Annual Talks
School of Mathematics

February 7 – 8, 2019

Schedule and Abstracts of Talks

Tata Institute of Fundamental Research

Title of Talks

Najmuddin Fakhruddin	*Hecke operators on the integral coherent cohomology of Shimura varieties*
Moni Kumari	*Distribution of Hecke eigenvalues of a Hecke-Maass form.*
Ankit Rai	*Lefschetz properties for noncompact orthogonal Shimura varieties*
Mahbub Alam	*Equidistribution on homogeneous spaces and the distribution of approximates in Diophantine approximation*
Indranil Biswas	*A characterization of finite vector bundles on Gauduchon astheno-Kahler manifolds*
Gianluca Faraco	*Complex projective structure with maximal number of Möbius transformations*
Abhishek Khetan	*Cheeger inequalities for graphons.*
Swarnava Mukhopadhyay	*Derived category of moduli of vector bundles on curves.*
Anupam Mondal	*Covering the plane by a sequence of discs with a constraint*
Sandeep Varma	*Some homogeneity results for spherical characters*
Anish Ghosh	*Rational points, spectra and ergodic theorems*
Ravitheja Vangala	*p-adic Rankin product L-functions.*
Sampat Kumar Sharma	*Homotopy and commutativity principle*
Nitin Nitsure	*The tree of life – a demographic model for the development of organisms.*
Siddhartha Bhattacharya	*Translational tilings of the plane*

Abstracts

Thursday, 07 February 2019 (09:30-10:30)

Speaker : Najmuddin Fakhruddin
Title : *Hecke operators on the integral coherent cohomology of Shimura varieties*

For X a smooth projective scheme over $\mathbb{Z}_p$, results of Faltings and Fontaine (generalising a result of Mazur) imply that for any i, the Newton polygon associated to the action of Frobenius on the i-th crystalline cohomology of the special fibre of X lies above the Hodge polygon associated to the i-th de Rham cohomology of the generic fibre of X. An analogue of this result in the setting of automorphic forms was first considered by Clozel. His results were generalised to cohomological automorphic forms (corresponding to Betti cohomology of locally symmetric spaces) on arbitrary split reductive groups by V. Lafforgue. In work in progress with Vincent Pilloni, we consider this for automorphic forms associated to the coherent cohomology of automorphic vector bundles on Shimura varieties. The main work consists in defining suitably normalised Hecke operators on integral coherent cohomology of vector bundles on (integral models of) Shimura varieties. In this talk, after explaining the background, I will discuss some of our results in the unitary PEL case.

Thursday, 07 February 2019 (10:50-11:20)

Speaker : Moni Kumari
Title : *Distribution of Hecke eigenvalues of a Hecke-Maass form.*

In order to study the distribution of eigenvalues of a holomorphic modular form, J. P. Serre in 1992, proved that there are infinitely many primes p such that $|\lambda_f(p)| < d$ for some positive constant d.

In this talk, I will address an analogous result for eigenvalues of a Hecke-Maass cups form. This talk is based on an ongoing project with Prof. J. Sengupta.

Thursday, 07 February 2019 (11:30-12:00)

Speaker : Ankit Rai
Title : *Lefschetz properties for noncompact orthogonal Shimura varieties*

We will discuss an injectivity result for restriction of the cohomology of a noncompact orthogonal Shimura variety to a orthogonal Shimura subvariety.

Thursday, 07 February 2019 (12:10-12:40)

Speaker : Mahbub Alam
Title : *Equidistribution on homogeneous spaces and the distribution of approximates in Diophantine approximation*

We will talk about equidistribution results for certain flows on homogeneous spaces and related questions in Diophantine approximation. We state a quantitative version of Sullivan's theorem on Diophantine approximation in $\mathbb{C}$ by Gaussian integers $\mathbb{Z}[i]$ and Schmidt's theorem on Diophantine approximation in $\mathbb{R}$ by integers $\mathbb{Z}$. We will briefly explain how these results can be proved using equidistribution results of flows. We then state the problem of Diophantine approximation with respect to rationals in a fixed number field and "spiraling" results for the distribution of directions of approximates of Diophantine inequalities in number fields. This generalizes the work of Athreya, Ghosh and Tseng as well as Kleinbock, Shi and Weiss.

