breaking news from the hot-dense land:

how far can we go? how high can we reach?

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how far can we go?

QCD phase boundary & radius of convergence in μ_B

HotQCD: Phys. Lett. B795, 15 (2019)
Mukherjee & Skokov: arXiv:1909.04639

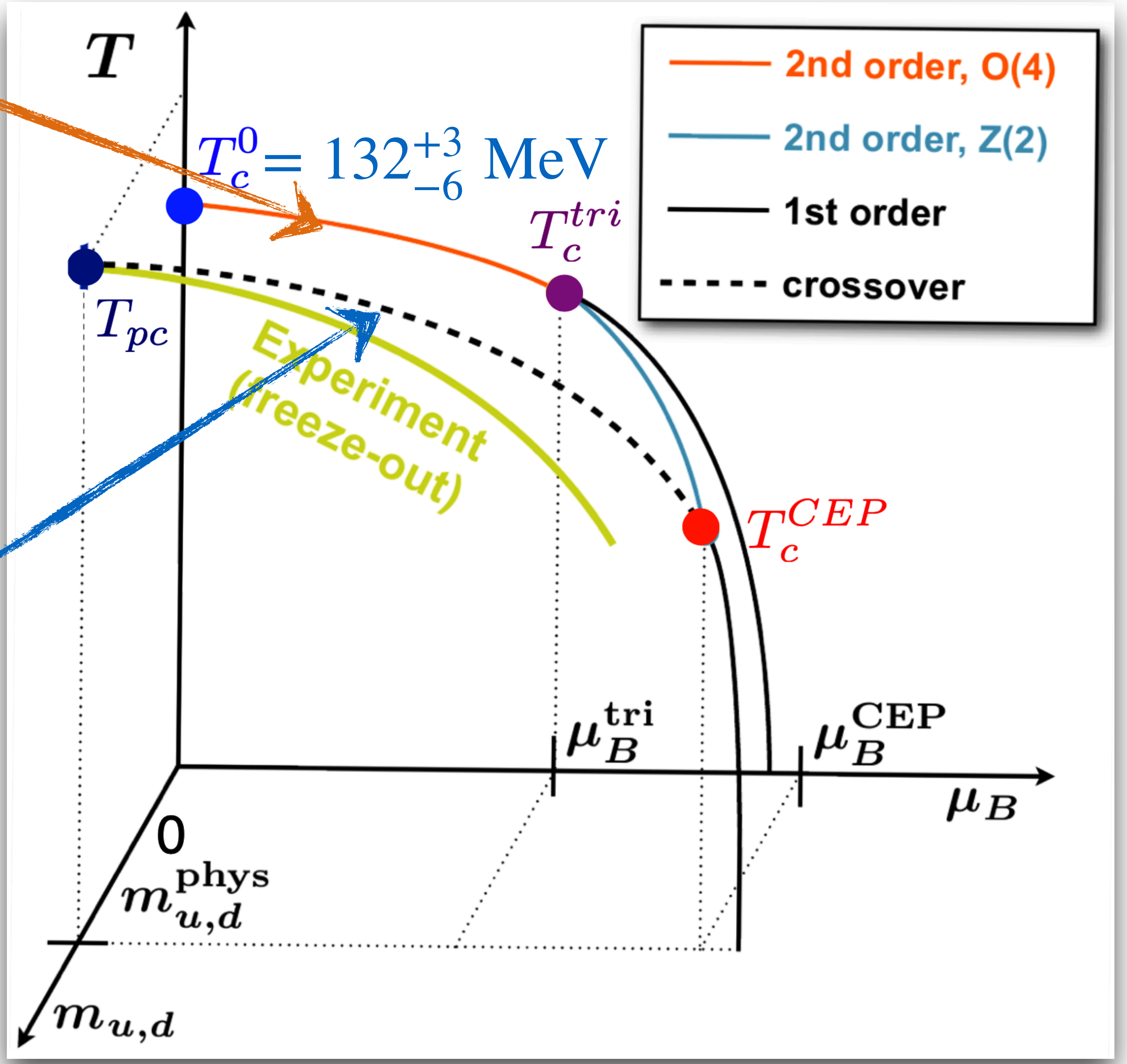
chiral transition, 2nd order, 3-d O(4)

$(m_u = m_d \to 0)$

chiral crossover

$(m_u = m_d = m_l^{\mathrm{phys}})$

$$T_{\mathrm{pc}}(\mu_B) = T_{\mathrm{pc}}(0)\left[1 - \kappa_2^B\left(\frac{\mu_B}{T_{\mathrm{pc}}(0)}\right)^2 - \kappa_4^B\left(\frac{\mu_B}{T_{\mathrm{pc}}(0)}\right)^4\right]$$



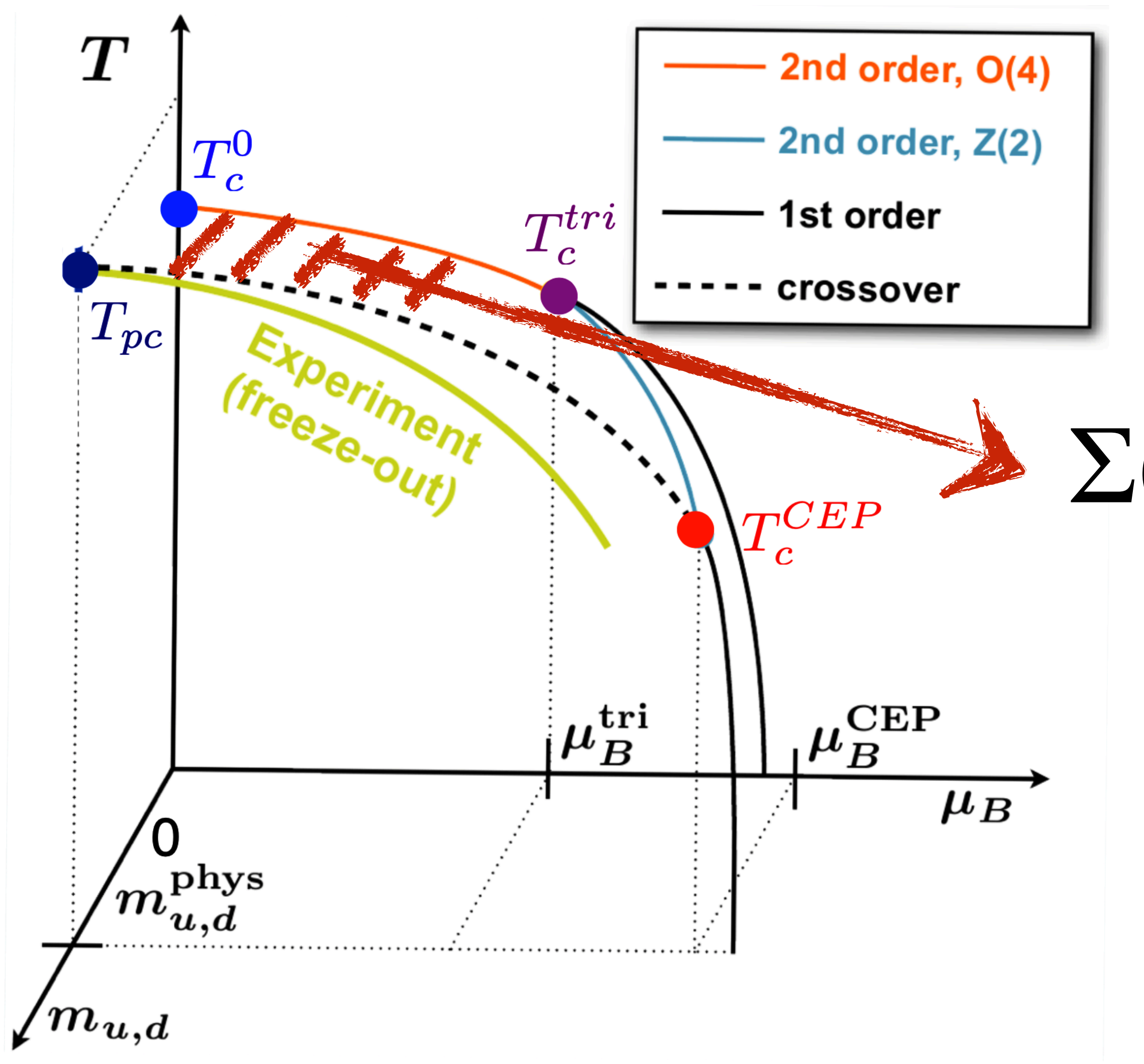
- chiral order parameter: $$\Sigma(T,\mu_B) = \frac{1}{f_K^4}\left[m_s\langle \bar{u}u + \bar{d}d\rangle - (m_u+m_d)\langle \bar{s}s\rangle\right]$$ quark condensate

- disconnected chiral susceptibility: $$\chi(T,\mu_B) = \frac{m_s^2}{f_K^4}\left[\langle(\bar{u}u+\bar{d}d)^2\rangle - (\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)^2\right]$$ light quark condensate fluctuation

- order parameter susceptibility: $$\chi^{\Sigma}(T,\mu_B) = m_s\left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_u}\right)\Sigma(T,\mu_B)$$ order parameter fluctuation

- Taylor expansion in chemical potential:

$$\Sigma(T,\mu_B) = \sum_{n=0}^{\infty}\frac{C_{2n}^{\Sigma}(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n} \qquad \chi(T,\mu_B) = \sum_{n=0}^{\infty}\frac{C_{2n}^{\chi}(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n}$$

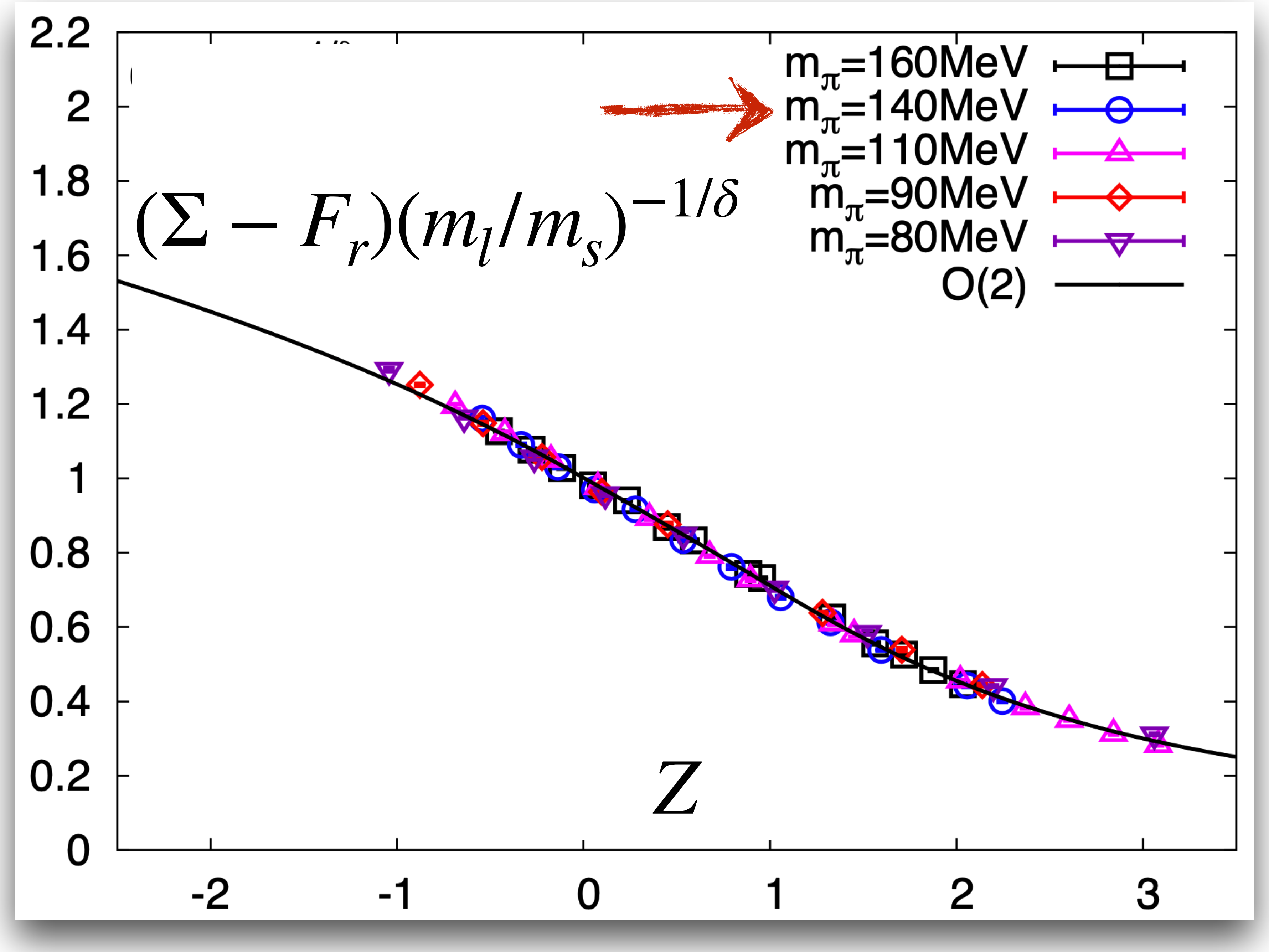


universal scaling function

$$\Sigma(T,\mu_B) = (m_l/m_s)^{1/\delta} \; f_G(z) \; + F_r(m_l,T)$$

non-universal analytic corrections

$$(m_l/m_s)\left[a_0 + a_1(T-T_c^0)/T_c^0 + a_2((T-T_c^0)/T_c^0)^2\right]$$



HotQCD: PoS LATTICE2016, 372 (2017)

scaling variable:

$$z = z_0 \, (m_l/m_s)^{-1/\beta\delta} \left[(T-T_c^0)/T_c^0 + \kappa_2^B \, (\mu_B/T_c^0)^2 \right] \longrightarrow \frac{\partial}{\partial T} \equiv \kappa_2^B \frac{\partial^2}{\partial(\mu_B/T)^2}$$

non-universal constants

$$\partial_T C_0^{\Sigma}(T), C_2^{\Sigma}(T) \sim m_l^{(\beta-1)/\beta\delta}\ \partial_z f_G(z)$$

$$\partial_T^2 C_0^{\Sigma}(T) = 0 \qquad \partial_T C_2^{\Sigma}(T) = 0$$

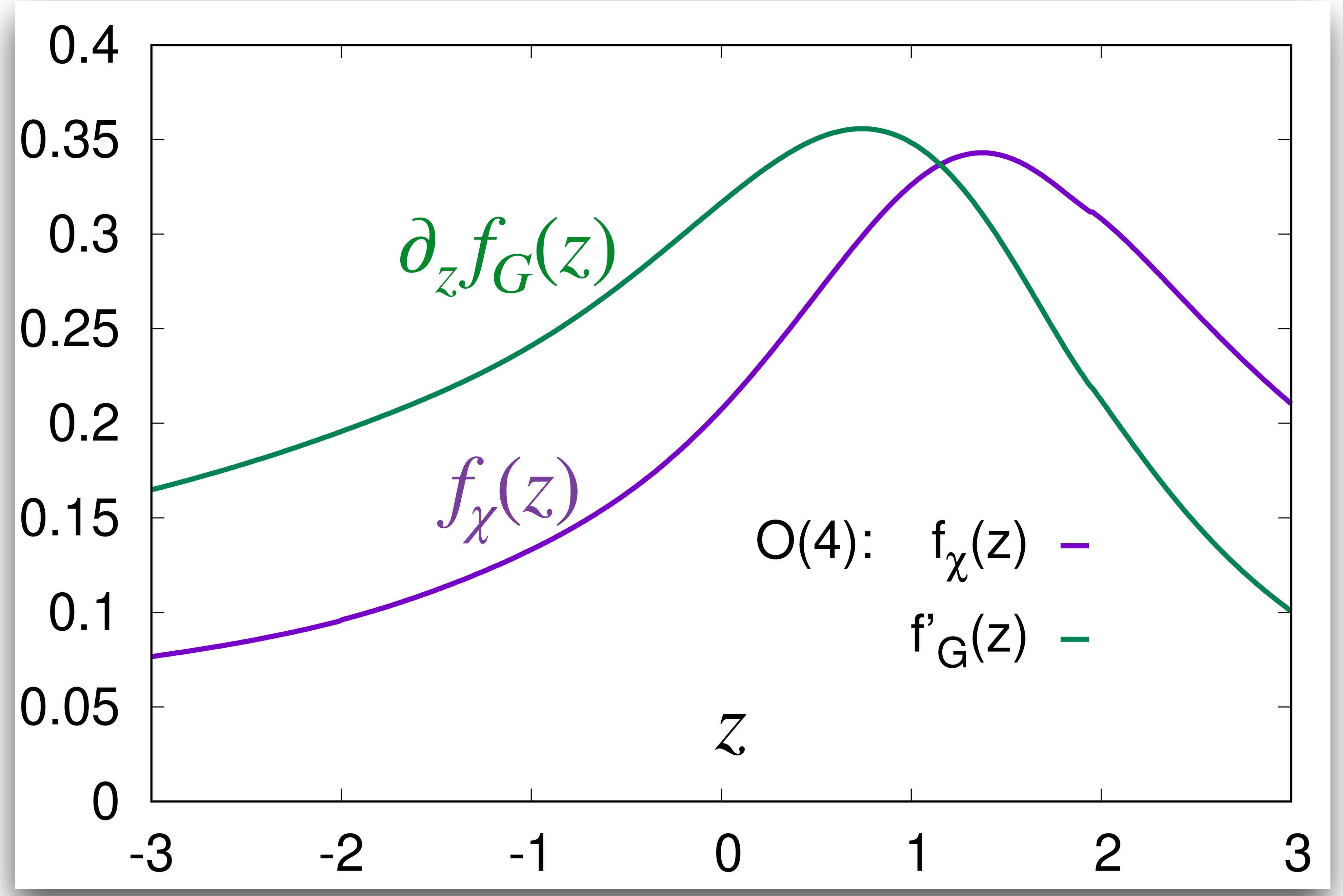
similarly:

$$\chi(T,\mu_B), \chi^{\Sigma}(T) \sim m_l^{(1-\delta)/\delta}\ f_\chi(z)$$

$$\partial_T \chi^{\Sigma}(T), \partial_T C_0^{\chi}(T), C_2^{\chi}(T) \sim m_l^{(\beta-\beta\delta-1)/\beta\delta}\ \partial_z f_\chi(z)$$

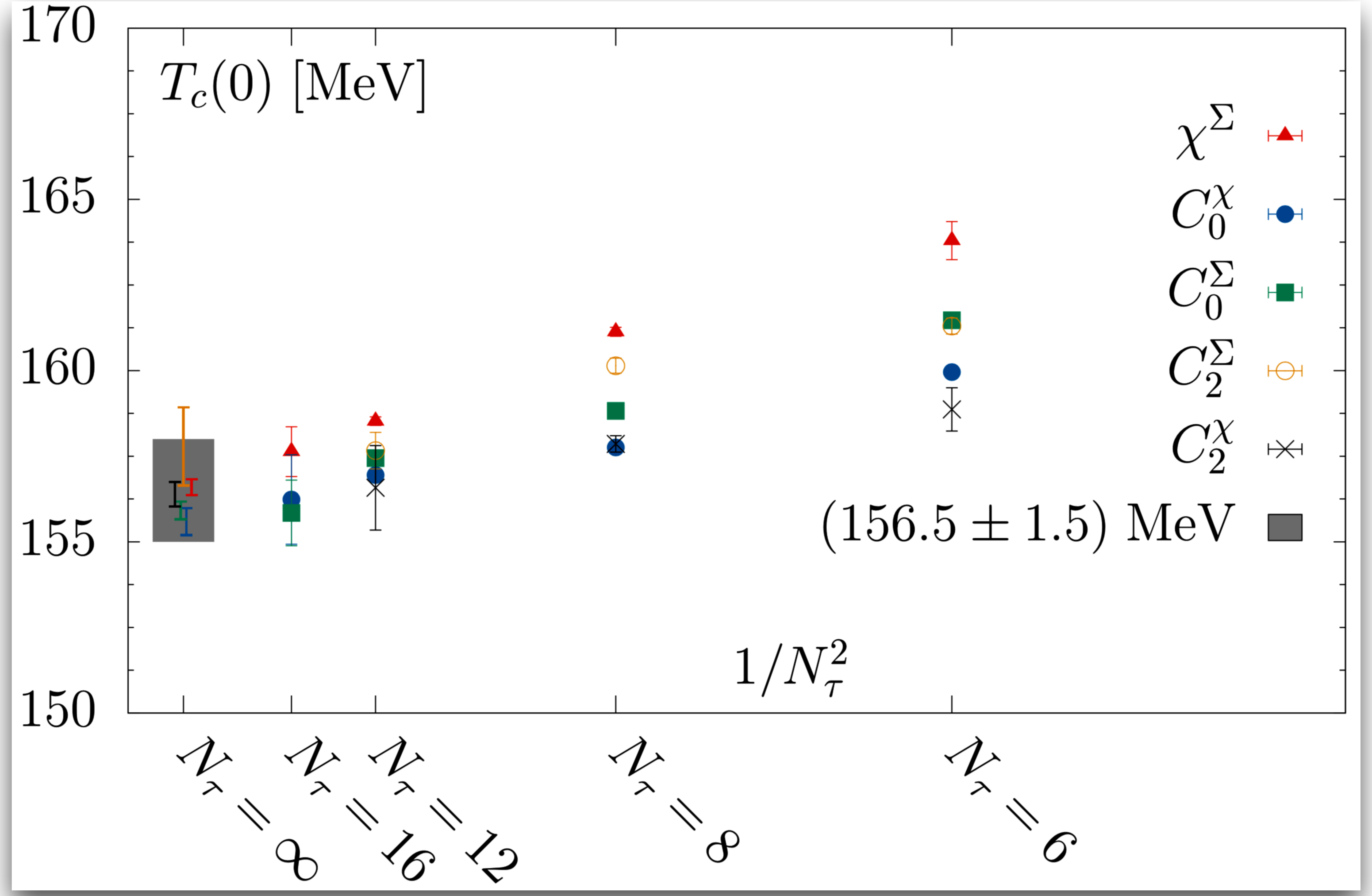
$$\partial_T C_0^{\chi}(T) = 0 \qquad C_2^{\chi}(T) = 0$$

$$\partial_T \chi^{\Sigma}(T) = 0$$

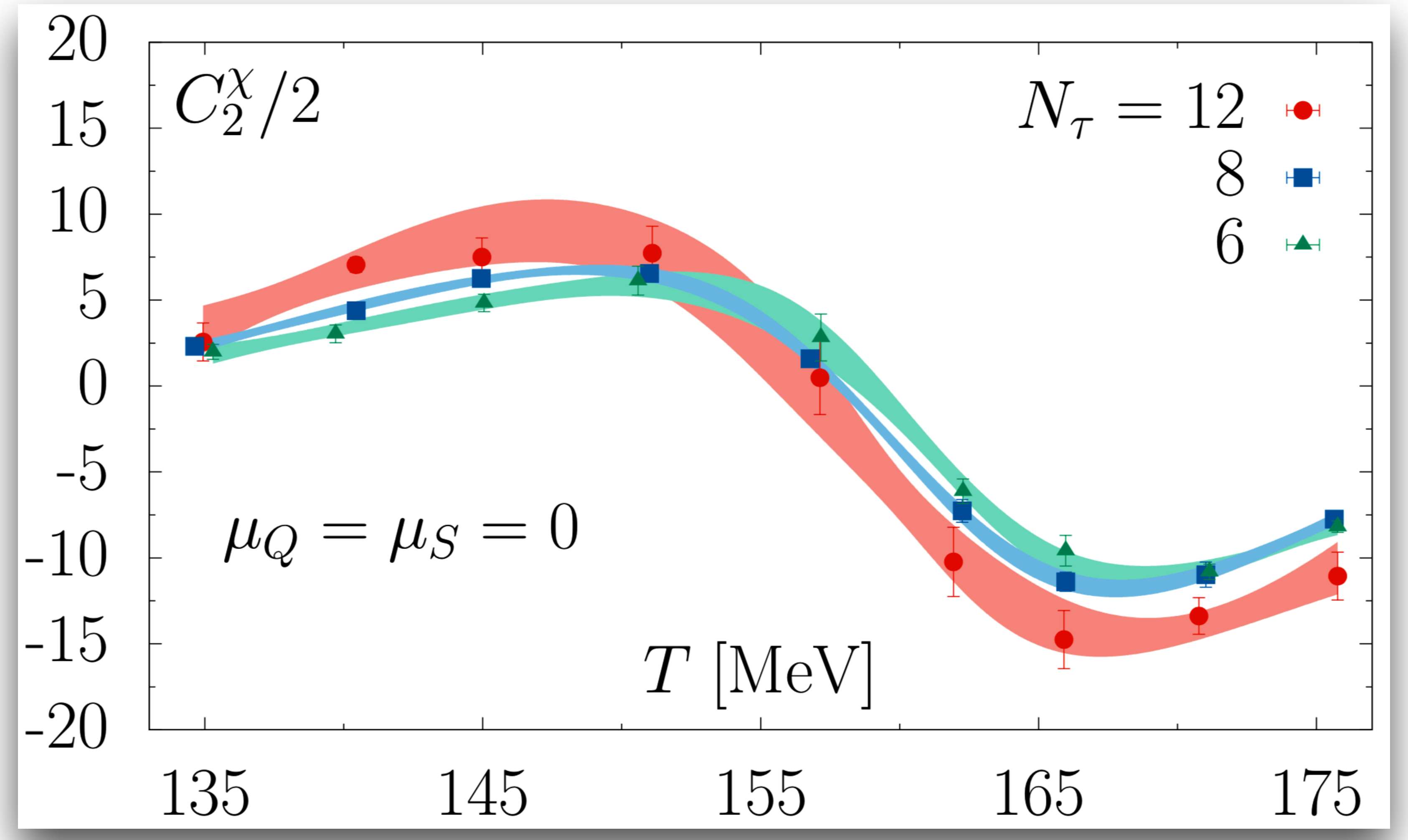


- m > 0: crossover, different susceptibilities can lead to different crossover temperatures

