The onset of the bipolar flavor conversion of supernova neutrinos

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May 28, 2010

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Free Meson Seminar

Outline of the Talk

1. Introduction
2. A brief review of the vacuum oscillation
3. MSW Effect
4. Collective oscillations with three phases
5. Onset of the bipolar oscillations
6. Conclusions

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- Supernova neutrino study involves an interesting non-linear collective oscillation phenomena.
- At small r neutrino and antineutrino densities (n_ν and $\overline{n}_\nu$, respectively) are high enough to make the self-interaction effect important.
- Collective effect fully develop before MSW effect.
- The average energies of different flavors are as follows:

$$E_{\nu_e} = 10 - 12\ MeV$$
$$E_{\bar{\nu}_e} = 13 - 16\ MeV$$
$$E_{\nu_x} = 15 - 25\ MeV \qquad (x = \mu \text{ or } \tau)$$

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- In the earlier work incorporating the refractive index of the neutrino-neutrino forward scattering, the off diagonal refractive indices were left out. This was rectified by Pantaleone. [Phys. Rev. D **46**, 510 (1992)]
- Here the only relevant mixing angle is $\theta_{13}(=\Theta)$, governing the oscillation amlitude in the following channels.

$$\nu_e \longrightarrow \nu_x \text{ and } \bar{\nu}_e \longrightarrow \bar{\nu}_x \ (x = \mu \text{ or } \tau)$$

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- Flavor eigen states are the superpositions of the mass eigen states, given by the relation

$$\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$\Theta \longrightarrow$ small vacuum mixing angle

$\nu_i \longrightarrow$ physical field with mass m_i $\quad (i = 1, 2)$

Vacuum Oscillation

The evolution equation for the vacuum oscillation in flavor basis:

$$i\partial_t \begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = [k + \frac{m_1^2 + m_2^2}{4k} + \frac{(\triangle m)^2}{4k} \begin{pmatrix} -\cos 2\Theta & \sin 2\Theta \\ \sin 2\Theta & \cos 2\Theta \end{pmatrix}] \begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix}$$

where, $(\triangle m)^2 = m_2^2 - m_1^2$

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Vacuum Oscillation: density matrix formalism

Let us take

$$\rho = \begin{pmatrix} \nu_e^* \\ \nu_x^* \end{pmatrix} \begin{pmatrix} \nu_e & \nu_x \end{pmatrix}$$

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Let us further take

$$\rho = \frac{1}{2}(1 + \mathbf{P}.\sigma)$$

Vacuum Oscillations: Stodolsky equation

The EOM becomes

$$\partial_t \mathbf{P} = \omega(\mathbf{B} \times \mathbf{P})$$

followed by Stodolsky [Phys. Rev. D, **36**, 2273 (1987)]

where, $\mathbf{B} = (\sin 2\Theta, 0, -\cos 2\Theta)$ $\qquad \omega = \frac{\triangle m^2}{2k}$

$\mathbf{P} = (P_1, P_2, P_3)$ is the polarization vector

$P_1 = 2Re(\nu_e \nu_x^*)$ $\qquad P_2 = 2Im(\nu_e \nu_x^*)$ $\qquad P_3 = |\nu_e|^2 - |\nu_x|^2$

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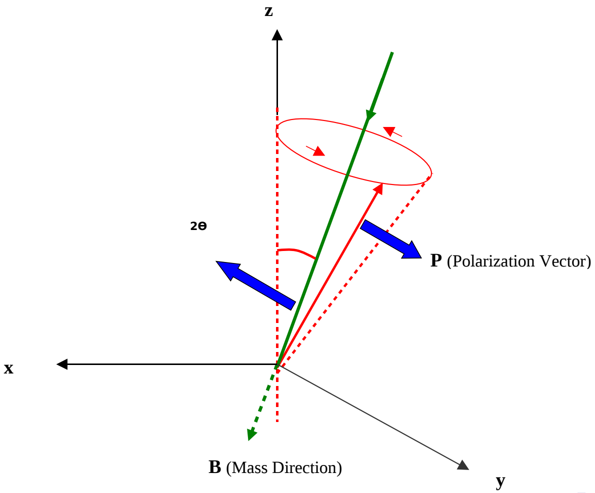
$P = 1 \longrightarrow$ pure state
$P = 0 \longrightarrow$ completely incoherent mixture of both flavors

