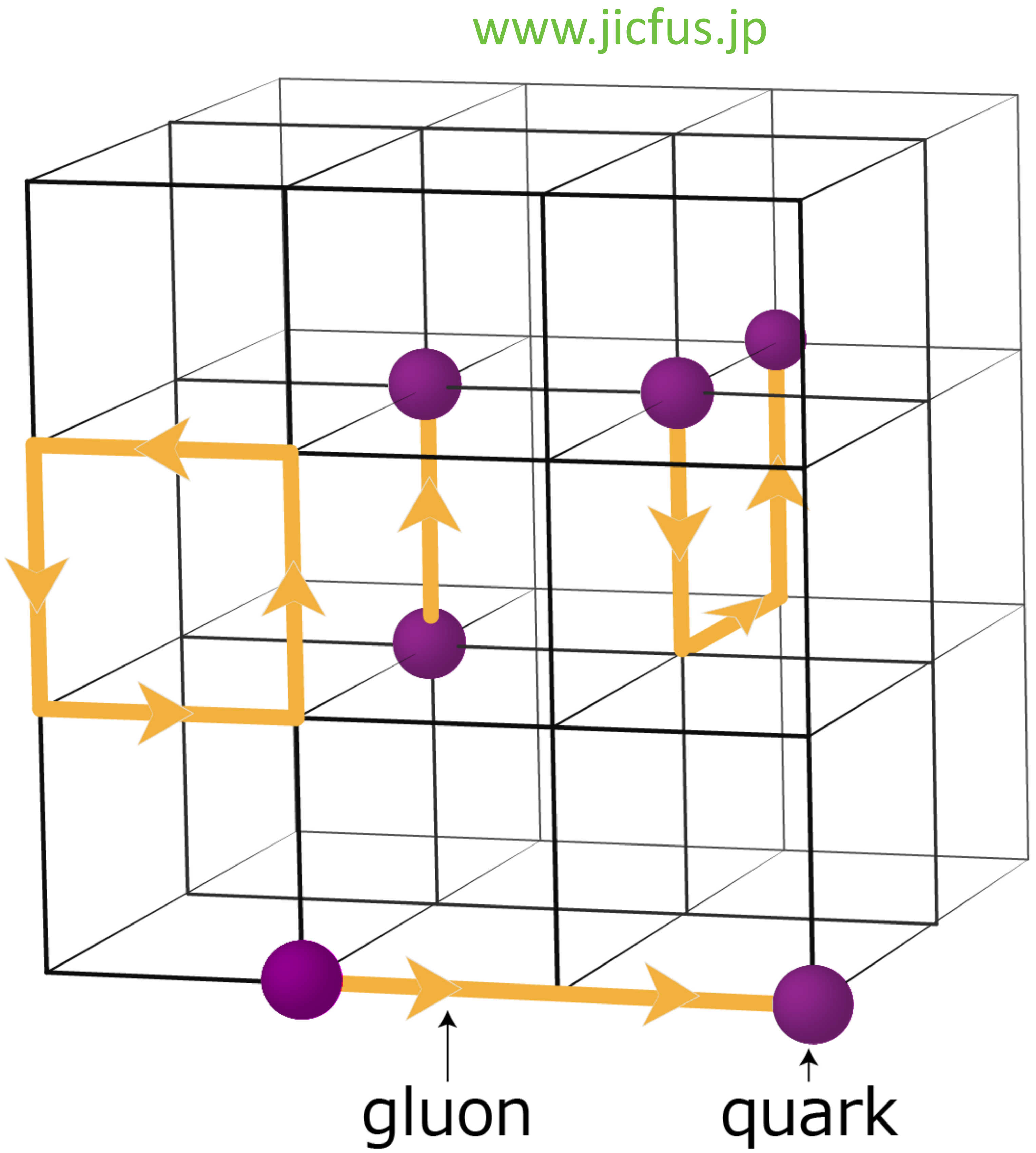
1. Phenomenological: Modeling the QGP expansion and subsequent freeze out: Relativistic hydrodynamics + Cooper-Frye.

2. Models: Understanding the properties of the QGP semi-quantitatively through various models such as the (Polyakov)Nambu-Jona-Lasinio ((P)NJL) model,

3. pQCD: Perturbative QCD at finite temperature and/or density. This requires a resummation of various diagrams in order to improve the convergence. Nevertheless, it is known that perturbation theory at finite temperature breaks down beyond sixth order. Valid at very high to moderately high temperatures.

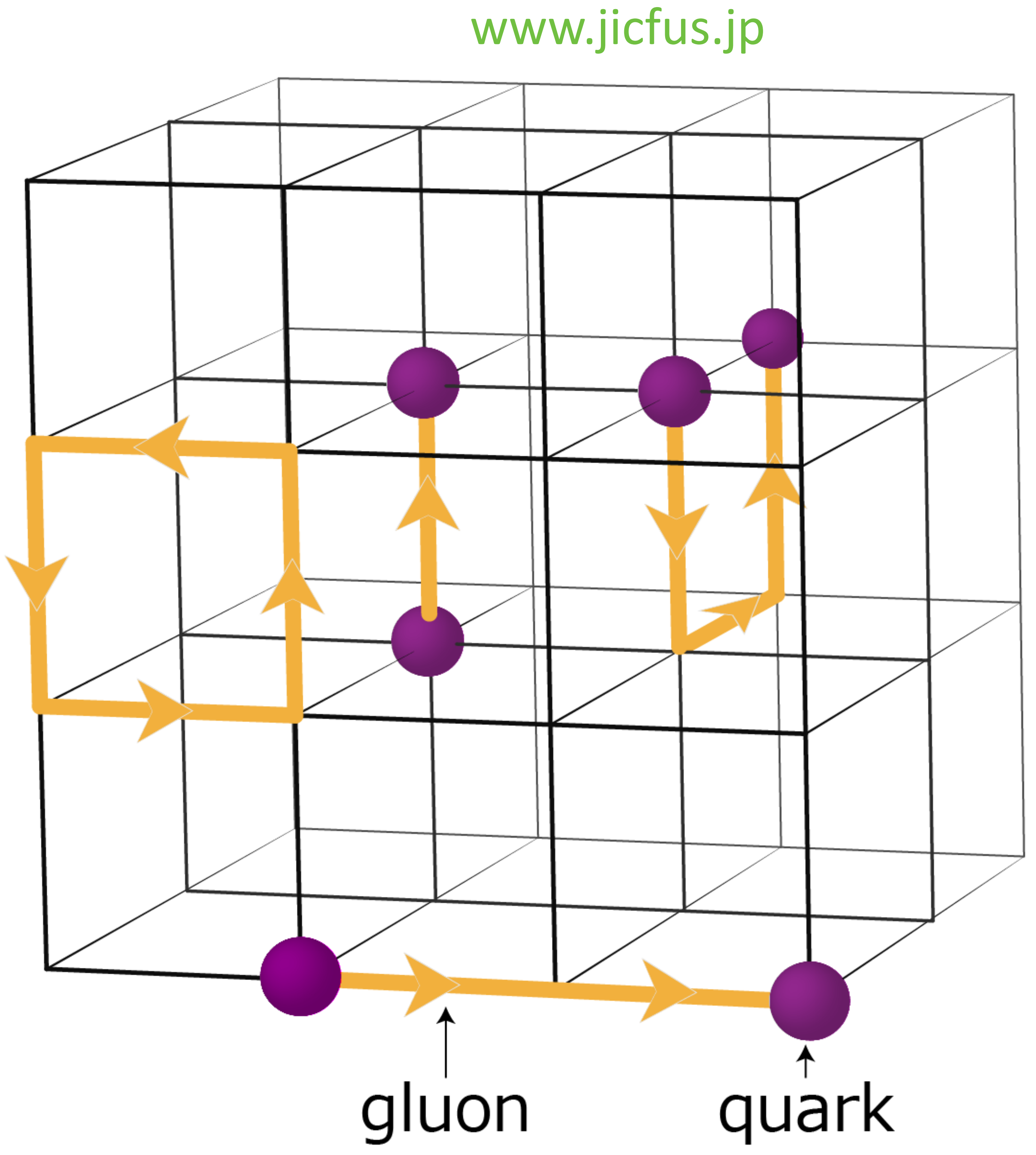
4. Ab initio: Direct calculation of the QGP properties from the underlying theory of QCD. Requires powerful supercomputers to solve the QCD path integral. Computationally challenging. Works where other methods break down.

Lattice QCD: Non-perturbative QCD from first principles



- Four dimensional formulation of QCD in Euclidean space.
- Quarks live on lattice sites where gluons are represented by SU(3) matrices that live on the links connecting two sites.
- Path integral gets replaced by a (highly) multi-dimensional integral that is evaluated using Monte Carlo techniques.

Lattice QCD: Non-perturbative QCD from first principles



- A typical calculation proceeds by first generating a set of gauge configurations ("snapshots" of the QCD vacuum). In the next step, the desired observables are "measured" on each of these configurations.
- By averaging over these measurements, one obtains estimates for the values of these observables. The Monte Carlo method correctly generates configurations in proportion to their relative weights in the path integral.

Lattice QCD on supercomputers around the world

Cori @ NERSC, USA

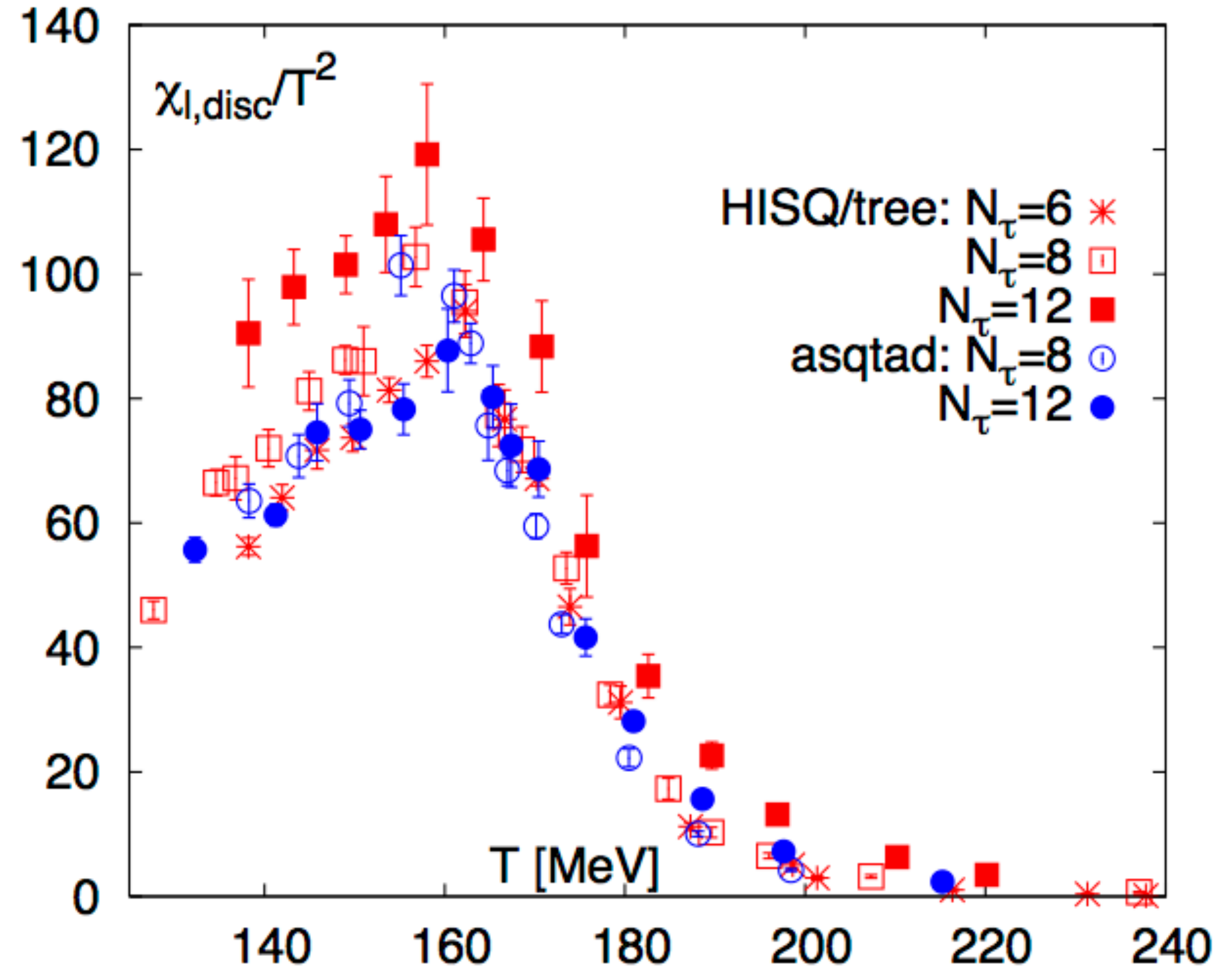
Titan @ Oak Ridge, USA

K-Computer, Japan

Tianhe-2, China

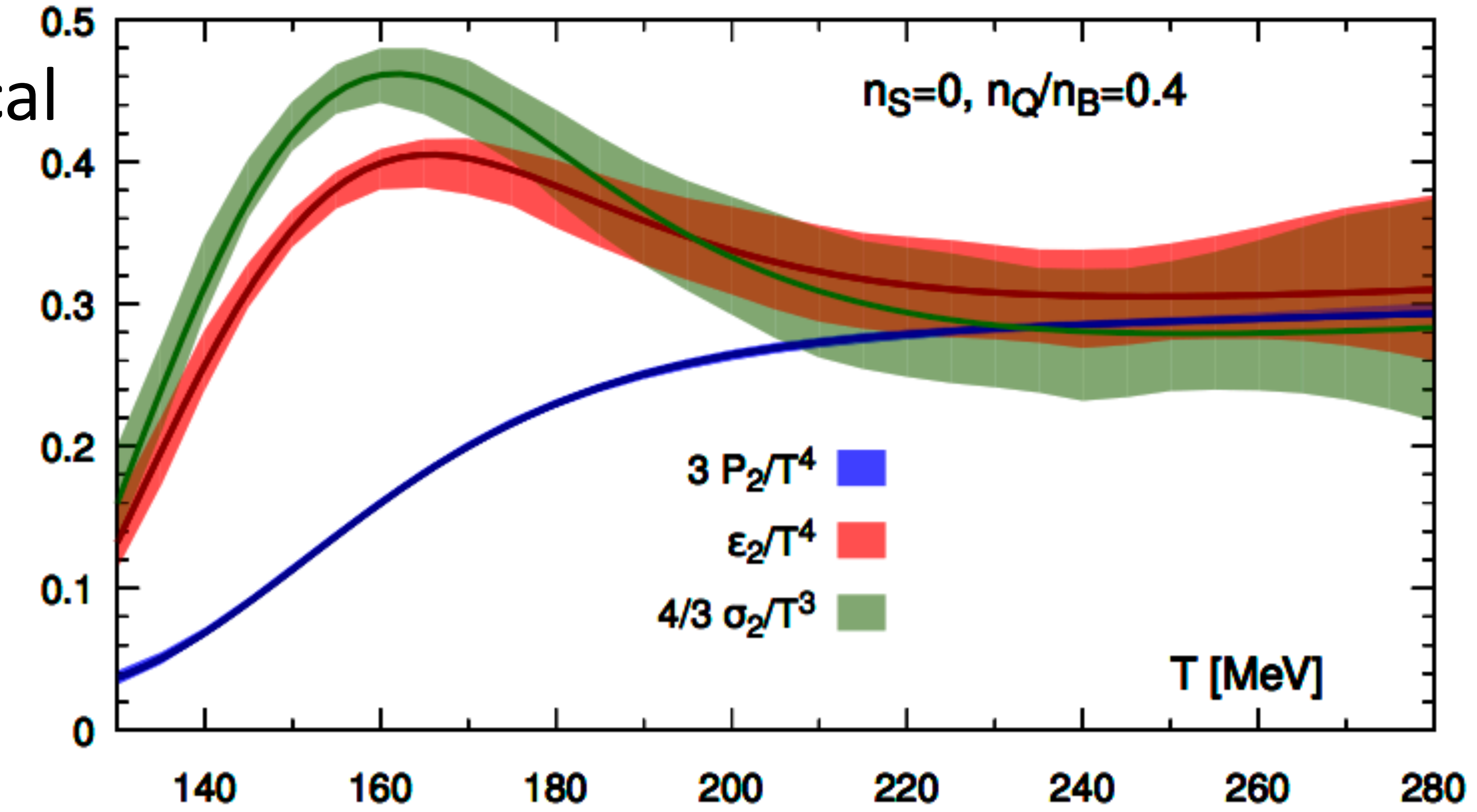
Lattice QCD calculations are currently carried out at 6 out of the 10 fastest supercomputers in the world.

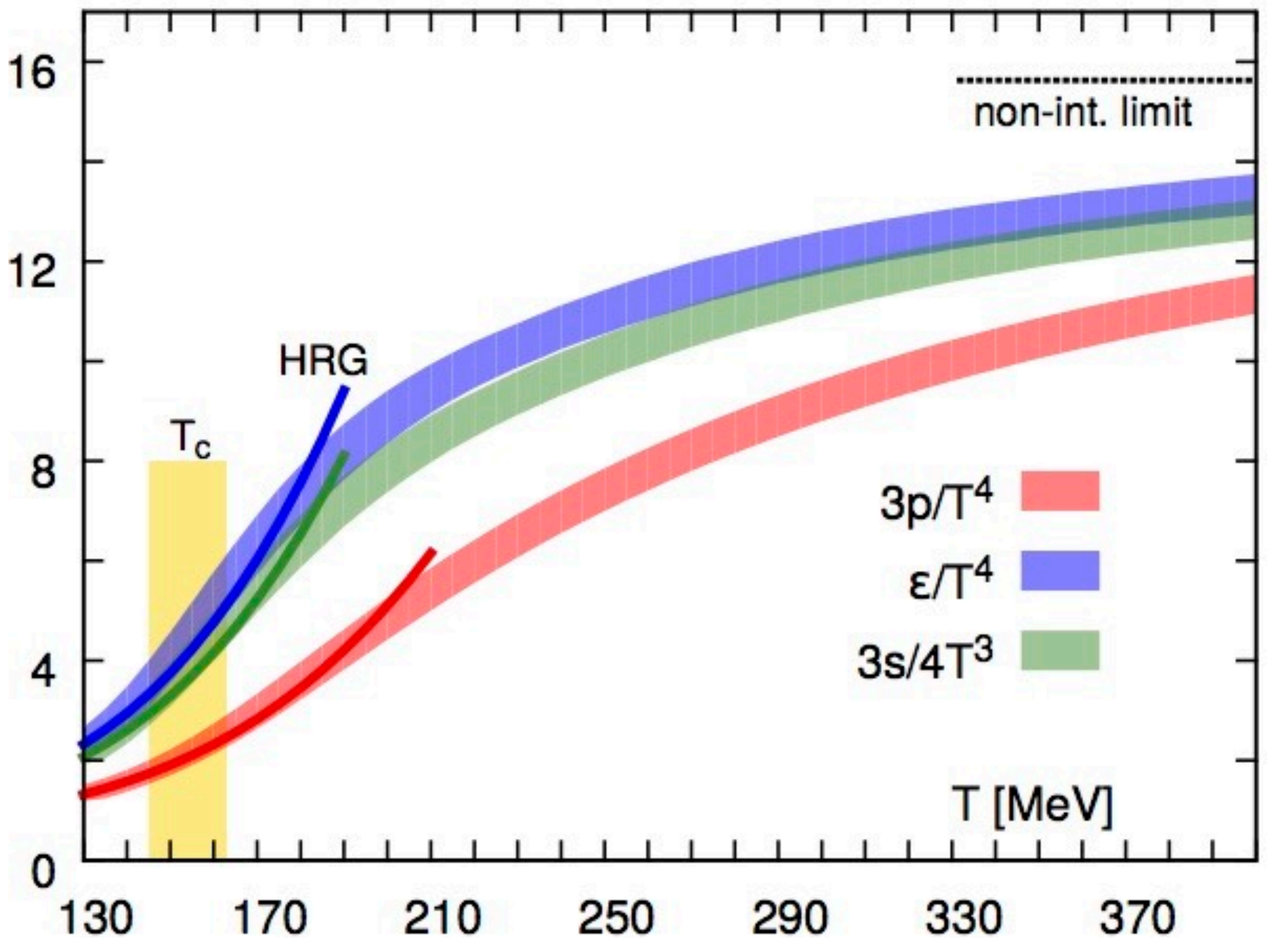
What can we calculate using lattice QCD?



The nature of the chiral phase transition.

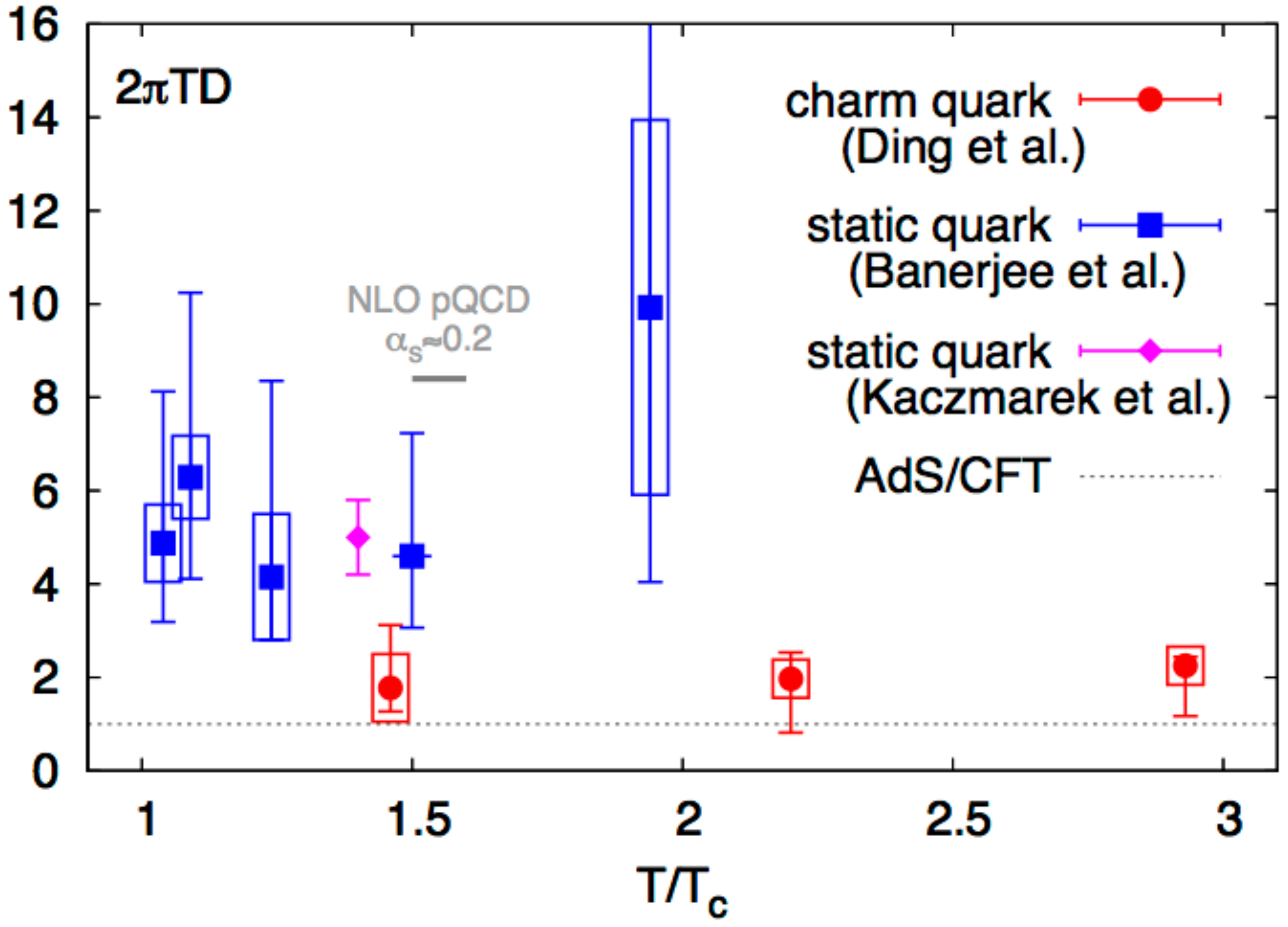
QGP at small chemical potentials.



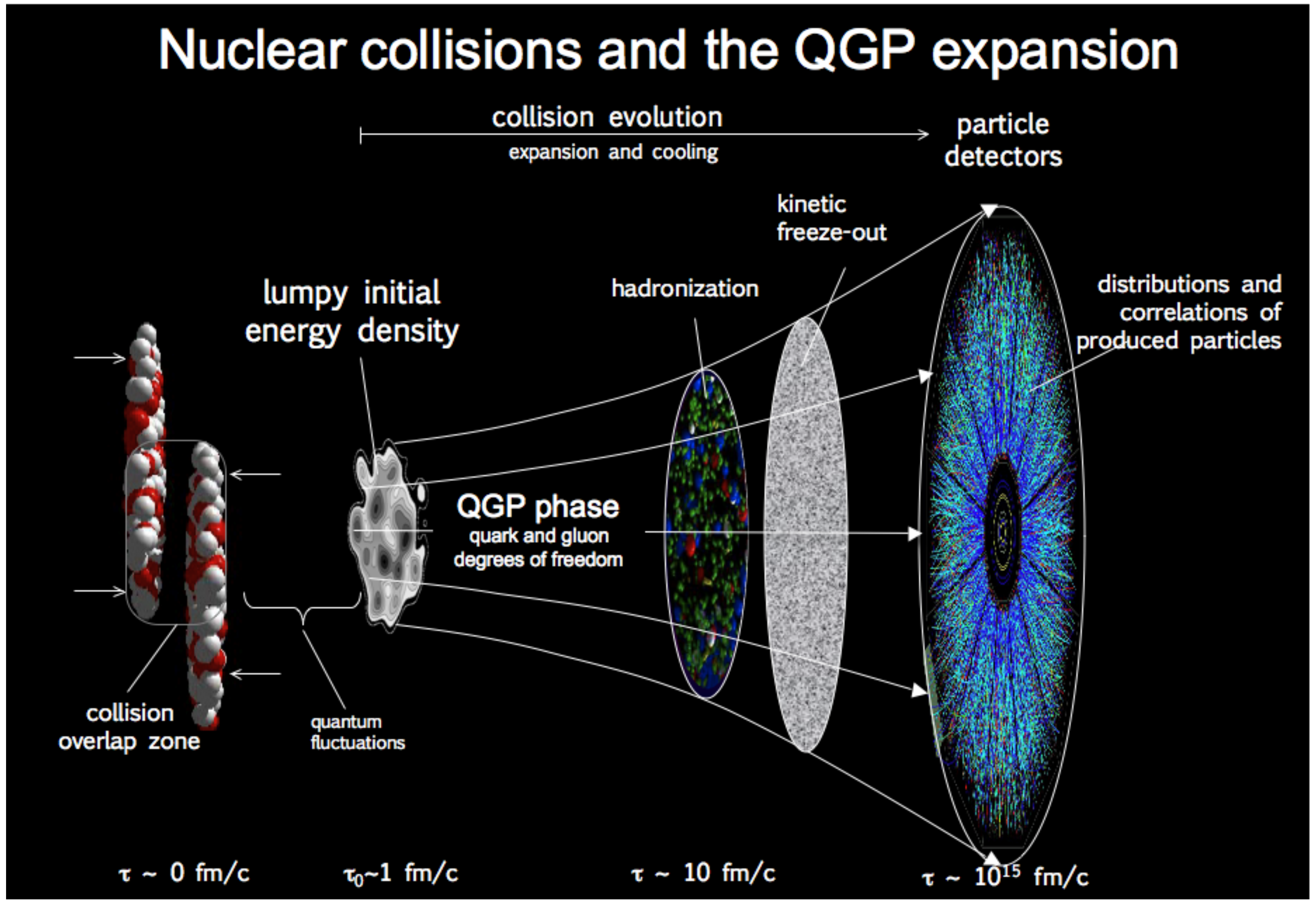


The QCD equation of state

Spectral properties (conductivity, bound states, etc.)