$T_{pc}(\mu_B = 0) = 156.5 \pm 1.5$ MeV



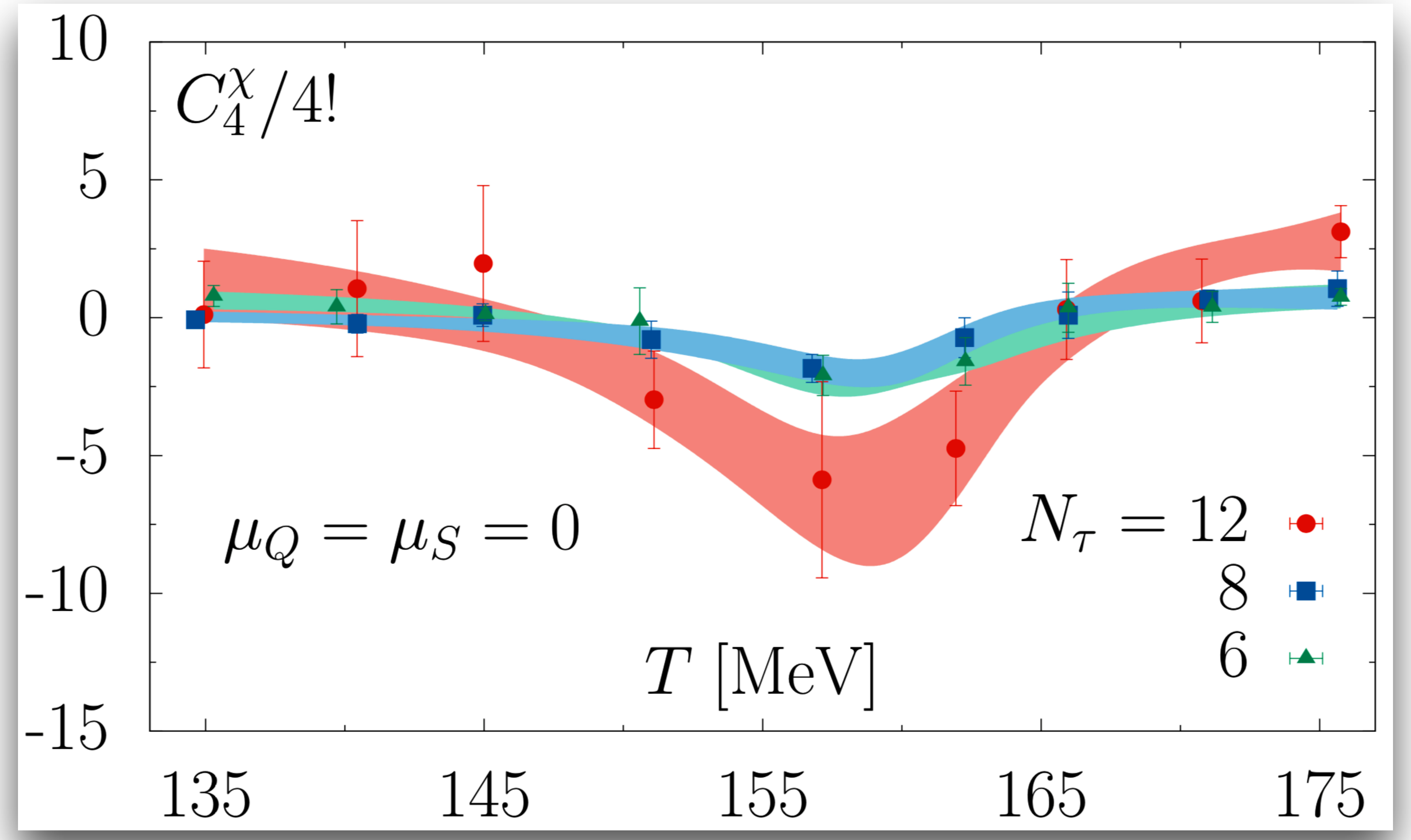
$$T_{\mathrm{pc}}(\mu_B) = T_{\mathrm{pc}}(0)\left[1 - \kappa_2^B\left(\frac{\mu_B}{T_{\mathrm{pc}}(0)}\right)^2 - \kappa_4^B\left(\frac{\mu_B}{T_{\mathrm{pc}}(0)}\right)^4\right]$$



$$\partial_T \chi(T,\mu_B) = \partial_T C_0^\chi(T) + \partial_T C_2^\chi(T) + \partial_T C_4^\chi(T) + \cdots = 0$$

$$\kappa_2^X = \frac{1}{2T^2\partial_T^2 C_0^\chi}\left[T\partial_T C_2^\chi - 2C_2^\chi\right]$$

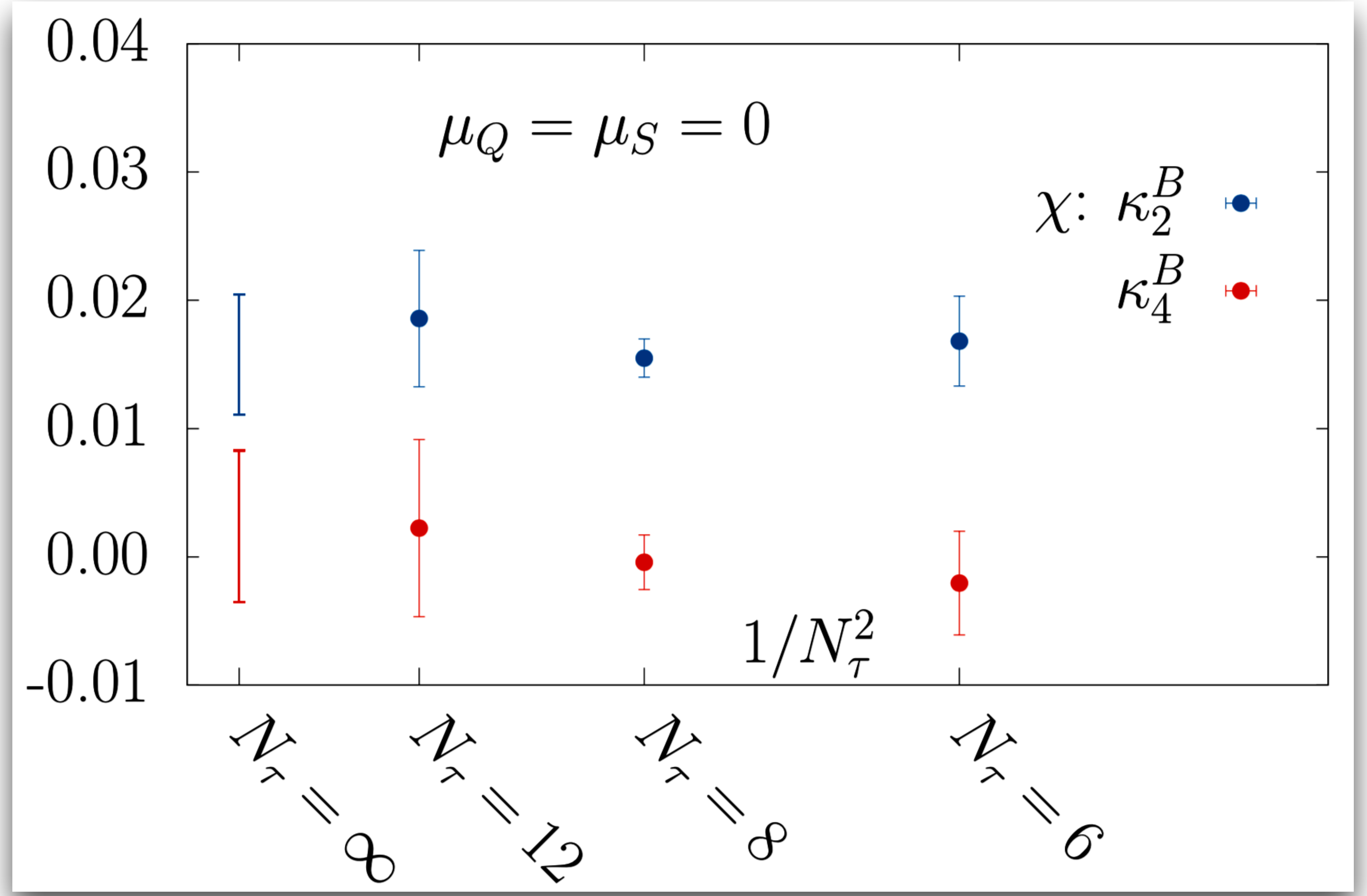
$$\kappa_4^X = \frac{1}{24T^2\partial_T^2 C_0^\chi}\Big[-72\kappa_2^X C_2^\chi - 4C_4^\chi + T\partial_T C_4^\chi + 12\kappa_2^X\left(4T\partial_T C_2^\chi - T^2\partial_T^2 C_2^\chi + \kappa_2^X T^3 \partial_T^3 C_0^\chi\right)\Big]$$



$n_S = 0,\ n_Q/n_B = 0.4$

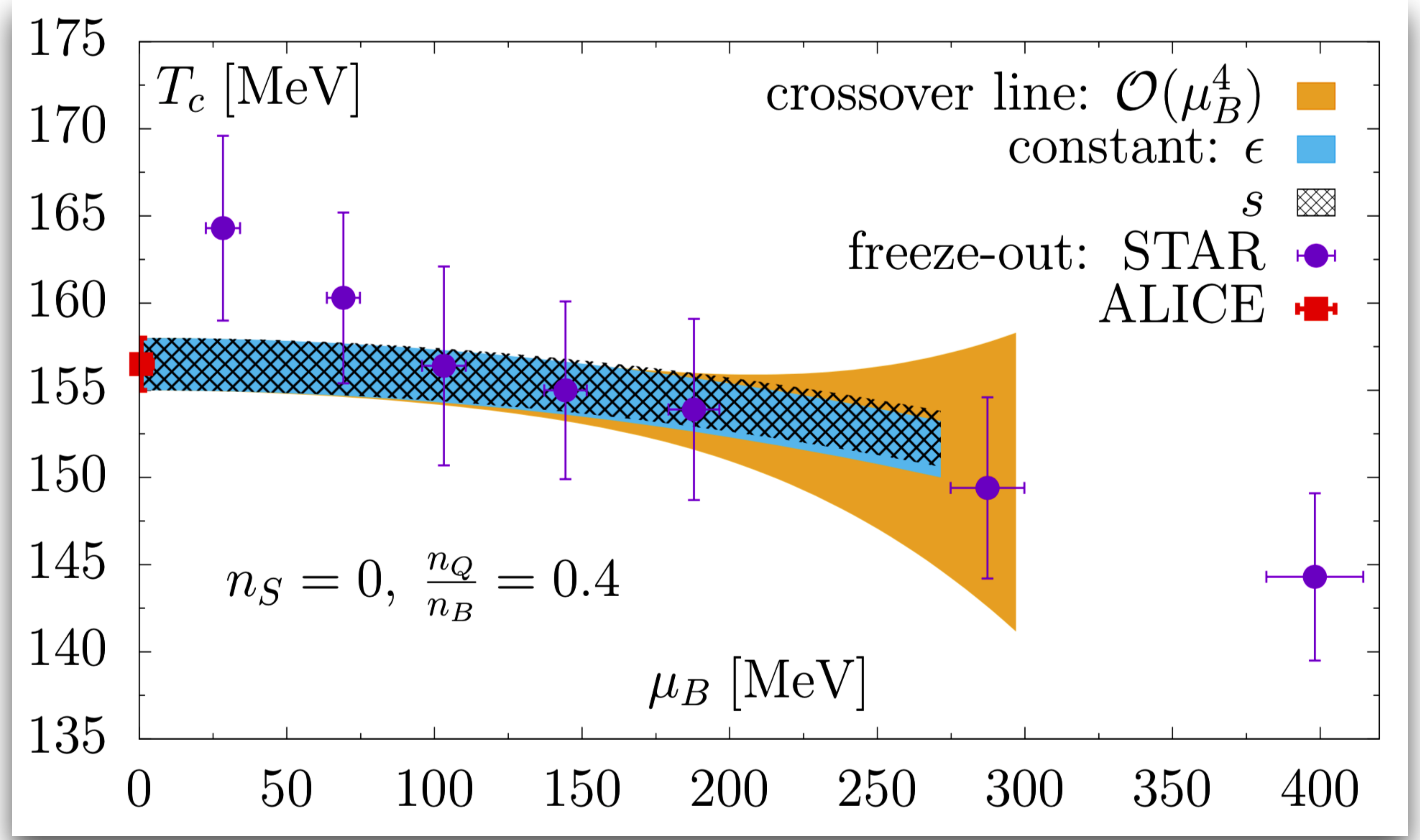
$\kappa_2^B = 0.012(4)$

$\kappa_4^B = 0.000(4)$



QCD phase boundary

- freeze-out line coincides with the chiral crossover
- along the chiral crossover energy density & entropy density remains constant
- sings of enhanced fluctuations around the phase boundary ?



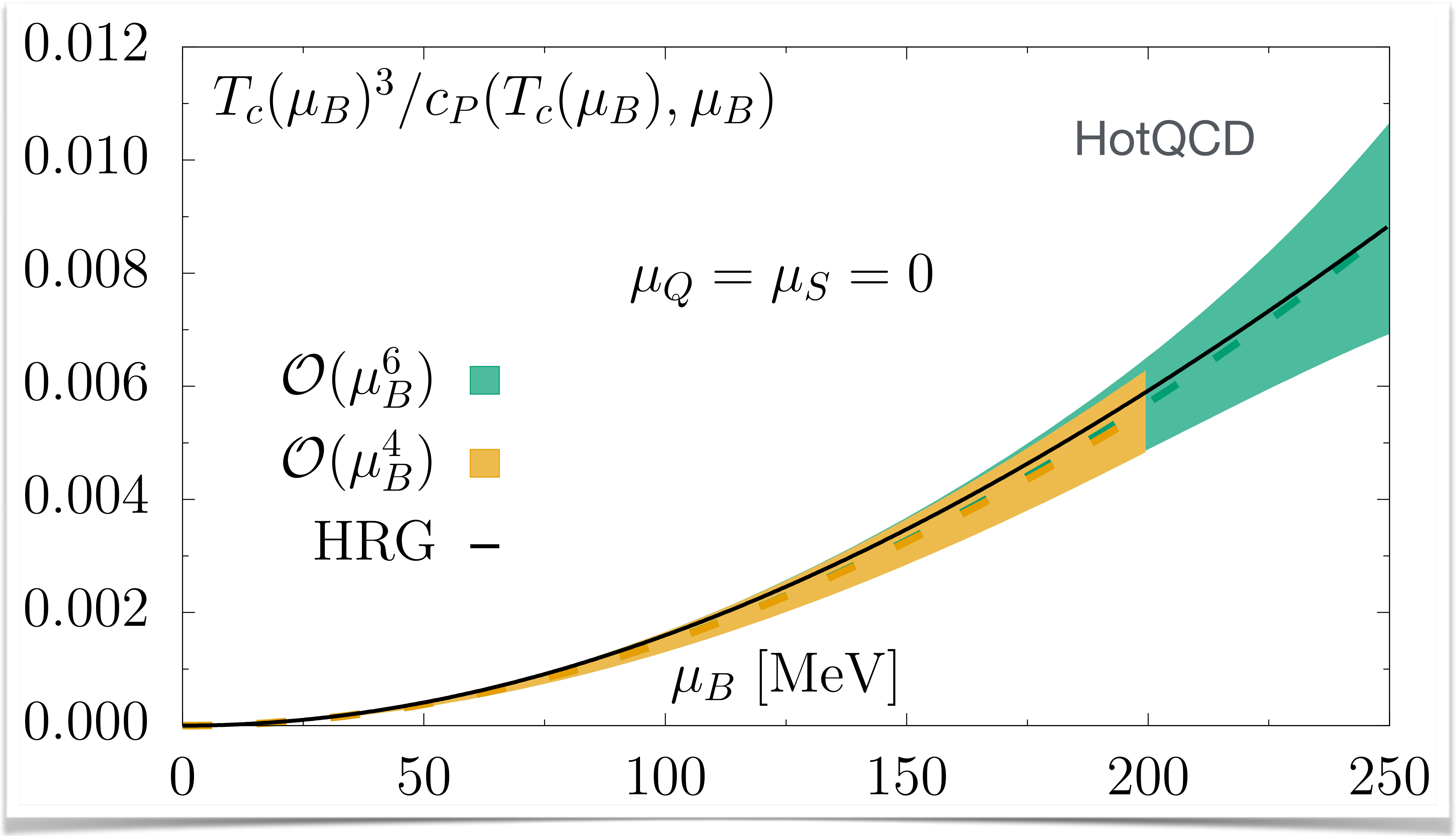
HotQCD: Phys. Lett. B795, 15 (2019)

(inverse) specific heat @ constant pressure

$$c_p = \frac{T}{(s/n_B)} \left[\frac{\partial (s/n_B)}{\partial T} \right]_p$$

- no increase above HRG

along $T_c(\mu_B)$

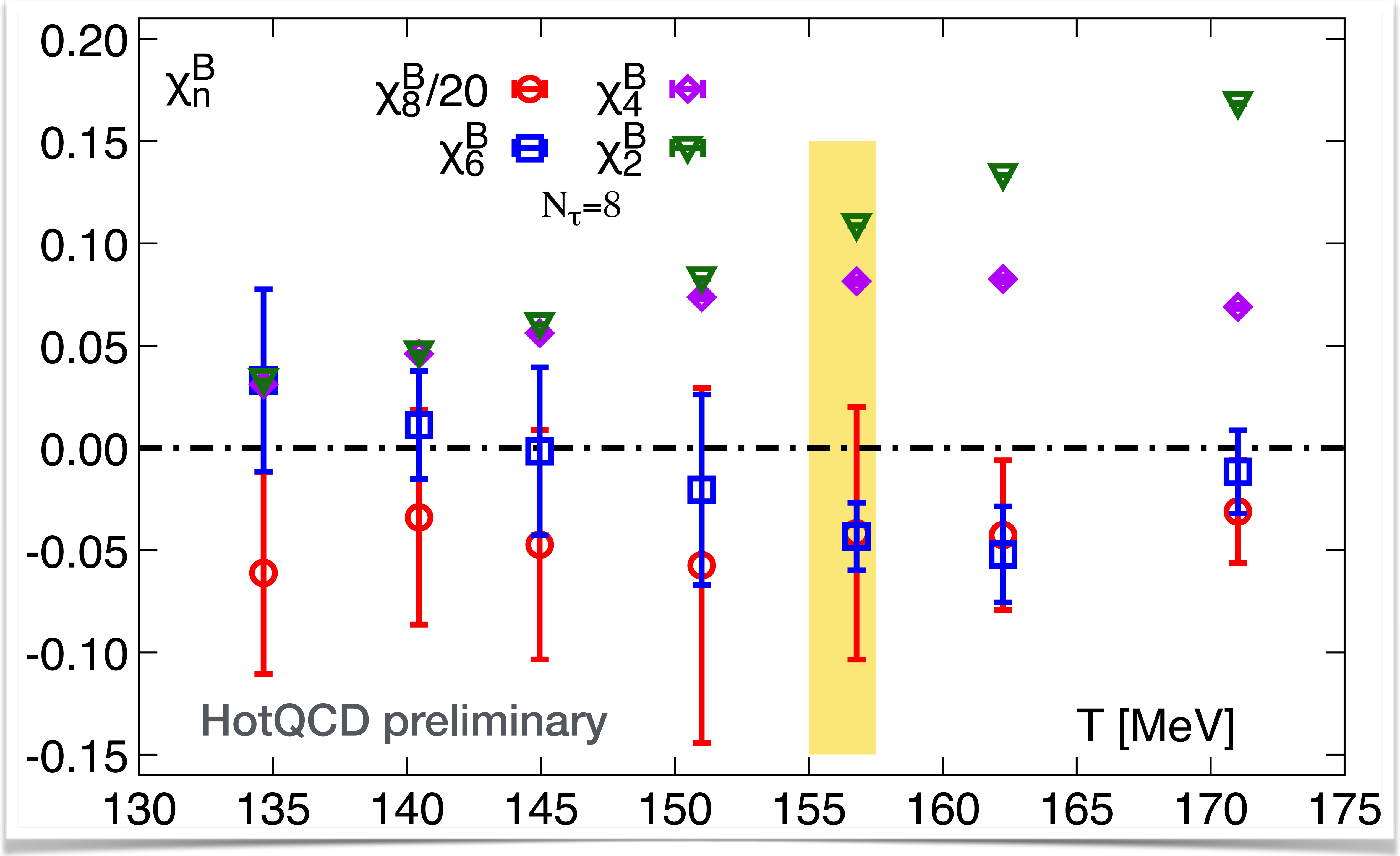


constrain location of QCD CEP from radius of convergence of Taylor expansion ?

- radius of convergence for baryon number susceptibility:

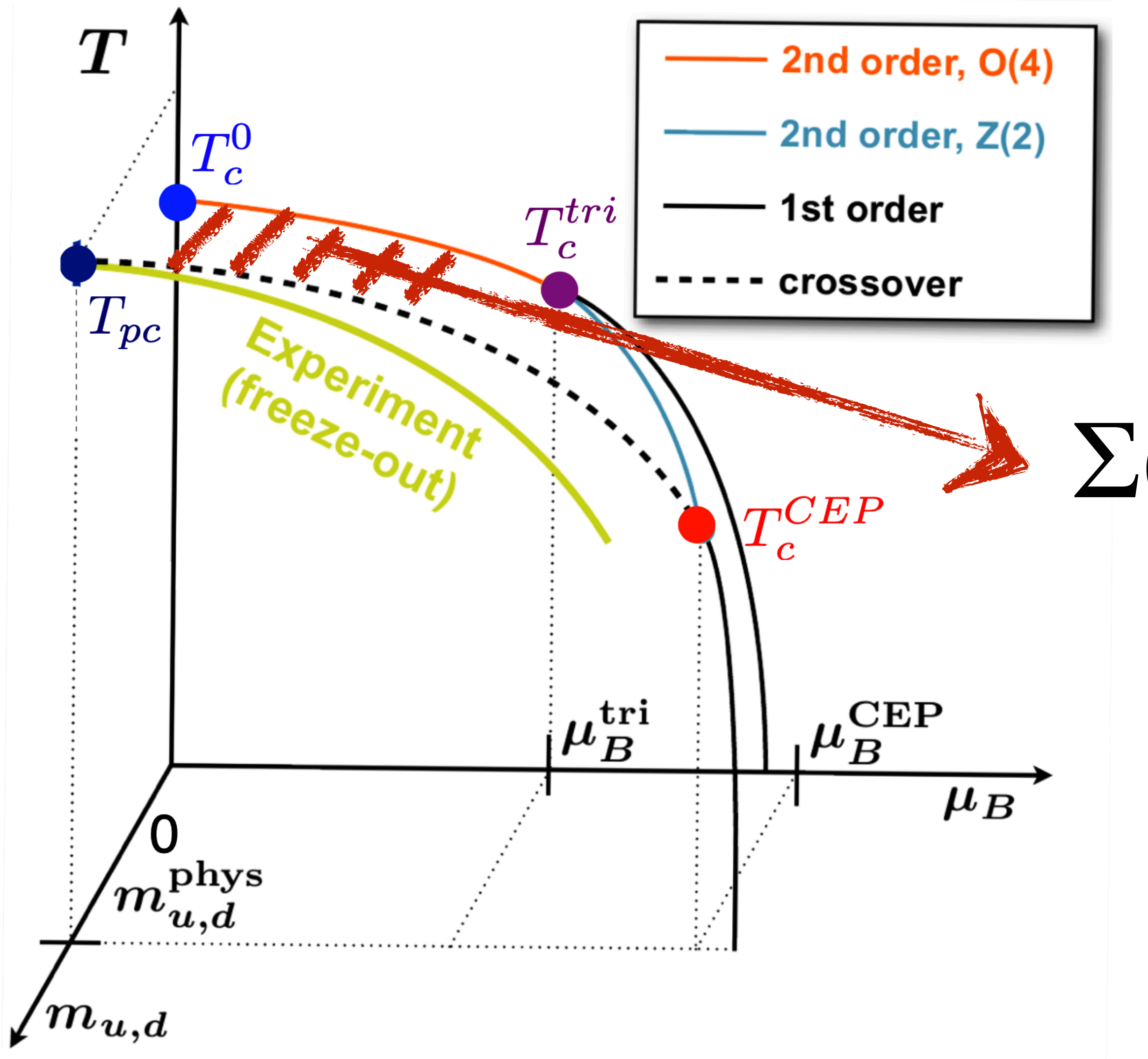
$$r_{2n}^{\chi} = \sqrt{2n(2n-1)\left|\frac{\chi_{2n}^B}{\chi_{2n+2}^B}\right|}$$

$T \geq 135$ MeV : $\chi_8^B < 0$?