Pendulum in Flavor Space



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- Now if we add this MSW the EOM:

$$\partial_t \mathbf{P} = (\omega \mathbf{B} + \lambda \mathbf{z}) \times \mathbf{P}$$

$\mathbf{z} \longrightarrow$ flavor direction
$\lambda = \sqrt{2} G_F n_e$ [$n_e \longrightarrow$ electron density]

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  $\mathbf{z} \rightarrow$ flavor direction
  $\lambda = \sqrt{2} G_F n_e$ [$n_e \rightarrow$ electron density]

- We consider a corotating frame which rotates with angular velocity $\lambda \mathbf{z} \longrightarrow$ removes the term $\lambda \mathbf{z}$

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Collective oscillation

In the density matrix formalism EOM

$$i\partial_t \rho_{\mathbf{k}} = [\frac{M^2}{2k}, \rho_{\mathbf{k}}] + \sqrt{G_F}[L, \rho_{\mathbf{k}}] + \sqrt{G_F} \int \frac{d^3\mathbf{k}'}{(2\pi)^3}(1 - \cos\theta_{\mathbf{k}\mathbf{k}'})[(\rho_{\mathbf{k}'} - \bar{\rho}_{\mathbf{k}'}), \rho_{\mathbf{k}}]$$

$$i\partial_t \bar{\rho}_{\mathbf{k}} = -[\frac{M^2}{2k}, \bar{\rho}_{\mathbf{k}}] + \sqrt{G_F}[L, \bar{\rho}_{\mathbf{k}}] + \sqrt{G_F} \int \frac{d^3\mathbf{k}'}{(2\pi)^3}(1 - \cos\theta_{\mathbf{k}\mathbf{k}'})[(\rho_{\mathbf{k}'} - \bar{\rho}_{\mathbf{k}'}), \bar{\rho}_{\mathbf{k}}]$$

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In terms of Stodolsky equation

$$\partial_t \mathbf{P}_\omega = \omega(\mathbf{B} \times \mathbf{P}_\omega) + \lambda(\mathbf{z} \times \mathbf{P}_\omega) + \mu(\mathbf{D} \times \mathbf{P}_\omega)$$

$$\partial_t \overline{\mathbf{P}}_\omega = -\omega(\mathbf{B} \times \overline{\mathbf{P}}_\omega) + \lambda(\mathbf{z} \times \mathbf{P}_\omega) + \mu(\mathbf{D} \times \overline{\mathbf{P}}_\omega)$$

where, $\lambda = \sqrt{2}G_F n_e \quad \mu = \sqrt{2}G_F(n_\nu + \overline{n}_\nu) \longrightarrow$ self-interaction energy

- **Synchronized Oscillation**

  S.Samuel, Phys. Rev D, **48**, 1462 (1993)
  S. Pastor, G. G. Raffelt and D. V. Semikoz, *Phys. Rev. D* **65**, 053011 (2002)

- **Bipolar Oscillation**

  S. Hannestad, G. G. Raffelt, G. Sigl and Y. Y. Y. Wong, *Phys. Rev. D* **74**, 105010 (2006)
  G. L Fogli, E. Lissi, A. Marrone and A. Mirizzi,*JCAP* **12**, (2007)

- **Spectral Split**
  (Will not be discussed here)

  G. G. Raffelt and A. Y. Smirnov *Phys. Rev. D* **74**, 105010 (2006)
  B. Dasgupta, A. Dighe, G. G. Raffelt and A. Y. Smirnov *Phys. Rev. Lett.* **103**, 051105 (2009)

Synchronized Oscillation

Consider oscillation of j-th mode of neutrino. The corresponding EOM becomes

$$\partial_t \mathbf{P}_j = \omega_j(\mathbf{B} \times \mathbf{P}_j) + \mu(\mathbf{J} \times \mathbf{P}_j)$$

where, $\mathbf{J} = \sum_{j=1}^{N} \mathbf{P}_j \longrightarrow$ Polarization vector for entire ensembles.

