Universality and critical behavior in QCD

– The chiral phase transition in (2+1)-flavor QCD –

Frithjof Karsch
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सादगी सार्वभौमिकता का सार है।

मो. क. गांधी

Simplicity is the essence of universality.

M. K. Gandhi

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CRC-TR 211 Strong-interaction matter under extreme conditions

DFG Deutsche Forschungsgemeinschaft

Universality and critical behavior in QCD

UNIVERSALITY IN FINITE TEMPERATURE LATTICE QCD

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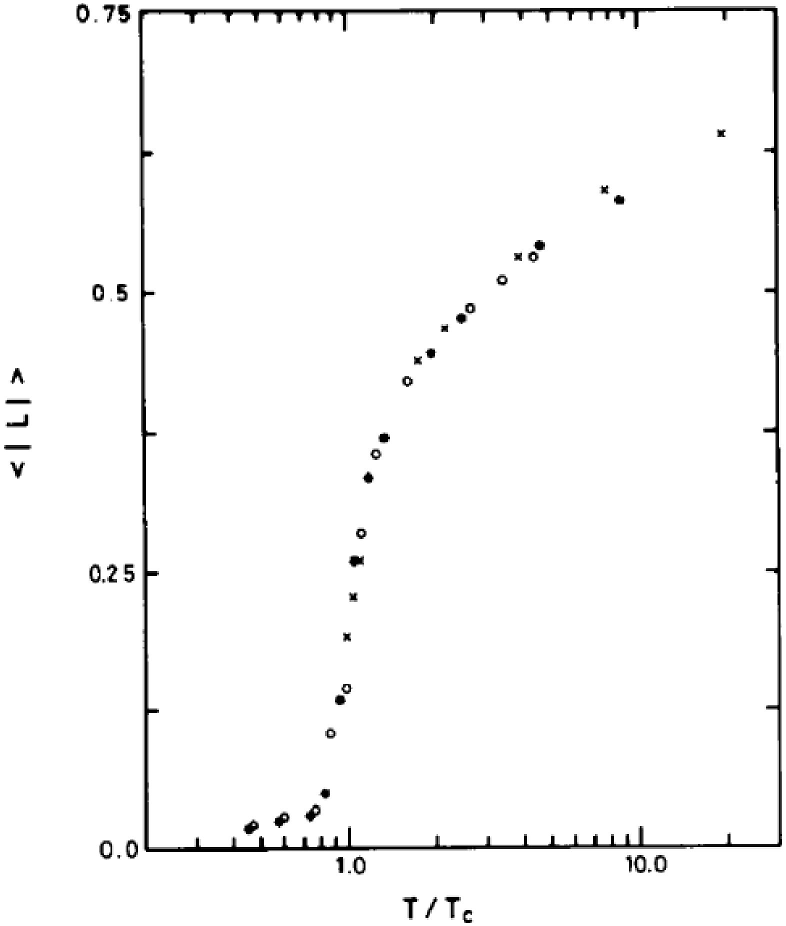

Fig. 1. Thermal Wilson loop as a function of T/T_c, calculated on a 3×10^3 lattice for Wilson action (×), Manton action (•) and Villain action (○). Here T is the temperature of the SU(2) gluon matter while T_c is the deconfinement temperature.


ARCTIC SCHOOL OF PHYSICS 1982
ÄKÄS-HOTELLI

Universality and critical behavior in QCD

– The chiral phase transition in (2+1)-flavor QCD –

Frithjof Karsch
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Universität Bielefeld

- **Universality and critical behavior** in the limit of vanishing light quark masses

  --- **the chiral PHASE TRANSITION**

- Higher order cumulants of **conserved charge fluctuations**

  --- making contact to fluctuation data from **heavy ion experiments**

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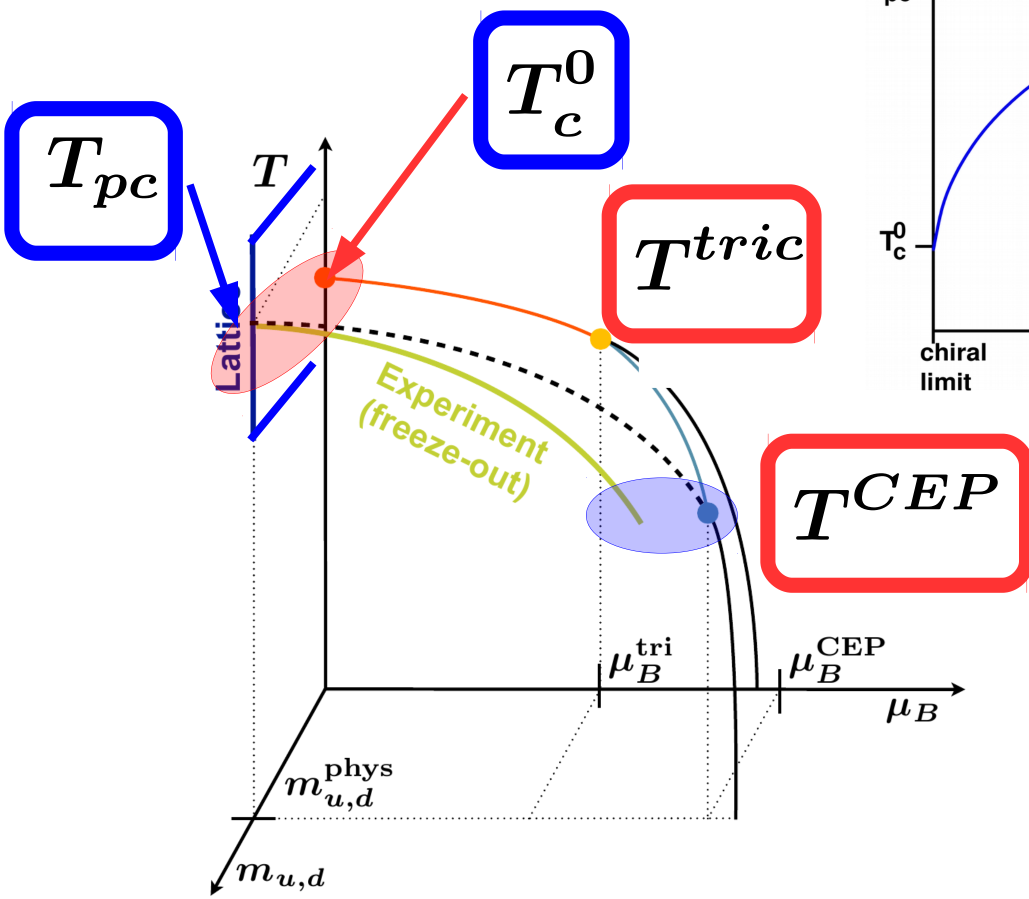
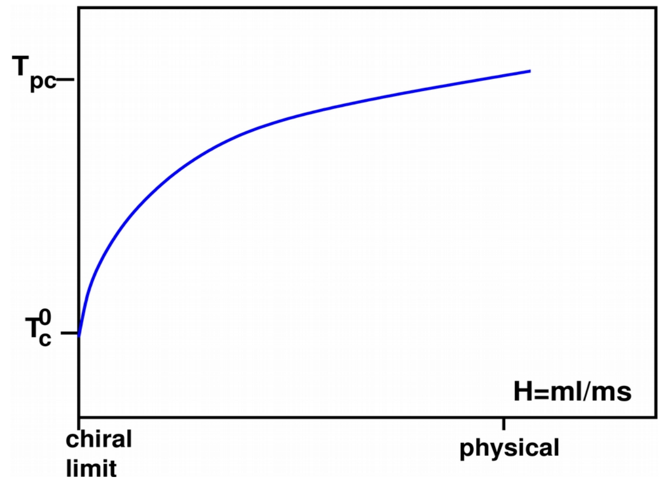
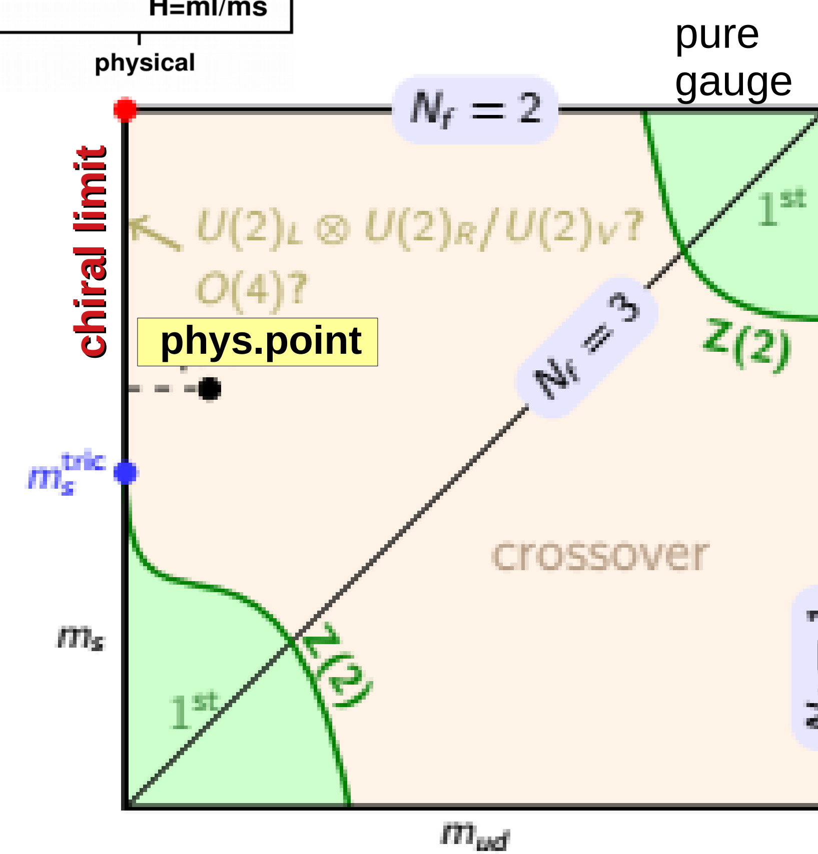
Phases of strong-interaction matter





this talk focuses on

$\mu_B = 0$



$$T_{pc} > T_c^0 > T^{tric} > T^{CEP}$$

– determination of the critical temperature

(and the order of the transition)