The QCD equation of state



Initial lumpy configuration quickly thermalizes into the quark-gluon plasma. Its subsequent expansion is modelled using relativistic hydrodynamics. The QCD-specific part is the QCD equation of state.

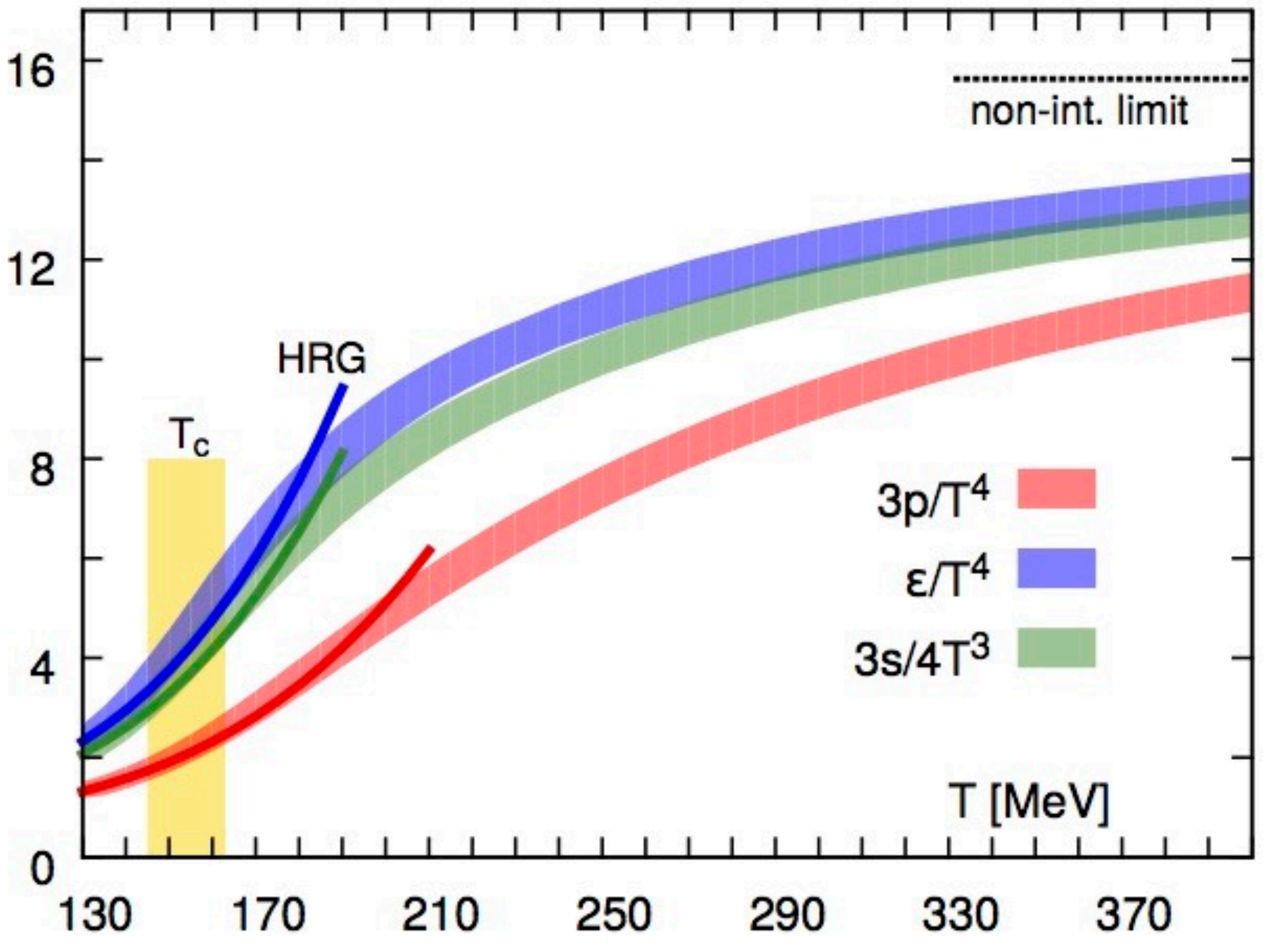
QGP formed in collisions and observed on the lattice. Strongly-coupled system in both cases. Perturbation theory is not directly applicable, unless some form of resummation is carried out to improve convergence [Haque *et al.* 2016].

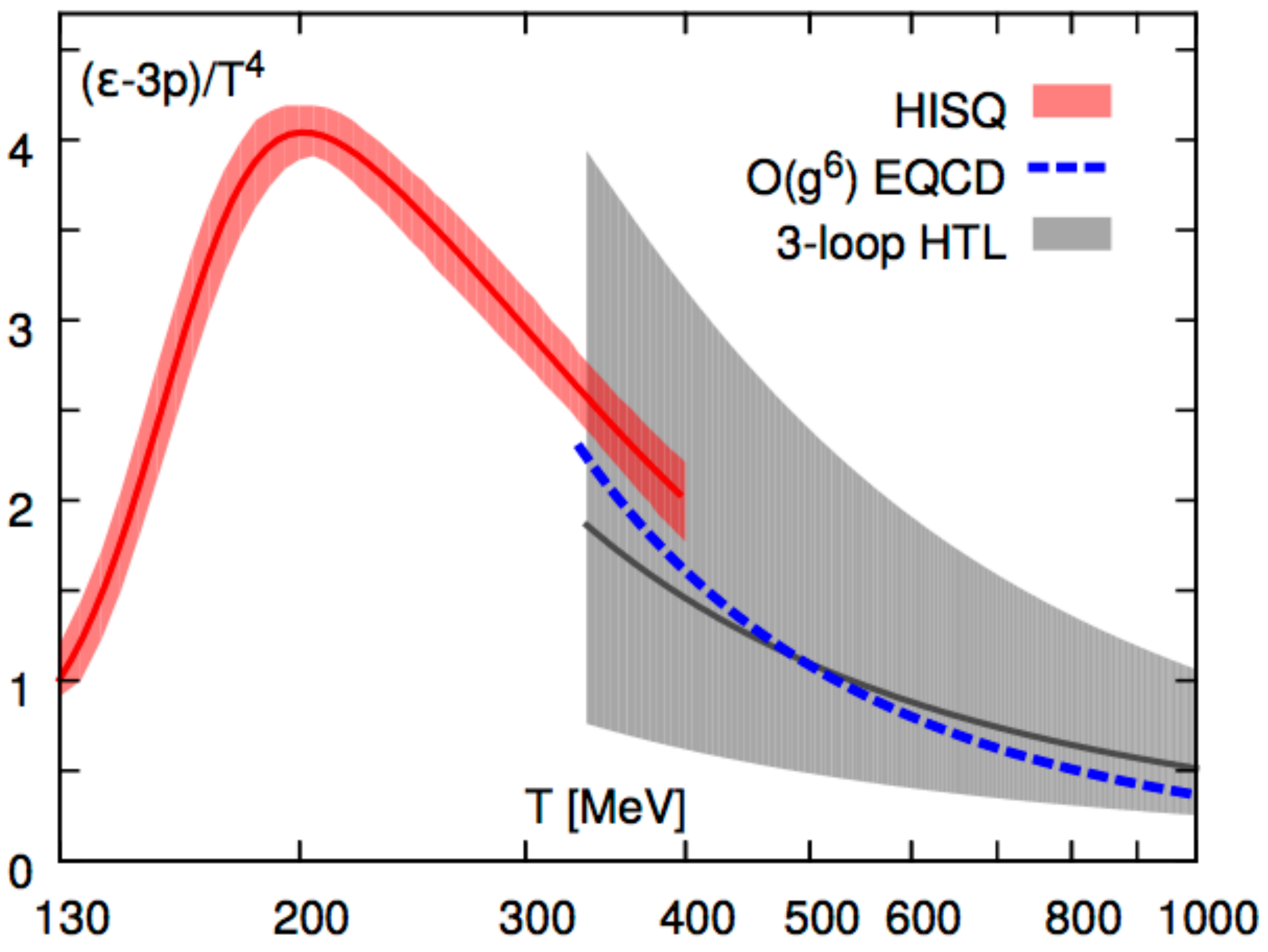
Moreover perturbation theory also suffers from a severe problem of infrared divergences [Linde 1983], which renders it impossible to proceed beyond sixth order.

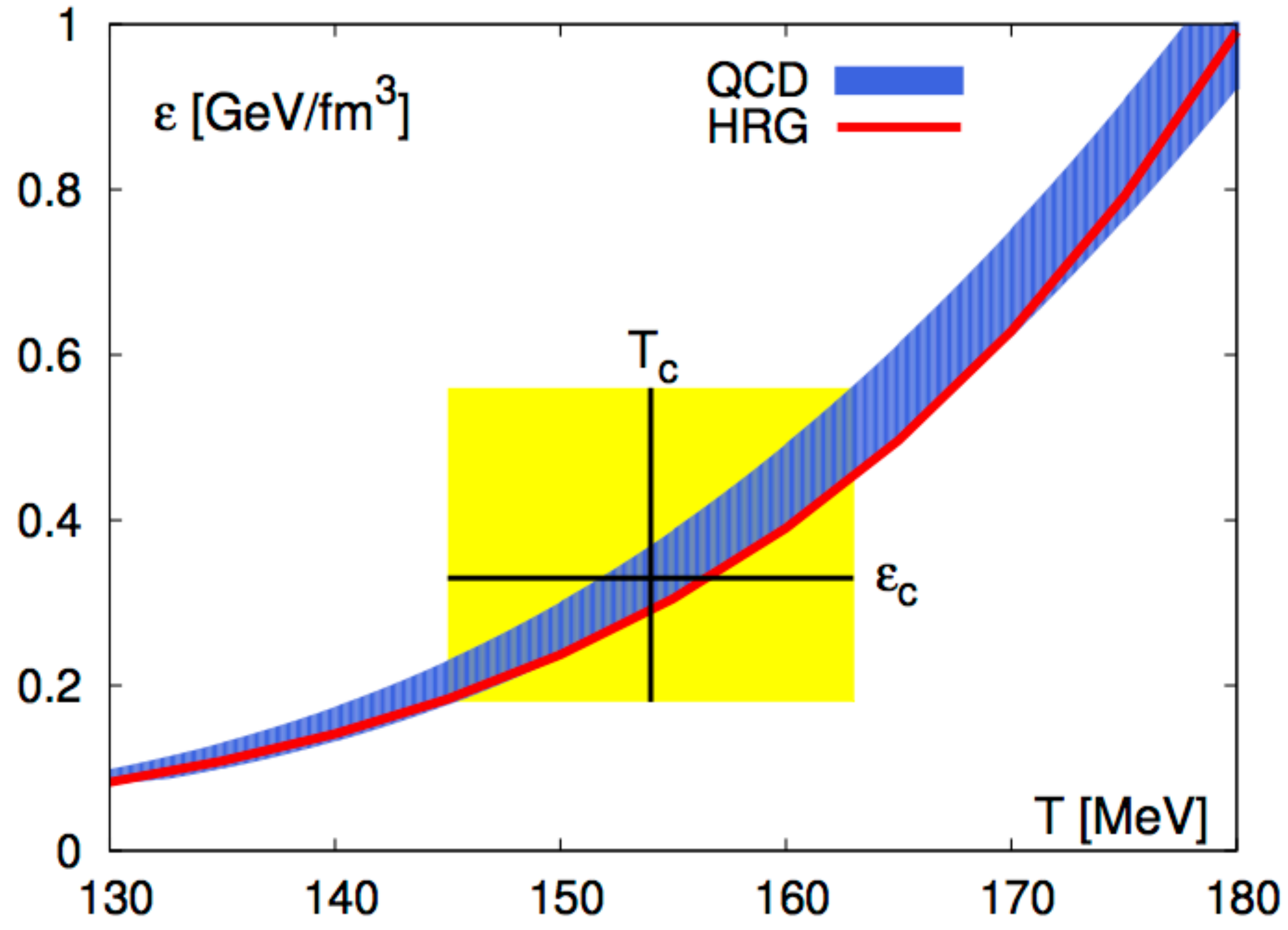
The lattice provides a parameter-free non-perturbative determination of the QCD equation of state directly from first principles QCD.

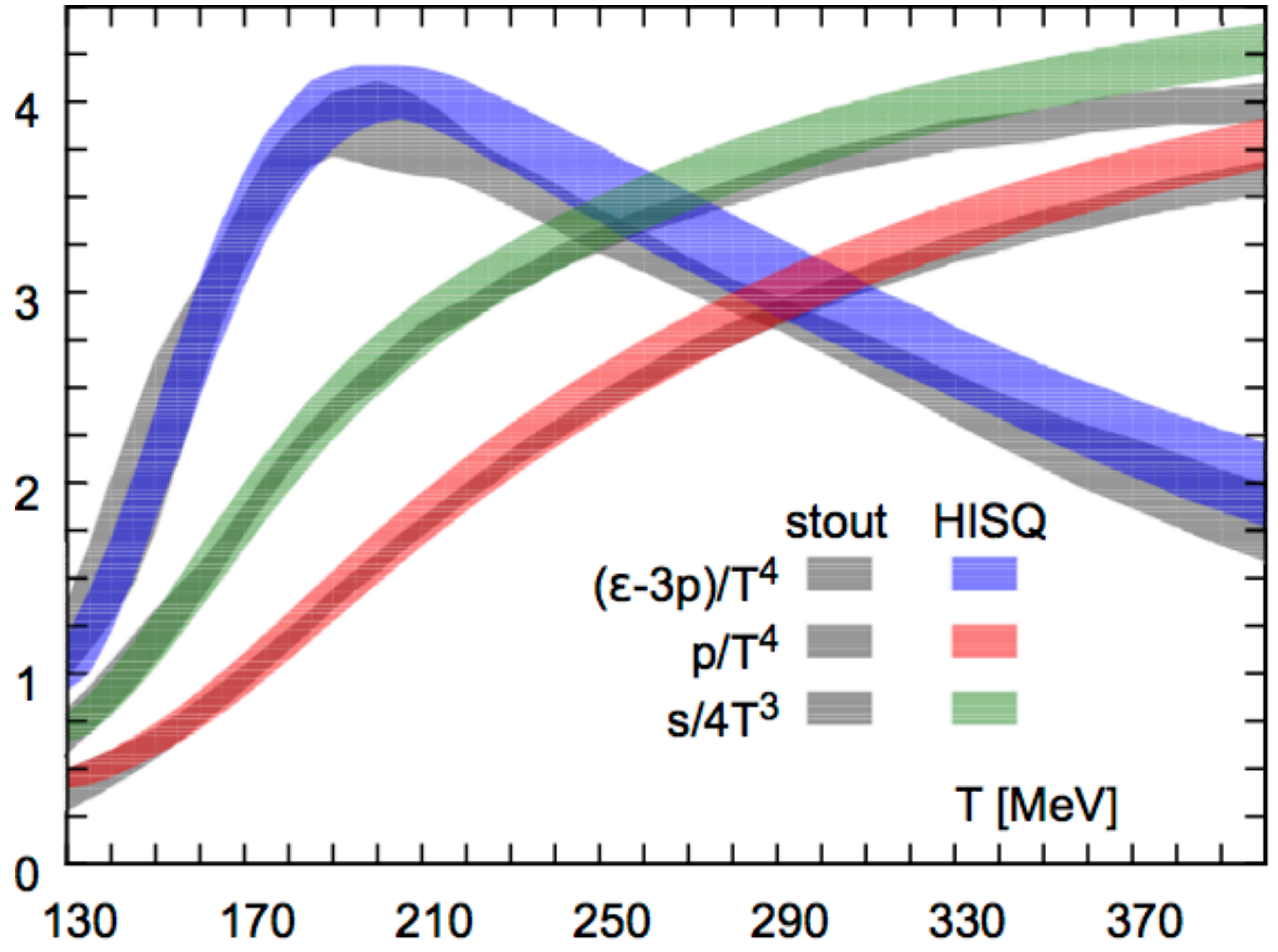
A state of the art EoS at zero chemical potential exists [Borsanyi *et al.* 2012, Bazavov *et al.* 2014 (HotQCD)]. Current research is about extending these results to finite density, as we will see.

