Dissipationless Nernst effects

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D. Bergman and V. O., PRL 104, 066601 (2010)
V. O., S.Sondhi, D.Huse, PRB, 73, 094503 (2006)
V. O. and I. Ussishkin, PRB 70, 054503 (2004)

outline

- What is Nernst effect and why study it – experiments

- “Standard” theories:

  linear response from Drude-Boltzmann, classical and quantum critical kinetics and hydrodynamics;

- New results from Landau levels:

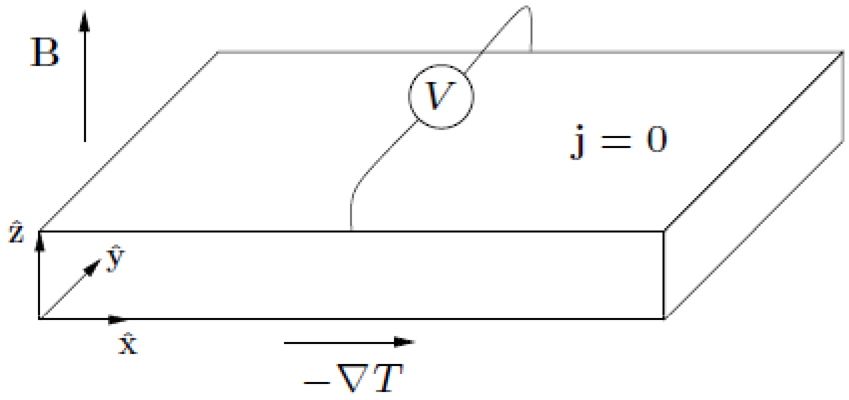
  edge theory, disorder in 2D, irrelevant but important interactions; 3D

- Open questions and afterthoughts

Nernst thermopower

Gradients and currents:

$$\mathbf{J} = \sigma \cdot \mathbf{E} - \alpha \cdot \nabla T \,,$$
$$\mathbf{J}^{\mathbf{Q}} = T\alpha \cdot \mathbf{E} - \kappa \cdot \nabla T \,,$$



Thermopower tensor:

$$J = 0 \rightarrow S = \sigma^{-1} \cdot \alpha$$

Nernst thermopower and coefficient, ν:

$$S_{xy} = \rho_{xx}\alpha_{xy} + \rho_{xy}\alpha_{yy} \equiv B\nu$$

Why study Nernst effect?

Why study Nernst?

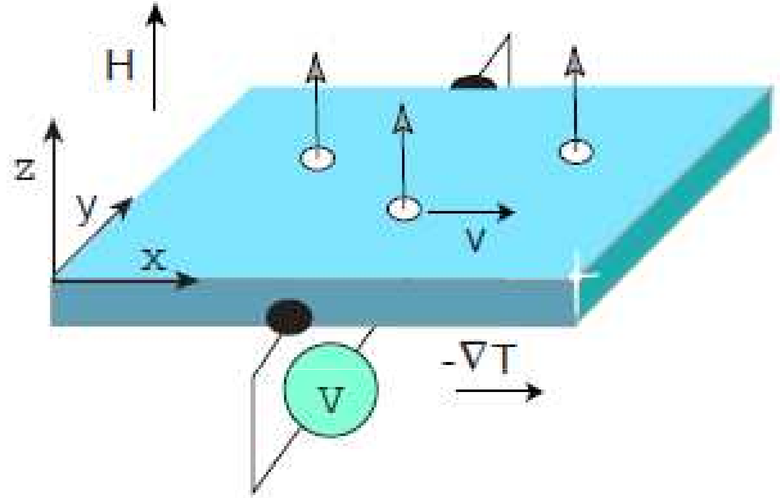
Voltage without current? Must be vortices!

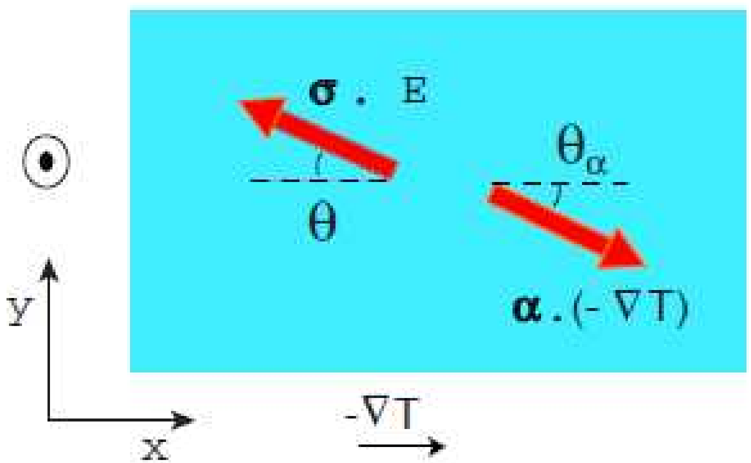
<u>keywords:</u>
entropic force,
vortex drift,
vortex entropy

<u>key person:</u>
N.P.Ong

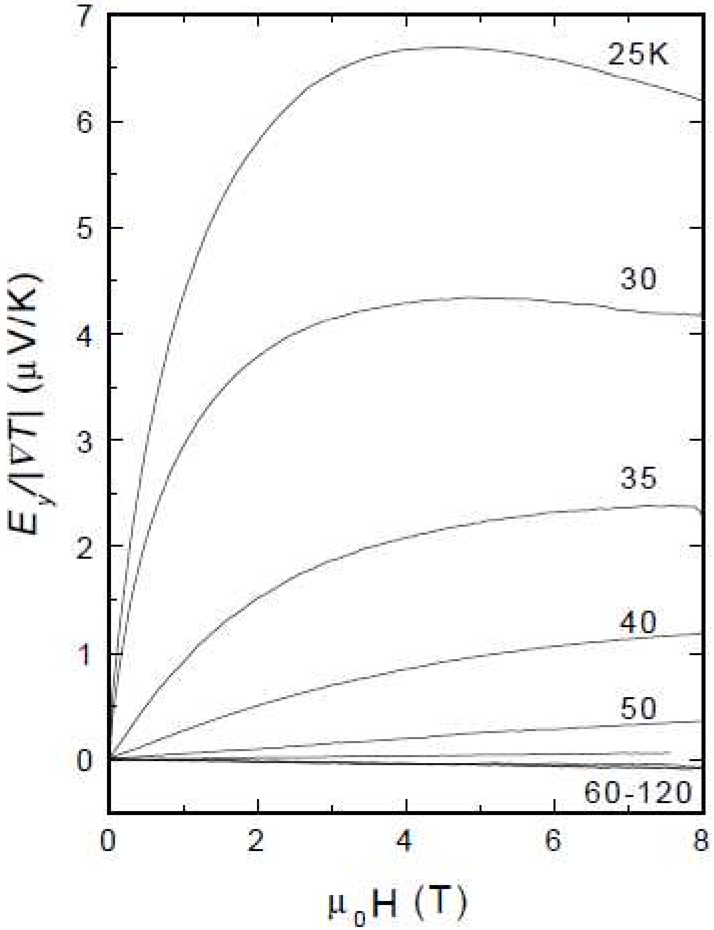
No established <u>key theory</u>
based on vortices

