Joint and Threshold Resummation in Prompt Photon Production : Soft Collinear Effects

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Resummation

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- Organization of large logarithms in perturbative expansion
- Construction from a subset of terms in a finite order perturbative series of an all order expression whose expansion gives at least those terms back

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Factorization

- Cross section for a process involving two hadrons A and B can be factorized into convolutions

$$\sigma = \Sigma_{a,b} \int dx_a \phi_{a/A}(x_a, \mu) \int dx_b \phi_{b/B}(x_b, \mu) \hat{\sigma}_{ab \to FX}$$

  $\phi_{a/A}$:distribution function for parton a with momentum fraction x_a in A
  $\hat{\sigma}$; partonic cross section
- Collinear singularities are factorized order by order and absorbed into pdf's
- $\hat{\sigma}$ is IR safe to all orders(Collins, Soper, Sterman 1988)

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- $\hat{\sigma}$ can be calculated order by order in perturbation theory

$$\hat{\sigma} = \Sigma c_n \alpha_s^n + R_n$$

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- LO: Lowest order usually needs tree graphs
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- Sometimes the series contains powers of some numerically large logarithm L

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- Single logs

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Resummation of large logs

Resummation is

- technology that organizes these logs in perturbative expansions
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$$\hat{\sigma} = \hat{\sigma}_0 exp\left[Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots\right] C(\alpha_s)$$

$g_1, g_2, \cdots$ are computable functions and lead to leading log (LL), next-to-leading-log(NLL) .....accuracy

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- What are these large logs?
- Depends on the observable in question
- For thrust T distribution $A = 1 - T$
- For $\frac{d\sigma(p\bar{p} \to Z+X)}{dp_T^Z}$, $A = \frac{M_Z}{p_T^Z}$
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Origin of large logs

- IR divergences cancel between real and virtual corrections
- For virtual corrections, integration is over all energies
- For real corrections, kinematics determines the phase space
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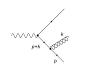
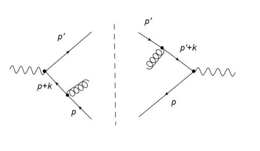


- propagator with $p^2 = k^2 = 0$. Singularities: $E_g = 0 \longrightarrow$ *soft*; $\theta_{qg} = 0 \longrightarrow$ *collinear*.

$$\frac{1}{(p+k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_g E_q(1 - \cos\theta_{qg})}\,.$$

- phase space integration

$$\alpha_s \int \frac{d^4k}{(2\pi)^4} \frac{p \cdot p'}{p \cdot k\, p' \cdot k} \sim \alpha_s \int \frac{dE_g}{E_g} \int \frac{d\theta_{qg}}{\theta_{qg}} \sim \alpha_s \ln^2(\dots)\,.$$



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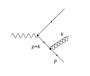

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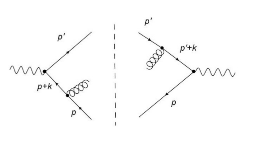

When dimensionally regulating this integral

$$\alpha_s \int \frac{d^{4-2\epsilon}k}{(2\pi)^4} \frac{p\cdot p'}{p\cdot k\, p'\cdot k} \sim \alpha_s \int^K \frac{dE_{\rm g} E_{\rm g}^{-\epsilon}}{E_{\rm g}} \int \frac{d\theta_{\rm qg} \sin^{-\epsilon}\theta_{\rm qg}}{\theta_{\rm qg}}$$
$$\sim \alpha_s \left(\frac{1}{\epsilon^2} + \ln^2(K)\right). \quad (1)$$

we see that double logs are directly linked to IR and COL divergences

How to gain all-order control over IR and COL divergences?

