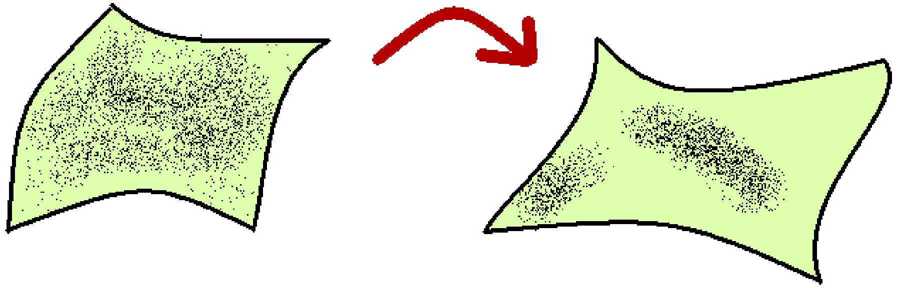
Fluctuations and Order

Mustansir Barma

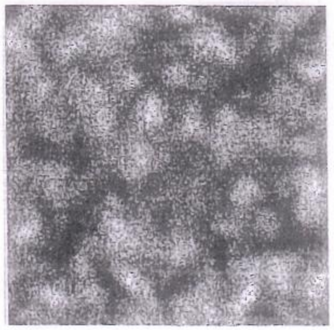
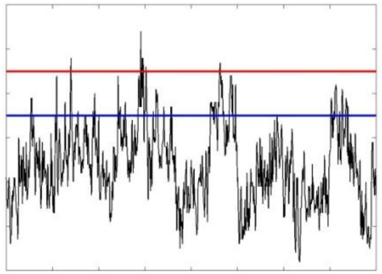
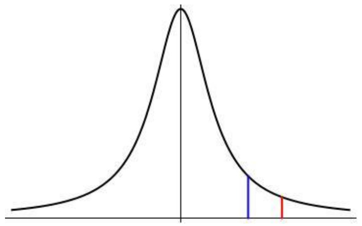
TIFR Centre for Interdisciplinary Sciences,

Tata Institute of Fundamental Research, Hyderabad, India



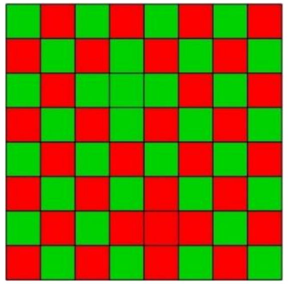
Fluctuations:

Irregular rising and falling

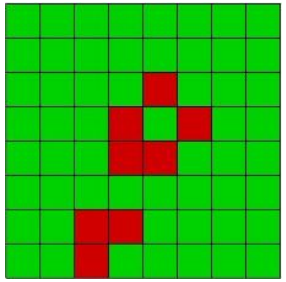




Order:

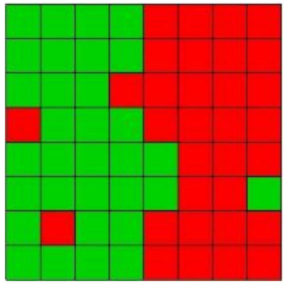
Arrangements extending to large distances: Organization on a large scale



Alternation



Single Colour



Separation

Can giant fluctuations coexist with an ordered state ?

Fluctuations and Response

- Fluctuations: Spontaneous deviations from an average value

- Response: How a system responds to an external influence.

Interestingly, the response can be figured out ***without*** applying an external field.

All one needs to do is monitor ***fluctuations*** in the system.

RESPONSE FUNCTION		FLUCTUATION
$Compressibility$	$\propto$	$\frac{<(\Delta N)^2>}{V}$
$Susceptibility$	$\propto$	$\frac{<(\Delta M)^2>}{V}$
$Specific\ Heat$	$\propto$	$\frac{<(\Delta E)^2>}{V}$

Fluctuations and Order

Normally

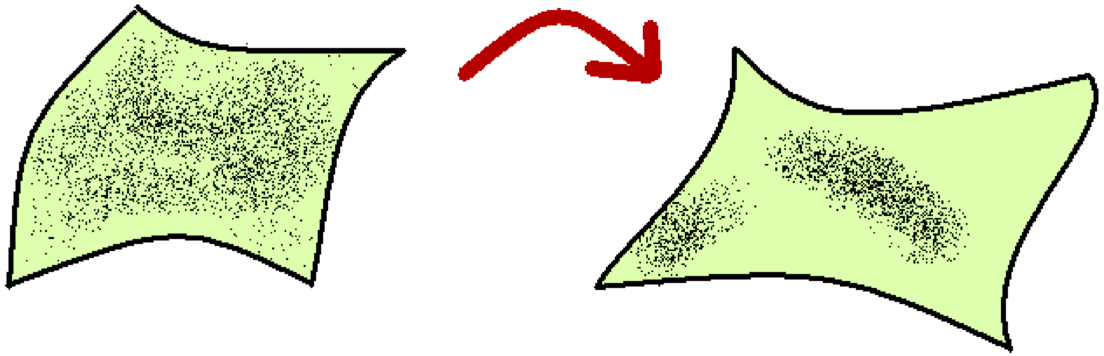
$$\frac{RMS\ fluctuation}{Average\ value} \sim \frac{1}{\sqrt{V}} \quad \rightarrow \quad 0 \quad as \quad V \rightarrow \infty$$

But sometimes

$$\frac{RMS\ fluctuation}{Average\ value} \sim O(1)$$

Can order survive in such a situation?

Particles on a Fluctuating Surface



Particles sliding on a fluctuating surface tend to collect in local valleys.

But valleys and hills are constantly re-forming.

What happens to the particles as time passes?

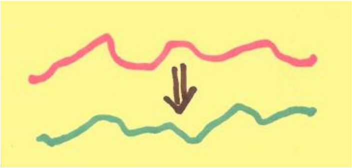
[T. Bohr, A. Pikovsky (1993); B. Drossel, M. Kardar (2000); D. Das, M. Barma (2000); C.-S. Chin (2002)]

Particles sliding down a fluctuating surface

The shared environment induces strong correlations

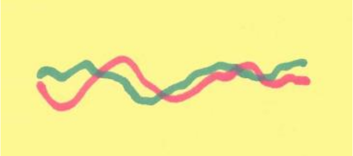
The degree of clustering depends on dynamics

(i)