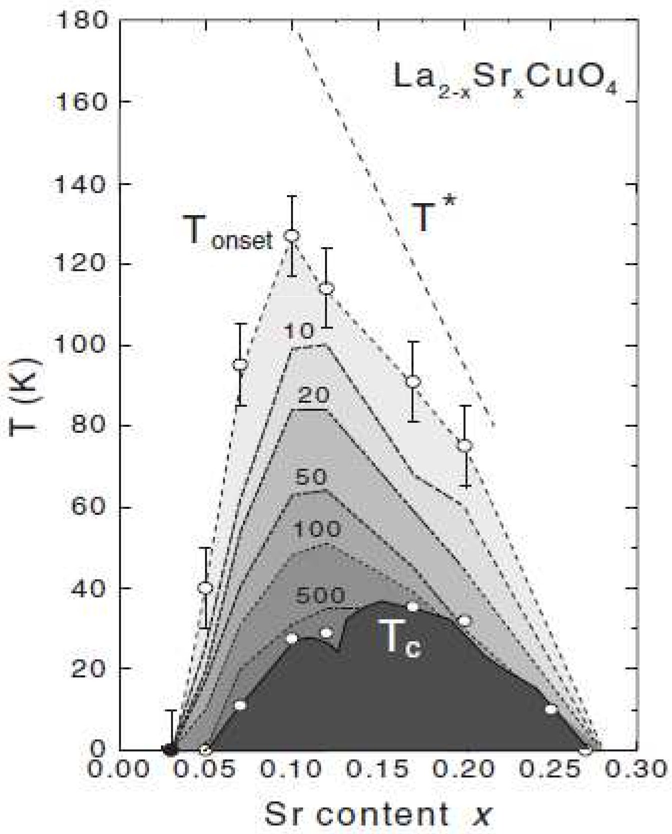
<u>key question:</u>
Do vortices exist
<u>above</u> Tc in cuprates?
i.e. does superconductivity
survive in some form outside the
superconducting state?





Clear and unambiguous Nernst signal inside the "pseudogap" regime, smoothly connecting to the vortex signal below Tc

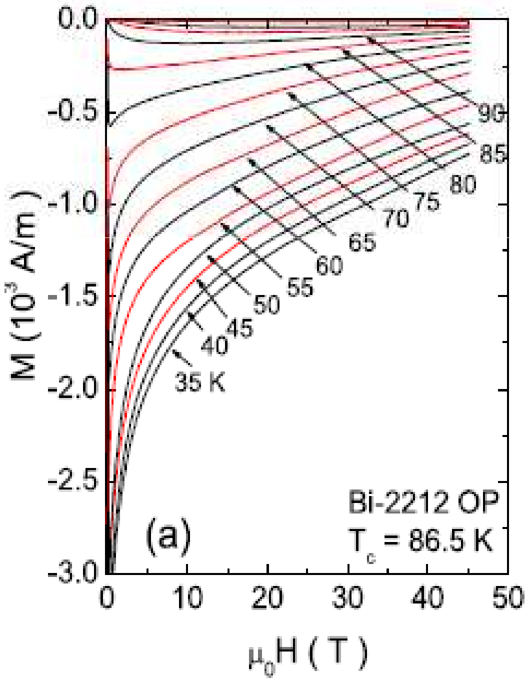


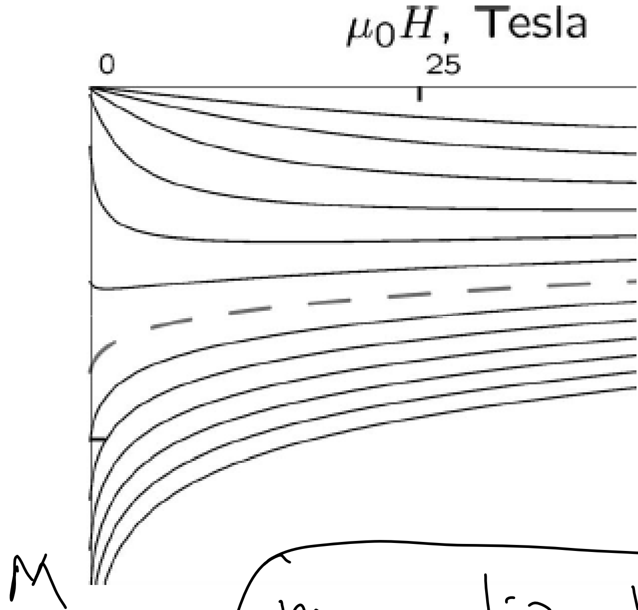


Ong, Wang, Li, etal, 1999 onwards

Are these strong superconducting fluctuations?

- No clear fluctuation conductivity is observed in cuprates.
- Relatively recently strong fluctuation diamagnetism was observed using





Theory based on vortices: Kosterlitz-Thouless + B (V.O. etal 2006)

magnetization

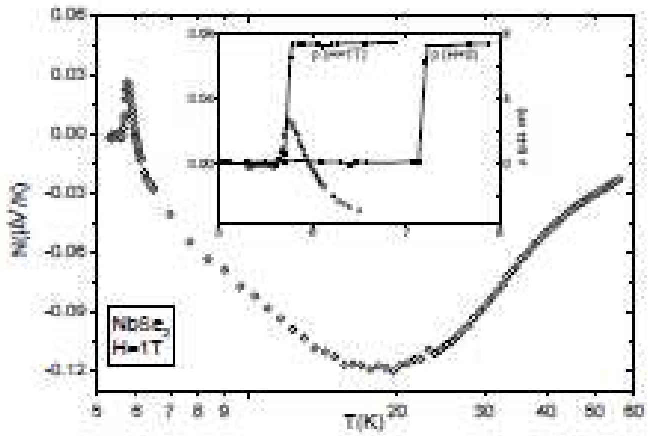
$$m = -\frac{\partial F}{\partial B}$$

- Nernst effect has also been seen in the fluctuation regime in dirty low Tc SC film superconductors (exp. - Behnia etal; theory Ussishkin etal, Galitski etal, Michaeli/Finkelstein)

