Color-flavor center symmetry of QCD and its order parameter

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Order parameter and QCD : The common lore

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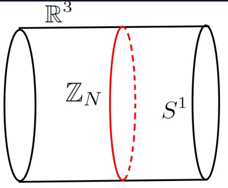

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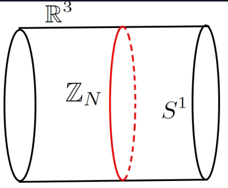

- Adding fundamental fermions spoils center symmetry $\longrightarrow$ Polyakov loop not an order parameter any more.

Recap : Confinement

credit : Nobel Prize Committee 2004

Center symmetry

Is there a symmetry whose realization can work as an indicator of confinement ?

Only for pure YM on $\mathcal{R}_3 \times \mathcal{S}_1$.

Take YM, compactify the time direction,

Use periodic boundary conditions for the gauge fields.
$\rightarrow$ thermal physics with extent of time direction = inverse temperature.

Only periodic gauge transformation are allowed in order to preserve periodicity of gauge fields in the temporal direction. But there is one exception !

Center symmetry contd..

Exception is center transformation.

Center transformations are periodic up to an element of the center of the group : the set of elements in a group that commute with all other elements of the group.

$A'_\mu \to U A_\mu U^\dagger - U \partial_\mu U^\dagger$

with $U(x, t+\beta) = C U(x,t), C \in$ Center. Easy to check that periodicity of gauge field A' is preserved by this transformation. Looks like a gauge transformation, but is not one unless $C = 1$.

$\to$ a legitimate symmetry of the theory.

Order parameter

There is a gauge invariant object that transforms under this symmetry : Polyakov loop

$$\langle \text{tr } \Omega \rangle = \langle \mathcal{P} e^{i \int_0^t dx_0 A_0} \rangle$$

Under a center transformation $\langle \text{tr } \Omega \rangle \to C \langle \text{tr } \Omega \rangle$.

$\langle \text{tr } \Omega \rangle = 0$ stands for confinement where as $\langle \text{tr } \Omega \rangle \neq 0$ stands for deconfinement.

This nice story falls apart the moment one introduces fundamental quarks in the problem.

Catastrophe with fundamental quarks

The boundary condition for fundamental quarks have to anti-periodic in the time direction.

Under center

$$q'(x,t) = U(x,t)q(x,t)$$

If $q(t+\beta) = -q(t)$, then $q'(t+\beta) = -Cq'(t)$.

Hence center symmetry does not preserve the boundary conditions and is no longer a symmetry of the theory.

Order parameter and QCD : The common lore

Exploring phase transitions in QCD involves subtleties :-

- For massless quarks there is $G = SU(n_{\rm f})_V \times SU(n_{\rm f})_A \times U(1)_Q$ and the chiral condensate $\langle \sum_a \bar{q}_a q_a \rangle$ is an order parameter.

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- Massive quarks break chiral symmetry explicitly $\longrightarrow$ $\langle \sum_a \bar{q}_a q_a \rangle$ no longer an order parameter.
- Hence QCD with massive dynamical fermions lacks non-trivial order parameter at zero baryon density.

Can we do better ?

Possible with degenerate quark flavors !

- On $\mathcal{R}_4$ there is $U(n_\mathrm{f})_V$ flavor symmetry as well as flavor permutation symmetry.

- Consider the theory on $\mathcal{R}_3 \times \mathcal{S}_1$.

- Use flavor-twisted boundary condition on quarks ($\mathbb{Z}_{n_\mathrm{f}}$ symmetric)

$$q_a(x_1+L) = \mathcal{U}^{ab}\, q_b(x_1)\,.$$

H. Kouno, Y. Sakai, T. Makiyama, K. Tokunaga, T. Sasaki, M. Yahiro, K. Kashiwa, T. Iritani, E. Itou, T. Misumi, T. Hirakida, J. Takahashi, T. Hirakida, J. Sugano

Can we do better ?

- Choose $\mathbb{Z}_{n_\mathrm{f}}$ symmetric BC where $N = \#$ of colors, $n_\mathrm{f} = \#$ of flavors,

$$\mathcal{U} = \mathrm{diag}(1, \nu, \cdots, \nu^{n_\mathrm{f}-1}), \qquad \nu \equiv e^{2\pi i/n_\mathrm{f}}\,, \tag{1a}$$

or

$$\mathcal{U} = \mathrm{diag}(\nu^{1/2}, \nu^{3/2}, \cdots, \nu^{n_\mathrm{f}-1/2})\,. \tag{1b}$$

- key observation : For

$$d \equiv \gcd(n_\mathrm{f}, N) > 1\,, \tag{2}$$

center transformation followed by a $\mathbb{Z}_d$ cyclic flavor permutation is a symmetry of the theory $\equiv$ CFC symmetry.

