Meson Screening Masses from 2+1-Flavor Lattice QCD

Prasad Hegde
Centre for High Energy Physics
Indian Institute of Science, Bangalore, India

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The Non-Perturbative Nature of QCD

- The Standard Model provides an Effective description of all the forces known in nature except the gravitational force.
- Of these, only QCD (described by $SU(3)_c$) is non-perturbative in nature.

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{f=1}^{N_f} \bar{\psi}_f(x) \left(i\not{D} - m_f \right) \psi_f(x) - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 .$$

- Currently, the only way to obtain the observed spectrum from the QCD Lagrangian is through numerical simulations.

Lattice QCD

- Path-integral approach to QCD formulated on a discrete lattice in Euclidean spacetime.

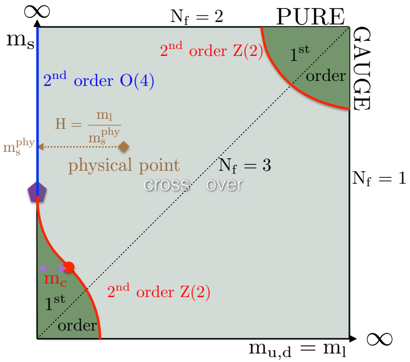
$$\mathcal{Z}_{\mathrm{QCD}} = \int \mathcal{D}\left[\Psi, \bar{\Psi}, U\right] e^{-S_F - S_G}.$$

- Instead of gauge fields $A_\mu^a(x)$, one has $SU(3)_c$ group elements $U_\mu(x, x+\hat{\mu})$ that are defined on the links connecting adjacent sites.
- Path-integral phase factor replaced by a Boltzmann factor e^{-S}. If S is real and positive-definite, then a probabilistic interpretation is possible, allowing for a Monte Carlo calculation of the path-integral.
- Number of degrees of freedom: For a $32 \times 32 \times 32 \times 8$ lattice: $32^3 \times 8 \times 3$ colors $\times$ 4 spins $\approx 10^8$(!) Use Monte Carlo to evaluate the path integral.

Quark-Gluon Plasma and Chiral Symmetry Restoration

- The *quark-gluon plasma* is a new phase of a strongly-interacting matter that exists at extremely high temperatures.
- In this phase, nuclear matter is deconfined and chirally symmetric.
- The nature of the transition has been established as a crossover for physical quark masses [Y. Aoki *et al.* (2006)].
- Recently the crossover temperature too has been determined to be $T_{\mathrm{cross}} \simeq 156.5(1.5)$ MeV [HotQCD collaboration (2019)].

The Chiral Phase Transition



- The two-flavor chiral transition is 2^{nd} order belonging to the $3d$-$O(4)$ universality class in the limit $m_l \to 0$.
- The transition temperature for this transition too has been determined recently, to be $T_c = 132^{+3}_{-6}$ MeV. [HotQCD collaboration (2019)].

Screening Correlators

- Screening correlators carry important information about the degrees of freedom of QCD at finite temperature, especially in the quark-gluon plasma phase [R. Gavai *et al.* (1987), C. DeTar (1987), R. Gavai, S. Gupta & P. Majumdar (2001)].

- The meson screening correlators are defined by

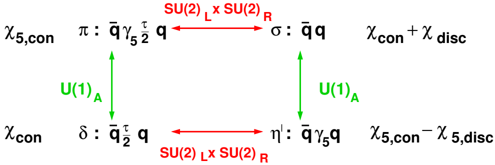
$$G_\Gamma(z) = \int_0^\beta d\tau \int dxdy \left\langle \mathcal{M}_\Gamma(x,y,z,\tau)\overline{\mathcal{M}_\Gamma}(0,0,0,0)\right\rangle,$$

  where $\mathcal{M}_\Gamma \equiv \bar{\psi}(\Gamma \otimes t^a)\psi$ is a meson operator and β is the inverse temperature.

- The large-distance fall-off of these correlators is controlled by the respective screening masses viz.

$$G_\Gamma(z) \sim \exp(-m_\Gamma(T)\, z), \qquad z \to \infty.$$

Meson Correlators and Symmetry Restoration



- The restoration of various symmetries manifests itself as a degeneracy among various correlation functions.

- In the case of $2+1$-flavor QCD, it suffices to study two-point functions, *i.e.*, meson screening functions.

- Chiral symmetry restoration identifies the vector and axial vector isotriplet correlators while $U_A(1)$ restoration identifies the scalar and pseudoscalar isotriplet correlators.