Universality and Critical behavior in QCD

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal scaling function

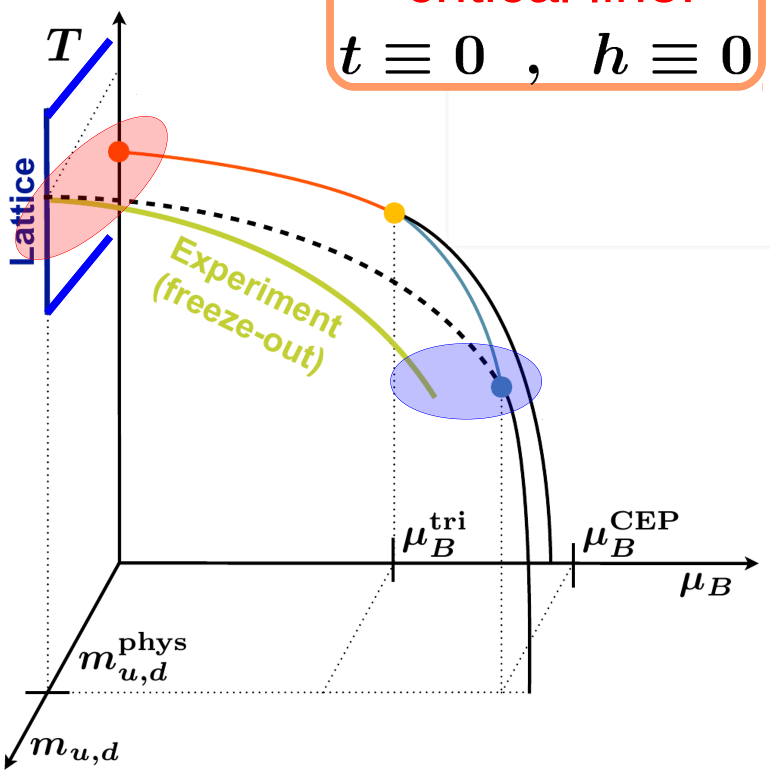
singular regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

critical line: $t \equiv 0 \ , \ h \equiv 0$

$$t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_q}{T}\right)^2 \ , \quad h \sim \frac{m_q}{T_c}$$

"thermal" coupling "magnetic" coupling



here only: $\mu_q \equiv 0$

question: Where is the chiral PHASE TRANSITION for $m_u = m_d = 0$ located?
What is its influence on observables at the pseudo-critical temperature?

Universality and Critical behavior in QCD

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal scaling function

singular — regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

$$t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_q}{T}\right)^2$$

$$h \sim \frac{m_q}{T_c}$$

Pseudo-critical temperatures

response functions
2nd order cumulants

magnetic	mixed	thermal
$\frac{\partial^2 \ln Z}{\partial h^2}$	$\frac{\partial^2 \ln Z}{\partial h\, \partial t}$	$\frac{\partial^2 \ln Z}{\partial t^2}$
$\sim \left(\frac{m_l}{T_c}\right)^{1/\delta-1}$	$\sim \left(\frac{m_l}{T_c}\right)^{(\beta-1)/\beta\delta}$	$\sim \left(\frac{m_l}{T_c}\right)^{-\alpha/\beta\delta}$
~ -0.79	~ -0.34	$\sim +0.11$
strong	moderate	none

divergence: strong, moderate, none

O(4) critical exponents

$$\alpha = -0.21$$
$$\beta = 0.38$$
$$\delta = 4.82$$

Chiral observables in QCD

– chiral condensate: $\langle\bar{\psi}\psi\rangle_q = \frac{\partial P/T}{\partial m_q/T}$, $\langle\bar{\psi}\psi\rangle_l = (\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d)/2$

– chiral order parameter: $M = \frac{2}{f_K^4}\left[m_s\langle\bar{\psi}\psi\rangle_l - m_l\langle\bar{\psi}\psi\rangle_s\right]$

$$m_l = (m_u + m_d)/2$$

– chiral susceptibility: $\chi_M = m_s\left(\frac{\partial M}{\partial m_u} + \frac{\partial M}{\partial m_d}\right)$ **magnetic**

– mixed chiral susceptibility: $\chi_t = T\frac{\partial M}{\partial T}$ **mixed**

– conserved charge fluctuations: $\chi_X = T^2\frac{\partial^2 P/T^4}{\partial\mu_X^2}\Big|_{\mu_X=0}$ **thermal**

$$X = B,\ S, \ldots$$

Scaling in the thermodynamic (infinite volume) limit

– approaching the chiral limit –

some definitions

$$z = \frac{t}{h^{1/\beta\delta}}$$

$$t \equiv \frac{1}{t_0}\frac{T - T_c}{T_c}$$

$$h = \frac{1}{h_0} H$$

$$H \equiv \frac{m_l}{T_c}$$

$$z_0 = h_0^{1/\beta\delta}/t_0$$

– order parameter M and its susceptibility

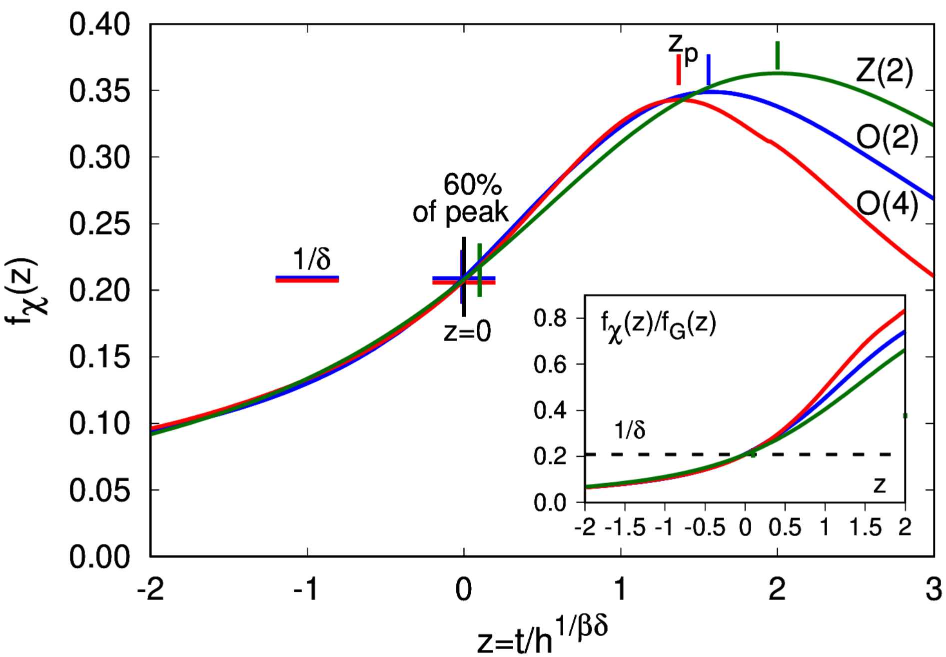
$$M = h^{1/\delta} f_G(z) + f_{sub}(T,H)$$

$$\chi_M = h_0^{-1} h^{1/\delta - 1} f_\chi(z) + \tilde{f}_{sub}(T,H)$$

for ANY fixed z:

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z}{z_0} H^{1/\beta\delta}\right) + \text{sub leading}$$

– corrections-to-scaling
– regular terms

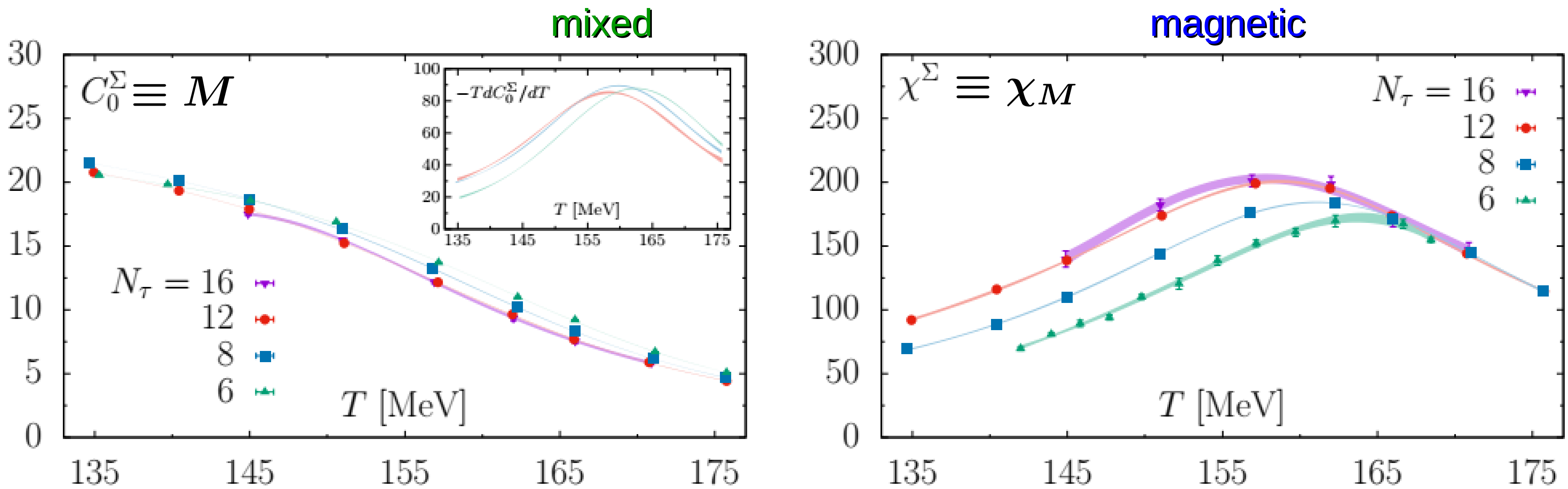


scaling functions $f_\chi(z)$ for some 3-d universality classes:

	δ	$1/\beta\delta$	z_p	z_{60}	z_δ
Z(2)	4.805	0.640	2.00(5)	0.10(1)	0
O(2)	4.780	0.599	1.58(4)	-0.005(9)	0
O(4)	4.824	0.545	1.37(3)	-0.013(7)	0

