Exploring aspects of QCD from Quantum Link Models

Debasish Banerjee

Saha Institute of Nuclear Physics, Kolkata

November 19, 2019

QCD in the non-perturbative regime

TIFR, Mumbai

Outline

Non-perturbative gauge fields in Nature

Abelian link models: toy meson physics

Non-abelian link models: toy QGP, nuclear physics

Outlook

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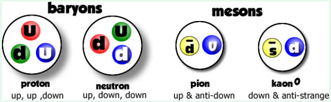
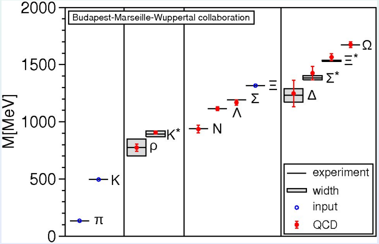
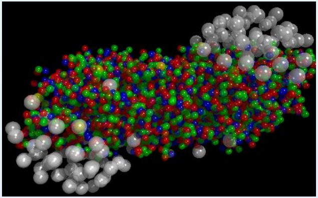
Strong Interactions

Properties of protons, neutrons and other particles (hadrons) made of quarks and gluons explained by quantum chromodynamics (QCD).





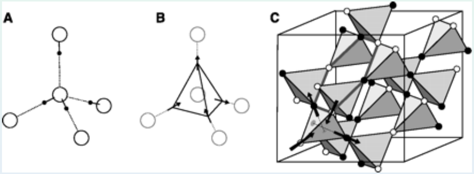
Hadron spectrum



Quark Gluon Plasma

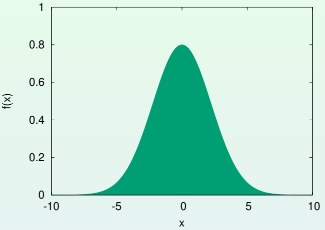
Frustrated Magnets

- Emergent gauge fields describe many condensed matter systems.
- Degenerate ground states in water-ice (H_2O) and spin-ice (pyrochlore materials, e.g. $Ho_2Ti_2O_7$) $\longrightarrow$ ice states.

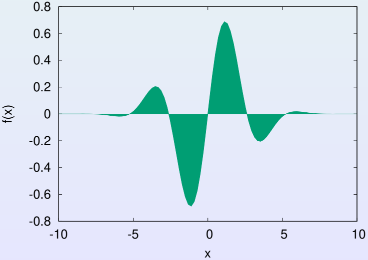


- Tunneling between two ice states via loop operators
  10^2 mK $\sim 10^{-10}$ MeV .
- Low energy spin liquid phases admit gauge theory description.

Success stories of classical computers

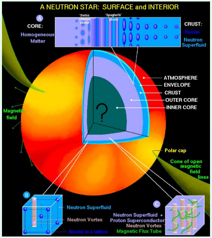


Monte Carlo methods on fast, reliable supercomputers.

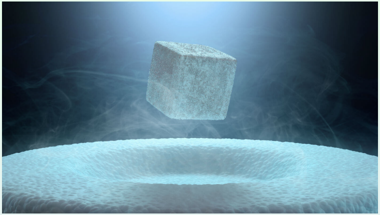


Importance sampling breaks down for rapidly oscillating integrands.

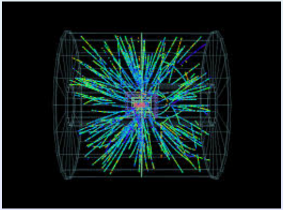
Classical nightmares for sign failures



Neutron Star



Superconductivity



Heavy-Ion Collisions

Hinders first principle studies of
finite density systems,
non-equilibrium phenomena.

New Models, New Tools

- Quantum Link Models (QLMs) are ideal versatile candidates.
- Horn(1981); Orland, Rohrlich(1990); Chandrasekharan,Wiese(1997)
- Rokshar, Kivelson(1988); Moessner, Sondhi, Fradkin (2002)
- Microscopic descriptions need not be identical to produce the same infra-red physics.

Classical Simulators for Quantum Gauge Matter.

- Reformulations $\longrightarrow$ efficient Monte Carlo methods.
- Controlled variational methods $\longrightarrow$ Tensor networks.
- Effective field theory $\longrightarrow$ analytic understanding.

New Results

- Explore static and dynamic properties.
- Gauge matter in simpler models of condensed matter, particle and astro-particle physics.
- Increasing complexity to approach actual systems in Nature.

Quantum Simulators for Quantum Gauge Matter.

- Benchmark quantum simulator platforms.
- Quantum circuits for quantum computers.
- Topological properties for use in quantum hardware.