Other reasons to measure Nernst

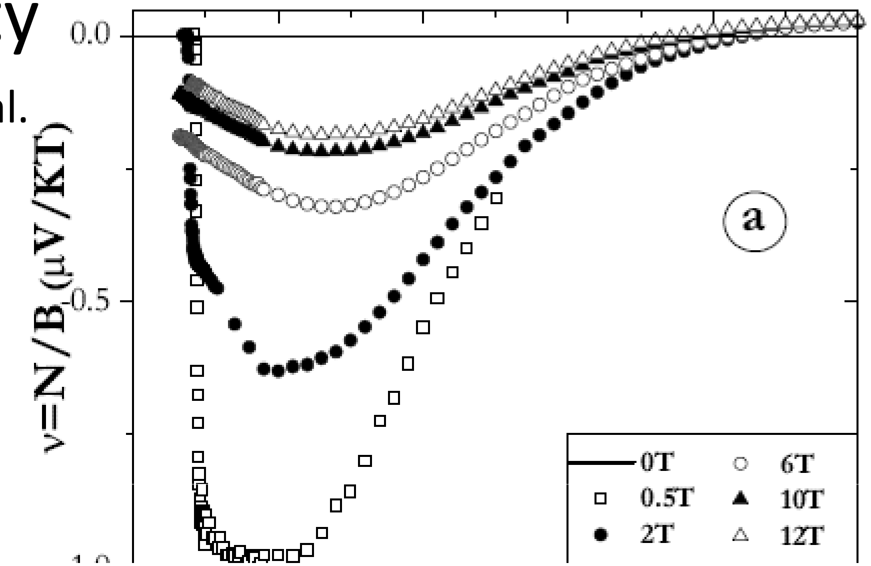
- α_{xy} is traditionally difficult to measure well because it is small and not strongly varying, e.g. with temperature or field.
- Other than in superconductors, why would anyone study it? What qualitative physics is there to learn?
- Experimentalists took different, hands-on approach: measure first, question later

Nernst effect often shows strong field and temperature dependences in correlated electron materials, stronger than conventional conductivity



CDW metal -- $NbSe_2$

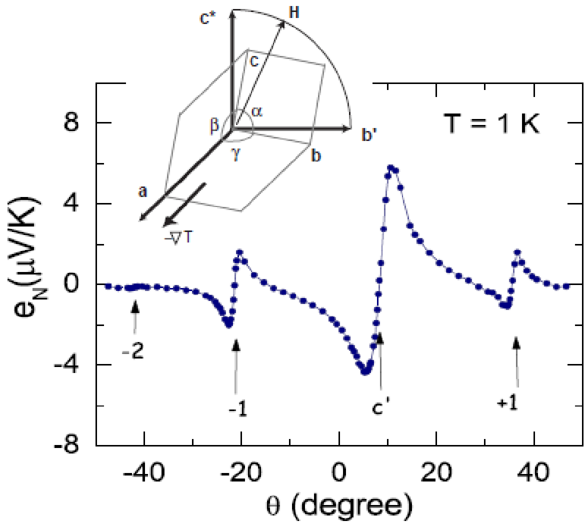
Behnia et. al.



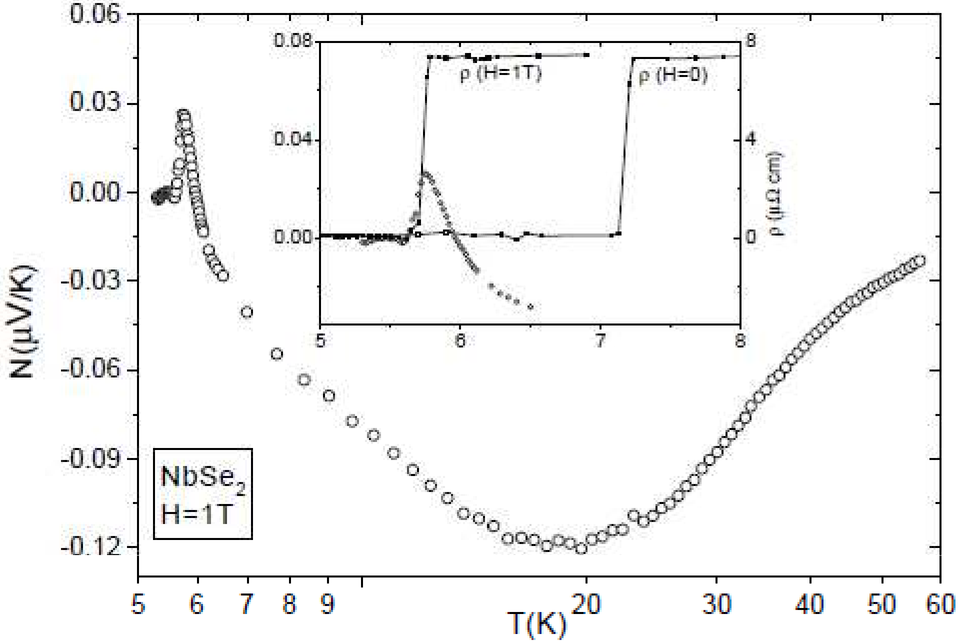
$CeCoIn_5$ -- near critical Kondo lattice

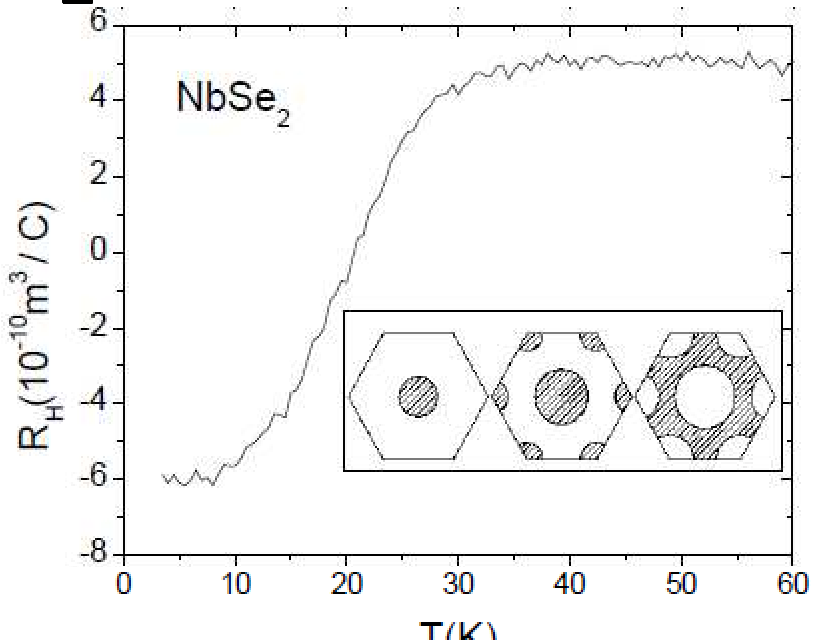
Motivation is not always clear but results are compelling…what do they mean?