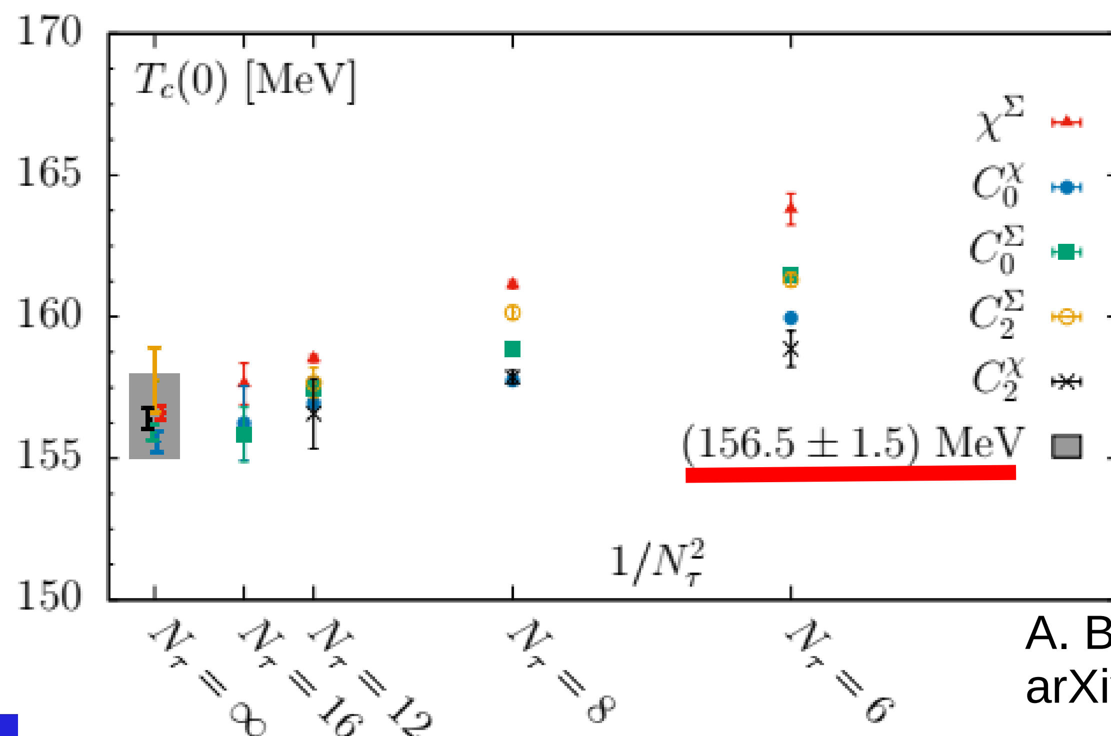
characteristic points on the scaling function $f_\chi(z)$

Pseudo-critical temperatures from chiral observables



physical
light & strange
quark masses;

continuum
extrapolated



$\chi^\Sigma : \partial_T \chi_M = 0$

$C_0^\chi : \partial_T \chi^{disc} = 0$

$C_0^\Sigma : \partial_T^2 M = 0$

$C_2^\Sigma : \partial_T \partial_\mu^2 M = 0$

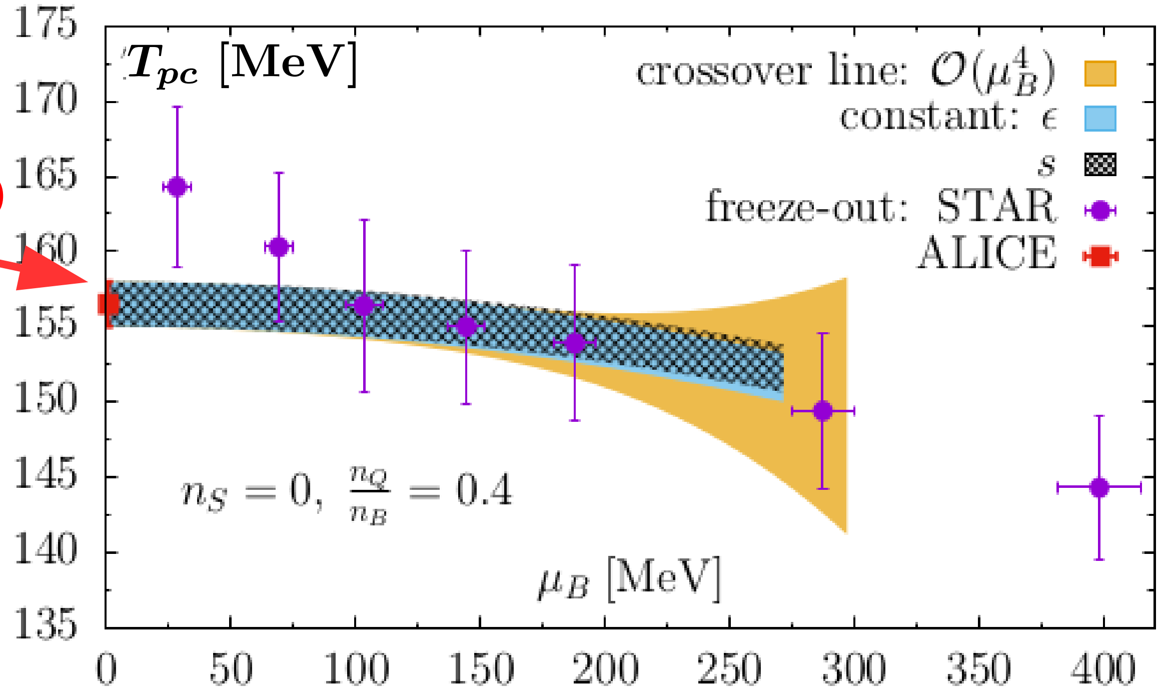
$C_2^\chi : \partial_\mu^2 \chi^{disc} = 0$

A. Bazavov et al [HotQCD], arXiv:1812.08235

Pseudo-critical (crossover) temperature

$$T_{pc}(\mu_B) = T_{pc}^0 \left(1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4 + \dots\right)$$