Surface fluctuates and moves ↓

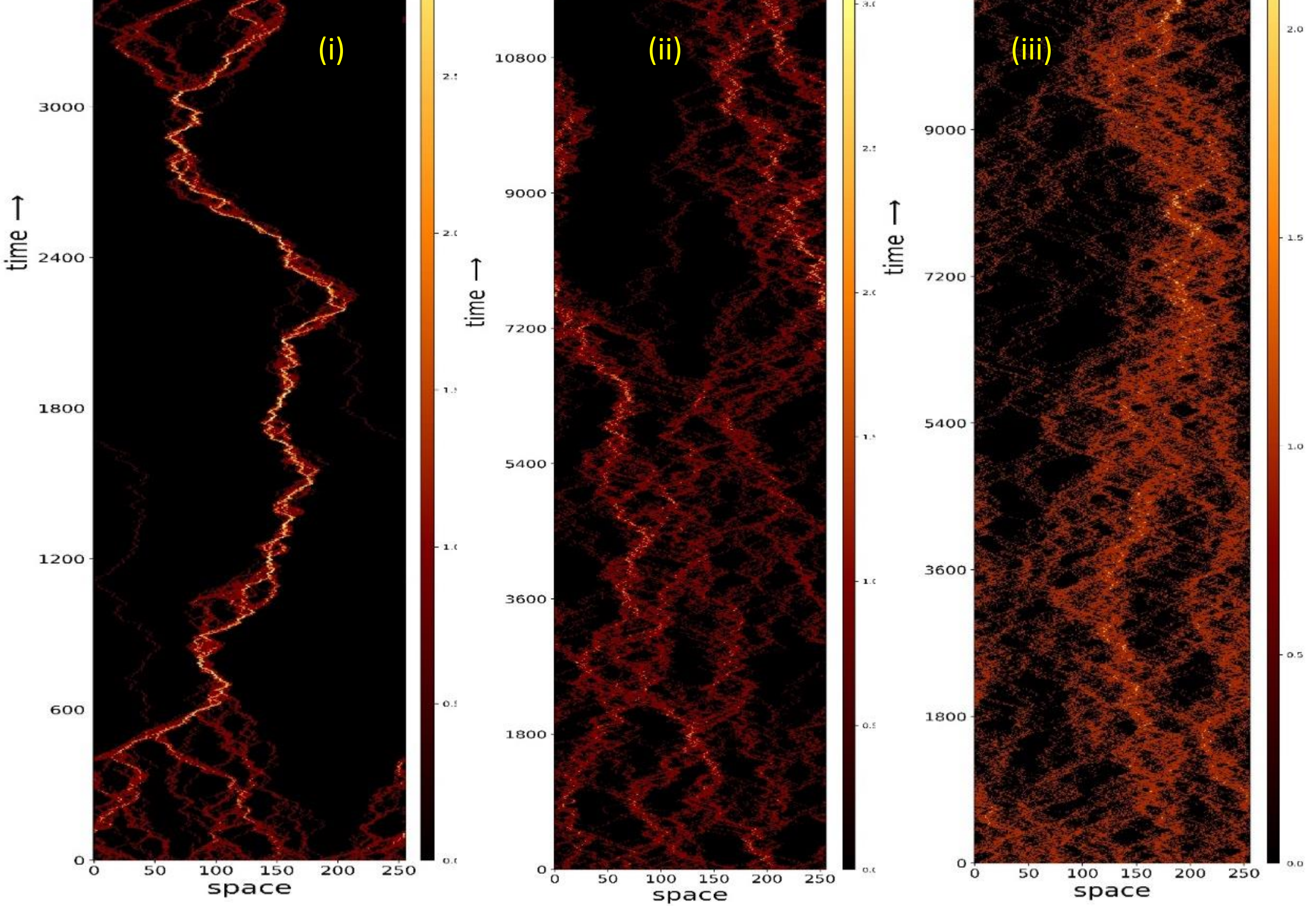
(ii)



Surface fluctuates

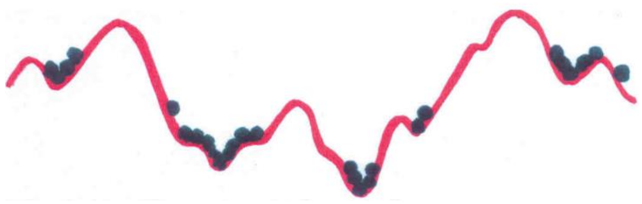
(iii)

Surface fluctuates and moves ↑

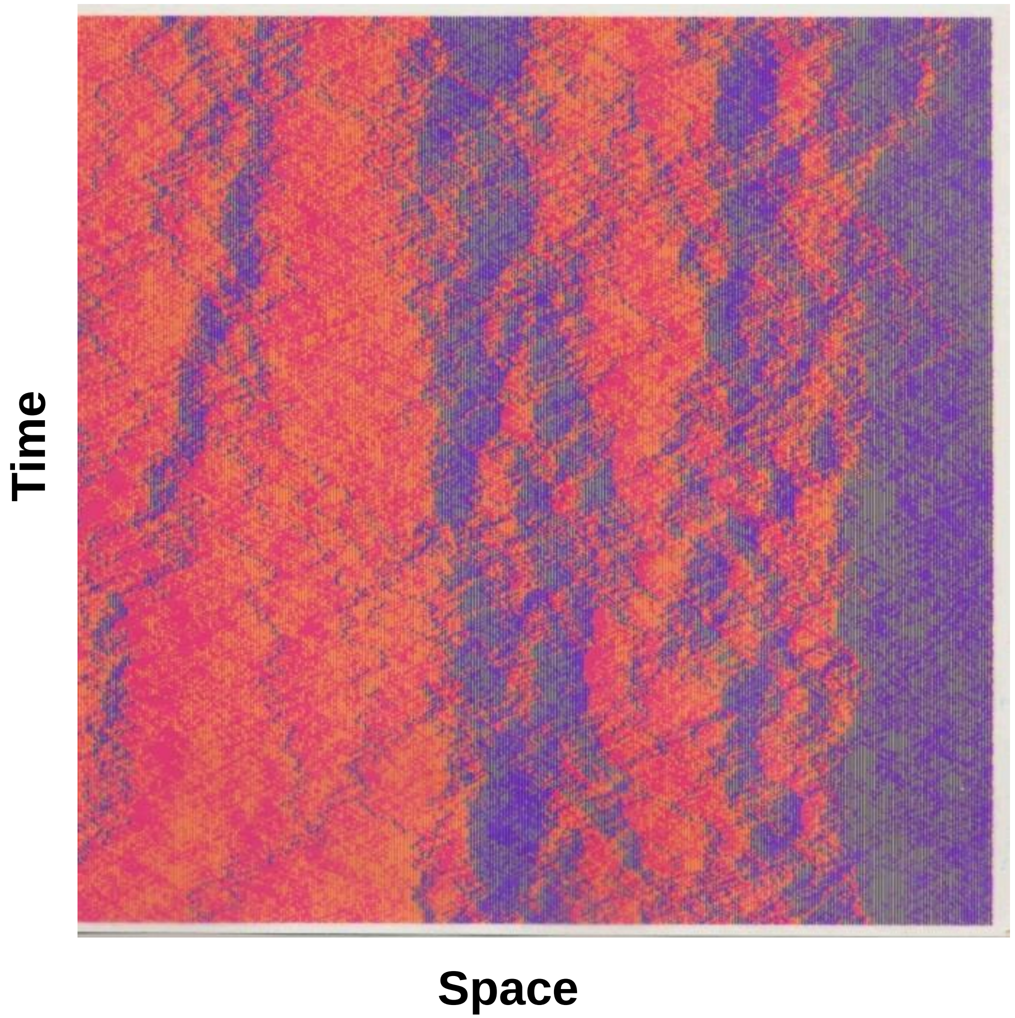


Interacting particles sliding down a fluctuating surface

Mutual exclusion changes the pattern of large-scale clustering



- Clusters spread out --- resembles phase separation
- But the region between large clusters is fragmented
  --- i.e. interfaces are not sharp

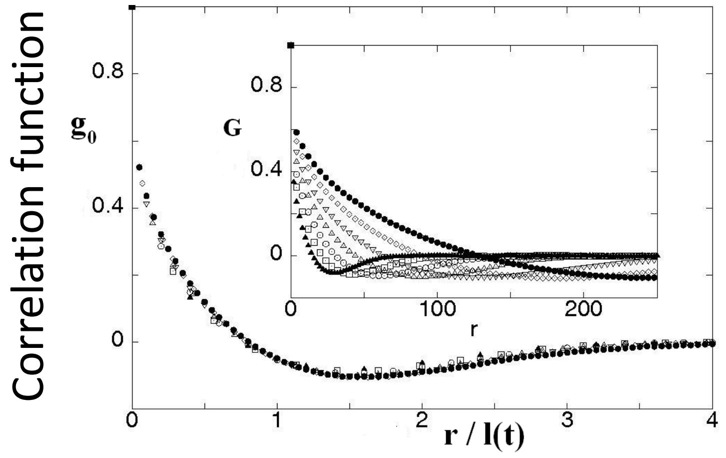


Fluctuation-dominated Phase Ordering: Hallmarks

- Cusp Singularity in Scaled 2-point Correlation Function

Coarsening

- Particles on a fluctuating surface tend to cluster.
- Typical size cluster size $l(t)$ grows in time $\Longrightarrow$ Scaling