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When $\mu J \gg \omega_j$

All modes are coupled to each other by their strong 'internal magnetic field' $\mathbf{J}$ and as a result

$$\partial_t \mathbf{J} = \omega_{syn} (\mathbf{B} \times \mathbf{J})$$

$\omega_{syn} = \frac{1}{J} \sum \omega_j (\mathbf{P}_j . \widehat{\mathbf{J}}) \longrightarrow$ synchronized frequency.

Synchronized Oscillation: Neutrino-Antineutrino Case

$\mathbf{P}_j \longrightarrow$ Polarization vector of jth mode of neutrino
$\overline{\mathbf{P}}_k \longrightarrow$ Polarization vector of kth mode of antineutrino

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The corresponding EOMs:

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$\mathbf{D} = \mathbf{P} - \overline{\mathbf{P}} \longrightarrow$ Internal magnetic field $\quad \mathbf{P} = \sum \mathbf{P}_j, \qquad \overline{\mathbf{P}} = \sum \overline{\mathbf{P}}_k$

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In case of synchronized motion $\quad (\mu D \gg \omega_j)$

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$\mathbf{P}(0) = (0, 0, 1)$ and $\overline{\mathbf{P}}(0) = \alpha(0, 0, 1)$

where, $0 \leq \alpha \leq 1$

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The EOMs for **S** and $\overline{\mathbf{D}}$ $\longrightarrow$

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$$\mathbf{Q} = \mathbf{S} - \frac{\omega}{\mu}\mathbf{B}$$

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$$\mathbf{Q} = \mathbf{S} - \frac{\omega}{\mu}\mathbf{B}$$

It implies

$$\mathbf{D} = \frac{\mathbf{q} \times \partial_t \mathbf{q}}{\mu} + \sigma \mathbf{q}$$

With $\sigma = \frac{\mathbf{D} \cdot \mathbf{Q}}{Q} \longrightarrow$ lepton asymmetry

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1. $\frac{\mathbf{q} \times \partial_t \mathbf{q}}{\mu} \longrightarrow$ Orbital angular momentum
2. $\sigma \mathbf{q} \longrightarrow$ Spin angular momentum

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Thus for a small mixing angle the suitable mass hierarchy can cause a complete flavor conversions (**Bipolar Oscillation**)

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When $\frac{\omega}{\mu} \ll 1$, $\sigma \approx (1-\alpha)$, $Q \approx (1+\alpha)$, $\frac{\mathbf{q} \times \partial_t^2 \mathbf{q}}{\mu} \ll \sigma \partial_t \mathbf{q}$

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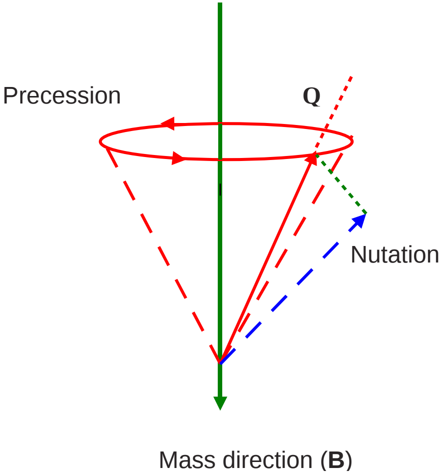
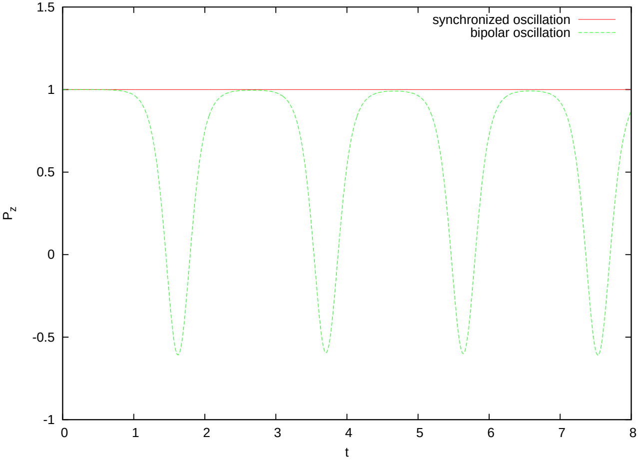
Both the cases are extreme. In general, the **precession** and **nutation** occurs simultaneously $\longrightarrow$ **Bipolar effect**