The QCD equation of state at zero μ_B





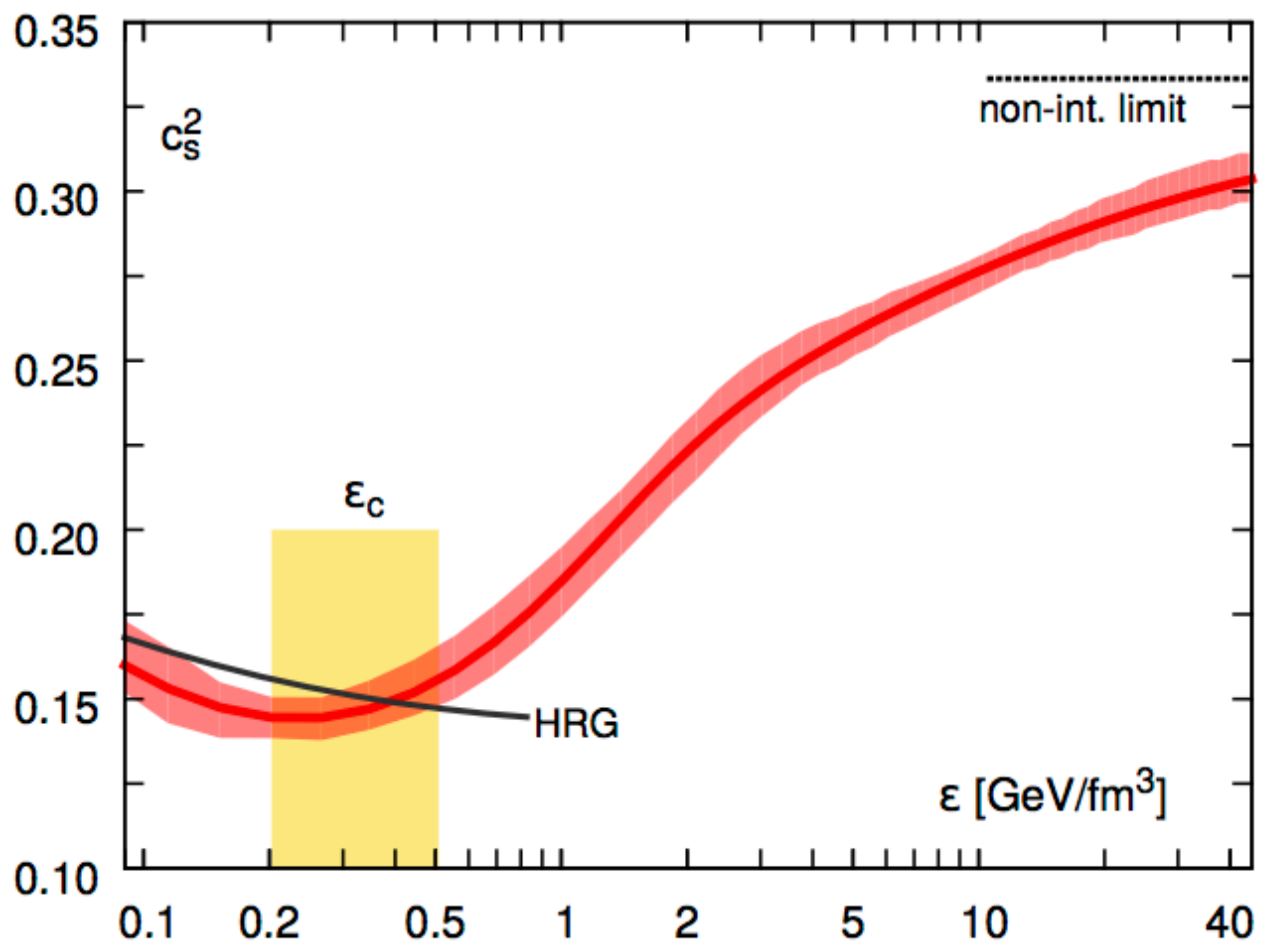




1. Good agreement between the results from HotQCD as well as Wuppertal-Budapest collaborations.

2. Broad agreement with Hard Loop Thermal (HTL) calculations above Tc.

3. Smooth transition as expected from a crossover.



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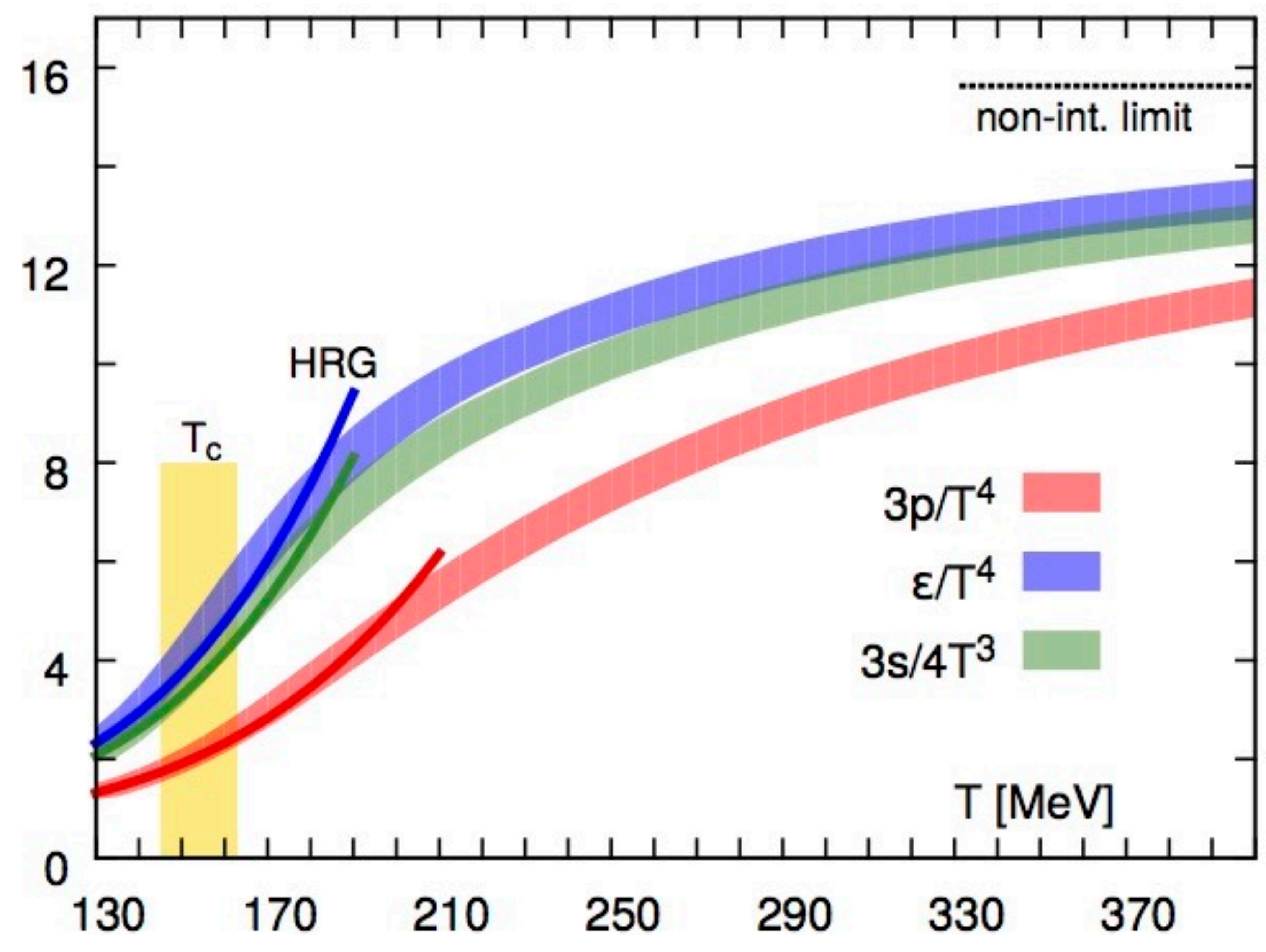
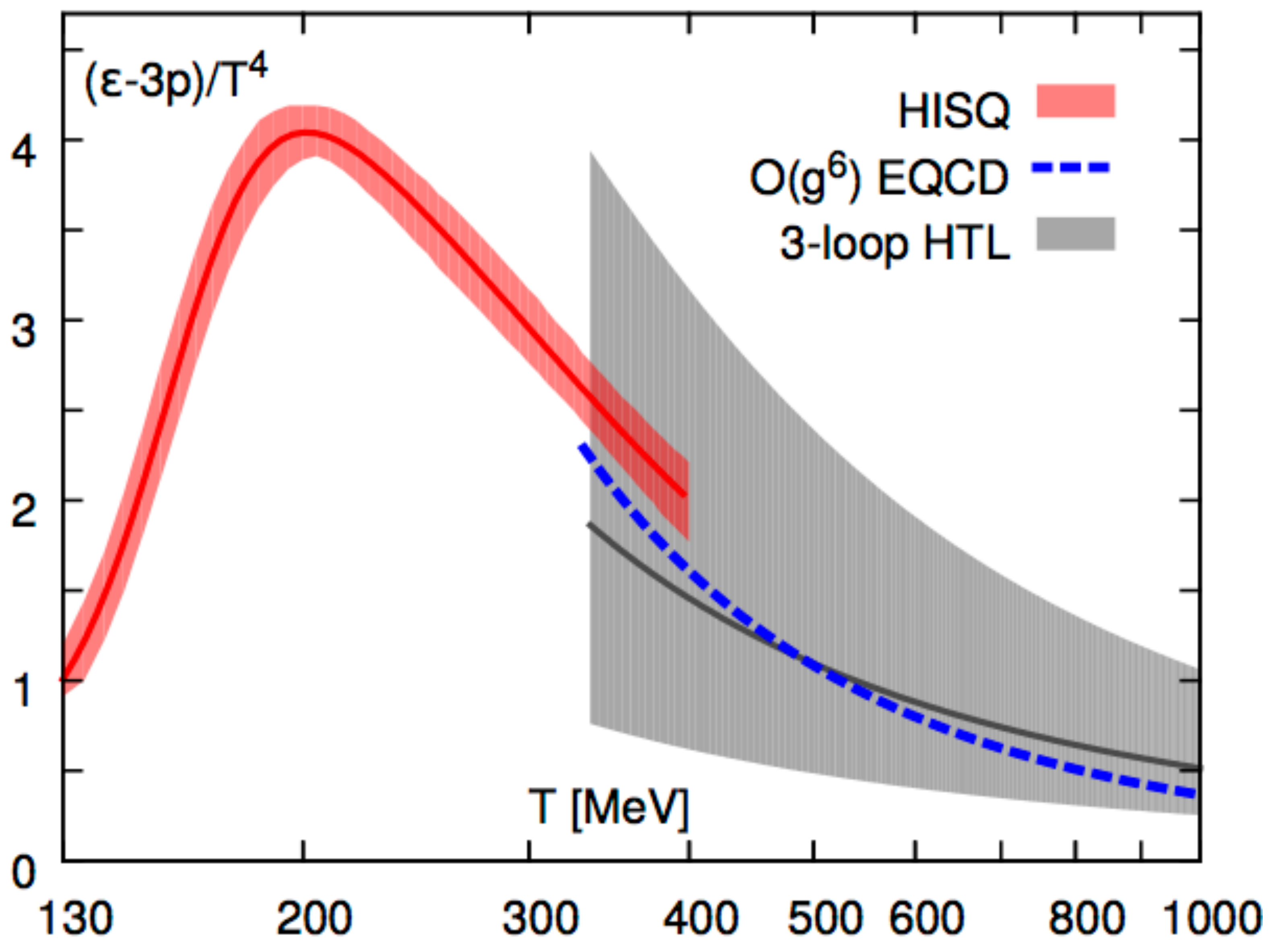
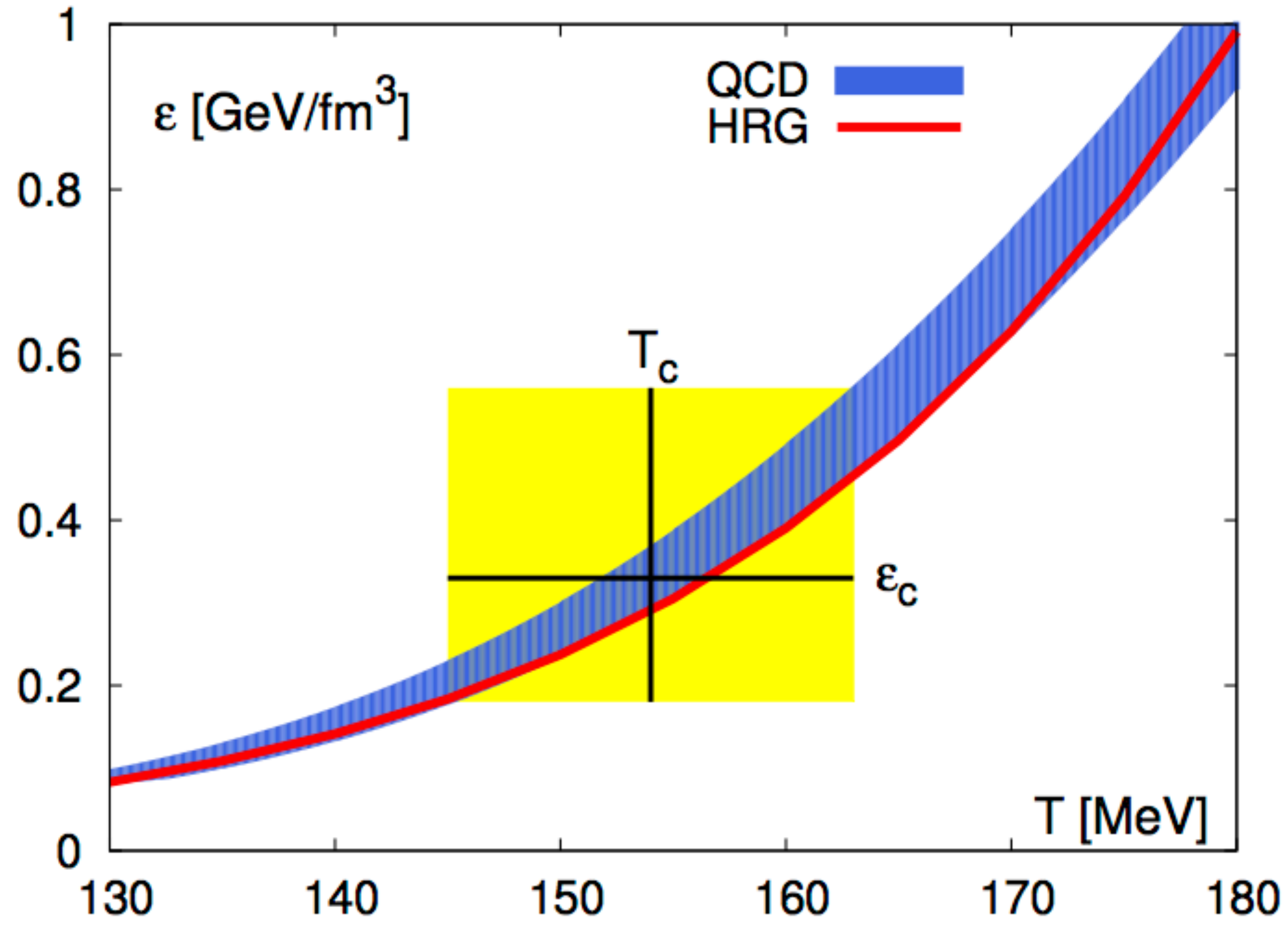
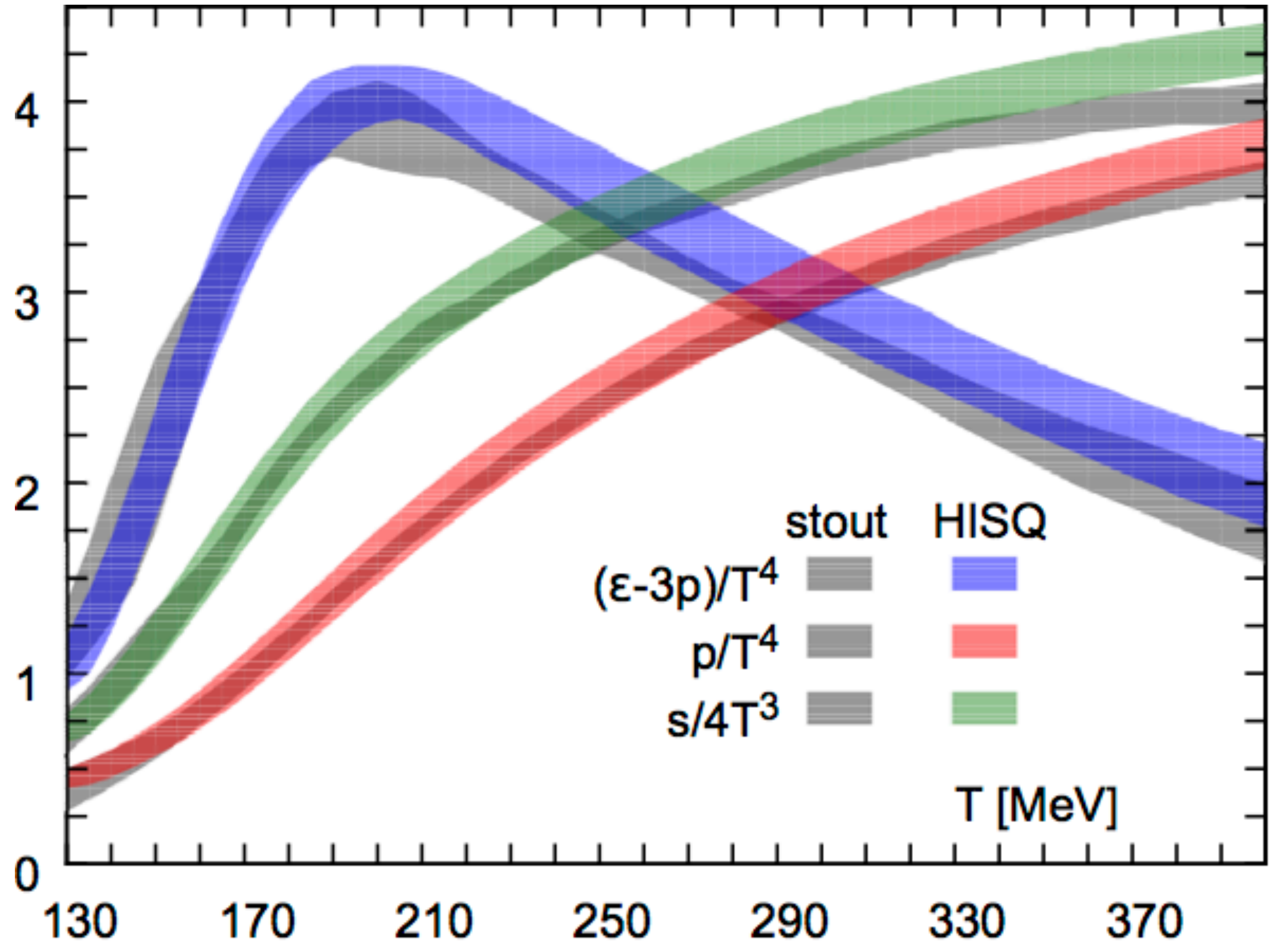


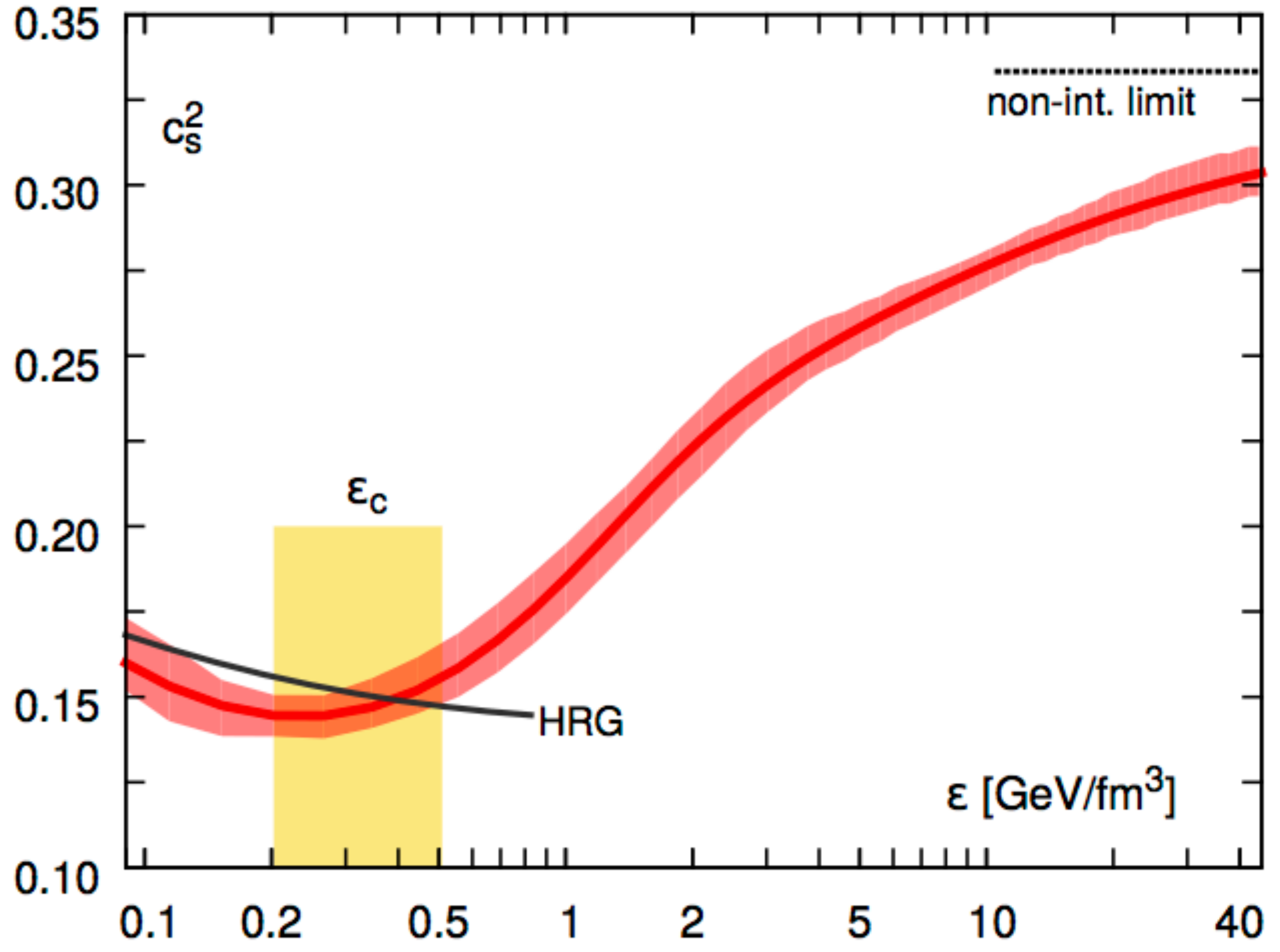




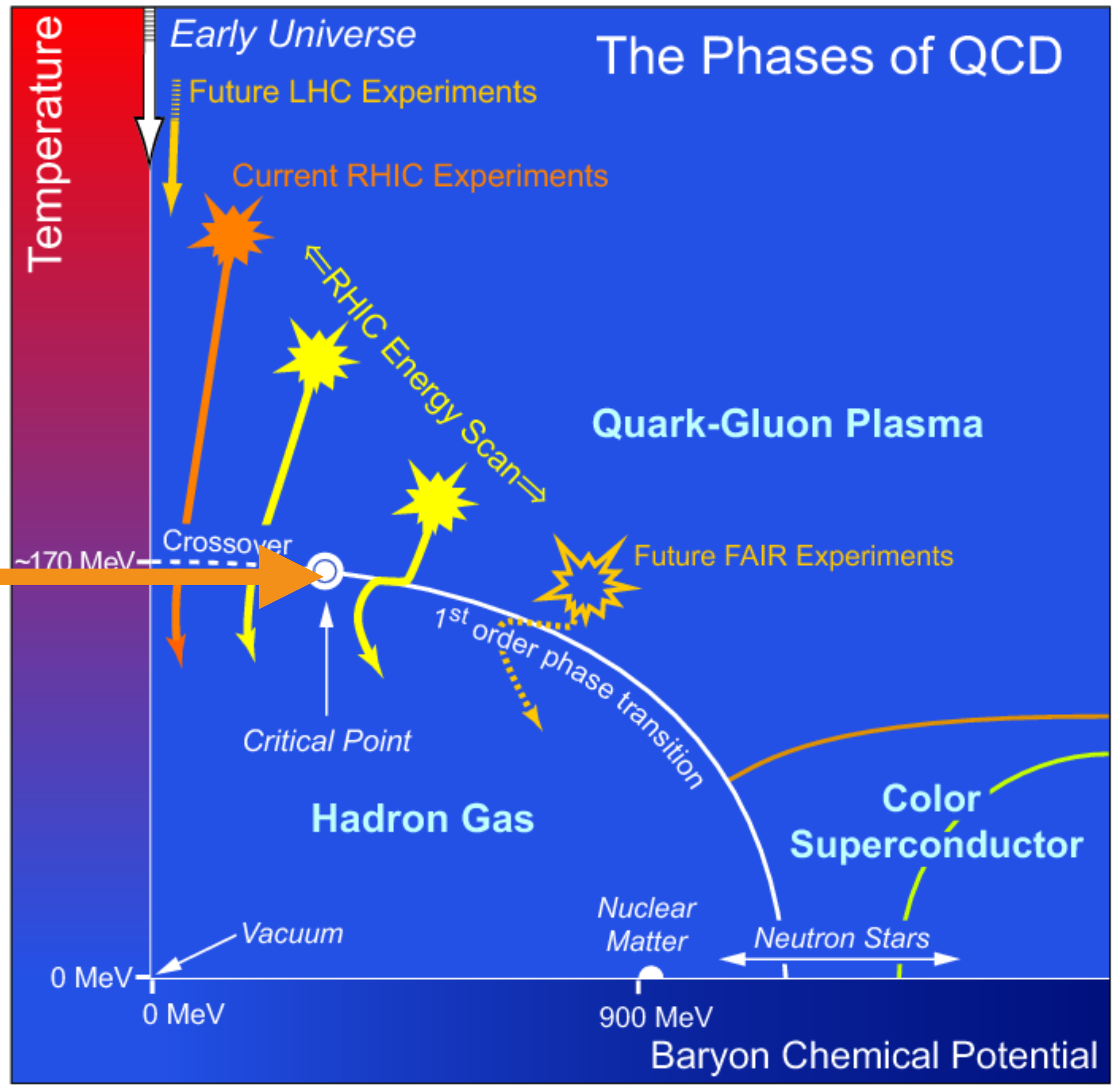


4. The energy density at $T=T_c$ is of the same order as normal nuclear density.

5. The speed of sound reaches its minimum ("the softest point") around $T=T_c$. Speed of sound not really expected to go to zero since the O(4) specific heat exponent α is negative.

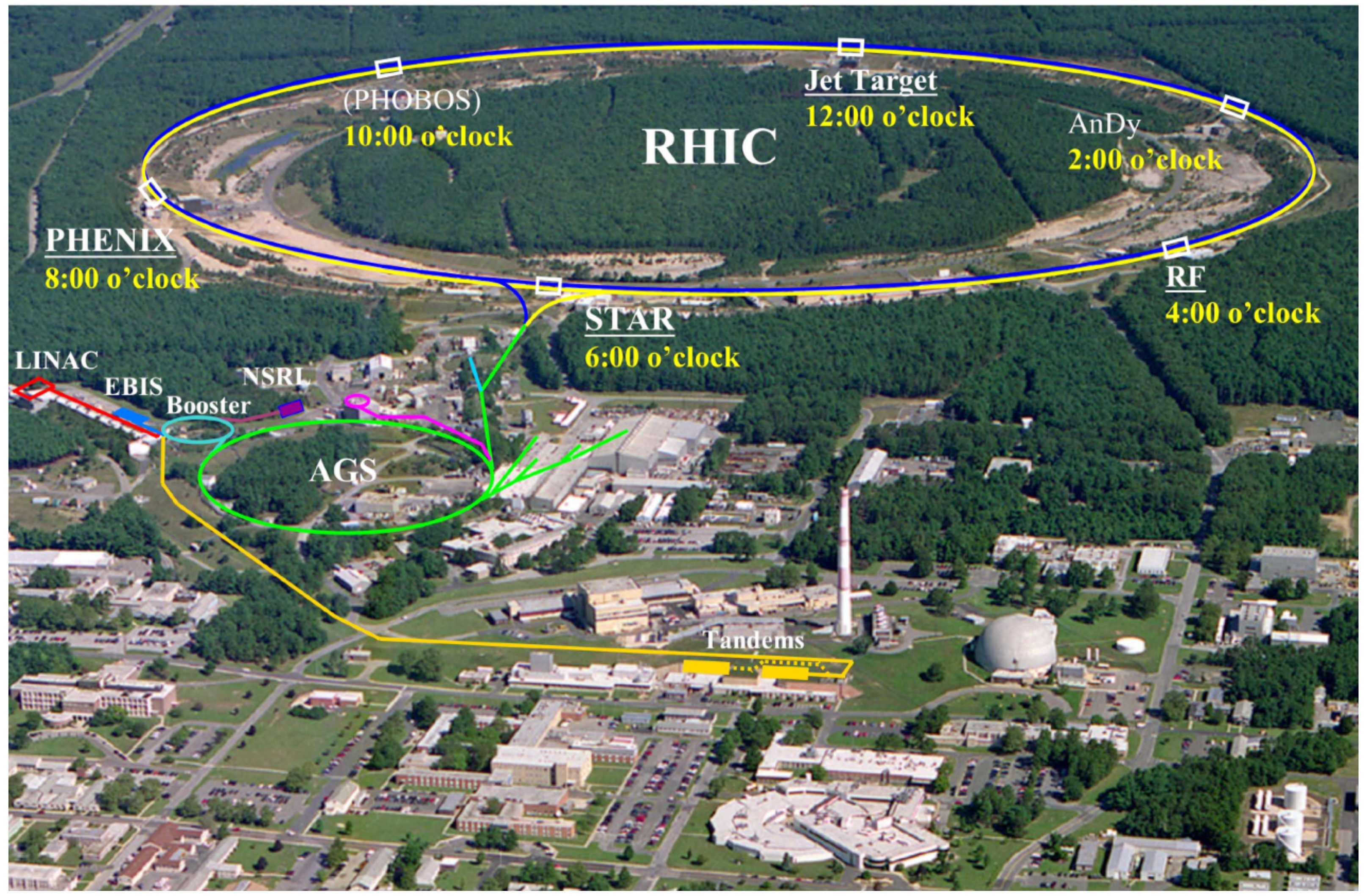


The Beam Energy Scan Program at RHIC



It has been conjectured that the QCD crossover transition turns first-order at large baryon chemical potentials ($\mu <= \mu_B$). At $\mu = \mu_B$, the first-order line is capped by a critical point.

The Beam Energy Scan (BES) program at RHIC commenced in 2010. Its goal is to look for the QCD critical point by creating the QGP at larger densities.



The collision energy is varied down from its top energy of 200A-GeV down to ~5.5A-GeV. This is because the quark-gluon plasma created at lower energies is more dense (higher μ_B).

It has been estimated [Cleymans et al. 1999, Andronic et al. 2006] that (T^f, μ_B^f) ~ (150 MeV, 450-500 MeV) at freeze-out at the lowest energies. Thus we will need an equation of state valid up to μ_B/T ~ 3-3.5.

Extending lattice observables to finite density

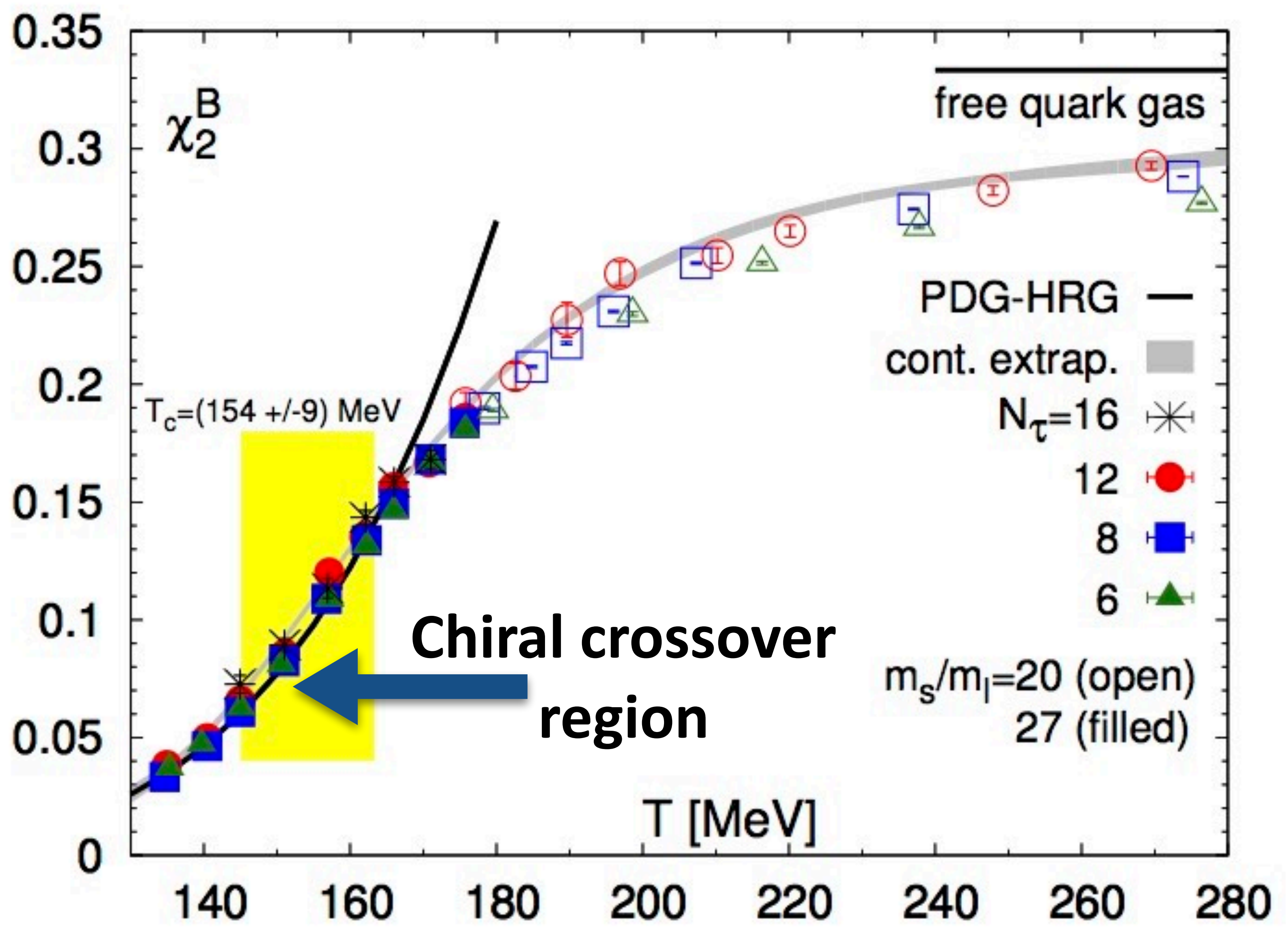
- Lattice QCD suffers from the infamous sign problem at $\mu_B > 0$. This prevents a direct simulation at finite density.

- One way to work around the problem is by Taylor-expanding the partition function around $\mu_B = 0$ [Allton et al. 2002; Gavai-Gupta 2002]. The Taylor coefficients, known as quark number susceptibilities (QNS), are defined at $\mu_B = 0$ and can be calculated using the techniques of lattice QCD.

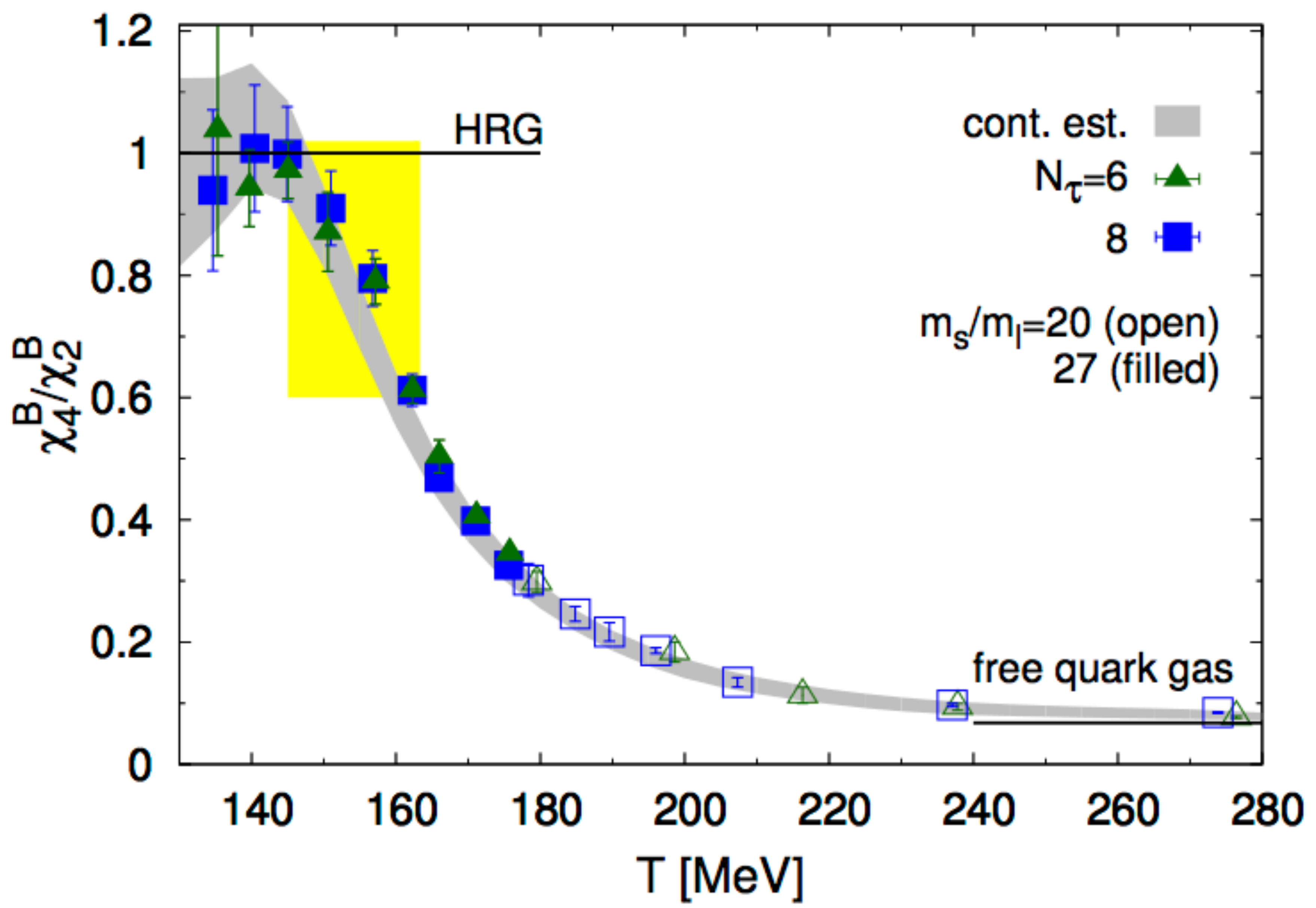
- The sign problem however now manifests itself as a "noise problem": The signal-to-noise ratio quickly degrades as one tries to calculate higher orders [P. de Forcrand, Lattice 2008].