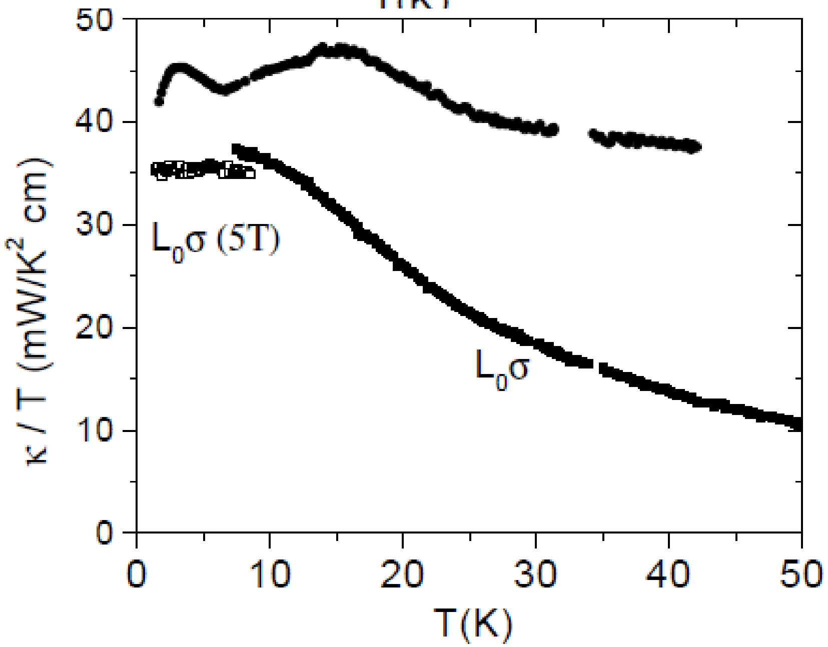
Quasi 1D organics (Chaikin et al)



$NbSe_2$

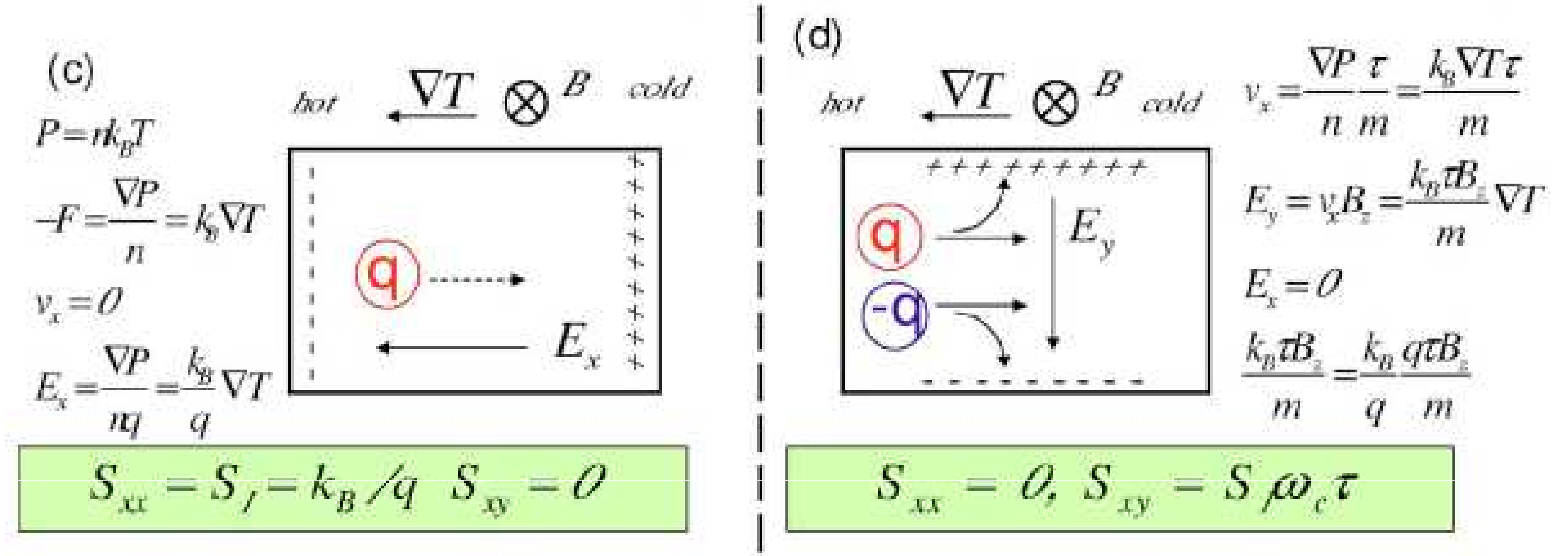






Nernst effect shows
an apparently broad
"fluctuation" regime
well above Tcdw=34K (Tsc=4K)

Nernst from drude theory



Nerst thermopower vanishes in a simple kinetic theory of one type of carriers, need two (Wu,Chaikin, 2005)

This is an artefact –
an exact cancellation in: $$S_{xy} = \rho_{xx}\alpha_{xy} + \rho_{xy}\alpha_{yy}$$

Oftentimes (in experiments) only first $\rho_{xx}\alpha_{xy}$, term is appreciable

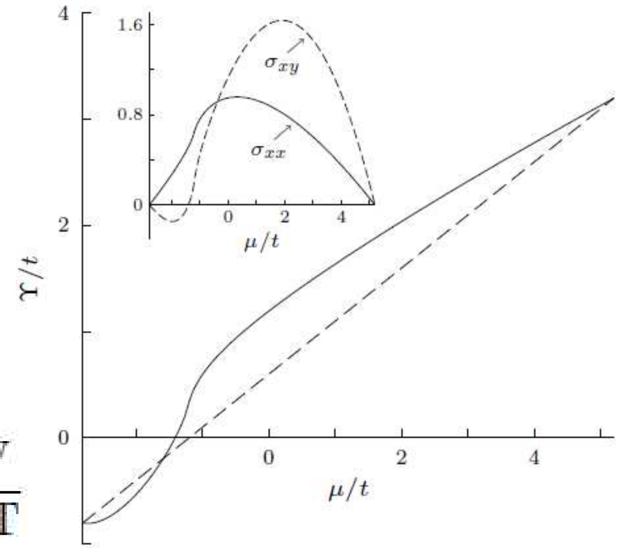
Nernst from Boltzmann equation – band curvature can be quantitatively significant!

Relaxation time approx, steady state distribution, the cross-term:

- $\delta n_k = \tau^2 e \frac{\partial n}{\partial \epsilon_k} \vec{E}_k \cdot \frac{1}{m} \cdot \vec{v}_k \times \vec{B} \;\rightarrow\; \langle \vec{j} \rangle = \int dk e \vec{v}_k \delta n_k$
- $\vec{v}_k = \partial \epsilon_k / \partial \vec{k}$
- $\vec{E}_k = e\vec{E} - \vec{\nabla}\mu - \frac{\epsilon_k - \mu}{k_B T} \vec{\nabla} T$
- Simple low T expansion, the Mott formula: $\alpha = -\frac{\pi^2}{3} \frac{k_B^2 T}{e} \frac{\partial \sigma}{\partial \mu}$
- From $\nu = -\frac{\pi^2 k_B^2 T}{3eB} \frac{\partial \Theta_H}{\partial \mu}$ we can do dimensional analysis/phenomenology
- $B\nu = -\frac{\pi^2}{3} \frac{k_B}{e} \frac{a^2}{\ell_B^2} \frac{k_B T \tau}{\hbar} \frac{\partial \Upsilon}{\partial \mu}$
- $k_B/e \approx 86\mu V/K$ is the Planck constant;

  everything else is dimensionless,

  On average $\langle \frac{\delta \Upsilon}{\delta \mu} \rangle = \frac{1}{W} \frac{\hbar^2}{a^2} (\frac{1}{m_e} + \frac{1}{m_h}) \approx \frac{1}{2}$



Simple dimensional analysis does NOT preclude large Nernst effect:

$$\nu \approx -100 \frac{k_B T \tau}{\hbar} \frac{\mathrm{nV}}{\mathrm{K\,T}}$$

Strong correlations and Nernst effect: critical hydrodynamics

- General intuition: “entropy currents”, “vortex entropy”
- Classical critical dynamics, TDGL: Ussishkin (Gaussian fluct.)
- TDGL and vortex liquid numerics: Mukerjee et. al., Podolsky et.al.