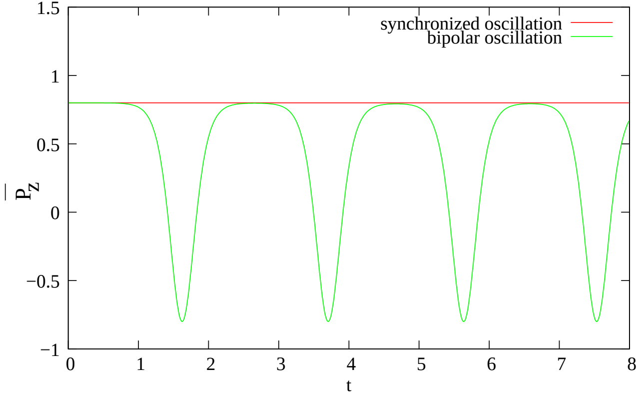
Precession and nutation



Synchronized ($\mu = 200$) and bipolar oscillations ($\mu = 10$) for neutrinos



Synchronized ($\mu = 200$) and bipolar oscillations ($\mu = 10$) for antineutrinos



Outline of the talk

1 Introduction

2 A Brief Review of the vacuum oscillation

3 MSW Effect

4 Collective Oscillations with three phases

5 Onset of the bipolar oscillation

6 Conclusions

Overview

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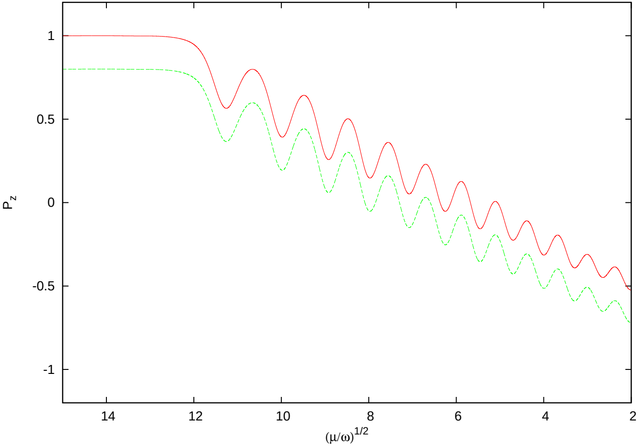
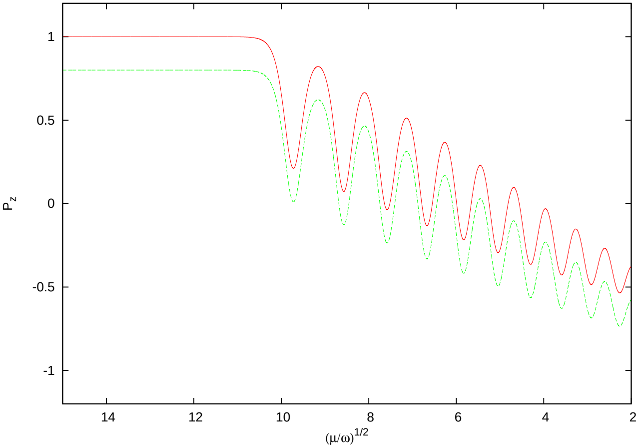
- How and when the nutation (bipolar effect) starts from a purely precession (synchronized effect)?
- Adiabatic change of μ plays a crucial role.
- The form of μ is taken as