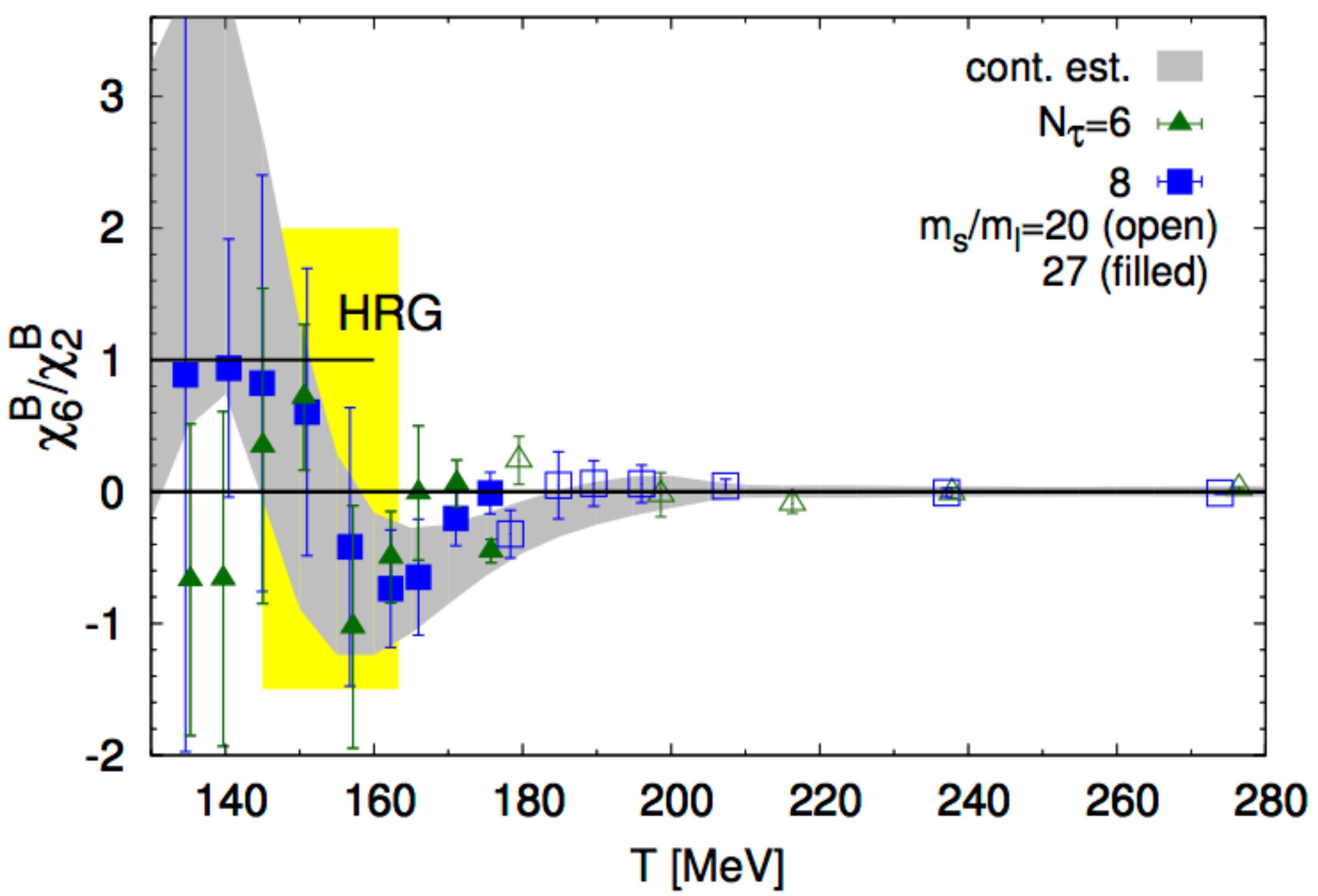
- Apart from this, QNS are interesting in their own right. They can be compared to the observed fluctuations of conserved charges [V. Koch & S. Jeon, 2000], used to determine the temperature and chemical potential at freeze-out [A. Bazavov *et al.* (BNL-Bielefeld) 2012] and most importantly, used to estimate the location of the QCD critical point [Gavai-Gupta 2003].

Extending lattice observables to finite density



Chiral crossover region
Tc = 154 +/- 9 MeV
[M. Cheng *et al.* (HotQCD) 2012]

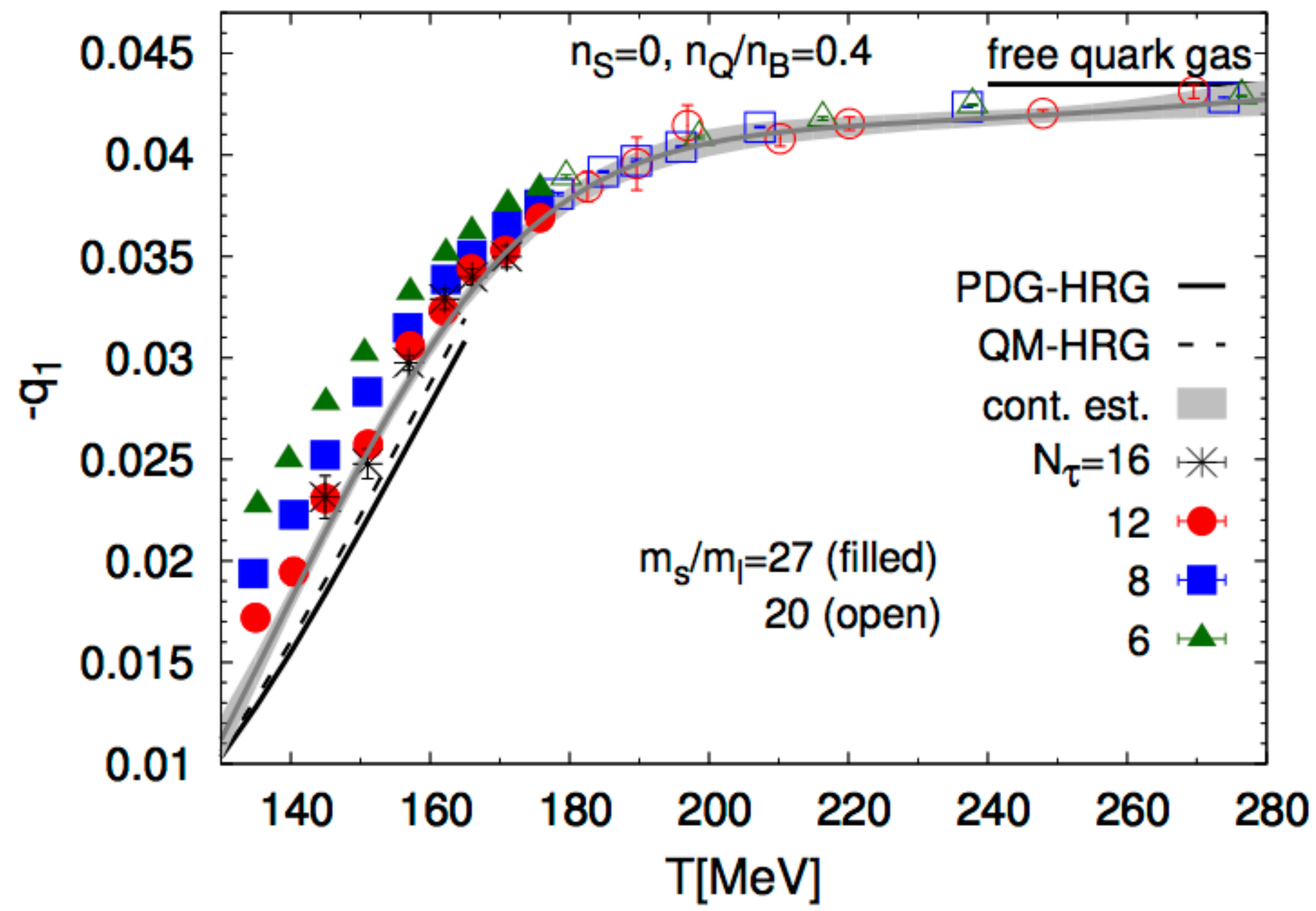


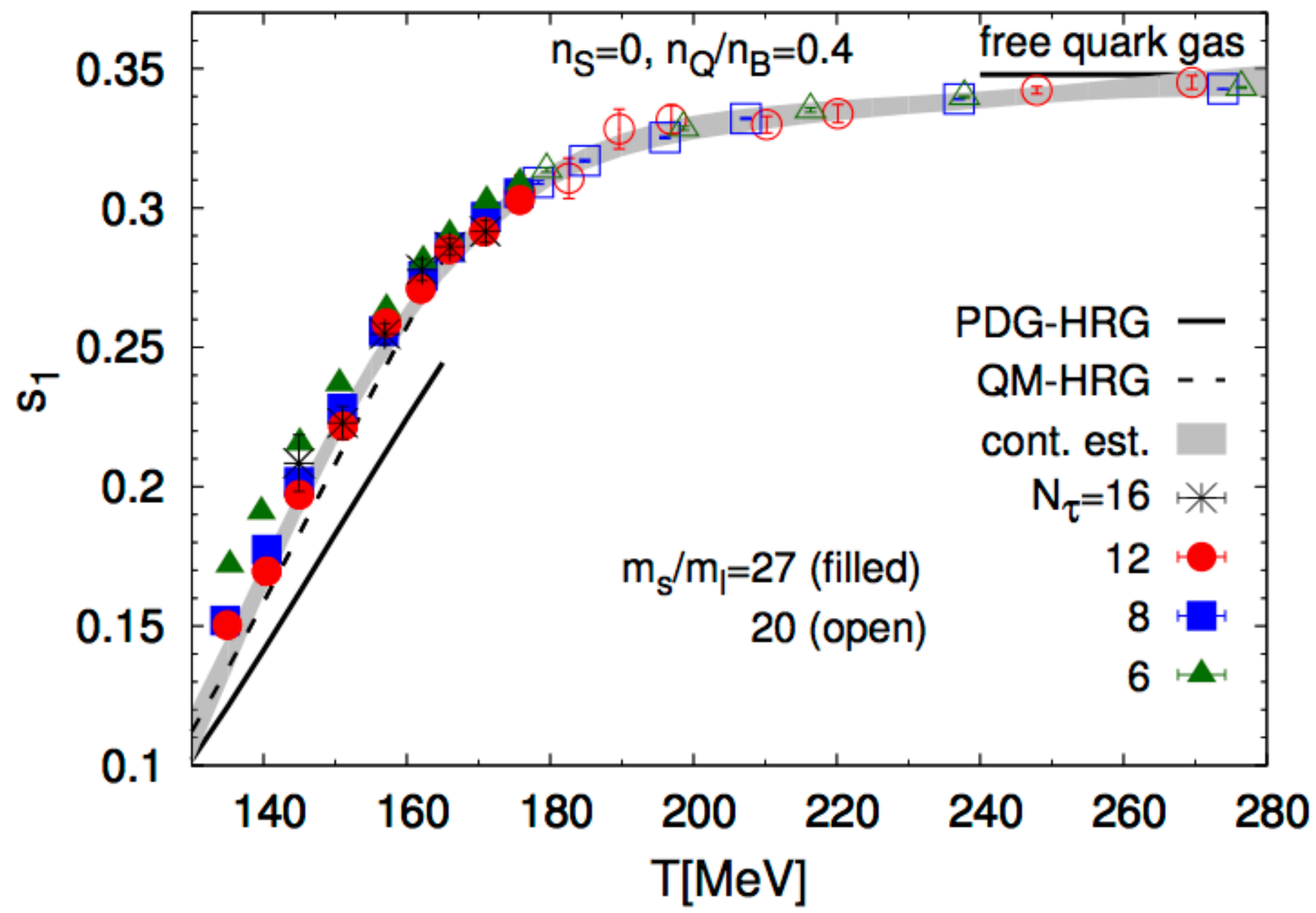


Our results are based on approximately 50-100,000 trajectories, generated using the HISQ action at two quark masses $m_l = m_s/20$ and $m_s/27$ and at various lattice spacing N_τ = 6, 8, 12 and 16 for several temperatures in the range 135 MeV - 280 MeV. The use of multiple lattice spacings allowed us to make a continuum extrapolation.

We use the exponential formalism [Hasenfratz & Karsch 1984] to calculate up to the fourth derivatives of the quark matrix. From sixth order onwards, we use the linear-μ formalism [Gavai & Sharma 2010].

Strangeness neutrality and initial conditions in heavy-ion collisions

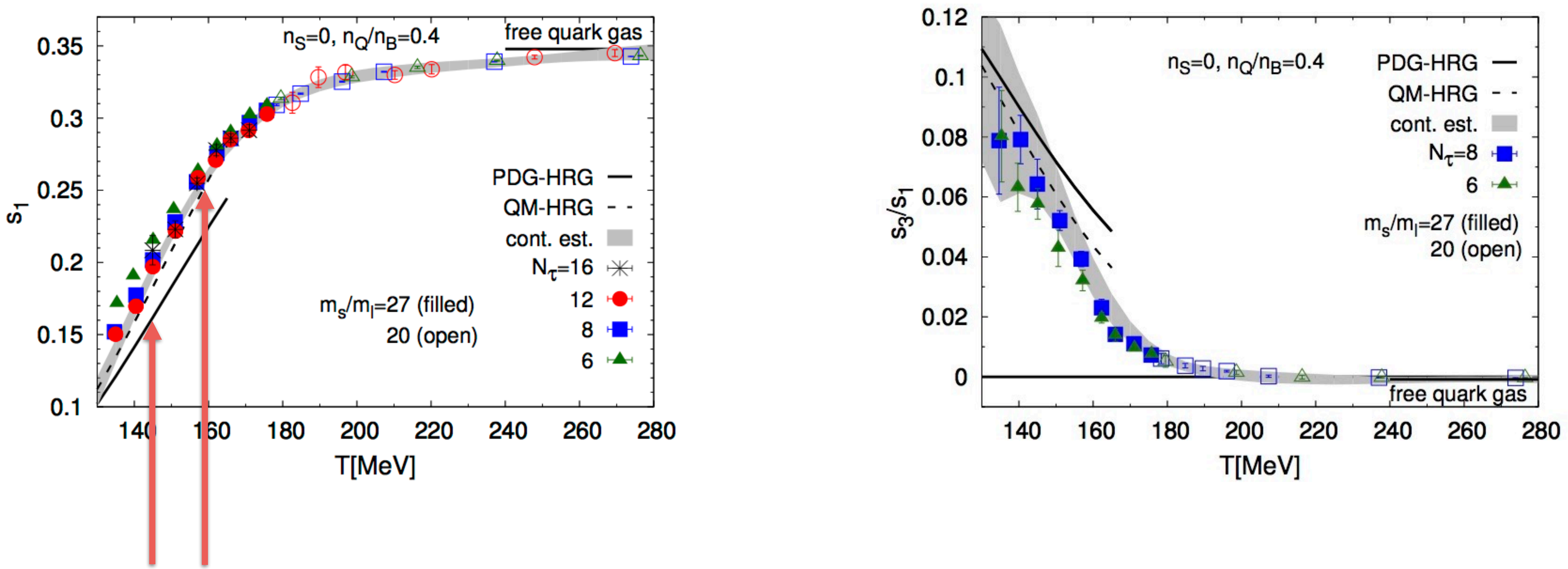




The initial conditions in a heavy-ion collision are i) $n_S = 0$ (net strangeness zero), and ii) n_Q/n_B = const. (fixed proton-to-neutron ratio).

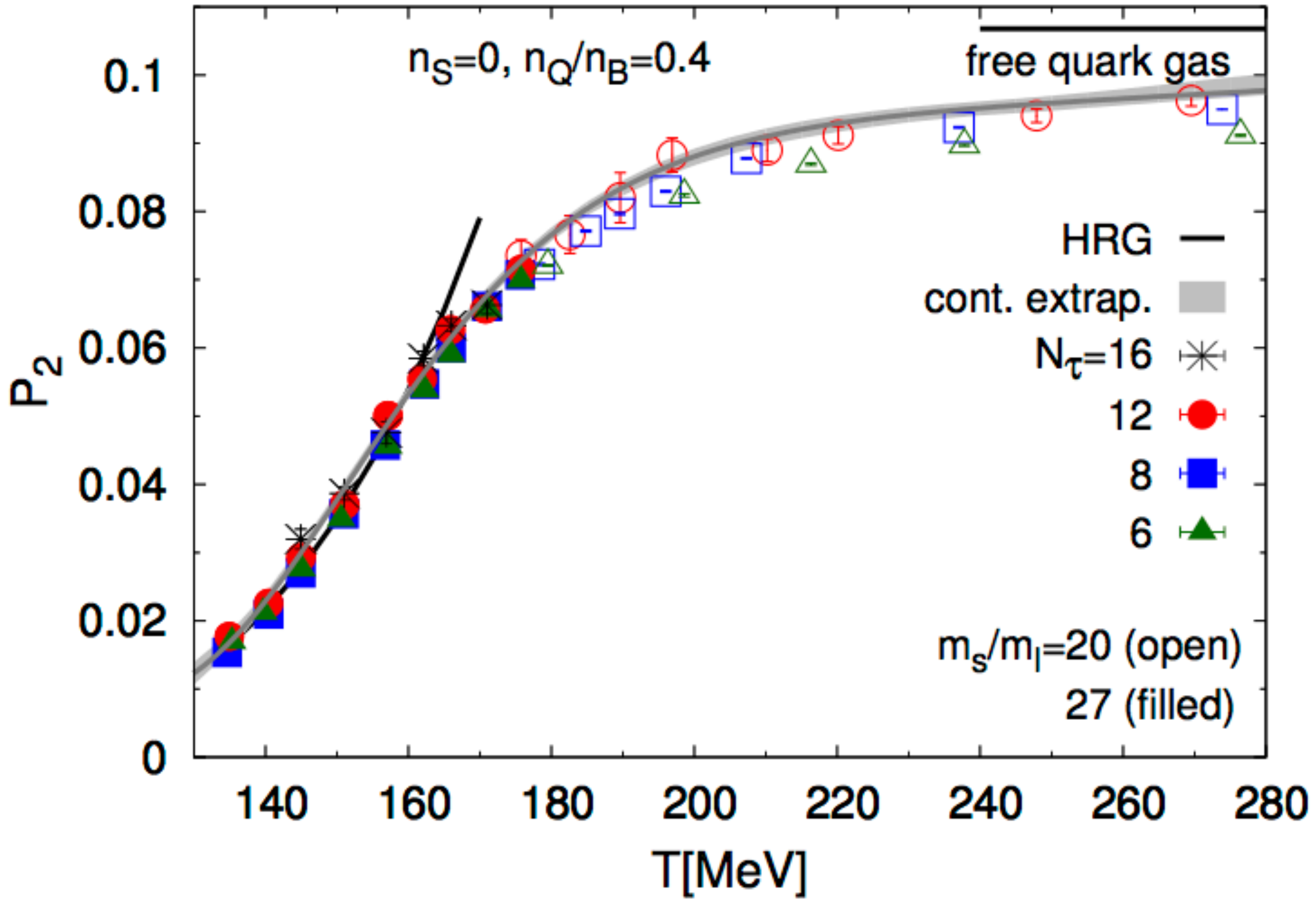
These conditions imply that μ_Q and μ_S are nonzero whenever μ_B is. Using the QNS, they can be determined order-by-order in μ_B.

Unseen resonances in the strange sector?

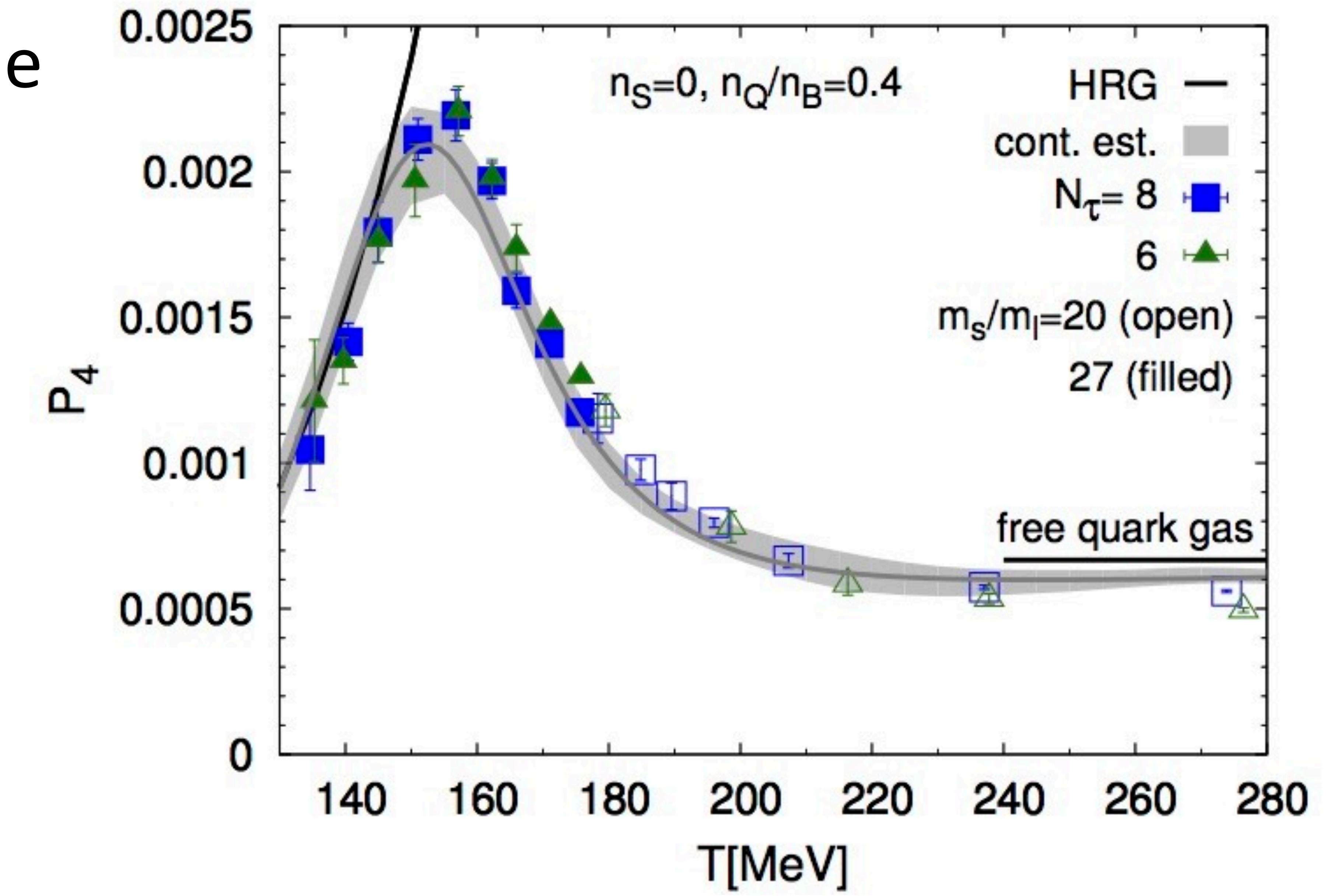


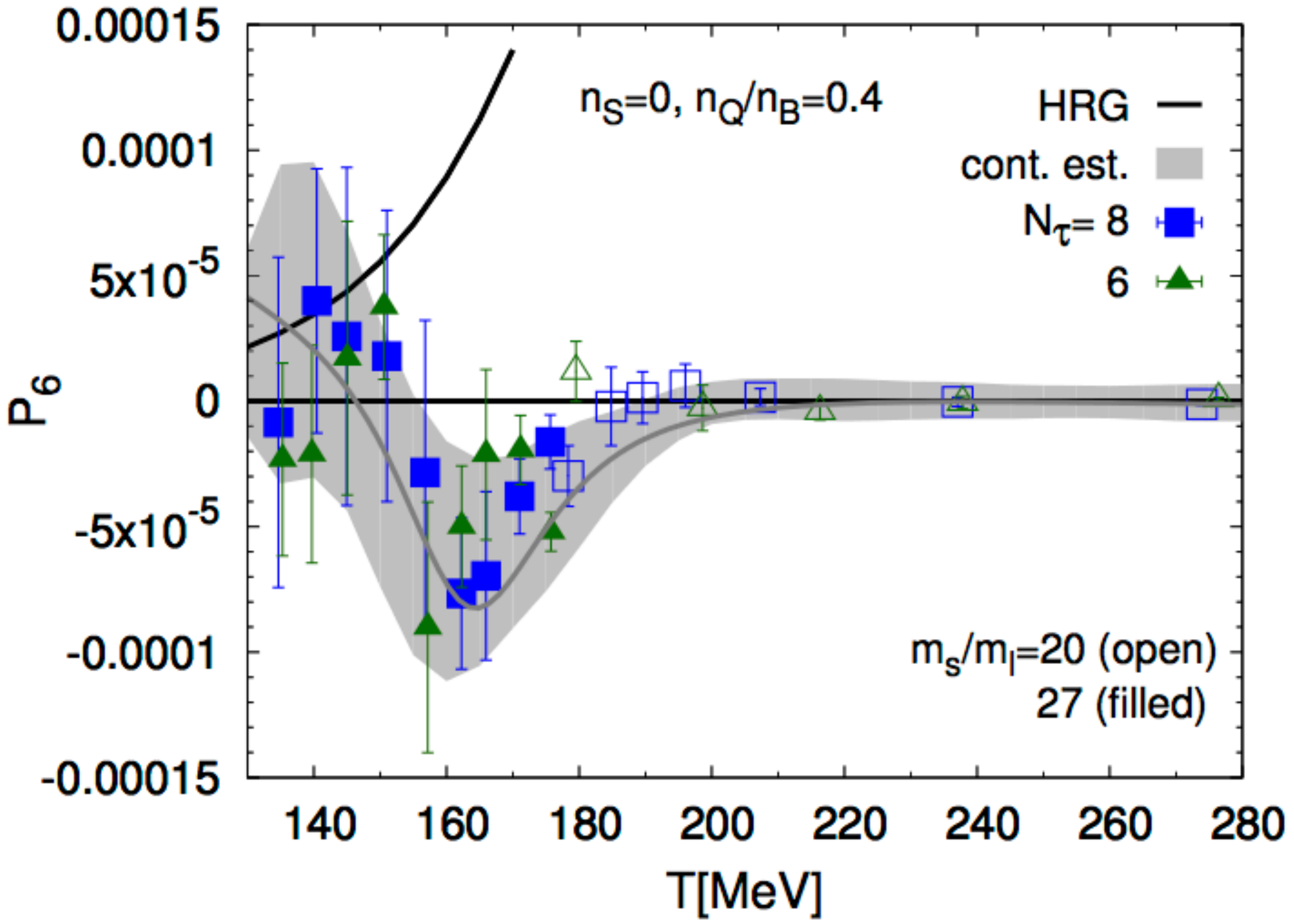
There is a discrepancy between the continuum-extrapolated results for s_1 and the predictions of PDG-HRG. Consequence of missing states? [BNL-Bielefeld-CCNU 2014; S.Borsanyi *et al.* 2017].