A. Andronic et al., Nature 561 (2018) 321



A. Bazavov et al [HotQCD], arXiv:1812.08235

$$T_{pc}^0 = (156.5 \pm 1.5)\ \mathrm{MeV}$$

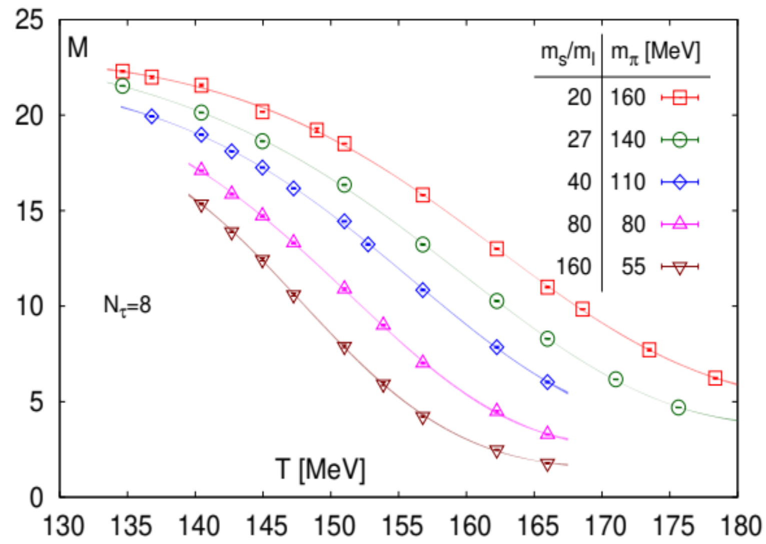
$$\kappa_2 = 0.012(4)$$

$$\kappa_4 = 0.000(4)$$

average over 5 definitions of a pseudo-critical temperature

The Chiral **PHASE TRANSITION** in (2+1)-flavor QCD: $m_l/m_s \to 0$

$$M \sim m_s \frac{\partial \ln Z}{\partial m_l}$$



$$m_l = (m_u + m_d)/2$$
$$\Rightarrow\ 0$$

m_s fixed, physical

"magnetic" susceptibility

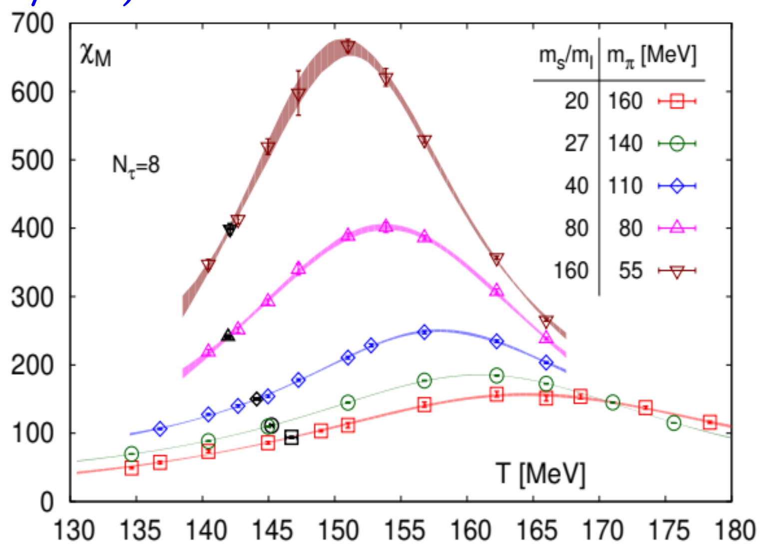
$\sim (m_s/m_l)^{0.79}$

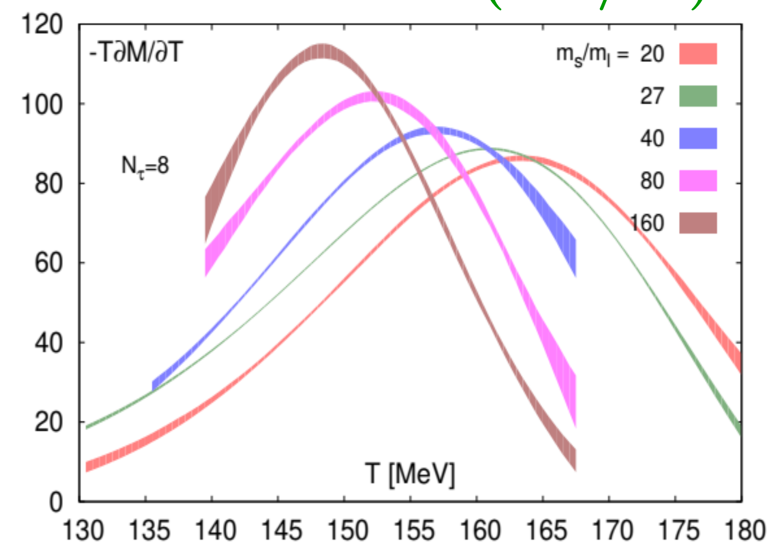
$(160/27)^{0.79} \sim 4$

"mixed" susceptibility

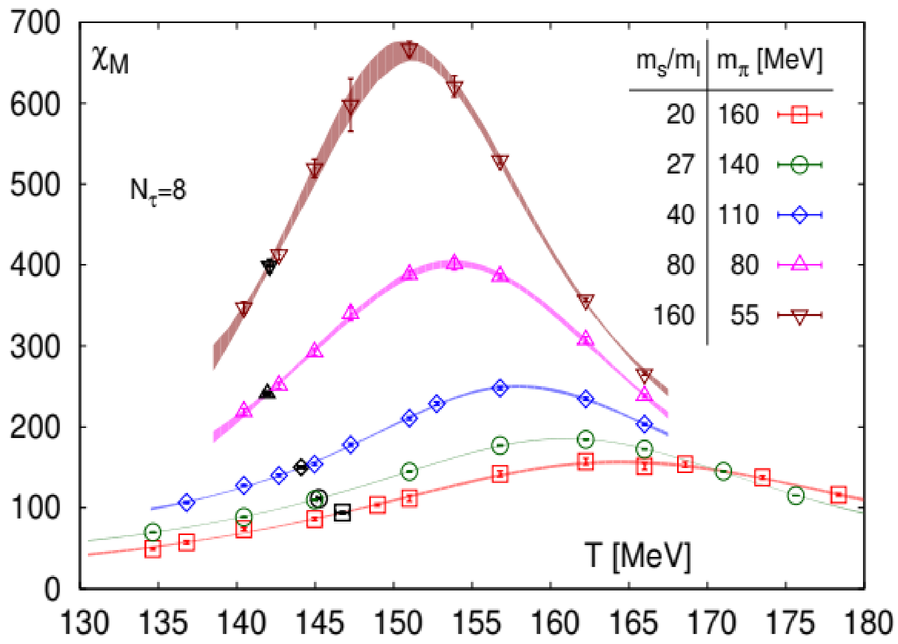
$\sim (m_s/m_l)^{0.34}$

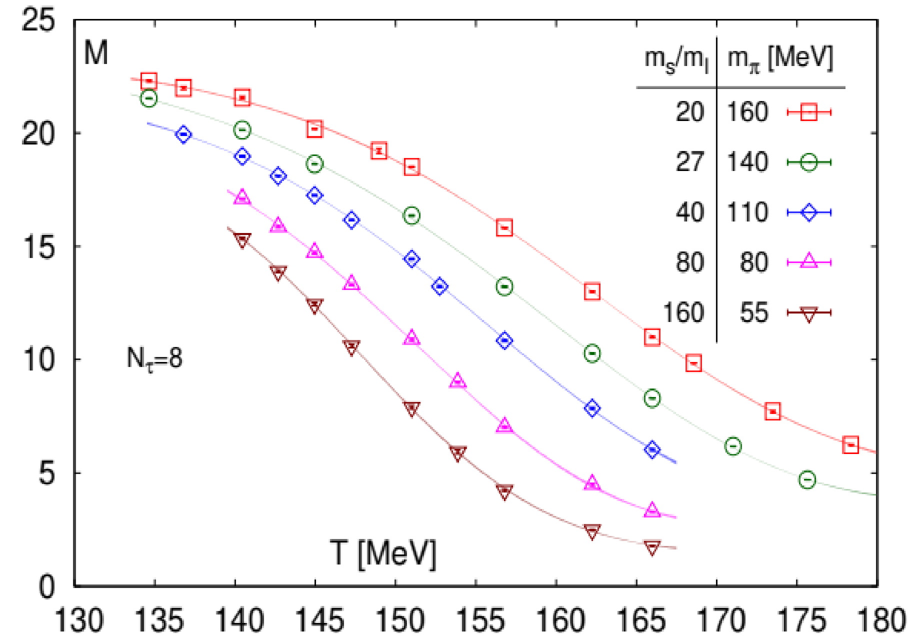
$(160/27)^{0.34} \sim 1.8$





Chiral PHASE TRANSITION





a universal ratio at any fixed z

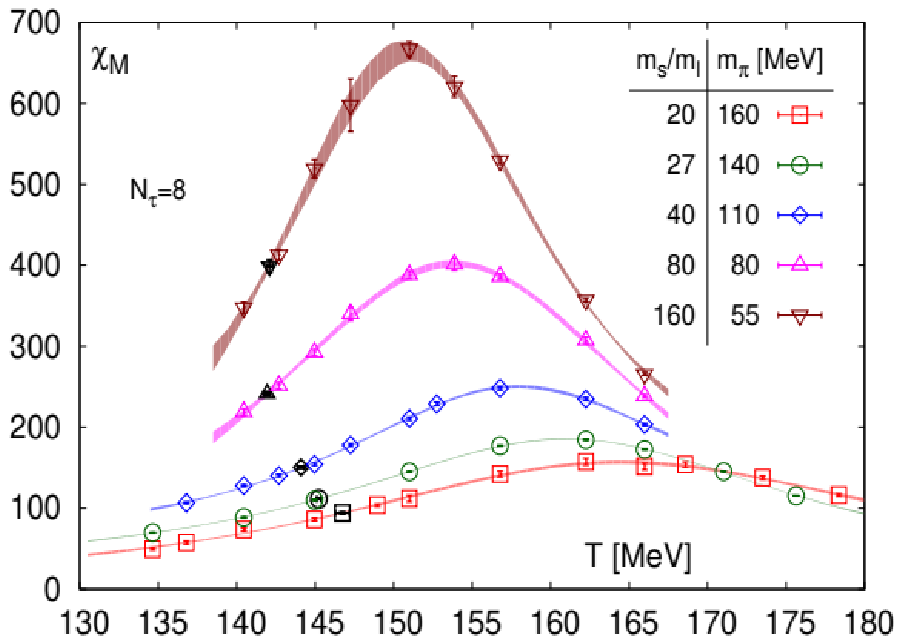
$$\frac{H\chi_M}{M} = \frac{f_\chi(z)}{f_G(z)} + \text{regular}$$

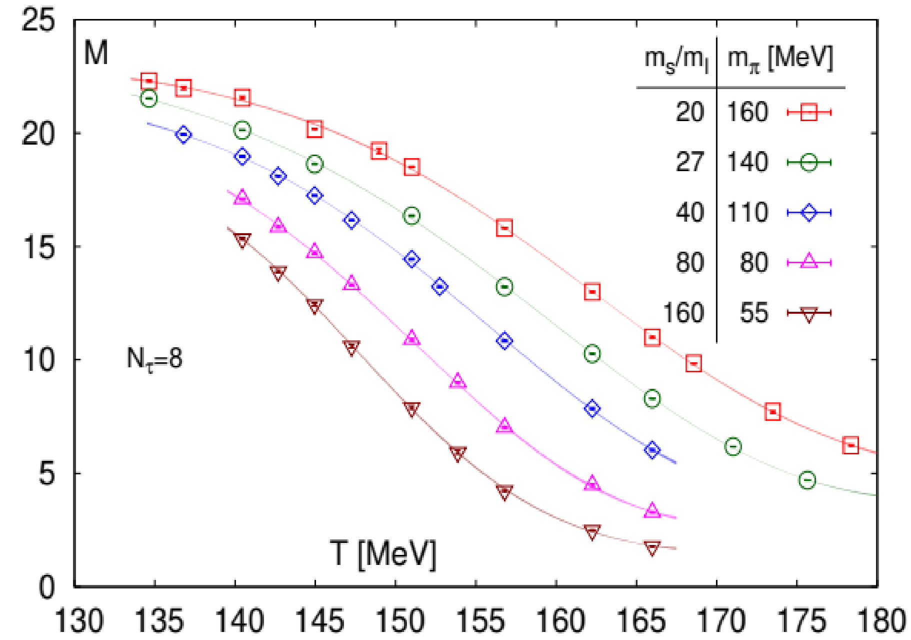
estimators for $\sim T_c^0$

characteristic points on the scaling function $f_\chi(z)$

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z}{z_0} H^{1/\beta\delta}\right) + \text{sub leading}$$

Chiral PHASE TRANSITION





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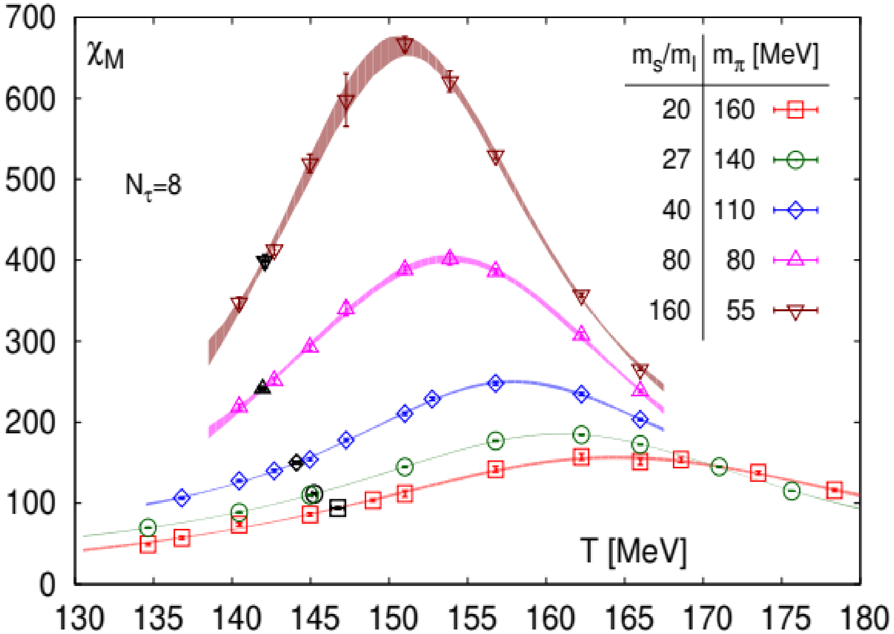
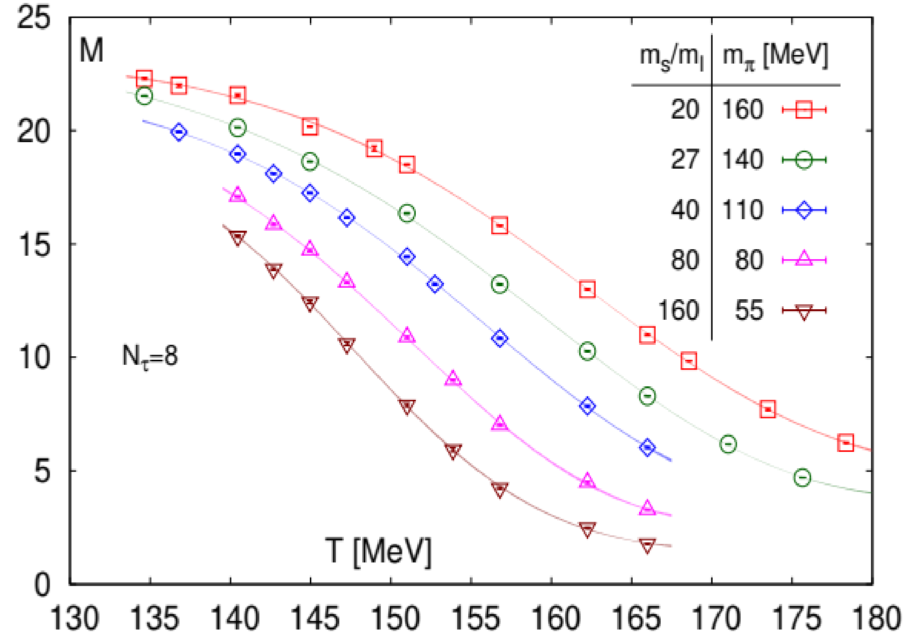
Chiral PHASE TRANSITION