$T \geq$ 135 MeV : nearest singularity in complex μ_B ?

consistent with $T_c^{\mathrm{CEP}} < T_c^0 = 132_{-6}^{+3}$ MeV

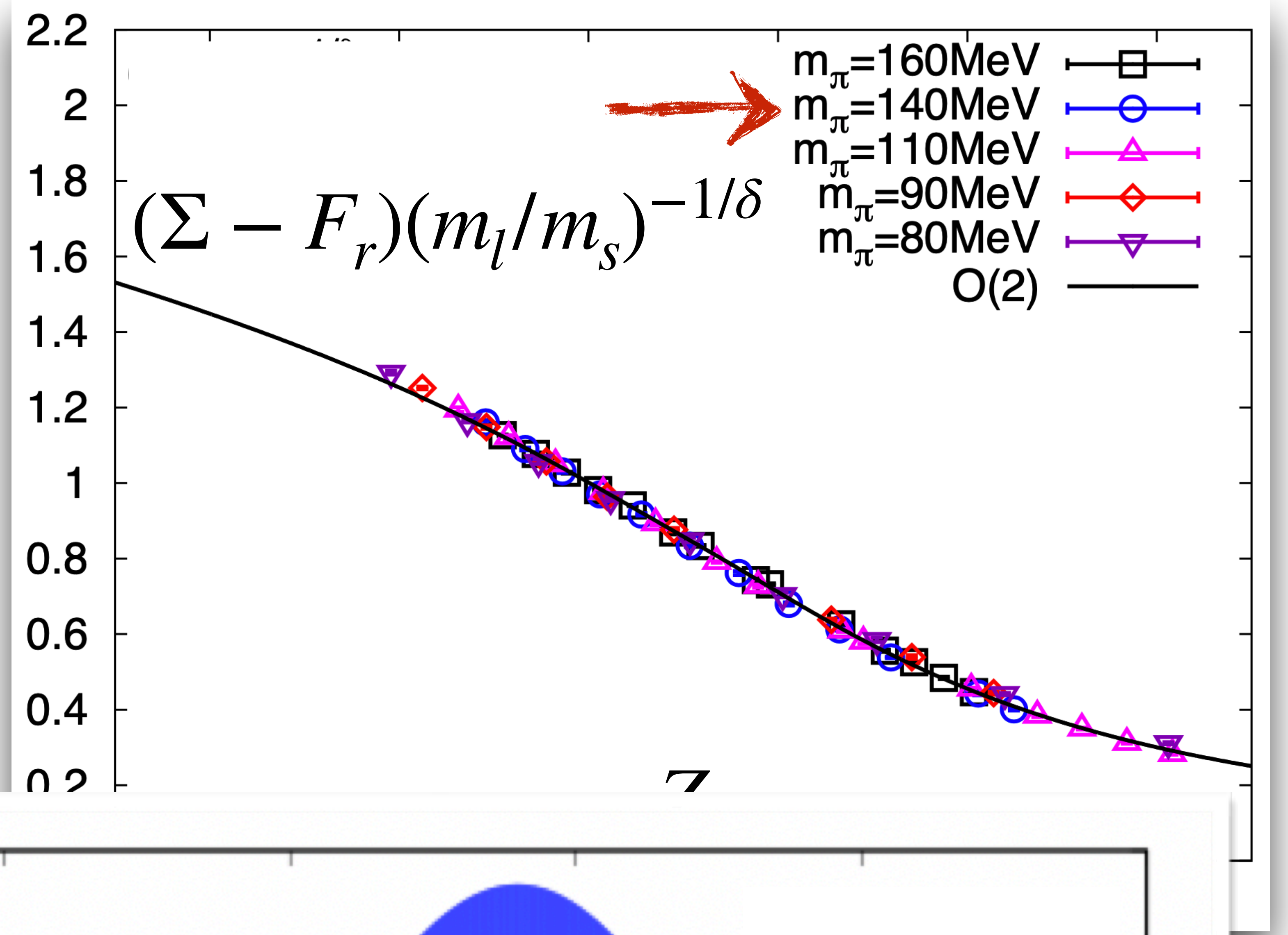


universal scaling function

$$\Sigma(T,\mu_B) = (m_l/m_s)^{1/\delta}\; f_G(z) \; + F_r(m_l, T)$$

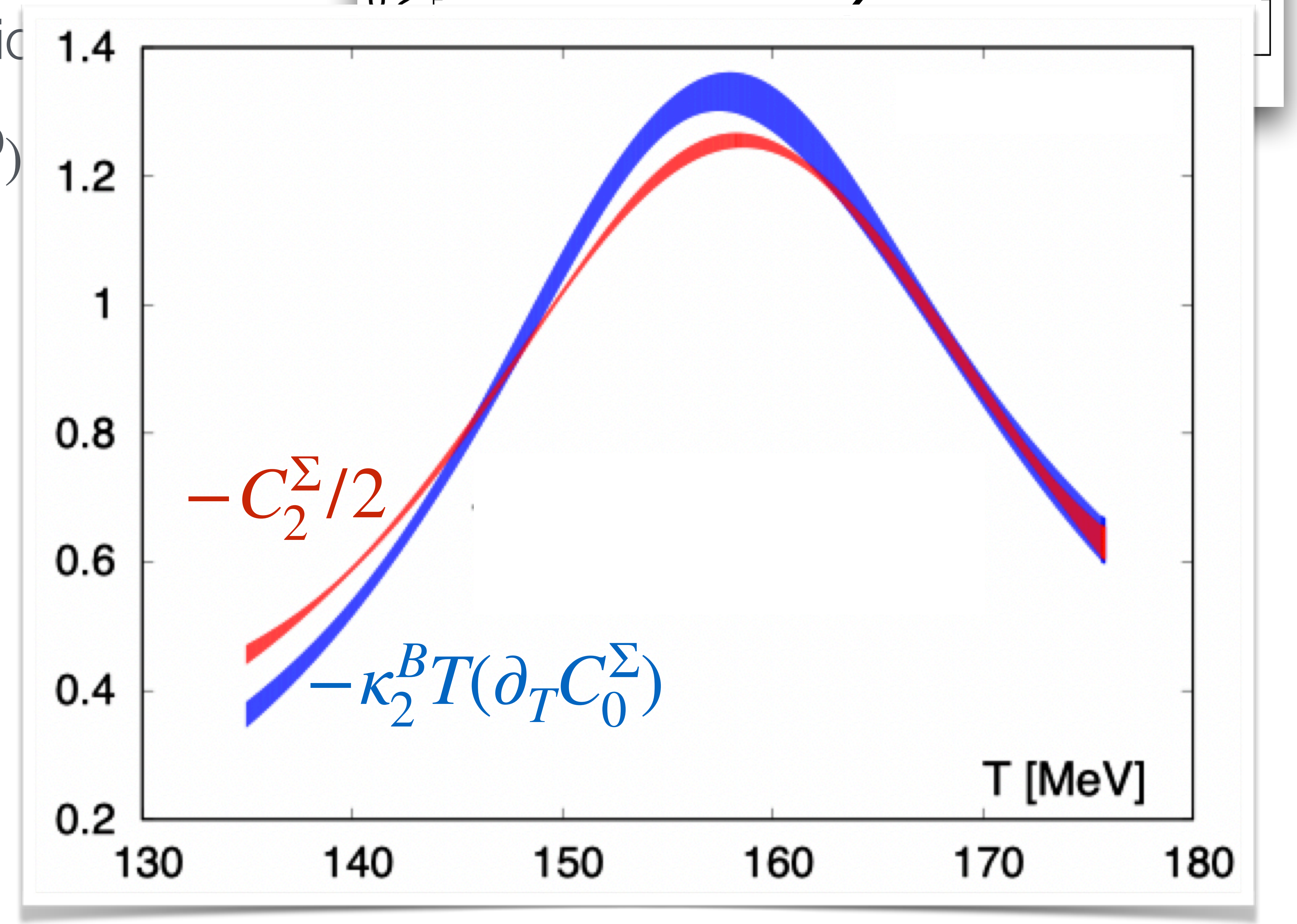
non-universal analytic correctio

$$(m_l/m_s)\left[a_0 + a_1(T-T_c^0)/T_c^0 + a_2((T-T_c^0)\right.$$

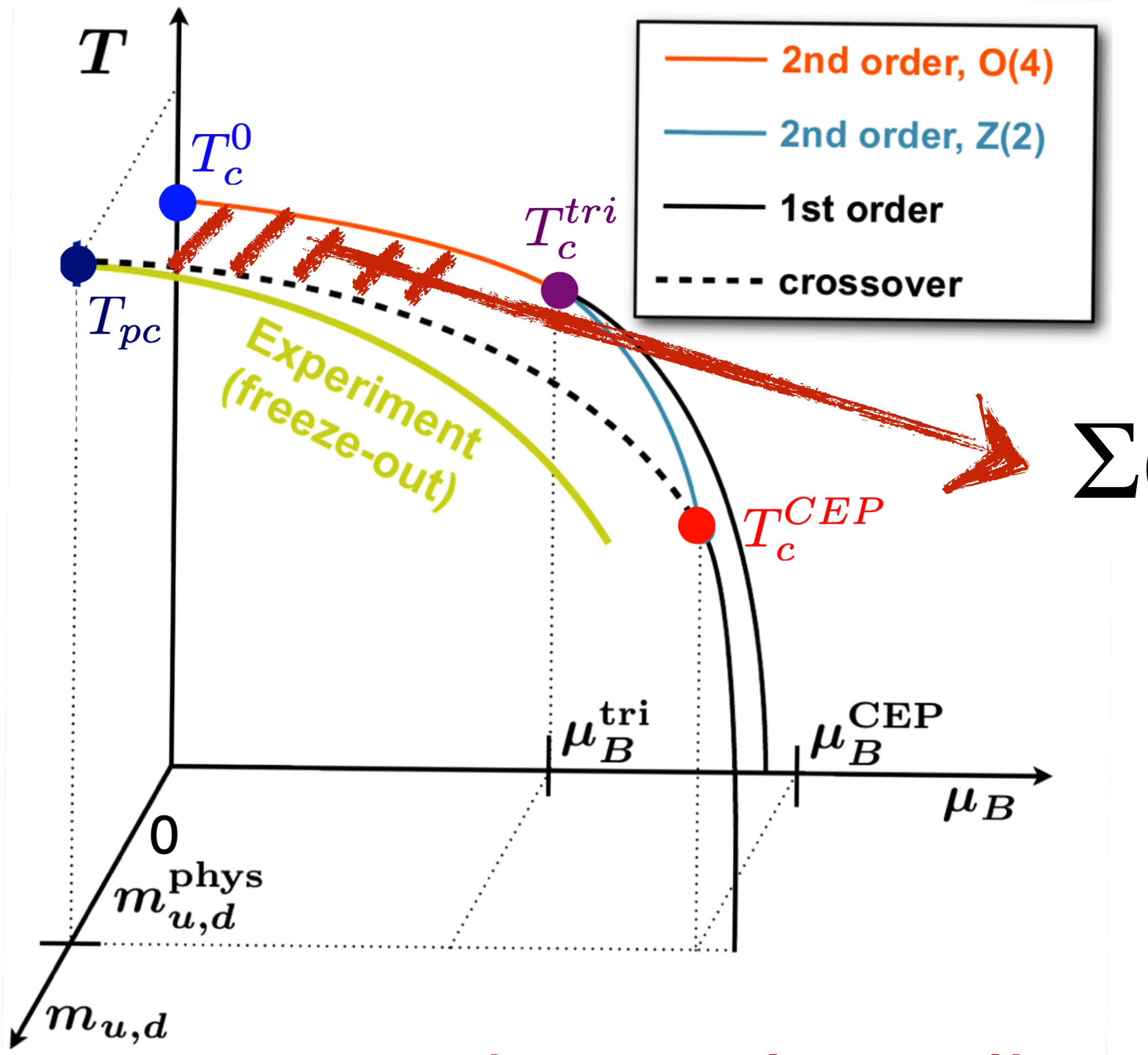


$$z = z_0\,(m_l/m_s)^{-1/\beta\delta}\left[\,(T-T_c^0)/T_c^0 \;+\; \kappa_2^B\,(\mu_B/T_c^0)^2\,\right]$$

$$\frac{\partial}{\partial T} \equiv \kappa_2^B \frac{\partial^2}{\partial(\mu_B/T)^2}$$



Mukherjee & Skokov: arXiv:1909.04639



$$\Sigma(T,\mu_B) = (m_l/m_s)^{1/\delta}\, f_G(z) + F_r(m_l,T)$$

non-universal analytic corrections

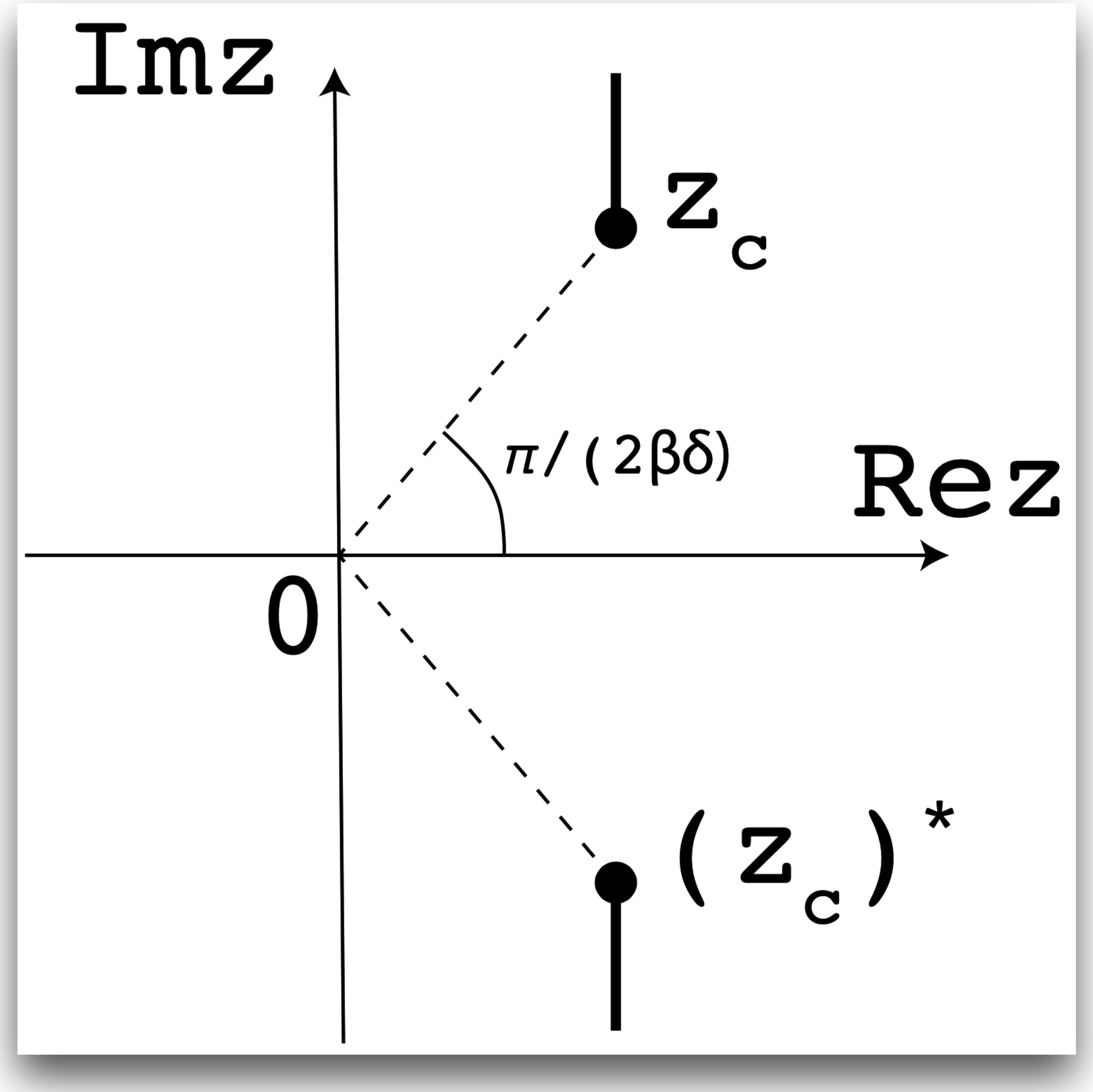
$$(m_l/m_s)\left[a_0 + a_1(T-T_c^0)/T_c^0 + a_2((T-T_c^0)/T_c^0)^2\right]$$

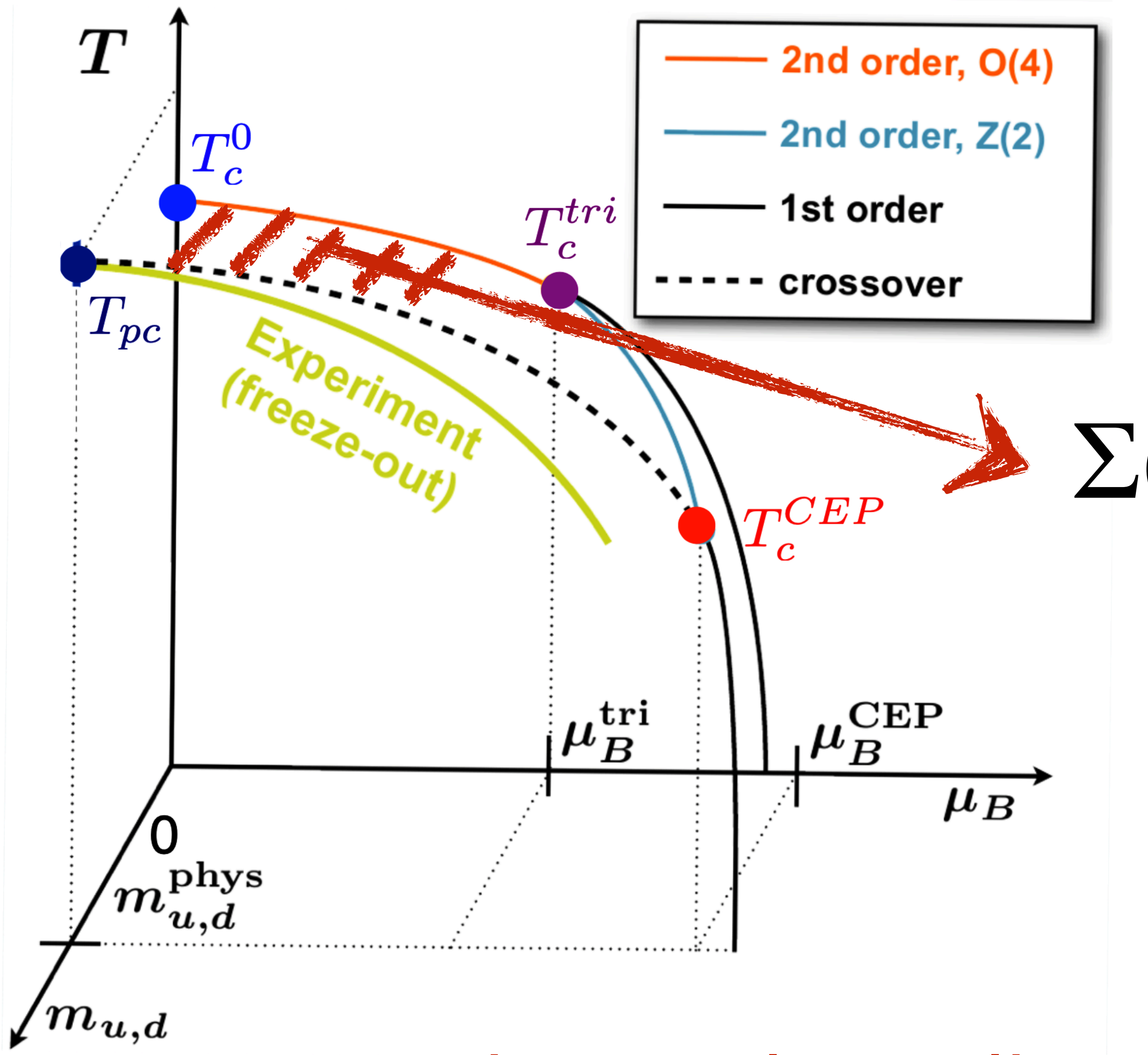
universal scaling function is non-analytic in complex z

Yang-Lee edge singularity: $f_G(z) \sim (z-z_c)^\sigma$ O(4): $\sigma \approx 0.1$

$z_c = |z_c|\, e^{i\pi/\beta\delta}$ → singularity in complex μ_B

$$z = z_0\,(m_l/m_s)^{-1/\beta\delta}\left[\,(T-T_c^0)/T_c^0 \;+\; \kappa_2^B\,(\mu_B/T_c^0)^2\,\right]$$





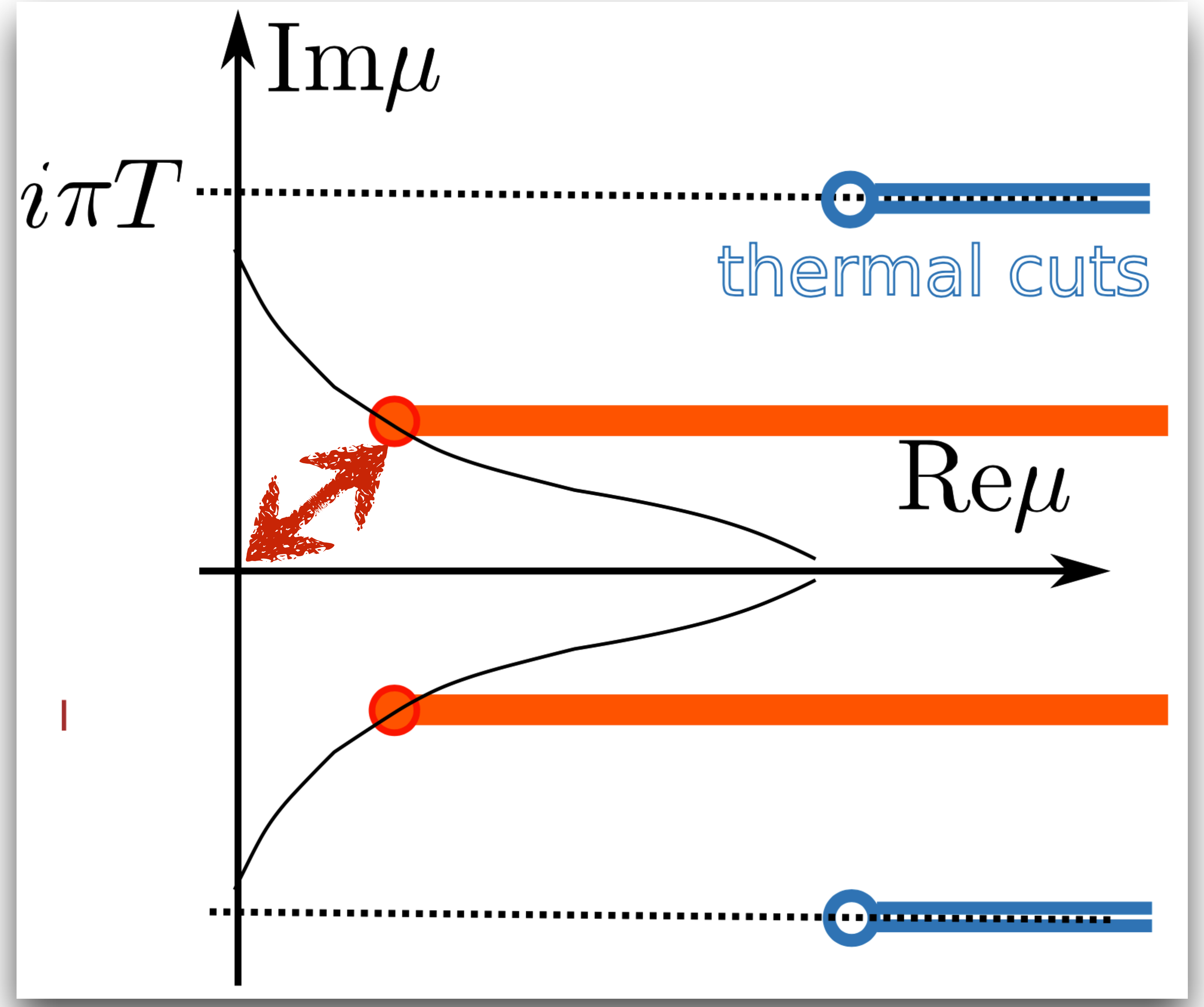
$$\Sigma(T,\mu_B) = (m_l/m_s)^{1/\delta}\, f_G(z)$$

universal scaling function is non-analytic in complex z

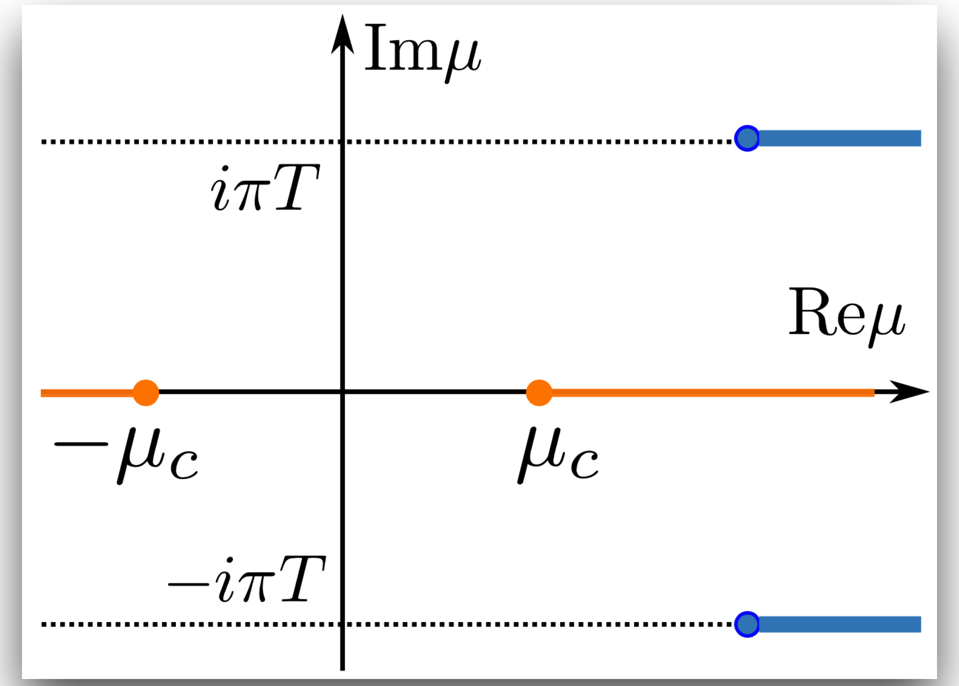
Yang-Lee edge singularity: $f_G(z) \sim (z - z_c)^{\sigma}$

$z_c = |z_c| e^{i\pi/\beta\delta}$ → singularity in complex μ_B

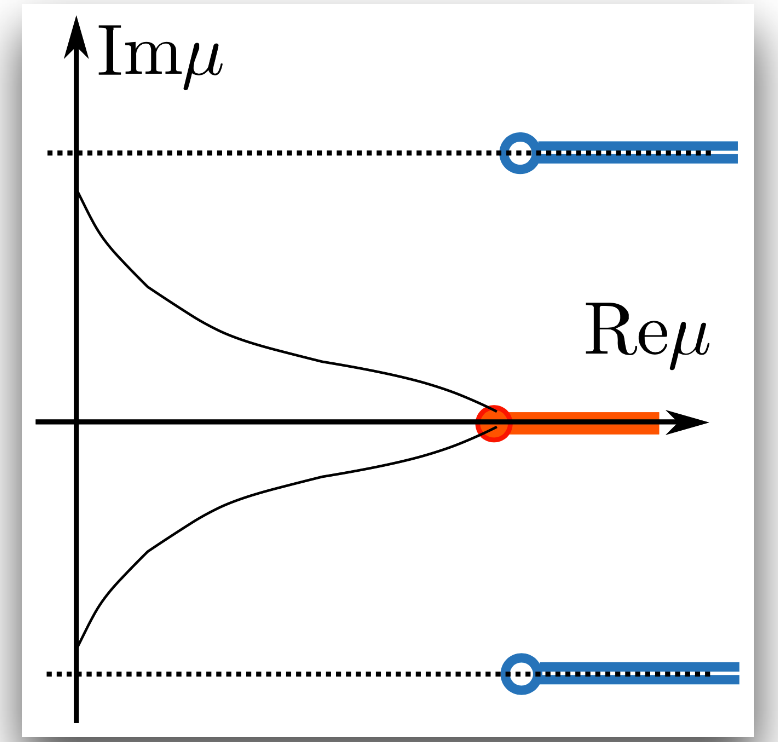
$m_l > 0,\ T_{pc}(\mu_B) \sim T > T_c^{CEP}$