Scaled separation

$$G(r,t) \equiv \langle n(0,t)n(r,t)\rangle - \langle n\rangle\langle n\rangle$$

$$= g_0\left(\frac{r}{l(t)}\right)$$

Cusp: $g(y) \approx m_0{}^2 - cy^{\alpha}$ as $y = r/l(t) \rightarrow 0$

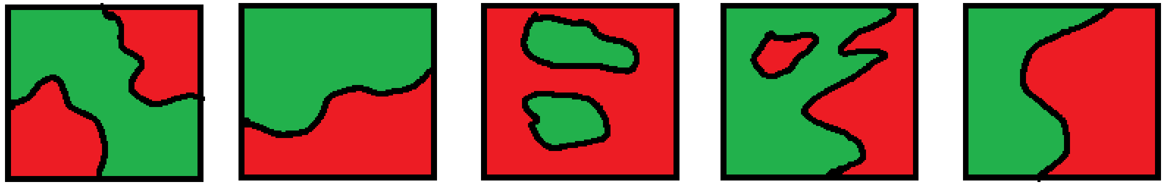
Singularity $\Longrightarrow$ The Porod Law fails to hold

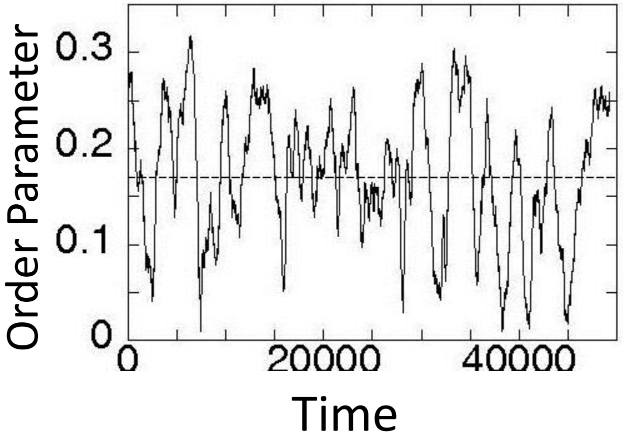
Interfaces are not sharp, but diffuse and broad.

Fluctuation-dominated Phase Ordering: Hallmarks

- Giant Fluctuations of Order Parameter

Steady State





Fluctuations: As large as the mean

Distributions remain broad as L→∞

Large macroscopic clusters break and re-form without disintegrating into tiny pieces. ⟹ Multiple order parameters required

Fluctuation-dominated Phase Ordering

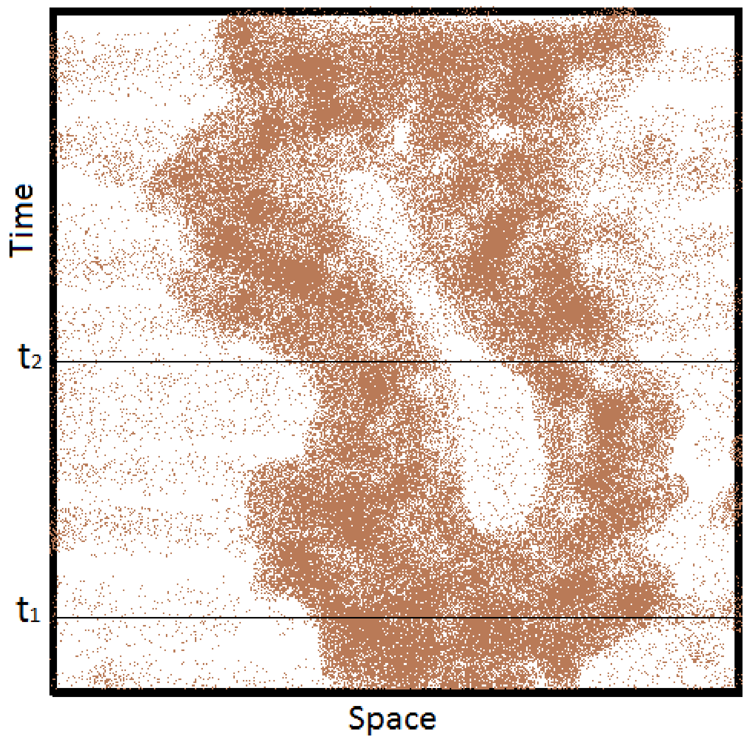
Fluctuations carry the system through states with a different number of macroscopic clusters.

The system circulates in the subset of 'ordered states', spending a finite fraction of time in each.

It never goes to completely disordered states.

This sort of 'loose ordering' was not characterized earlier.

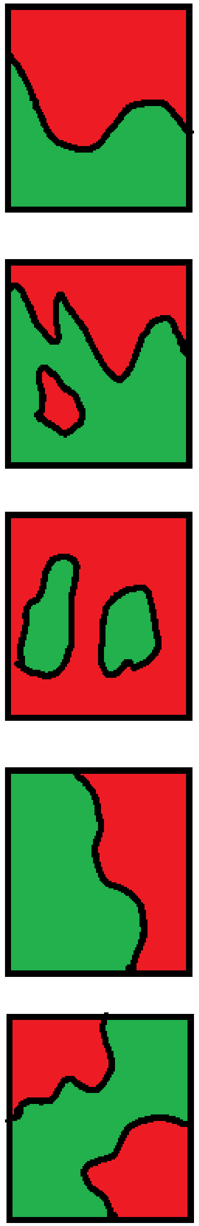
Interestingly, it is found in several types of systems.



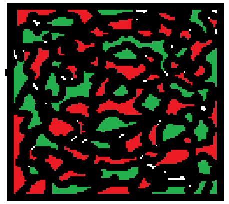
Nature of Ordering

System drawn to an attractor of ‘ordered’ configurations
with macroscopic segments of variable size

Schematic depiction of system evolution



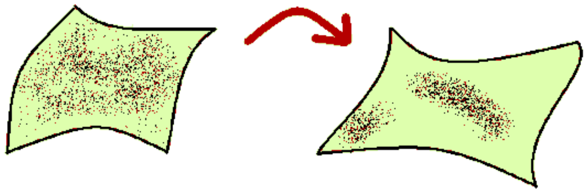
Ordered



Disordered

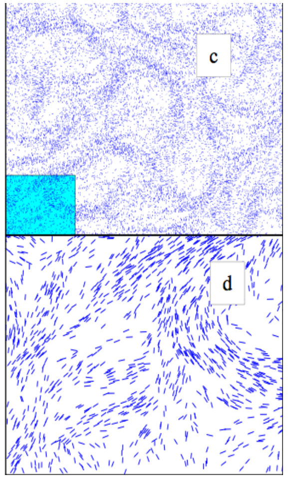
Systems showing FDPO

Passive scalars



Particles sliding down a fluctuating potential
Passive scalars advected by a Burgers fluid

[D. Das et al (PRL, 2000); G. Manoj et al (JSP, 2003);
A. Nagar et al (PRL, 2005); S. Chatterjee et al (PRE, 2006)]