- Incomplete intuition: modifying models to increase entropy should suppress diamagnetism (and it does in the simulation) but it should produce more Nernst (which is also suppressed in the simulation)

- Quantum criticality - quantum critical kinetics (Bhaseen etal), quantum critical hydrodynamics and black holes etc. (Mueller/Sachdev/McGreevy et. al.)

Summary of review:

- Strange looking thermoelectric probe, α_{xy}, appears to show enhanced sensitivity to interesting electronic behavior, e.g. fluctuation superconductivity (and, perhaps, CDW formation)
- Theory tends to be "difficult" and not transparent intuitively. What does a Nernst measurement tell us, qualitatively?

- Next – some new theory (and experiments) in high magnetic fields

High magnetic field – Landau levels (in the Landau gauge)

$$H = -\frac{\hbar^2}{2m}[(\partial_y - x/\ell^2)^2 + (\partial_x)^2]$$

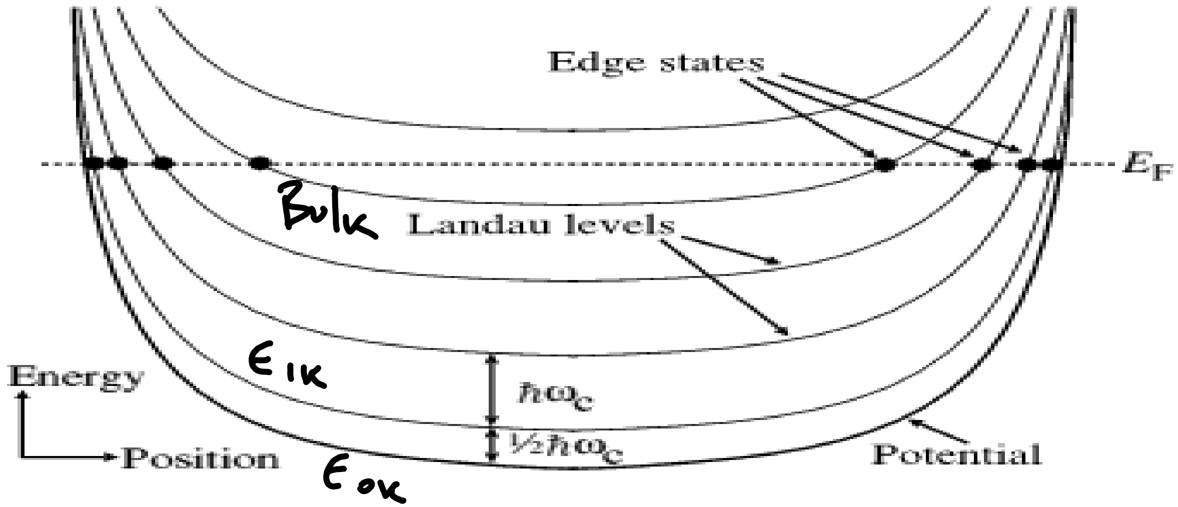
$$\psi_n(k_y, r) \sim \phi_n^{SHO}(x - \ell^2 k_y)e^{ik_y y}$$

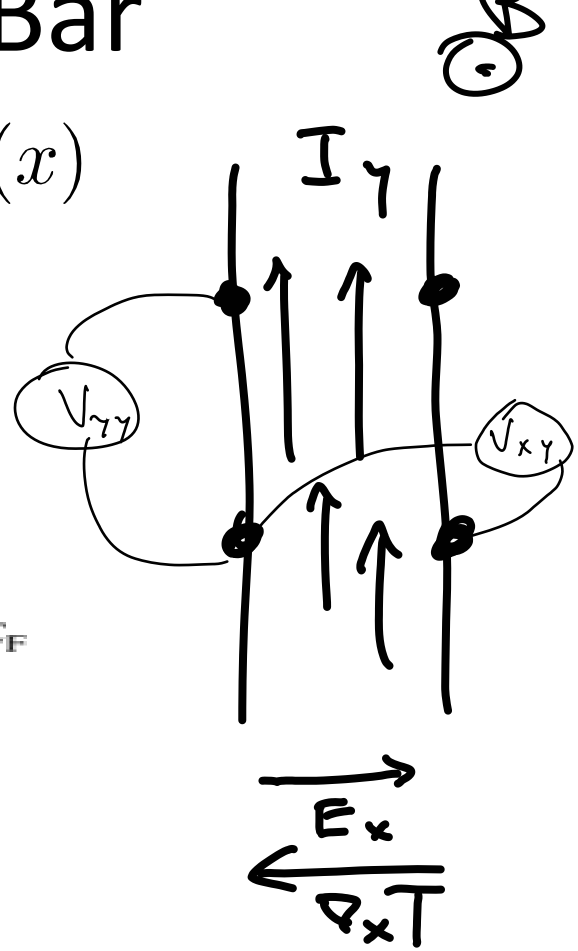
Landau level states are waves with a definite momentum along y-direction but localized in the x-direction on scale set by the magnetic length, $\ell = \sqrt{\hbar/(eB)}$

Landau levels in a Hall Bar

$$H = -\frac{\hbar^2}{2m}[(\partial_y - x/\ell_B^2)^2 + (\partial_x)^2] + V(x)$$

$$\psi_n(k_y, r) \sim \phi_n^{SHO}(x - \ell^2 k_y)e^{ik_y y}$$





$$\langle I_y \rangle = \frac{L_y}{2\pi}\int dk \sum_n \frac{-e}{\hbar L_y}\frac{\partial \epsilon_{n,k}}{\partial k} f(\epsilon_{n,k})$$

$$f(\epsilon) = 1/(1 + e^{\beta(\epsilon - \mu)})$$

Simple linear response theory – purely off-diagonal transport

Halperin '81, Girvin/Jonson '82
Bergman/V.O. '10

Linear response via edge theory

$$\sigma_{xy} = -\frac{e^2}{h} C_0; \alpha_{xy} = \frac{k_B e}{h} C_1; \kappa_{xy} = -\frac{k_B^2 T}{h} C_2;$$

$$C_q = -\sum_n \int_{\hbar\omega_n}^{\infty} d\epsilon \left(\frac{\epsilon-\mu}{k_B T}\right)^q \frac{\partial f(\epsilon)}{\partial \epsilon}$$