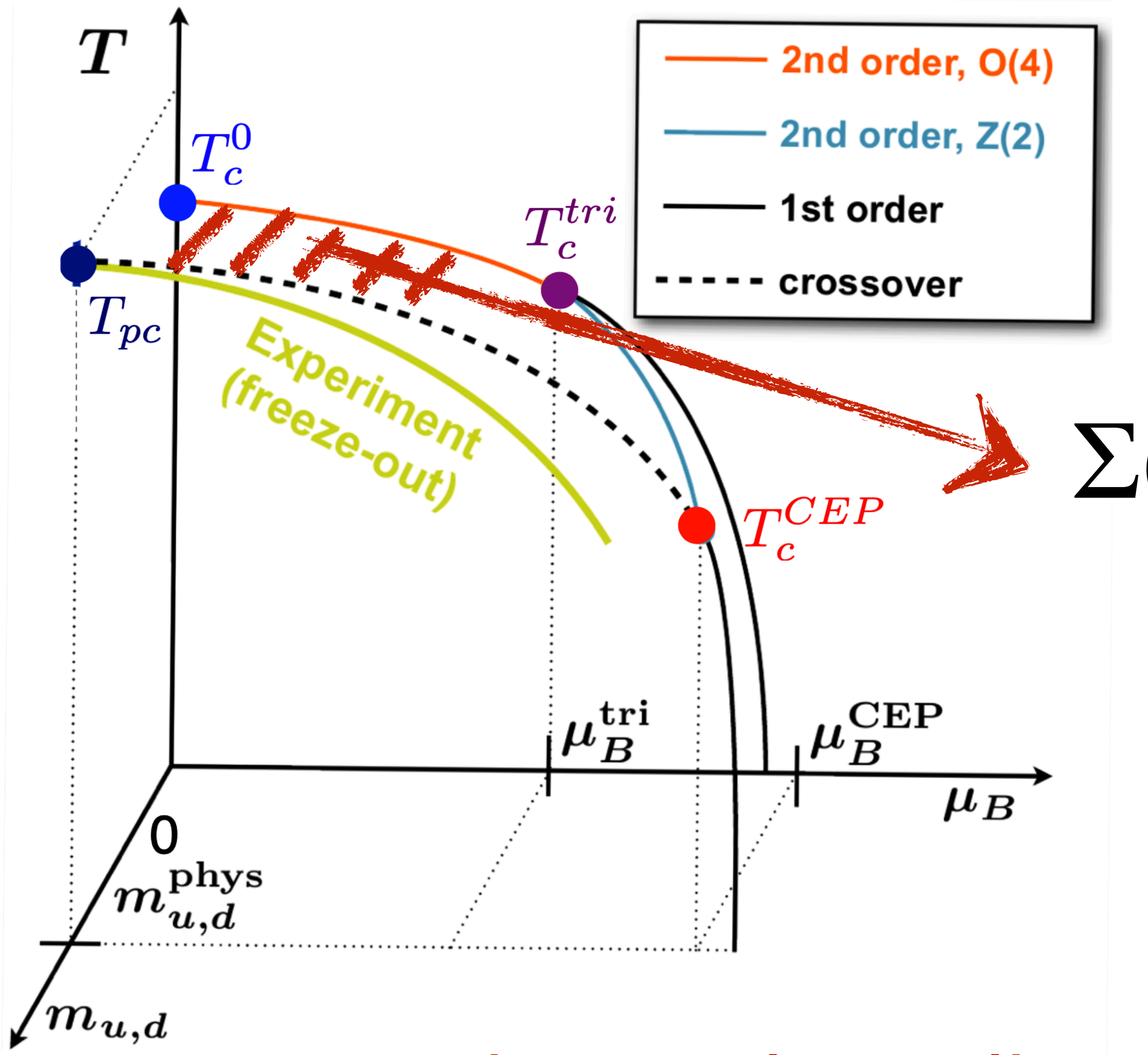


$m_l = 0,\ T_c^{tri} < T < T_c^0$



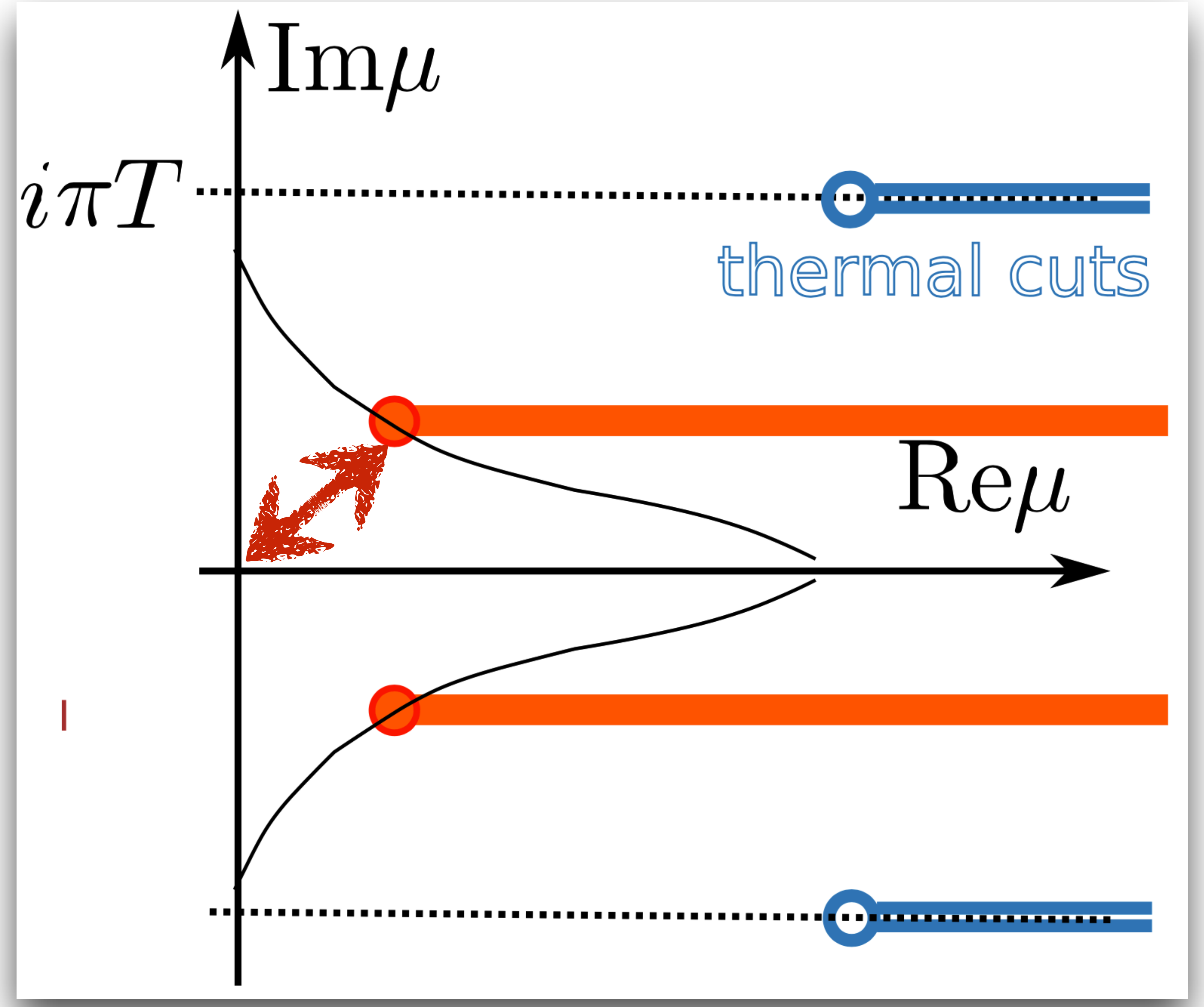
$m_l > 0,\ T = T_c^{CEP}$





$$\Sigma(T,\mu_B) = \left(m_l/m_s\right)^{1/\delta} f_G(z)$$

$m_l > 0,\ T_{\rm pc}(\mu_B) \sim T > T_c^{\rm CEP}$



universal scaling function is non-analytic in complex z

Yang-Lee edge singularity: $f_G(z) \sim (z - z_c)^\sigma$

$z_c = |z_c| e^{i\pi/\beta\delta}$ → singularity in complex μ_B

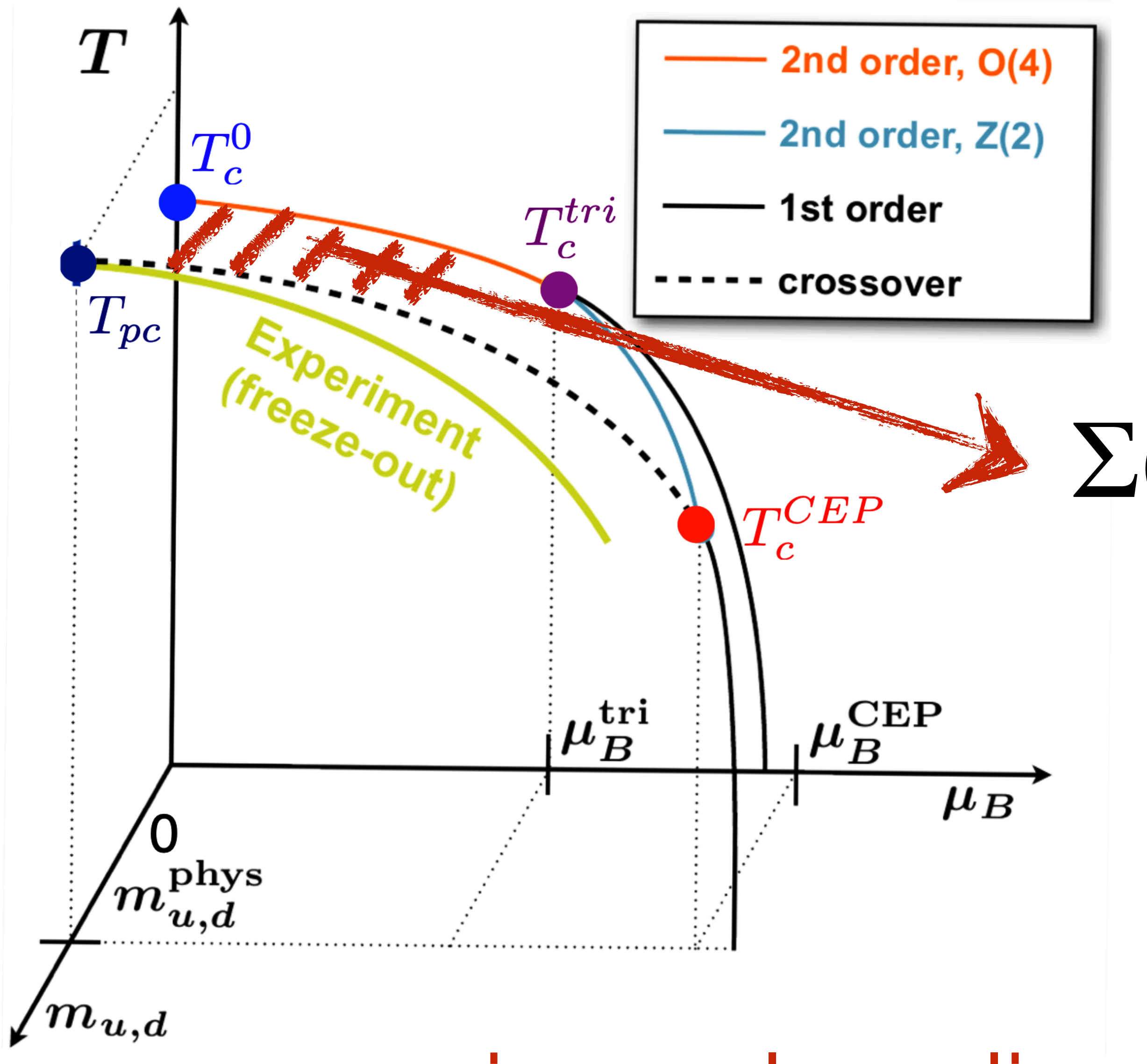
completely determined by the universality class

$|z_c| \approx 1.89$ … mean field

$|z_c| \approx 1.65$ … $O(N \to \infty)$

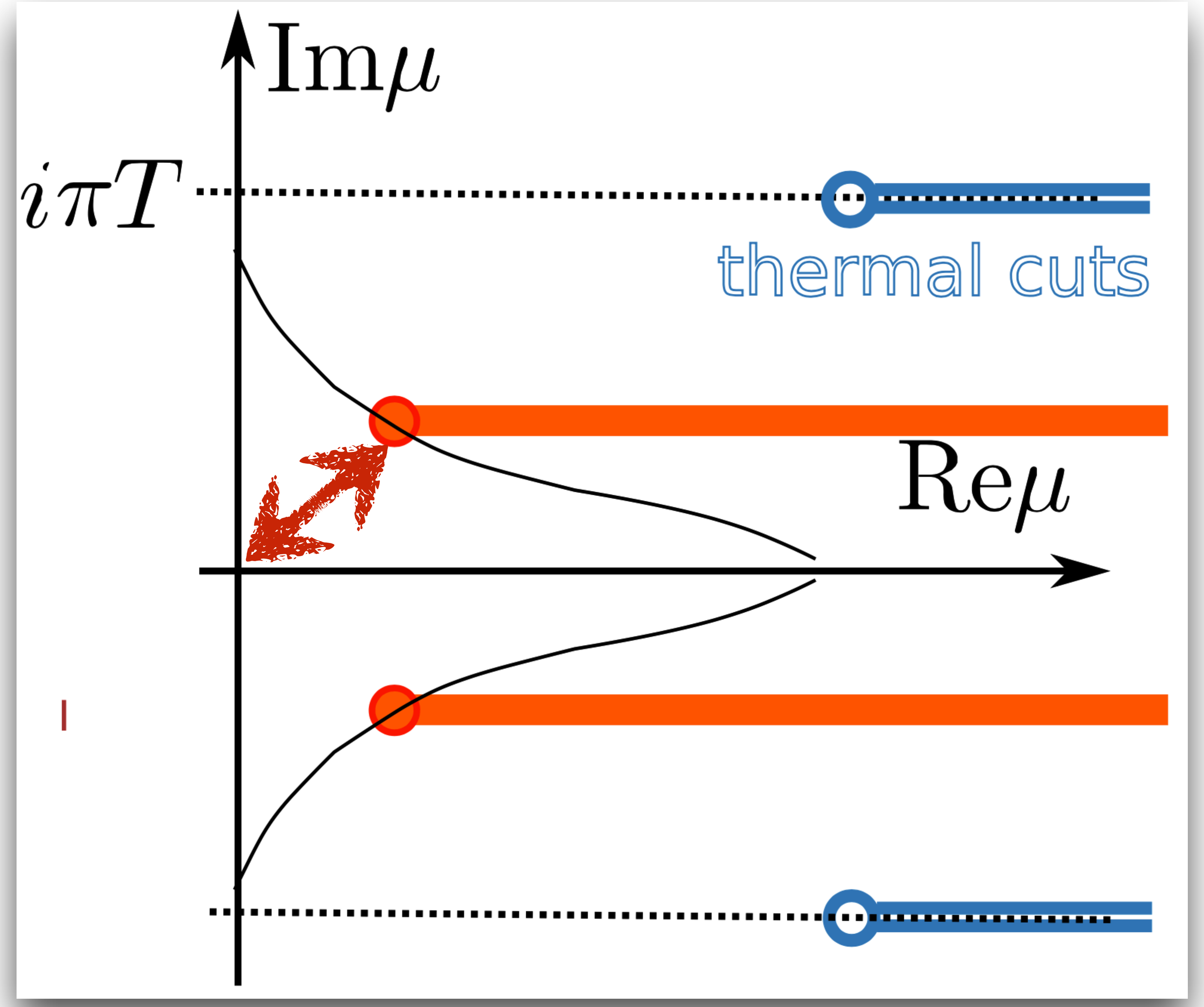
$|z_c| \approx 1.68$ … $O(4)$

quark-meson mode: functional renormalization group method (Connelly, Johnson & Skokov: to appear)



$$m_l > 0,\ T_{\rm pc}(\mu_B) \sim T > T_c^{\rm CEP}$$

$$\Sigma(T,\mu_B) = \left(m_l/m_s\right)^{1/\delta}\ f_G(z)$$



universal scaling function is non-analytic in complex z

Yang-Lee edge singularity: $f_G(z) \sim (z - z_c)^{\sigma}$

$z_c = |z_c| e^{i\pi/\beta\delta}$ → singularity in complex μ_B

$z \ \leftrightarrow\ \mu_B$

non-universal constants from (L)QCD

$$z = z_0\ (m_l/m_s)^{-1/\beta\delta}\ \left[\ (T - T_c^0)/T_c^0\ +\ \kappa_2^B\ (\mu_B/T_c^0)^2\ \right]$$

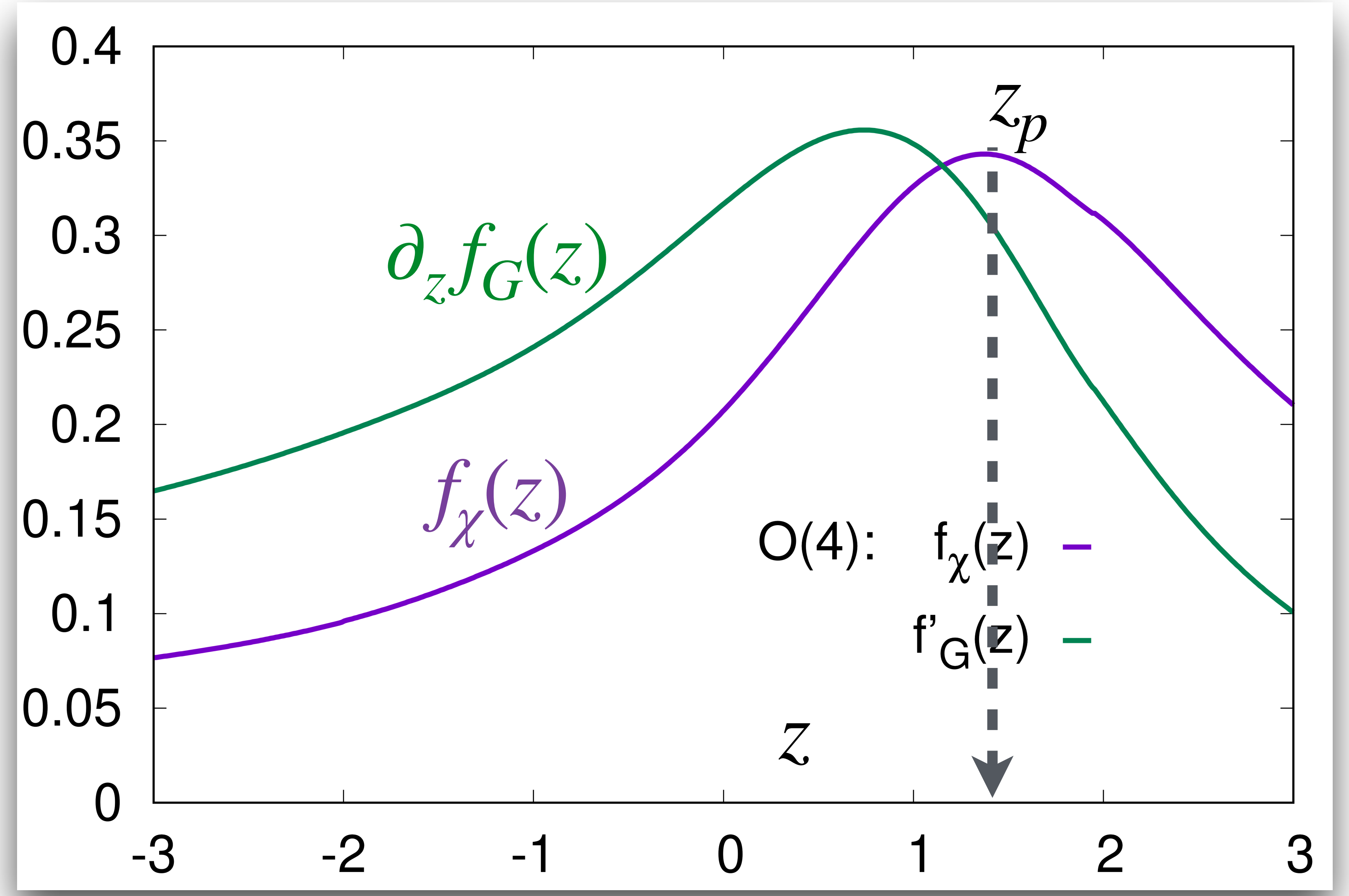
$T_c^0 = 132^{+3}_{-6}\ MeV$ $\quad \kappa_2^B = 0.012(4)$

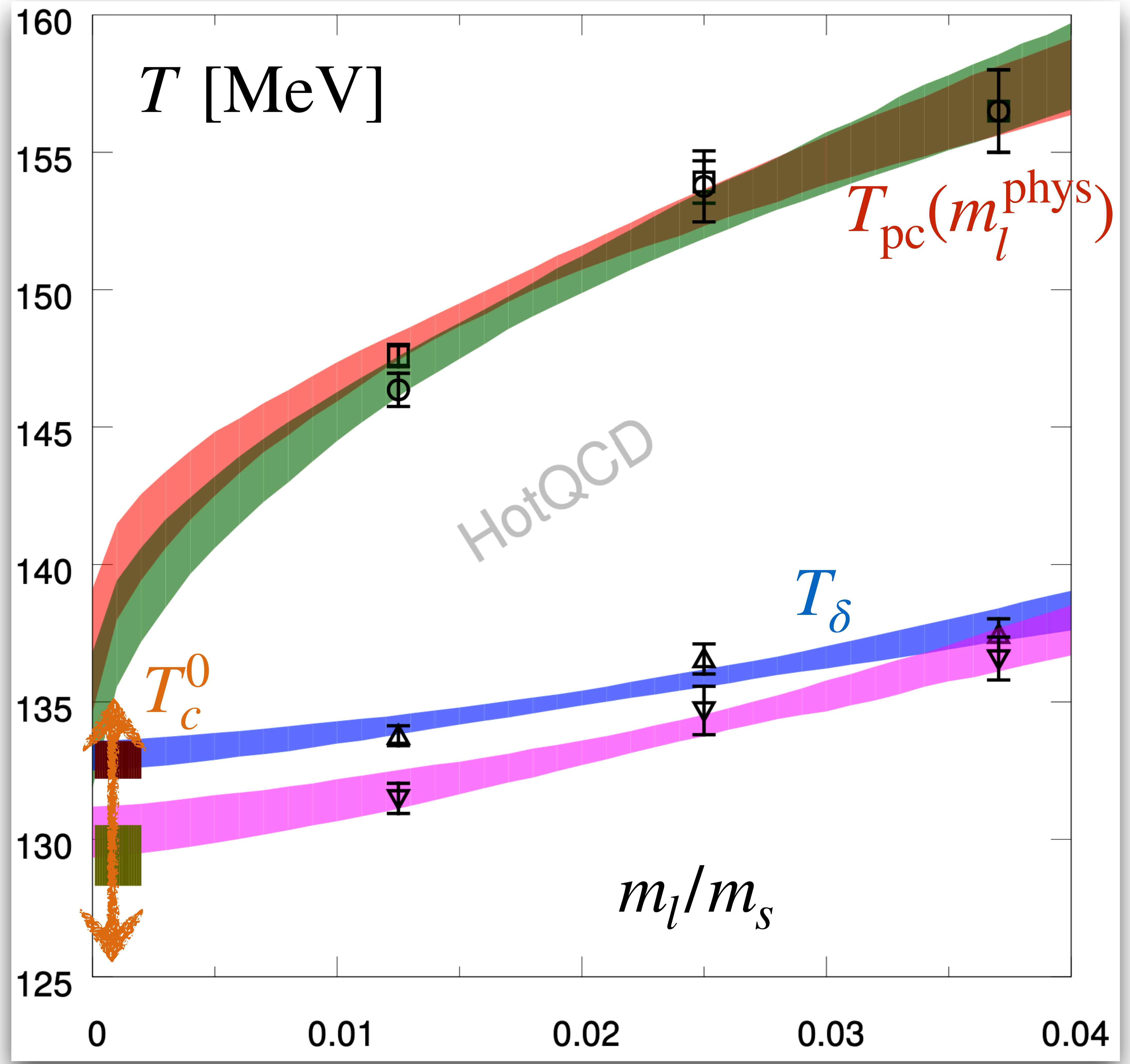
$\kappa_4^B = 0.000(4)$

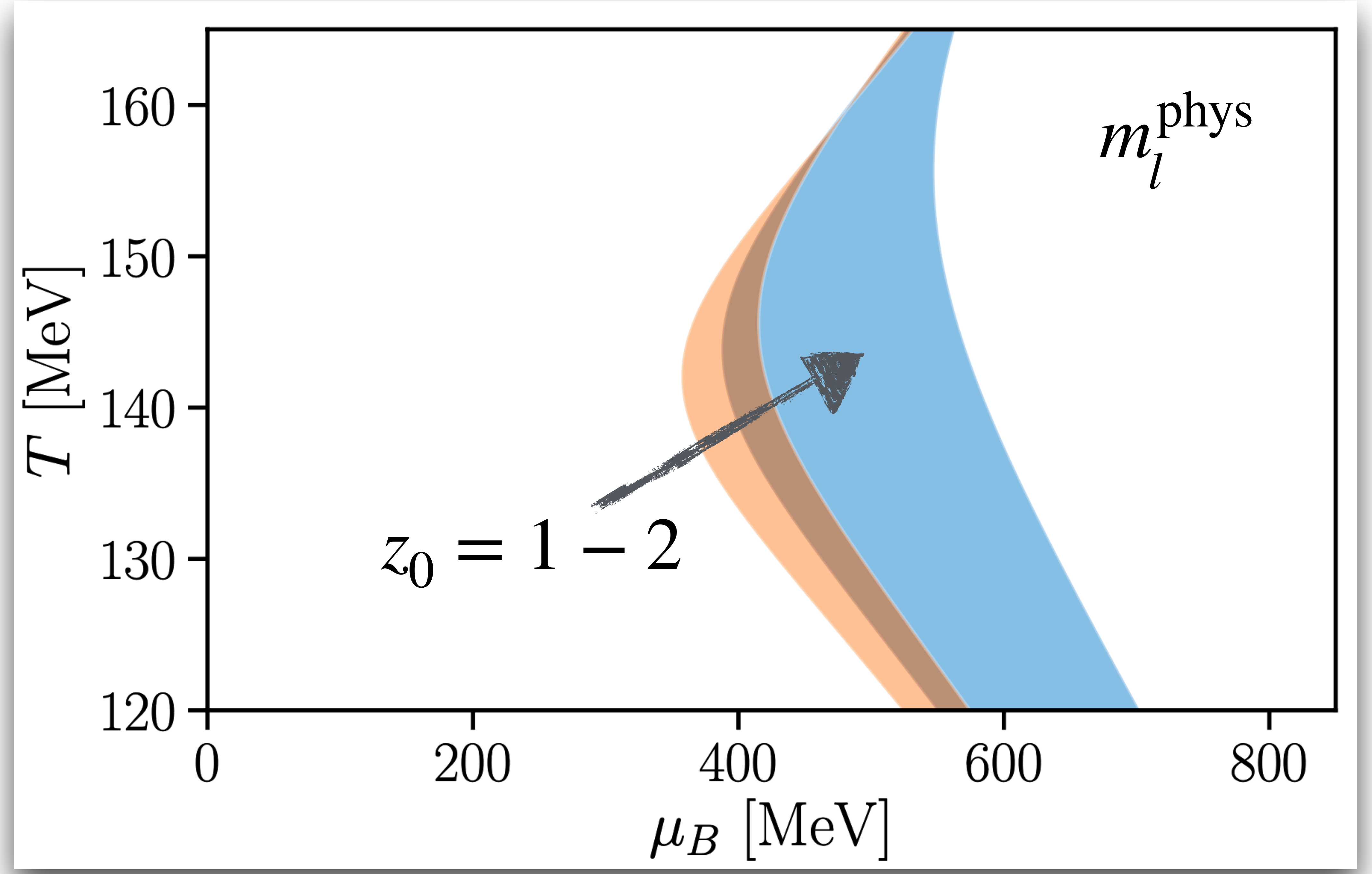
$$T_{\rm pc}(m_l) = T_c^0 \left[1 + \frac{z_p}{z_0} \left(\frac{m_l}{m_s} \right)^{1/\beta\delta} \right]$$