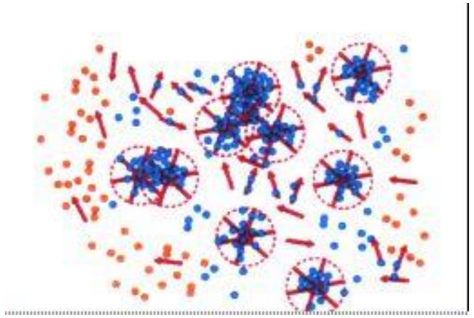


Active nematics

Particles advected by nematic angle field
[S. Mishra, S. Ramaswamy (PRL, 2006)]

Movement of apolar rods depends on orientation
[H. Chaté F Ginelli, R. Montaigne (PRL, 2006)]

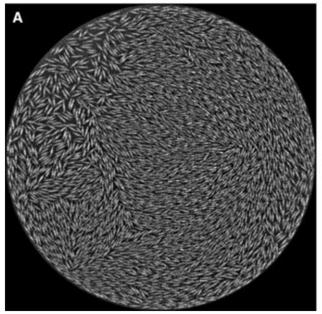
Correlation functions and fluctuations analyzed
[S. Dey, D. Das, R. Rajesh (PRL, 2012)]



Actin-stirred membrane

Phase segregation with strong fluctuations
[A. Das, A. Polley, Madan Rao (PRL, 2016)]

Systems showing FDPO



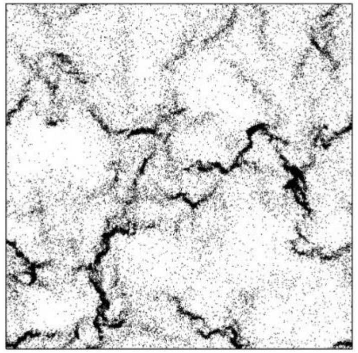
Vibrated rods

Giant number fluctuations

[V. Narayan, N. Menon, S. Ramaswamy (Science, 2007)]

Correlation functions extracted

[S. Dey et al (PRL, 2012)]

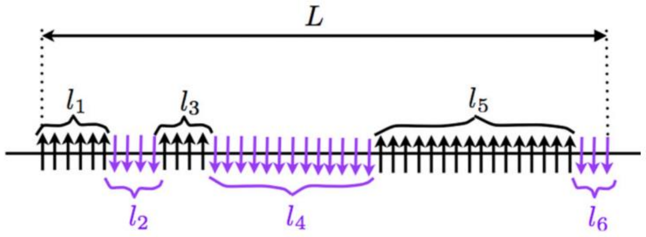


[I. Goldhirsch, G. Zanetti, (PRL, 1993)]

Freely cooling granular gases

Inelastically colliding particles in 1-d with velocity-dependent restitution

[M. Shinde, D. Das, R. Rajesh (PRL, 2007)]



Equilibrium Ising model with long-range interactions

Cluster-wise interactions induce mixed order transitions

[A. Bar, S. N. Majumdar, G. Scher, D. Mukamel (PRE, 2016)]

Exact evaluation of correlation function $\Longrightarrow$ Cusp singularity

[S. N. Majumdar, D. Mukamel, M. Barma (J. Phys. A, 2019)]

Conclusions

Fluctuations and Order

Fundamental attributes of a statistical system.

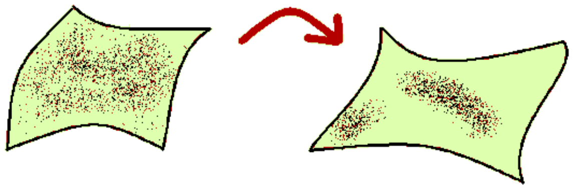
In several Systems with Long-range Interactions

Giant fluctuations coexist with order

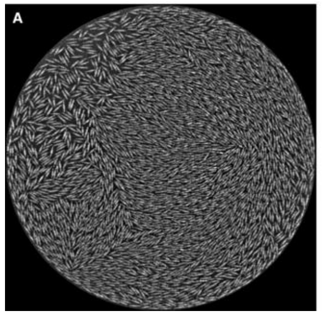
Fluctuation-dominated phase ordering

Key Signature:

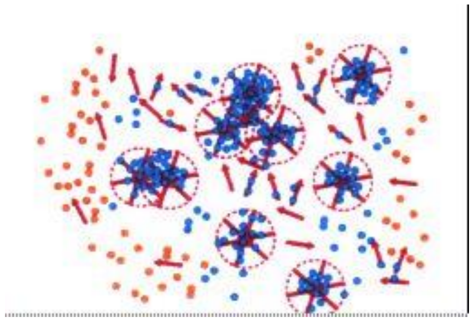
Cusps in scaled 2-point correlation functions



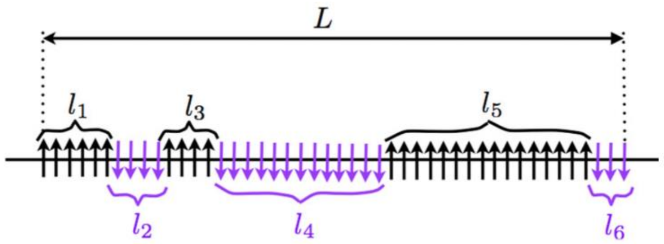
Passive scalars



Vibrated rods



Actin-stirred membrane



Spins with long-range interactions

Collaborators

- Dibyendu Das (IIT Bombay)
- Satya Majumdar (Université Paris-sud)
- Apoorva Nagar (IIST Trivandrum)
- Sakuntala Chatterjee (S N Bose Centre, Kolkata)
- Shauri Chakraborty (S N Bose Centre, Kolkata)
- G. Manoj (IIT Madras)
- Rajeev Kapri (IISER Mohali)
- Malay Bandyopadhyay (IIT Bhubaneswar)
- Tapas Singha (TCIS, Hyderabad)
- Samvit Mahapatra (UM-DAE CEBS, Mumbai)
- David Mukamel (Weizmann Institute, Rehovot)

Passive Scalar Problem

One driven system drives another … but no back effect

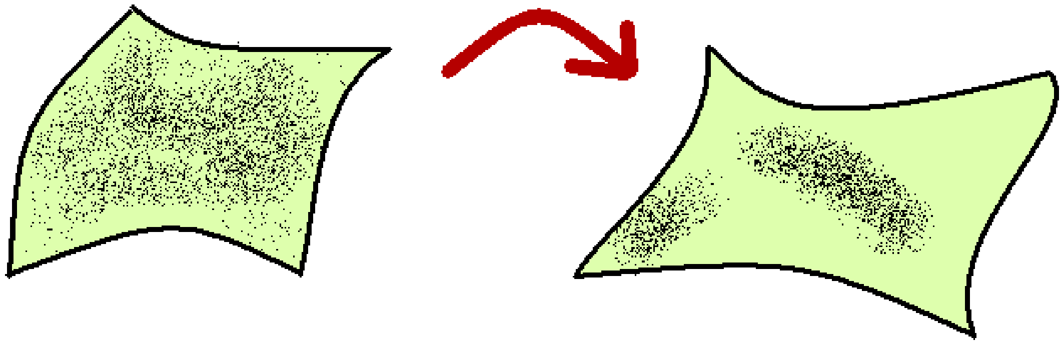
Fluctuating surface

The height field $h(x,t)$ evolves stochastically

$$\frac{\partial h}{\partial t} = \mu\nabla^2 h \quad + \quad \frac{\lambda}{2}(\nabla h)^2 \quad + \quad \vartheta$$