Setup of the Calculation

- We calculated meson screening masses in 2+1-flavor QCD for temperatures 140 MeV $\lesssim T \lesssim$ 1 GeV.
- Our lattices were generated using the 2+1-flavor Highly Improved Staggered Quark action (HISQ).
- Our strange quark was tuned to its physical value, while the light quark mass was set to one of two values: $m_l = m_s/20$ (nearly physical, high temperatures) and $m_l = m_s/27$ (physical, low temperatures).
- We calculated the screening masses for $N_\tau = 6$, 8, 10 (only for $m_l = m_s/20$), 12 and 16 (only for $m_l = m_s/27$). This allowed us to take the continuum limit.

Staggered Fermions

- Not easy to put fermions on the lattice: The well-known "fermion doubling" problem [L. Karsten & J. Smit (1978), H. Nielsen & M. Ninomiya (1981)].
- Staggered fermions: Reduce the number of doubler flavors from 16 to 4 by spin-diagonalising the Dirac operator:

$$\psi(n) \to \gamma_1^{n_1}\gamma_2^{n_2}\gamma_3^{n_3}\gamma_4^{n_4}\,\psi(n), \qquad \mathcal{S}^{-1}\gamma_\mu\mathcal{S} \to (-1)^{n_1+\cdots+n_{\mu-1}}.$$

- One-component Dirac spinors, hence inexpensive to simulate. However non-trival relation to continuum Dirac action.

Staggered Meson Operators

- A staggered meson operator is given by

$$\mathcal{M}(x) = \sum_n \phi(x)\bar{\chi}(x)\chi(x+n),$$

  where $\phi(x)$ is an x-dependent phase factor and n points to one or more vertices of the unit hypercube based at x.

- If $n = 0$, the operator is said to be a local operator.

- The connection between these correlators and the continuum meson correlators is complicated, but is known from group theory [M. Goltermaan (1986), S. Gupta (1999)].

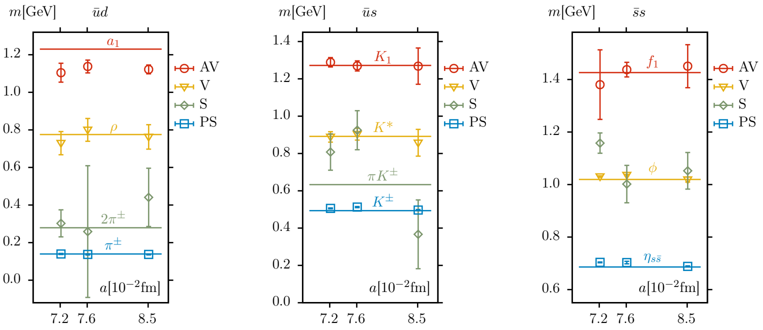
Staggered Meson Operators

- Each staggered meson comes in sixteen flavors, known as tastes. These tastes are degenerate in the continuum. At finite lattice spacing however, this degeneracy is broken at $\mathcal{O}(a^2)$.
- A staggered correlator couples to two mesons of opposite parities:

$$G(n_\sigma) = \sum_{i=0,1,2,\ldots} A_i^{(-)} \cosh\left(am_i^{(-)}\left(n_\sigma - \frac{N_\sigma}{2}\right)\right) - (-1)^{n_\sigma} \sum_{j=0,1,2,\ldots} A_j^{(+)} \cosh\left(am_j^{(+)}\left(n_\sigma - \frac{N_\sigma}{2}\right)\right).$$

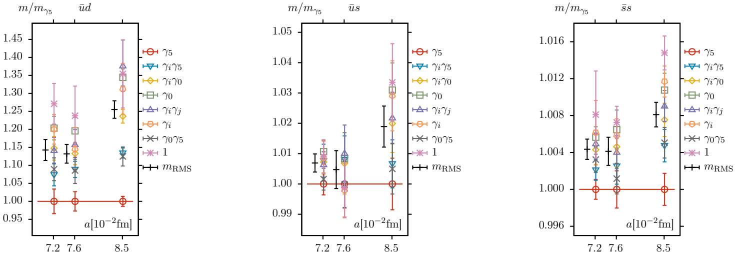
- For example, the vector correlator that we study here couples to both the vector as well as to one of the tastes of the axial-vector mesons.

Spectrum at $T = 0$



- No determination of the flavored scalar meson ($a_0(980)$).
- This is because the staggered scalar decays to two pions [S. Prelovsek (2005)].
- Unphysical contribution from the various taste sectors cancels out in the continuum; more on this later.