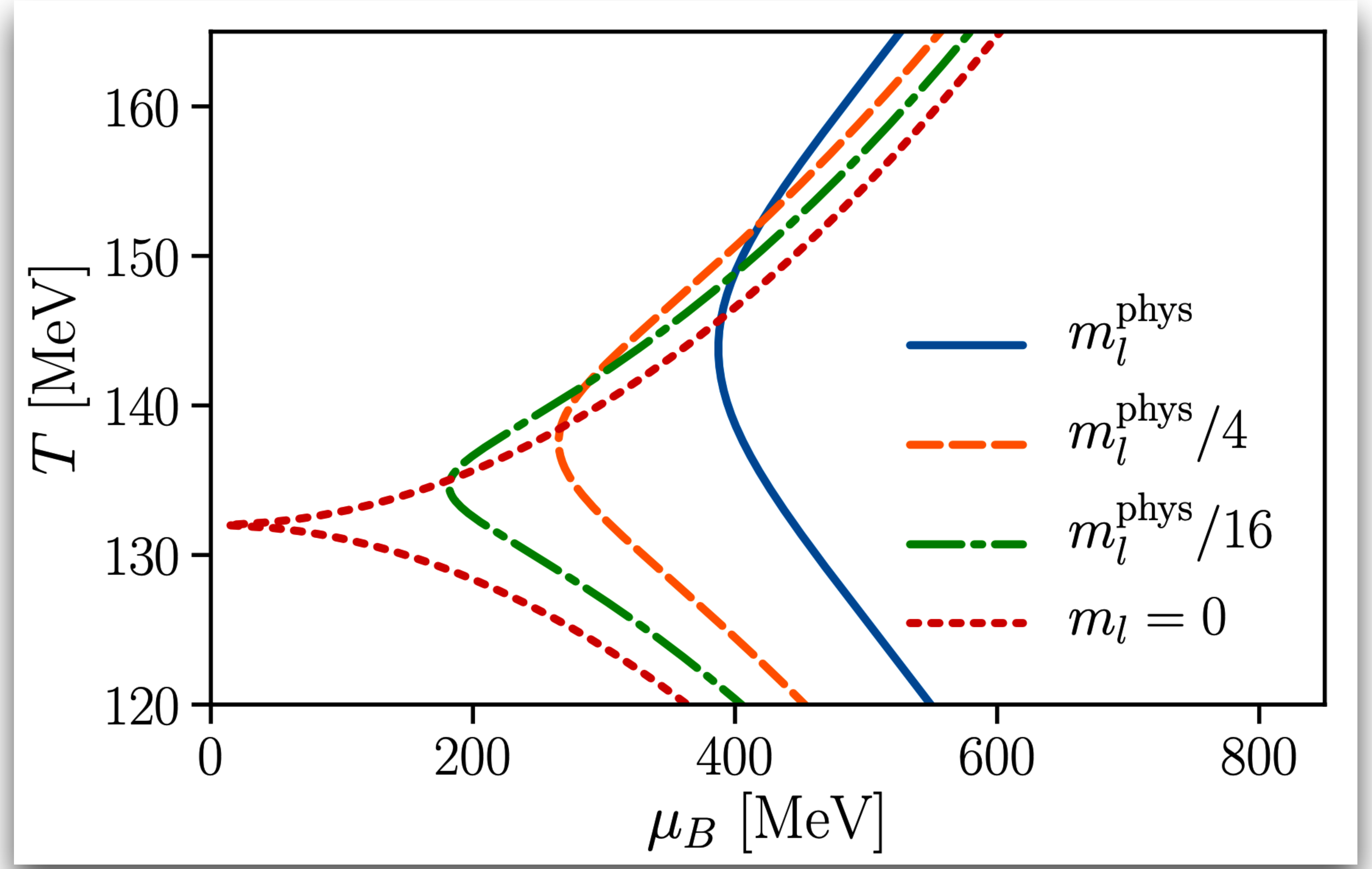
1-2

z_p: peak-location of scaling function, $f_\chi(z)$, for χ









Mukherjee & Skokov: arXiv:1909.04639

radius of convergence in μ_B

based only on universality & (L)QCD inputs

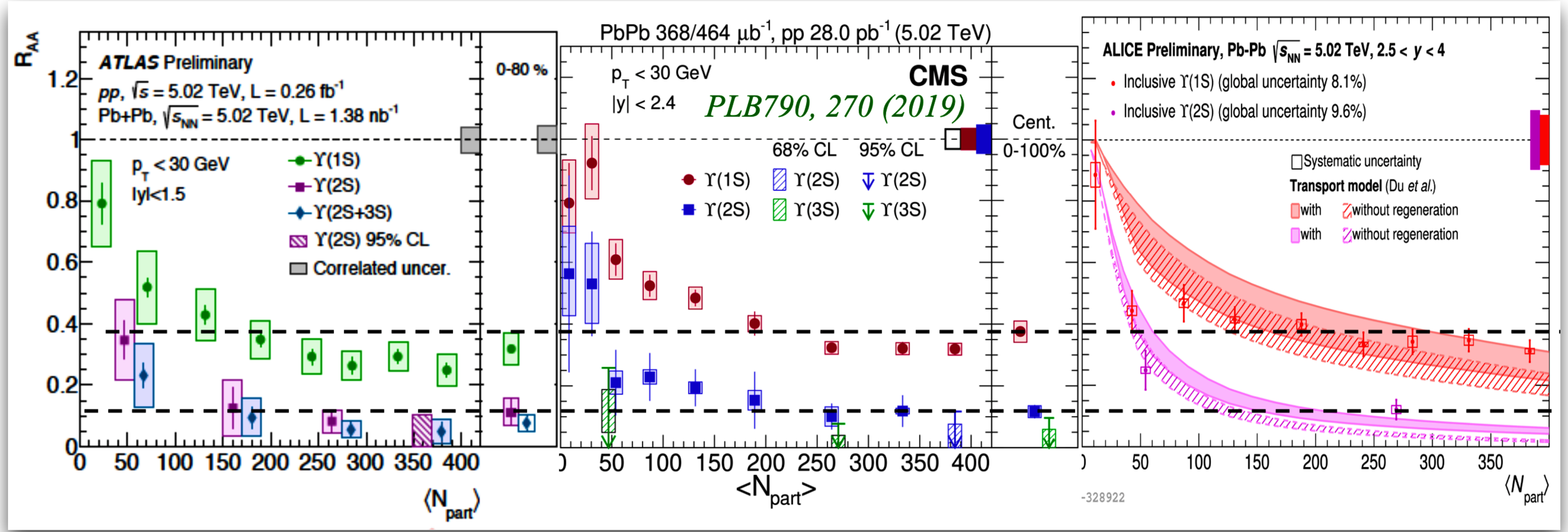
within O(4) scaling regime $T \sim T_{\text{pc}}(\mu_B) \sim T_c^0$

assuming QCD CEP is not in-between

how high can we reach?

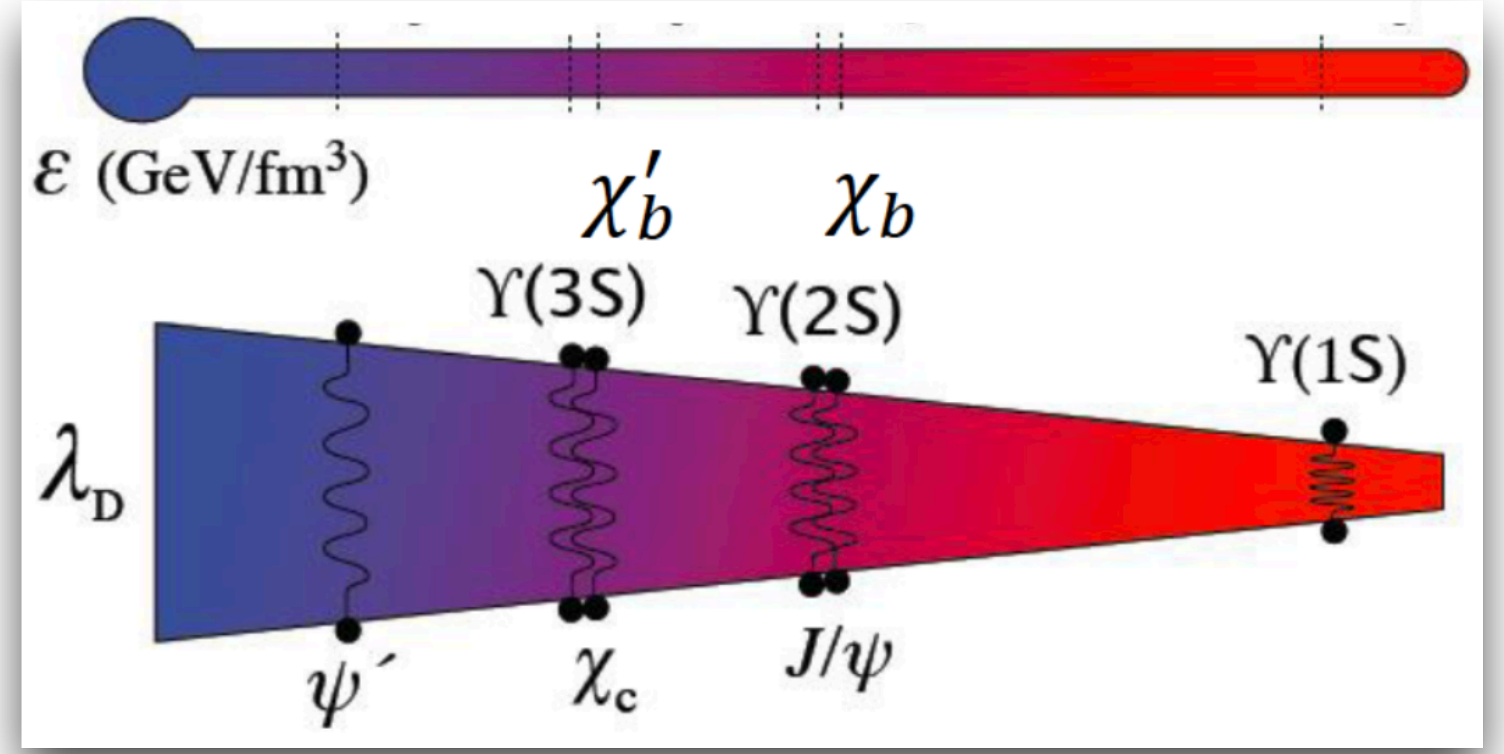
excited bottomonia in QGP: 1, 2, 3 …

Larsen, Meinel, Mukherjee & Petreczky: Phys. Rev. D100, 074506 (2019)
Larsen, Meinel, Mukherjee & Petreczky: arXiv:1910.07374 (Phys. Lett. B???)

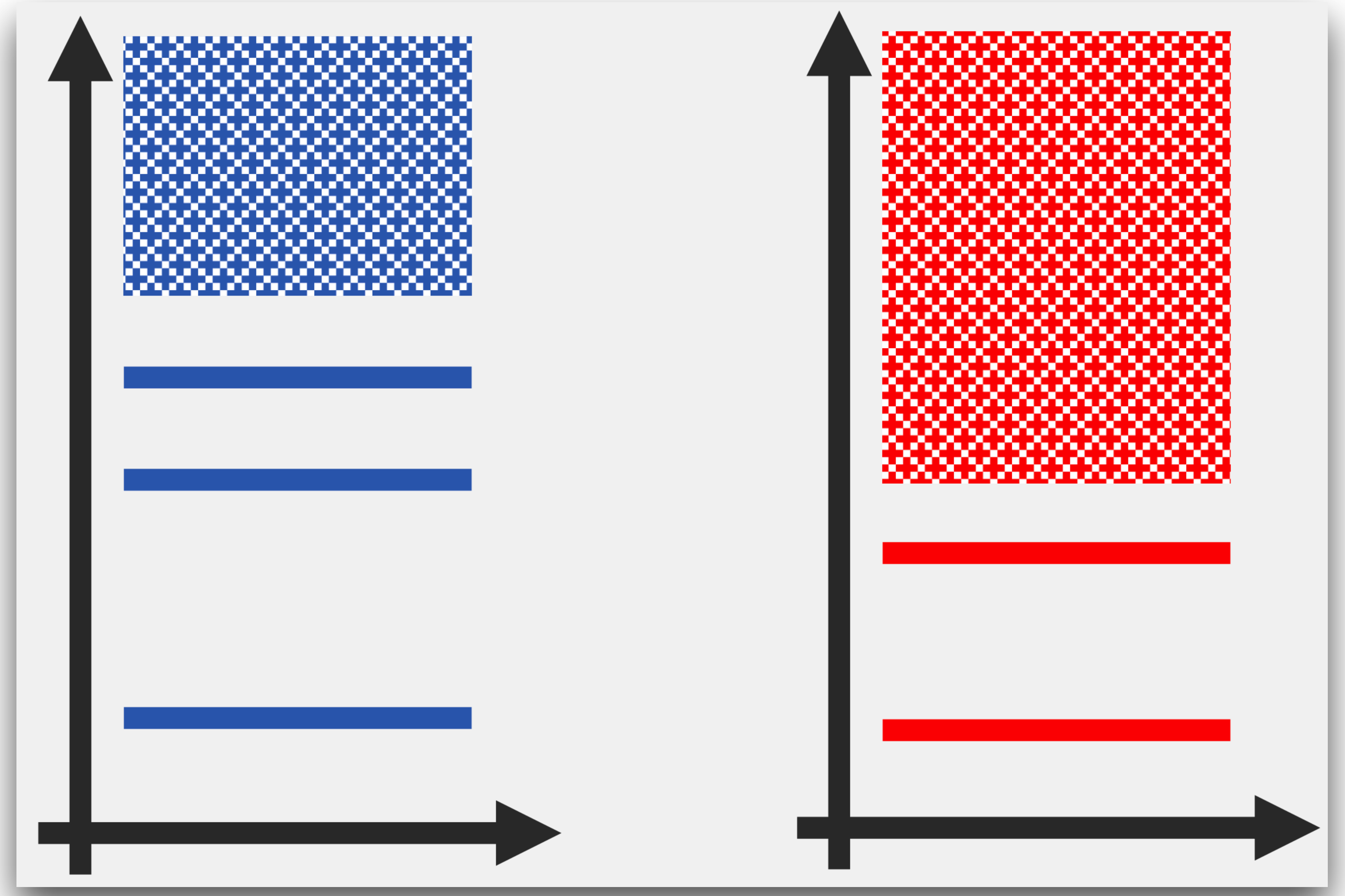


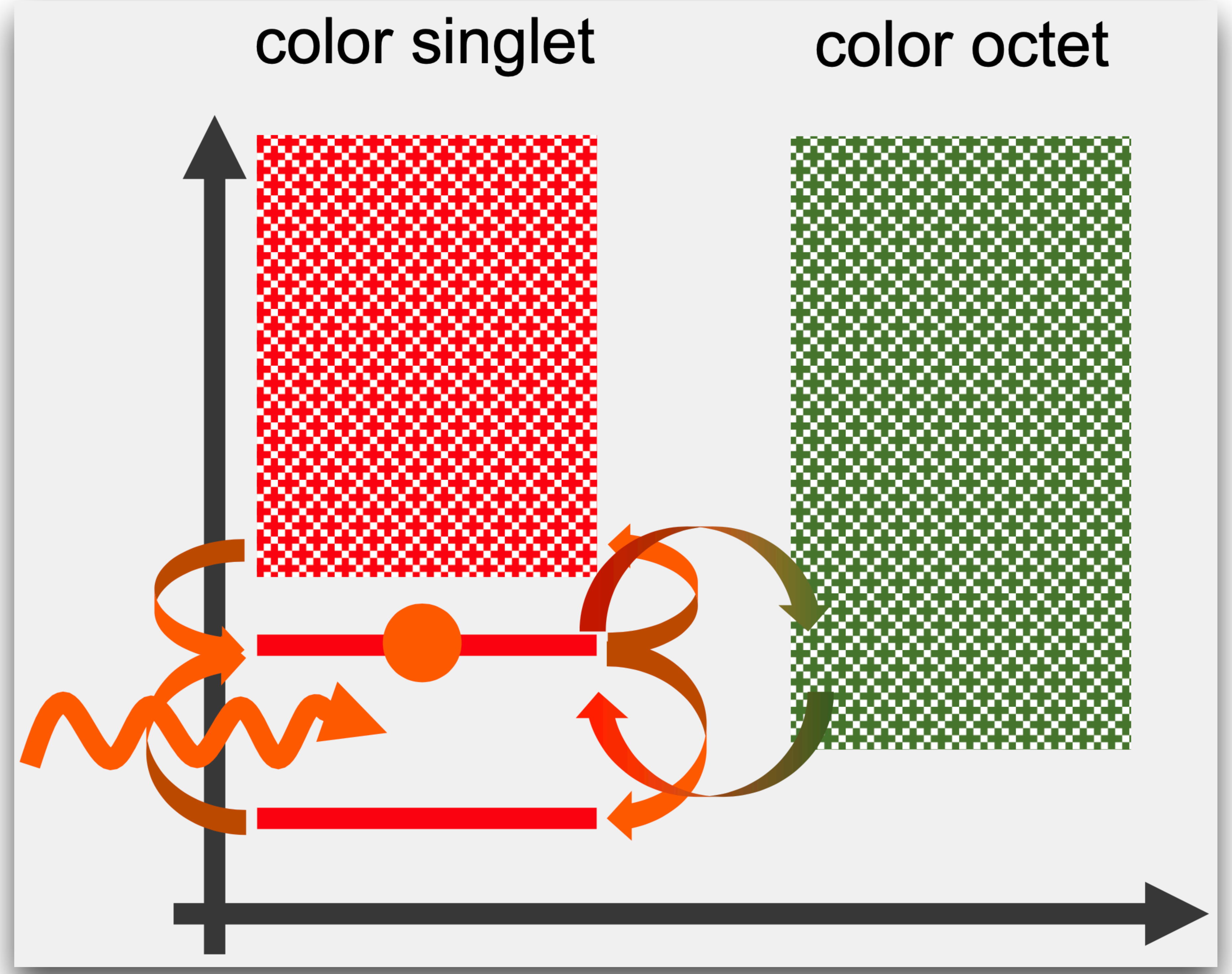
Zebo Tang: Quark Matter 2019

sequential suppression of upsilons in QGP ?



how does this happen:





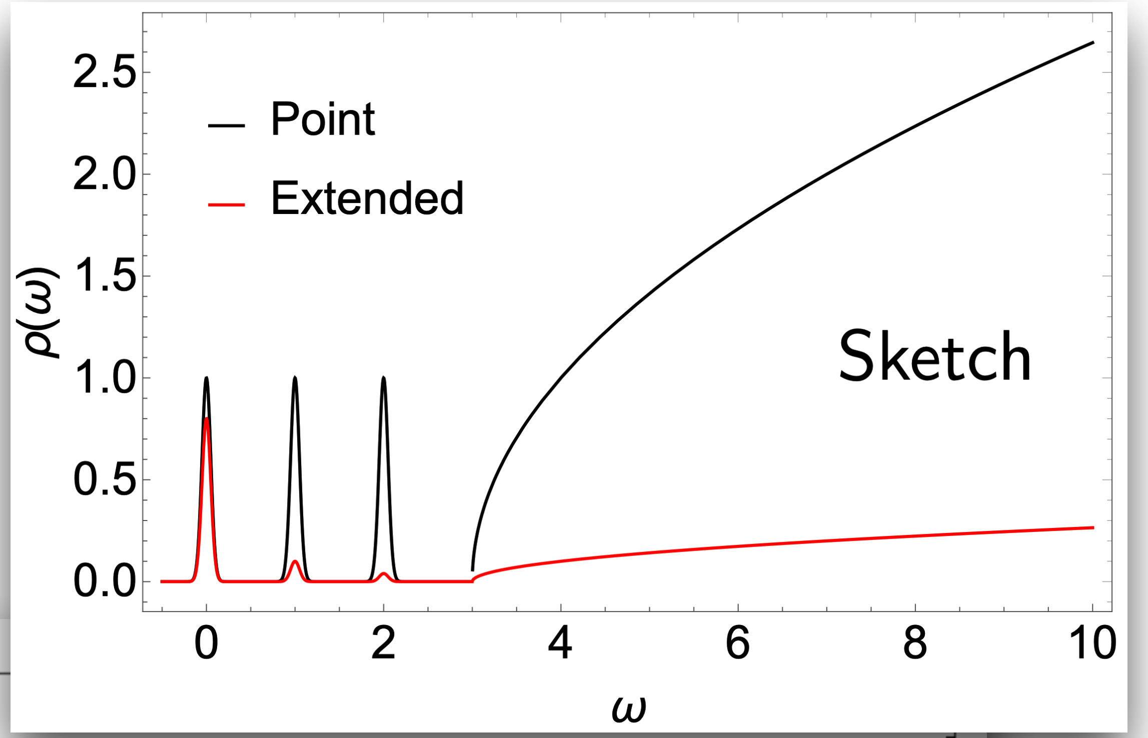
$\Re(V)$ **!?**

$\Im(V)$ **!?**

$\Re(V) + \Im(V)$ **!?**

$m_b > \Lambda_{\rm QCD}$, $m_b > \pi T$: lattice NRQCD

- expansion in m_b^{-1} : $\mathcal{O}(v^4)$ + $\mathcal{O}(v^6)$ spin, tree-level tadpole improved
- HISQ: $N_f = 2+1$, $m_\pi = 160$ MeV, $m_K^{\rm phys}$, 12×48^3

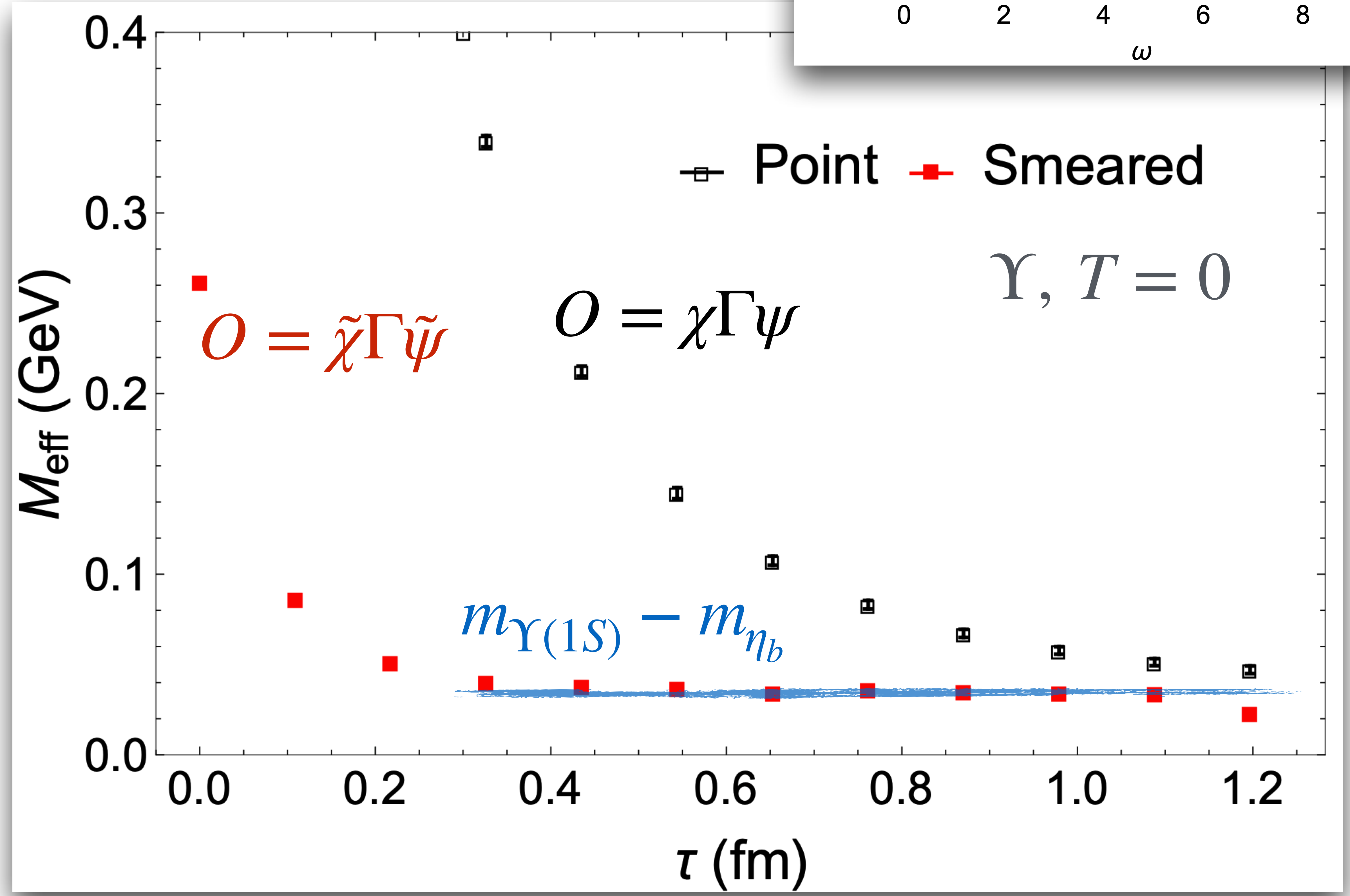


extended bottomonium operators

$$\tilde{\chi} = W\chi,\ \tilde{\psi} = W\psi,\ W = \left[1 + \sigma^2\Delta^{(2)}/(4N)\right]^N$$

$$R_{\rm rms} = \sqrt{3}\sigma/2 \approx 0.21\ \text{MeV}$$

large reductions in the continuum contributions



χ, ψ : NRQCD (anti-) quark fields, 2-component spinors

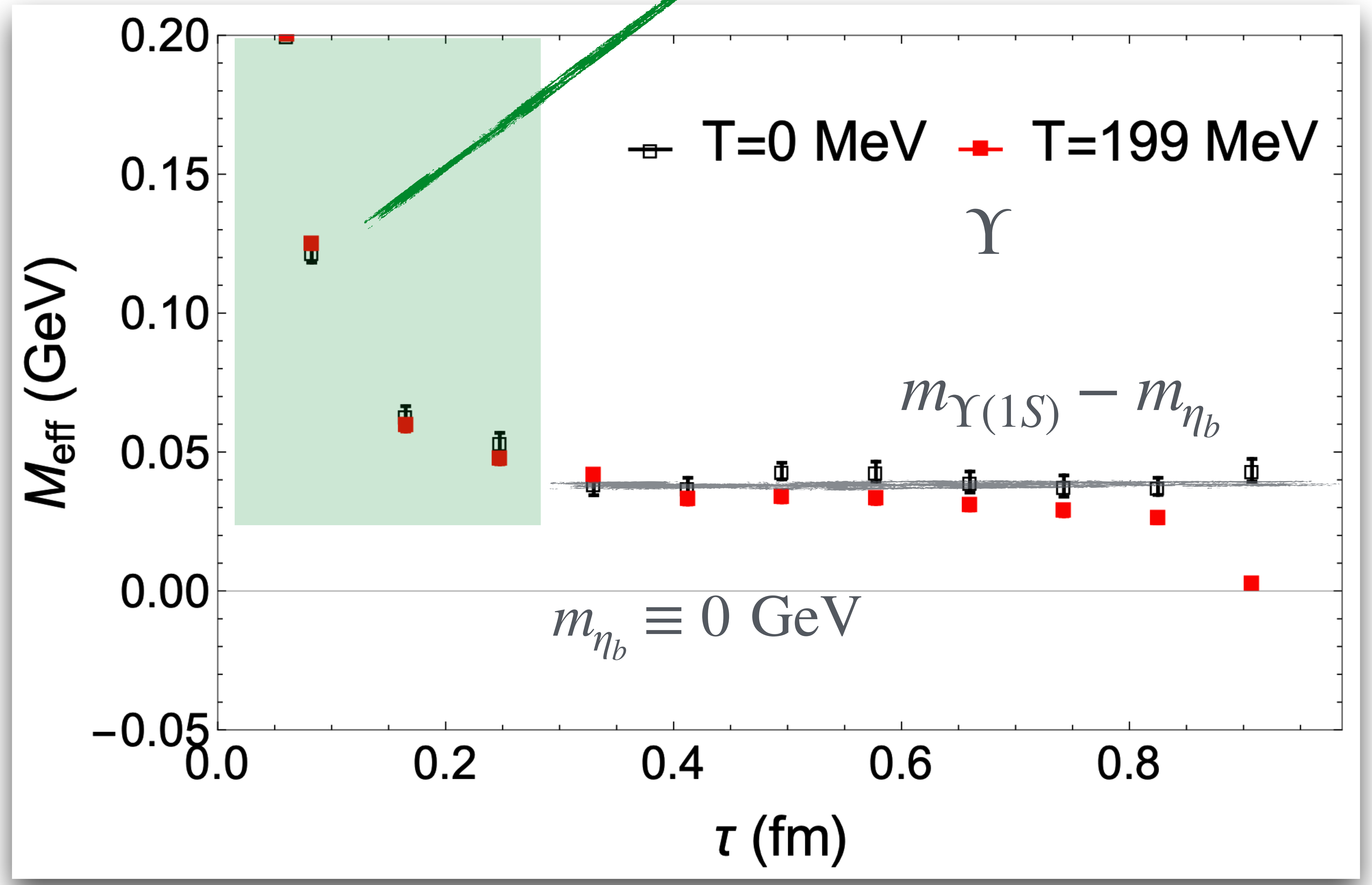
$\Delta^{(2)}$: covariant lattice Laplacian