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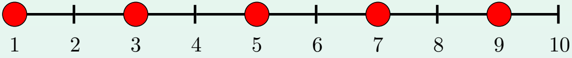
Quantum links in $(1+1)-$d

$$H = \frac{g^2}{2} \sum_{xy} E_{xy}{}^2 - \kappa \sum_{xy} \left[\psi_x^\dagger U_{xy} \psi_y + \text{h.c.} \right] + m \sum_x (-1)^x \psi_x^\dagger \psi_x$$



- Quantum links:
  $U = S^+; \; U^\dagger = S^-; \quad E = S^z$
- Finite dimensional gauge invariant representations.

$$[E_{xy}, U_{xy}] = U_{xy}$$
$$[E_{xy}, U_{xy}^\dagger] = -U_{xy}^\dagger$$
$$[U_{xy}, U_{xy}^\dagger] = 2E_{xy}$$

- Gauge symmetry:
  $[G_x, H] = 0;$ where
  $G_x = (E_{xy} - E_{wx}) - \rho_x$

$$V = \prod_x \exp\left(iq\theta_x G_x \right)$$
$$\tilde{H} = VHV^\dagger = H$$

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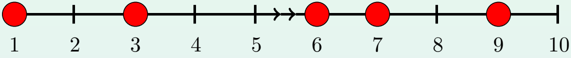

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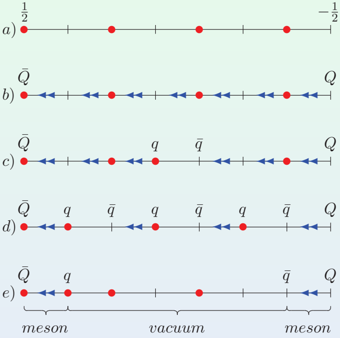
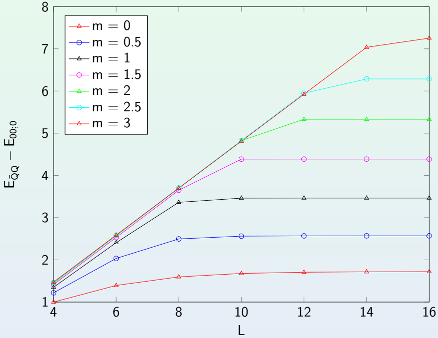
String breaking

Using $S = 1$ links, a confining string can be realized.

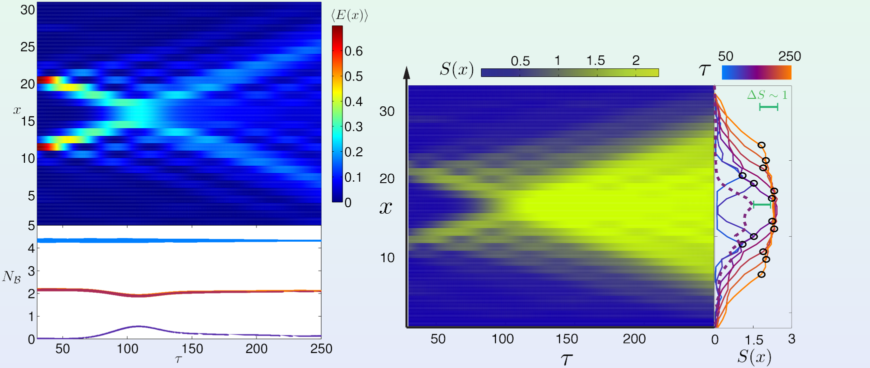




- String breaking in a quantum simulator. Banerjee, Dalmonte, Rico Ortega, Stebler, Wiese, Zoller (2012, 2013).
- Dynamical Quantum Phase Transitions. Huang, Banerjee, Heyl (2018).

Scattering toy mesons

- Collision of toy meson wave-packets in real time.
  Pichler, Dalmonte, Rico, Zoller, Montanegro (2016).



Closer connections with the same theory in Euclidean time with efficient algorithms?

Abelian Rishons

- Quantum links: $U = S^+;\ U^\dagger = S^-;\ \ E = S^z$
- Finite dimensional gauge invariant representations.

$$[E_{xy}, U_{xy}] = U_{xy}$$
$$[E_{xy}, U^\dagger_{xy}] = -U^\dagger_{xy}$$
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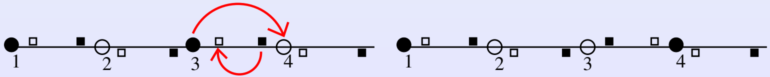
Commutations satisfied with Schwinger bosons

$$U_{xy} = b^\dagger_{y,-} b_{x,+};\quad U^\dagger_{xy} = b^\dagger_{x,+} b_{y,-};$$
$$E_{xy} = \frac{1}{2}\left(b^\dagger_{y,-} b_{y,-} - b^\dagger_{x,+} b_{x,+}\right);$$
$$\mathcal{N}_{xy} = b^\dagger_{y,-} b_{y,-} + b^\dagger_{x,+} b_{x,+} = 2S = N$$