Taste-Splitting in the Pion Sector



Our results may be compared to earlier results on taste-splittings for the HISQ action [HotQCD collaboration, Lattice 2010].

List of Meson Operators

	$\phi(\mathbf{x})$	Γ		J^{PC}	
		NO	O	NO	O
$\mathcal{M}1$	$(-1)^{x+y+\tau}$	$\gamma_3\gamma_5$	1	0^{-+}	0^{++}
$\mathcal{M}2$	1	γ_5	γ_3	0^{-+}	0^{+-}
$\mathcal{M}3$	$(-1)^{y+\tau}$	$\gamma_1\gamma_3$	$\gamma_1\gamma_5$	1^{--}	1^{++}
$\mathcal{M}4$	$(-1)^{x+\tau}$	$\gamma_2\gamma_3$	$\gamma_2\gamma_5$	1^{--}	1^{++}
$\mathcal{M}5$	$(-1)^{x+y}$	$\gamma_4\gamma_3$	$\gamma_4\gamma_5$	1^{--}	1^{++}
$\mathcal{M}6$	$(-1)^{x}$	γ_1	$\gamma_2\gamma_4$	1^{--}	1^{+-}
$\mathcal{M}7$	$(-1)^{y}$	γ_2	$\gamma_1\gamma_4$	1^{--}	1^{+-}
$\mathcal{M}8$	$(-1)^{\tau}$	γ_4	$\gamma_1\gamma_2$	1^{--}	1^{+-}

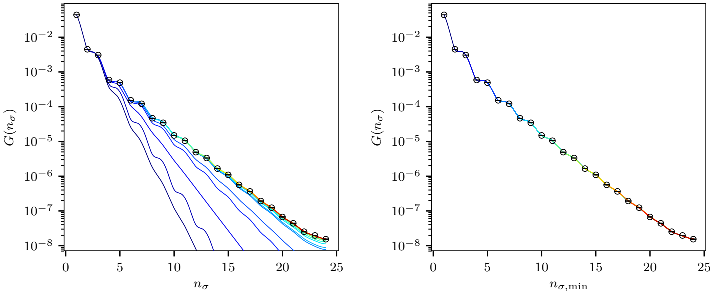
In this study, we used only local operators, and we studied the screening masses for spin-0 and spin-1 mesons of both parities.

Fitting the Correlators

Multi-state fits tend to be highly unstable. The number of fit parameters grows and the # degrees of freedom decreases quickly.

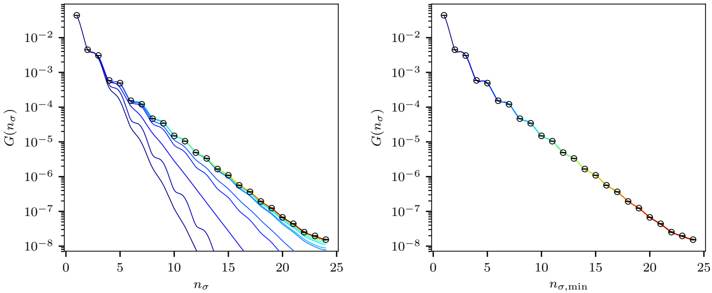
- One-state fits in a narrow fit window $[N_\sigma/2 - \tau, N_\sigma/2 + \tau]$: n.d.f. much reduced. Also, we found that this was not sufficient for all cases.
- Corner wall sources were found to work best for the vector and axial vector correlators below $T \sim 300$ MeV. Comparable results to point wall sources in other cases.
- Effective mass estimators [S. Mukherjee *et al.* (2014)] Split the correlator into oscillating and non-oscillating parts and solve analytically for the effective mass. Only works for one-state fits.
- Bayesian fits [Lepage (2001)] Need prior information (screening masses and amplitudes), which we did not have.

Akaike Information Criterion



- Akaike Information Criterion [H. Akaike 1971, 1974] Provides a criterion for measuring the goodness-of-fit of a given model to the data. We actually used a corrected version of AIC (AICc), which is used when the sample size is small.