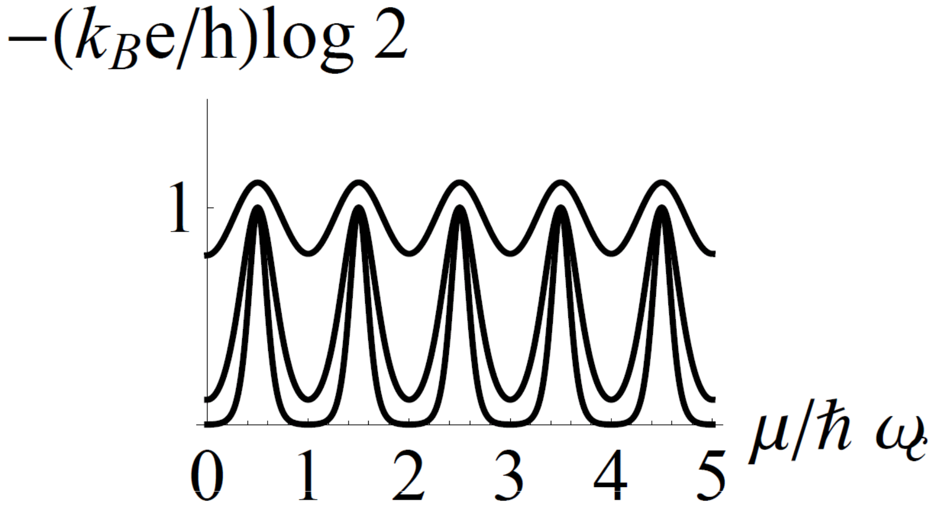
Note: celebrated "universality" of the edge theory comes from absorbing the details of the confining potential into the integration variable, i.e. by switching from k to energy integration (Halperin '81)

BUT, we can finish these integrals quite generally in a closed form by a further variable change:

$$C_q = -\sum_n \int_{f_n}^{0} df \log(1/f - 1)^q$$
$$C_0 = \sum_n f_n$$
$$C_1 = \sum_n \left[f_n \log f_n + (1 - f_n) \log(1 - f_n)\right]$$
$$C_2 = \sum_n \left[\frac{\pi^2}{3} + f_n \log^2(1/f_n - 1) - \log^2(1 - f_n) 2Li_2(1 - f_n)\right]$$

Bergman/V.O. '10

Off-diagonal thermoelectric conductivity:



- (e/h) times entropy per carrier: $\alpha_{xy} = \frac{k_B e}{h} \sum_n [f_n \log f_n + (1-f_n)\log(1-f_n)]$
- at plateaux transitions: temperature independent value of $-(ek_B/h)\log 2$;
- on plateaux activated but with a large $\sim 1/T$ prefactor;
- no usual (derivative) Mott relation
- **Is this universal?** E.g. disorder, interactions, experiments (contacts, phonons, etc)?

Disorder – yes, it is universal!

Proof:

1) Generalized Mott formula (exact in infinite samples)
(Obraztsov '65,Smrcka/Streda '77, Jonson/Girvin '84)

$$C_1(T,\mu) = -\int_{-\infty}^{\infty} d\epsilon \frac{\partial f}{\partial \epsilon} \frac{\epsilon-\mu}{k_B T} C_0(T=0,\epsilon)$$

2) Universal quantization of Hall conductivity,
except at mobility edges, μ_{Cn}

$$\sigma_{xy}(T=0,\mu) = \frac{e^2}{h} \sum_n \Theta(\mu - \mu_{Cn})$$

3) Plug (2) into (1) get

$$\alpha_{xy} = \frac{k_B e}{h} \sum_n \left[\tilde{f}_n \log \tilde{f}_n + (1-\tilde{f}_n) \log(1-\tilde{f}_n) \right]$$

Tildes are important:

with disorder there are no longer Landau levels but mobility edges,
i.e. only extended states' entropy is counted

Further 2D comments:

- Dirac fermions – no problem

- Universality requires localization, i.e. “all orders” treatment of disorder (SCBA based works of Jonson/Girvin ’84, Hatano ’05, etc. are wrong)

- Response is activated vs. distance (B or μ) to mobility edges – <u>only entropy of extended states counts</u>

$$\alpha_{xy} = \frac{k_B e}{h} \sum_n \left[\tilde{f}_n \log \tilde{f}_n + (1 - \tilde{f}_n) \log(1 - \tilde{f}_n) \right]$$

  … but that is the only true (dynamical?) entropy there is … the localized states are “stuck” and should be omitted when <u>computing</u> entropy.

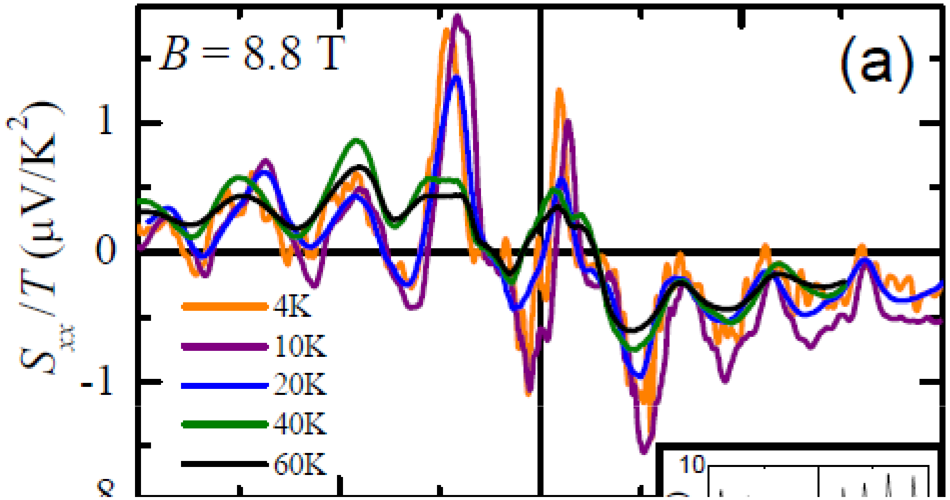
- What if (or rather when) experiments do not see this?

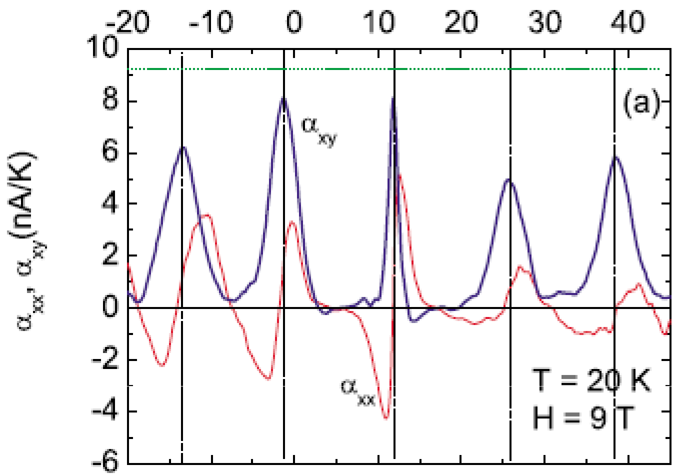
  the experiment is “wrong”

  OR

  we are seeing finite size, interaction and/or phonon effects outside of simple integer Hall physics– thermoelectric response may be better suited to study these than conductivity.