Smoothening — Nonlinearity (if ↑ and ↓ are distinguished) — Noise

(Kardar-Parisi-Zhang Equation)



Sliding particles

For the m'th particle,

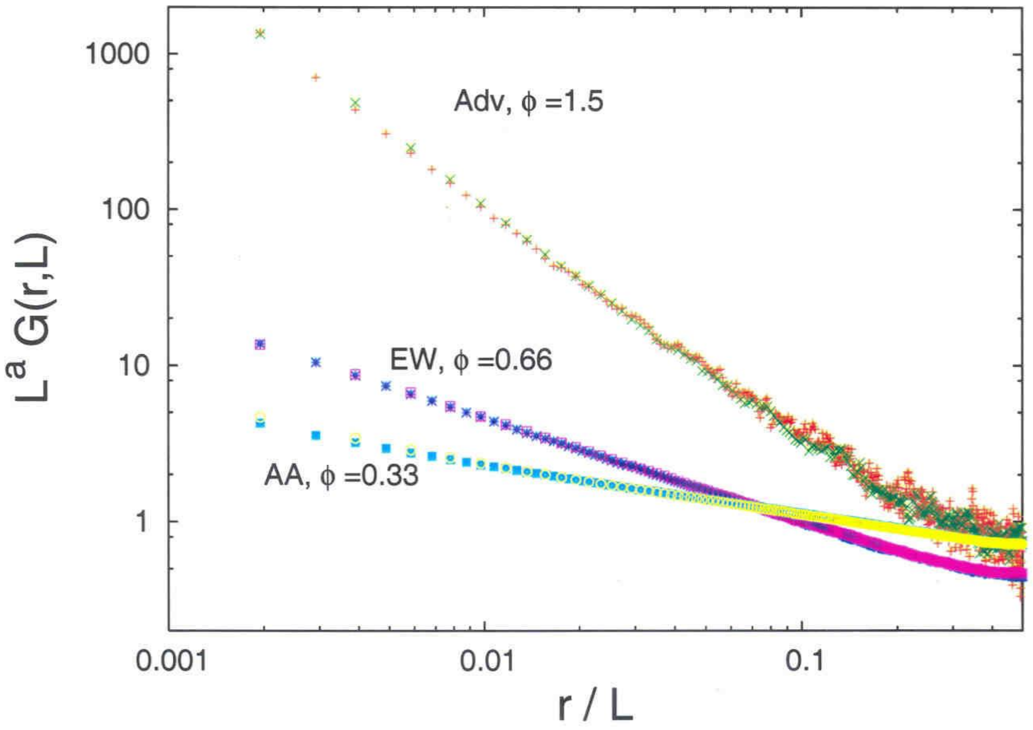
$$\frac{dx_m}{dt} = -a\nabla h|_{x_m} + \vartheta_m$$

In addition, there are exclusion effects between particles.

Non-Interacting Particles

Two-point correlation functions

(for KPZ Advection, Edwards-Wilkinson, KPZ Anti-advection)



Divergent Scaling Functions

$$L^a\ G \sim (r/L)^{-\varphi} \quad \text{as } r/L \to 0$$

[A. Nagar et al (2006)]

Connection with quenched disorder

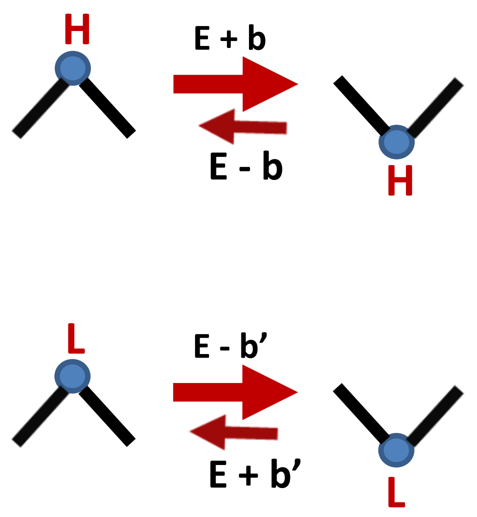
Adiabatic limit → A problem with quenched disorder (Sinai problem)

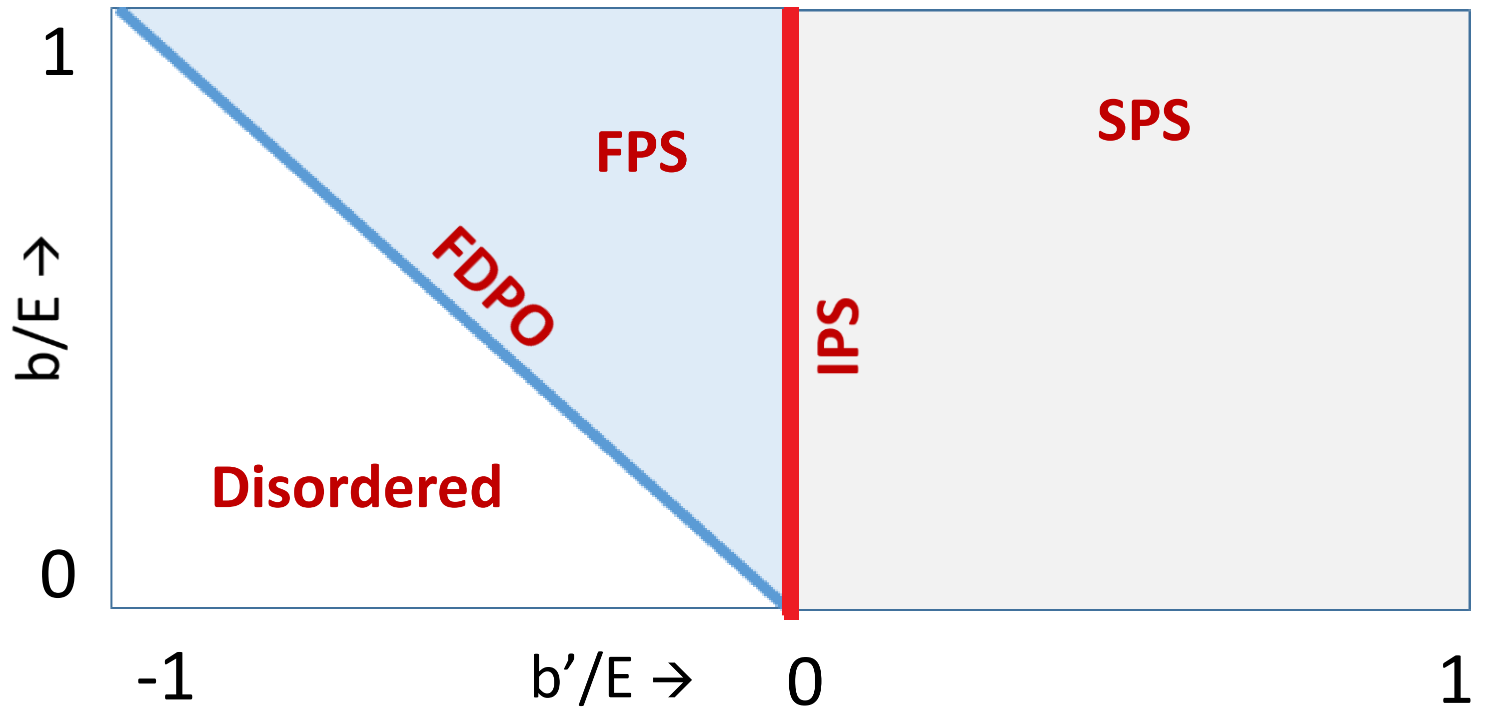
$$G(r,L) = (2\pi\beta^2 L)^{-1/2}\left[\frac{r}{L}\left(1-\frac{r}{L}\right)\right]^{-3/2}$$

[A. Comtet , C. Texier (1997)]

Fits KPZ advection data remarkably well.