Factorization and Resummation

- Factorization Theorems→ a factorization of degrees of freedom

$$\sigma = J(col) \times J(col) \times S(Soft) \times G(off - shell)$$

- Factorization leads to resummation: an example(UV case)

$$G_B(p, \Lambda, g_B) = Z(\frac{\mu}{\Lambda}, g(\mu)) G_R(\frac{p}{\mu}, g_R(\mu))$$

- This implies

$$\frac{d}{d\mu} lnG_R(\frac{p}{\mu}, g_R(\mu)) = -\frac{d}{d\mu} lnZ(\frac{\mu}{\Lambda}) = \gamma(g_R(\mu))$$

- which leads

$$G_R(\frac{p}{\mu}, g_R(\mu)) = G_R(1, g_R(p)) exp\left[\int_p^\mu \frac{d\lambda}{\lambda}\gamma(g_R(\lambda))\right]$$

- One can do something analogous to IR + Collinear case above leading to exponentiation of large logs (Collins, Soper , Sterman)

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Prompt Photon Production

The process of interest

$$H_A + H_B \to \gamma + X$$

The lowest order QCD processes producing the prompt photon at

partonic cm energy $\sqrt{s}$ are

$$q(p_a) + \bar{q}(p_b) \to \gamma(p_c) + g(p_d)\,,\, g(p_a) + q(p_b) \to \gamma(p_c) + q(p_d)\,.$$

Pointlike coupling of photon to quark provides a clean em probe of QCD hard scattering

Threshold Resummation For Prompt Photon Production

In perturbative QCD, rapidity integrated cross section at fixed E_T is given by the following factorised formula

$$\begin{aligned}\frac{d\sigma_\gamma(x_T, E_T)}{dE_T} &= \frac{1}{E_T^3}\sum_{a,b}\int_0^1 dx_1\ f_{a/H_1}(x_1,\mu_F^2)\int_0^1 dx_2\ f_{b/H_2}(x_2,\mu_F^2) \\ &\cdot \int_0^1 dx \left\{\delta\left(x-\frac{x_T}{\sqrt{x_1x_2}}\right)\hat{\sigma}_{ab\to c\gamma}(x,\alpha_s(\mu^2);E_T^2,\mu^2,\mu_F^2,\mu_f^2)\right. \\ &+ \sum_c\int_0^1 dz\ z^2\ d_{c/\gamma}(z,\mu_f^2) \\ &\cdot \left.\delta\left(x-\frac{x_T}{z\sqrt{x_1x_2}}\right)\hat{\sigma}_{ab\to c}(x,\alpha_s(\mu^2);E_T^2,\mu^2,\mu_F^2,\mu_f^2)\right\}\end{aligned}$$

where $x_T = 2\frac{E_T}{\sqrt{S}}$ (Direct + Fragmentation terms)

The partonic cross sections $\hat{\sigma}_{ab\to cd}$ and $\hat{\sigma}_{ab\to c\gamma}$ are computable in QCD perturbation theory

$$\hat{\sigma}_{ab\to c\gamma} = \alpha\alpha_s[\hat{\sigma}^{(0)}{}_{ab\to c\gamma(x)} + \Sigma\alpha_s^n(\mu^2)\hat{\sigma}^n_{ab\to c\gamma}]$$

$$\hat{\sigma}_{ab\to cd} = \alpha\alpha_s[\hat{\sigma}^{(0)}_{ab\to cd(x)} + \Sigma\alpha_s^n(\mu^2)\hat{\sigma}^n_{ab\to cd}]$$

$\hat{\sigma}$ has been computed to NLO in perturbation theory.
Discrepancy between theory and data needs explanation
Possible remedy: Include resummation effects

Threshold Resummation

- Near partonic threshold, $E_T^\gamma \to \sqrt{x_1 x_2 S}$
- $\hat{\sigma}^{(n)}(x) \sim \ln^{2n}(1-x)$ due to soft gluon radiation leading to corrections to $\frac{d\hat{\sigma}}{dp_T}$ as large as $\alpha_s^k ln^{2k}(1-\hat{x}_T^2)\hat{\sigma}^{Born}$ at order α_s^k in perturbation theory
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Threshold resummation is performed by going over to Mellin-transform space or N-space

$$\sigma_{\gamma,N}(E_T) = \int_0^1 dx_T^2 (x_T^2)^{N-1} E_T^3 \frac{d\sigma_\gamma(x_T, E_T)}{dE_T}$$