$$\mu = \mu_I e^{-kt}$$

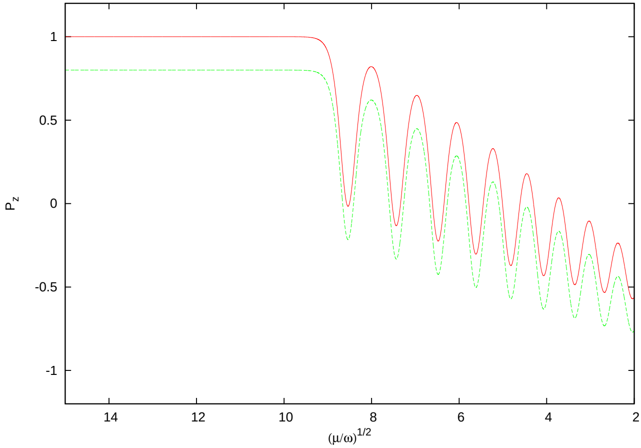
Evolution of P_z and $\bar{P}_z$ against $(\frac{\mu}{\omega})^{\frac{1}{2}}$ at $\theta_0 = 10^{-2}$



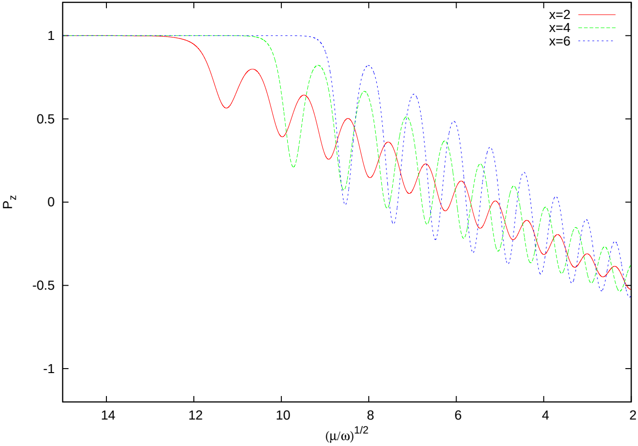
Evolution of P_z and $\bar{P}_z$ against $(\frac{\mu}{\omega})^{\frac{1}{2}}$ at $\theta_0 = 10^{-4}$



Evolution of P_z and $\bar{P}_z$ against $(\frac{\mu}{\omega})^{\frac{1}{2}}$ at $\theta_0 = 10^{-6}$



Evolution of P_z against $\left(\frac{\mu}{\omega}\right)^{\frac{1}{2}}$ at different $\theta_0 = 10^{-x}$



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- The evolution of P_z and $\bar{P}_z$ are plotted against $(\frac{\mu}{\omega})^{\frac{1}{2}}$ with three different vacuum mixing angles.
- A sharp deviation of P_z from its initial value denotes the onset.
- It is observed that the onset decreases with decreasing the mixing angle. That indicates the longer time is required to acheive the onset with smaller mixing angle.
- It was also observed that the onset value of $(\frac{\mu}{\omega})^{\frac{1}{2}}$ would decrease logarithmically with decreasing small mixing angle. We shall examine it.

Spinning top

The Lagrangian of a top is given by

$$L = \frac{I_1}{2}(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta) + \frac{I_3}{2}(\dot{\psi} + \dot{\phi}\cos\theta)^2 - mgl\cos\theta$$

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There is no torque either along the spin axis or along the veritical axis as both of them are perpendicular to the line of nodes.