estimators for $\sim T_c^0$





T_{60} black symbols: 60% of peak height

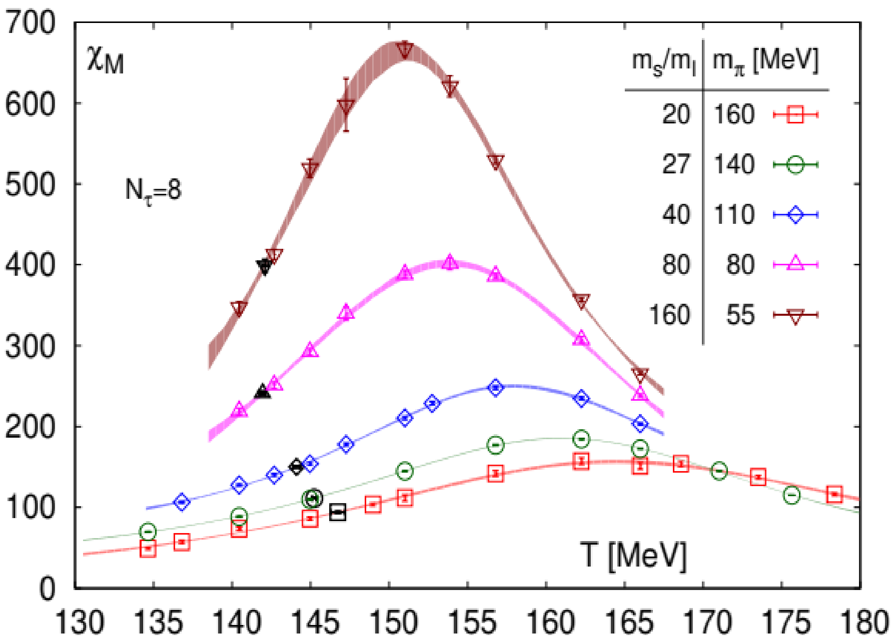
a universal ratio at any fixed z

$$\frac{H\chi_M}{M} = \frac{f_\chi(z)}{f_G(z)} + \text{regular} \qquad z = t/h^{1/\beta\delta}$$

$$= \begin{cases} 1/\delta & , \; z = 0 \quad \Rightarrow T_\delta \\ \sim 0.5 & , \; z = z_p \end{cases}$$

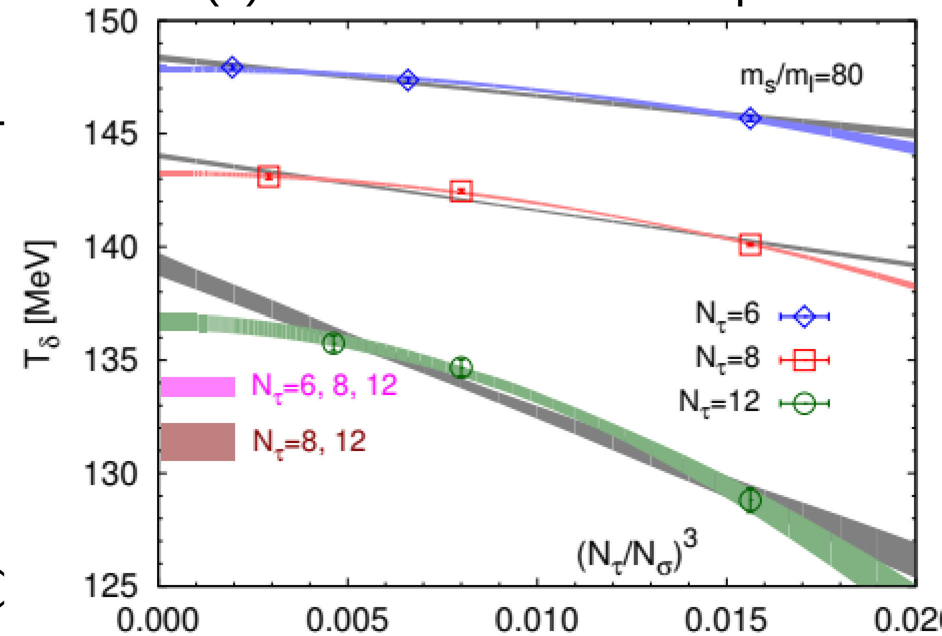
!! **holds approx. for** $m_s/m_l = 27 - 160$ →

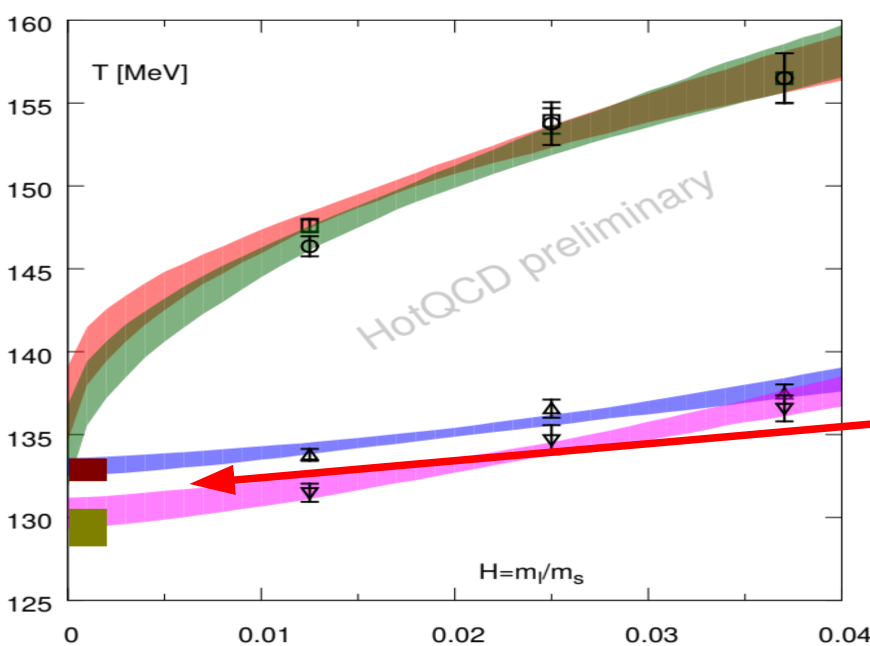
Chiral PHASE TRANSITION temperature



(1) **infinite volume** extrapolation

(2) **continuum limit** extrapolation





$$T_{pc}^{phys} = (156.5 \pm 1.5)\ \mathrm{MeV}$$

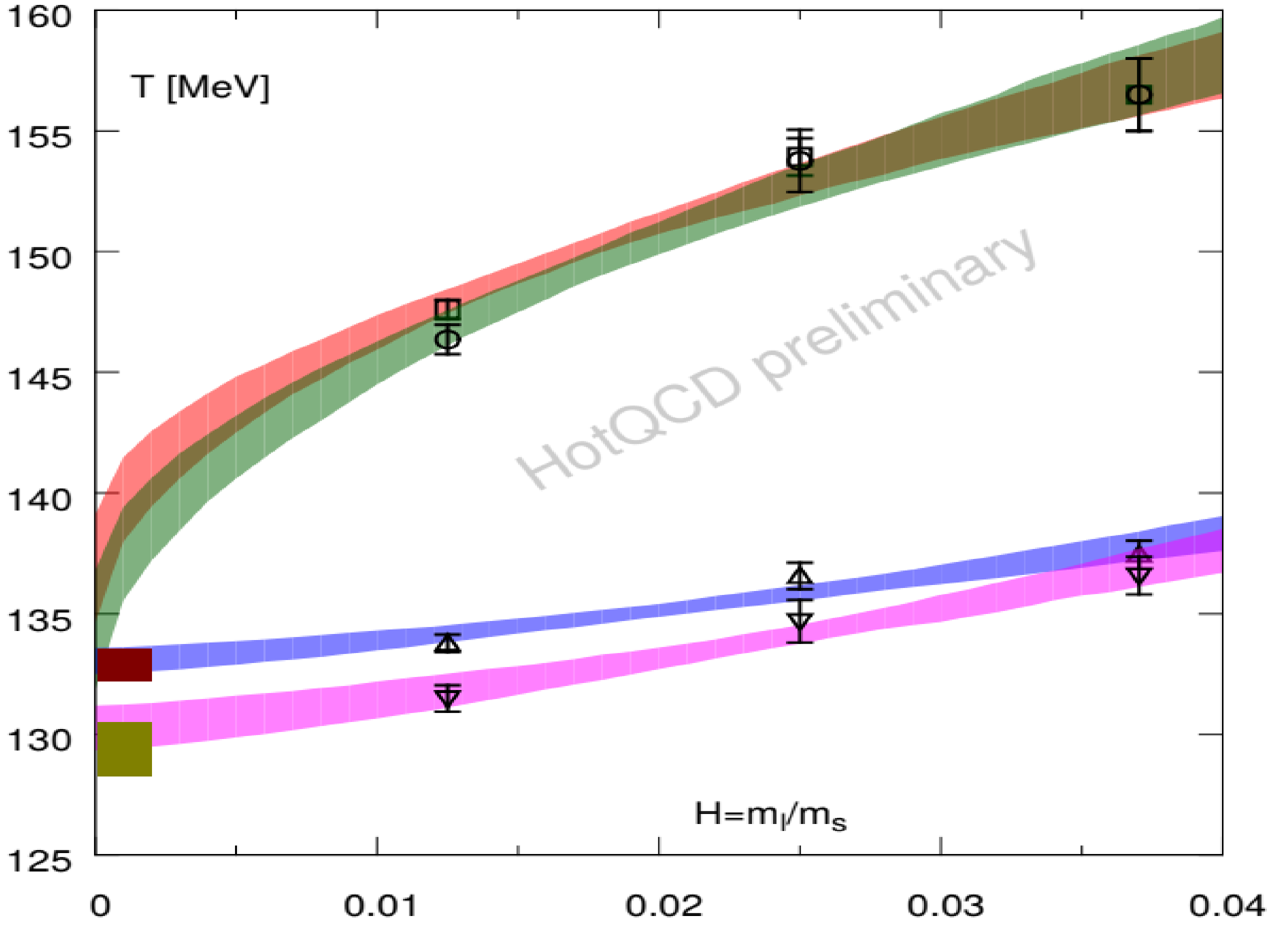
A. Bazavov et al [HotQCD], arXiv:1812.08235

(3) **chiral limit** extrapolation

$$T_c^0 = 132^{+3}_{-6}\ \mathrm{MeV}$$

H.-T. Ding et al [HotQCD], PRL 123 (2019) 062002 arXiv:1903.04801

Chiral PHASE TRANSITION temperature



T_{pc}^{phys}

(156.5 ± 1.5) MeV

$\Delta T \simeq 25$ MeV

$T_c^0 = 132^{+3}_{-6}$ MeV

Critical behavior and higher order cumulants
– Taylor expansion and universality –