Mellin moments of plus distributions give rise to powers of lnN in the Mellin N-moment expressions
In Mellin space

- Convolutions $\Rightarrow$Ordinary products
- $limit(\hat{x}_T^2 \to 1) \Rightarrow limit(N \to \infty)$
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Convolution in factorized cross section is converted to ordinary products in Mellin space leading to simple factorised form

$$\begin{aligned}
\sigma_{\gamma,\,N}(E_T) &= \sum_{a,b} f_{a/H_1,\,N+1}(\mu_F^2)\; f_{b/H_2,\,N+1}(\mu_F^2) \\
&\cdot \Big\{\hat{\sigma}_{ab\to\gamma,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2) \\
&+ \sum_c \hat{\sigma}_{ab\to c,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2)\; d_{c/\gamma,\,2N+3}(\mu_f^2)\Big\} \quad ,
\end{aligned}$$

in terms of the moments of each of the functions.

- Leading soft gluon correction terms $\sim \alpha_s^k ln^{2k} N$
- Leading log(LL): $\alpha_s^k ln^{2k} N$
- Next to leading log(NLL): $\alpha_s^k ln^{2k-1} N$
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All-order resummed expressions

All order resummed expressions (log-enhanced threshold contributions)

$$\hat{\sigma}^{(\mathrm{res})}_{q\bar{q}\to\gamma,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2) = \alpha\,\alpha_s(\mu^2)\,\hat{\sigma}^{(0)}_{q\bar{q}\to g\gamma,\,N}\; C_{q\bar{q}\to\gamma}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) \cdot \Delta^{q\bar{q}\to g\gamma}_{N+1}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2)\;,$$

and similar expressions for $\hat{\sigma}^{(\mathrm{res})}_{q\bar{g}\to\gamma,\,N}$ and $\hat{\sigma}^{(\mathrm{res})}_{\bar{q}g\to\gamma,\,N}$

$C_{ab\to\gamma}$ ⇒ N independent and hence constant for N large

$\Delta^{q\bar{q}\to g\gamma}_{N+1}$ ⇒ Radiative factors ⇒ ln N dependence fully embodied in these.

The Radiative factors

Depend on the flavour of the partons a, b, d of the LO process
They have an exponential form

$$\begin{aligned}\Delta_N^{ab\to d\gamma}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) &= \Delta_N^a(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) \\ &\cdot \Delta_N^b(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) \\ &\cdot J_N^d(\alpha_s(\mu^2), Q^2/\mu^2) \\ &\cdot \Delta_N^{(\mathrm{int})\, ab\to d\gamma}(\alpha_s(\mu^2), Q^2/\mu^2) \ .\end{aligned}$$

$$ln\Delta_N^a = lnNh_a^{(1)}(\lambda) + h_a^{(2)}(\lambda)$$

$$h_a^{(1)} = \frac{A_a^{(1)}}{2\pi b_0)\lambda}[2\lambda + (1-2\lambda)ln(1-2\lambda)]$$

with

$$\lambda = b_0\alpha_s(\mu^2)lnN$$

- $h_a^{(1)}(\lambda)$ and $h_a^{(2)}(\lambda)$do not depend separately on α_s and but are functions of expansion variable $b_0\alpha_s(\mu^2)lnN$.
- Thus all DL terms $c_{n,2n}\alpha_s^n ln^{2n}N$ terms in $\hat{\sigma}_N^{(n)}$ are produced by exponentiating LO contribution $c_{1,2}\alpha_s ln^2 N$.
  $h_a^{(1)}(\lambda)$ – LL exponent
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Recoil Effects