Experiments on graphene





- α_{xy} appears universal – also, appears to be a “cleaner” probe than S (at least at T=20K)
- More questions:

T dependence – non-interacting theory predicts none at the peak!

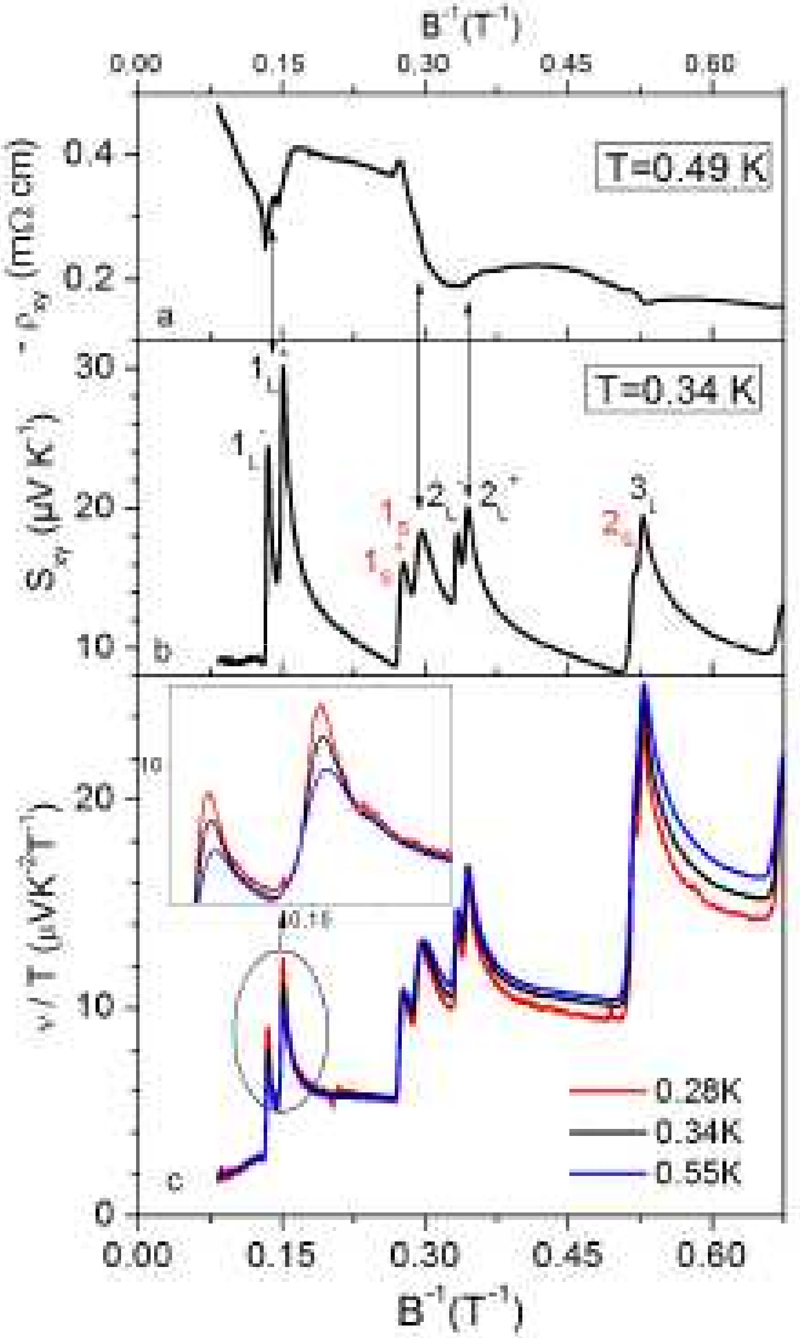
Also finite α_{xx}? Are these (irrelevant) interaction effects? slope vs. T; wiggles – UCF?

What about 3D?

Experiments on graphite (Behnia 2009):

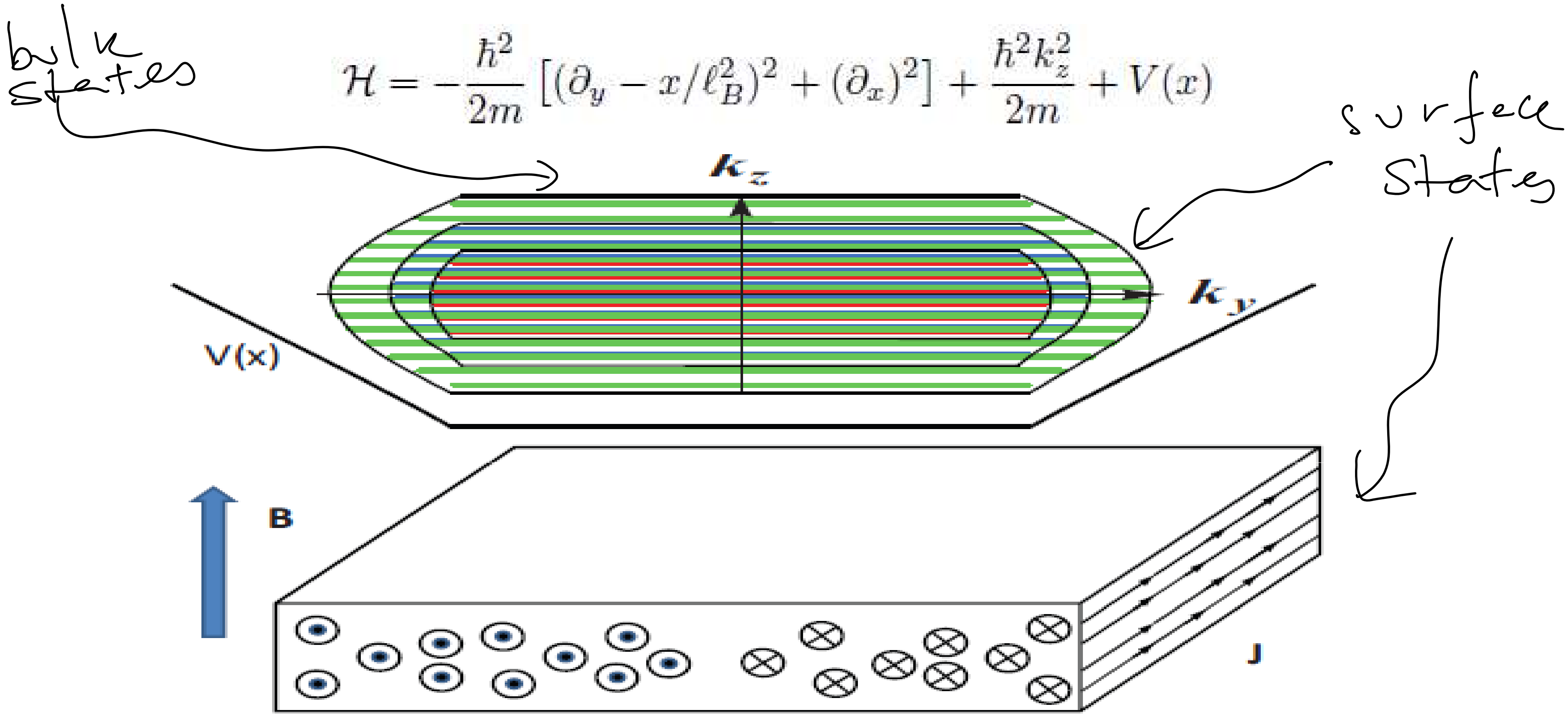
pronounced singularity structure inherited from Landau levels in 3D;