What is it goof for ?

CFC will tell us nothing about confinement. But it has something to say about the QCD phase diagram and conformal window studies.

QCD phase diagram recap : Consider QCD phase diagram in temperature T vs baryon chemical potential μ plane.

- Low T, low μ corresponds to hadronic matter.
- High T region is quark-gluon plasma (QGP).
- High μ and low T color superconducting $\rightarrow$ CFL.

QCD phase diagram ?

keep in mind that

- Twisting thermal bc will tamper with thermal physics - so don't.
- Compactify one of the spatial directions (Length= L) and apply flavor twist along it.
- Ask about the Polyakov loop along the spatial circle.
- We need to make L large to comment on the phase diagram $L \gg \frac{1}{\mu}$, $L \gg \frac{1}{T}$.

$\mu \gg \Lambda$ and $g\mu \gg T$

Specialize to three color three flavor case. Set quark mass to zero.

one-loop effective potential for Ω at weak coupling :
$V_{\text{eff}}(\Omega) = V_{\text{g}}(\Omega) + V_{\text{f}}(\Omega)$

where gluon contribution :

$$V_{\text{g}}(\Omega) = -\frac{1}{L^4} \sum_{n=1}^{\infty} f_n^g \left(|\text{tr}\, \Omega^n|^2 - 1 \right).$$

gluons pick Meissner mass (m_g) due to CFL pairing.

Hence $f_n^g \sim e^{-n m_{\text{g}} L}$, when $m_{\text{g}} L \gg 1$.

Fermion contribution :

$$V_f(\Omega) \sim \sum_{n=1}^{\infty} (\text{tr}\,\mathcal{U}^n \text{tr}\Omega^n + \text{h.c.})\#(n) \tag{3}$$

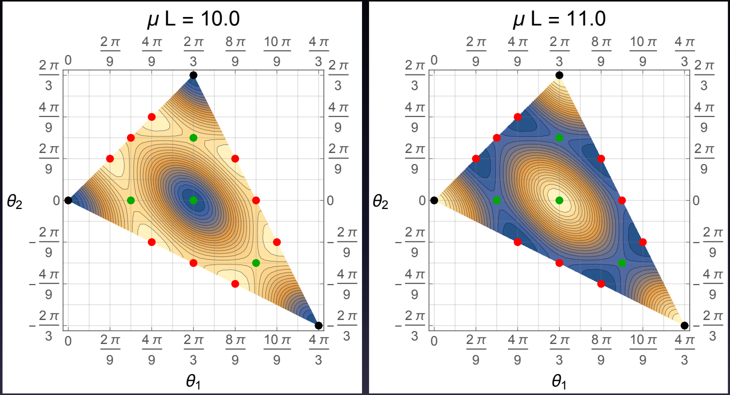
where $\#(n) \sim e^{-nTL}$ and the leading contribution to the fermion effective potential is

$$V_{\rm f}(\Omega) = \frac{\pm 2T\, e^{-n_{\rm f}\pi LT}}{n_{\rm f}\pi L^2}\left[\mu \sin(n_{\rm f}\mu L) + \pi T \cos(n_{\rm f}\mu L)\right]$$
$$\times\,(\text{tr}\,\Omega^{n_{\rm f}} + \text{h.c.}) + (\text{holonomy-independent}),$$

$V_{\rm eff}(\Omega) \propto \mathrm{Re}\,\mathrm{tr}\,\Omega^{n_{\rm f}} \times$ oscillating function of $n_f \mu L$. Four categories of extrema :

- center-symmetric extremum at $\Omega = \mathrm{diag}\,(1, e^{2\pi i/3}, e^{4\pi i/3})$
- three center-broken extrema with $\Omega = \mathrm{diag}\,(e^{(2k-1)i\pi/3}, e^{2ki\pi/3}, e^{(2k+1)i\pi/3})$, $k = 0, 1, 2$;
- nine center-broken "$SU(2) \times U(1)$" extrema $\Omega = \mathrm{diag}\,(e^{ki\pi/9}, e^{ki\pi/9}, e^{-2ki\pi/9})$ with $k \bmod 6 = 2$, 3 or 4;
- three center-broken "$SU(3)$" extrema, $\Omega = \mathrm{diag}\,(e^{2ki\pi/3}, e^{2ki\pi/3}, e^{2ki\pi/3})$, $k = 0, 1, 2$.