- Another source of higher order corrections - Recoil effects
- A soft gluon may be emitted before the hard scattering
- Outgoing pair recoils against soft gluon
- $\hat{\sigma}$ in $\frac{d\sigma_{AB\to\gamma X}}{d^2Q_T dp_T}$ is singular upto $\alpha_s^n[\frac{1}{Q_T^2} ln^{(2n-1)}(\frac{Q_T^2}{Q^2})]_+$.Here, Q_T is the transverse momentum of γq pair in Compton scattering for example.
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- In Colins-Soper formalism, the two different types of logs are summed by choosing appropriate cutoffs in the evaluation of soft gluon corrections.
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Joint Resummation

Proposed by Laenen, Sterman, Vogelsang 2000
Generalize threshold resummation to include recoil effects.

- Based on generalization of threshold resummation
- Reoganizes logs within collinear factorization - standard parton distributions are used.
- Derived from refactorization of partonic cross sections
- Reproduces threshold resummation when recoil effects are neglected
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JR expression for resummed cross section is

$$p_T^3 \frac{d\sigma_{AB\to\gamma}}{dp_T} = \sum_{ij} \frac{p_T^4}{8\pi S^2} \int_{\mathcal{C}} \frac{dN}{2\pi i}\, \tilde{\phi}_{i/A}(N)\tilde{\phi}_{j/B}(N) \int_0^1 d\tilde{x}_T^2 (\tilde{x}_T^2)^N \, \frac{|M_{ij}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}}$$

$$C_\delta^{(ij\to\gamma k)}(\alpha_s, \tilde{x}_T^2) \int \frac{d^2\mathbf{Q}_T}{(2\pi)^2}\, \Theta\left(\bar{\mu} - Q_T\right) \left(\frac{S}{4{\mathbf{p}'_T}^2}\right)^{N+1} P_{ij}(N, \mathbf{Q}_T, Q)$$

$\tilde{x}_T^2 = 4|\mathbf{p}_T - \mathbf{Q}_T/2|^2/\hat{s}$,
$\tilde{\phi}_{i/A}(N)$ Mellin moments of the parton distributions,
$|M_{ij}|^2$ the Born amplitudes,
$\bar{\mu}$ a cut-off restricting $\mathbf{Q}_T$ to sufficiently small values for resummation to be relevant.
$C_\delta^{(ij\to\gamma k)}$:infrared safe coefficient functions, which include short-distance dynamics at the scale Q.

$P_{ij}(N, \mathbf{Q}_T, Q)$:Profile function is a profile of $\mathbf{Q}_T$ -dependence for fixed N.

$$P_{ij}(N, \mathbf{Q}_T, Q) = \int d^2\mathbf{b} e^{-i\mathbf{b}\cdot\mathbf{Q}_T} e^{E_{ij\to\gamma k}}$$

where $e^{E_{ij\to\gamma k}}$ is the perturbative exponent given by

$$\begin{aligned} E^{\rm PT}_{ij\to\gamma k}(N, b, Q, \mu, \mu_F) &= E^{\rm PT}_i(N, b, Q, \mu, \mu_F) + E^{\rm PT}_j(N, b, Q, \mu, \mu_F) \\ &+ F_k(N, Q, \mu) + G_{ijk}(N, \mu)\,. \end{aligned}$$

Integral form of the initial state NLL exponent can be written as

$$E_i^{PT}(N, b, Q, \mu, \mu_F) = -\int_{\frac{Q^2}{\chi^2}}^{Q^2} \frac{dk_T^2}{k_T^2}[A_i(\alpha_s(k_T)) ln(\frac{Q}{\tilde{N}k_T})] \\ -2ln\tilde{N}\int_{\mu_F^2}^{Q^2} \frac{dk_T^2}{k_T^2} A_i(\alpha_s(k_T))$$

The function $\chi(N, b)$ defines the N- and b-dependent scale of soft gluons to be included in the resummation, and is chosen as

$$\chi(N, b) = \tilde{b} + \frac{\tilde{N}}{1 + \frac{\eta\tilde{b}}{\tilde{N}}},$$

$N \to \infty, b = 0$ reproduces threshold resummation.
$b \to \infty, N = 0$ reproduces recoil resummation.