Phase Diagram





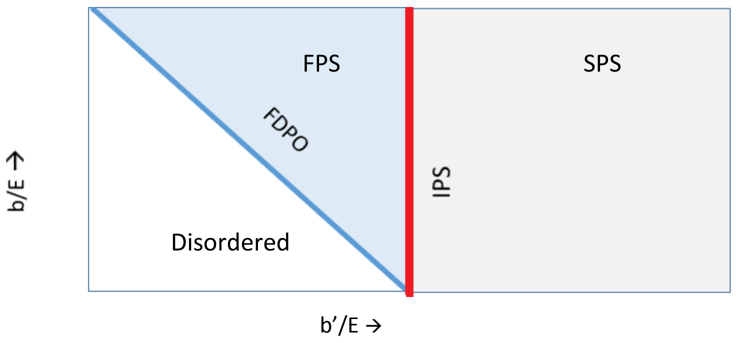
SPS: Strong Phase Separation

IPS: Infinitesimal fall with Phase Separation

FPS: Fast fall with Phase Separation

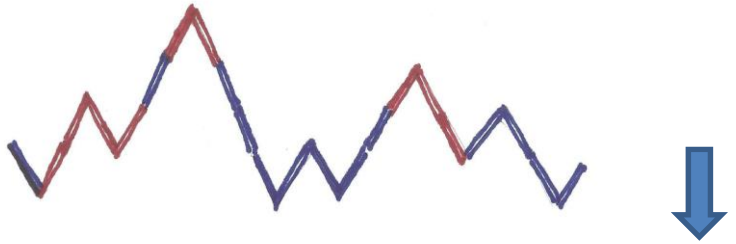
FDPO: Fluctuation-Dominated Phase Ordering

Typical Configurations in Phases



L particles push the surface upwards
[Studied by Lahiri et al (1997, 2000)]
SPS: Strong Phase Separation

L particles are neutral
IPS: Infinitesimal fall with Phase Separation



L and H particles push surface equally
[Reduces to passive scalar problem, Das et al (2000)]
FDPO: Fluctuation-Dominated Phase Ordering

L particles push surface down, but less than H particles
FPS: Fast fall with Phase Separation

FDPO in an Ising model with long-range interactions

$$H = -J_{NN} \sum_{i=1}^{L} \sigma_i \sigma_{i+1} - \sum_{i<j} J(i-j)\sigma_i\sigma_j \, I(i \sim j)$$

$J(r) \approx \frac{C}{r^2}$

Indicator function = 1 if $i, j \epsilon$ same cluster
= 0

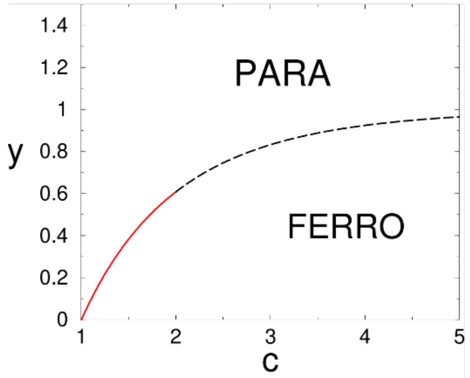
Cluster Representation: $H_{eff} = C \sum_{n=1..N} \ln(l_n) + \Delta N$

Chemical potential for domain walls; Fugacity $y = \exp(-\frac{\Delta}{T})$

The model shows a "mixed-order" transition *i.e.* Jump of magnetization + Divergence of correlation length

[A. Bar, D. Mukamel (PRL, 2014); A. Bar et al (PRE, 2016)]

Along the critical locus,



- Normal critical behaviour provided $c \equiv C/T > 2$ $\quad G(r) \sim \frac{1}{r^{d-2+\eta}}$
- Fluctuation-dominated phase ordering (FDPO) if $1 < c < 2$, $G(r, L) \approx m_0{}^2 - a \left|\frac{r}{L}\right|^{\alpha}$

The cusp exponent $\alpha = 2 - c$ varies continuously.

[MB, S. N. Majumdar and D. Mukamel, submitted to J. Phys. A]

Shows: FDPO can arise in an equilibrium system.

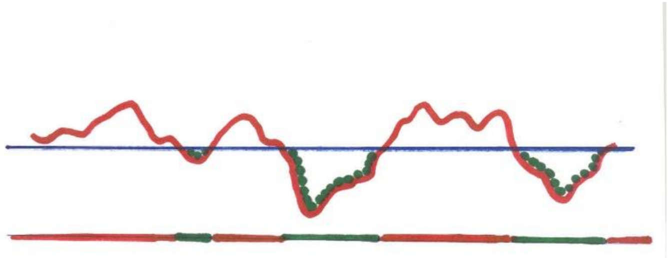
Understanding the Cusp Singularity

In steady state, $\mathcal{L}(t) \rightarrow$ System size L

$$G(r,L) \equiv < n(0)n(r) > - < n >< n > \approx g_0(\frac{r}{L})$$

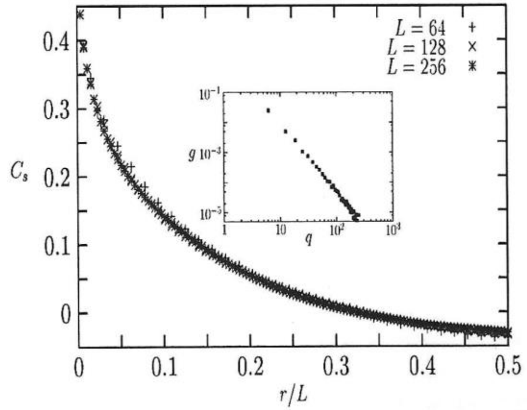
Random Walks and the Cusp Singularity

- In the extreme adiabatic limit, particles settle to the minimum energy state.
- Surface → Random walk path → $Prob(segment\ length = l) \sim l^{-3/2}$
- $G(r,L)$ can be found: $G(r,L) \approx 1 - a\,(\frac{r}{L})^{1/2} \ldots$



Higher Dimensions

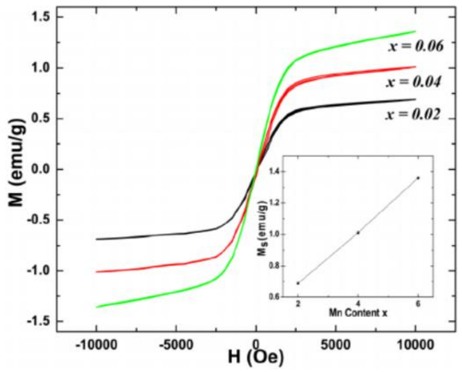
- $G(r,L)$ shows a cusp for particles driven by a 2d surface as well.



[G. Manoj et al (JSP, 2003)]

Fluctuations and Response

How does a system respond when conditions are changed?



S. Hussain et al (2012)

Measures of Response:

Magnetization change when field is applied:

Susceptibility $\sim \frac{\partial m}{\partial h}$

Density change when pressure is applied:

Compressibility $\sim \frac{\partial v}{\partial p}$

Heat required to raise the temperature:

Specific heat $\sim T\frac{\partial s}{\partial T}$

Interestingly, these can be found ***without*** applying an external influence.