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$$p_\psi = \frac{\partial L}{\partial \dot{\psi}} = I_3(\dot{\psi} + \dot{\phi}\cos\theta) = constant = I_1 a$$

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Taking $u = \cos\theta$ we can obtain an expression as follows:

$$f(u) = \dot{u}^2 = (1-u^2)(\alpha - \beta u) - (b - au)^2$$

where,

$$\alpha = \frac{2E_c}{I_1}, \qquad \beta = \frac{2mgl}{I_1}$$

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- Now, if u_0 is the double root the third one cannot enter into the interval $(-1, 1)$ and the top merely continues its spin.
- The nutation will be possible only when $u = u_0$ will be no longer a double root and the top nutates between two roots of $f(u) = 0$ in $(-1, 1)$.

Spinning top

Using double root condition at $u = u_0$

$$f(u) = 0 \qquad \frac{df}{du} = 0 \quad \Rightarrow \quad \cos\theta_0 \dot{\phi}^2 - a\dot{\phi} + \frac{\beta}{2} = 0$$

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The double root condition holds so far the equation has real solution i.e., for

$$\frac{a^2}{2\beta} \geq \cos\theta_0 \;\; (precession)$$

and violates

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Thus the onset is at

$$\frac{a^2}{2\beta} = \cos\theta_0 \approx 1$$

At the onset

$$\dot{\phi} = \frac{a}{2\cos\theta_0} \approx \frac{a}{2}$$

Comparison: Spinning top and Collective Oscilation

$\mathbf{B} = (\sin\theta_0, 0, \cos\theta_0) \longrightarrow$ inverted hierarchy $\qquad (2\Theta \rightarrow \theta_0)$

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Let us now compare the various terms

Comparison: Spinning top and Collective Oscilation

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$q = l = 1; \qquad ml^2 = I_1 = I_3 = \mu^{-1}$

$\mathbf{B} \rightarrow$ positive vertical axis; $\qquad \mathbf{q} \rightarrow$ positive spin axis

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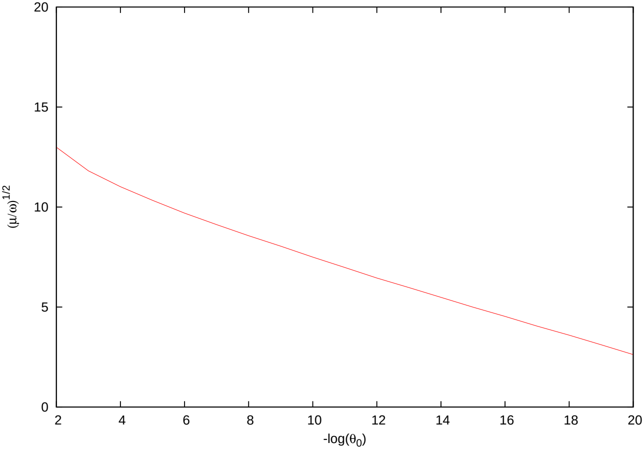
Thus the onset condition becomes

$$\frac{\mu}{\omega} \approx 4\frac{Q}{\sigma^2}$$

for the small θ_0.

The same onset condition was observed analytically by Hannested et al. [Phys. Rev. D **74**, 105010 (2006)]

The onset values of $(\frac{\mu}{\omega})^{\frac{1}{2}}$ (obtained numerically) are plotted against $-\log\theta_0$



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- What to be fixed then?
- Let us consider the situation just after the onset, where

$$\mu = \mu_0 e^{-kt} \approx \mu_0(1 - kt)$$

$$\mu_0 \approx 4\omega \frac{Q}{\sigma^2}$$

Variation of onset with θ_0

$$L = \frac{1}{2\mu}(\dot{\theta}^2 + \dot{\phi}^2\theta^2) + \frac{1}{2\mu}(\dot{\psi} + \dot{\phi})^2 - \omega Q\mu$$