LL and NLL exponents can be written in terms of functions $h^{(1)}$ and $h^{(2)}$ as before where now

$$h_a^{(1)}(\lambda, \beta) = \frac{A_a^{(1)}}{2\pi b_0 \beta}[2\beta + (1-2\lambda) ln(1-2\beta)]$$

etc, with
$\lambda = b_0 \alpha_s(\mu) ln(\tilde{N})$
$\beta = b_0 \alpha_s(\mu) ln(\chi)$
Preliminary analysis based on this approach showed substantial effects for prompt photon production.

Soft Collinear Effects

Another important class of potentially large terms are of the form

$$\alpha_s^i \sum_j^{2i-1} d_{ij} \frac{\ln^j N}{N} \,. \tag{2}$$

which have soft-collinear origin.
Two kinds of sources for such terms

- The singular plus distributions $[\ln^{2j-1}(1-z)/(1-z)]_+$,
  Can be included by keeping the subleading terms in Mellin transform of plus distributions
- For example

$$\int_0^1 dz z^N \left[\frac{ln(1-z)}{1-z}\right]_+ = \frac{1}{2} ln^2 N - \frac{1}{2}(lnN+1)\frac{1}{N} + \cdot$$

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Effect incorporated in threshold resummation by the replacement

$$\frac{z^{N-1}-1}{1-z}A_i^{(1)} \to \left[\frac{z^{N-1}-1}{1-z} - p_i z^{N-1}\right] A_i^{(1)} + \mathcal{O}(\frac{1}{N^2})\,, \qquad (3)$$

in each of the radiative factors
($p_q = 1, p_g = 2$),
The replacement is equivalent to exchanging at order j one soft-collinear gluon (corresponding to one factor $\alpha_s \ln^2 N$) for a hard-collinear one (corresponding to a factor $\alpha_s \ln N/N$

$$\alpha_s^k \ln^{2k} N \to \alpha_s^k \frac{\ln^{2k-1} N}{N}\,.$$

E Laenen, S. Majhi, AM and R. Basu, 2004

$\frac{lnN}{N}$ terms in JR

The initial state related $\alpha_s^k \ln^{2k-1} N/N$ terms can be generated in the context of joint resummation by extending evolution of parton densities to a soft scale.
Based on the observation that in JR resummed expression

$$E_i^{\rm PT}(N,b,Q,\mu,\mu_F) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left\{ A_i\left((k_T)\right) \ln\left(\frac{Q}{k_T}\right) + B_i\left((k_T)\right)\right\}$$
$$+ \int_{\mu_F^2}^{Q^2/\chi^2} \frac{dk_T^2}{k_T^2} \left\{ -\ln \bar{N} A_i\left((k_T)\right) - B_i\left((k_T)\right)\right\} .$$

the second term represents flavor-conserving evolution to NLL accuracy (the integrand consists of the $\ln N$ and constant terms for the anomalous dimension matrix $\gamma_{i/j}(N)$ for $j = i$) from the hard scale μ_F to the soft scale Q/χ.

The first term in this expression leads to

$$E_i^{\mathrm{PT}}(N, b, Q, \mu) = \frac{1}{\alpha_s(\mu)} h_i^{(0)}(\beta) + h_i^{(1)}(\beta, Q, \mu))$$

In second term replace

$$(-A_i(\alpha_s)\ln(N) - B_i(\alpha_s)) \longrightarrow \gamma_{i/i}(N)(\alpha_s)$$

This includes the leading, flavor-diagonal $\ln N/N$ effects generated by the k_T integral (the $1/N$ part of $\gamma_{i/i}$ combines with the $\ln N$ terms). Can also include the off-diagonal contributions via the replacement

$$\delta_{ig} \exp\left[\frac{-A_g^{(1)} \ln \bar{N} - B_g^{(1)}}{2\pi b_0} s(\beta)\right] f_{g/H}(N, \mu_F) \longrightarrow \mathcal{E}_{ik}(N, Q/\chi, \mu_F)\, f_{k/H}(N, \mu_F)\,.$$

where $s(\beta) = \ln(1 - 2\beta)$ plus NLL corrections.