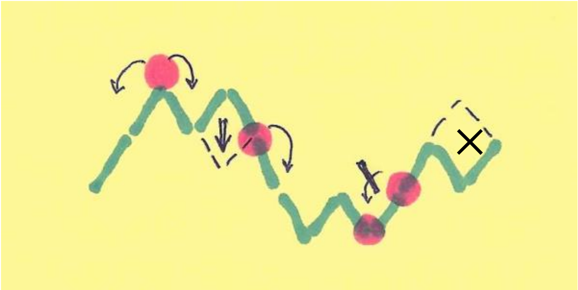
Instead, all one needs to do is measure the ***fluctuations*** in the system.

Passive Sliders on a 1D Fluctuating Lattice

Lattice: Autonomous evolution

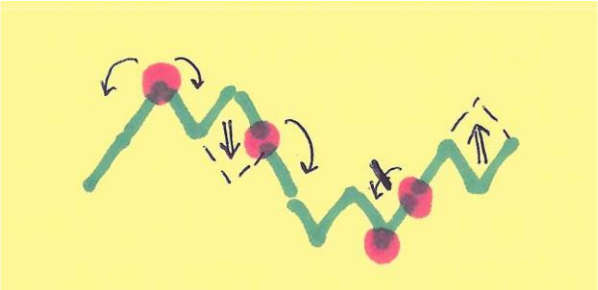
Sliders: Mutual exclusion ⇒ Single occupancy
Dynamics: Sliding stochastically down slopes

Models with stochastic evolution: At large length and time scales → Continuum description



Downward motion of fluctuating surface fluctuations

Kardar-Parisi-Zhang (KPZ)



Equilibrium surface

Edwards-Wilkinson (EW)

Particles move downward in either case, respecting exclusion.

Particles in Fluctuating Fields

Particles subject to random force fluctuations in space and time show interesting clustering properties.

The nature of clustering depends strongly on correlations of the driving force field.

- Passive particles in turbulent fluids

G. Falkovich, K. Gawedzki, M. Vergassola (Rev. Mod. Phys., 2001)

- Path coalescence of tracks

J. Deutsch (J. Phys. A, 1985)
M. Wilkinson, B. Mehlig (Phys. Rev. E, 2003)

- Proteins on actin-stirred cell membranes

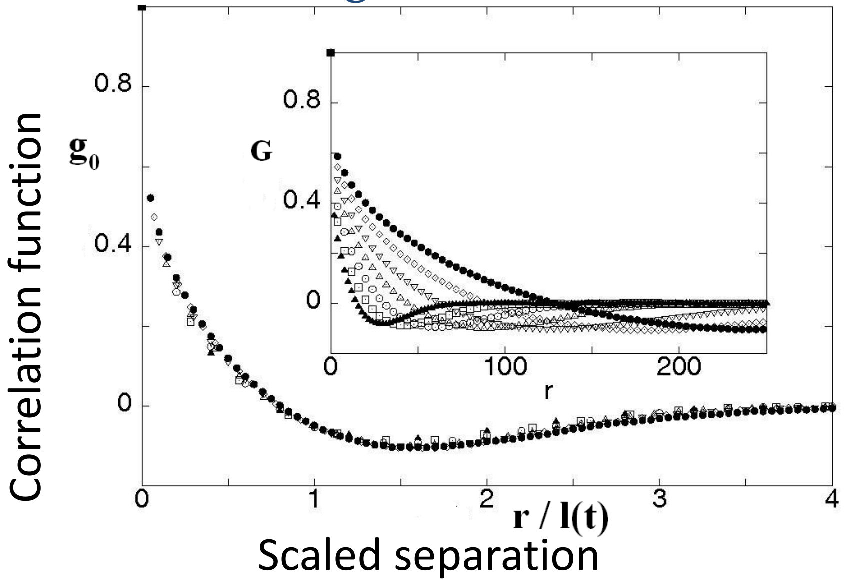
A. Das, A. Polley, Madan Rao (Phys. Rev. Lett., 2016)
F. Cagnetta, M. R. Evans, D. Marenduzzo (Phys. Rev. Lett., 2018)

Fluctuation-dominated Phase Ordering: Correlations

- Particles on a fluctuating surface tend to cluster.
- The typical size of a cluster $\mathcal{L}(t)$ grows in time $\Longrightarrow$ Scaling ensues

$$G(r,t) \equiv < n(0,t)n(r,t) > - < n >< n >$$

$$= g_0\left(\frac{r}{\mathcal{L}(t)}\right) \qquad \text{with } \mathcal{L}(t) \sim t^{1/z}$$



Cusp Singularity in Scaled Correlation Function

$$g_0(y) \approx C - ay^{\alpha} \quad \text{as } y = \frac{r}{\mathcal{L}(t)} \to 0; \quad \alpha < 1$$

$\alpha \cong \frac{1}{2}$ if the driving interface fluctuates

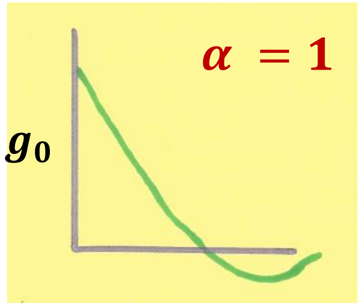
$\alpha \cong \frac{1}{4}$ if the driving interface fluctuates and moves ↓

[D. Das et al (PRL, 2000)]

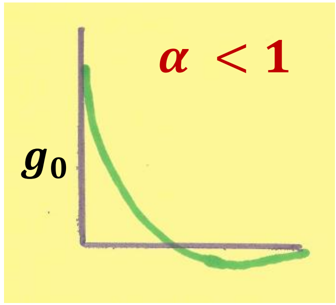
Cusps in the Correlation Function

Density-density correlation function shows Scaling

$G(r,t) \equiv \langle n(0,t)n(r,t) \rangle - \langle n \rangle \langle n \rangle \approx g_0(\frac{r}{\mathcal{L}(t)})$ with $\mathcal{L}(t) \sim t^{1/z}$



$r/\mathcal{L}(t)$



$r/\mathcal{L}(t)$

Porod Law

Valid

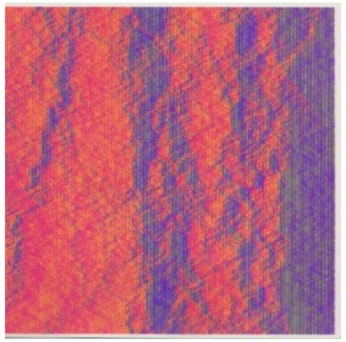
Breaks down

Scaling Function

As $y \to 0$, $\boldsymbol{g_0(y) \approx m_0{}^2 - a\,|y|^{\alpha}}$

- Intercept $m_0{}^2 = G_\infty$ measures Long-range order.
- Cusp Exponent $\alpha < 1$ indicates Very broad interfaces.