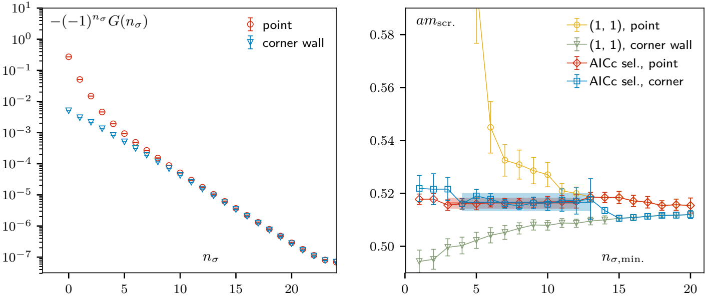
Akaike Information Criterion



(Left) One-state fits, no AICc. (Right) AICc-chosen fits.

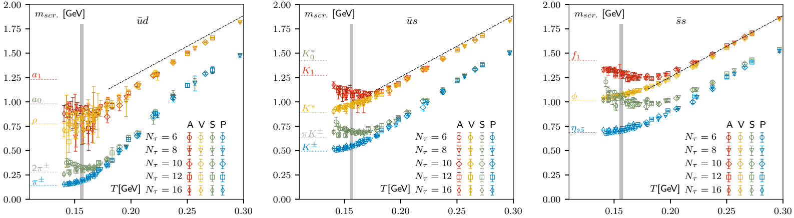
Multi-state fits for multiple fit windows; allow AICc to pick the best fit for each window.

Point versus Corner Wall sources



- Select effective mass plateaus by hand.
- We found that point and corner wall fits performed comparably.
- We used corner wall sources for vector and axial vector correlators below $T \sim 300$ MeV, and point sources in all other cases.

Screening Masses: 140 MeV $\lesssim T \lesssim$ 300 MeV

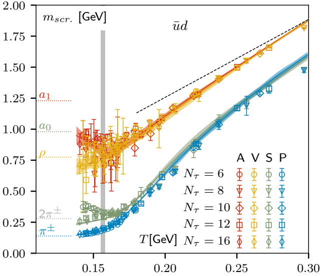


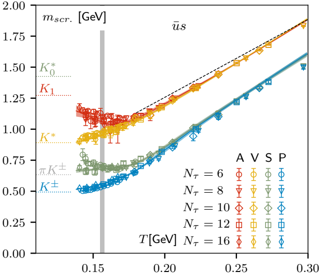
- The screening masses tend to the mass of the respective $T = 0$ mesons as the temperature is decreased.
- However, this is not true for the case of the scalar screening mass.

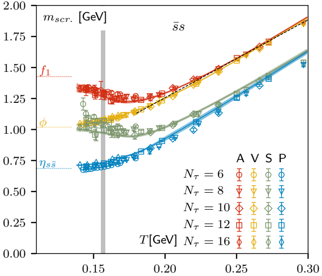
The staggered scalar correlator

- The scalar mass tends to $2m_\pi$, rather than m_{a_0}, at low temperatures.
- As already noted, this is because the staggered a_0 can undergo the unphysical decay $a_0 \to \pi\pi$.
- The decay arises from contributions of various tastes beyond tree level to the staggered correlator [S. Prelovsek (2006), S. Prelovsek *et al.* (2004)].
- These contributions cancel out in the continuum limit. In our case however, we calculate the screening mass first and then take the continuum limit.
- Beyond the question of screening masses, this also poses questions regarding $U_A(1)$ restoration.

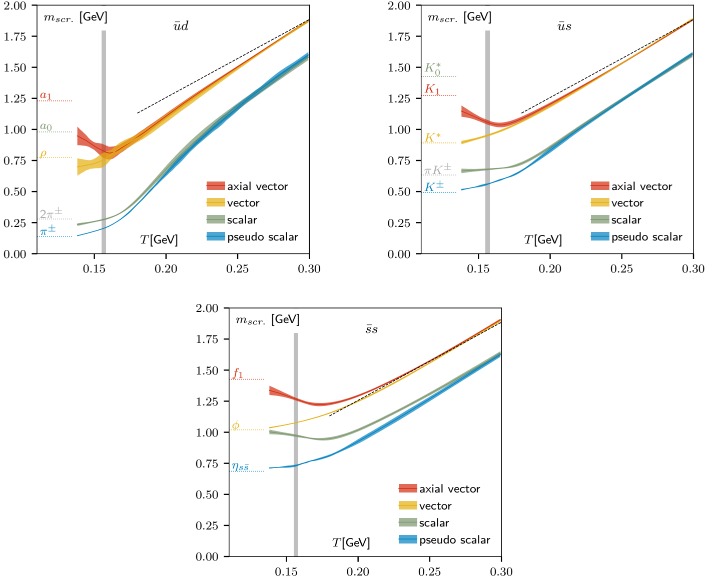
Continuum-Extrapolated Results







Continuum-Extrapolated Results



The question of $U_A(1)$ Symmetry Restoration

- Not known whether $U_A(1)$ symmetry is also restored at the chiral phase transition [E. Shuryak (1994), M. Birse, T. Cohen and J. McGovern (1996), S. Lee and T. Hatsuda (1996), N. Evans, S. Hsu and M. Schwetz (1996), S. Aoki *et al.* (2012)].