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} P/T^4}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k} \right|_{\mu_{B,Q,S}=0} \quad , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

Critical behavior and higher order cumulants

– the breakdown of the HRG model description in the "vicinity of Tc" becomes obvious in properties of higher order cumulants,

pressure: $$\frac{p}{T^4} = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

	alpha
O(4)	-0.213
Z(2)	+0.107

$$t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_q}{T}\right)^2 \qquad h \sim \frac{m_q}{T_c}$$

$$\frac{\partial}{\partial T} \simeq \frac{\partial^2}{\partial(\mu_B/T)^2}$$

FK et al., arXiv:1009.5211

chemical potentials are "**thermal couplings**"

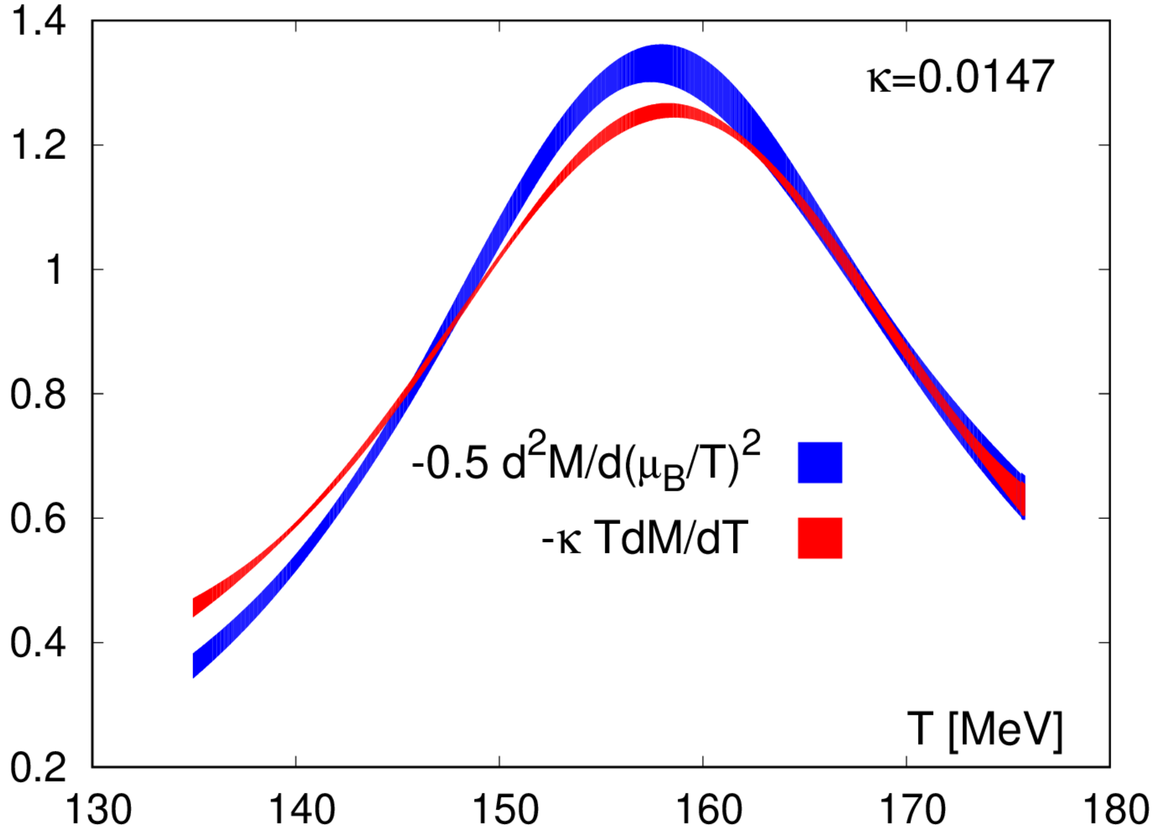
T-derivative ⟺ two μ-derivatives

Critical behavior and higher order cumulants

critical behavior in chiral observables: the T-derivative of the chiral condensate

a mixed susceptibility

$$\Delta_{ls}(T,\mu_B) = \Delta_{ls}(T,0) + \frac{1}{2}\frac{\partial^2 \Delta_{ls}}{\partial(\mu_B/T)^2}\bigg|_{\mu_B=0}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$



$$\frac{\partial^2}{\partial(\mu_B/T)^2} \simeq \frac{\partial}{\partial T}$$

$$t \sim \frac{T-T_c}{T_c} + \kappa_2\left(\frac{\mu_B}{T}\right)^2$$

curvature of crossover line

$\kappa_2 = 0.012(4)$

Critical behavior and higher order cumulants

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$$\frac{\partial}{\partial T} \simeq \frac{\partial^2}{\partial(\mu_B/T)^2}$$

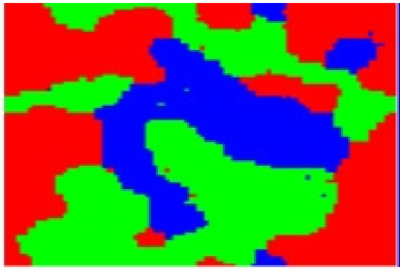
chemical potentials are "**thermal couplings**"

T-derivative ⟺ two μ-derivatives

cumulants:

$$\chi_X^{(2n)} = \left.\frac{\partial^{2n} p/T^4}{\partial(\mu_X/T)^{2n}}\right|_{\mu_X=0} = m_q^{(2-\alpha-n)/\beta\delta} f_f^{(n)}(t/h^{1/\beta\delta}) \ , \ X = B, S, ...$$

O(4): singular terms dominate only for $2n \geq 6$

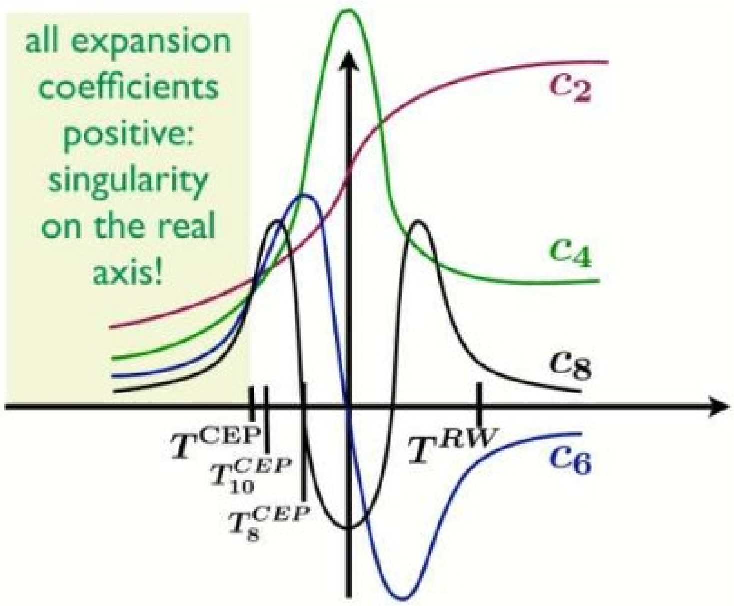


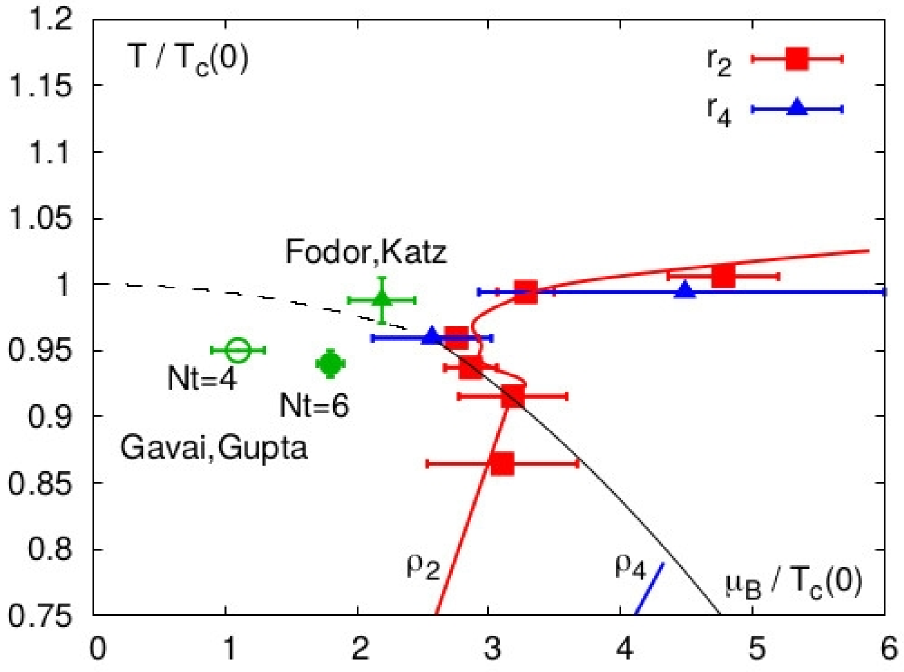
Estimating the location of the critical point: (T^{CEP}, μ_c)

i) T^{CEP}: find the largest temperature for which all c_n stay positive

ii) μ_c/T^{CEP}: estimate $\lim_{n\to\infty} r_n$

$$r_n \equiv \left(\frac{\mu_c}{T^{CEP}}\right)_n = \sqrt{\frac{c_n}{c_{n+2}}}$$





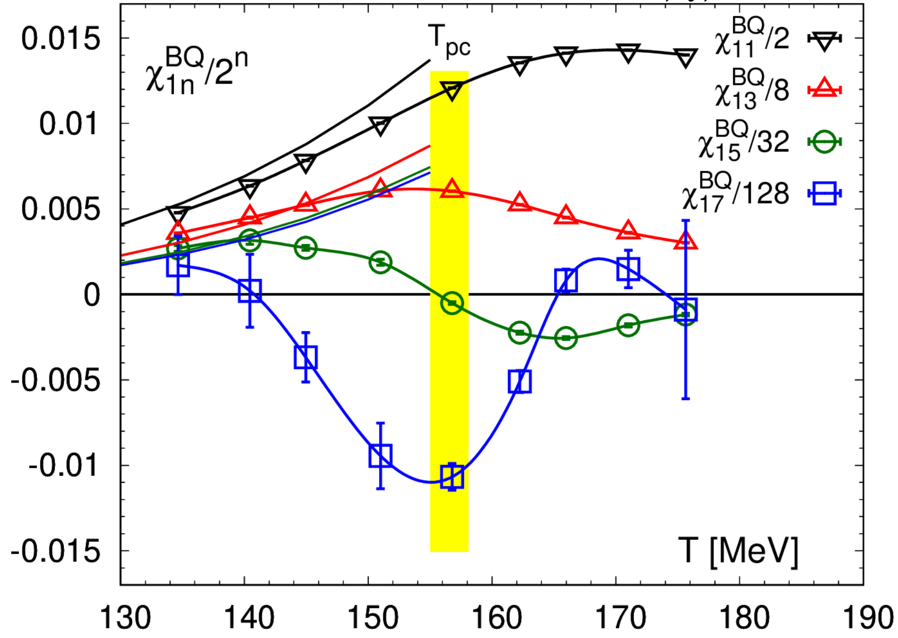
- $\mathcal{O}(a^2)$ improved action; slight quark mass dependence; weak cut-off dependence
- first non-trivial estimate of T^{CEP} requires 8^{th} order for c_n: $\Rightarrow \quad r_6$
- already $\mathcal{O}(c_6)$ requires more statistics