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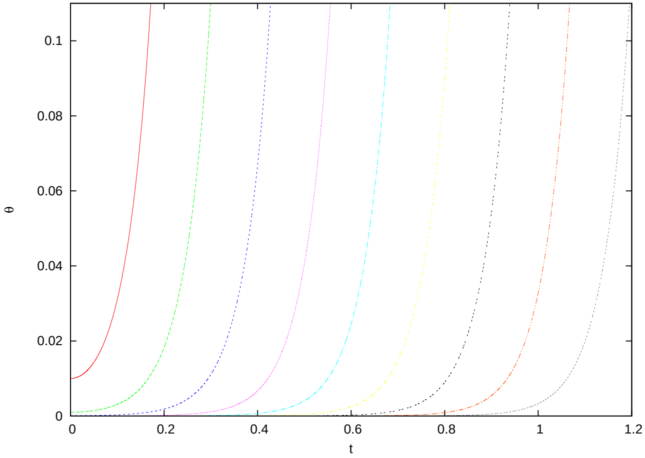
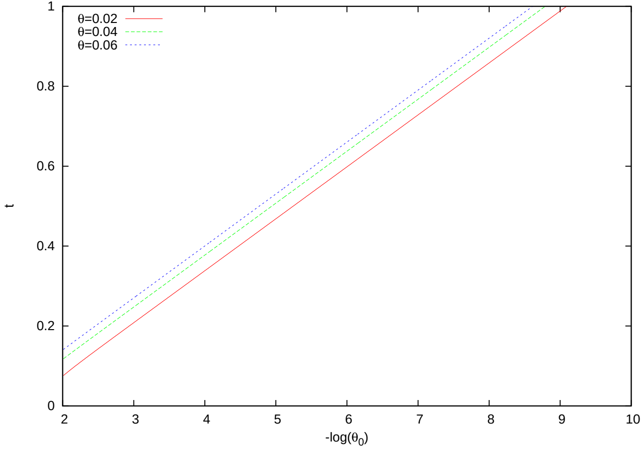
which implies

$$\frac{\mu_0\sigma t}{2} = \ln\theta - \ln\theta_0 + \ln(1 + \sqrt{(}1 - \frac{\theta_0^2}{\theta^2}))$$

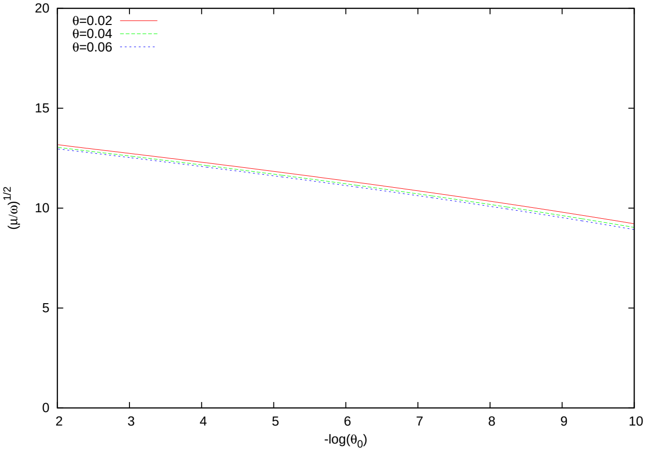
The change of θ with t at different θ_0



The dependence of t on θ_0 at different θ



The dependence of $\sqrt{\frac{\mu}{\omega}}$ on θ_0 at different θ



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- The adiabatic change of $\mu = \mu_I e^{-kt}$ does not change the dynamics of the top.
- It is important to fix the adiabatic parameter k properly. In our numerical calculations we have chosen $k = 0.5$.

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- Any fixed point (in terms of θ) nearer to the true onset exhibits same kind of logarithmic dependence. Therefore, pseudo onset may not be a fixed point.
- The adiabatic change of $\mu = \mu_I e^{-kt}$ does not change the dynamics of the top.
- It is important to fix the adiabatic parameter k properly. In our numerical calculations we have chosen $k = 0.5$.
- In our numerical calculations we have assumed $\alpha = 0.8$.

Conclusions

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- In our numerical calculations we have assumed $\alpha = 0.8$.
- The work is still in progress.

Thank you.