Thursday, 07 February 2019 (01:50-02:20)

Speaker : Indranil Biswas
Title : *A characterization of finite vector bundles on Gauduchon astheno-Kahler manifolds*

A vector bundle E on a projective variety X is called finite if it satisfies a nontrivial polynomial equation with integral coefficients. A theorem of Nori implies that E is finite if and only if the pullback of E to some finite etale Galois covering of X is trivial. We prove the same statement when X is a compact complex manifold admitting a Gauduchon astheno-Kahler metric.
Joint with Vamsi P. Pingali

Thursday, 07 February 2019 (02:30-03:00)

Speaker : Gianluca Faraco
Title : *Complex projective structure with maximal number of Möbius transformations*

In this talk we consider the group of projective automorphisms for a complex projective structure on a Riemann surface. Projective automorphisms are in particular biholomorphisms for the underlying conformal structure, therefore their number subject to the classical Hurwitz bound. We show the existence of structures achieving the maximal possible number of projective automorphisms, i.e. the Hurwitz bound can not be improved. This leads to adopt a relative point view and look for which projective structures the group of projective automorphisms coincides with the group of biholomorphisms. We show that Galois Belyi; curves are precisely those Riemann surfaces admitting a unique complex projective structure invariant under the full group of biholomorphisms. This is a joint work with Lorenzo Ruffoni.

Speaker **:** Abhishek Khetan
Title **:** *Cheeger inequalities for graphons.*

One way to quantify connectedness of a graph is via its Cheeger constant. The classical Cheeger inequalities for graphs give an estimate of the Cheeger constant of a graph G in terms of the second eigenvalue of the Laplacian of G and conversely. These inequalities first arose in the context of Riemannian manifolds and were subsequently developed for the discrete case of graphs. In this talk we will discuss Cheeger inqualities for graphons, which are "limit objects for a sequence of dense graphs."

Speaker **:** Swarnava Mukhopadhyay
Title **:** *Derived category of moduli of vector bundles on curves.*

The bounded category of a smooth variety is an important invariant that has applications to various areas of mathematics. Decomposing a derived category into simpler triangulated sub categories is a fundamental question. Fano varieties always admits a non trivial semiorthogonal decomposition. A natural class of Fano varieties come from the moduli space of vector bundles of on a curve with fixed determinant and coprime degree. In this talk, we will discuss natural subcategories of the derived category of these moduli spaces and give a conjectural semiorthogonal decomposition in rank 2 and provide evidence towards the conjecture. This is a joint work with Pieter Belmans and Sergey Galkin.

Speaker **:** Anupam Mondal
Title **:** *Covering the plane by a sequence of discs with a constraint*

We are interested in the following problem of covering the plane with a sequence of congruent (circular) discs with a constraint on the distance between consecutive discs. Let $(D_n)_{n \in \mathbb{N}}$ be a sequence of unit circular discs covering the plane with the condition that for $n \geq 2$, centre of the disc D_n lies in D_{n-1}. What is a "most economical" way of placing D_n for all $n \in \mathbb{N}$? We answer this question in the case where no "sharp" turn is allowed, i.e. if C_n is the centre of the disc D_n, then for all $n \geq 2$, $\angle C_{n-1}C_nC_{n+1}$ is not very small.

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Speaker : Sandeep Varma
Title : *Some homogeneity results for spherical characters*

Let (π, V) be an irreducible admissible representation of $G(F)$, G a connected reductive group over a finite extension F of $\mathbb{Q}_p$.