F. Karsch, APS-Denver, May 2009 – p.21/24

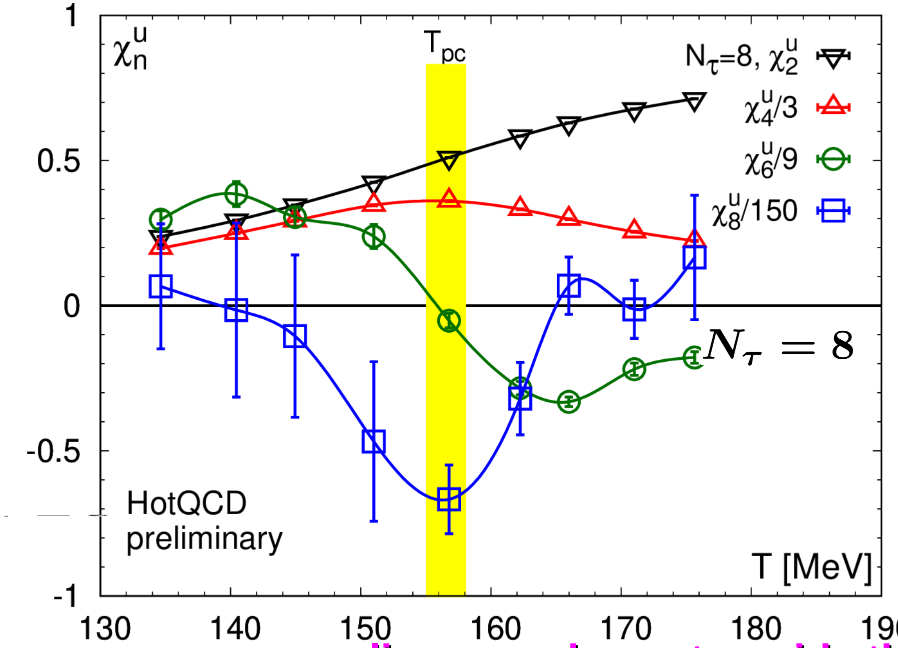
also INT, Seattle 2008,& Paris 2009

Critical behavior and higher order cumulants

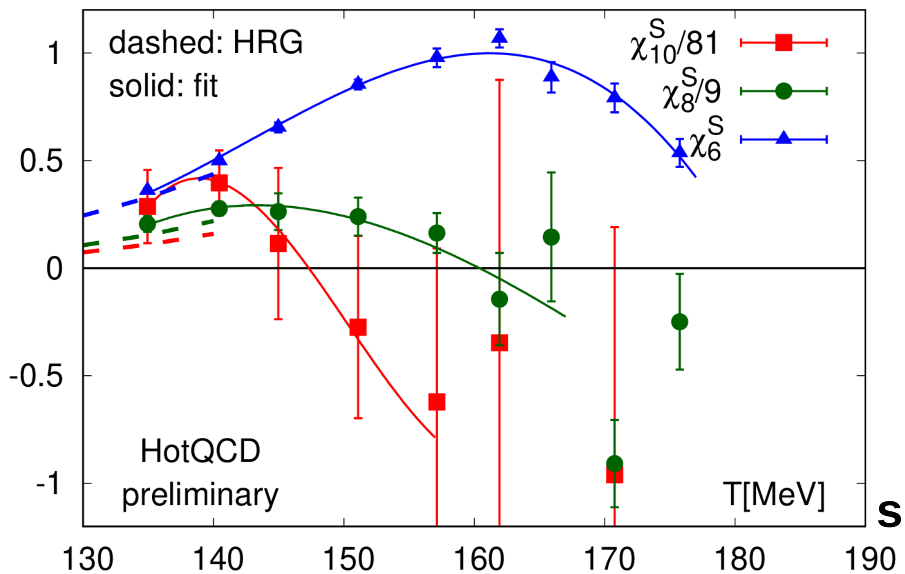
$$\chi_{1n}^{BQ} = \left.\frac{\partial^{n+1} P/T^4}{\partial\hat{\mu}_B \partial\hat{\mu}_Q^n}\right|_{\mu_{B,Q,S}=0}$$



$$\chi_n^u = \left.\frac{\partial^n P/T^4}{\partial\hat{\mu}_u^n}\right|_{\mu_{u,d,s}=0}$$



lines are drawn to guide the eye



$$\frac{\partial^2}{\partial(\mu_X/T)^2} \simeq \frac{\partial}{\partial T}$$

– expected from structure of O(N) scaling fields

many 8th order cumulants turn negative for

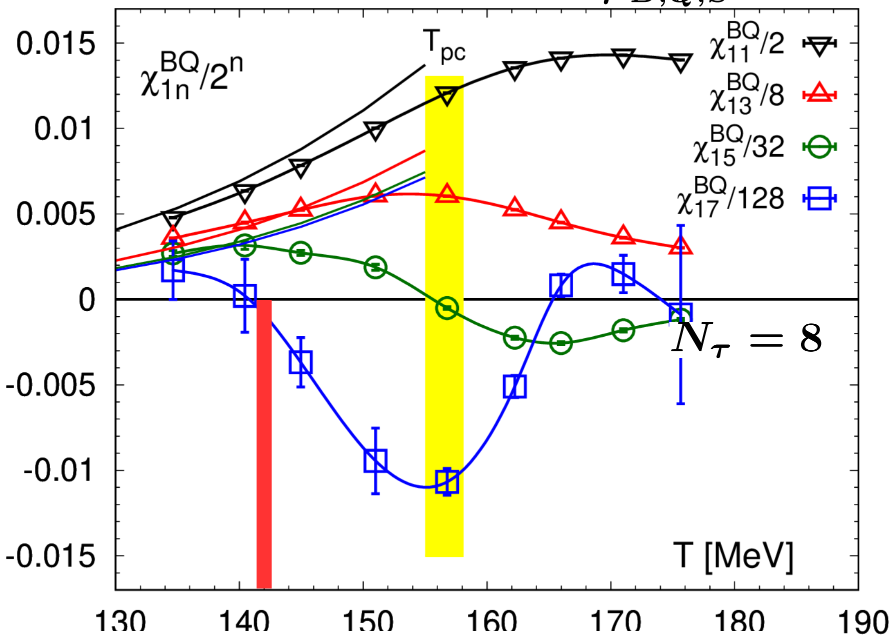
$$T^- \gtrsim (140-145)\ \mathrm{MeV}$$

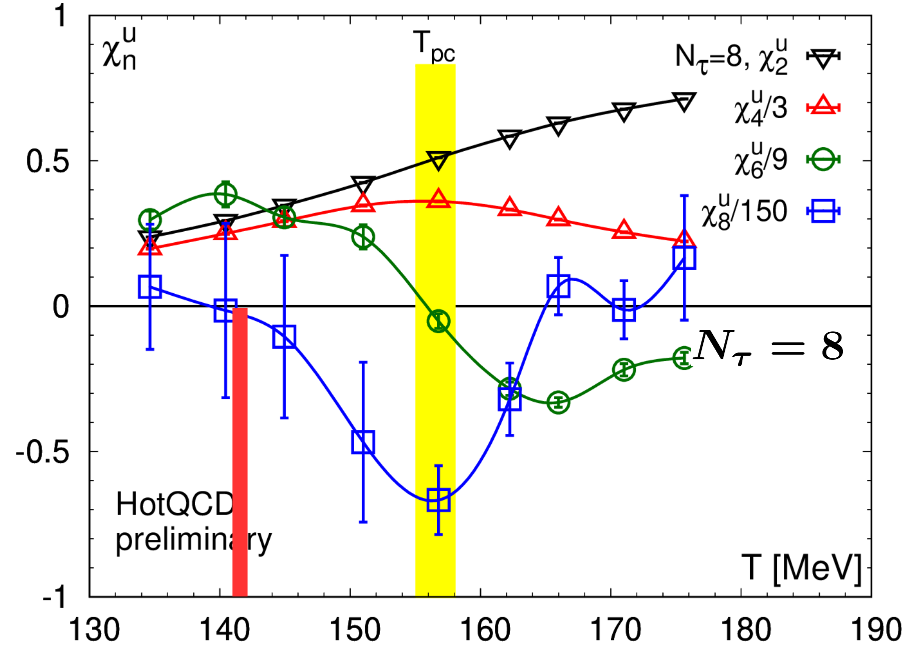
suggests zeroes in complex plane → no phase transition

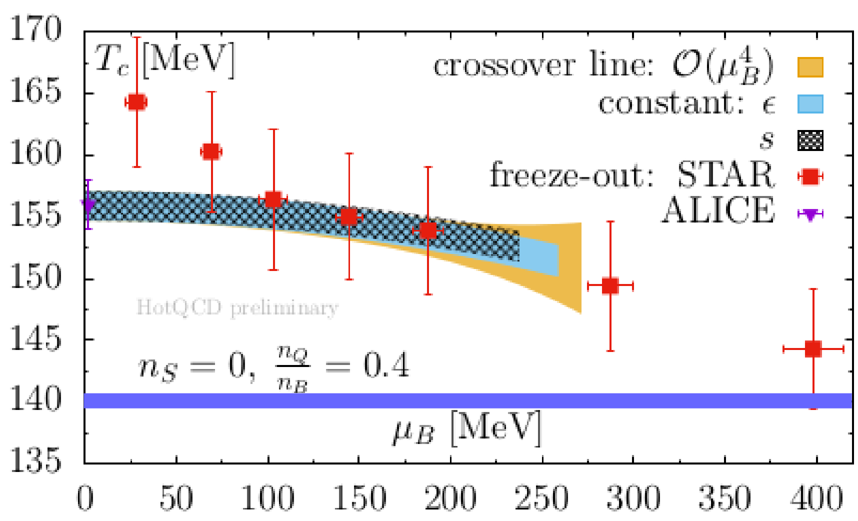
Critical behavior and higher order cumulants

$$\chi_{1n}^{BQ} = \frac{\partial^{n+1} P/T^4}{\partial\hat\mu_B \partial\hat\mu_Q^n}\Big|_{\mu_{B,Q,S}=0}$$

$$\chi_n^u = \frac{\partial^n P/T^4}{\partial\hat\mu_u^n}\Big|_{\mu_{u,d,s}=0}$$







many 8th order cumulants turn negative for

$$T^- \gtrsim (140-145)\ \text{MeV}$$

suggests zeroes in complex plane → no phase tr.