Corrections to the pressure: Strangeness-neutral case



Similar to corrections for the case $\mu_Q = \mu_S = 0$ but about 20% smaller in magnitude.

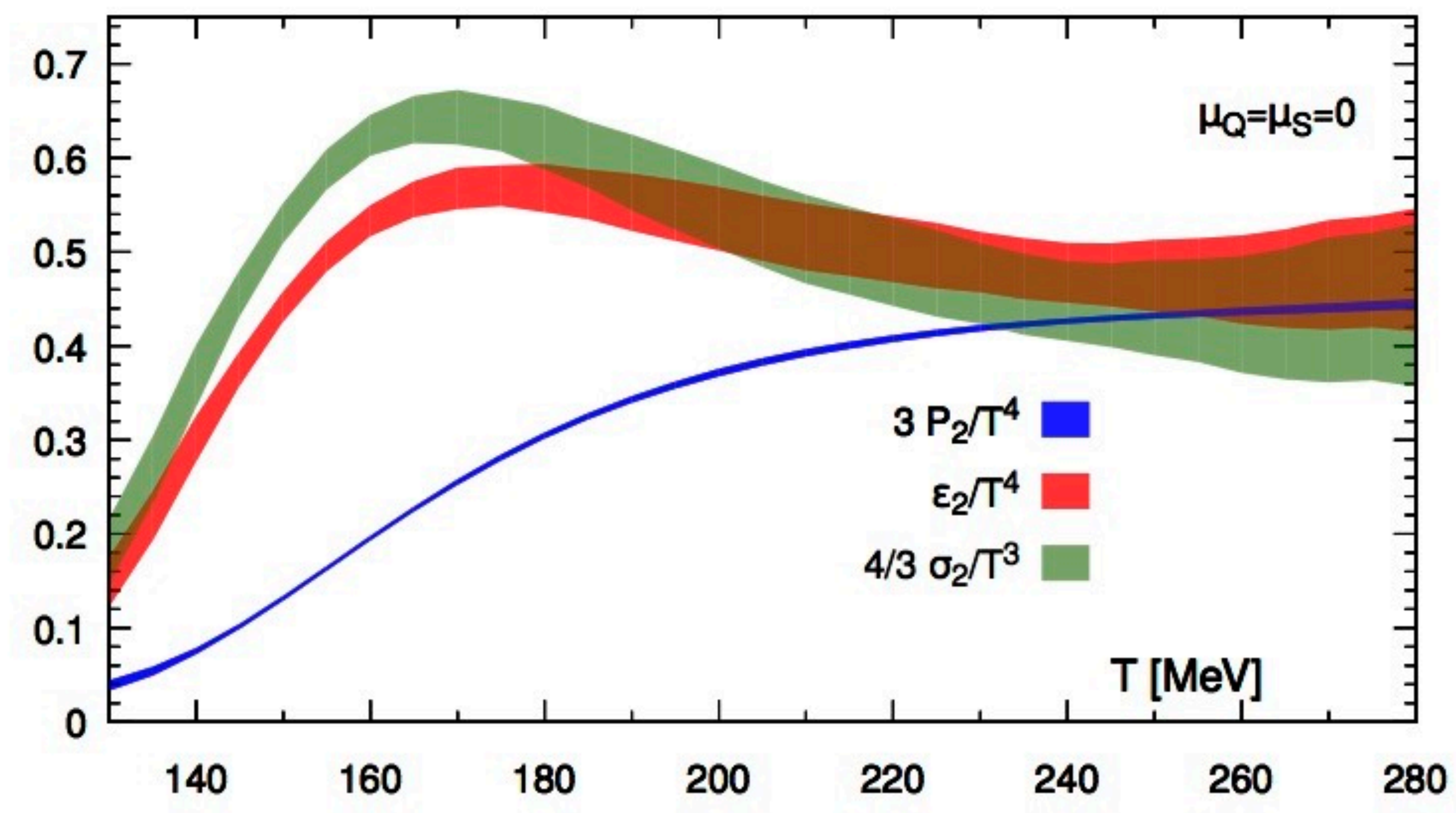


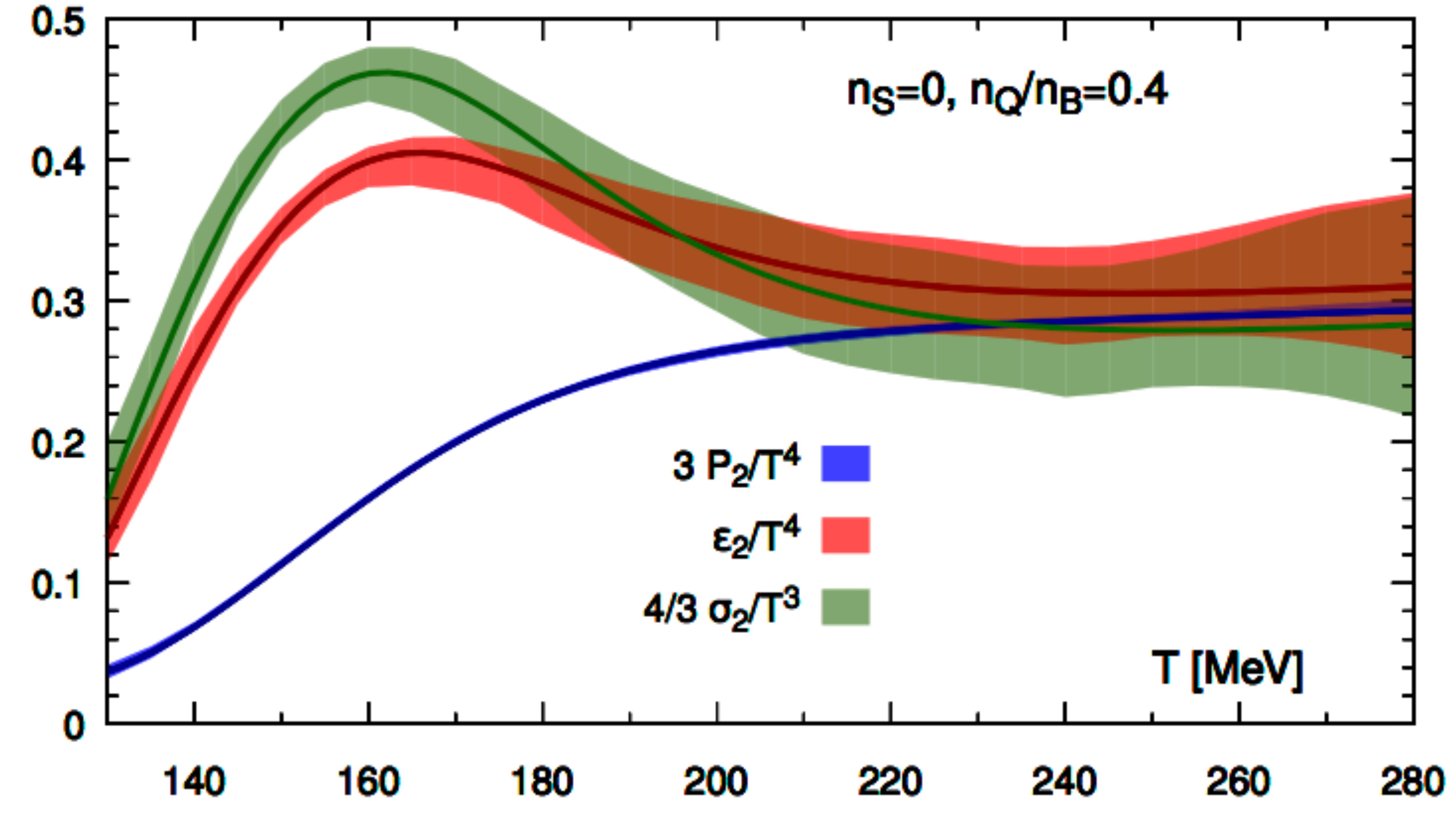


Shapes can be qualitatively understood as due to the 2nd-order O(4) transition in the chiral limit.

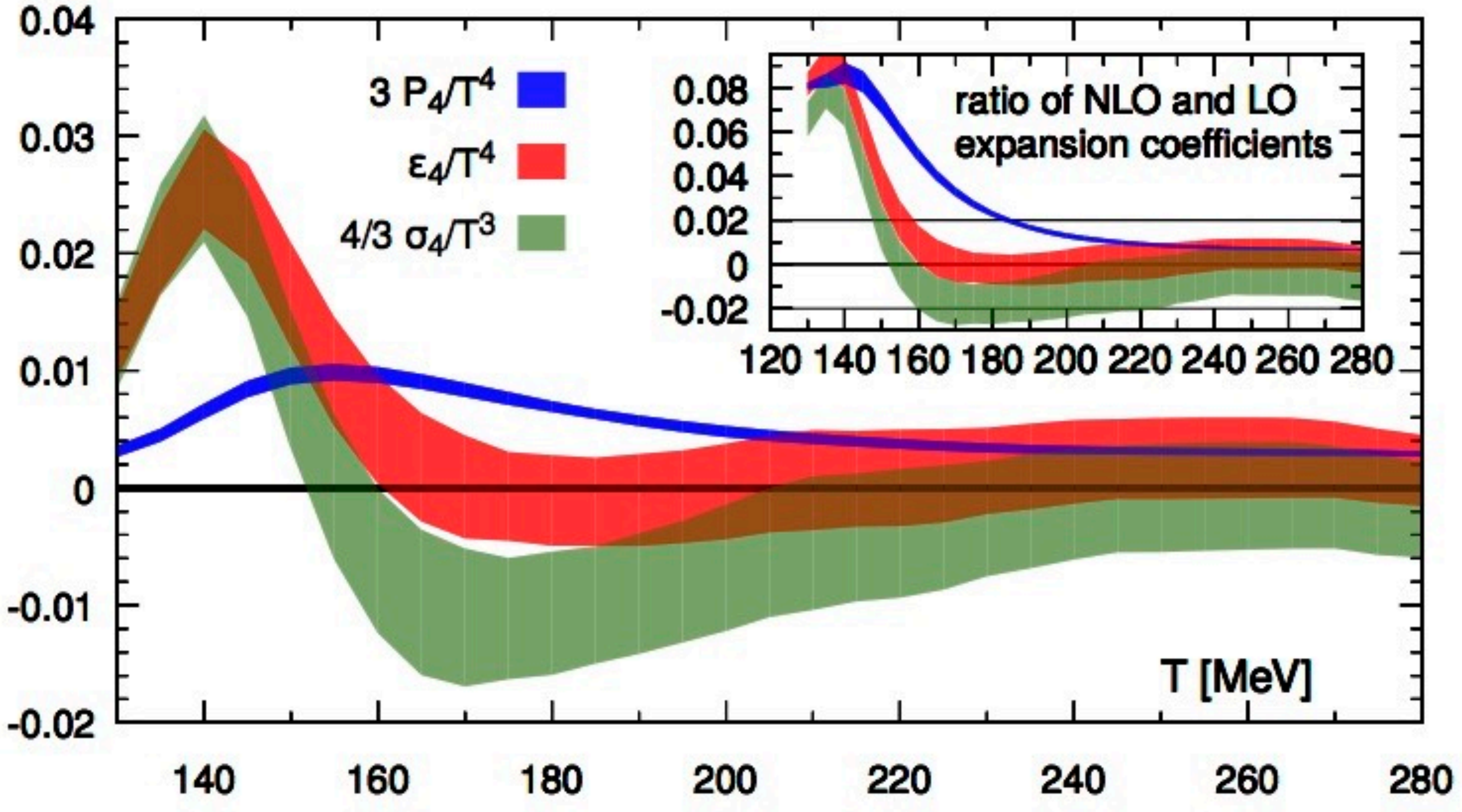
BNL-Bielefeld-CCNU, Phys. Rev. **D86**, 054504 (2017)

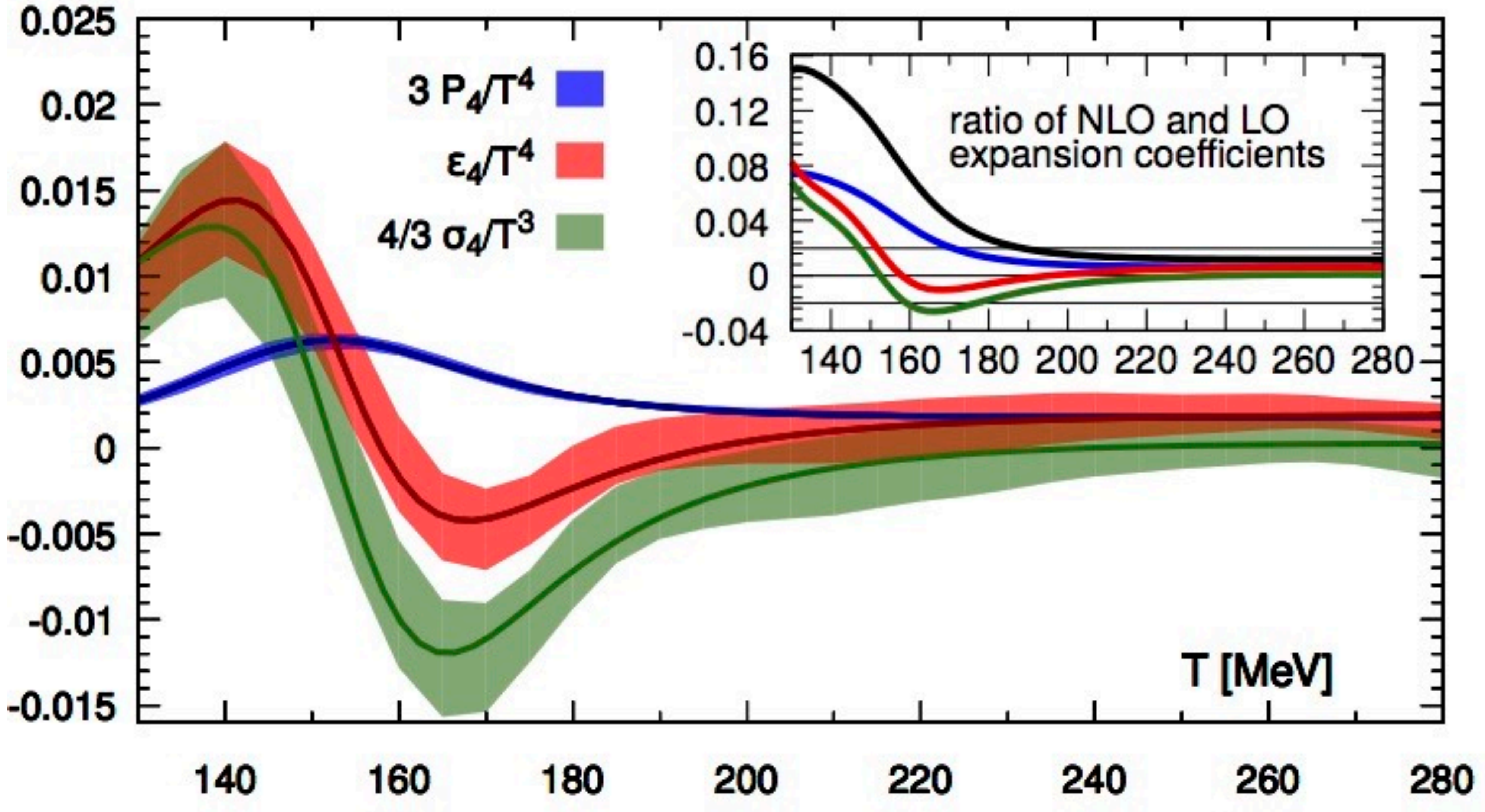
Pressure, energy and entropy corrections



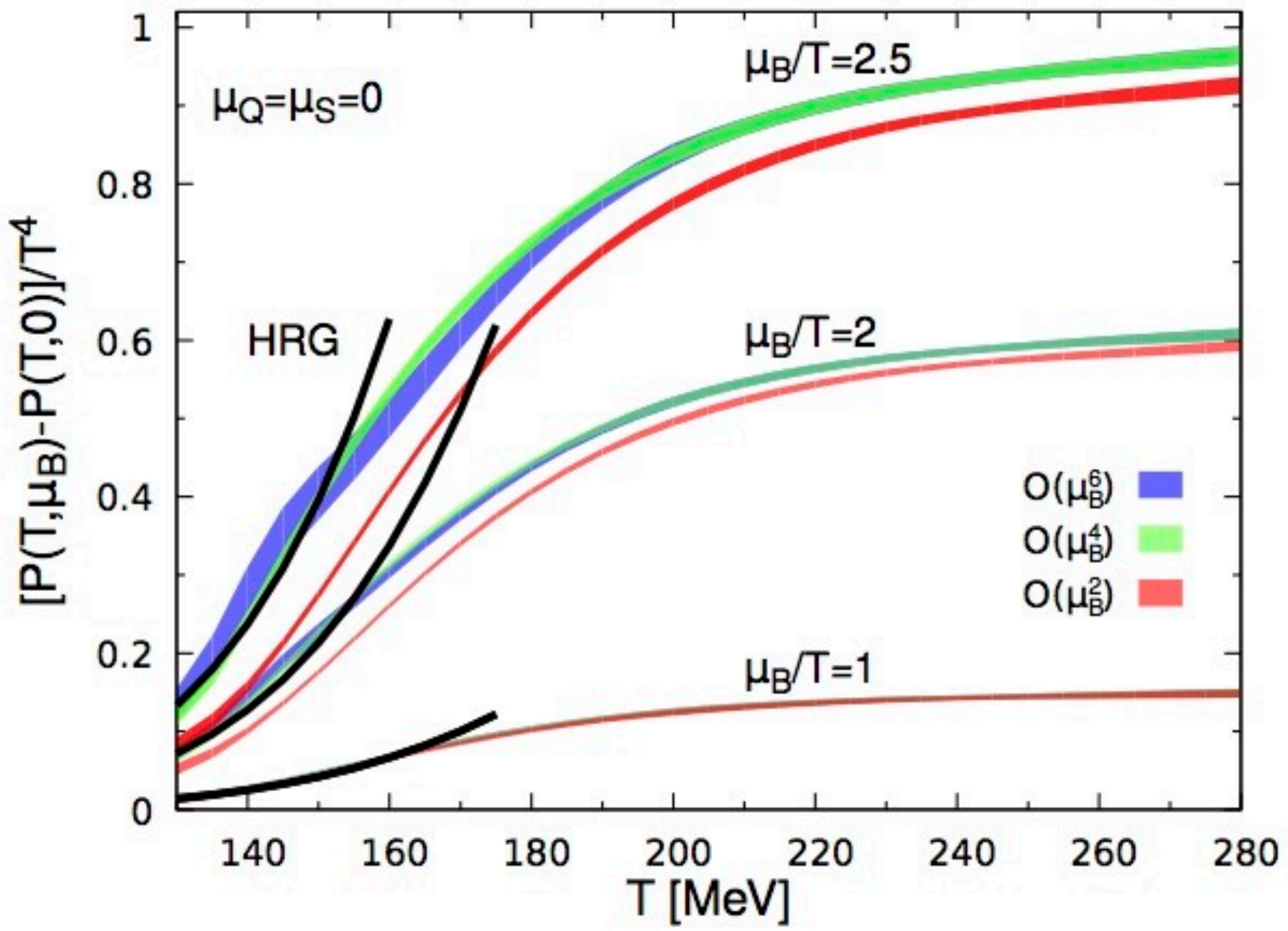


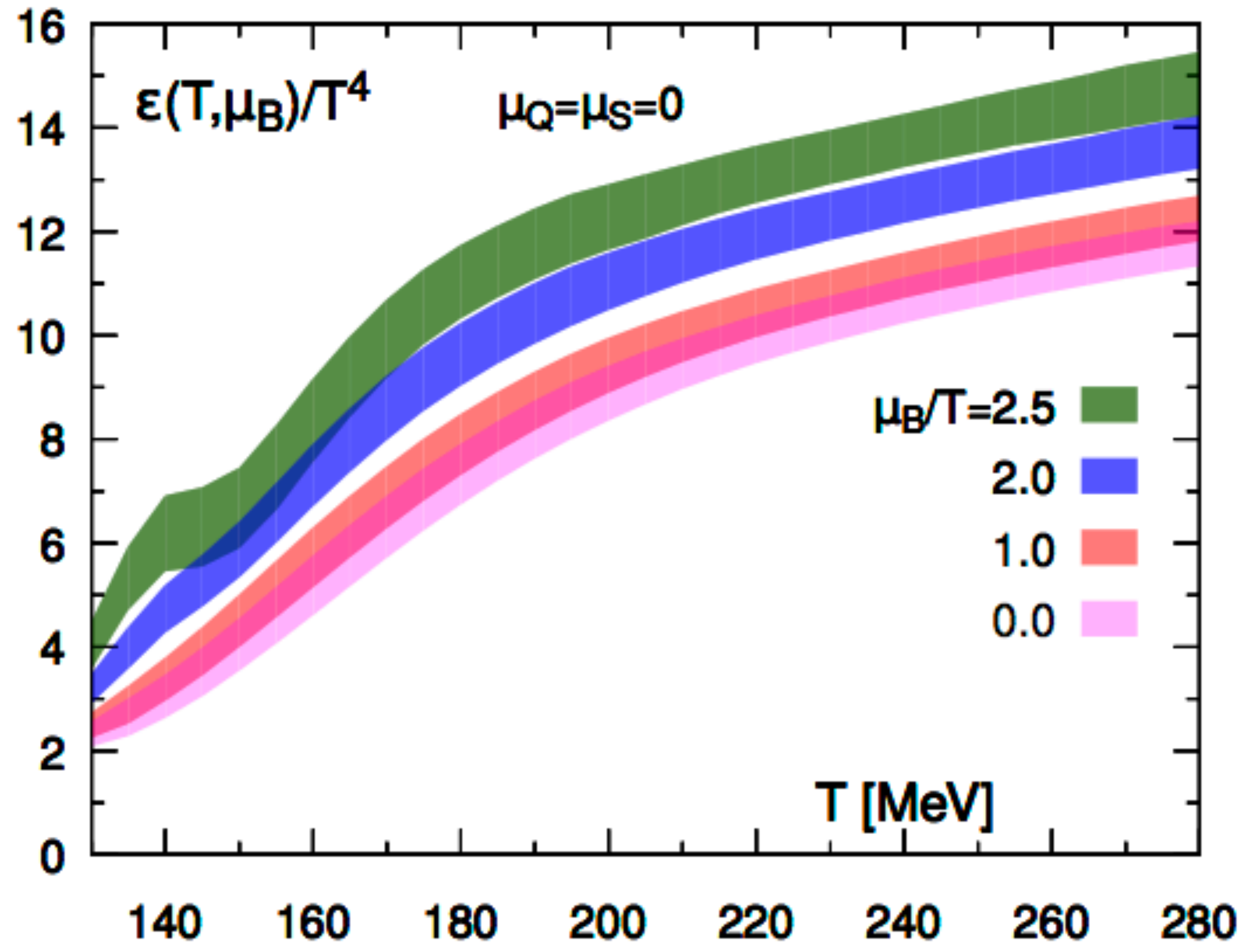
BNL-Bielefeld-CCNU, Phys. Rev. **D86**, 054504 (2017)