Harish-Chandra showed that one can actually talk of the character of π as a function $g \mapsto \Theta_\pi(g)$ on a dense open subset G_{reg} of $G(F)$: while π is usually infinite dimensional so that one cannot define $\Theta_\pi(g) = \text{trace}(\pi(g))$, $\Theta_\pi(g)$ can be defined by requiring that:

$$\int_{G_{reg}} f(g)\Theta_\pi(g)\,dg = \text{trace}\left(\int_{G(F)} f(g)\pi(g)\,dg\right),$$

for all locally constant compactly supported functions f on $G(F)$. Further, he proved the existence of a certain 'asymptotic expansion' to describe the behavior of the function Θ_π near the identity element of $G(F)$, in terms of objects associated to the Lie algebra of G.

Much later, J.-L. Waldspurger and S. DeBacker, assuming $p \gg 0$, proved some explicit 'homogeneity results for invariant distributions on $G(F)$', that imply that Harish-Chandra's asymptotic expansion is valid in an explicit, 'large' neighborhood of the identity element of $G(F)$ (depending on π through its 'depth' in the sense of A. Moy and G. Prasad). In this talk, I will describe work in progress jointly with J. Adler and E. Sayag, on analogous (but partial) homogeneity results for symmetric spaces $H \backslash G$.

I will try to make most of the talk accessible to students in the second and more senior years.

Speaker : Anish Ghosh
Title : *Rational points, spectra and ergodic theorems*

I will discuss some problems concerning the distribution of rational points on varieties and how they can be studied using ergodic theoretic methods. Our ergodic approach uses certain spectral information. Thus spectral information can be profitably used to understand arithmetic. If time permits, I will discuss if arithmetic can be profitably used to understand spectra. Joint work with Alexander Gorodnik and Amos Nevo.

Friday, 08 February 2019 (11:30-12:00)

Speaker : Ravitheja Vangala
Title : *p-adic Rankin product L-functions.*

We describe a construction of the *p*-adic Rankin product *L*-function due to A. Panchishkin. We noticed an error in the proof of the Kummer congruences. In this talk we correct the proof by tracing the error back to a sign in an Eisenstein series. This is a joint work with E.Ghate.

Friday, 08 February 2019 (12:10-12:40)

Speaker : Sampat Kumar Sharma
Title : *Homotopy and commutativity principle*

I will give an evidence for the principle that a (special linear, symplectic) matrix over a commutative ring which is homotopic to the identity will commute up to an elementary matrix with all elements of the (respective) group. We can generalize this principle that a (special linear, symplectic) matrix over a commutative ring which is homotopic to identity will commute up to an elementary matrix with all $(n \times m)$ right invertible matrices.

Friday, 08 February 2019 (01:50-02:20)

Speaker : Nitin Nitsure
Title : *The tree of life – a demographic model for the development of organisms.*

In developmental biology, it is of importance to have a good estimate of the time periods that are required for the various stages of the development of an organism. For the case under study – the development of spermatozoa of fruitflies – it was not practical to determine these periods by continuously observing a fixed individual as it passes through its various stages. Instead, a simple probabilistic model in the form of a 'weighted tree of life' was proposed, which allowed the use of fixed time demographic data to determine the relations between the periods of successive stages. These turn out to be a succession of Mobius transformations. Sophesticated experiments involving death and decomposition led to the estimation of the coefficients of these elements of $PSL_2(R)$. The knowledge of the periods thus gained uncovered an unforseen aspect of the development, namely, the transition between dividing and differentiating occurs sharply after the first two stages, instead of a gradual progress towards differentiation while successive divisions go on.

This is joint work with Purna Gadre and Krishanu Ray of the Department of Biological Sciences, TIFR.

Speaker : Siddhartha Bhattacharya
Title : *Translational tilings of the plane*

A finite set $F \subset \mathbb{Z}^2$ is said to be a translational tile if $\mathbb{Z}^2$ can be expressed as a disjoint union of translates of F. In this talk we will explore the link between such tiles and ergodic theory. Using ergodic theoretic techniques we will show that for a given finite set F, the question whether F tiles $\mathbb{Z}^2$ is decidable.