Link "gauge" invariance: $[H, \mathcal{N}_{xy}] = 0$.

$$H = -\kappa \sum_{xy} \left[\psi^\dagger_x b^\dagger_{y,-} b_{x,+} \psi_y + \text{h.c.}\right] + \frac{g^2}{2} \sum_{xy} \frac{1}{4} \left(b^\dagger_{y,-} b_{y,-} - b^\dagger_{x,+} b_{x,+}\right)^2$$



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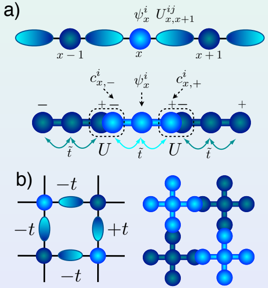
Non-Abelian QLMs

$$H = -t \sum_{xy} \left(s_{xy} \psi_x^{i\dagger} U_{xy}^{ij} \psi_y^j + \text{h.c.} \right) + \frac{g^2}{2} \sum_{xy} \left(L_{xy}^2 + R_{xy}^2 \right),$$

$$G_x^a = \psi_x^{i\dagger} \lambda_{ij}^a \psi_x^j + \sum_k \left(L^a_{x,x+\hat{k}} + R^a_{x-\hat{k},x} \right).$$

Link operators can be expressed as fermionic rishons:

$$L_{xy}^a = c_{x,+}^{i\dagger} \lambda_{ij}^a c_{x,+}^j;$$
$$R_{xy}^a = c_{y,-}^{i\dagger} \lambda_{ij}^a c_{y,-}^j;$$
$$U_{xy}^{ij} = c_{x,+}^i c_{y,-}^{j\dagger}$$

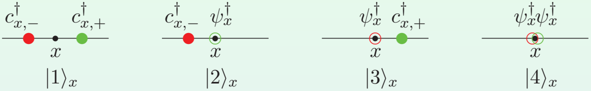


Color degrees can be summed to obtain color singlets:

$$\psi_x^{i\dagger} U_{xy}^{ij} \psi_y^j = \psi_x^{i\dagger} c_{x,+}^i c_{y,-}^{j\dagger} \psi_y^j = \mathcal{Q}_x^\dagger \mathcal{Q}_y; \quad \mathcal{Q}_{y,\pm k} = c_{y,\pm k}^{j\dagger} \psi_y^j$$

Gauge Invariant States

- A $U(2)$ QLM with staggered fermions in $(1+1)$-d has 4 gauge invariant states ($\mathcal{N}_{xy} = 1$ rishon per link):



$$|1\rangle_x = \frac{1}{\sqrt{2}}\left(c^{\dagger 1}_{x,-} c^{\dagger 2}_{x,+} - c^{\dagger 2}_{x,-} c^{\dagger 1}_{x,+}\right)|0\rangle_x; \quad |2\rangle_x = \frac{1}{\sqrt{2}}\left(c^{\dagger 2}_{x,-} \psi^{\dagger 1}_x - c^{\dagger 1}_{x,-} \psi^{\dagger 2}_x\right)|0\rangle_x;$$

$$|3\rangle_x = \frac{1}{\sqrt{2}}\left(c^{\dagger 2}_{x,+} \psi^{\dagger 1}_x - c^{\dagger 1}_{x,+} \psi^{\dagger 2}_x\right)|0\rangle_x; \quad |4\rangle_x = \psi^{\dagger 2}_x \psi^{\dagger 1}_x |0\rangle_x$$

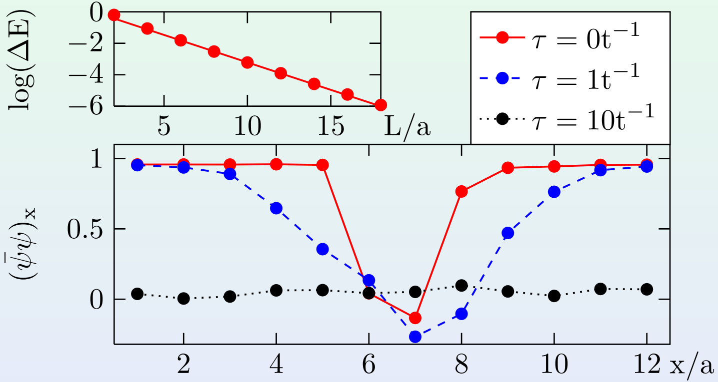
- Using the basis states $\{|1\rangle_x, |2\rangle_x, |3\rangle_x, |4\rangle_x\}$

$$\mathcal{Q}_{x,+} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathcal{Q}_{x,-} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Banerjee, Bögli, Dalmonte, Rico, Stebler, Wiese, Zoller (2013)

Expansion of toy fireball

The $\mathbb{Z}(2)$ chiral symmetry breaks spontaneously with $m = 0$ and $V = -6t$.