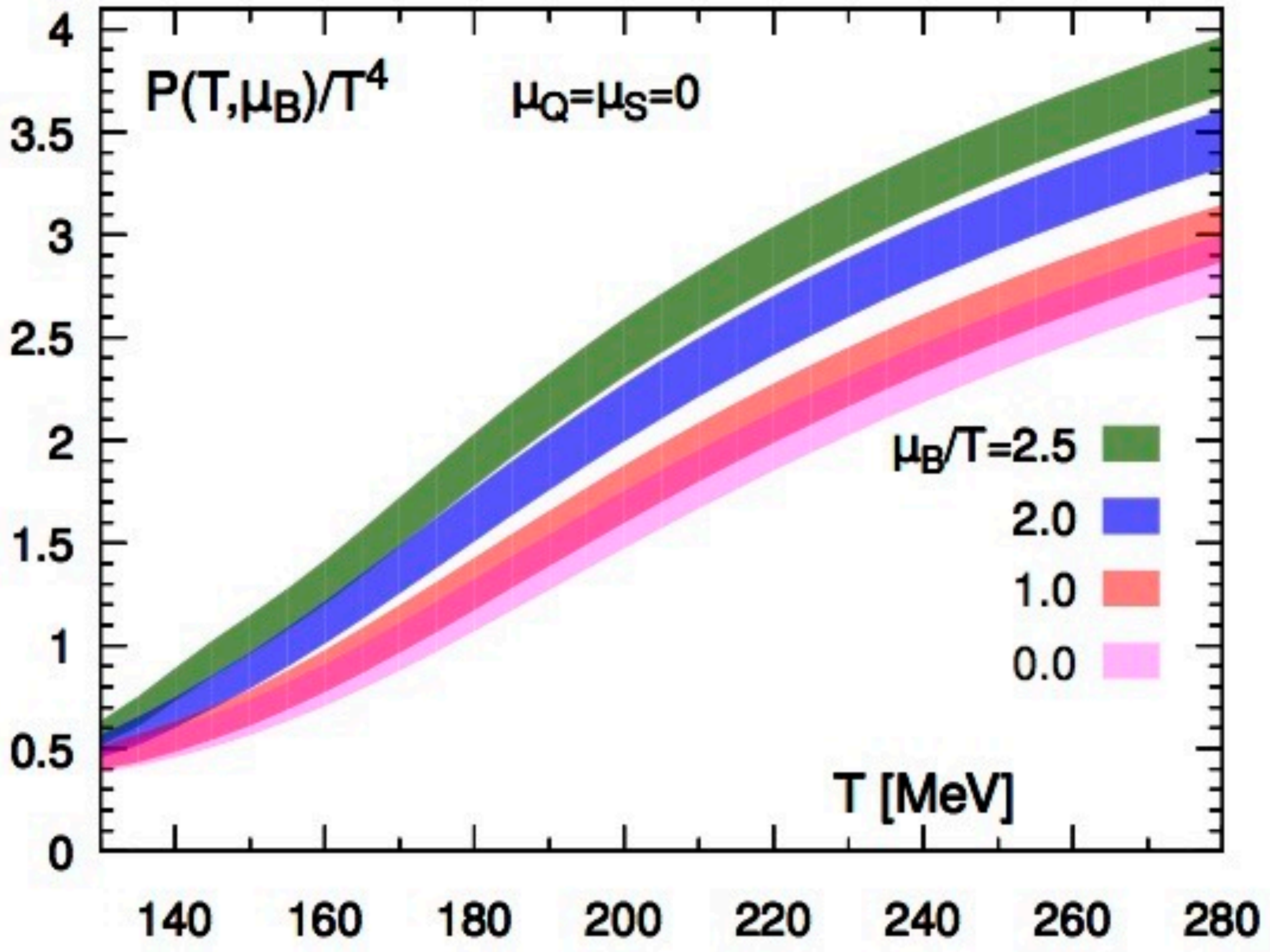


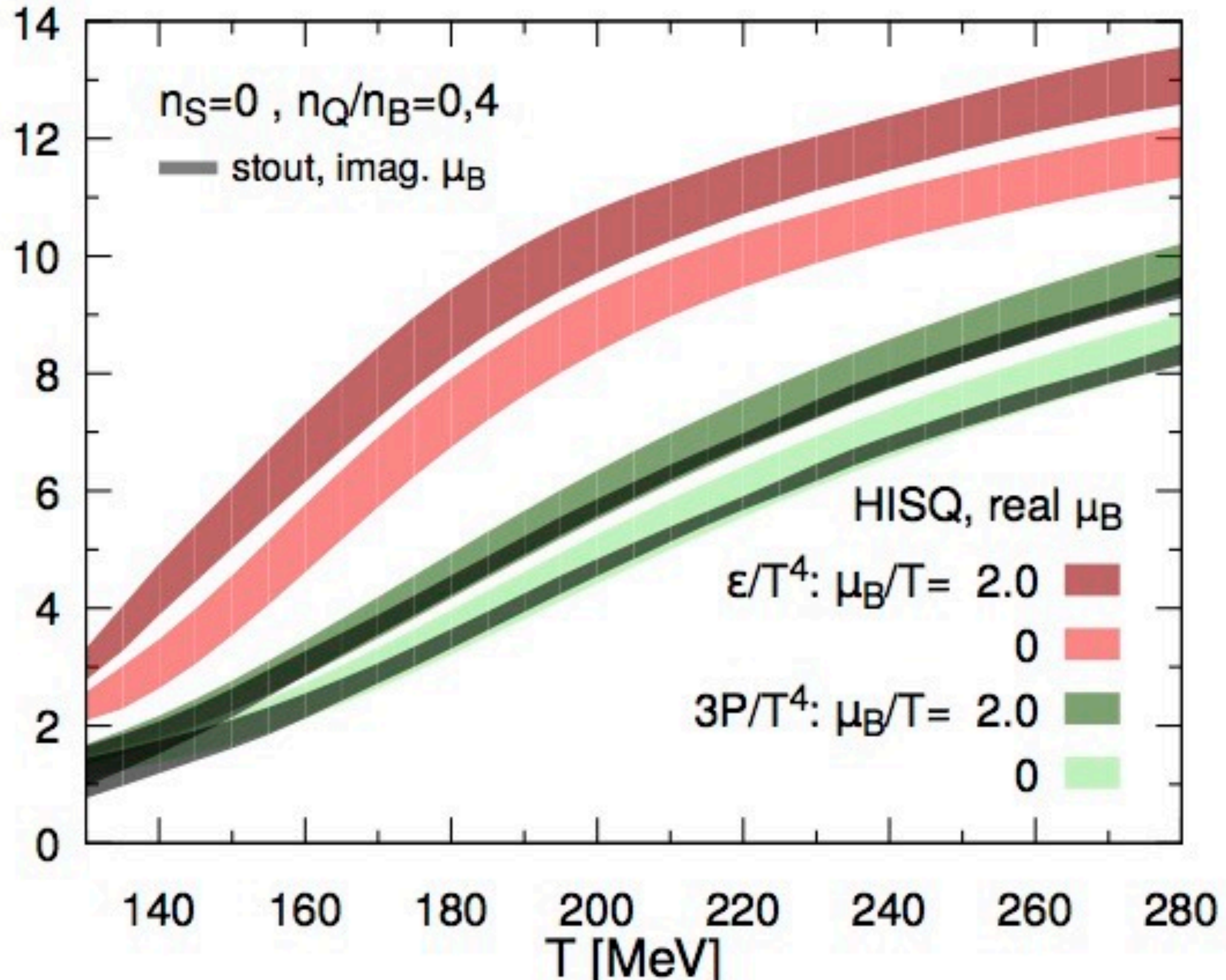


Pressure, energy and entropy: Corrections and total









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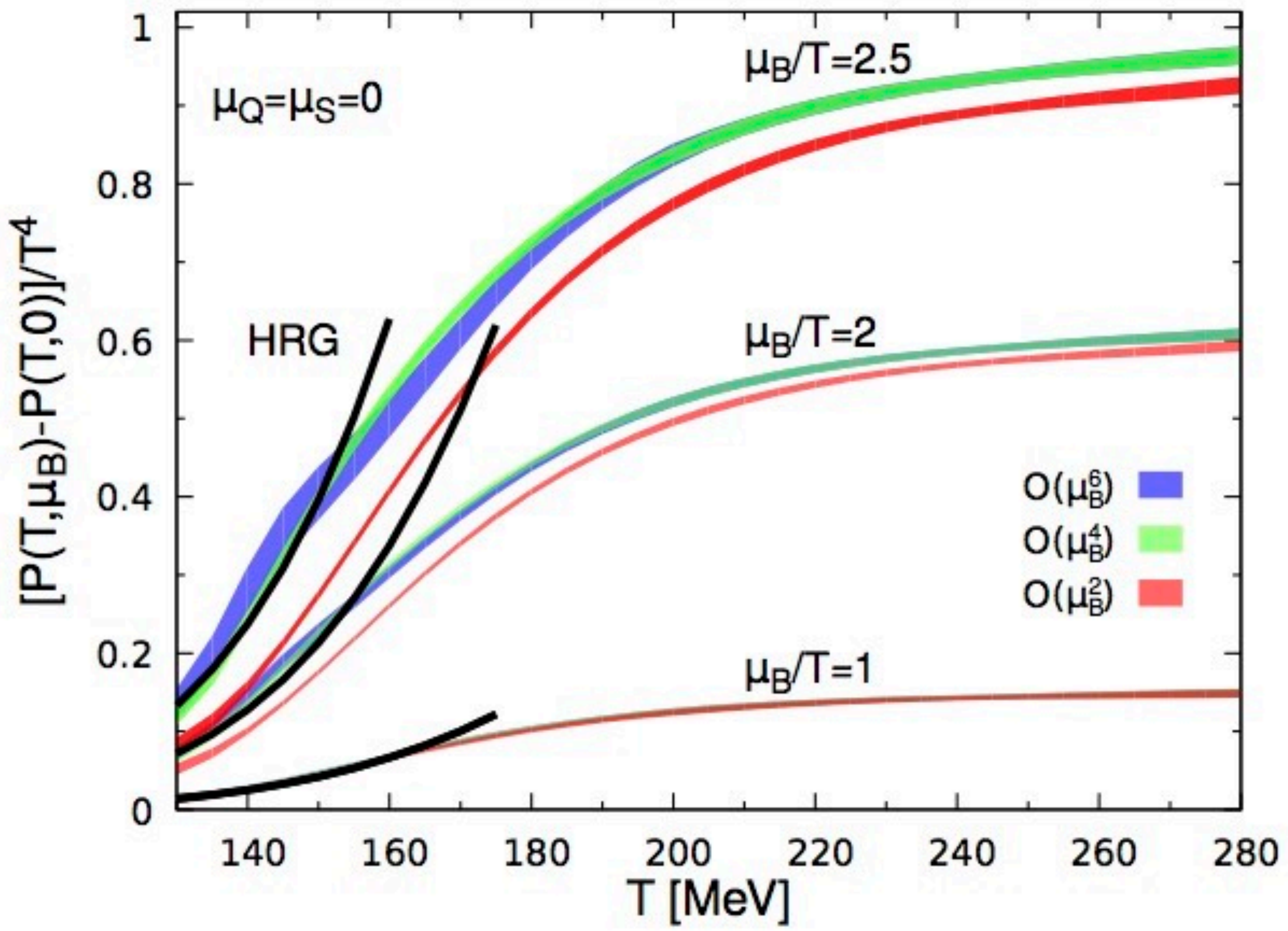
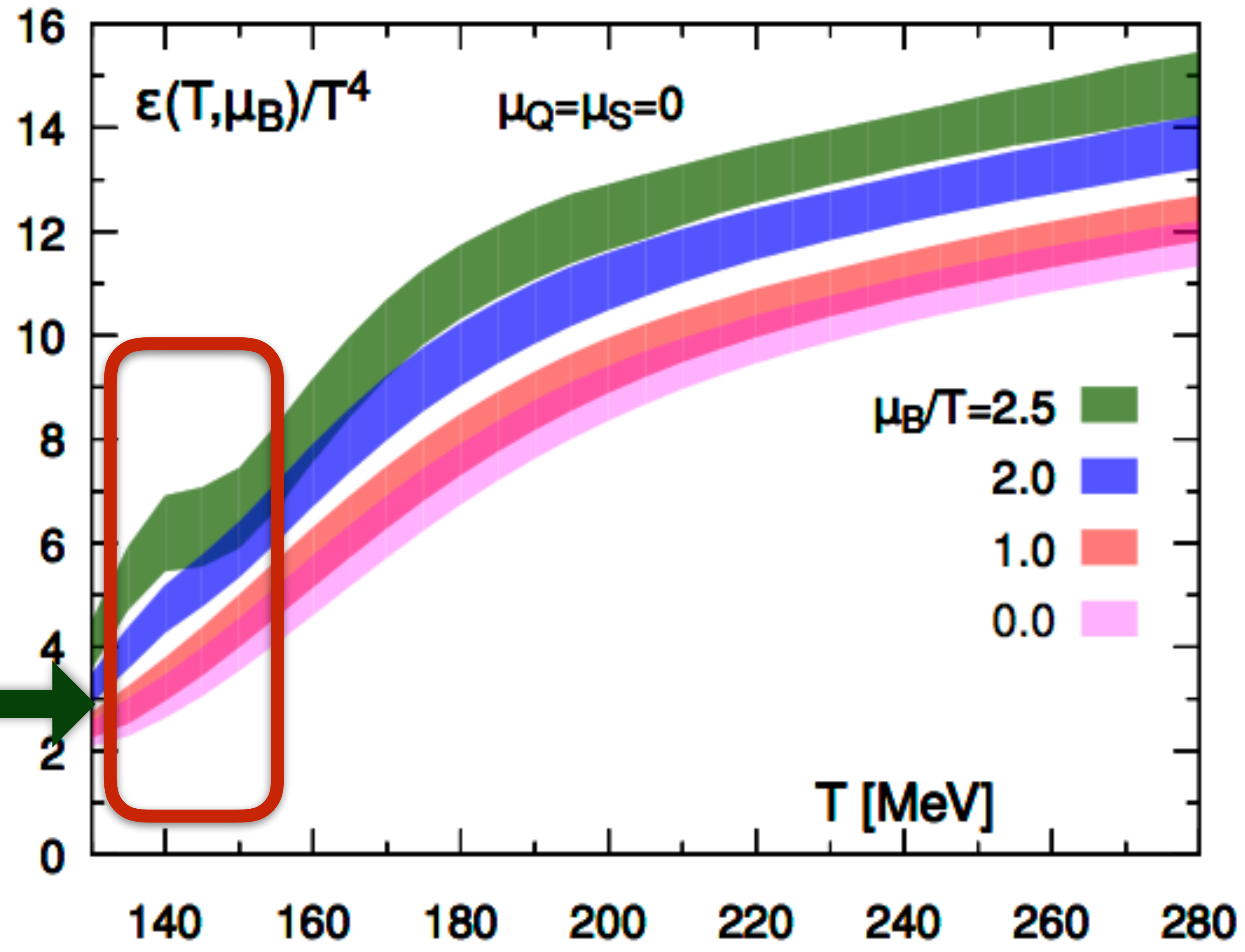
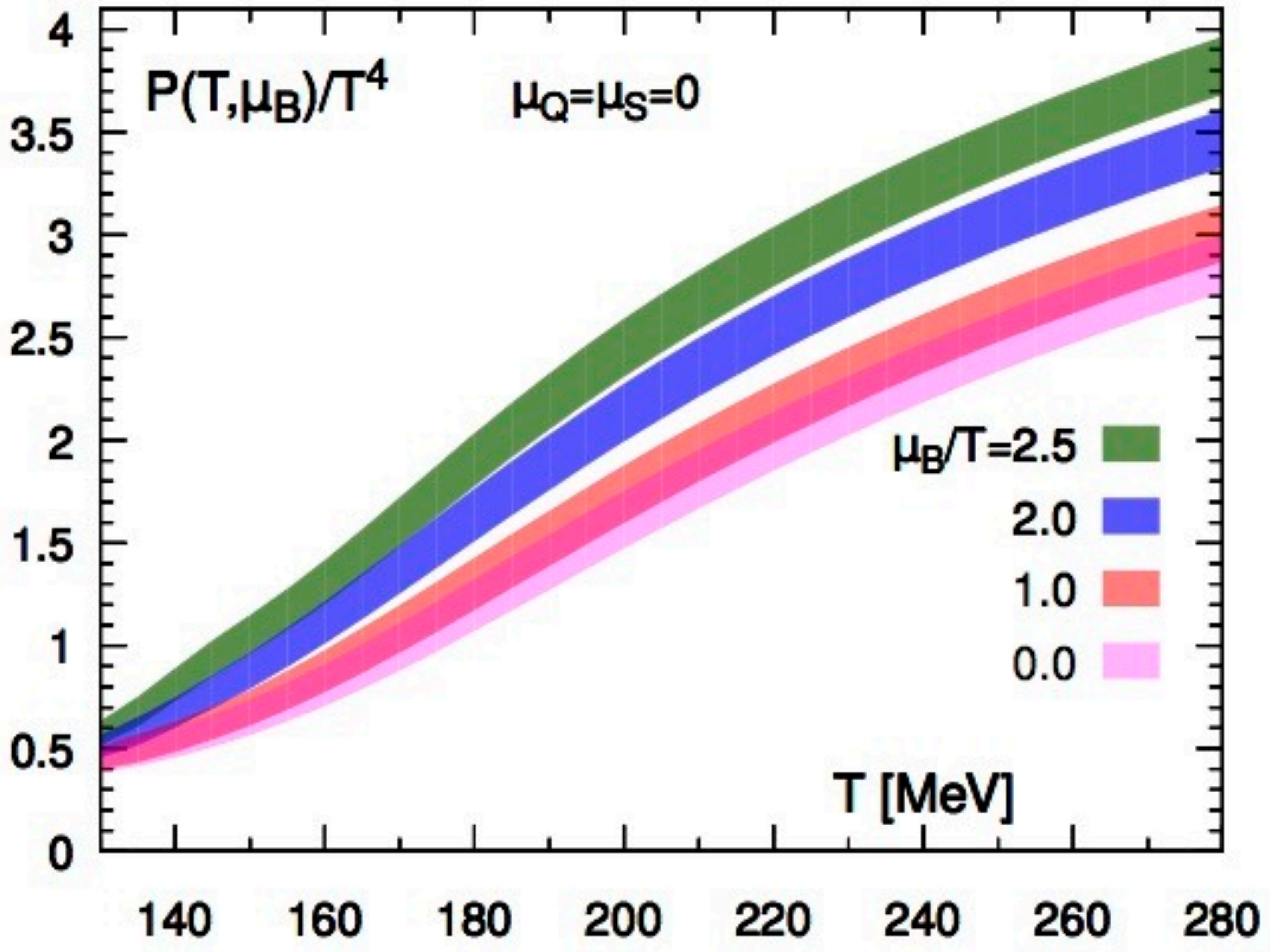
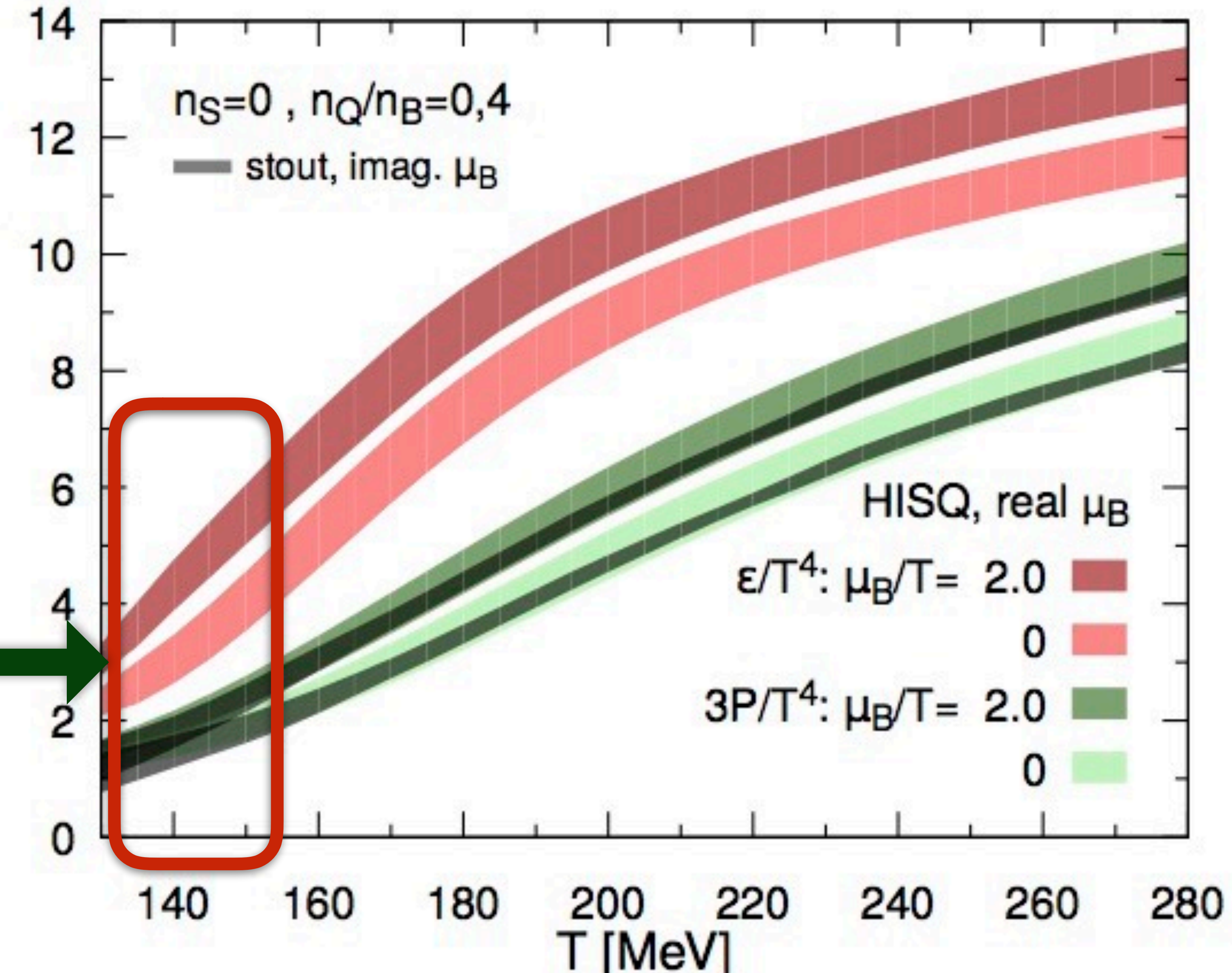




6th-order contributions more pronounced for the $\mu_Q = \mu_S = 0$ case.





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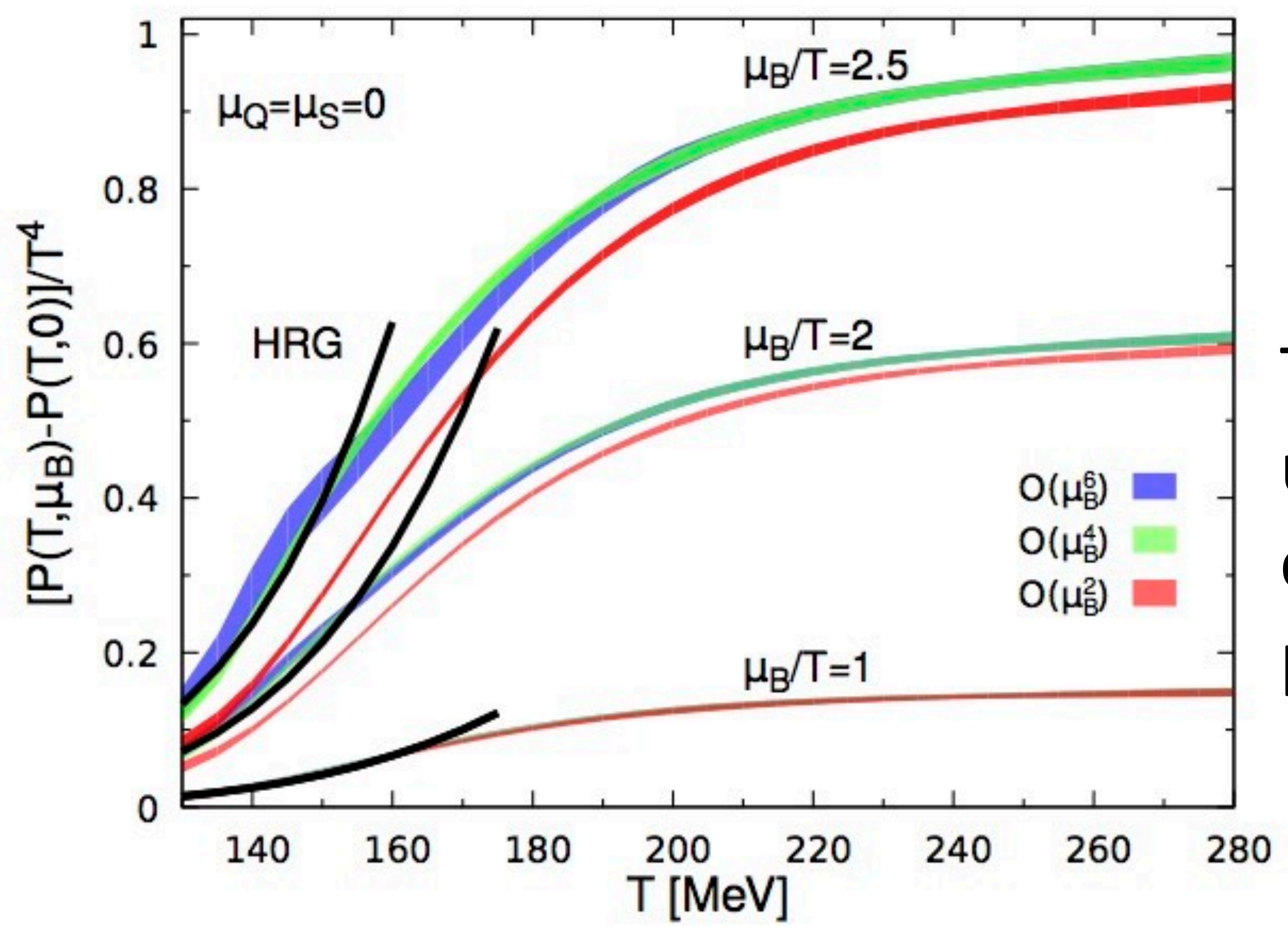
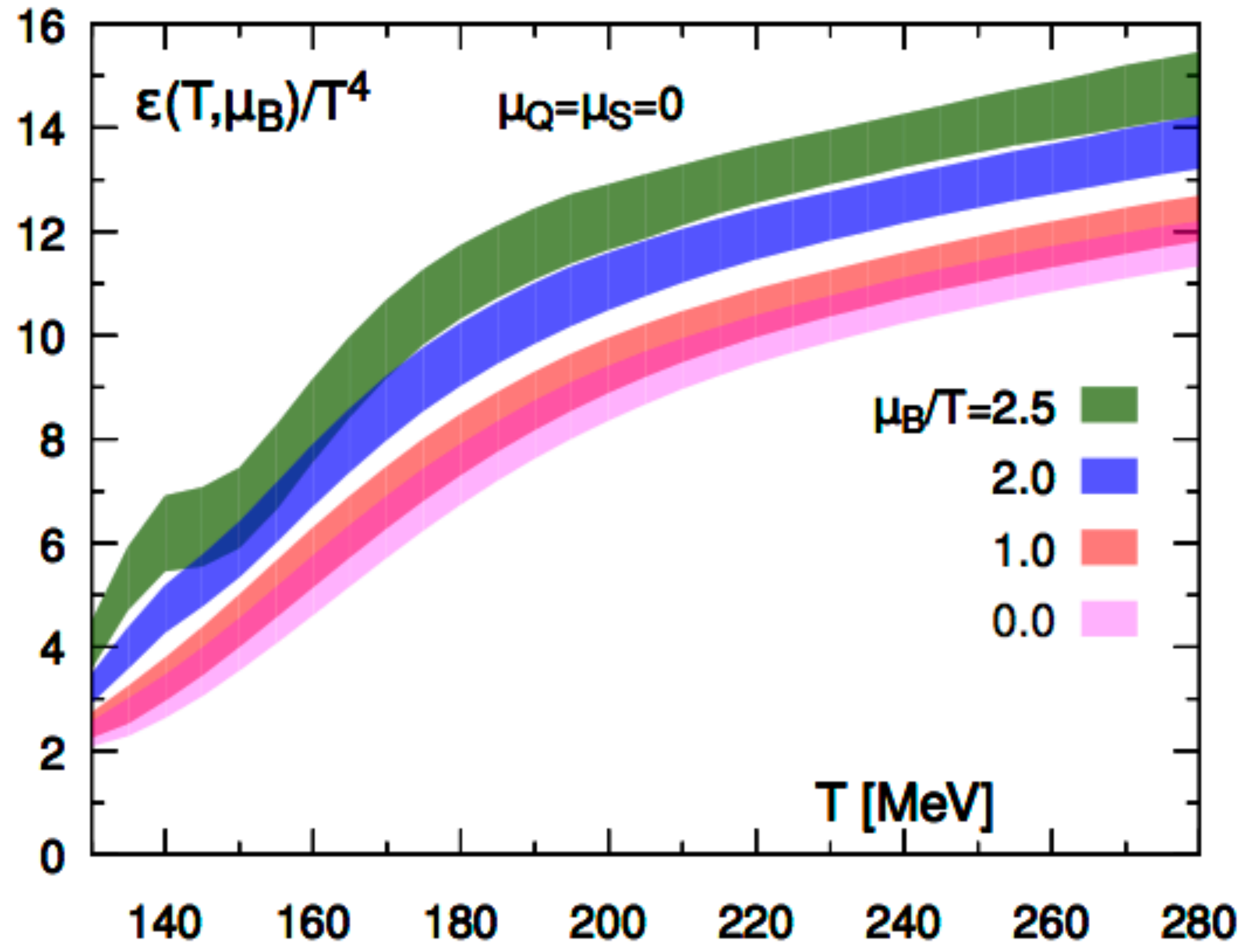
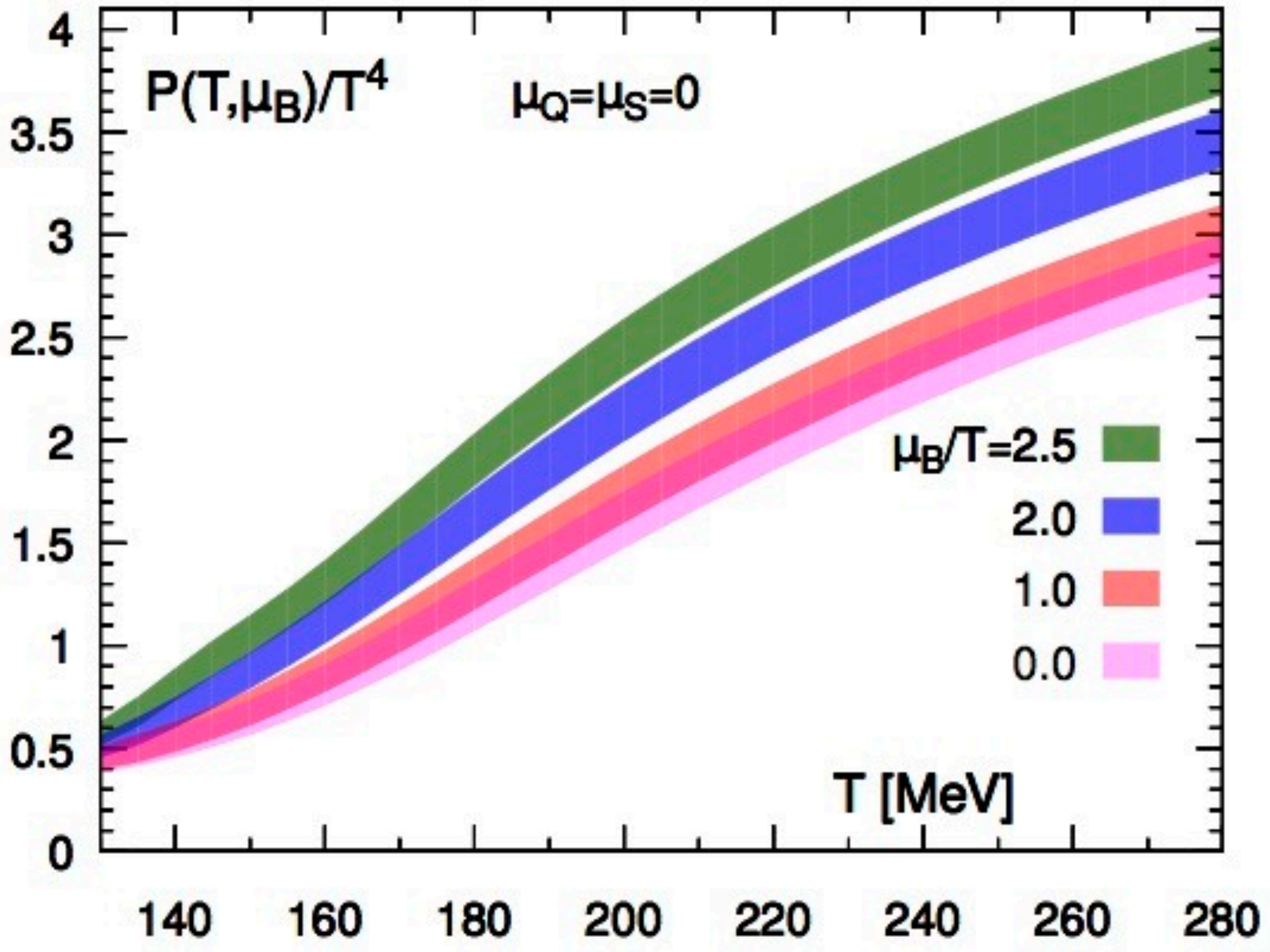
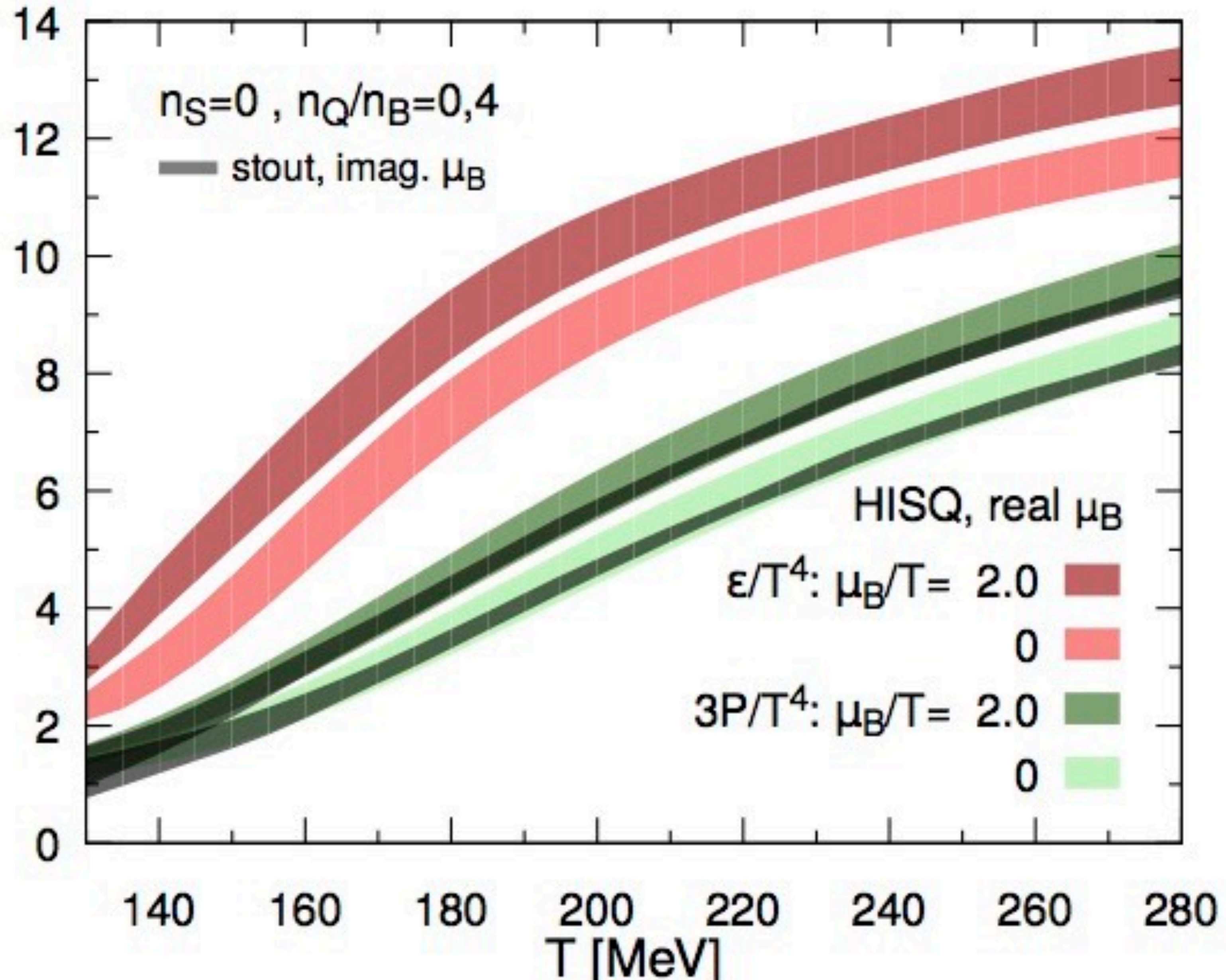