strong residual temperature dependence despite dirty materials



Landau levels of the Hall "brick"

$$\mathcal{H} = -\frac{\hbar^2}{2m}\left[(\partial_y - x/\ell_B^2)^2 + (\partial_x)^2\right] + \frac{\hbar^2 k_z^2}{2m} + V(x)$$



$$\alpha_{xy} = \frac{k_B e}{h L_z}\sum_n \sum_{k_z}\left[f_n \log f_n + (1-f_n)\log(1-f_n)\right]$$

$$\alpha_{xy} = -2\pi \ell_B^2 \frac{e}{h} s = -\frac{s}{B}$$

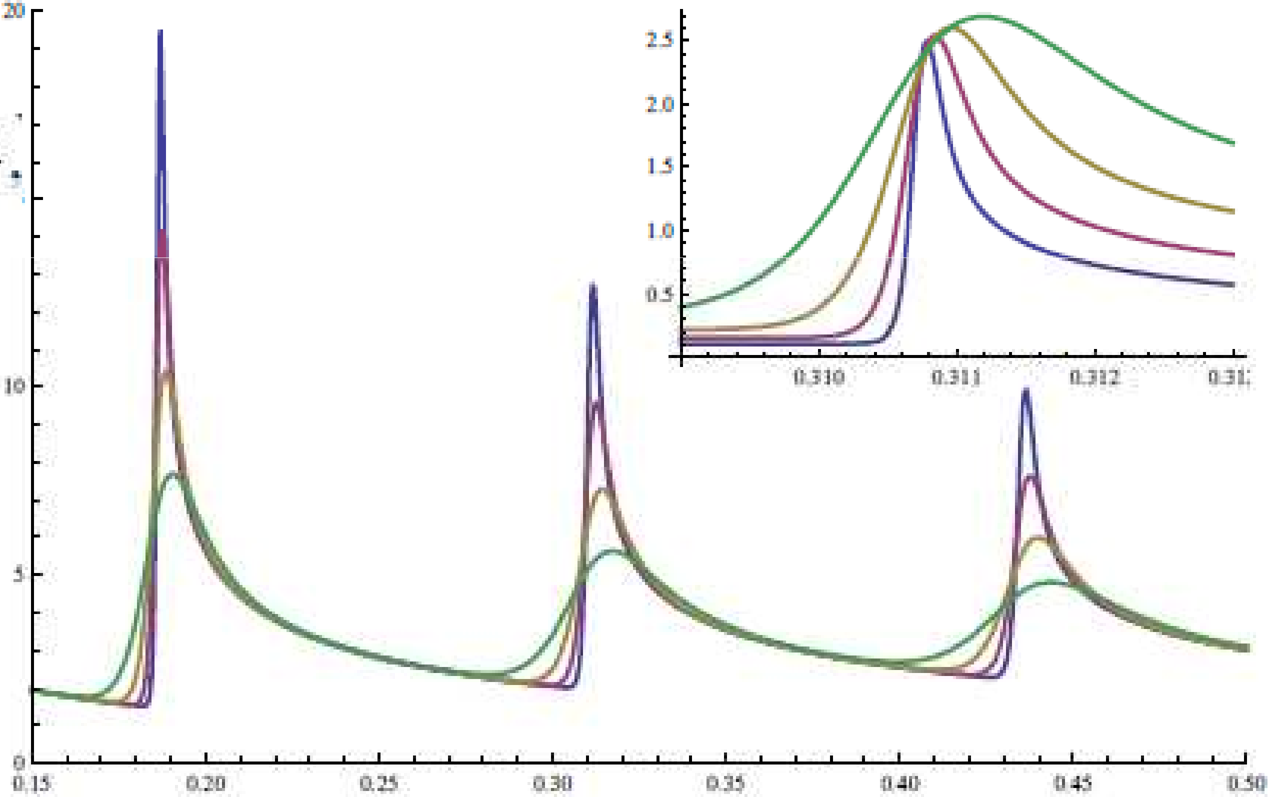
Results (no disorder or interactions):

two types of behavior: linear T and sq. root T

$B\alpha/(T\alpha_0)$

$$\alpha_{xy} \approx -2(e/h)(k_B/\lambda_{Tz})$$

$$\alpha_0 \hat{=} -\frac{k_B e}{h\lambda},$$



1/B

Analytic results – LL "quantum criticality":

General scaling formalism:

$$\alpha_{xy} = -\frac{ek_B}{h\lambda_{Tz}}\sum_n F(b_n),$$

$$F(b) \equiv -\int_{-\infty}^{\infty} dz\, f_z \log f_z + (1-f_z)\log(1-f_z)$$

$$F(b) = \begin{cases} \sim 1/\sqrt{b} \text{ for } b \to \infty \\ \sim e^{-b} \text{ for } b \to -\infty \\ \approx 2 \text{ at } b = 0 \\ \approx 2.4 \text{ for } b = b_{min} \approx 1.3 \end{cases}$$

Linear T Regime:

$$\alpha_{xy} = -\frac{ek_B}{h}\frac{\pi^2}{3}\frac{\ell_B}{\lambda_{Tz}\lambda_T}\sum_{n=0}^{n_{\max}}\frac{1}{\sqrt{\nu - n}}$$

$$= i\frac{ek_B}{h}\frac{\ell_B}{\lambda_{Tz}\lambda_T}\left[\xi(1/2,-\nu) - \xi(1/2, 1 + n_{\max} - \nu)\right]$$

"Quantum critical" regime:

$$\alpha_{xy} \approx -2(e/h)(k_B/\lambda_{Tz})$$

Summary + outlook

So, why should we study Nernst effect?

Appears to be a sensitive probe of…

…carrier entropy, EVEN with disorder. Immobile excitations are ignored

How general is this result outside "dissipationless" regime? In 3D with disorder or interactions? Mesoscopic effects?

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Low field

$$\alpha_{xy} = -s/B$$

HgSe:Fe