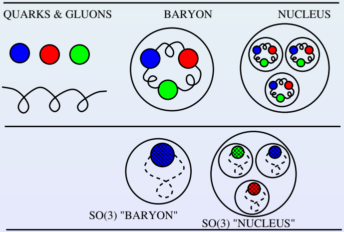
Real-time evolution of the order parameter profile $(\overline{\psi}\psi)_x(\tau) = s_x \langle \psi_x^{i\dagger}\psi_x^i - \frac{N}{2} \rangle$ for $L = 12$, mimicking the expansion of a hot quark-gluon plasma.

Toy Nuclear Physics from link models

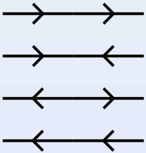
$(d+1)$–d SO(3) QLM with adjoint triplet fermions:

$$H = -t \sum_{xy,ab} s_{xy} \left[\psi_x^{a\dagger} O_{xy}^{ab} \psi_y^b + \text{h.c.} \right] + m \sum_x s_x \psi_x^{a\dagger} \psi_x^a$$
$$= -t \sum_x \left(B_{x,+}^\dagger B_{y,-} + \text{h.c.} \right) + m \sum_x (-1)^x M_x$$

Baryons are fermionic color singlets of fermion and link fields.



$$O_{xy}^{ab} = \sigma_{x,+}^a \otimes \sigma_{y,-}^b$$



Rico, Dalmonte, Zoller, Banerjee, Bögli, Stebler, Wiese (2018).

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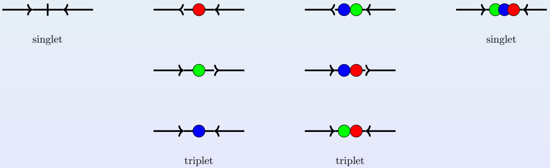
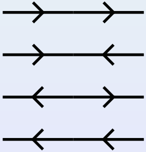




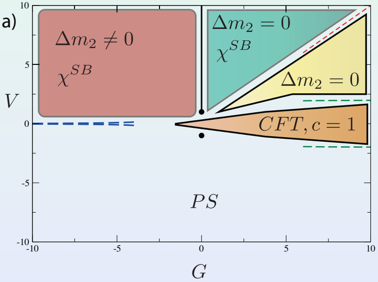
Rico, Dalmonte, Zoller, Banerjee, Bögli, Stebler, Wiese (2018).

Toy Nuclear Physics from link models

With 4-fermi couplings, rich physics in $d = 1$ dimensions:

$$G \sum_x \left(M_x - \frac{3}{2} \right)^2 ; \quad V \sum_x \left(M_x - \frac{3}{2} \right) \left(M_{x+1} - \frac{3}{2} \right)$$

	3-d QCD	1-d $SO(3)$	2-d $SO(3)$
gauge symmetry	$SU(3)$	$SO(3)$	$SO(3)$
chiral symmetry	$SU(2)_L \times SU(2)_R$	$\mathbb{Z}_2$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
flavor symmetry	$SU(2)_{L=R}$	$\mathbb{I}$	$\mathbb{Z}_2$
baryon symmetry	$U(1)$	$U(1)$	$U(1)$
charge conjugation	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
parity	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$



Nuclear binding and χ-symmetry restoration at finite μ_B.

Rico, Dalmonte, Zoller, Banerjee, Bögli, Stebler, Wiese (2018).

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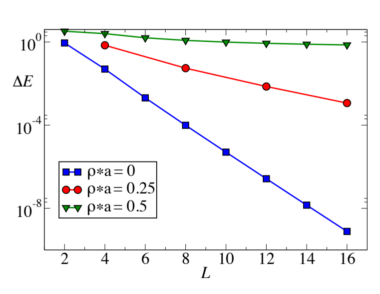

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- QLMs are capable of exhibiting many rich physical phenomena in particle physics.
- Applications to frustrated magnetism and high Tc superconductors are relevant for condensed matter physics.
- New algorithmic developments meron cluster methods (Banerjee, Huffman) to look out for.
- With tensor network methods, various dynamical aspects can also be studied.
- Using rishons and the gauge-invariant states are useful for both algorithmic developments as well as quantum simulators.

THANK YOU FOR YOUR ATTENTION