$$M_{\rm eff} = a^{-1}\partial_\tau \ln C(\tau)$$

$$C(\tau) = \int d^3x \langle O(\tau, x) O^\dagger(0,0)\rangle = \int_0^\infty e^{-\omega\tau}\rho(\omega)d\omega$$

separate the continuum …

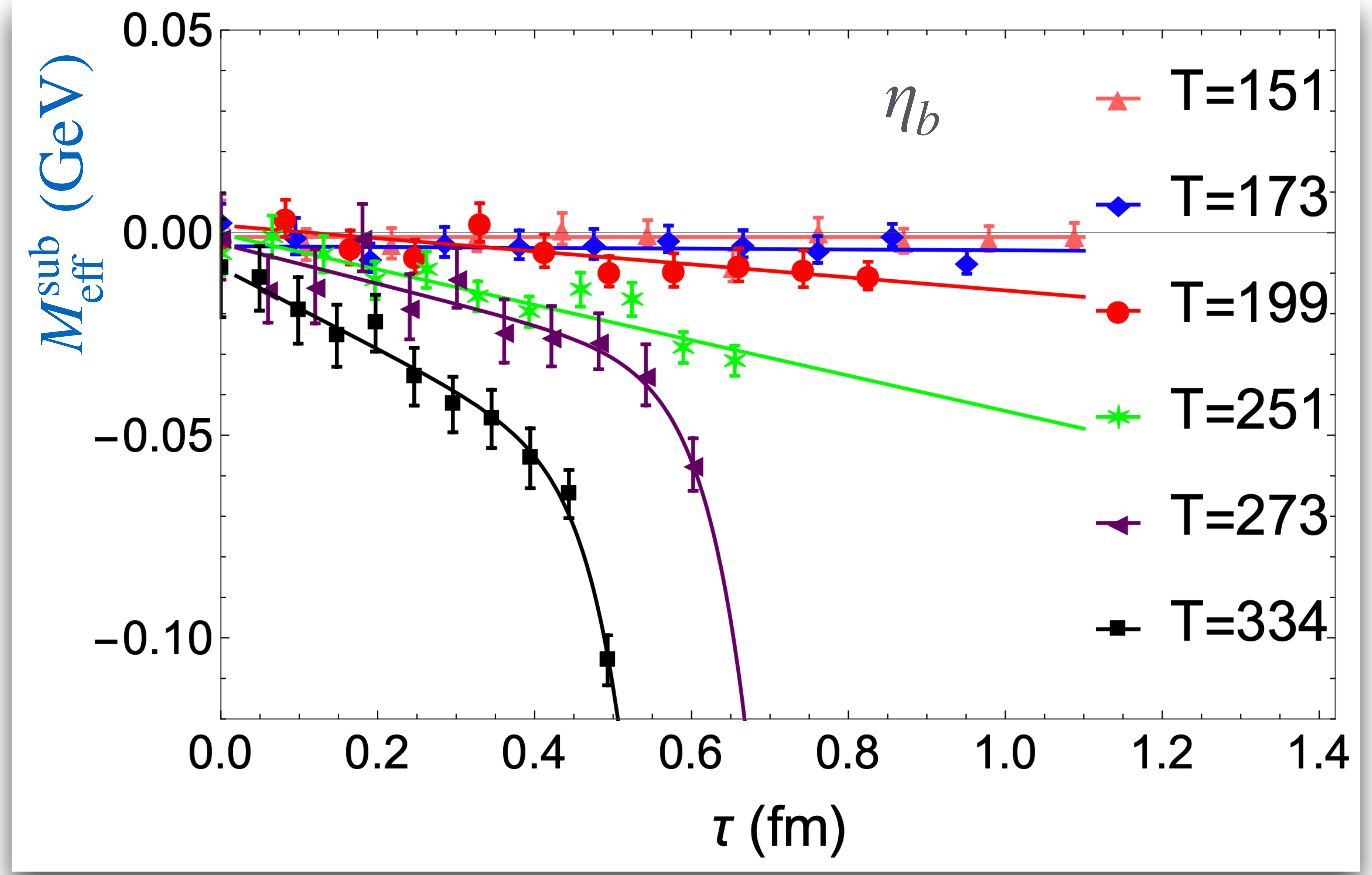
$$T = 0: \quad C^{\text{cont}}(\tau) = C(\tau) - e^{-m\tau}$$



… & get rid of it

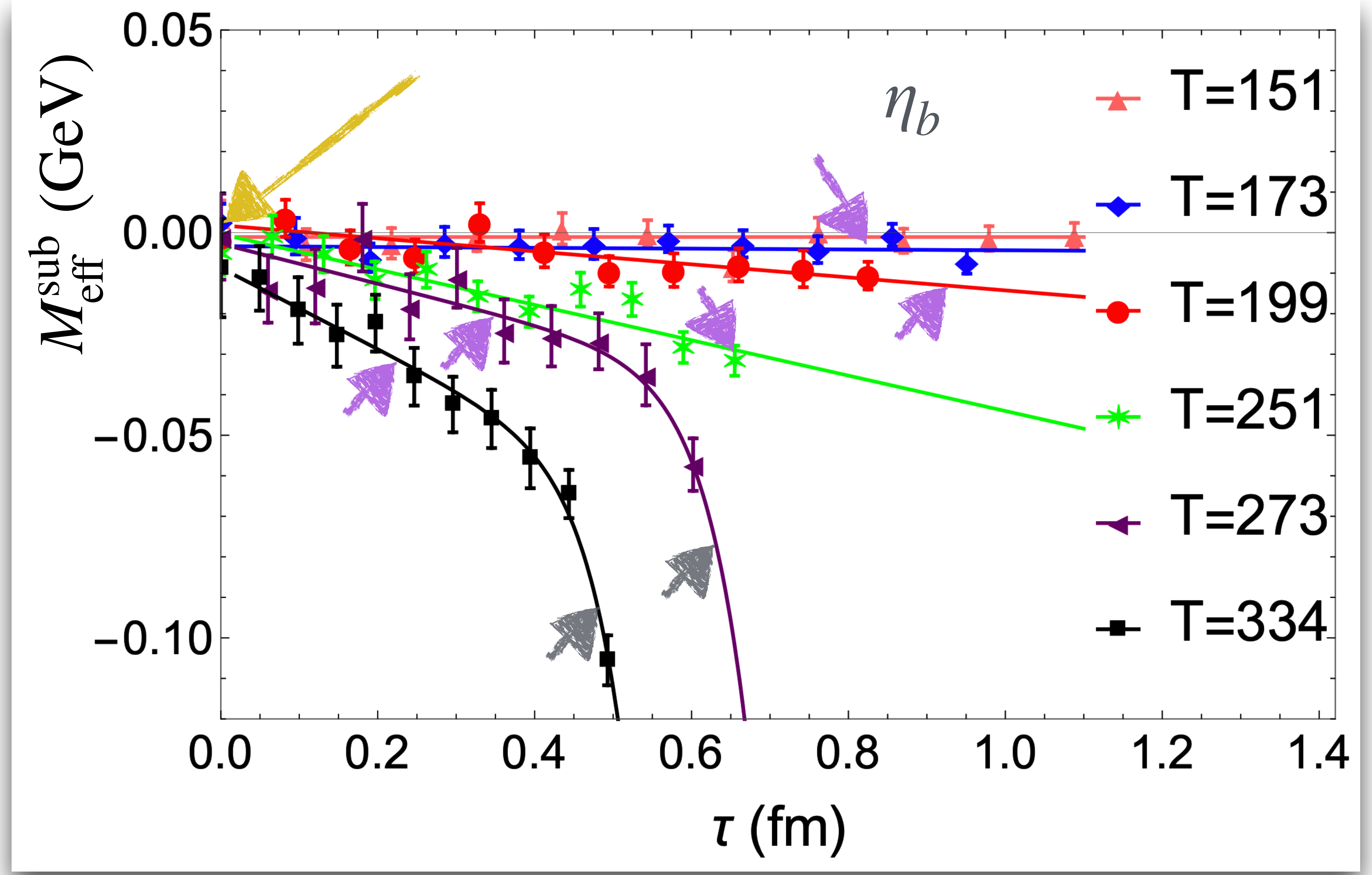
$$T > 0: \quad C^{\text{sub}}(\tau) = C(\tau) - C^{\text{cont}}(\tau, T=0)$$

$$M_{\text{eff}}^{\text{sub}} = a^{-1}\partial_\tau \ln C^{\text{sub}}(\tau)$$



$$M_{\mathrm{eff}}^{\mathrm{sub}} \propto \tau$$

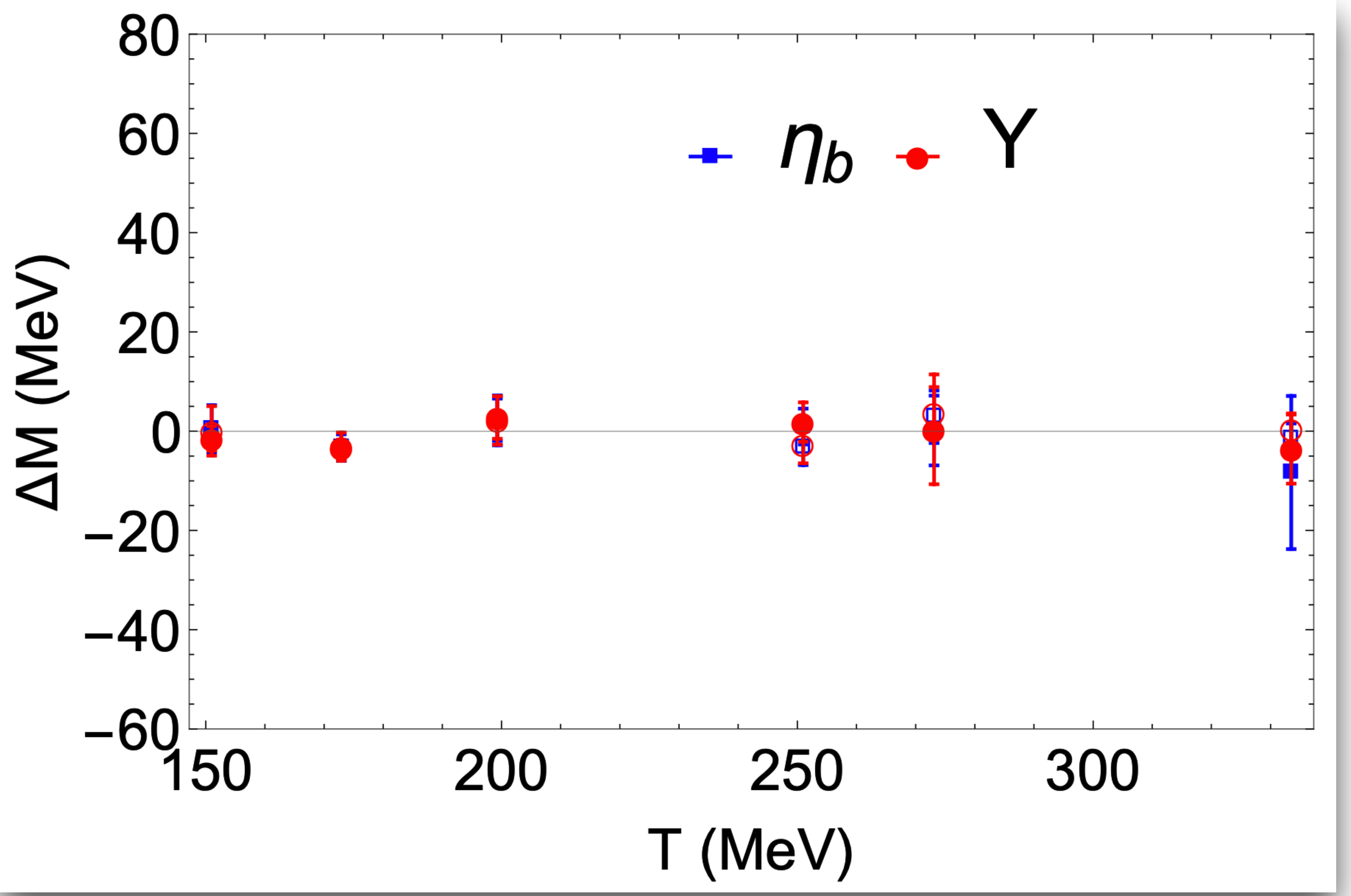
$$C^{\mathrm{sub}}(\tau) \sim \exp\left[-m\tau + \frac{1}{2}\Gamma^2\tau^2 + \mathcal{O}(\tau^3)\right]$$



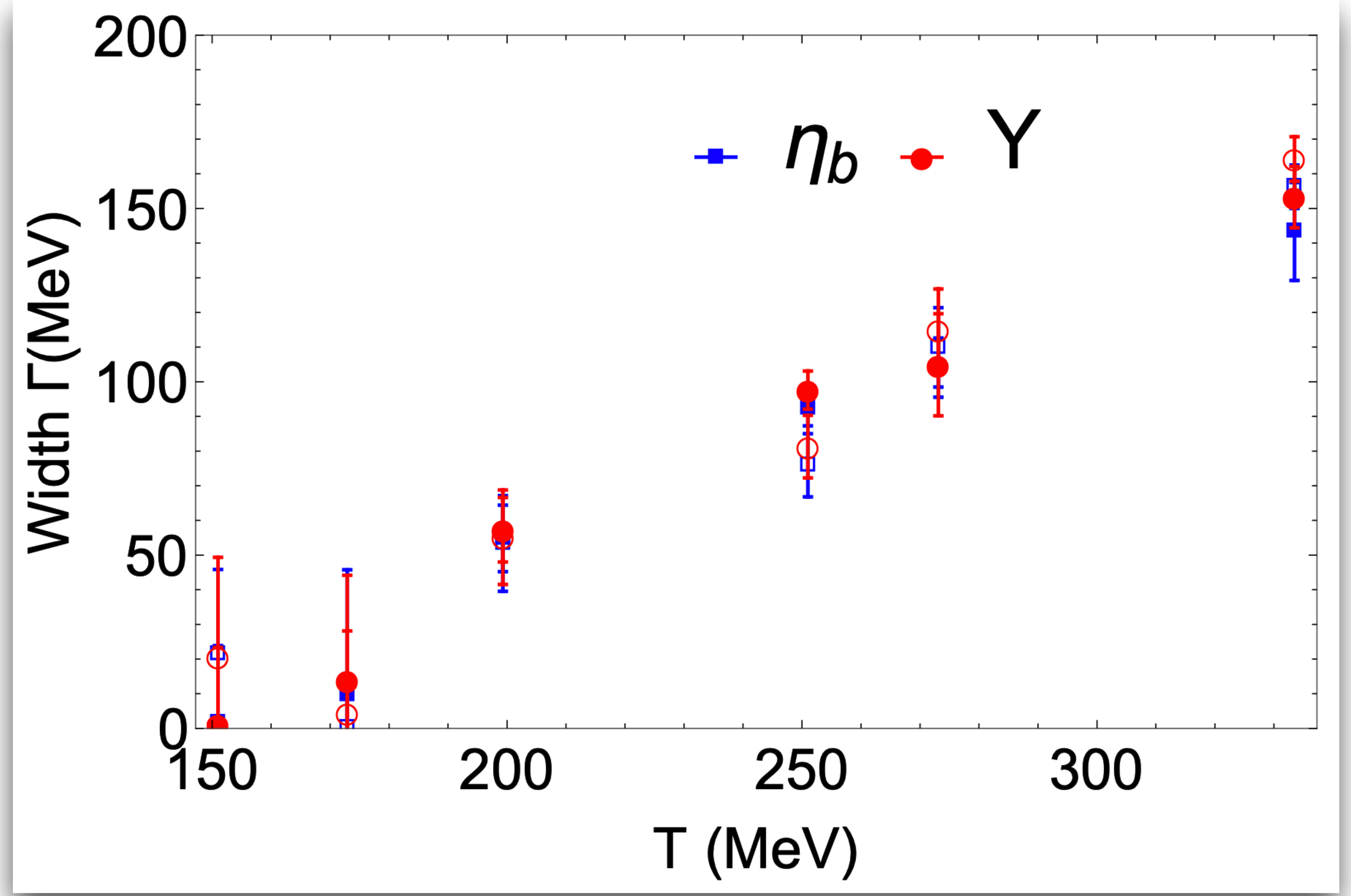
simplest spectral function consistent with this:

$$\rho(\omega) \sim A \exp\left[-\frac{(\omega - m)^2}{2\Gamma^2}\right] + A_{\mathrm{cut}}\delta(\omega - \omega_{\mathrm{cut}})$$