Contour plot for V_f



(e) V_f

Darker colors indicate lower values of $L^4 V_{\rm eff}$. The center-symmetric point $(\theta_1, \theta_2, \theta_3) = (0, 2\pi/3, 4\pi/3)$ lies at the center while the corners are the coinciding eigenvalue points $(0,0,0)$ and $\pm(2\pi/3, 2\pi/3, 2\pi/3)$. Dots denote critical points of $\widehat{V}_{\rm f}$.

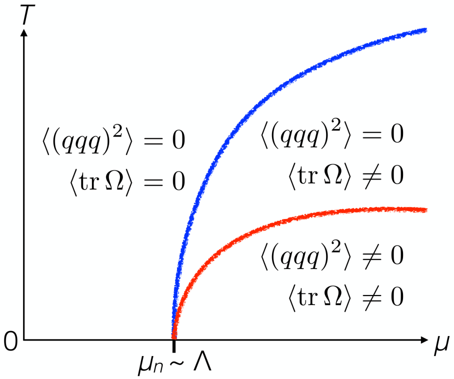
Quantum oscillation and multiphase point

- Quantum oscillations as a function of μL.
- Two groups of degenerate extrema :
  - center-symmetric extremum and three $SU(3)$ extrema
  - six $SU(2) \times U(1)$ extrema
- $L \to \infty$ is an accumulation point or multiphase point.
- The small residual gluon contribution to V_{eff} favors $SU(3)$ extrema. Hence multi-phase region $\longrightarrow$ broken CFC symmetry, with $\langle \text{tr}\, \Omega \rangle \neq 0$

Rest of the phase diagram

- Small T, small μ regime : lattice studies imply that $\langle\Omega\rangle = 0$.
- High T regime with $T \gg \max(\Lambda, \mu)$: the dynamics on spatial scales large compared to $(g^2 T)^{-1}$ are described by pure 3D YM theory which confines, so $\langle \mathrm{tr}\, \Omega\rangle = 0$.
- We expect the high-temperature region to be smoothly connected to the region near $T = \mu = 0$.

The full picture : Phase diagram



Sketch of a possible phase diagram of circle-compactified $SU(3)_V$ symmetric QCD at $m_q > 0$, as a function of T and μ, in the large L limit.

Conformal window

Let $x \equiv n_{\mathrm{f}}/N$, and $m_q = T = \mu = 0$.

- If $x > \frac{11}{2}$, gauge theory becomes an infrared-free theory.
- For x below some $x_\chi < \frac{11}{2}$, chiral symmetry is believed to be spontaneously broken.
- In the range $x \in (x_\chi, \frac{11}{2}) \longrightarrow$ non-trivial infrared (IR) fixed point, NO chiral symmetry breaking $\equiv$ "Conformal Window".

Conformal window at large N

- Choose N, n_{f} such that $d = \gcd(n_{\mathrm{f}}, N)$ is fixed and greater than 1, while the ratio $x = n_{\mathrm{f}}/N$ approaches a non-zero limit.
- If $\epsilon \equiv \frac{11}{2} - x \to 0^{+}$, $\exists$ an IR fixed point with a parametrically small coupling $\lambda_{\mathrm{IR}} = \frac{64}{75}\pi^2\epsilon \ll 1$.[1, 2]
- Compute the quantum effective potential $V_{\mathrm{eff}}(\Omega)$ for λ at all scales when $\epsilon \ll 1$: analysis valid for all L.
- Classically, $V_{\mathrm{eff}}(\Omega)$ is zero.

1. W. E. Caswell, Phys. Rev. Lett. 33, 244 (1974).
2. T. Banks and A. Zaks, Nucl. Phys. B196, 189 (1982).

Conformal window

At one loop $V_{\rm eff}(\Omega) = V_{\rm g}(\Omega) + V_{\rm f}(\Omega)$ with gluon and fermion contributions given by

$$V_{\rm g}(\Omega) = -\frac{2}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \left(|\mathrm{tr}\, \Omega^n|^2 - 1 \right),$$

and

$$V_{\rm f}(\Omega) = \frac{2}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \left(\mathrm{tr}\, \mathcal{U}^{-n}\, \mathrm{tr}\, \Omega^n + \mathrm{tr}\, \mathcal{U}^n\, \mathrm{tr}\, \Omega^{-n} \right)$$

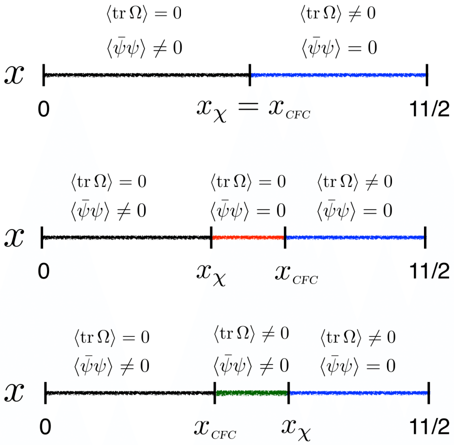
$$= \frac{2}{\pi^2 L^4 {n_{\rm f}}^3} \sum_{n=1}^{\infty} \frac{(\pm 1)^n}{n^4} \left(\mathrm{tr}\, \Omega^{n_{\rm f} n} + \mathrm{h.c.} \right) .$$

$V_{\rm g} = O(N^2)$ and $V_{\rm f} = O(N^{-2})$ $\longrightarrow$ $V_{\rm g}$ dominates setting $\Omega \propto 1$ $\longrightarrow$ breaks CFC.