As a result, we can replace the combination

$$f_{i/A}(\mu_F, N) f_{j/B}(\mu_F, N) \exp\left[E_i^{\mathrm{PT}}(N, b, Q, \mu, \mu_F) + E_j^{\mathrm{PT}}(N, b, Q, \mu, \mu_F)\right]$$

by

$$\mathcal{C}_{i/A}(Q, b, N)\, \mathcal{C}_{j/B}(Q, b, N)\ \exp\left[E_i^{\mathrm{PT}}(N, b, \mu, Q) + E_j^{\mathrm{PT}}(N, b, \mu, Q)\right]$$

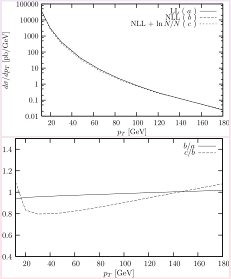
where

$$\mathcal{C}_{i/H}(Q, b, N) = \sum_k \mathcal{E}_{ik}\,(N, Q/\chi, \mu_F)\ f_{k/H}(N, \mu_F)\ .$$

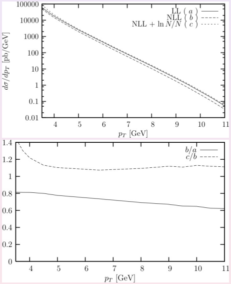
The matrix $\mathcal{E}$ implements evolution from scale μ_F to scale Q/χ, and is normalized to be the unit matrix if these two scales are equal.

Results

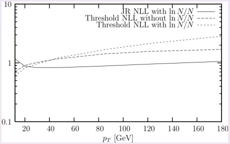
R.Basu, E.Laenen,AM,P.Motylinski2007 : Effect of including $\frac{lnN}{N}$ corrections in case of prompt photon
JR with and without $\frac{lnN}{N}$ *contribution*(*Tevatron*)



JR with and without $\frac{lnN}{N} contribution(E706)$



Threshold vs JR(Tevatron)



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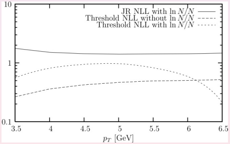

Frangmentation contribution

- Apart from the direct partonic sub processes, there are contributions from $2 \to 2$ hard scattering processes also in which one of the final state partons fragments into photon

$$q(p_a) + \bar{q}(p_b) \to q_(p_c) + \bar{q}(p_d) \,,$$

where the parton c subsequently fragments into a photon.

- Fagmentation component also contributes at $O(\alpha\alpha_s)$, expected to contribute to the cross section substantially (deFlorian et al 2005).

- Taking into account the fragmentation component, the p_T distribution of prompt photons in hadronic collisions is

$$\frac{p_T^3 d\sigma^{(\mathrm{resum})}_{AB\to\gamma+X}}{dp_T} = \frac{p_T^3 d\sigma^{(\mathrm{direct})}_{AB\to\gamma+X}}{dp_T} + \frac{p_T^3 d\sigma^{(\mathrm{frag})}_{AB\to\gamma+X}}{dp_T}$$