thermal broadening of $\Upsilon(1S)$, $\eta_b(1S)$

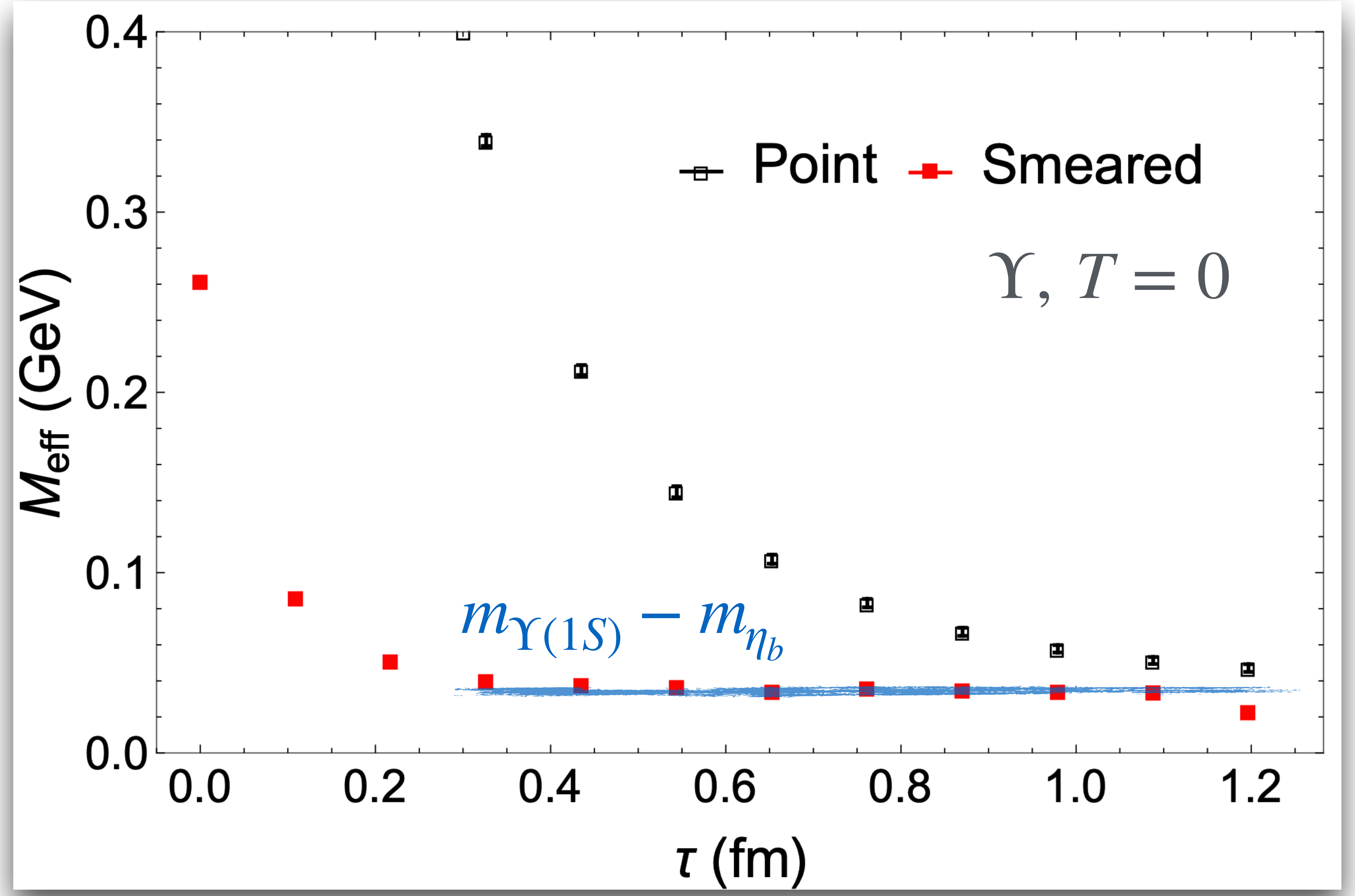


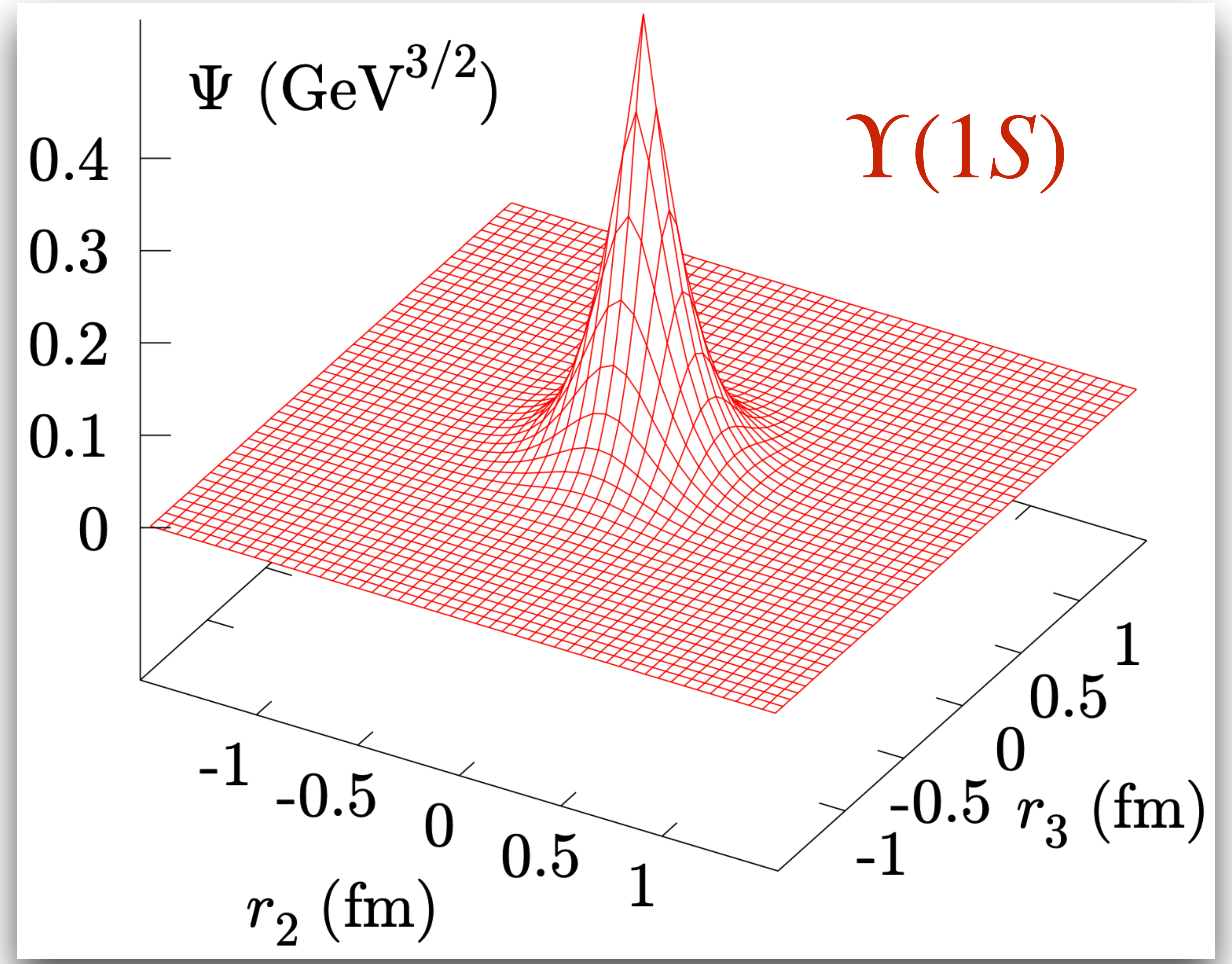
$$\Delta M = m(T) - m(T=0)$$

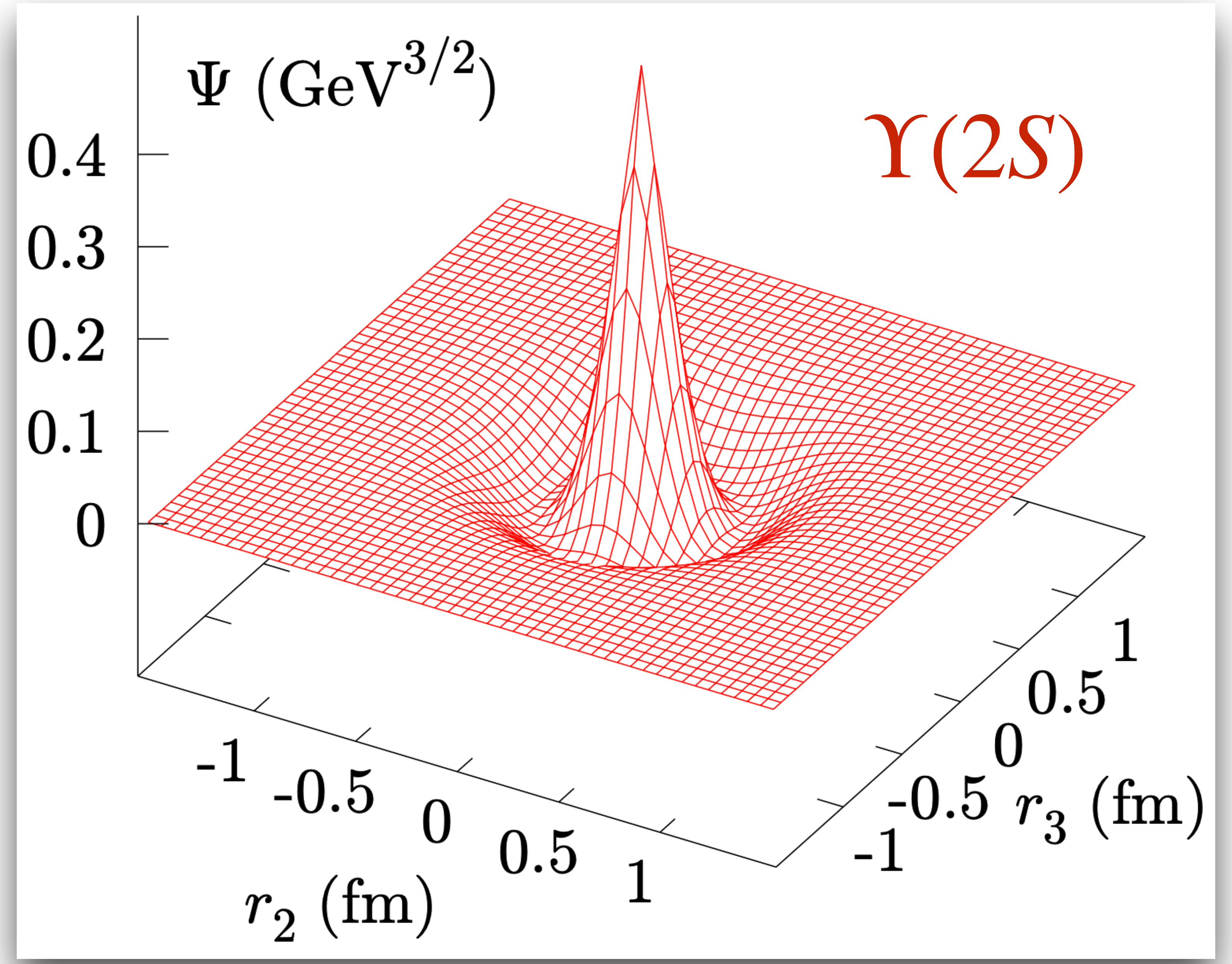


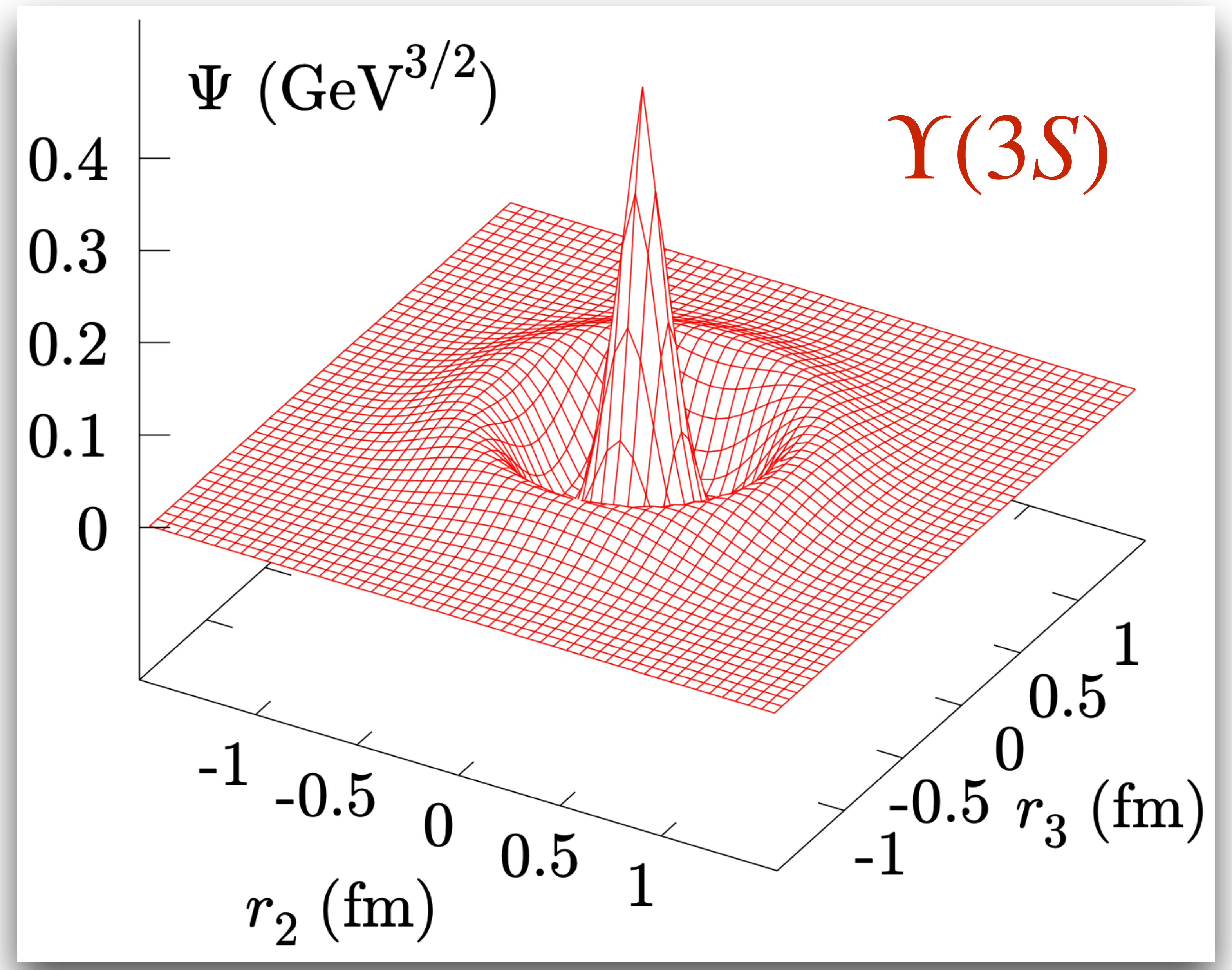
Larsen, Meinel, Mukherjee & Petreczky: Phys. Rev. D100, 074506 (2019) sequential modifications from thermal broadening ?

how to reach the higher states: 1, 2, 3 … ?









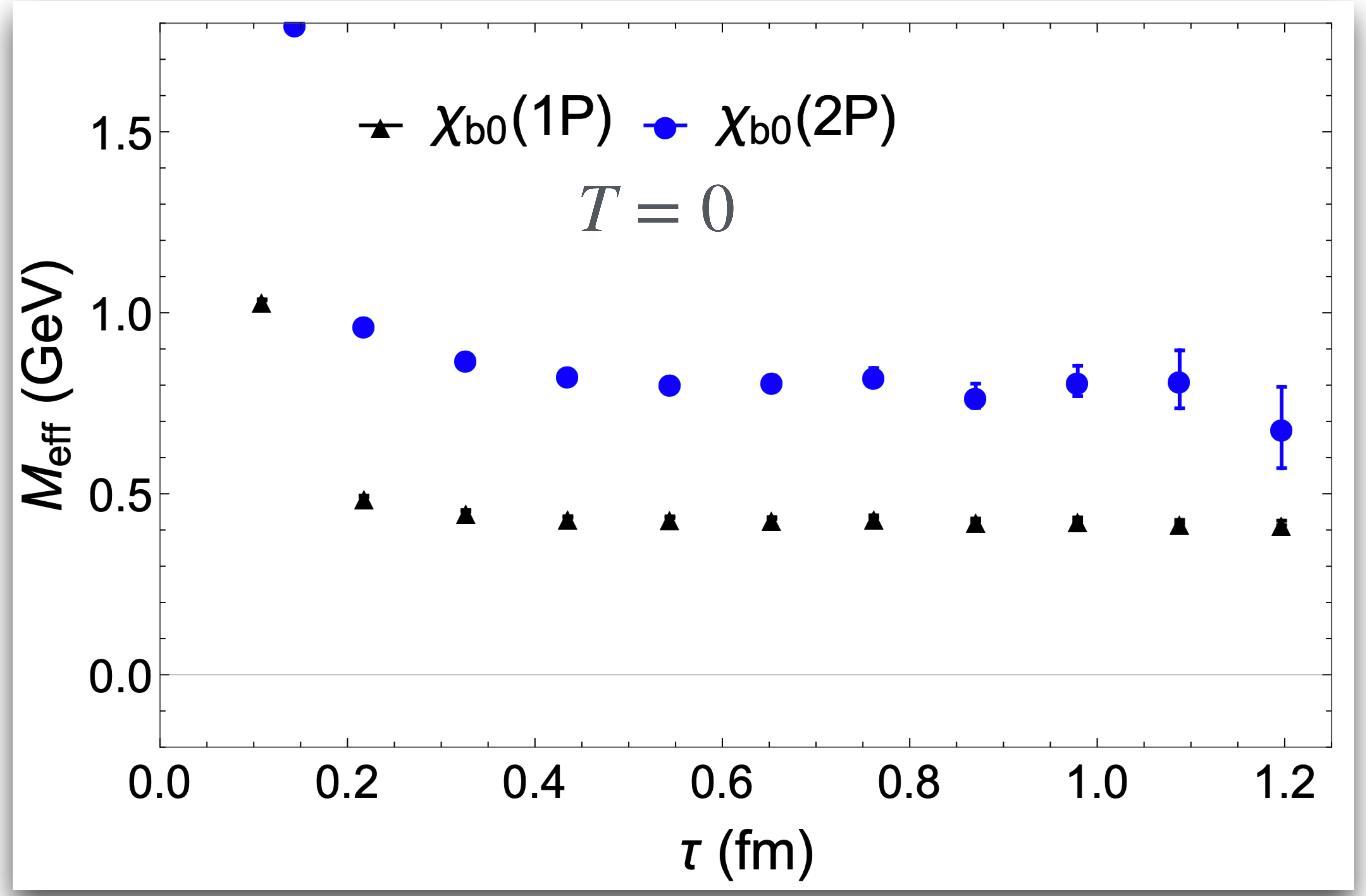
$$C_{\alpha\beta}(\tau) = \int d^3x \langle O_\alpha(\tau, x) O_\beta^\dagger(0,0) \rangle$$

$$O_\alpha(\tau, x) = \sum_r \Psi(r)\, \chi(\tau, x + r) \Gamma \psi(\tau, x)$$

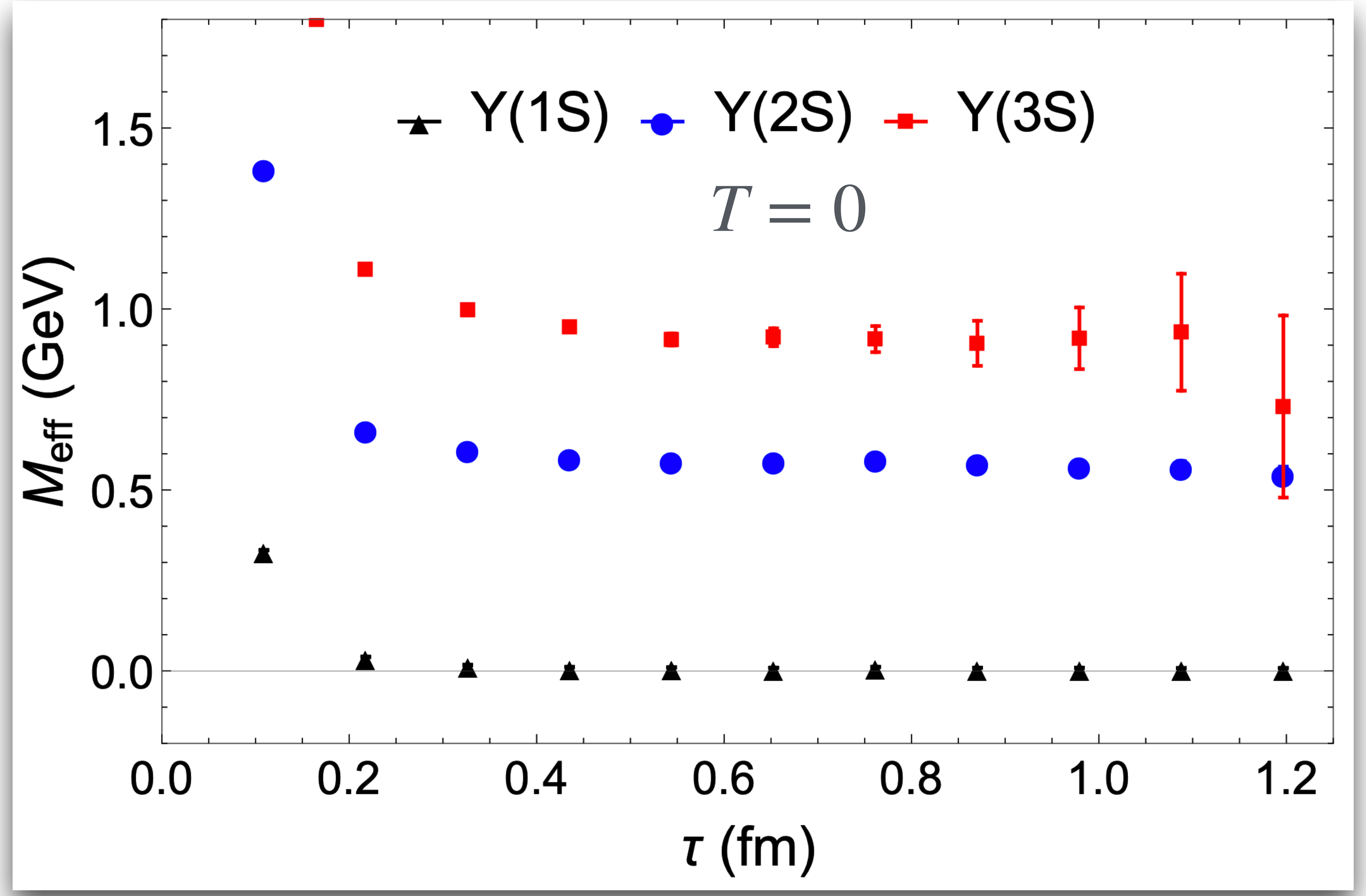
+ variational technique

energies by diagonalizing $C_{\alpha\beta}(\tau)$

solving discretized Schrodinger equation on a 3-dimensional lattice with a Cornell potential that reproduces T=0 spectrum

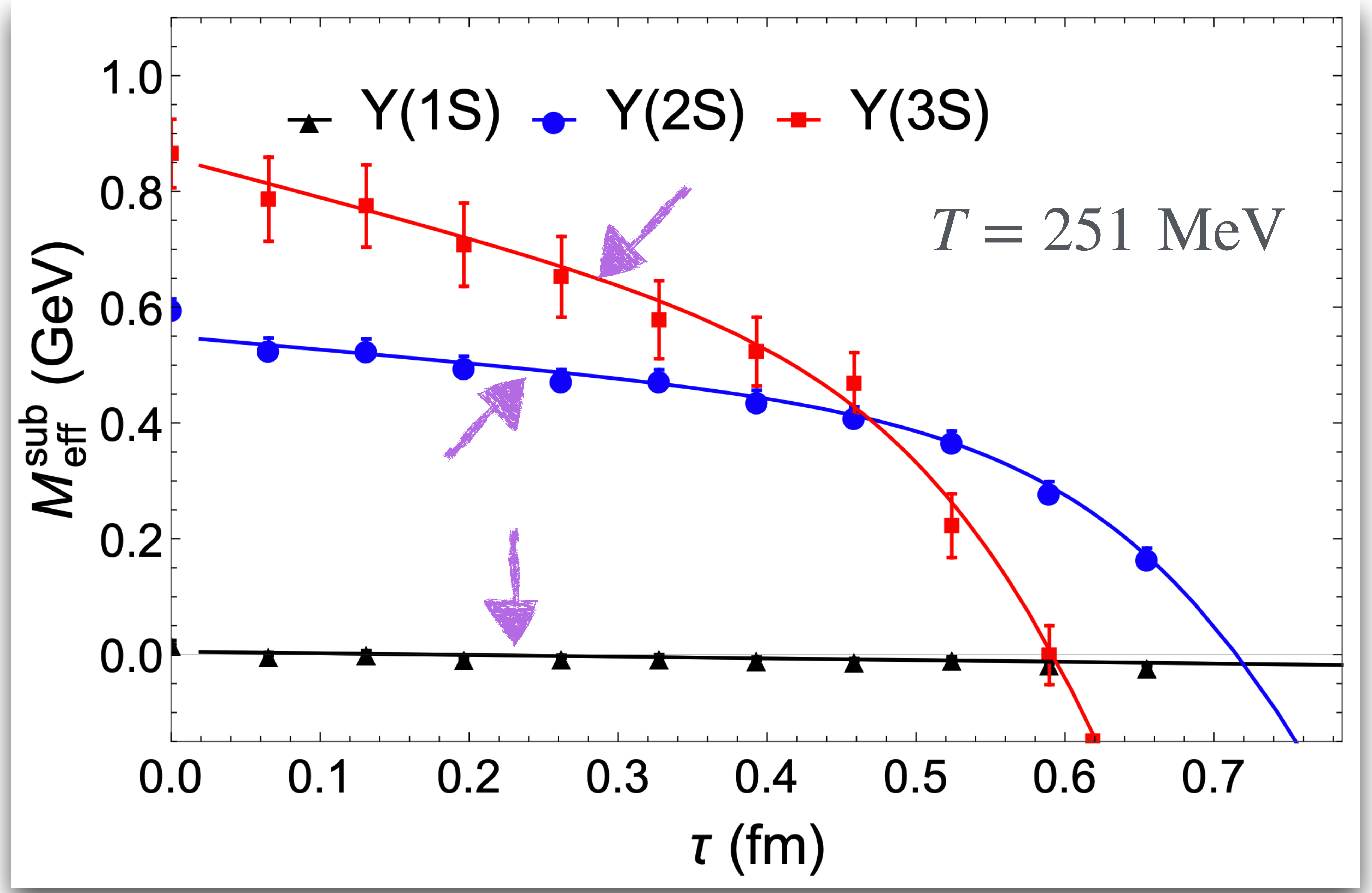


χ_{b0}(1P)
χ_{b0}(2P)
$T = 0$
$M_{\rm eff}$ (GeV)
τ (fm)



Υ(1S)
Υ(2S)
Υ(3S)
$T = 0$
$M_{\rm eff}$ (GeV)
τ (fm)

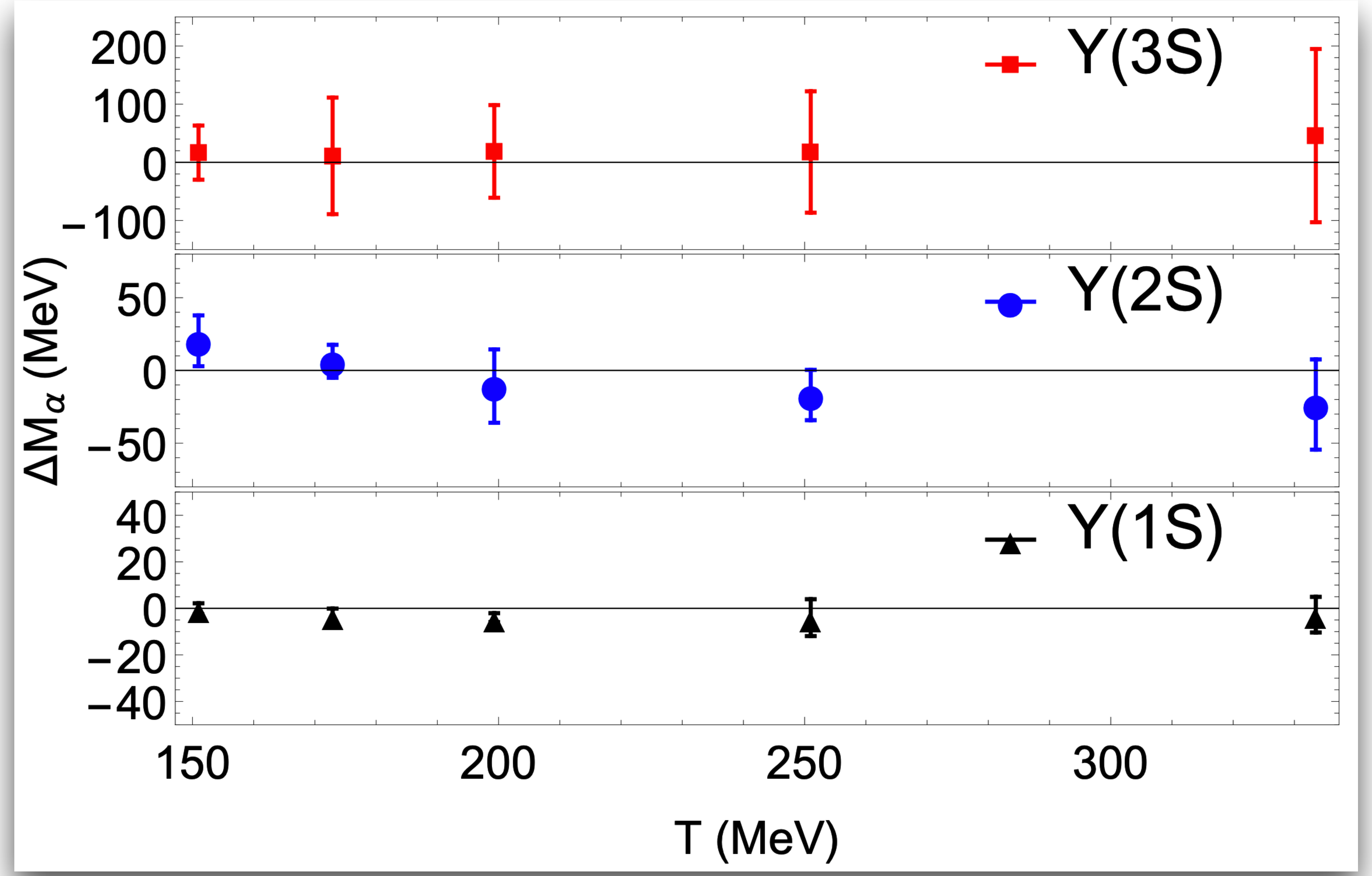
continuum-subtracted effective masses



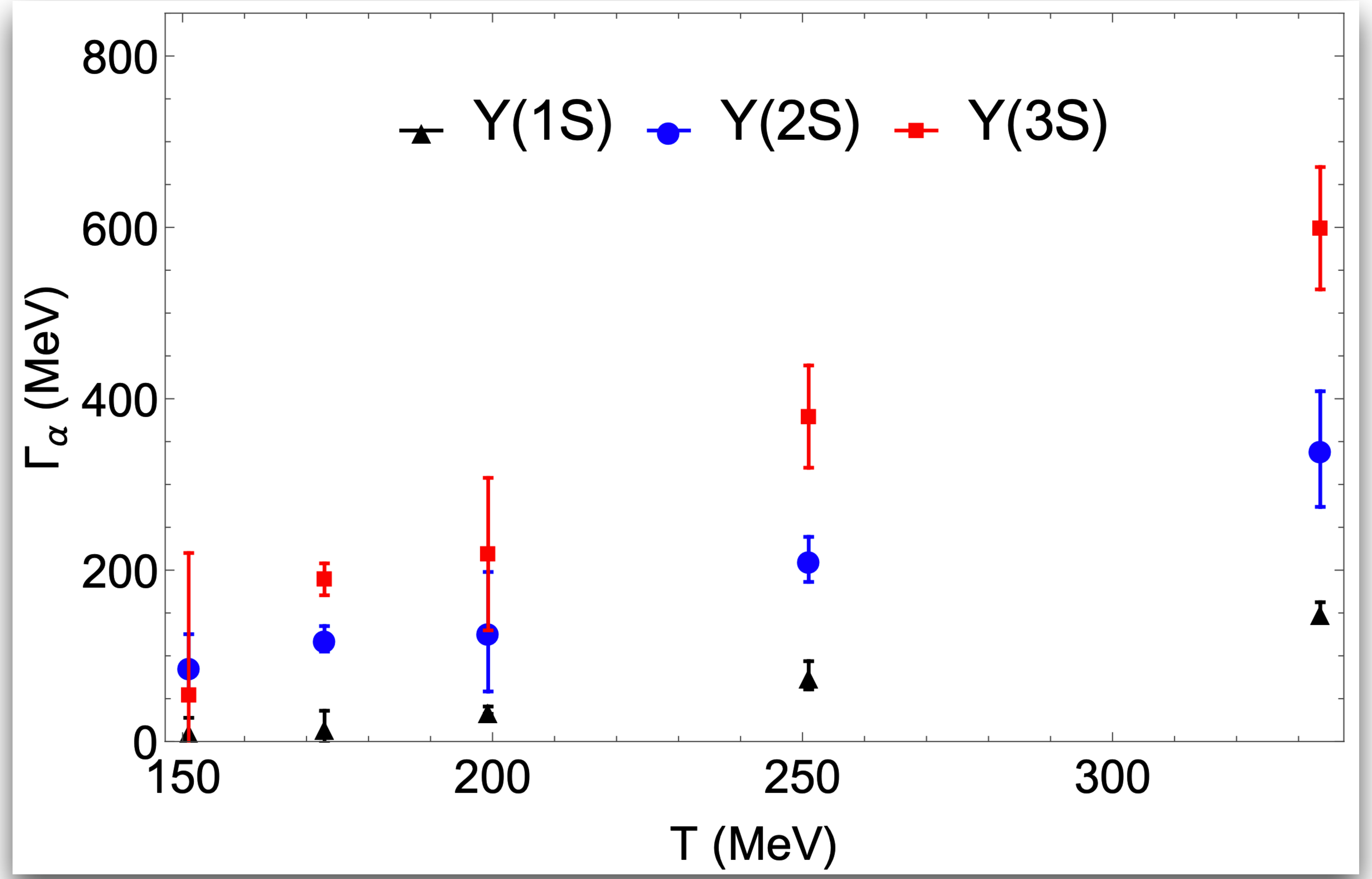
$M_{\rm eff}^{\rm sub} \propto \tau$

$$C_\alpha^{\rm sub}(\tau) \sim \exp\left[-m_\alpha \tau + \frac{1}{2}\Gamma_\alpha^2 \tau^2 + \mathcal{O}(\tau^3)\right]$$