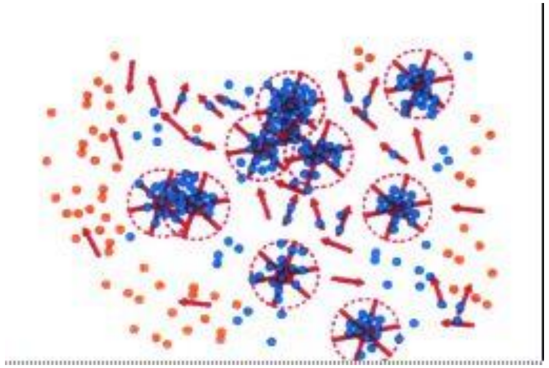
(Breakdown of Porod Law)



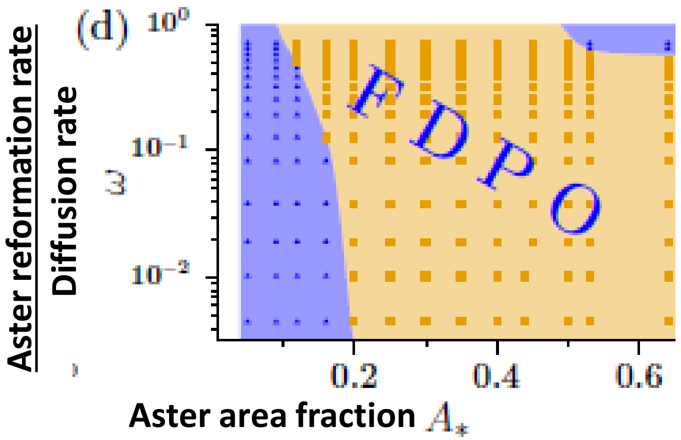
Molecular Clustering at Cell Surface

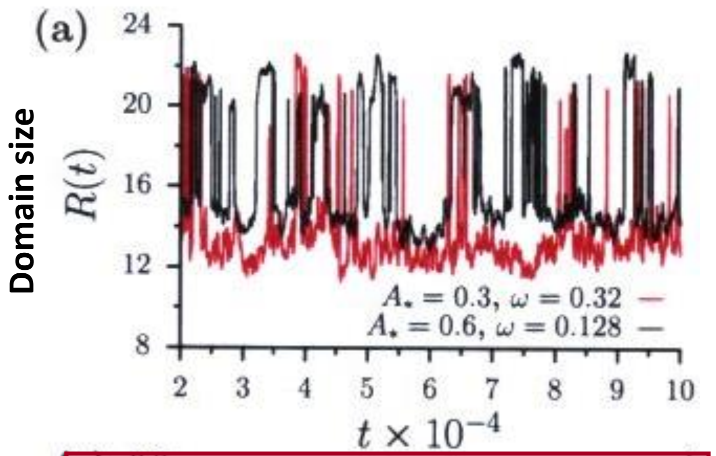
[A. Das, A. Polley, Madan Rao (PRL, 2016)]

Phase segregation driven by activity:
Membrane stirred by actin activity → Clustering of advected membrane molecules

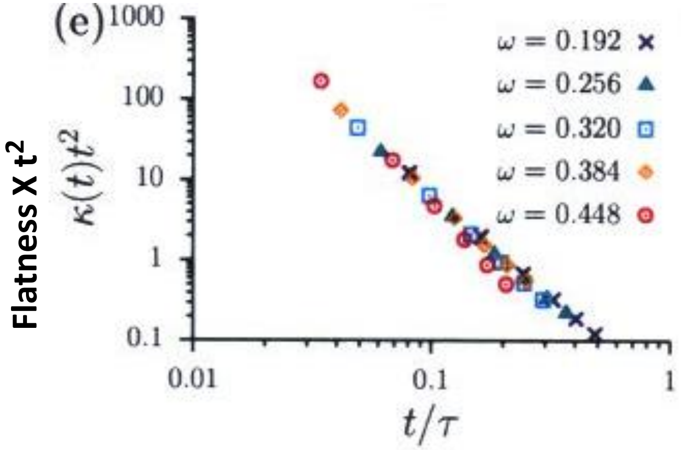


Find: Transition from micro-clustering to phase segregation with strong fluctuations





Intermittent domain sizes



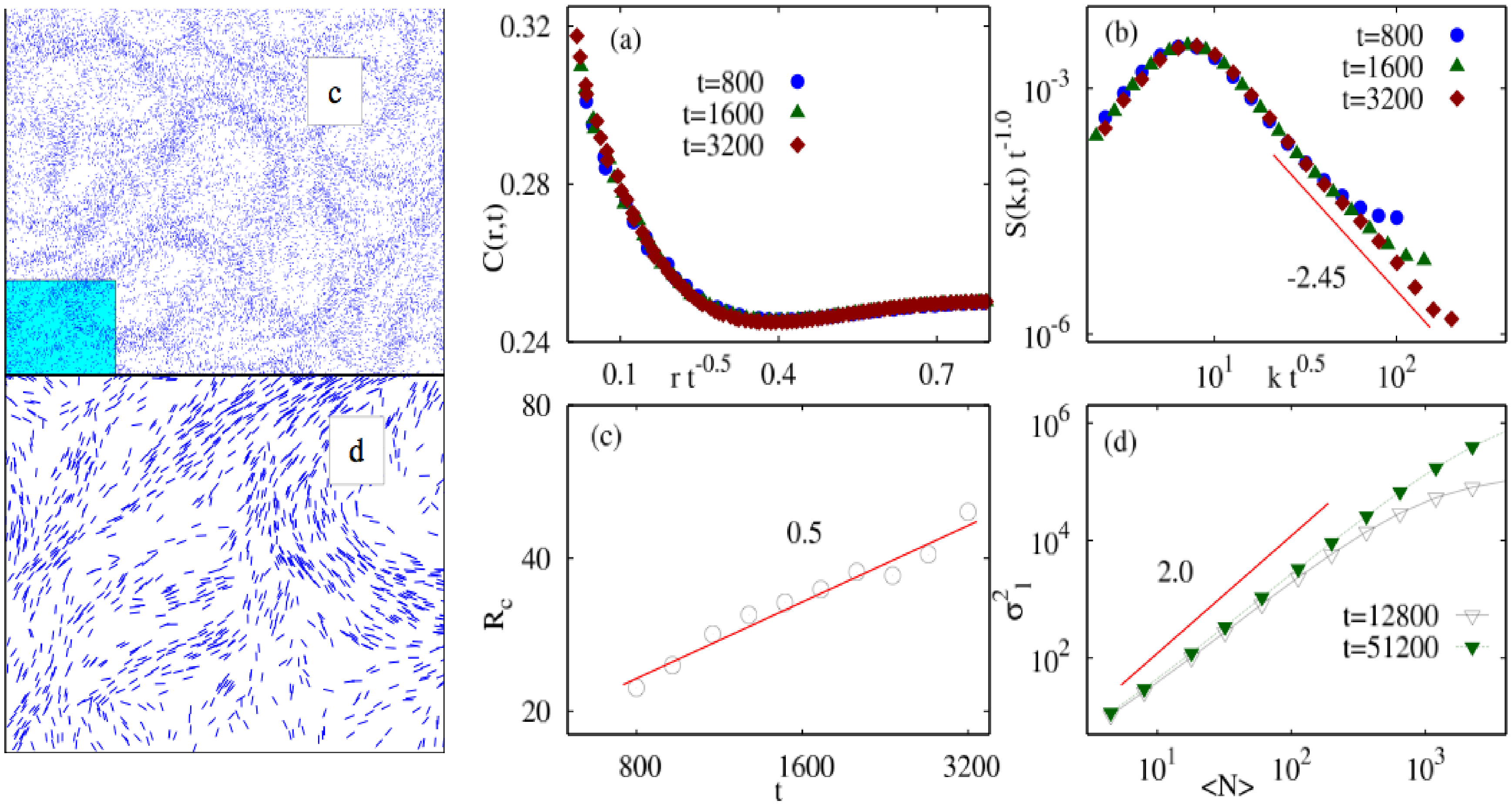
Active Nematics

Model Movement of apolar rods depends on orientation

[H. Chate, F Ginelli, R. Montaigne (PRL, 2006)]

Analyze Correlation functions and fluctuations

[S. Dey, D. Das, R. Rajesh (PRL, 2012)]



Cusp exponent $\cong 0.5$; Enters (sub-leading) in expression for σ^2