⇒ plausible scenario:

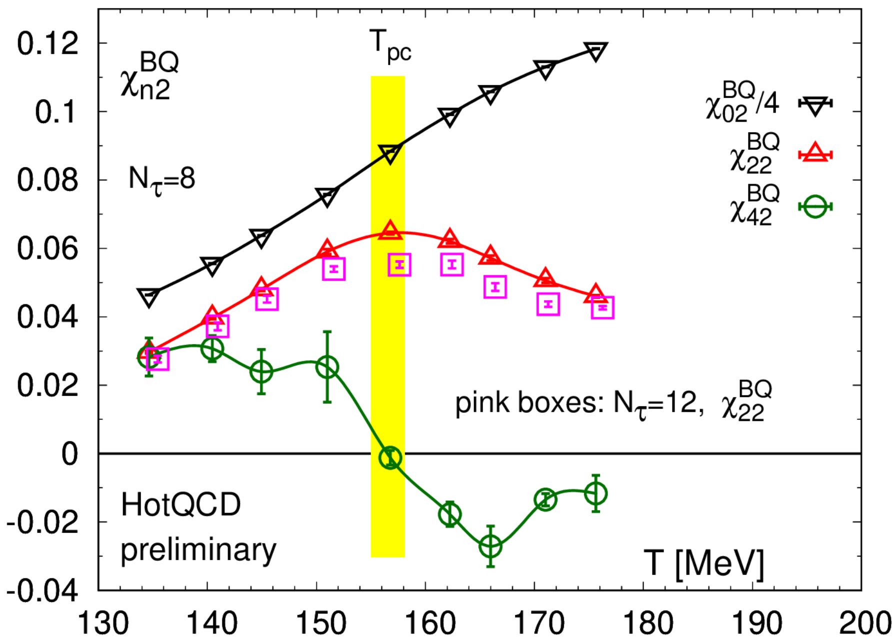
$$T_{cp} < 140\text{MeV}\ ,\ \mu_B^{cp} > 400\text{MeV}$$

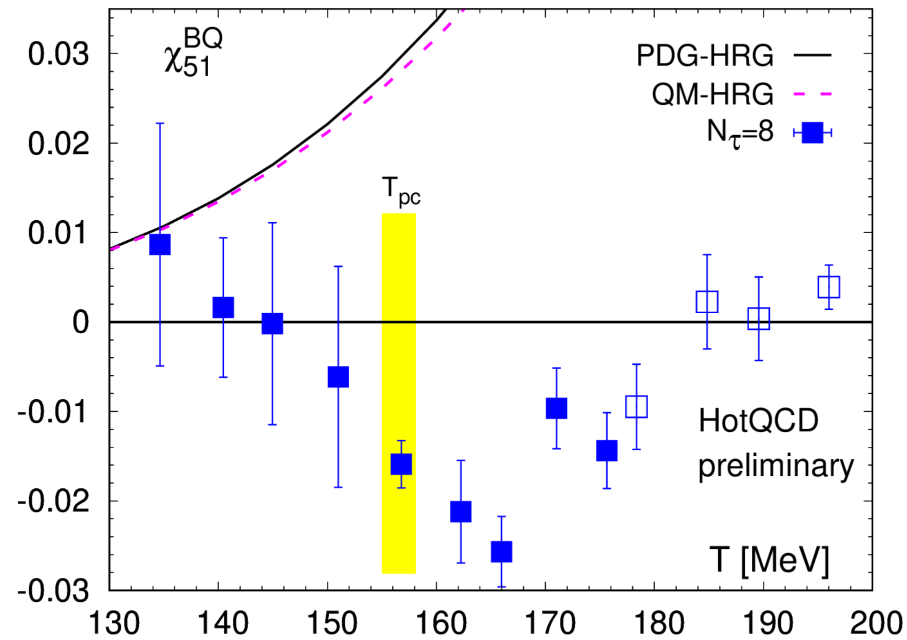
consistent with $T_c^0 = 132^{+3}_{-6}$ MeV

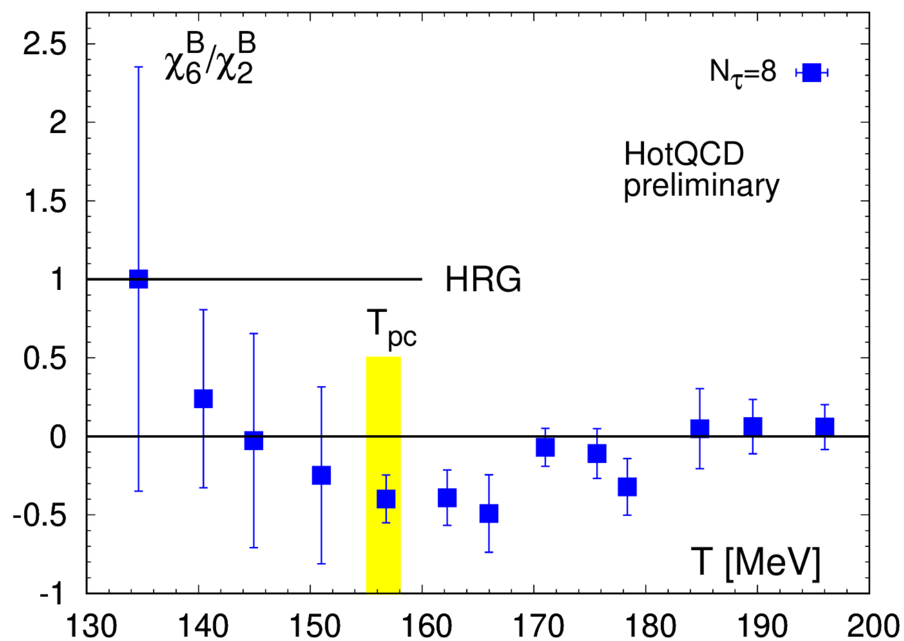
and the analytic structure of the O(4) scaling function

S. Mukherjee, V. Skokov, arXiv:1909.04639

some 6th order cumulants







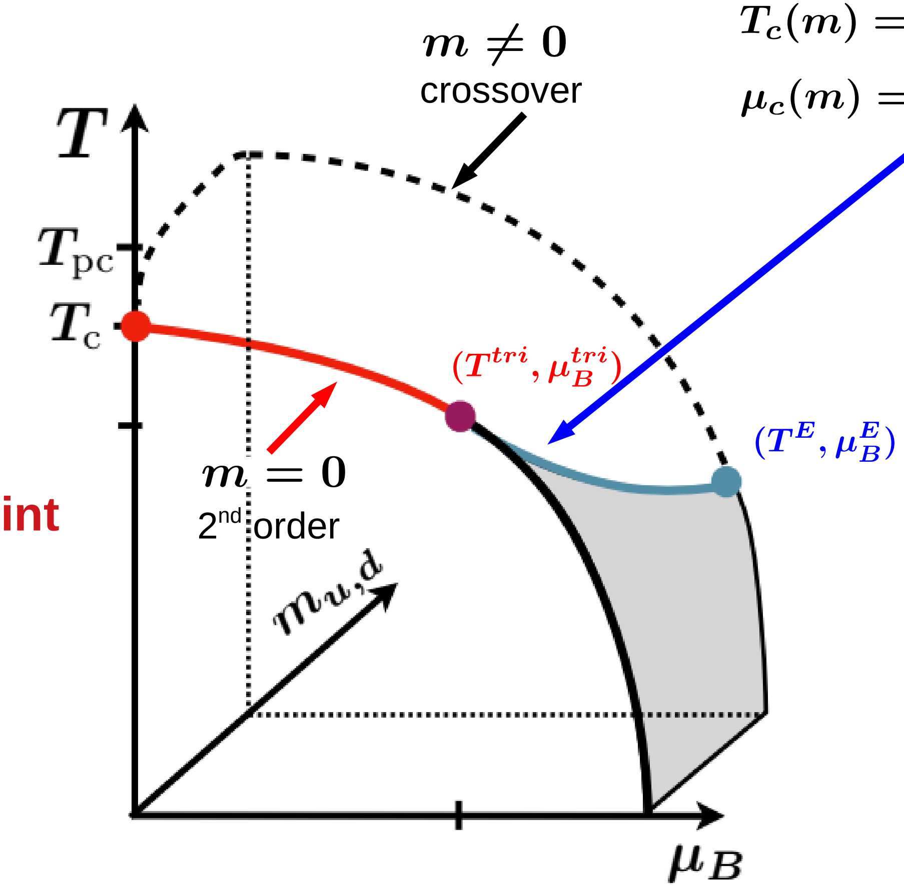
- 6th order cumulants of baryon number fluctuations and their correlations with electric charge are negative at the pseudo-critical temperature

- large deviations from the non-interacting HRG model

- still largely influenced by regular terms

Crossover, chiral phase transition at $\mu_B = 0$ and the (tri)-critical point at $\mu_B > 0$

$$T_c(m) = T^{tri} - C_t m^{2/5} + \mathcal{O}(m^{4/5})$$
$$\mu_c(m) = \mu^{tri} + C_\mu m^{2/5} + \mathcal{O}(m^{4/5})$$



a possible tri-critical point will show up only at

$T < 135$ MeV

the critical point is likely to be located at $T < 130$ MeV

Random Matrix Model — A. Halasz, A.D. Jackson, R.E. Shrock, M.A. Stephanov, J.J.,M. Verbaarschot, Phys. Rev. D58 (1998) 096007

MF — Y. Hatta, T. Ikeda , Phys. Rev. D67 (2003) 01028

QCD — M. Stephanov, Phys. Rev. D73 (2006) 094508

NJL — M. Buballa, S. Carignano, Phys. Lett. B791 (2019) 361

Conclusions

– the chiral phase transition is located at a temperature T_c^0 about 25 MeV lower than the pseudo-critical temperature T_{pc} at physical values of the quark masses

– for $T \simeq T_{pc}$ chiral (magnetic) observables at physical values of the quark masses are sensitive to critical behavior in the chiral limit.

– higher order cumulants show large deviations from non-interacting (point-like) HRG model calculations even below T_{pc}

negative 6th & 8th order cumulants as well as the low chiral phase transition temperature suggest that a possibly existing critical point may be found only for

$$\mu_B^{cp} > 400\mathrm{MeV}\ ,\ T_{cp} < (130 - 140)\mathrm{MeV}$$

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