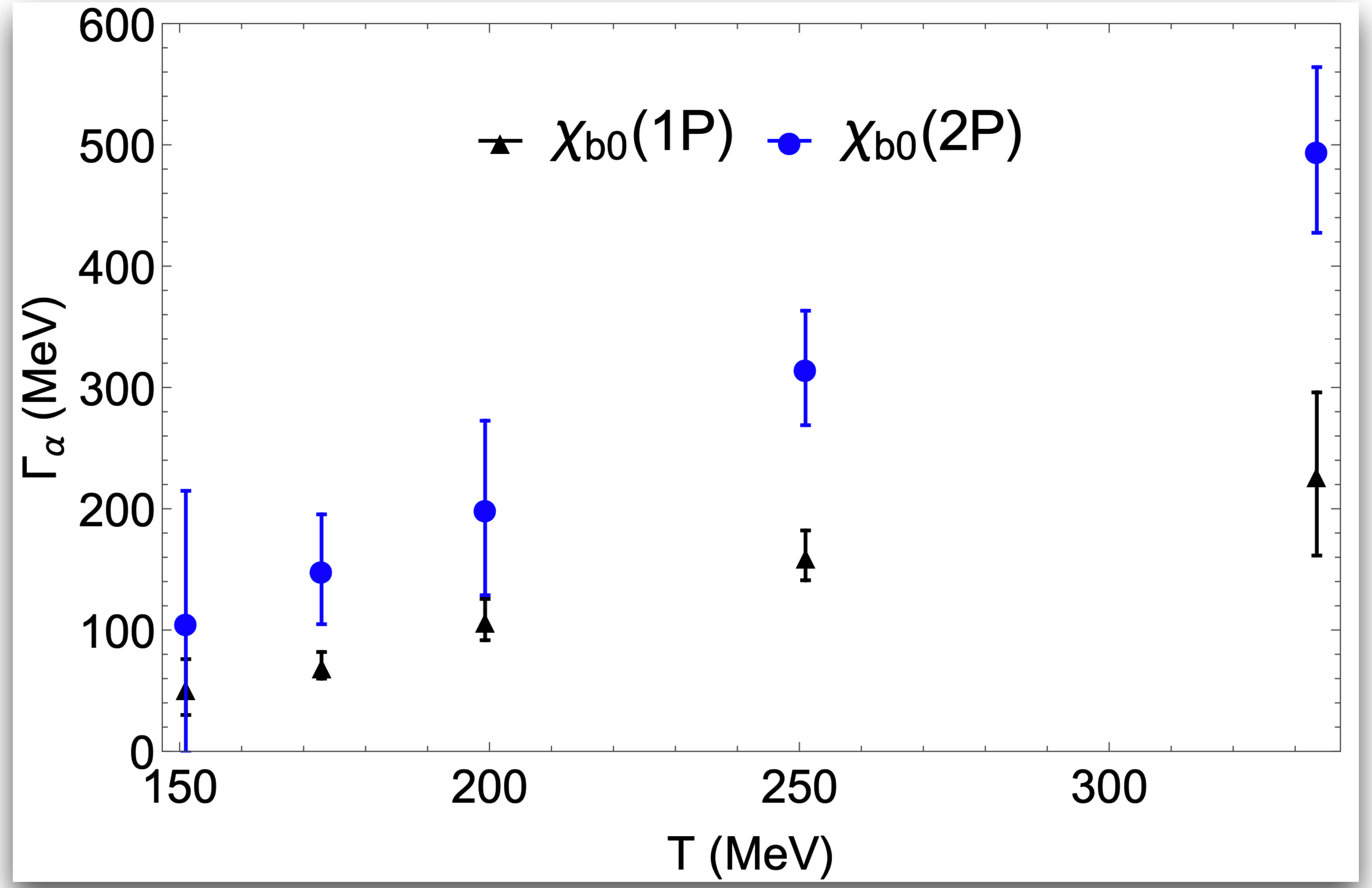
$$\rho_\alpha(\omega) \sim A_\alpha \exp\left[-\frac{(\omega - m)^2}{2\Gamma_\alpha^2}\right] + A_{\rm cut}\delta(\omega - \omega_{\rm cut})$$

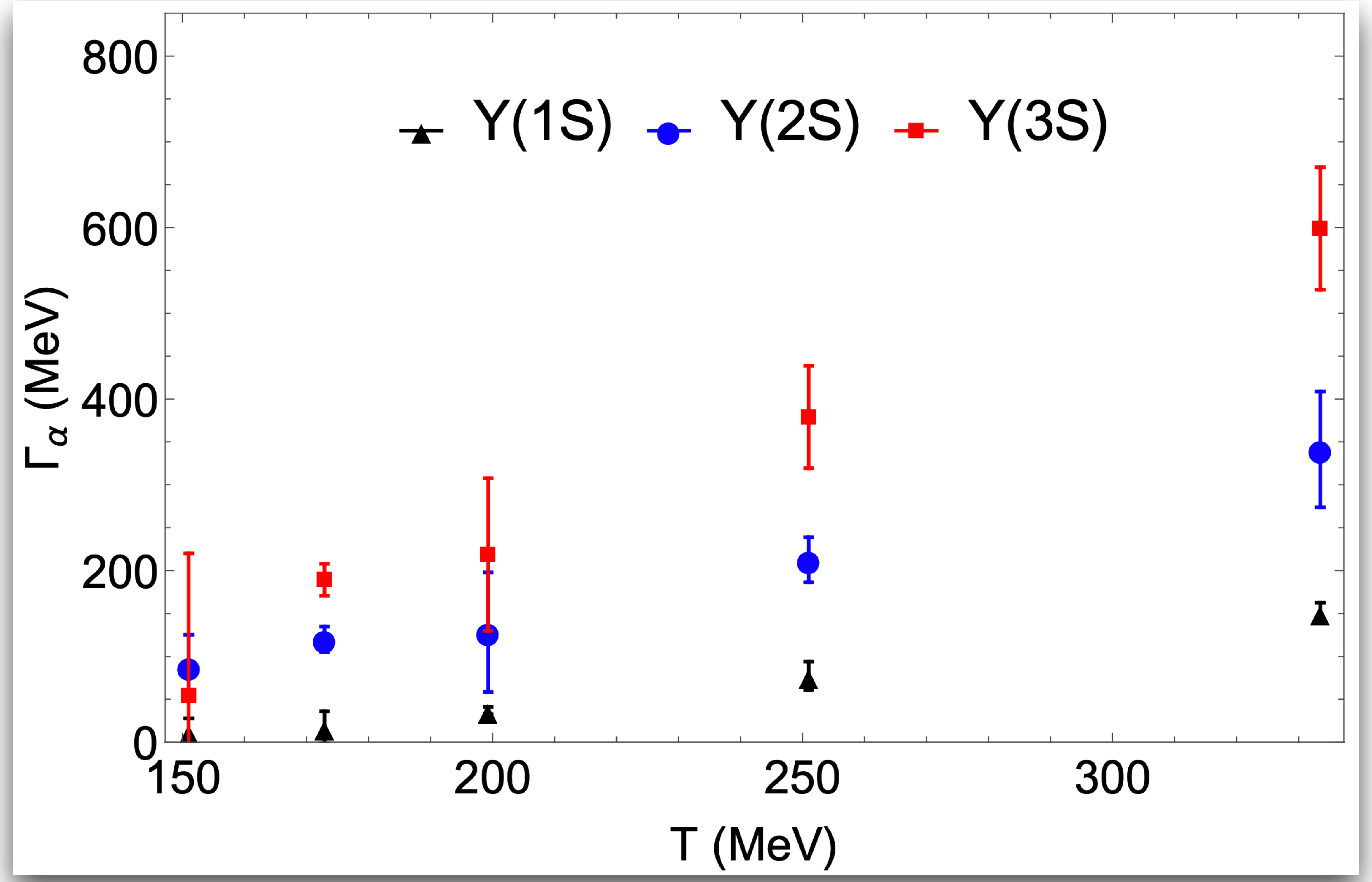


$$\Delta M_\alpha = m_\alpha(T) - m_\alpha(T=0)$$



thermal broadening

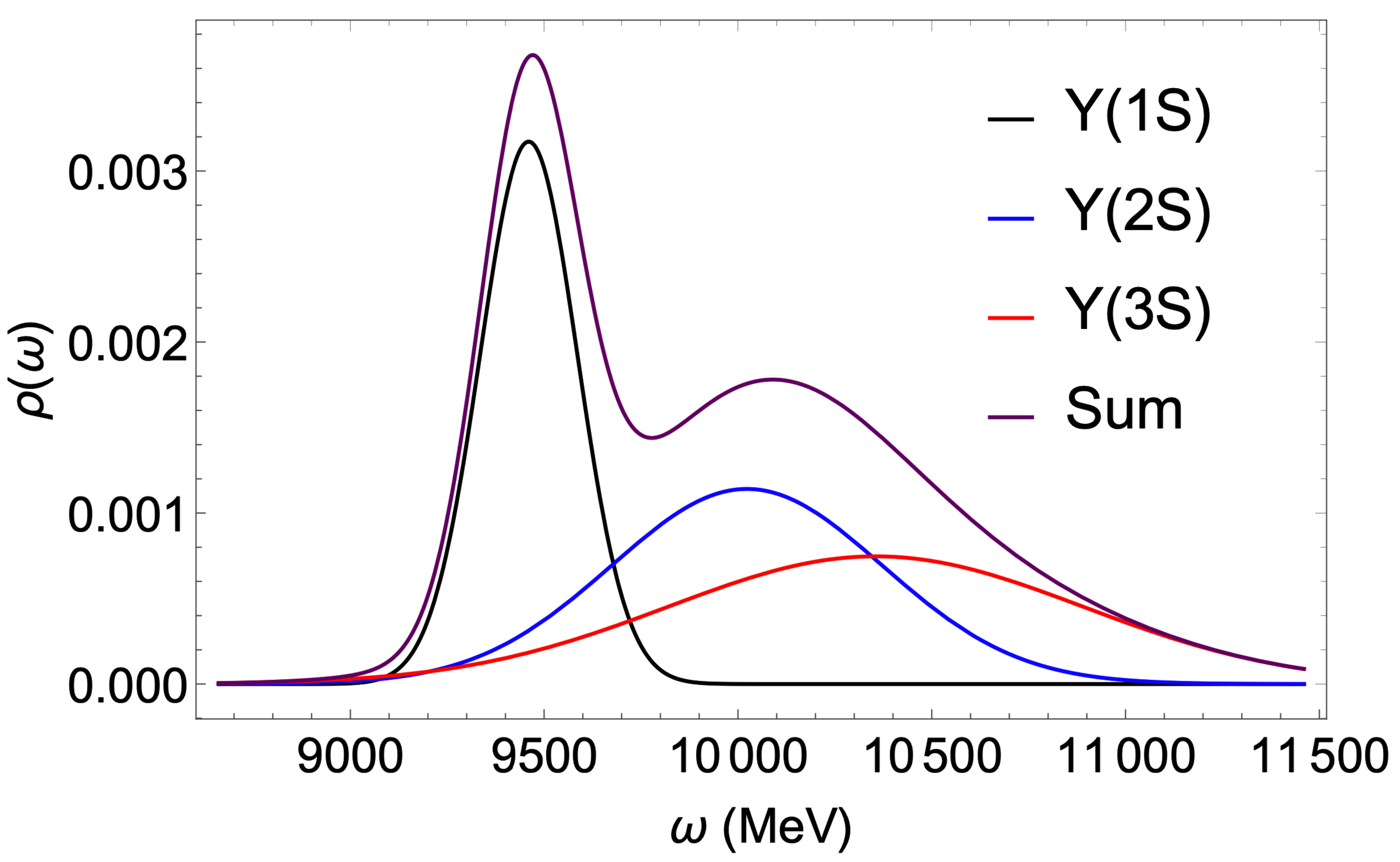


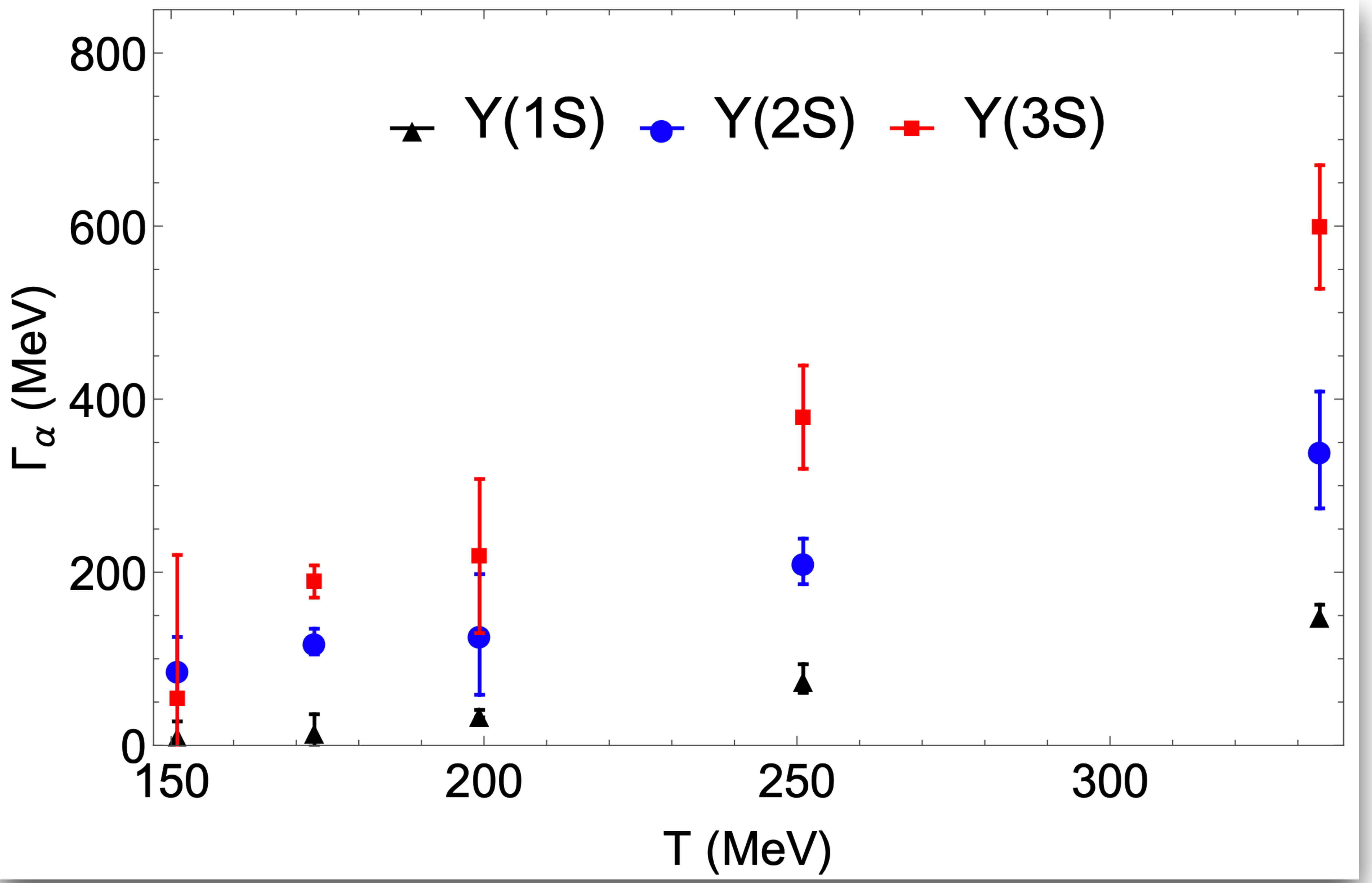


$$\Gamma_{1S} < \Gamma_{1P} < \Gamma_{2S} < \Gamma_{2P} < \Gamma_{3S}$$

sequential hierarchical pattern according to increasing size

Larsen, Meinel, Mukherjee & Petreczky: arXiv:1910.07374 (Phys. Lett. B???)





$$T \gtrsim 200 \text{ MeV}: \;\; \Gamma_{3S} \gtrsim M_{3S} - M_{2S}$$

$$\Gamma_{2P} \gtrsim M_{2P} - M_{1P}$$

Larsen, Meinel, Mukherjee & Petreczky: arXiv:1910.07374 (Phys. Lett. B???)

heavy quark momentum diffusion coefficient

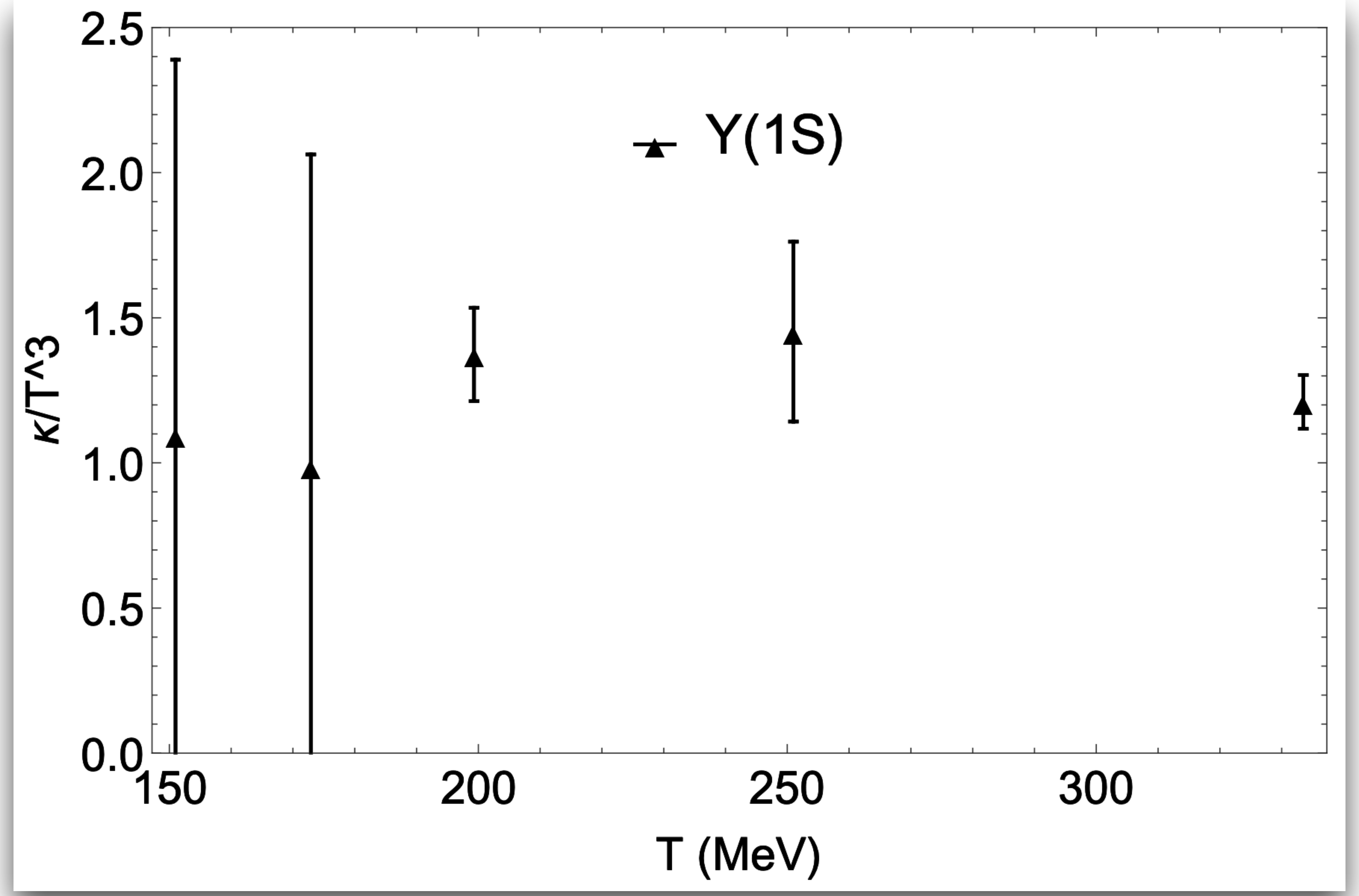
$$\Gamma_{1S} = 3a_0^2\kappa$$

pNRQCD for open quantum system

$$a_0^{-1} \gg T \sim m_D \gg E$$

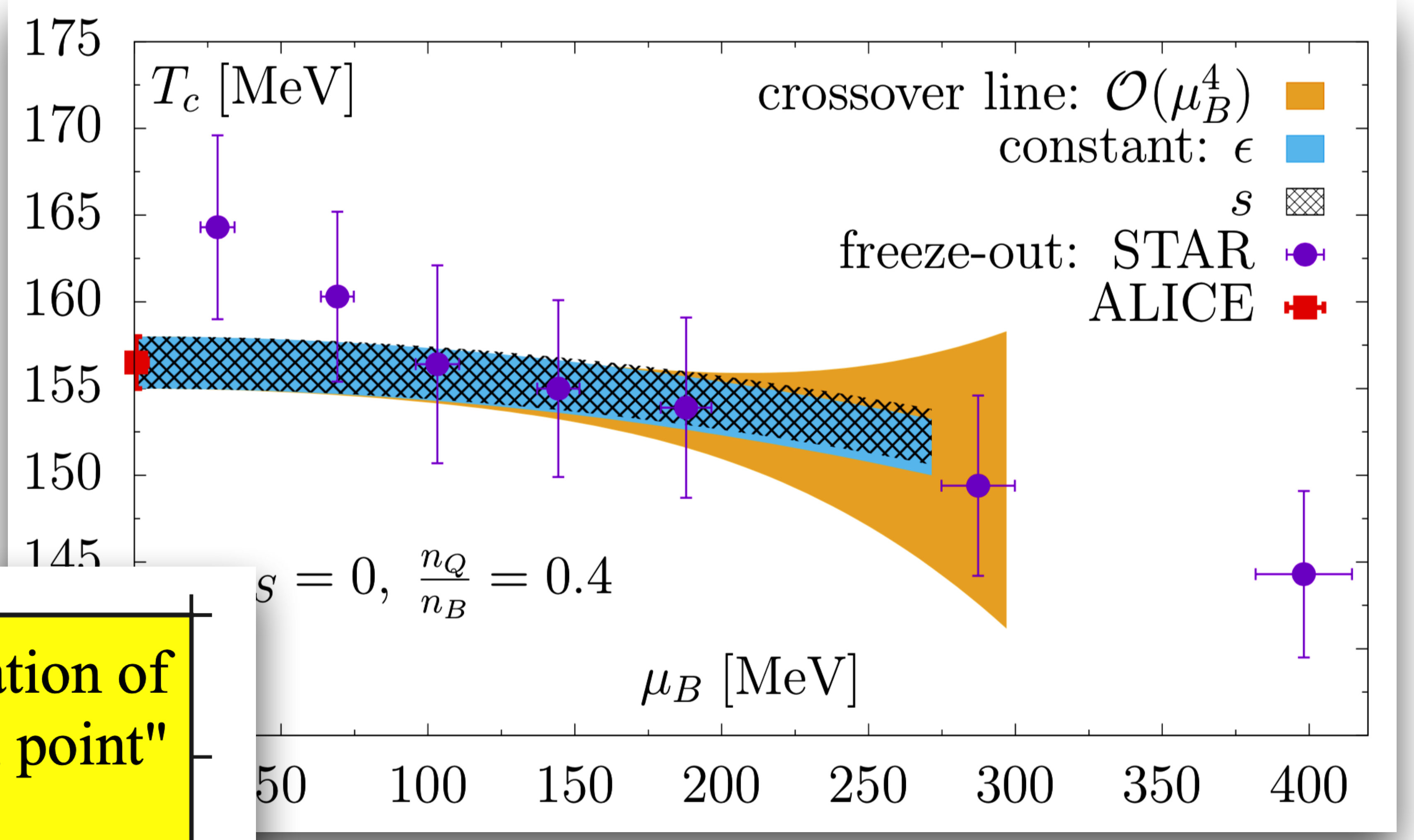
$$a_0 = 0.21 \text{ fm}$$

Brambilla, Escobedo, Soto, Vairo: Phys. Rev. D97, 074009 (2018)

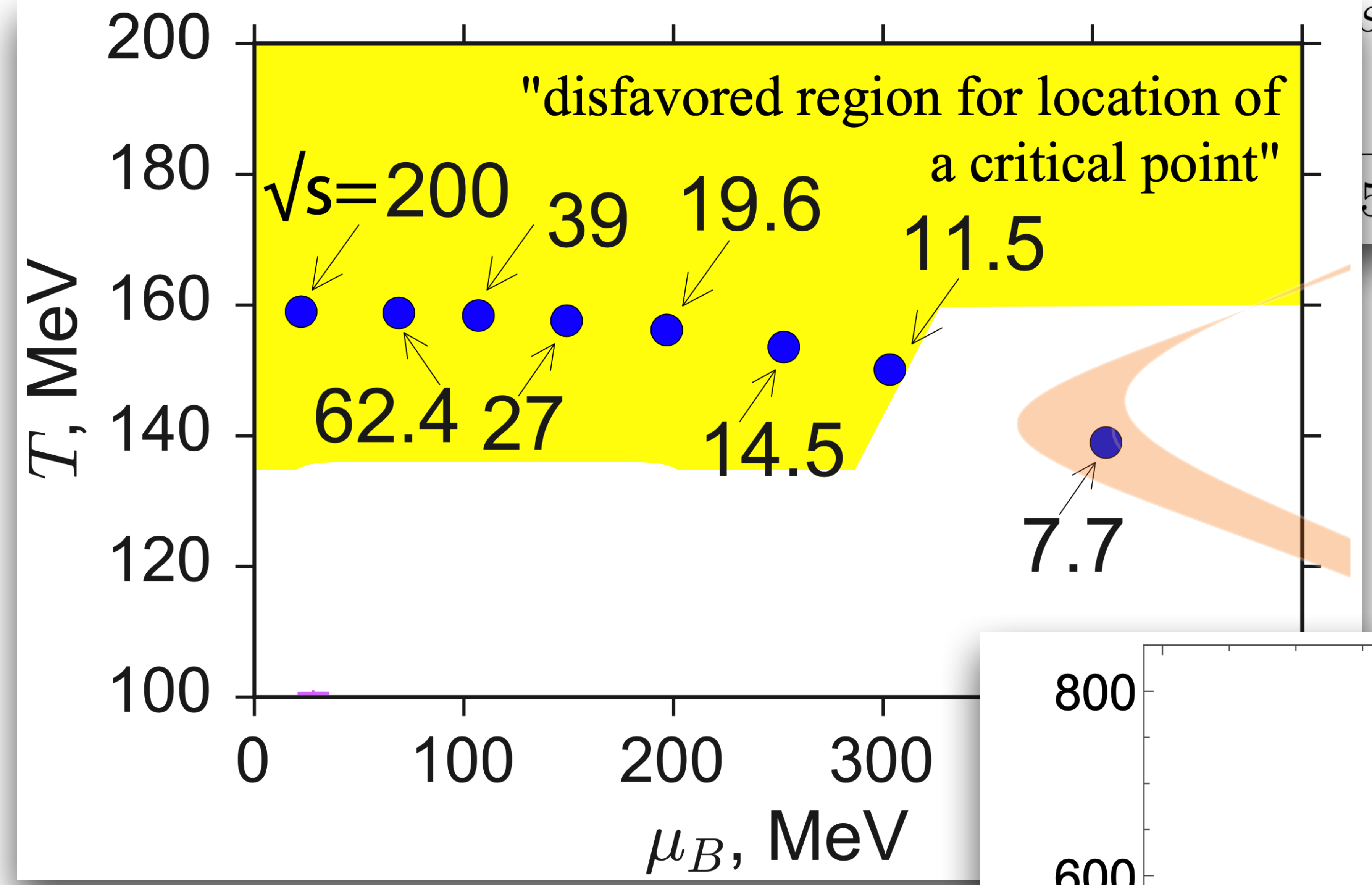


summary

- QCD phase boundary in $T-\mu_B$



- radius of convergence in μ_B



- excited bottomonia in QGP