The equation of state is under control for the constrained case up to μ_B/T ~ 2.5.







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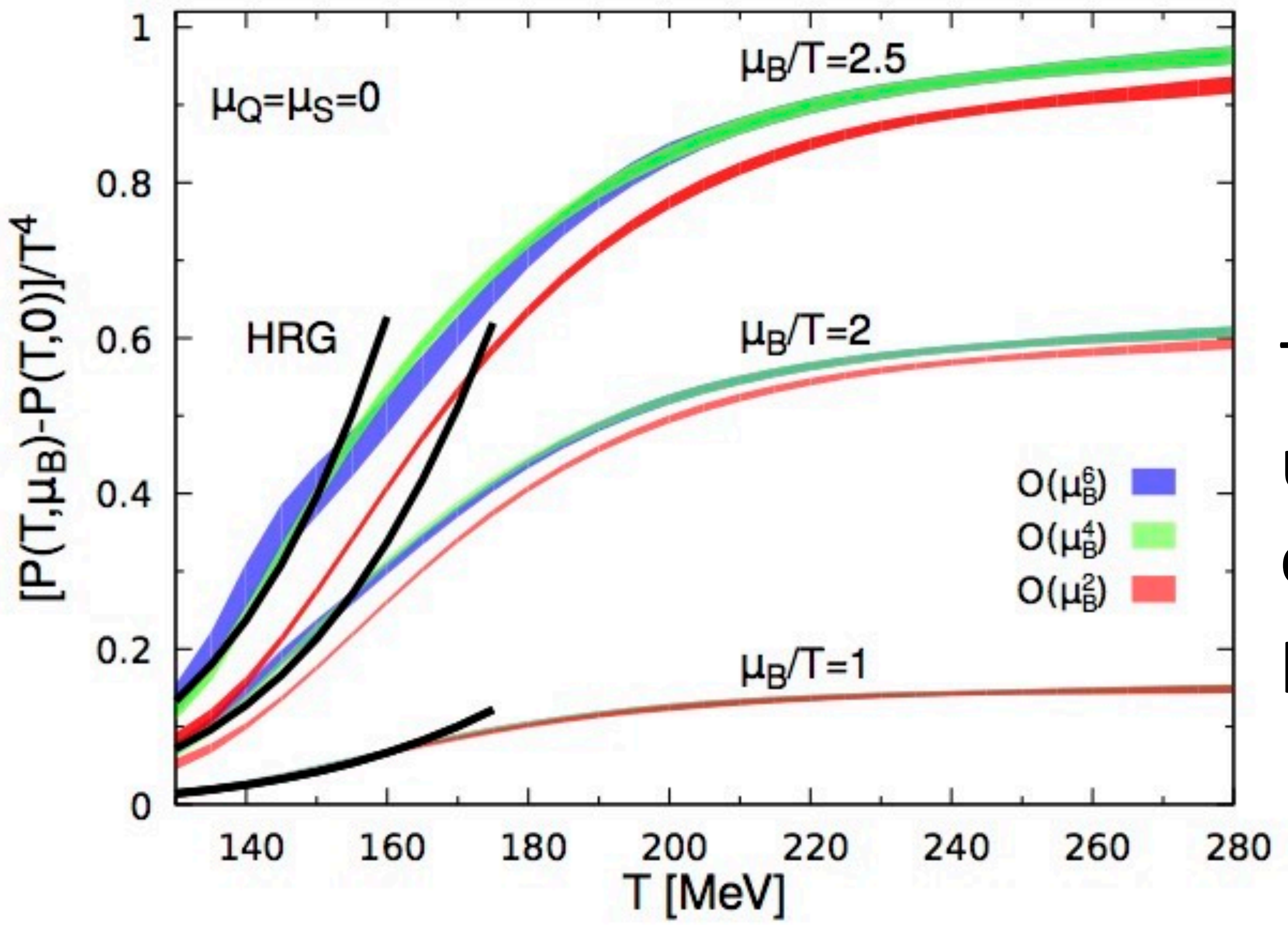

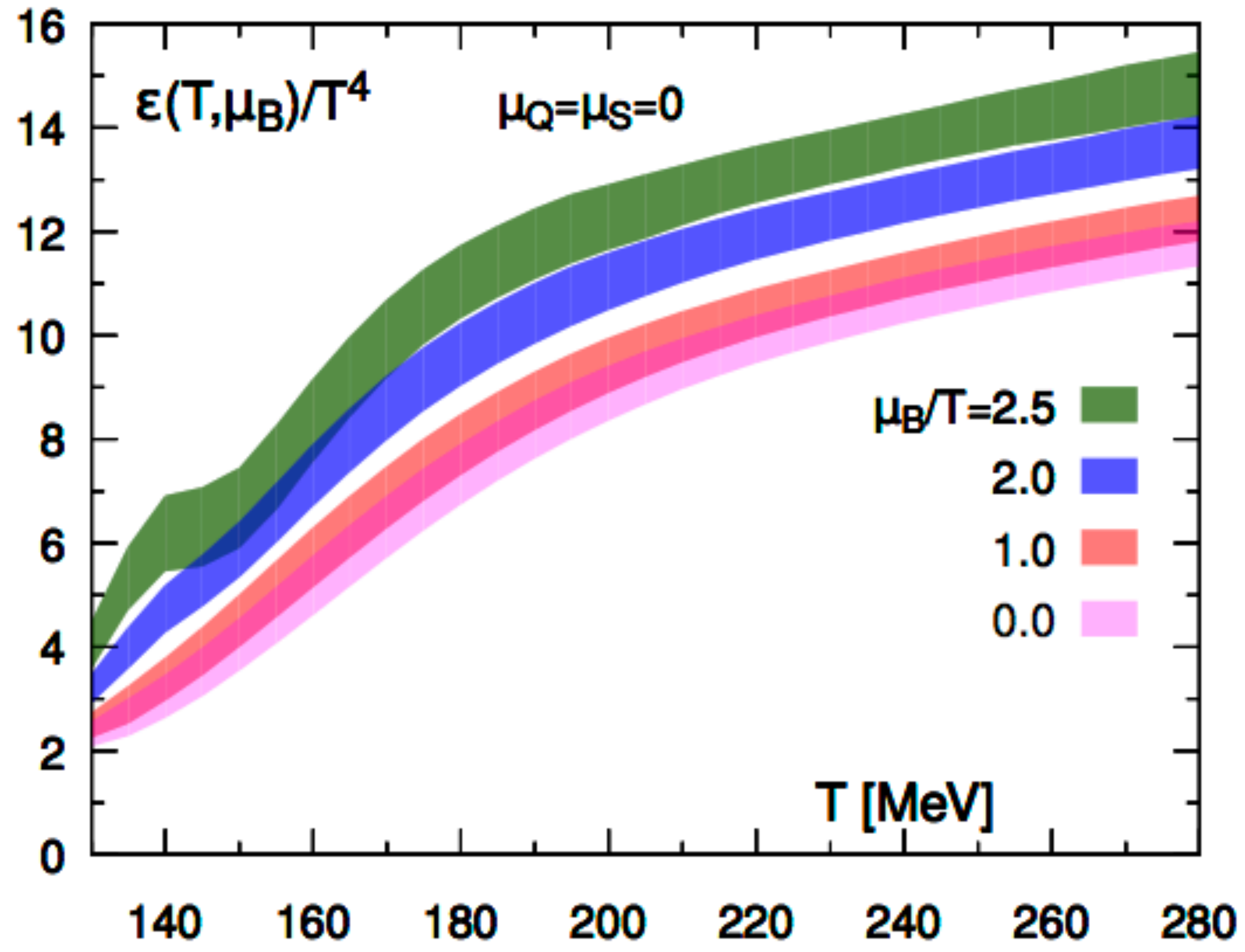

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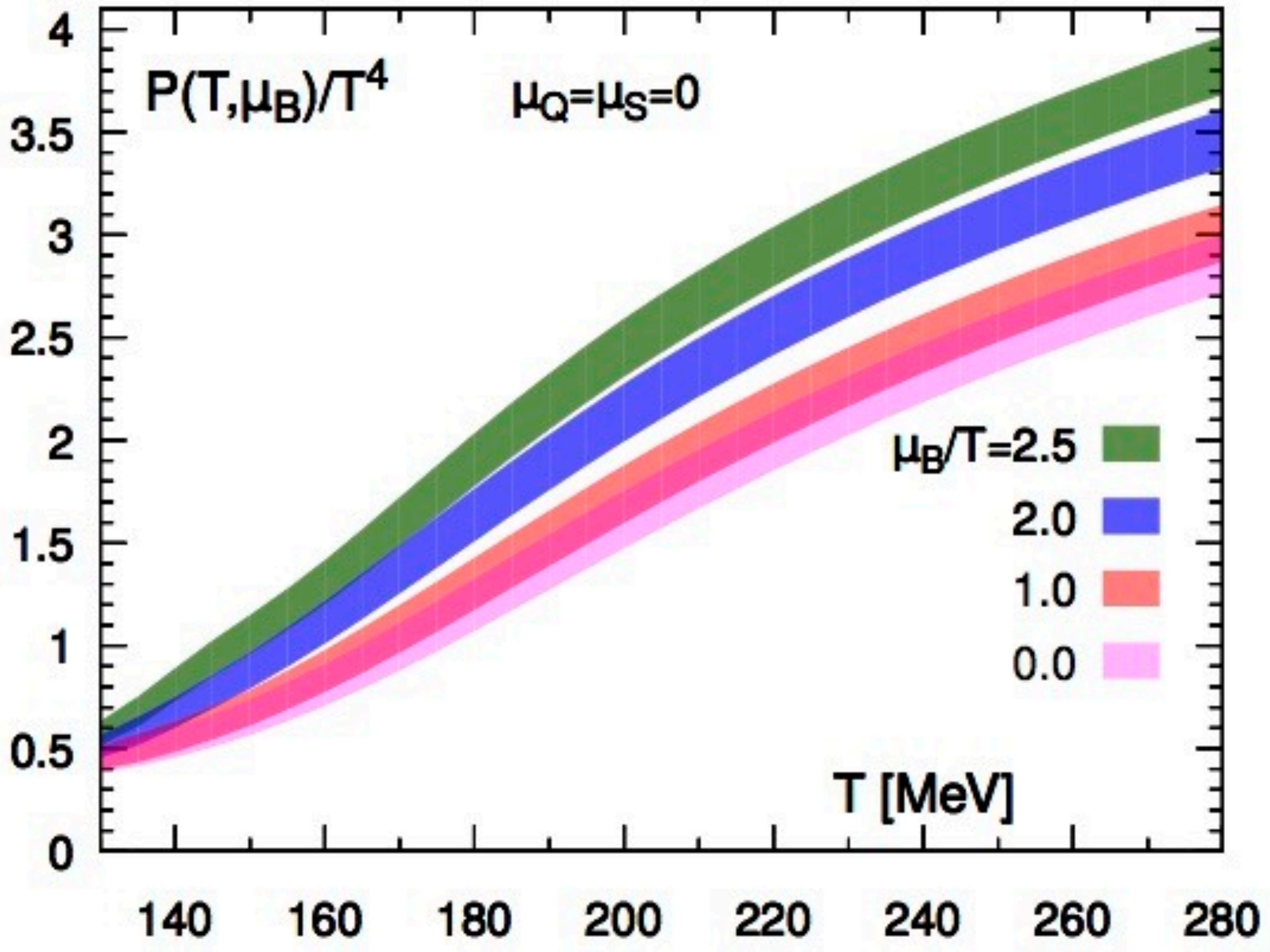
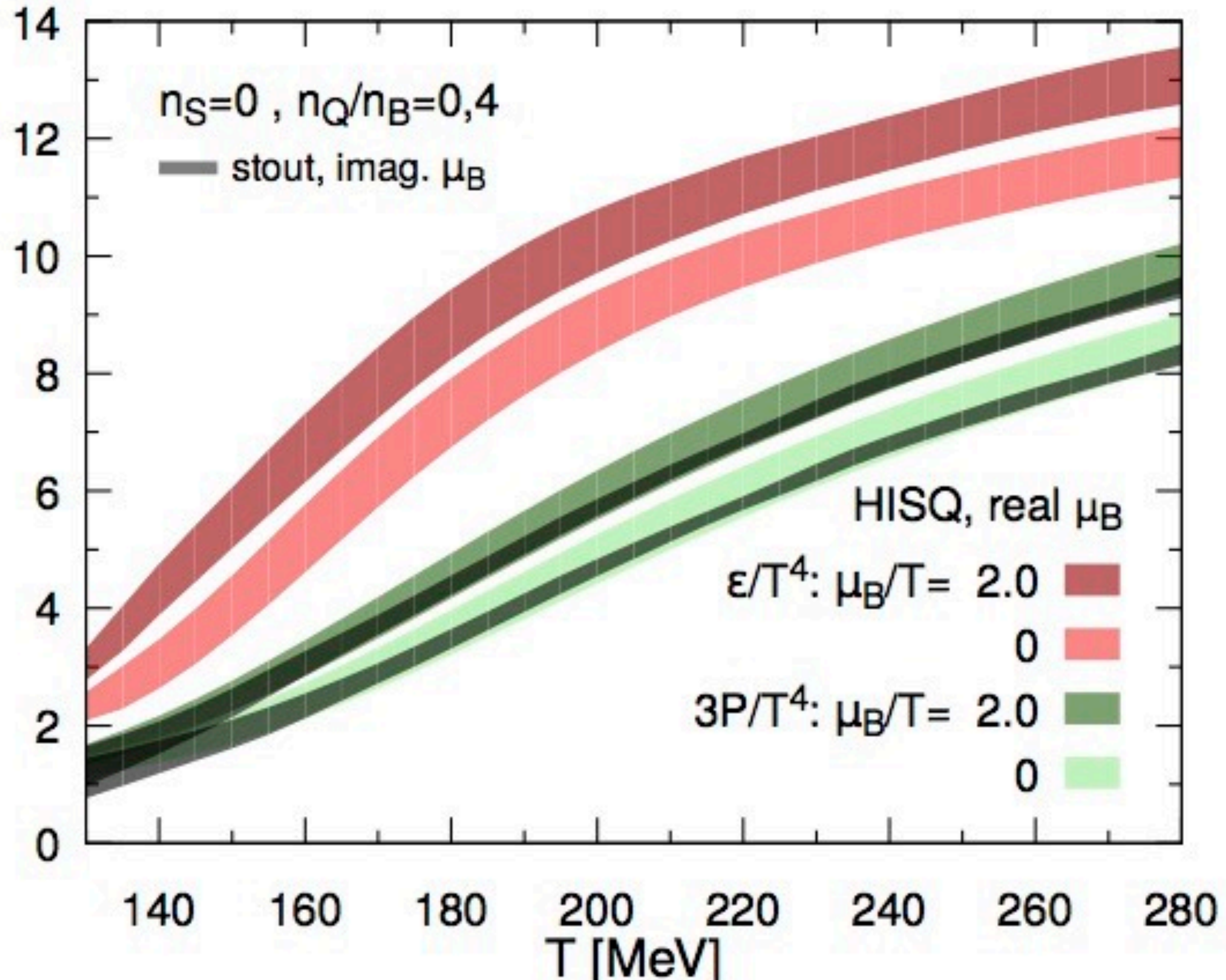
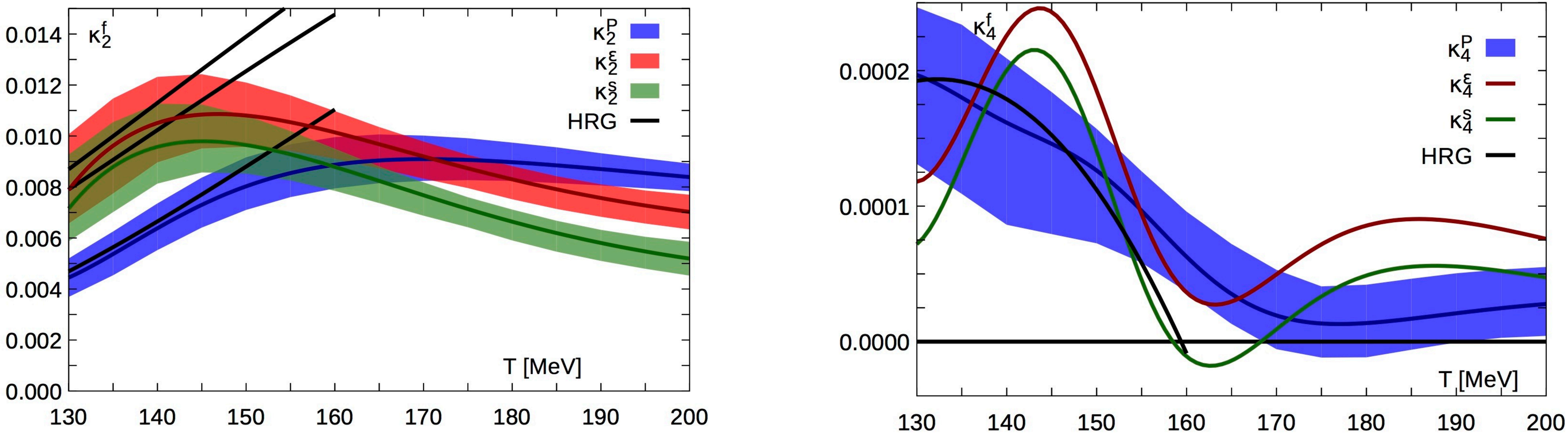


However, BES reaches up to μ_B/T = 3-3.5. An eighth-order expansion is needed (in progress)!