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$$\times \int_0^1 d\tilde{x}_T^2 \left(\tilde{x}_T^2\right)^N \frac{|M_{ab\to cd}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}} \; C^{(ab\to cd)}((\mu),\tilde{x}_T^2)$$
$$\times \int \frac{d^2\mathbf{Q}_T}{(2\pi)^2} \, \Theta\left(\bar{\mu} - Q_T\right) \left(\frac{S}{4\mathbf{p}'^{\,2}_T}\right)^{N+1}$$
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- Additional radiative factor for the final state parton c which fragments into photon
- Radiative term for fragmenting parton is similar to the initial state parton
- The underlying hard process is $2 \to 2$ scattering involving only partons and hence involves, unlike direct case any color tensor that may be constructed from color representations of the incoming partons. The different color structures may mix due to soft gluon emission hence the radiative factor for wide angle soft radiation is a matrix in color space.
- In a color basis, where the soft anomalous dimension matrix Γ_S is diagonal, the path-ordered exponentials of matrices reduce to a sum of simple exponentials

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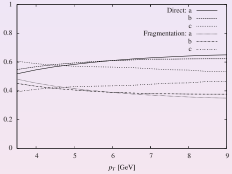
Results : Direct vs Fragmentation : Rahul Basu, E . Laenen, AM, P. Motylinski, hep-ph 1204.2503

- Direct vs fragmentation : E706 kinematics



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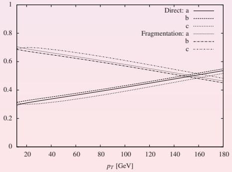

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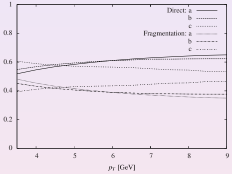

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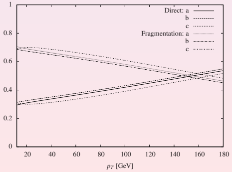
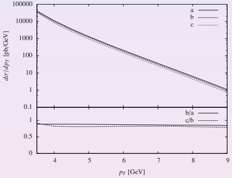


- Fragmentation only LL vs NLL vs lnN/N : E706 kinematics



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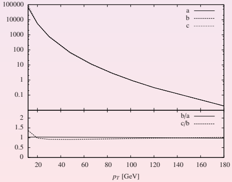

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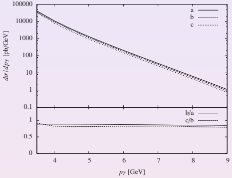

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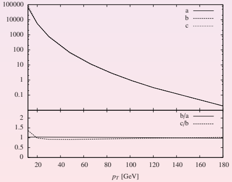
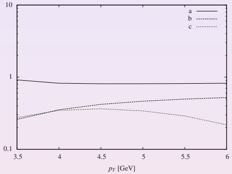


- Comparison of JR without lnN/N with JR with lnN/N (a), Threshold without lnN/N (b) and Threshold with lnN/N(c) : E706 kinematics



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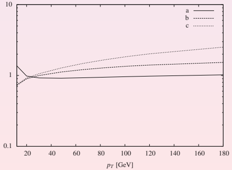

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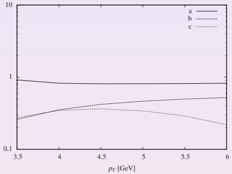

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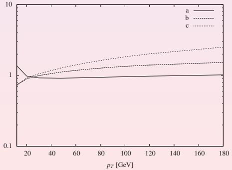

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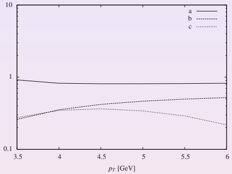

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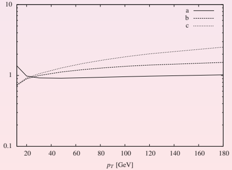
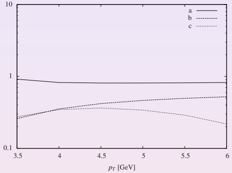


- Comparison of JR without lnN/N with JR with lnN/N (a), Threshold without lnN/N (b) and Threshold with lnN/N(c) : E706 kinematics



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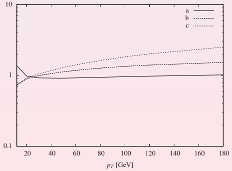