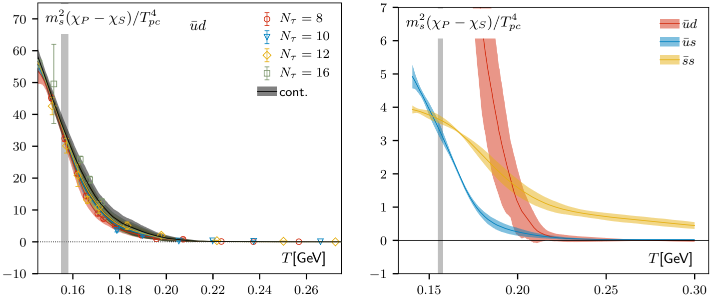
- Lattice studies can provide information by looking for a degeneracy between the π and a_0 (δ) correlators [HotQCD Collaboration (2012), M. Buchoff *et al.* (2013), G.Cossu *et al.* (2012, 2013, 2017), R. Gavai, S. Gupta and R. Lacaze (2001), T.-W. Chiu *et al.* (2013)].

- Easier to determine the degeneracy between the corresponding susceptibilities viz.

$$\chi_\pi = \sum_{n_\sigma=0}^{N_\sigma-1} \mathcal{M}2(n_\sigma), \qquad \chi_\delta = -\sum_{n_\sigma=0}^{N_\sigma-1} (-1)^{n_\sigma} \mathcal{M}1(n_\sigma).$$

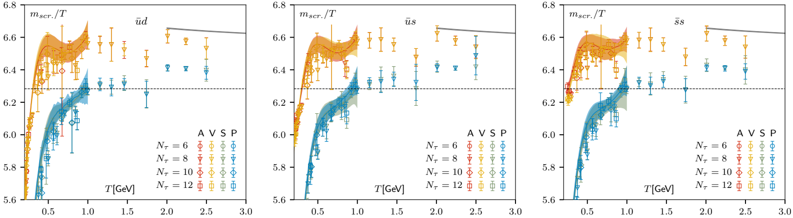
(The oscillating phase factor is only needed in the staggered case).

$U_A(1)$ Symmetry Restoration on the Lattice



- Taking the continuum limit of the susceptibilities is equivalent to taking the continuum limit of the correlators.
- We find that $m_s^2(\chi_\pi - \chi_\delta)$ goes to zero very slowly and not at the chiral crossover temperature itself.
- Note however that the question of $U_A(1)$ restoration only makes sense in the chiral limit. A systematic chiral extrapolation needs to be carried out before the question can really be addressed.

Screening Masses: 300 MeV $\lesssim T \lesssim$ 1000 MeV



- Quark-gluon plasma known to be non-perturbative just above the chiral phase transition. It is not known at what temperature the system becomes perturbative[E. Laermann & F. Pucci (2012), S. Gupta & N. Karthik (2013), C. Rohrhofer *et al.* (2019)].
- We compare our results to the predictions of dimensionally reduced QCD [M. Laine and M. Vepsalainen (2003), M. Laine and Y. Schroeder (2005)].
- We find a difference between our results and EQCD predictions out to $T \sim 1$ GeV. In any case, the spin-0 and spin-1 masses are very different, whereas all masses receive the same corrections in perturbation theory.

Conclusions

- We calculated meson screening masses in $2+1$-flavor QCD for temperatures 140 MeV $\lesssim T \lesssim$ 1 GeV.
- We were able to take the continuum limit owing to having results for multiple lattice spacings.
- We compared these results to predictions from resummed perturbation theory at high temperatures. We found that the system remained non-perturbative up to temperatures $T \sim 1$ GeV.
- The low-temperature limit of the vector, axial vector and pseudoscalar screening masses was as expected. The scalar mass had the wrong $T \to 0$ limit due to staggered artifacts. These artifacts disappear when the continuum limit of the correlator is taken first. We calculated the continuum limit of $\chi_\pi - \chi_\delta$ and found that the difference goes to zero well above the chiral crossover temperature.