Conformal Window

- When $\epsilon = \frac{11}{2} - x \ll 1$ the CFC symmetry is spontaneously broken at any L.
- At the pure Yang-Mill point, $x = 0$, center symmetry is certainly expected to be unbroken at large L.
- There must be at least one transition at some $x = x_{\mathrm{CFC}}$ where the realization of the CFC symmetry changes.
- This point may or may not coincide with the point x_χ where the chiral symmetry realization changes.

The full picture : Conformal window



Possible phase structures of massless QCD as a function of $x = n_{\mathrm{f}}/N$. The chiral and CFC symmetry realizations change at some $x = x_\chi$ and $x = x_{\mathrm{CFC}}$, respectively.

CFC and Local Order parameter

- CFC has local order parameters as well :

  $\mathcal{O}_\Gamma^{(p)} \equiv \sum_{a=1}^{n_\mathrm{f}} \nu^{-ap}\, \bar{q}_a \Gamma q_a$, where Γ is an arbitrary Dirac matrix, $p \bmod d \neq 0$ and $\nu \equiv e^{2\pi i/n_\mathrm{f}}$.

The action of the $\mathbb{Z}_d$ CFC symmetry is

$$\mathrm{tr}\, \Omega^p \;\to\; \omega^{Np/d}\, \mathrm{tr}\, \Omega^p \,, \quad \mathcal{O}_\Gamma^{(p)} \;\to\; \nu^{n_\mathrm{f} p/d}\, \mathcal{O}_\Gamma^{(p)} \,.$$

Center and confinement

- Consider

$$\langle \mathrm{tr}\, \Omega(\vec{x})\, \mathrm{tr}\, \Omega^\dagger(0) \rangle_{\mathrm{conn}} \equiv e^{-F(r)} \,, \qquad r = |\vec{x}| \,.$$

- Suppose $\exists$ non-zero lower bound E on the energy of states that contribute to the correlator, so $F(r) \sim E\, r$ as $r \to \infty$.

- For $n_f = 0$ $\mathbb{Z}_N$ symmetric ground state :

  - No intermediate local operator contributes to the correlator.

  - All contributions must involve flux tubes which wrap the compactified dimension, with $E = L\sigma$ with σ the string tension. $\longrightarrow$ "Confinement of static quarks by flux tubes".

Contrast between CFC and center

- Of course with $n_f \neq 0$ center is broken explicitly $\longrightarrow$ local operators ccontribute to the correlator $\longrightarrow$ string breaking : "deconfinement".

- Local order parameters transforming under CFC exist even when CFC is not broken explicitly or spontaneously.

- For example, states created by $\mathcal{A} \equiv \sum_{a=1}^{n_\mathrm{f}} \nu^{-ap}\, \bar{q}_a \gamma_1 D_1 q_a$ and $\mathcal{B} \equiv \sum_{a=1}^{n_\mathrm{f}} \nu^{-ap}\, \bar{q}_a \gamma_1 q_a$ can contribute to correlators of $\mathrm{Re}\,\mathrm{tr}\,\Omega^p$ and $\mathrm{Im}\,\mathrm{tr}\,\Omega^p$, respectively.

- In summary, for $n_\mathrm{f} > 0$ No relation between the presence of a non-zero string tension and the existence, or realization, of CFC.

Summary

- Well-defined and non-trivial order parameters for quantum and thermal phase transitions exists in QCD compactified on a circle with degenerate massive flavors of quarks.
- A calculation of the one-loop gluon contribution to $V_{\text{eff}}(\Omega)$ in the hard dense loop approximation would give a better estimate for the CFC symmetry restoration temperature $T_{\text{CFC}}(\mu)$.
- The role of explicit $SU(3)_V$ symmetry breaking should be explored.
- It would be interesting to study local order parameters for CFC symmetry.

references

H. Kouno, Y. Sakai, T. Makiyama, K. Tokunaga, T. Sasaki, M. Yahiro, K. Kashiwa, T. Iritani, E. Itou, T. Misumi, T. Hirakida, J. Takahashi, T. Hirakida, J. Sugano