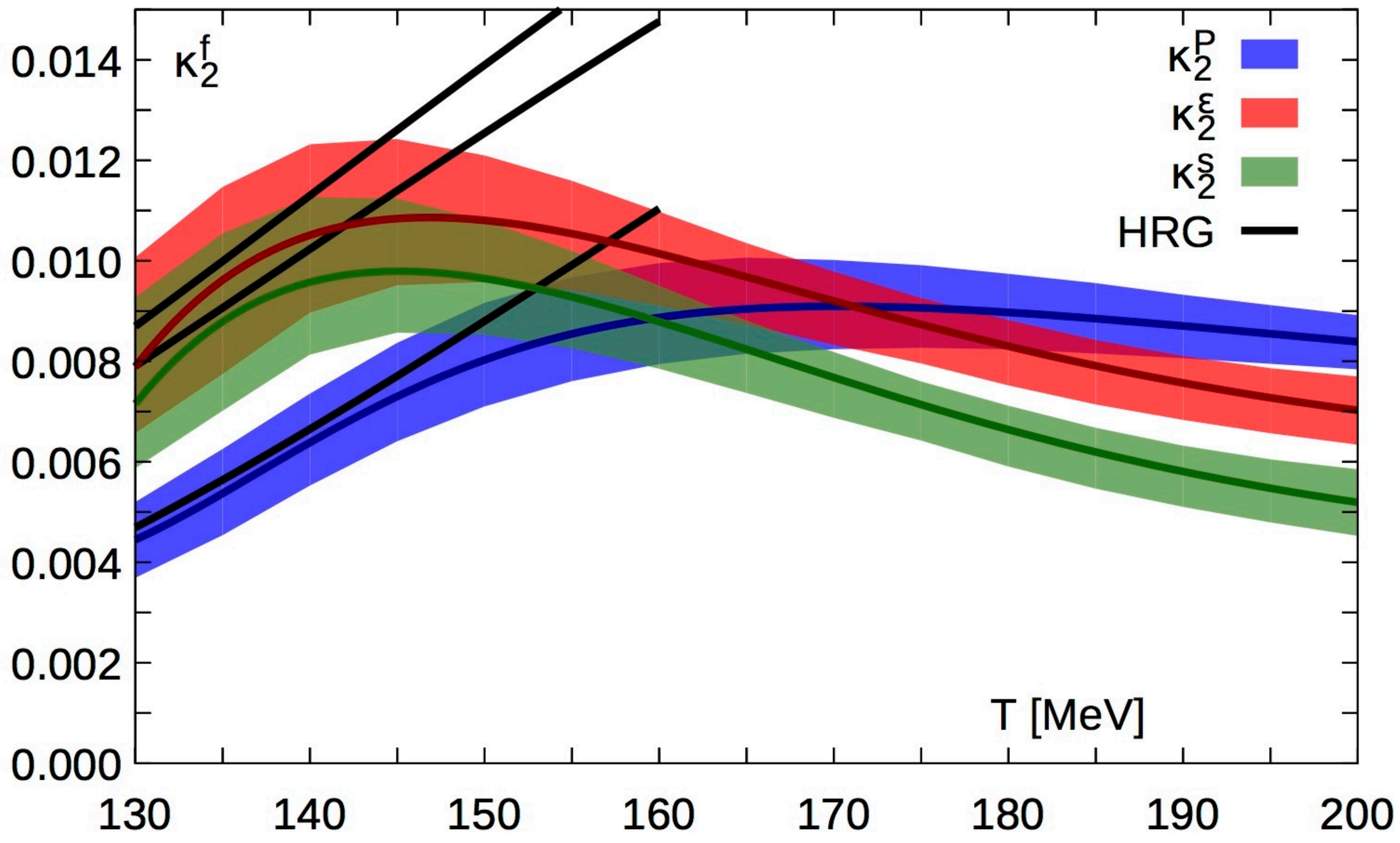
Lines of constant physics and the curvature of the freeze-out line

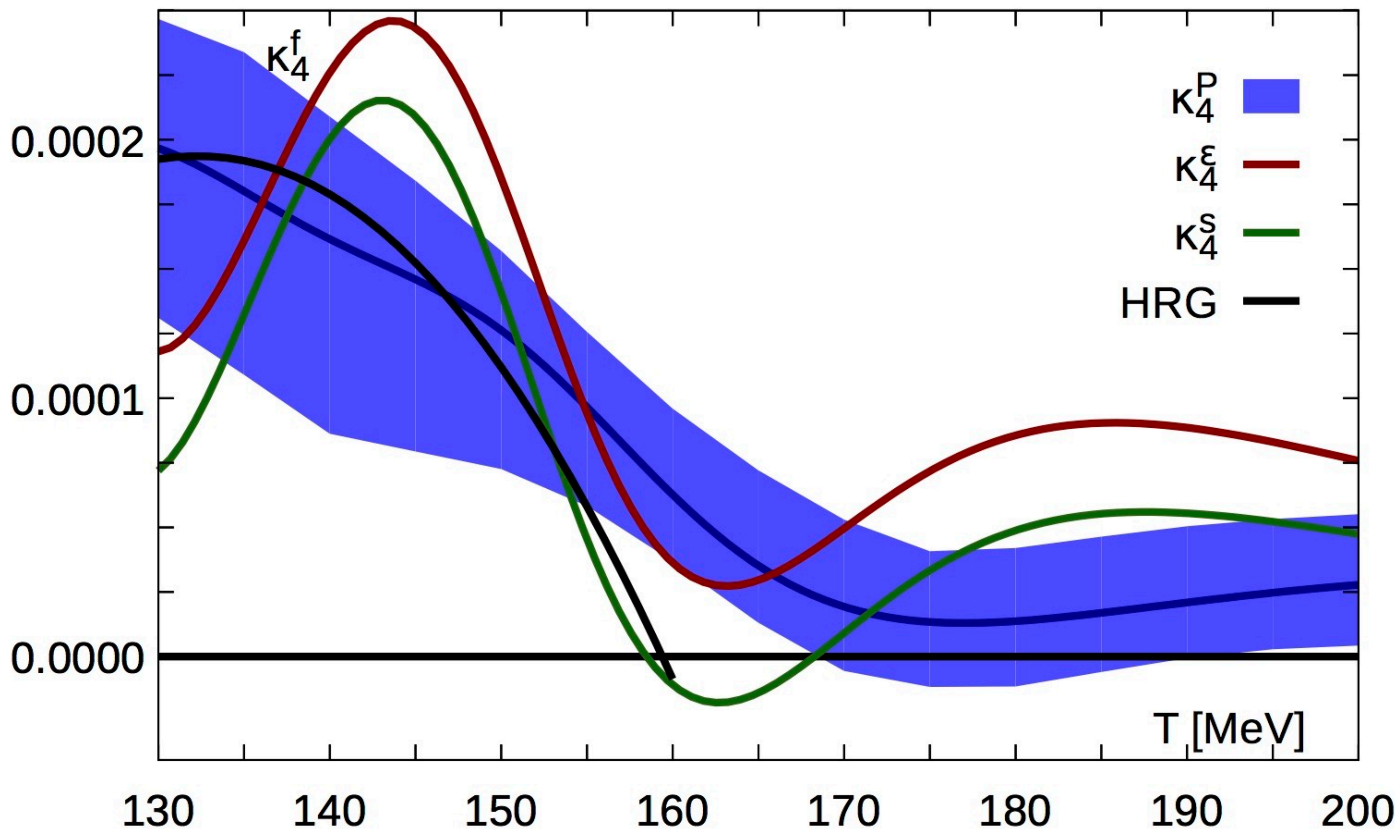


Lines of constant p, σ or ε are curves in the T-μ_B plane. For small μ_B, we can parametrize: $T(\mu_B) = T_0 + \kappa_2(\mu_B/T)^2 + \kappa_4(\mu_B/T)^4 + \ldots$

We determine κ_2 and κ_4 from our 2nd and 4th-order Taylor expansions. κ_4 is smaller than κ_2 by an order of magnitude. Our current statistics do not permit an accurate determination of κ_6.

Lines of constant physics and the curvature of the freeze-out line

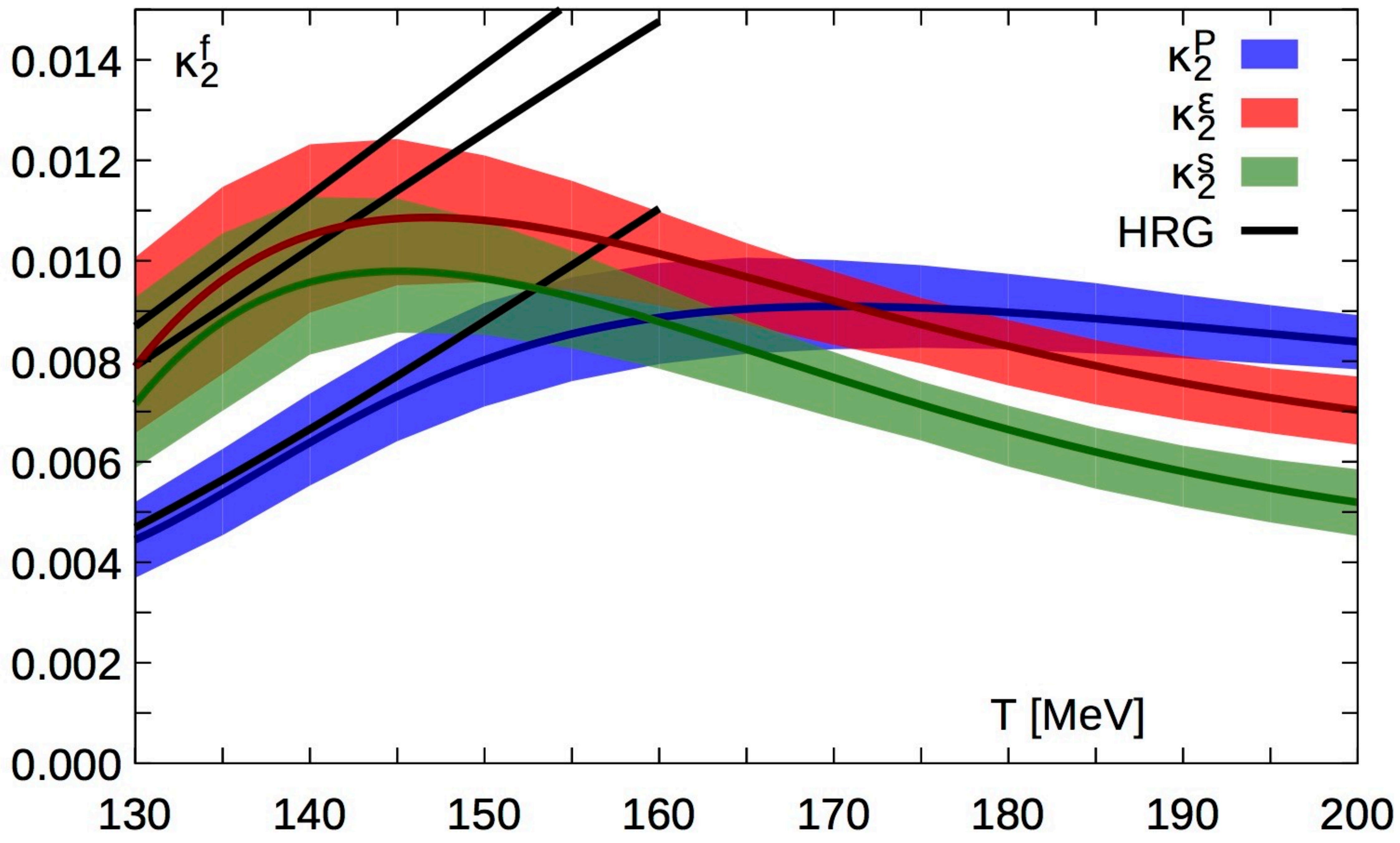


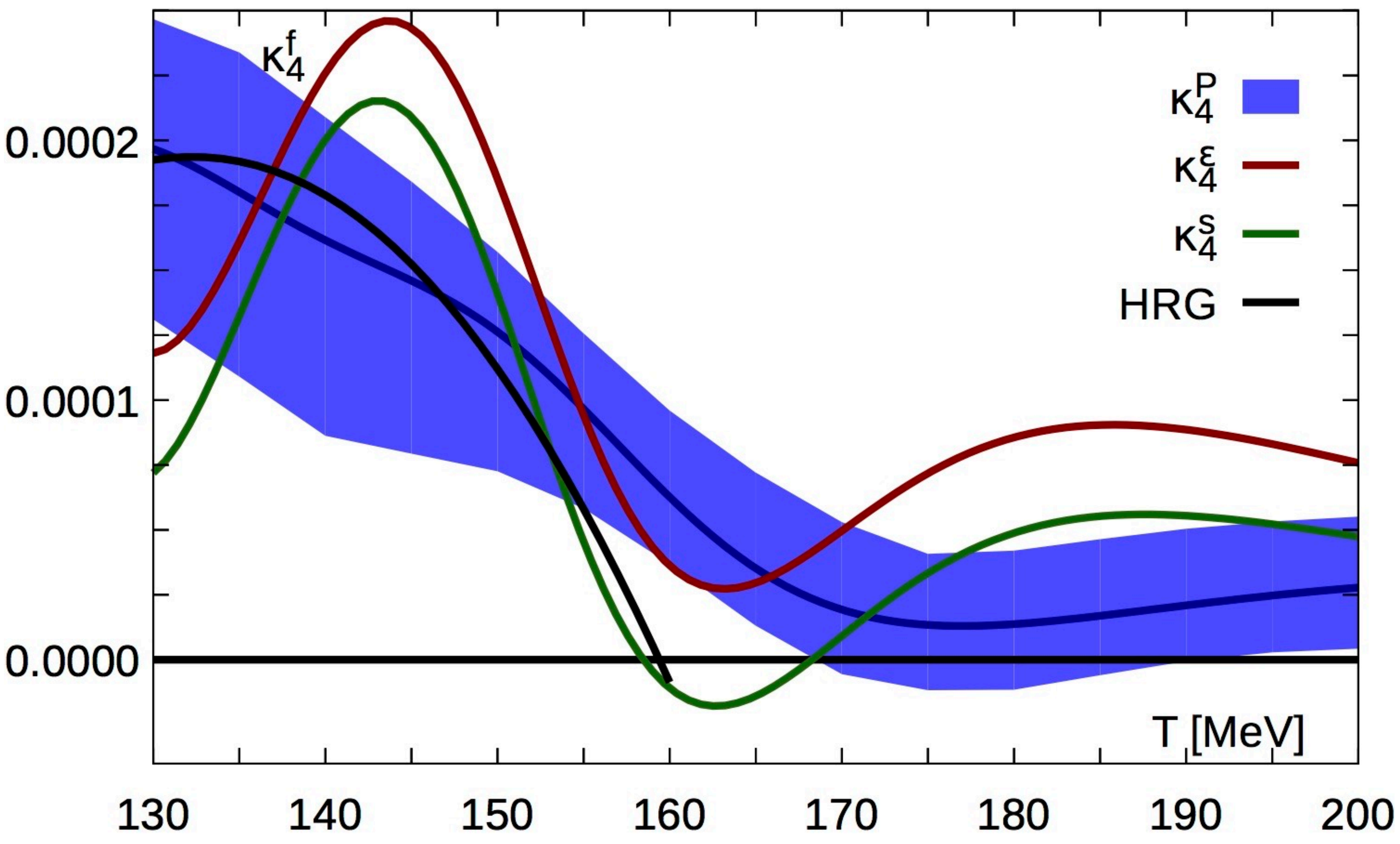


Phenomenologically, freeze-out has been conjectured to occur along such lines of constant ε or σ [Cleymans and Redlich 1999].

For T between 145 and 165 MeV, $0.0064 <= \kappa_2^p <= 0.0101$ and $0.0087 <= \kappa_2^\varepsilon <= 0.012$ [S. Sharma, QM2017]. This is in agreement with estimates for the curvature of the line of the chiral transition temperature [BNL-Bielefeld 2010; BW 2012, 2015; D'Elia *et al*. 2015; Cea *et al*. 2015].

Lines of constant physics and the curvature of the freeze-out line

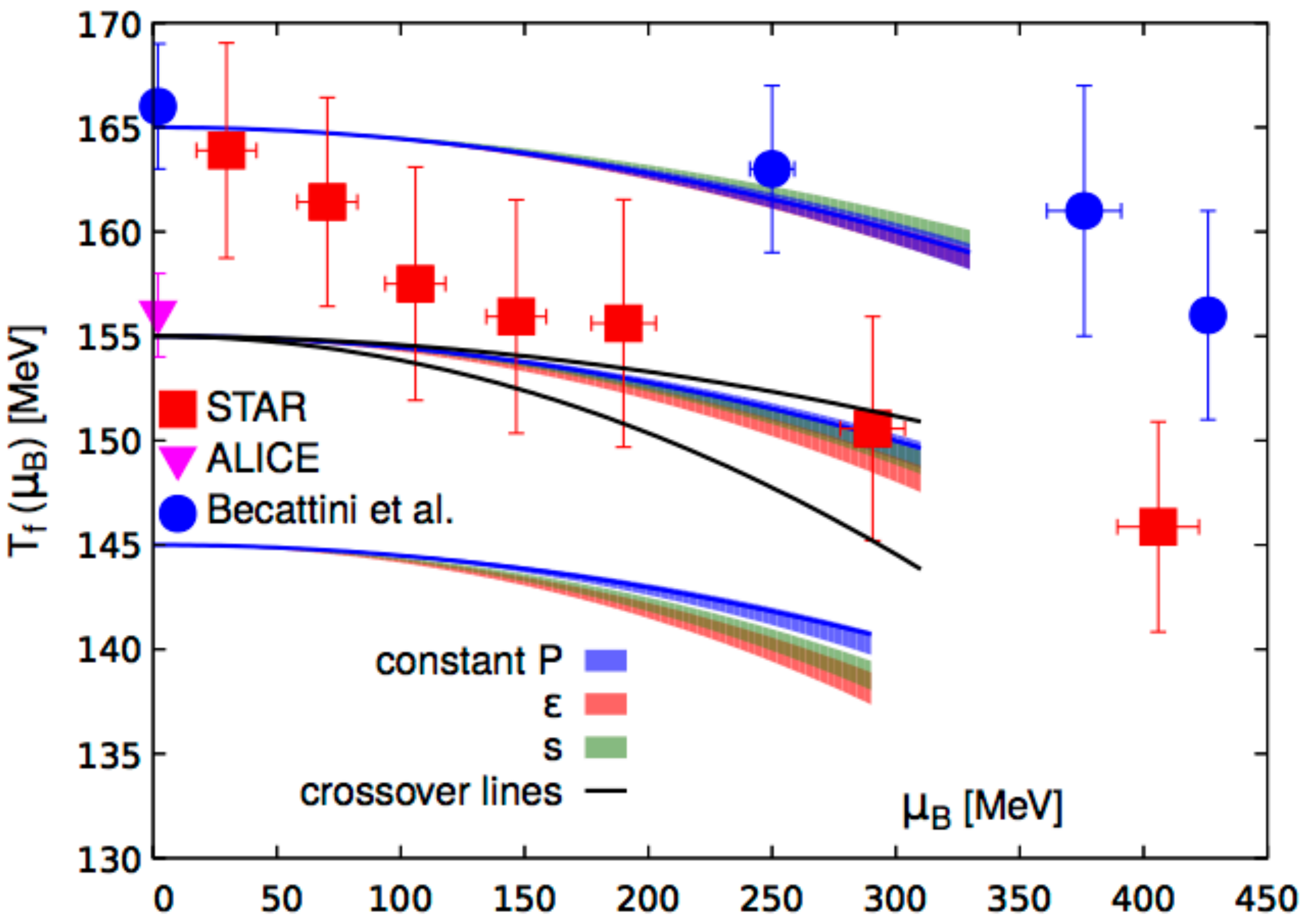




Phenomenologically, freeze-out has been conjectured to occur along such lines of constant ε or σ [Cleymans and Redlich 1999].

The 4th order correction to these curves is negligible for p, σ and ε up to $\mu_B/T = 2$.

Lines of constant physics: Comparison with experiment

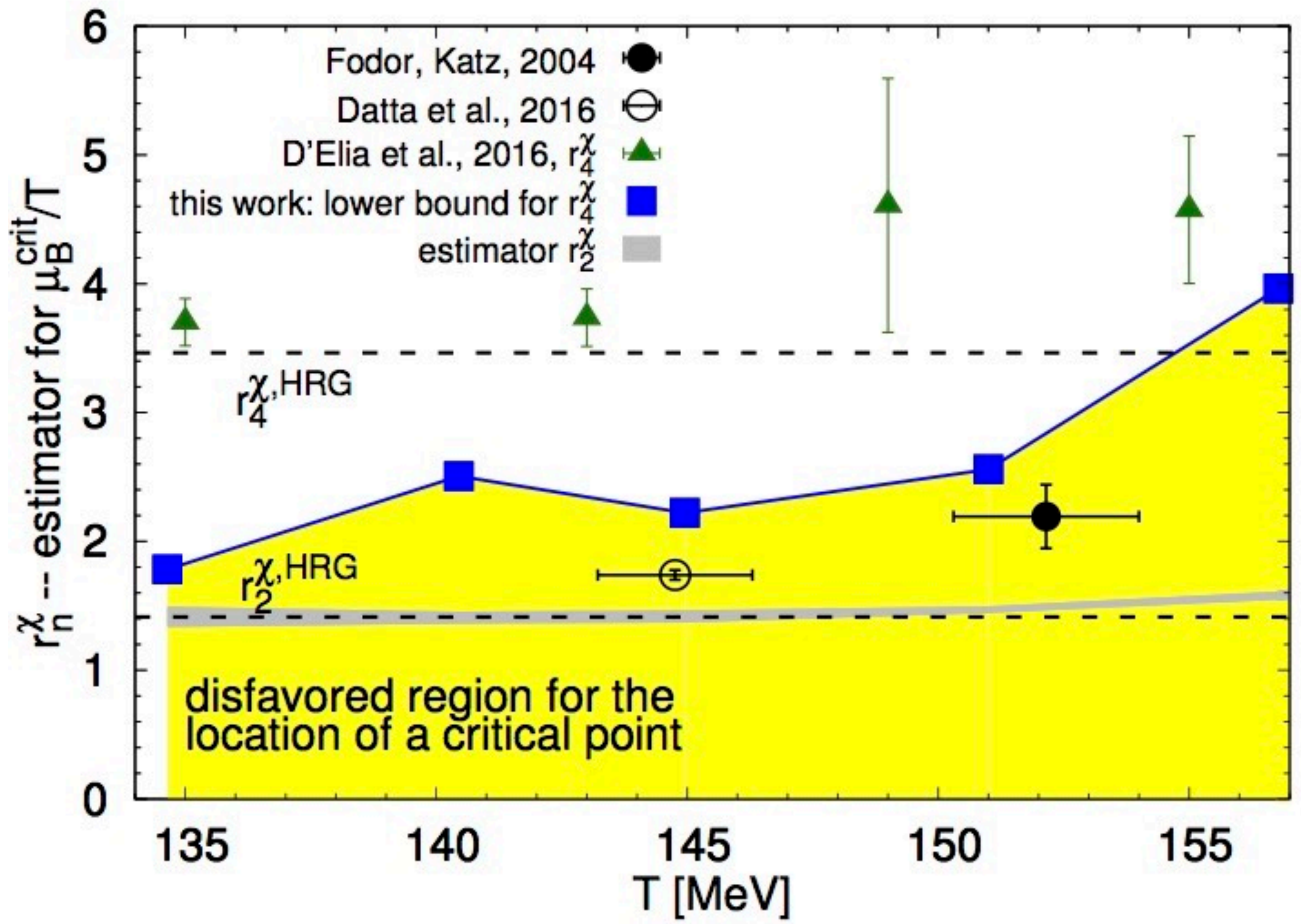


The temperature and baryochemical potential at freeze-out can be extracted from a comparison to HRG models [P. Braun-Munzinger and J. Stachel 2005]. These values have been quoted by both STAR and ALICE, as well as by various theorists.

BNL-Bielefeld-CCNU 2017; S.Sharma, Quark Matter 2017

Our lines of constant physics compare well with both experimental data as well as the various parametrizations of the freeze out curve provided by different workers.

Searching for the QCD Critical Point



[Phys. Rev. D95, no. 5, 054504 (2017)]

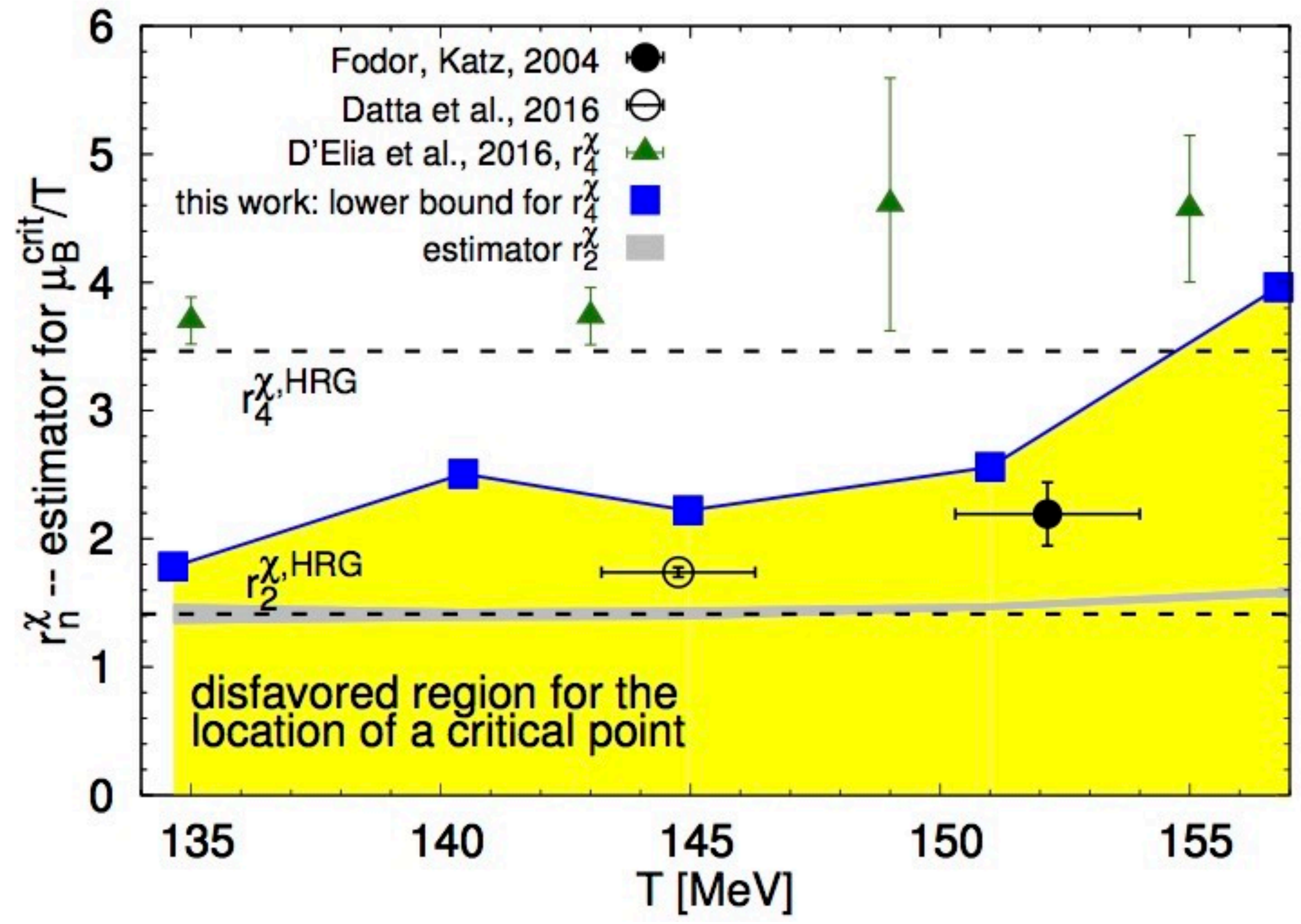
Our calculations seem to indicate that our expansions are under control for $\mu_B/T <= 2$. That is, the corrections strictly obey $P_2 >> P_4 >> P_6 >>$ …

Every Taylor series has a radius of convergence (which could be infinite), which is also the distance to the nearest singularity. In our case, this singularity would be the QCD critical point.

Close to the singularity, contributions from different orders start to become equal, leading to a breakdown of the expansion. This distance can be estimated from the formula:

$$\rho = \lim_{n\to\infty} \sqrt{\frac{P_n}{P_{n+2}}}$$

Searching for the QCD Critical Point



[Phys. Rev. D95, no. 5, 054504 (2017)]

It has also been pointed out that the baryon number susceptibility χ_2^B diverges at the critical point. The Taylor expansion for χ_2^B is given by:

$$\chi_2^B(\hat{\mu}) = \chi_2^B + \frac{\chi_4^B}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_6^B}{24}\left(\frac{\mu_B}{T}\right)^4 + \ldots$$

Similar to the pressure, the radius of convergence can be estimated by taking

$$\rho_\chi = \lim_{n\to\infty} r_{2n}^\chi = \lim_{n\to\infty}\sqrt{\frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B}}$$

Our results seem to suggest that ρ_χ for 135 MeV <= T <= 160 MeV is greater than 1.8 – 2. This conclusion was drawn from the behavior of the first two ratios r_2 and r_4.

To sum up…

Lattice QCD has made many invaluable contributions towards the study and deeper understanding of the quark-gluon plasma (QGP).

It remains the only way to calculate quantities directly from QCD in the difficult non-perturbative regime.

Much progress in the last few years: Pinning down the transition temperature, state-of-the-art equation of state for $\mu_B = 0$, $O(\mu_B^6)$ corrections to the equation of state, etc.

Still not enough to cover the entire range of beam energies scanned in BES-I and BES-II. Will need an equation of state to order $O(\mu_B^8)$ or more, valid for $\mu_B/T = 3\text{-}3.5$.

Beyond that: Much harder calculations involving spectral functions (transport coefficients, J/ψ suppression), clarifying the nature of the QCD phase diagram, finding the QCD critical point. Clearly much more work remains to be done!