- So far, evolution of final state fragmentation function for parton fragmenting into photon not included
- Evolution method tricky - singularity at $\mathbf{p_T} = \frac{\mathbf{Q_T}}{2}$
- Singularity dealt with by putting a cutoff, but the resummed expression should hold
- Alternative way : Approximate the kinematical factor $\left(\frac{S}{4(\bar{p}_T - \frac{\bar{Q}_T}{2})^2}\right)^{N+1} = \left(\frac{4p_T^2}{S}\right)^{-N-1} \left(1 - \frac{\bar{p}_T \cdot \bar{Q}_T}{p_T^2} - .\right)^{-N-1}$
- R.Basu, W. Benakker, E.Laenen, AM, P. Motylinski, M. van Beekweld, In preparation

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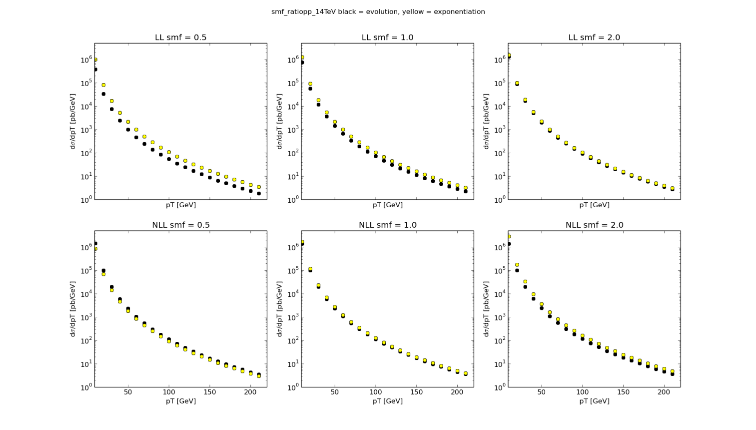
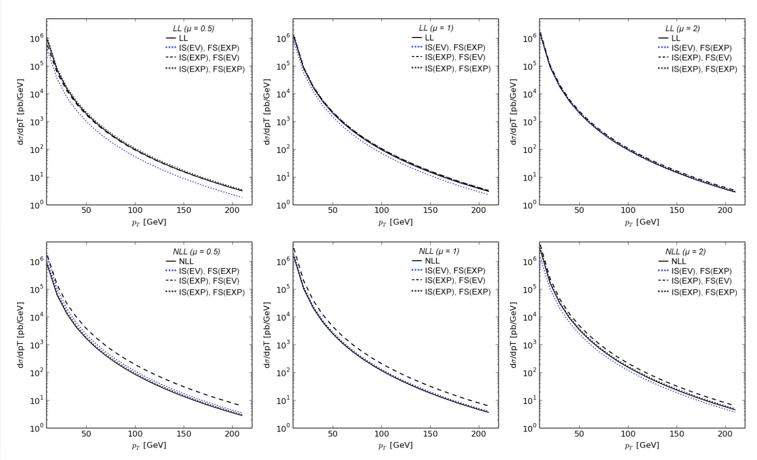
Preliminary Results




LL (μ = 0.5)
LL
IS(EV), FS(EXP)
IS(EXP), FS(EV)
IS(EXP), FS(EXP)
LL (μ = 1)
LL (μ = 2)
NLL (μ = 0.5)
NLL
NLL (μ = 1)
NLL (μ = 2)
dσ/dpT [pb/GeV]
p_T [GeV]

Summary

- Joint resummation extends QCD's predictive power beyond LO, NLO, NNLO...
- Many improvements possible in joint resummation, summing purely collinear enhancements ($\ln^i N/N$))
- Effect of including the leading term in resummed cross section is substantial
- Including fragmnetation contribution in lnN/N accuracy, substantial contribution in threshold but small effct in JR - corrections due to recoil effects overshadow soft-collinear effects
- It may be worthwhile to include sub leading terms of teh kind $\ln^i N/N$ and assess their impact